

FLEXIBLE BUS SCHEDULING WITH OVERLAPPING TRIPS

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Abstract

We investigate simplified forms of the flexible bus scheduling problem for one bus. It is motivated by practical needs of removing overlapping trips from the real bus schedule. The goal is to minimize the total overlaps of the sequence of trips of fixed running board. The possibility of using linear programming approach with integer solutions for optimistic and pessimistic models are considered.

1 INTRODUCTION

The topics discussed in this paper are focused on important operational problems confronting the management of competitive bus transportation system.

The basic bus scheduling problem consists of assigning buses to given set of trips in running board such that:

- each trip is performed exactly once,
- each bus must start and end its work day at the same depot,
- the number of buses is as low as possible minimum,
- the operational cost is minimum.

This problem has several variations with practical restriction (on number and types of depots, meal breaks, buses types, length of the running boards) studied in Slovakia by Palúch and his colleagues [1],[2],[3],[4],[5].

We will consider the Flexible Bus Scheduling Problem (FBSP) when the set of flexible trips is given for one bus (see table 1) and no practical restrictions are presumed. This paper aims to present and discuss mathematical programming formulations and the possibility of for solving the FBSP.

2 BASIC NOTATION

In this paper we will use the terminology from Hartley [6]. A flexible trip is a journey performed by bus between two places at specific time given by the flexible time-table.

Given set of flexible trips $\mathcal{S} = \{S_1, S_2, \dots, S_n\}$, each flexible trip S_i is represented by an arbitrary ordered quintuple

Departure		Arrival		Trip	Deadhead	Overlapping
place	time	place	time	minutes	minutes	minutes
A	5:30	B	5:55	25	0	
B	6:00	A	6:23	23	0	3
A	6:20	B	6:45	25	5	
C	6:50	A	7:20	30	0	
A	7:30	C	8:00	30	0	5
C	7:55	A	8:25	30	0	2
A	8:23	B	8:48	25	0	3
B	8:45	A	9:08	23	0	
:	:	:	:	:	:	:

Figure 1: Running board of bus with overlapping trips

$$S_i = (m_i^d, t_i^d, m_i^a, \tau_i, \tau_i^-, \tau_i^+) \quad (1)$$

where

- m_i^d - the departure place,
- t_i^d - the scheduled departure time,
- m_i^a - the arrival place,
- τ_i - the trip time,
- τ_i^- - the most likely departure time,
- τ_i^+ - the latest likely departure time.

We will assume that $\tau_i^- \leq t_i^d \leq \tau_i^+$. The bus models are of the discrete-time rather than of continuous-time variety. We will suppose that $t_i^d, \tau_i^-, \tau_i^+ \in \{1, 2, \dots, 1440\}$. Note that for flexible trip S_i the scheduled arrival time is equal $t_i^d + \tau_i$, the likely arrival time is equal $\tau_i^- + \tau_i$ and the latest likely arrival time is equal $\tau_i^+ + \tau_i$.

Let $\delta_{i,i+1}$ the times of the idle trip from place m_i^a to place m_{i+1}^d are given. The running board of the bus \mathcal{T} with n flexible trips is sequence

$$\mathcal{T} = S_1 \prec \dots \prec S_i \prec S_{i+1} \prec \dots \prec S_n,$$

where we write $S_i \prec S_{i+1}$ iff the bus arrives at i -th trip could start $(i+1)$ -th trip

$$\tau_i^- + \tau_i + \delta_{i,i+1} \leq \tau_{i+1}^+. \quad (2)$$

The overlapping time z_i (see figure 2) between trips S_i and S_{i+1} with scheduled departure times t_i^d and t_{i+1}^d is defined by value

$$z_i = \max\{0, t_i^d + \tau_i + \delta_{i,i+1} - t_{i+1}^d\}. \quad (3)$$

The problem is when in given running board \mathcal{T} exists trips S_i with positive z_i . It is possible to find real departure times such that running board of the bus is without overlapping times?

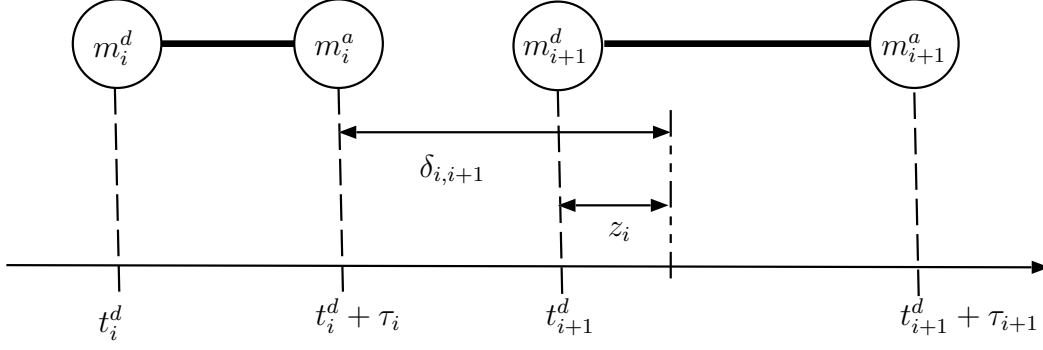


Figure 2: Overlapping time z_i between scheduled trips S_i and S_j

3 OPTIMISTIC OPTIMIZATION

The first we will optimistic assume that it is possible find real departure times

$$t_i : \tau_i^- \leq t_i \leq \tau_i^+, i = 1, 2, \dots, n, \quad (4)$$

that given running board \mathcal{T} is without overlapping times i. e.

$$\max\{0, t_i + \tau_i + \delta_{i,i+1} - t_{i+1}\} = 0 \text{ for } i=1,2,\dots,n-1. \quad (5)$$

However in practical conditions it is possible to find more solutions (4) conforming with constraints (3). One option is to find solution with minimum offset of scheduled times and we obtain the following mathematical programming problem (OOP1):

$$\begin{aligned} \sum_{i=1}^n \max |t_i - t_i^d| &\rightarrow \min \\ t_i + \tau_i + \delta_{i,i+1} - t_{i+1} &\leq 0 \quad i = 1, 2, \dots, n-1 \\ \tau_i^- \leq t_i \leq \tau_i^+ &\quad i = 1, 2, \dots, n \end{aligned}$$

It is easy see that if we set $t_i = t_i^d - x_i^- + x_i^+$ where variable $x_i^- \geq 0$ and $x_i^+ \geq 0$ are negative, and positive offset of the scheduled departure time than the OOP1 can be formulated as the following linear problem (OOP2):

$$\begin{aligned} \sum_{i=1}^n x_i^- + x_i^+ &\rightarrow \min \\ x_i^+ + x_{i+1}^- - x_i^- - x_{i+1}^+ &\leq t_{i+1}^d - t_i^d - \tau_i - \delta_{i,i+1} \\ 0 \leq x_i^- &\leq t_i^d - \tau_i^- \\ 0 \leq x_i^+ &\leq \tau_i^+ - t_i^d \end{aligned}$$

Note that the problem OOP2 has always integer solution x_i^-, x_i^+ if exists and so the problem OOP1 has integer solution $t_i = t_i^d - x_i^- + x_i^+$.

4 PESSIMISTIC OPTIMIZATION

The models OOP1 or OOP2 don't need to have a solution. Then we can pessimistic assume that it is possible find almost real departure times t_i only via the mathematical

programming problem (POP1):

$$\sum_{i=1}^{n-1} \max\{0, t_i + \tau_i + \delta_{i,i+1} - t_{i+1}\} \rightarrow \min$$

$$\tau_i^- \leq t_i \leq \tau_i^+ \quad i = 1, 2, \dots, n$$

If we set $y_i^+ - y_i^- = t_i + \tau_i + \delta_{i,i+1} - t_{i+1}$ where where variable $y_i^- \geq 0$ and $y_i^+ \geq 0$ are negative and positive offset of the overlapping time than the POP1 can be formulated as the following linear problem (POP2):

$$\sum_{i=1}^{n-1} y_i^+ \rightarrow \min$$

$$y_i^+ - y_i^- - t_i + t_{i+1} = \tau_i + \delta_{i,i+1} \quad i = 1, 2, \dots, n-1$$

$$\tau_i^- \leq t_i \leq \tau_i^+ \quad i = 1, 2, \dots, n$$

$$y_i^-, y_i^+ \geq 0 \quad i = 1, 2, \dots, n-1$$

Note that analogical as for the problem OOP2 the problem POP2 has always integer solution t_i, y_i^-, y_i^+ if exists and so the problem POP2 has integer solution t_i .

5 OPEN QUESTIONS

The bus companies schedule running boards of many buses. In the system KASTOR [5] exists a choice for minimum fleet size if fixed maximum overlapping of trips is feasible provisionally. Then scheduler handle with trips is necessary for design realistic running boards with computed fleet size.

It is possible to use solutions solving the optimistic a the pessimistic models presented in this paper. Open questions are following finer optimum criterion for the running boards. We can find a minimum of

- weighted sum of offsets for trips i.e. $\sum_i \alpha_i x_i^- + \beta_i x_i^+$,
- weighted sum of overlapping trips i.e. $\sum_i \gamma_i z_i$,
- longest overlapping trips i.e. $\max_i z_i$,
- longest offset of departure times i.e. $\max_i \{x_i^-, x_i^+\}$,
- irregularity of departure offsets i.e. $\sum_i (x_i^- + x_i^+)^2$,
- irregularity of overlapping trips i.e. $\sum_i z_i^2$.

The other questions deal with number of running boards in models. We hope that we can find a basic linear models – optimistic and pessimistic – for two busses too.

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References

- [1] Palúch, S.: *A Graph Theory Approach to Bus Scheduling with Types of Buses*, Studies of the Faculty of Management Science and Informatics, Volume 9, pp. 69-73, ISBN 80-7100-959-8, 2001
- [2] Peško, Š.: *The Minimum Fleet Size Problem for Flexible Bus Scheduling*, Studies of the Faculty of Management Science and Informatics, Vol. 9, University of Žilina, pp. 59-65, 2001
- [3] Palúch, S.: *Bus Scheduling as a Graph Coloring Problem*, Komunikácie, Vedecké listy Žilinskej univerzity, 5. ročník, 4/2003, pp. 16-20, ISSN 1335-4205, 2003
- [4] Czimmermann, P., Peško, Š.: *Autobusové rozvrhy s dvoma typmi vozidiel*, 3th International Conference APLIMAT, Bratislava, pp. 321-326, ISBN 80-227-1995-1, 2004
- [5] Palúch, S., Majer, T.: *K optimalizácii mestskej a prímestskej pravidelnej osobnej dopravy*, Vysoká škola jako facilitátor rozvoje spoločnosti a regionu, II. mezinárodní konference 2007, sborník, 26. ledna 2007, Kunovice, Česká republika, Evropský polytechnický institut, 245-248, ISBN 80-7314-107-8, 2007
- [6] Hartley T.: *A Glossary of Terms in Bus and Crew Scheduling*, In: Wren (ed.): *Computer Scheduling of Public Transport*, Nort-Holland Publishing Company, pp. 353-399, 1981

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