Two objective problems of time and capacity overloaded centers

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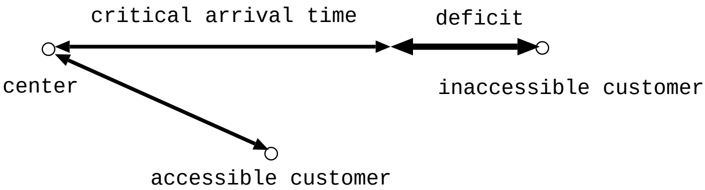
About what ?

We study the problem motivated by a task of placing a limited number of ambulances providing health services, with the effort to exceed critical ambulance arrival time only in exceptional cases. This issue is being intensively addressed by my departmental colleagues. A similar problem can occur in different services, not just in rescue ones.

The basic model of overloaded centers detects in a transport network of q-service centers with p-vehicles (with given capacity of customers / vehicles). The goal is to minimize the number of inaccessible customer locations (those with exceeded critical arrival time from an assigned center).

Accessible and inaccessible customers

In the other two models we consider the fairest approach to inaccessible customers. First we formulate a lexicographic problem, where the first criterion is the number of inaccessible customers, and the second is the total amount of time of customers that are unavailable, with capacity constraints. Next we propose its heuristic version, where we reduce the space of feasible solutions.



Notation

The transport network is represented by a sparse matrix $T = (t_{ij}), i \in I, j \in J$ where

- J is a set of customers where $n = |J|$ is a number of places (nodes) in the network,
- I is a set of feasible centers, $I \subset J$,
- t_{ij} is driving time from center i to customer j in minutes,
- w_j is weight of a customer $j \in J$,
- q is a number of vehicles,
- κ is capacity of vehicles,
- τ is critical arrival time in minutes,
- $A_i = \{j \in J : t_{ij} \leq \tau\}$ is a set of accessible customers from center $i \in I$.

Basic variables

We define for $i \in I, j \in A_i$ the binary variable

$$x_{ij} = \begin{cases} 1 & \text{if accessible customer } j \text{ is assigned to center } i, \\ 0 & \text{otherwise} \end{cases}$$

In addition, the value of the variable $x_{ii} = 1$ means that i is the center.

The variable $y_i \in \{0, 1, \ldots, q\}, i \in I$ indicates the number of vehicles in the center i. The selection variable $z_j \in \{0, 1\}, j \in J$ decides whether is a customer accessible $z_j = 0$ or inaccessible $z_j = 1$.

Model of customer time availability – CTA

We consider three parameters, the number of required centers q, the critical arrival time τ (in A_i) and weights of the customers w.

$$LEX\left(\sum_{j\in J} w_j z_j, \sum_{i\in I}\sum_{j\in A_i} w_j t_{ij} x_{ij}\right) \to \min \tag{1}$$

s.t.

$$\sum_{i\in I} x_{ii} \le q \tag{2}$$

$$\sum_{i\in I: j\in A_i} x_{ij} + z_j = 1 \qquad \forall j \in J, \tag{3}$$

$$x_{ij} \le x_{ii} \qquad \forall i \in I, \forall j \in A_i - \{i\}, \tag{4}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in I, \forall j \in A_i, \tag{5}$$

$$z_j \in \{0,1\} \qquad \forall j \in J. \tag{6}$$

We add two parameters, the number of available vehicles p with the capacity of κ customers.

$$LEX\left(\sum_{j\in J} w_j z_j, \sum_{i\in I}\sum_{j\in A_i} w_j t_{ij} x_{ij}\right) \to \min \tag{7}$$

s.t.

$$\sum_{i\in I} y_i = p, \tag{8}$$

$$\sum_{j\in A_i} w_j x_{ij} \le \kappa y_i \qquad \forall i \in I, \tag{9}$$

$$q x_{ii} \ge y_i \ge x_{ii} \qquad \forall i \in I, \tag{10}$$

$$y_i \in \{0, 1, \ldots, p\} \qquad \forall i \in I, \tag{11}$$

$$(2) - (6).$$

Extended variables

We add index i in binary variable z_j to get the assignment variable $z_{ij}, i \in I, j \in J$

$$z_{ij} = \begin{cases} 1 & \text{if inaccessible customer } j \text{ is assigned to center } i, \\ 0 & \text{otherwise} \end{cases}$$

$$LEX\Big(\sum_{i\in I}\sum_{j\in J} w_j z_{ij}, \sum_{i\in I}\sum_{j\in A_i} w_j t_{ij} x_{ij}, \sum_{i\in I}\sum_{j\in J} w_j (t_{ij}-\tau) z_{ij}\Big) \to \min \qquad (12)$$

s.t.

$$\sum_{i\in I} x_{ii} \le q \qquad (13)$$

$$\sum_{i\in I: j\in A_i} x_{ij} + \sum_{i\in I} z_{ij} = 1 \qquad \forall j \in J, \qquad (14)$$

$$\sum_{i\in I} y_i = p, \qquad (15)$$

$$\sum_{j\in A_i} w_j x_{ij} + \sum_{j\in J} w_j z_{ij} \le \kappa y_i \qquad \forall i \in I, \qquad (16)$$

$$q x_{ii} \ge y_i \ge x_{ii} \qquad \forall i \in I, \qquad (17)$$

$$x_{ij} \le x_{ii} \qquad \forall i \in I, \forall j \in A_i - \{i\}, \qquad (18)$$

$$z_{ij} \le x_{ii} \qquad \forall i \in I, \forall j \in J - \{i\}, \qquad (19)$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in I, \forall j \in A_i, \qquad (20)$$

$$y_i \in \{0,1,\ldots,p\} \qquad \forall i \in I, \qquad (21)$$

$$z_{ij} \in \{0,1\}, \qquad \forall i \in I, \forall j \in J. \qquad (22)$$

Step 1: Reduction of set of feasible solutions

Here we assume that centers have a lot of weight and so we will choose them from set of nodes whose weight is bigger than their median.

Let I^* be a set of hot candidates for centers (reduced set of candidates)

$$I^* = \{i \in I : w_i \geq w_{med}\}$$

where w_{med} is median of a set $\{w_i : i \in I\}$.

RCCTA:

$$LEX\Big(\sum_{j\in J} w_j z_j, \sum_{i\in I^*}\sum_{j\in A_i} w_j t_{ij} x_{ij}\Big) \to \min \tag{23}$$

s.t.

$$\sum_{i\in I^*} x_{ii} \le q \tag{24}$$

$$\sum_{i\in I^*: j\in A_i} x_{ij} + z_j = 1 \qquad \forall j \in J, \tag{25}$$

$$x_{ij} \le x_{ii} \qquad \forall i \in I^*, \forall j \in A_i - \{i\}, \tag{26}$$

$$\sum_{i\in I^*} y_i = p, \tag{27}$$

$$\sum_{j\in A_i} w_j x_{ij} \le \kappa y_i \qquad \forall i \in I^*, \tag{28}$$

$$q x_{ii} \ge y_i \ge x_{ii} \qquad \forall i \in I^*, \tag{29}$$

$$y_i \in \{0, 1, \dots, q\} \qquad \forall i \in I^*, \tag{30}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in I^*, \forall j \in A_i, \tag{31}$$

$$z_j \in \{0, 1\} \qquad \forall j \in J. \tag{32}$$

Step 3: Assignment inaccessible customers to fixed centers

We fix the selection of centers in the set I_x by definition $I_x = \{i \in I^* : x_{ii} = 1\}$ from step 2. We need the following sets:

- $J_x = J - \{j : i \in I_x, j \in A_i : x_{ij} = 1\}$ – a set of inaccessible customers,
- $K_x = \{(i,j) \in I_x \times J_x : t_{ij} \leq \rho\tau\}$ – a set of feasible pairs (i,j), where ρ is maximum extension of a critical driving time (for example $\rho = 3$),
- $U_j = \{i \in I_x : (i,j) \in K_x\}, j \in J_x$,
  $V_i = \{j \in J_x : (i,j) \in K_x\}, i \in I_x$, – auxiliary index sets,
- $w_i^* = \sum_{j \in A_i} w_j, i \in I_x$ – cumulative weights of accessible customers assigned to the center.

$$LEX\Big(\sum_{(i,j)\in K_x} w_j z_{ij}, \sum_{(i,j)\in K_x} w_j(t_{ij} - \tau) z_{ij} \Big) \to \min \tag{33}$$

s.t.

$$\sum_{i\in I_x} y_i = p, \tag{34}$$

$$\sum_{i\in U_j} z_{ij} = 1 \qquad \forall j \in J_x, \tag{35}$$

$$\sum_{j\in V_i} w_j z_{ij} + w_i^* \leq \kappa y_i \qquad \forall i \in I_x, \tag{36}$$

$$z_{ij} \in \{0,1\} \qquad \forall (i,j) \in K_x, \tag{37}$$

$$y_i \in \{0,1,\dots,p\} \qquad \forall i \in I_x. \tag{38}$$

Experiments were conducted on PC Workstation (processor 8-core i7-5960X 3GHz, RAM 32GB) with OS Linux (Debian/stretch). We used Python-based tools and the Python interface to commercial mathematical programming solver Gurobi.

We tested it on seven self-governing regions of Slovak republic i.e Bratislava (BA), Banská Bystrica (BB), Košice (KE), Nitra (NR), Trenčín (TN), Trnava (TT), Žilina (ZA) and in throughout the region (SR).

- n – number of nodes in network,
- p – number of vehicles,
- q – number of centers,
- δ – number of inaccessible nodes,
- κ – capacity of vehicles,
- obj_1 – weighted number of inaccessible customers,
- obj_2 – weighted travel time of accessible customers,
- *Time* – computational time in minutes.

Table 1: Computational results for models CAT and CCAT

				CAT ($\tau = 12$)				CCAT ($\tau = 12, \kappa = 2500$)			
Inst.	n	p	q	δ	obj_1	obj_2	*Time*	δ	obj_1	obj_2	*Time*
BA	87	25	9	5	44	31561	< 0.009	5	44	31561	<0.009
BB	515	46	46	8	10	27376	0.04	22	34	28611	0.06
KE	460	48	46	0	0	24117	0.02	5	17	34540	0.09
NR	350	36	35	1	3	34072	0.05	1	3	34072	0.05
PO	664	44	44	0	0	24467	0.03	37	72	38910	0.06
TN	276	26	26	0	0	23476	0.01	1	1	29298	0.01
TT	249	22	22	1	4	27142	0.02	8	30	27635	0.03
ZA	315	36	32	6	17	33757	0.01	6	17	33757	0.01
SR	2916	283	260	36	58	277231	6.18	36	58	277231	6.30

- δ – number of inaccessible nodes,
- $\overline{\Delta} = obj_3/obj_1$ – mean of deficit time of inaccessible costumers,
- obj_1 – weighted number of inaccessible customers,
- obj_2 – weighted travel time of accessible customers,
- obj_3 – weighted time of inaccessible customers,
- *Time* – computational time in minutes.

Table 2: Computational results for models ECCAT and HCCAT

	ECCAT						HCCAT					
Inst.	δ	$\overline{\Delta}$	obj_1	obj_2	obj_3	*Time*	δ	$\overline{\Delta}$	obj_1	obj_2	obj_3	*Time*
BA	5	4.16	44	31790	183	0.02	5	3.51	51	40198	179	<0.009
BB	22	3.00	34	28611	102	0.72	26	3.09	45	59828	139	0.05
KE	5	3.18	17	34560	54	0.56	-	-	-	-	-	-
NR	1	2.00	3	34072	6	0.37	4	4.00	13	46143	52	0.02
PO	37	2.60	72	39291	187	0.97	41	2.84	90	97295	256	0.09
TN	1	2.00	1	29373	2	0.08	6	4.88	17	35102	83	0.02
TT	8	8.87	38	27635	266	0.23	11	5.68	41	47581	233	0.02
ZA	6	4.12	17	33757	70	0.12	16	3.19	57	53098	182	0.02
SR	35	4.12	58	277547	156	59.14	91	3.88	215	1152501	835	2.03

A great disappointment is the heuristics RCCAP&HCCAP proposed for us. In the case of the KE region, they did not even get an admissible solution. The reason was the fact that one potencial center did not appear in the hot centers, which led to an inadmissible solution of the HCCAP.

In further research we want to focus on an uncertainty capacity constraints via in micro-genetic algorithms. We would like to chat the future results of over solution in simulation models for large instances.