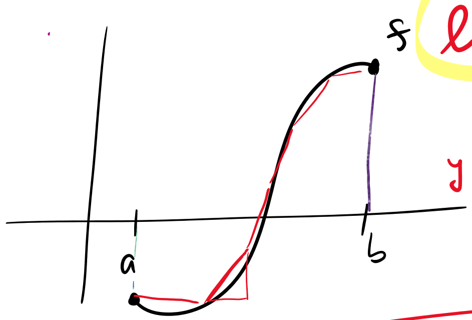
DĹŽKA KRIVKY



$$l = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

$$l = \int_\alpha^\beta \sqrt{[\varphi'(t)]^2 + [\psi'(t)]^2}\,dt$$

$$\int \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$$

$y = f(x)$, $x \in \langle a, b\rangle$

f' spojité na $\langle a, b\rangle$

SUBST: $x = \varphi(t)$ $t \in \langle \alpha, \beta\rangle$

$y = \psi(t)$, $dx = d\varphi(t) = \varphi'(t)dt$

$dx = |\varphi'(t)|\,dt$

f': derivovanie PARAMETRICKY

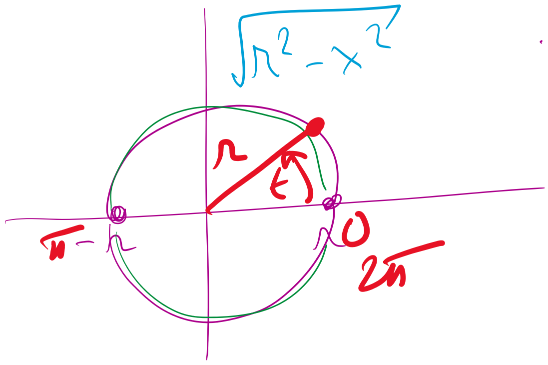
$$\frac{df(x)}{dx} = \frac{d\psi(t)}{d\varphi(t)} = \frac{\psi'(t)}{\varphi'(t)}$$

$$\sqrt{1+[f'(x)]^2} = \sqrt{1+\left[\frac{\psi'(t)}{\varphi'(t)}\right]^2} = \sqrt{1+\frac{[\psi'(t)]^2}{[\varphi'(t)]^2}} =$$

$$= \sqrt{\frac{[\varphi'(t)]^2 + [\psi'(t)]^2}{\varphi'(t)^2}} =$$

$$= \frac{\sqrt{[\varphi'(t)]^2 + [\psi'(t)]^2}}{|\varphi'(t)|}$$

OBVOD KRUŽNICE



$$x^2+y^2=r^2$$

$$f(x)=\sqrt{r^2-x^2}=(r^2-x^2)^{1/2}$$

$$f'(x)=\frac{1}{2}(r^2-x^2)^{-1/2}\cdot(-2x)$$

$$=\frac{x}{\sqrt{r^2-x^2}}$$

$r>0$

$$\sqrt{1+[f'(x)]^2}=\sqrt{1+\frac{x^2}{r^2-x^2}}=\sqrt{\frac{r^2}{r^2-x^2}}$$

$$o=2\int_{-r}^{r}\frac{r}{\sqrt{r^2-x^2}}\,dx=2r\left[\arcsin\frac{x}{r}\right]_{-r}^{r}=$$

$$=2r[\arcsin 1-\arcsin(-1)]$$

$$=2r\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right]=2\pi r$$

$$x=r\cdot\cos t \qquad x'=r\cdot(-\sin t)$$

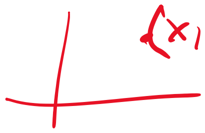
$$y=r\cdot\sin t \qquad y'=r\cdot\cos t$$

$$(x')^2+(y')^2=r^2\cdot\sin^2 t+r^2\cdot\cos^2 t=r^2$$

$$o=2\int_0^{\pi} r\,dt=2r\int_0^{\pi}dt=2r[t]_0^{\pi}=2\pi r$$

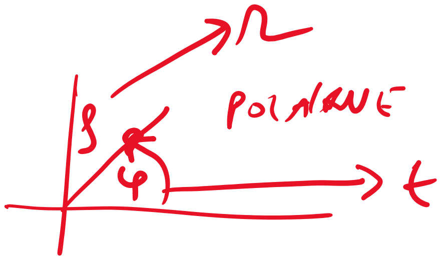
$$o=\int_0^{2\pi} r\,dt=r[t]_0^{2\pi}=2\pi r$$

| $\{x,y\}$ karteziánske



$[x,y]$ karteziánske

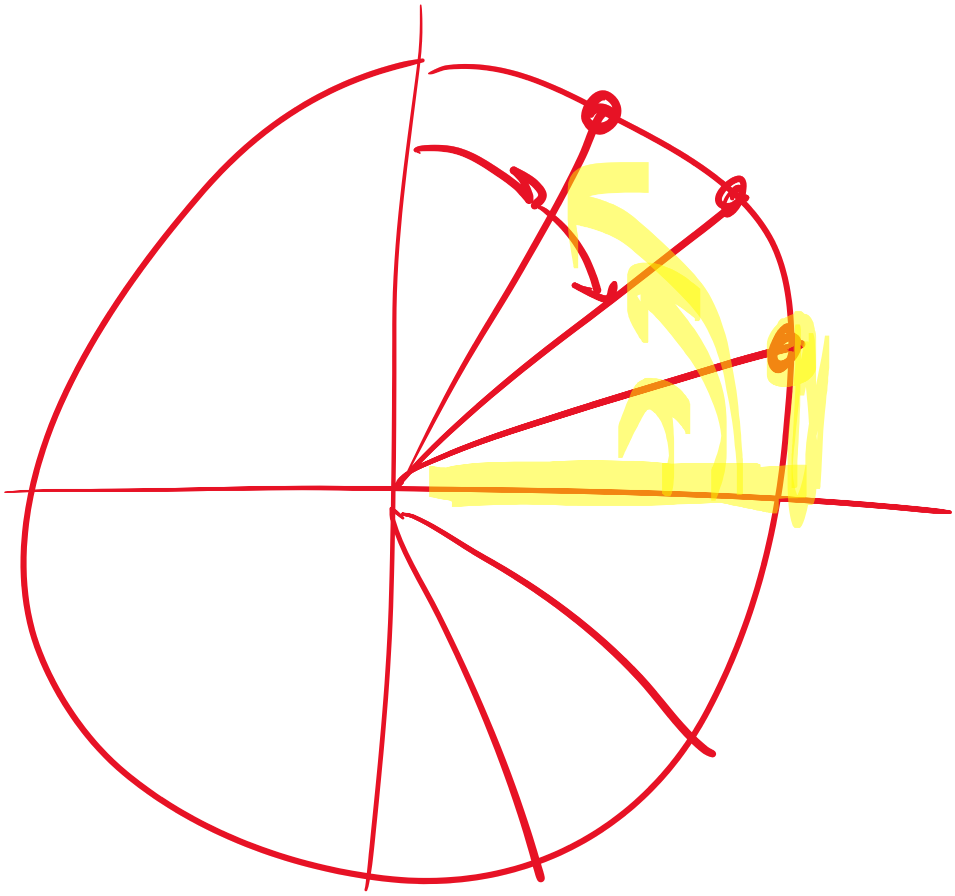
Kružnice



POLÁRNE

$$x = r.\cos t = \rho \cos \varphi$$
$$y = r.\sin t = \rho \sin \varphi$$

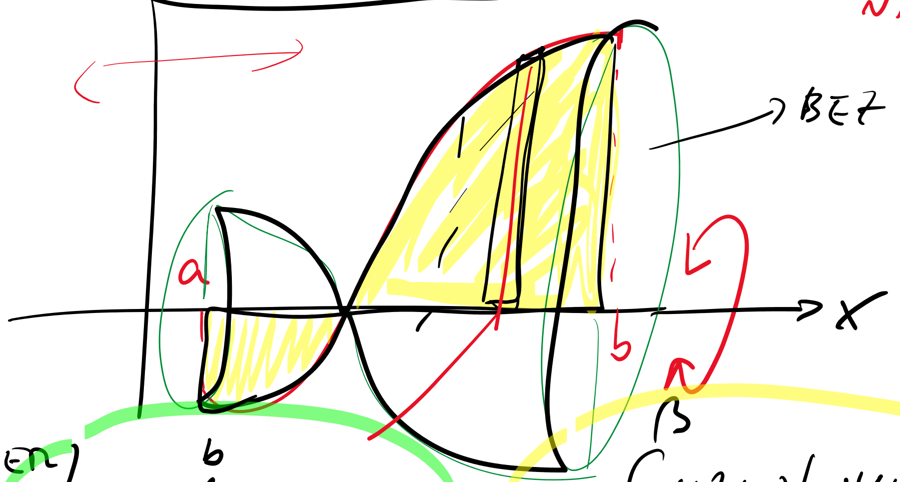
utorok 27. februára 2024 10:32



utorok 27. februára 2024 10:33

ROTÁCIA OKOLO OSI X

f SPOJITÁ NA $\langle a,b \rangle$



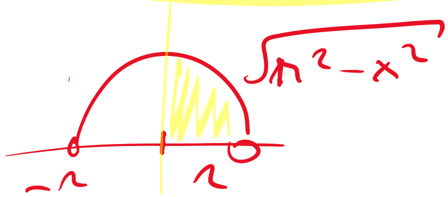
OBJEM

$$V = \pi \int_a^b f^2(x)\,dx = \pi \int_\alpha^\beta \psi^2(t)\,|\varphi'(t)|\,dt$$

POVRCH

$$S = 2\pi \int_a^b |f(x)| \sqrt{1+[f'(x)]^2}\,dx = 2\pi \int_\alpha^\beta |\psi(t)| \sqrt{[\varphi'(t)]^2+[\psi'(t)]^2}\,dt$$

OBJEM GULE



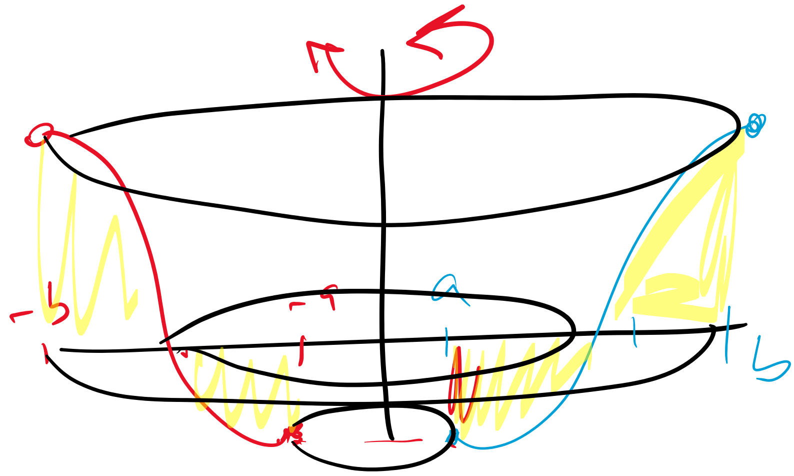
$$V = \pi \int_{-r}^{r} (r^2-x^2)\,dx$$

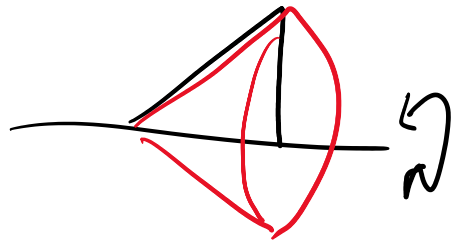
$$\pi = \left[r^2x - \frac{x^3}{3}\right]_{-r}^{r} = \pi\left[r^3 - \frac{1}{3}r^3 + r^3 - \frac{1}{3}r^3\right] = \frac{4}{3}r^3\pi$$

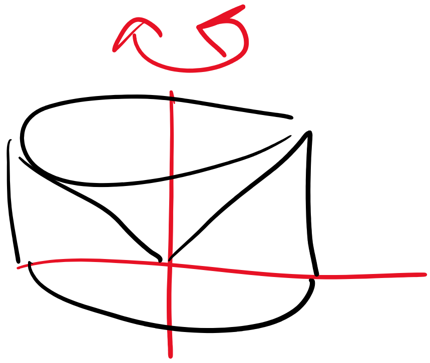
ROTÁCIA OKOLO OSI y

$$V = 2\pi \int_a^b |x f(x)|\, dx$$

$$= 2\pi \int_\alpha^\beta |\varphi(t)\psi(t)\varphi'(t)|\, dt$$

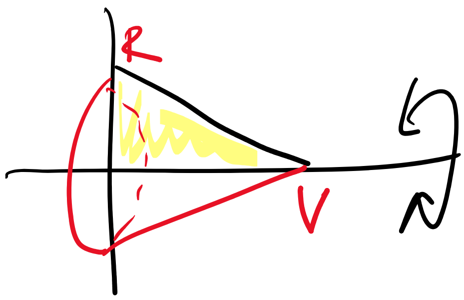


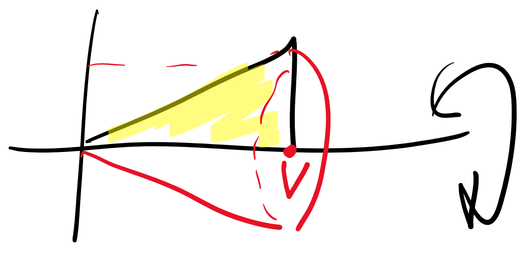


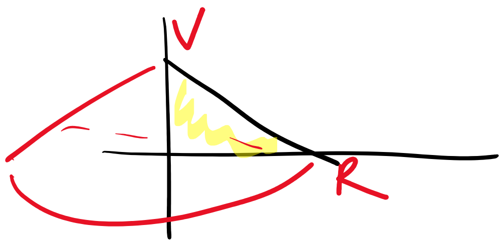


OKOLO OSI x

KUŽEĽ







KOMPLEXNÉ ČÍSLA

Reálne časť
Imaginárne

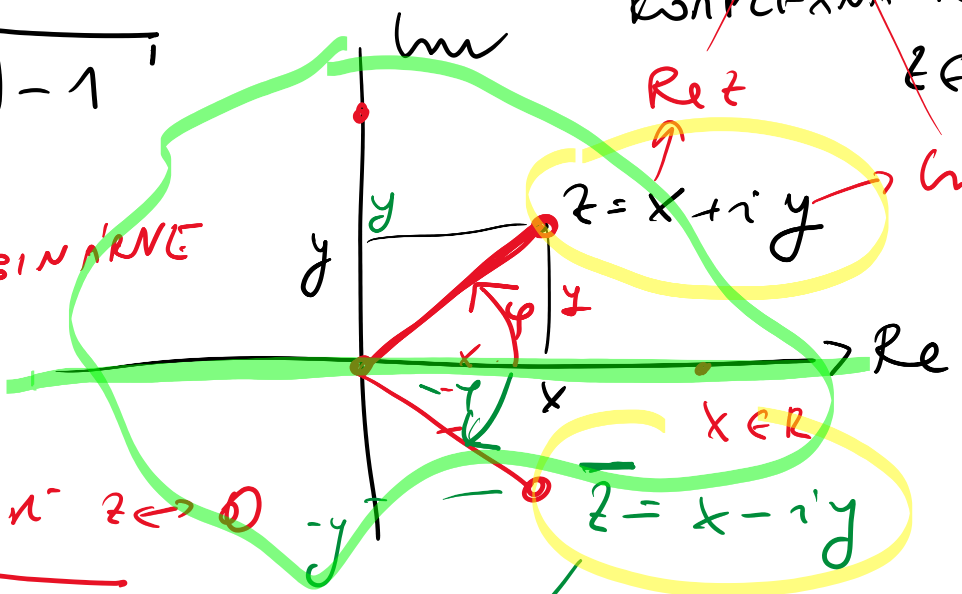
KOMPLEXNÁ ROVINA

$z \in \mathbb{C}$

$i = \sqrt{-1}$

$z = iy$

RÝDZO IMAGINÁRNE



vzdialenosť $z \leftrightarrow 0$

$|z| = \sqrt{x^2+y^2}$

ABSOLÚTNA HODNOTA

KOMPLEXNE ZDRUŽENÉ ku z

z a $\bar{z}$ .. $x \pm iy$

$|z| = |\bar{z}|$

$$\frac{z+\bar{z}}{2} = \operatorname{Re} z = \operatorname{Re} \bar{z}$$

$$\frac{z-\bar{z}}{2i} = \operatorname{Im} z = -\operatorname{Im} \bar{z}$$

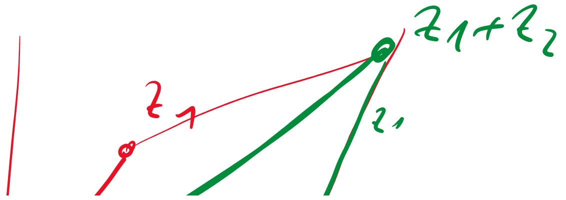
PLATÍ

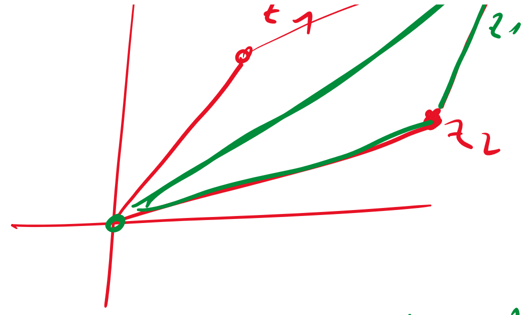
$$\overline{z_1+z_2} = \bar{z}_1 + \bar{z}_2$$

$$\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$$

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

$$|z_1+z_2| \leq |z_1| + |z_2|$$





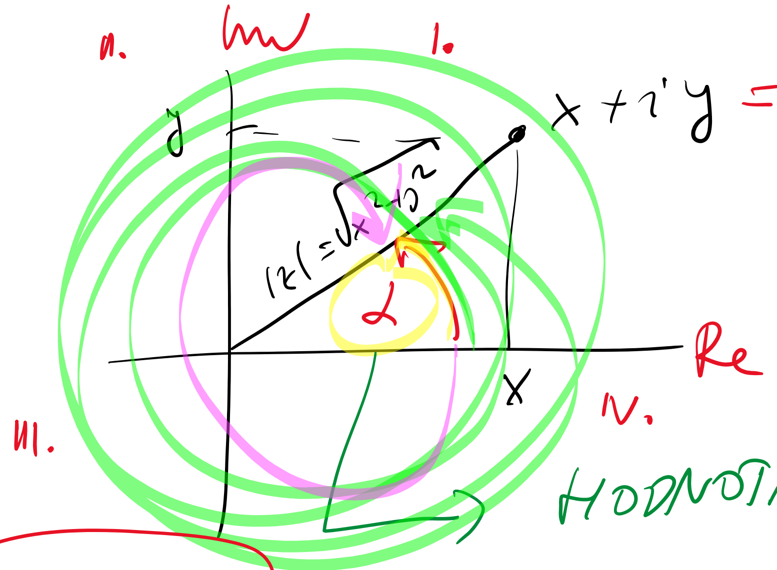
$$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \quad \text{pre } z_2 \neq 0$$

Ak $z = 0 \Rightarrow |z| = 0$ — Reálne číslo

$|z| = 0 \Rightarrow z = 0$ — Komplexné číslo

$$\mathbb{C} = \{ z = x + iy,\ x, y \in \mathbb{R} \}$$



$x + i y = \underbrace{|z|(\cos\alpha + i\cdot\sin\alpha)}$

Goniometrický tvar komplex. čísla

HODNOTA ARGUMENTU KOMPL. ČÍSLA z

$$\boxed{\begin{aligned} x &= |z|\cdot\cos\alpha \\ y &= |z|\cdot\sin\alpha \end{aligned}}$$

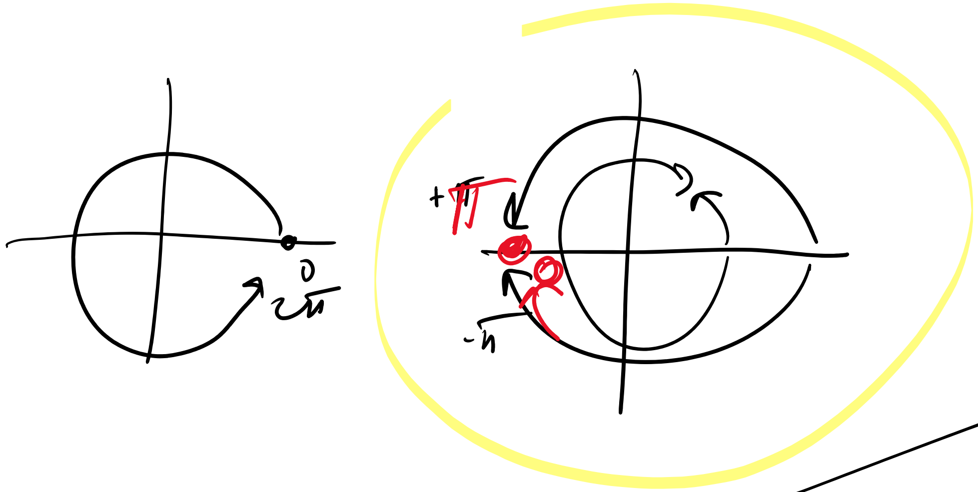
$$\underbrace{\boxed{\arg(z) = \alpha} + 2k\pi,\ k\in \mathbb{Z}}_{\text{argument}}$$

(vyberie sa 1 hodnota)

Hlavná hodnota argumentu

$$\operatorname{Arg} z \in (-\pi, \pi\rangle$$

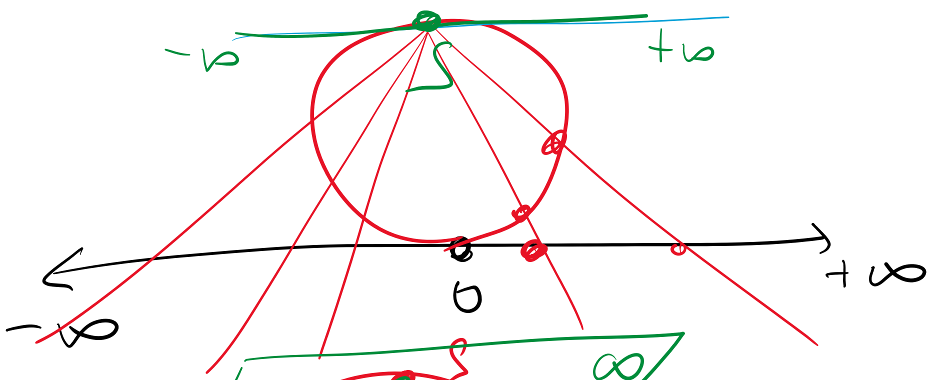
$\arg z = -\arg \bar{z}$

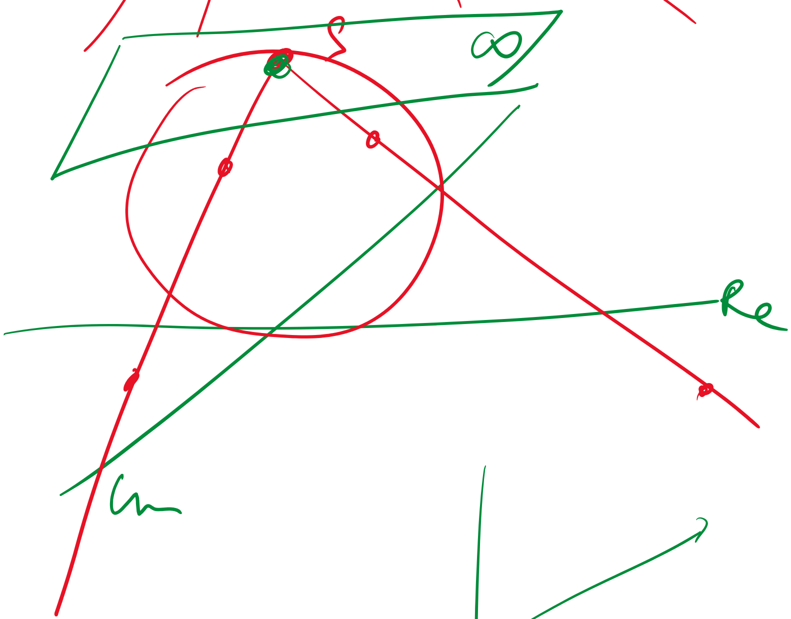


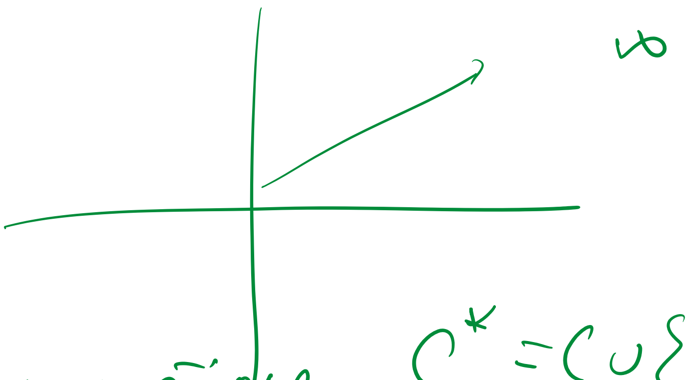
$\alpha = \arg z$

$z = x + i y = |z|\cdot(\cos\alpha + i\sin\alpha)$

$\arg z = \{ \operatorname{Arg} z + 2k\pi,\ k\in \mathbb{Z} \}$







ROZŠÍRENÁ MNOŽINA $\quad C^* = C \cup \{\infty\}$

$\Downarrow$

$$\infty + \infty = \infty$$
$$\infty - \infty = \infty$$

PLATÍ

$$z \pm \infty = \infty \pm z = \infty$$

$$\frac{\infty}{z} = \infty \quad \downarrow \quad z \neq 0$$

$$z \cdot \infty = \infty \cdot z = \infty$$

$$\frac{z}{\infty} = 0 \qquad \frac{z}{0} = \infty$$

$n \in N$

$$\infty^n = \infty \qquad \infty^{-n} = \frac{1}{\infty^n} = 0$$

$$0^n = 0 \qquad 0^{-n} = \frac{1}{0^n} = \frac{1}{0} = \infty$$

$\overline{\infty} = \infty$ $|\infty| = +\infty$

$\in \mathbb{C}^*$ $\in \mathbb{R}^*$

NEDEFINUJEME (limity)

$\frac{0}{0}$ $\frac{\infty}{\infty}$

$\infty \cdot 0$ $0 \cdot \infty$

OKOLIE

$O(z_0) = \{z, |z-z_0|<\varepsilon$

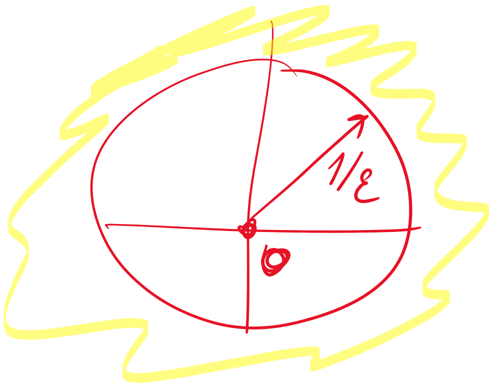
$O(z_0)$

PRSTENCOVÉ

$O(z_0) - \{z_0\}$

OKOLIE BODU ∞

$O(\infty) = \{z \in \mathbb{C}^*, |z| >$



$> \frac{1}{\varepsilon}\}$

KOMPLEXNÁ FUNKCIA KOMPLEX. PREMENNE

$f: C^k \to C^k$

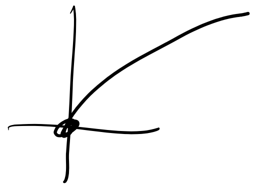
D(f)

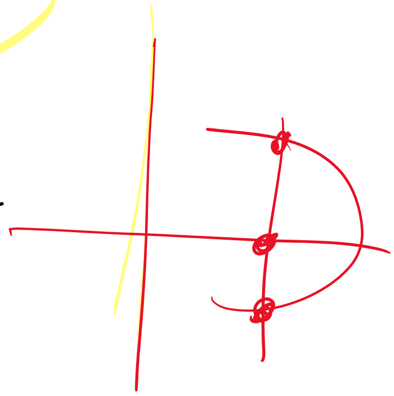
H(f)

f(z) PRE $z \in D(f)$

→ MÔŽE BYŤ - MNOŽINA

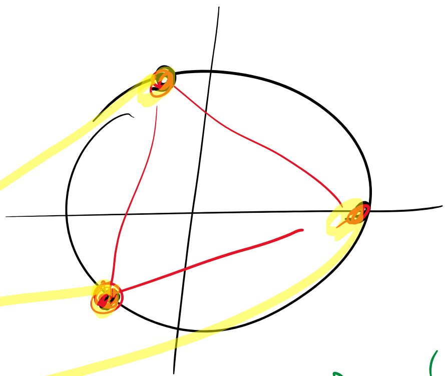
$y = \sqrt{x}$, $\langle 0, \infty) \to \langle 0, \infty)$





V KOMPLEXNOM OBORE MNOHOZNAČNÁ FUN

$$\sqrt{x} = \{ z \in C, \ z^2 = x \}$$



$$\sqrt{1} = \{-1, 1\}$$

$$\sqrt[3]{1} = \{ 1, \quad \}$$

2 - ZNAČNÁ

3 - " -

arg z ...

∞ ZNAČNÁ FUN

kce 1A

ce 1A

KONEČNÁ FUNKCIA .. $\forall z \in D(f)$ .. $z \in ($

utorok 27. februára 2024 11:21

$\mathbb{C}^* \to \mathbb{C}^*$

$z = x + iy$

$$f(z) = f(x+iy)$$

$$= \boxed{\operatorname{Re} f(z)} + i \boxed{\operatorname{Im} f(z)}$$

$$f_1(x+iy) + i\, f_2(x+iy)$$

$$f_1(x+iy) : \mathbb{C}^* \to \mathbb{R}^*$$

$$f_2(x+iy) : \mathbb{C}^* \to \mathbb{R}^*$$

LIMITA FUNKCIE $f: \mathbb{C} \to \mathbb{C}$

JE ROVNAKO DEFINOVANÁ AKO V $\mathbb{R}^n$

$$w = \lim_{z \to z_0} f(z)$$

ak 1. z_0 JE HROMADNÝ BOD $D(f)$

2. podmienka

$$\{z_n\}_{n=1}^{\infty} \to z_0 \implies \{f(z_n)\}_{n=1}^{\infty} \to w$$

PLATIA ROVNAKÉ PRAVIDLÁ

$\boxed{\text{SPOJITOSŤ}}$ ako v $\mathbb{R}^n$

f JE SPOJITÁ V BODE $z_0 \in D(f)$

$$\{z_n\}_{n=1}^{\infty} \to z_0$$

$$\implies \{f(z_n)\}_{n=1}^{\infty} \to f(z_0)$$

FUNKCIE $\mathbb{R} \to \mathbb{C}$

KOMPLEXNÁ FUNKCIA REÁLNEJ PREMENNEJ

$$f(t) = u(t) + i\,v(t), \quad t \in \mathbb{R}$$

$$f'(t) = u'(t) + i \cdot v'(t)$$

$$\int f(t)\,dt = \int u(t)\,dt + i \int v(t)\,dt$$

PLATÍ

$$\boxed{e^{x+iy} = e^x(\cos y + i \cdot \sin y)}$$

$$\int e^{\alpha t} \cos \beta t \, dt$$ ... per partes (2x

$$e^{\alpha t} \cos \beta t + i \cdot e^{\alpha t} \cdot \sin \beta t =$$

$$= e^{\alpha t}(\cos \beta t + i \sin \beta t)$$

$$= e^{\alpha t + i \beta t} = e^{(\alpha + i\beta)t} = e^{zt}$$

$$\boxed{\alpha + i\beta = z}$$

z -- ~~konstanta~~ 2

$$\int e^{zt}\,dt = \frac{e^{zt}}{z} + \text{konst.}$$

$$\int e^{\alpha t}\cos\beta t\,dt + i\int e^{\alpha t}\sin\beta t\,dt =$$

$$= \int e^{\alpha t}(\cos\beta t + i\sin\beta t)\,dt$$

$$= \int e^{zt}\,dt = \frac{e^{zt}}{z} + c$$

$\alpha + i\beta$

$$\frac{e^{\alpha t}\cos\beta t + i e^{\alpha t}\cdot\sin\beta t}{z} + c$$

$z = \alpha + i\beta$

$$\frac{e^{\alpha t}\cdot\cos\beta t}{\alpha + i\beta} + i\,\frac{e^{\alpha t}\cdot\sin\beta t}{\alpha + i\beta} + c \qquad \frac{\alpha - i\beta}{\alpha - i\beta}$$

$$\frac{e^{\alpha t}\cdot\cos\beta t\,(\alpha - i\beta) + i e^{\alpha t}\sin\beta t\,(\alpha - i\beta)}{\alpha^2 + \beta^2}$$

Re Im

$$z \cdot \bar{z} = (\alpha + i\beta)(\alpha - i\beta) =$$

$$= \alpha^2 + \beta^2 = |z|^2$$