Matematická analýza 1

2023/2024

6. Limita funkcie
Riešené príklady

Pre správne zobrazenie, fungovanie tooltipov, 2D a 3D animácií je nevyhnutné súbor otvoriť pomocou programu Adobe Reader (zásuvný modul Adobe PDF Plug-In webového prehliadača nestačí).

Kliknutím na text pred ikonou získate nápomoc.
Kliknutím na skratku v modrej lište vpravo hore sa dostanete na príslušný slajd, druhým kliknutím sa dostanete na koniec tohto slajdu.

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Obsah

Zoznam riešených limít – príklady 01–45

- **01.** $\lim\limits_{x\to 0}\frac{a^x-1}{x}$. • **02.** $\lim\limits_{x\to 2}\frac{x-2}{x^2-3x+2}$. • **03.** $\lim\limits_{x\to\infty}\left(\frac{5x^2}{x^2-1}+2^{\frac{1}{x}}\right)$. • **04.** $\lim\limits_{x\to 0}\frac{1-3^x}{x}$. • **05.** $\lim\limits_{x\to-\infty}\frac{1-3^x}{x}$.
- **06.** $\lim\limits_{x\to 1}\frac{1-3^x}{x}$. • **07.** $\lim\limits_{x\to\infty}\frac{1-3^x}{x}$. • **08.** $\lim\limits_{x\to 0}\frac{3x+\frac{2}{x}}{x+\frac{4}{x}}$. • **09.** $\lim\limits_{x\to\infty}\left(\frac{1-x}{x^2}+\frac{1-2x}{2-3x}\right)$.
- **10.** $\lim\limits_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$. • **11.** $\lim\limits_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^x$. • **12.** $\lim\limits_{x\to-\infty}\left(\frac{x+1}{x}\right)^x$. • **13.** $\lim\limits_{x\to 0}(1+3\,\mathrm{tg}^2\,x)^{\mathrm{cotg}^2\,x}$.
- **14.** $\lim\limits_{x\to 0}(1+3\,\mathrm{tg}^2\,x)^{\mathrm{tg}^2\,x}$. • **15.** $\lim\limits_{x\to\infty}(1-\frac{1}{x})^{x^2}$. • **16.** $\lim\limits_{x\to\infty}x\left[\ln(x+2)-\ln x\right]$.
- **17.** $\lim\limits_{x\to 0}\frac{x}{\ln(1+tx)}$. • **18.** $\lim\limits_{x\to 0}\frac{1-\sqrt{1-x}}{x}$. • **19.** $\lim\limits_{x\to 0}\frac{x}{\sqrt{1+x}-\sqrt{1-x}}$. • **20.** $\lim\limits_{x\to\infty}\frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$.
- **21.** $\lim\limits_{x\to\infty}\left(x-\sqrt{x^2-1}\right)$. • **22.** $\lim\limits_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}$. • **23.** $\lim\limits_{x\to a}\frac{a-x}{\sqrt{a}-\sqrt{x}}$. • **24.** $\lim\limits_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}$.
- **25.** $\lim\limits_{x\to 1}\frac{x^m-1}{x^n-1}$. • **26.** $\lim\limits_{x\to 1}\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$. • **27.** $\lim\limits_{x\to 1}\frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$. • **28.** $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}$. • **29.** $\lim\limits_{x\to 0}\frac{\sin 5x}{x}$.
- **30.** $\lim\limits_{x\to 0}\frac{\mathrm{tg}\,x}{x}$. • **31.** $\lim\limits_{x\to 0}\frac{\mathrm{arctg}\,x}{x}$. • **32.** $\lim\limits_{x\to 0}\frac{\mathrm{tg}\,x}{\sin 2x}$. • **33.** $\lim\limits_{x\to\pi}\frac{\mathrm{tg}\,x}{\sin 2x}$. • **34.** $\lim\limits_{x\to 0}\frac{\sin mx}{\sin nx}$. • **35.** $\lim\limits_{x\to\pi}\frac{\sin mx}{\sin nx}$.
- **36.** $\lim\limits_{x\to 2\pi}\frac{\sin mx}{\sin nx}$. • **37.** $\lim\limits_{x\to 0}\frac{1-\cos 2x}{x^2}$. • **38.** $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos 2x}{\cos x-\sin x}$. • **39.** $\lim\limits_{x\to 0}\frac{\sin 4x}{\sqrt{x+1}-1}$.
- **30.** $\lim\limits_{x\to 2}\frac{\arcsin(x-2)}{x^2-2x}$. • **41.** $\lim\limits_{x\to\infty}\mathrm{e}^x(2+\cos x)$. • **42.** $\lim\limits_{x\to 0}\frac{\sin x}{\sqrt{1-\cos x}}$. • **43.** $\lim\limits_{x\to 0}\frac{x^n-\sin^n x}{x-\sin x}$.
- **44.** $\lim\limits_{x\to a}\frac{\sin x-\sin a}{x-a}$. • **45.** $\lim\limits_{x\to a}\frac{\cos x-\cos a}{x-a}$.

Riešené limity – 01, 02, 03

$\lim_{x\to 0} \frac{a^x-1}{x}$ pre $a>0,\ a\neq 1$

$\lim_{x\to 2} \frac{x-2}{x^2-3x+2}$

$\lim_{x\to\infty} \left(\frac{5x^2}{x^2-1} + 2^{\frac{1}{x}}\right)$

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$\lim\limits_{x\to 0} \frac{a^x-1}{x}$ pre $a>0,\ a\neq 1$

- $= \left[\begin{array}{r|l|} \text{Subst. } z = a^x - 1 & x \to 0 \\ z+1 = a^x & z \to 0 \end{array} \right]$

$\lim\limits_{x\to 2} \frac{x-2}{x^2-3x+2}$

- $= \lim\limits_{x\to 2} \frac{x-2}{(x-2)\cdot(x-1)}$

$\lim\limits_{x\to\infty} \left(\frac{5x^2}{x^2-1} + 2^{\frac{1}{x}}\right)$

- $= \lim\limits_{x\to\infty} \frac{5}{1-\frac{1}{x^2}} + \lim\limits_{x\to\infty} 2^{\frac{1}{x}}$

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- $= \left[\begin{array}{r|c|r} \text{Subst. } z = a^x - 1 & x \to 0 & \ln(z+1) = \ln a^x = x \ln a \\ z+1 = a^x & z \to 0 & \Rightarrow x = \frac{\ln(z+1)}{\ln a} \end{array} \right]$

$\lim_{x\to 2} \frac{x-2}{x^2-3x+2}$

- $= \lim_{x\to 2} \frac{x-2}{(x-2)\cdot(x-1)} = \lim_{x\to 2} \frac{1}{x-1}$

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- $= \lim_{x\to\infty} \frac{5}{1-\frac{1}{x^2}} + \lim_{x\to\infty} 2^{\frac{1}{x}} = \frac{5}{1-\frac{1}{\infty}} + 2^{\frac{1}{\infty}}$

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  $= \lim\limits_{z\to 0} \frac{\ln a}{\ln(z+1)^{\frac{1}{z}}} = \frac{\ln a}{\lim\limits_{z\to 0} \ln(z+1)^{\frac{1}{z}}} = \left[z\to 0 \Rightarrow (z+1)^{\frac{1}{z}} \to \mathrm{e} \right] = \frac{\ln a}{\ln \mathrm{e}}$

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$\lim\limits_{x\to 0} \frac{a^x-1}{x} = \ln a$ pre $a>0,\ a\neq 1$

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$= \lim\limits_{z\to 0} \frac{\ln a}{\ln(z+1)^{\frac{1}{z}}} = \frac{\ln a}{\lim\limits_{z\to 0} \ln(z+1)^{\frac{1}{z}}} = \left[z\to 0 \Rightarrow (z+1)^{\frac{1}{z}} \to \mathrm{e} \right] = \frac{\ln a}{\ln \mathrm{e}} = \frac{\ln a}{1} = \ln a.$

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Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0} \frac{1-3^x}{x} = -\lim_{x\to 0} \frac{3^x-1}{x}$$

$$\lim_{x\to 1} \frac{1-3^x}{x}$$

$$\lim_{x\to -\infty} \frac{1-3^x}{x}$$

$$\lim_{x\to \infty} \frac{1-3^x}{x}$$

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- $= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z+1 = 3^x & z \to 0 \end{array}\right]$

$$\lim_{x\to 1}\frac{1-3^x}{x}$$

- $= \frac{1-3^1}{1}$

$$\lim_{x\to -\infty}\frac{1-3^x}{x}$$

- $= \frac{1-3^{-\infty}}{-\infty}$

$$\lim_{x\to \infty}\frac{1-3^x}{x}$$

- $= \lim_{x\to\infty}\left(\frac{1}{x} - \frac{3^x}{x}\right)$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x}$$

- $= \left[\begin{array}{r|l|} \text{Subst. } z=3^x-1 & x\to 0 \\ z+1=3^x & z\to 0 \end{array} \ln(z+1)=\ln 3^x = x\ln 3 \Rightarrow x=\frac{\ln(z+1)}{\ln 3} \right]$

$$\lim_{x\to 1}\frac{1-3^x}{x}$$

- $= \frac{1-3^1}{1} = \frac{1-3}{1}$

$$\lim_{x\to -\infty}\frac{1-3^x}{x}$$

- $= \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^\infty}}{-\infty}$

$$\lim_{x\to \infty}\frac{1-3^x}{x}$$

- $= \lim_{x\to\infty}\left(\frac{1}{x}-\frac{3^x}{x}\right) = \left[\lim_{x\to\infty}\frac{3^x}{x}=\infty\right]$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x}$$

- $= \left[\begin{array}{r|l|} \text{Subst. } z=3^x-1 & x\to 0 \\ z+1=3^x & z\to 0 \end{array}\ \ln(z+1)=\ln 3^x = x\ln 3 \Rightarrow x=\frac{\ln(z+1)}{\ln 3}\right] = -\lim\limits_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}}$

$$\lim_{x\to 1}\frac{1-3^x}{x}$$

- $= \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1}$

$$\lim_{x\to -\infty}\frac{1-3^x}{x}$$

- $= \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^{\infty}}}{-\infty} = \frac{1-0}{-\infty}$

$$\lim_{x\to \infty}\frac{1-3^x}{x}$$

- $= \lim\limits_{x\to\infty}\left(\frac{1}{x}-\frac{3^x}{x}\right) = \left[\lim\limits_{x\to\infty}\frac{3^x}{x}=\infty\right] = \frac{1}{\infty}-\infty$

Riešené limity – 04, 05, 06, 07

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$$\lim_{x\to 1}\frac{1-3^x}{x} = -2$$

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Riešené limity – 04, 05, 06, 07

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Riešené limity – 04, 05, 06, 07

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  $= -\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}}$

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Riešené limity – 04, 05, 06, 07

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  $= -\lim\limits_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim\limits_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = \left[z\to 0 \Rightarrow (z+1)^{\frac{1}{z}} \to \mathrm{e} \right]$

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- $= \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1} = -2.$

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$$\lim_{x\to \infty}\frac{1-3^x}{x} = -\infty$$

- $= \lim\limits_{x\to\infty}\left(\frac{1}{x} - \frac{3^x}{x}\right) = \left[\lim\limits_{x\to\infty}\frac{3^x}{x} = \infty \right] = \frac{1}{\infty} - \infty = 0 - \infty = -\infty.$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x}$$

- $= \left[\begin{array}{r|l|l} \text{Subst. } z=3^x-1 & x\to 0 & \ln(z+1)=\ln 3^x = x\ln 3 \Rightarrow x=\frac{\ln(z+1)}{\ln 3} \\ z+1=3^x & z\to 0 & \end{array}\right] = -\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0}\frac{z\ln 3}{\ln(z+1)}$

  $= -\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = \left[z\to 0 \Rightarrow (z+1)^{\frac{1}{z}} \to \mathrm{e} \right] = -\frac{\ln 3}{\ln \mathrm{e}}$

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$$\lim_{x\to \infty}\frac{1-3^x}{x} = -\infty$$

- $= \lim_{x\to\infty}\left(\frac{1}{x} - \frac{3^x}{x}\right) = \left[\lim_{x\to\infty}\frac{3^x}{x} = \infty \right] = \frac{1}{\infty} - \infty = 0 - \infty = -\infty.$

Riešené limity – 04, 05, 06, 07

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  $= -\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = \left[z\to 0 \Rightarrow (z+1)^{\frac{1}{z}}\to \mathrm{e}\right] = -\frac{\ln 3}{\ln \mathrm{e}} = -\frac{\ln 3}{1}$

$$\lim_{x\to 1}\frac{1-3^x}{x} = -2$$

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Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x} = -\ln 3$$

- $= \left[\begin{array}{r|l|l} \text{Subst. } z = 3^x - 1 & x\to 0 & \\ z+1 = 3^x & z\to 0 & \ln(z+1) = \ln 3^x = x\ln 3 \Rightarrow x = \frac{\ln(z+1)}{\ln 3} \end{array} \right] = -\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0}\frac{z\ln 3}{\ln(z+1)}$
  $= -\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = \left[z\to 0 \Rightarrow (z+1)^{\frac{1}{z}} \to \mathrm{e} \right] = -\frac{\ln 3}{\ln \mathrm{e}} = -\frac{\ln 3}{1} = -\ln 3.$

$$\lim_{x\to 1}\frac{1-3^x}{x} = -2$$

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$$\lim_{x\to \infty}\frac{1-3^x}{x} = -\infty$$

- $= \lim_{x\to\infty}\left(\frac{1}{x} - \frac{3^x}{x}\right) = \left[\lim_{x\to\infty}\frac{3^x}{x} = \infty \right] = \frac{1}{\infty} - \infty = 0 - \infty = -\infty.$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}}$$

$$\lim_{x\to \infty} \left(\frac{1-x}{x^2}+\frac{1-2x}{2-3x}\right)$$

$$\lim_{x\to 1} \left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}}$$

- $= \lim_{x\to 0} \left(\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x} \right)$

- $= \lim_{x\to 0} \frac{\frac{1}{x}\cdot(3x^2+2)}{\frac{1}{x}\cdot(x^2+4)}$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right)$$

- $= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right)$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$$

- $= \lim_{x\to 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)}$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}}$$

- $= \lim_{x\to 0} \left(\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right) = \lim_{x\to 0} \frac{3x^2+2}{x^2+4}$

- $= \lim_{x\to 0} \frac{\frac{1}{x}\cdot(3x^2+2)}{\frac{1}{x}\cdot(x^2+4)} = \lim_{x\to 0} \frac{3x^2+2}{x^2+4}$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right)$$

- $= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right) = 0 - 0 + \lim_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3}$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$$

- $= \lim_{x\to 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} = \lim_{x\to 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)}$

Riešené limity – 08, 09, 10

$\lim\limits_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}}$

- $= \lim\limits_{x\to 0} \left(\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x} \right) = \lim\limits_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4}$

- $= \lim\limits_{x\to 0} \frac{\frac{1}{x}\cdot(3x^2+2)}{\frac{1}{x}\cdot(x^2+4)} = \lim\limits_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4}$

$\lim\limits_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right)$

- $= \lim\limits_{x\to\infty} \frac{1}{x^2} - \lim\limits_{x\to\infty} \frac{1}{x} + \lim\limits_{x\to\infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim\limits_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3}$

$\lim\limits_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$

- $= \lim\limits_{x\to 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} = \lim\limits_{x\to 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)} = \lim\limits_{x\to 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)}$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

- $= \lim_{x\to 0} \left(\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x} \right) = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$

- $= \lim_{x\to 0} \frac{\frac{1}{x}\cdot(3x^2+2)}{\frac{1}{x}\cdot(x^2+4)} = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right) = \frac{2}{3}$$

- $= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$$

- $= \lim_{x\to 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} = \lim_{x\to 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)} = \lim_{x\to 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)}$

$$= -\lim_{x\to 1} \frac{(x+2)(x-1)}{(x-1)(1+x+x^2)}$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

- $= \lim\limits_{x\to 0} \left(\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x} \right) = \lim\limits_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$

- $= \lim\limits_{x\to 0} \frac{\frac{1}{x}\cdot(3x^2+2)}{\frac{1}{x}\cdot(x^2+4)} = \lim\limits_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right) = \frac{2}{3}$$

- $= \lim\limits_{x\to\infty} \frac{1}{x^2} - \lim\limits_{x\to\infty} \frac{1}{x} + \lim\limits_{x\to\infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim\limits_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$$

- $= \lim\limits_{x\to 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} = \lim\limits_{x\to 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)} = \lim\limits_{x\to 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)}$

$$= -\lim_{x\to 1} \frac{(x+2)(x-1)}{(x-1)(1+x+x^2)} = -\lim_{x\to 1} \frac{x+2}{1+x+x^2}$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

- $= \lim\limits_{x\to 0} \left(\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x} \right) = \lim\limits_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$

- $= \lim\limits_{x\to 0} \frac{\frac{1}{x}\cdot(3x^2+2)}{\frac{1}{x}\cdot(x^2+4)} = \lim\limits_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$

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- $= \lim\limits_{x\to\infty} \frac{1}{x^2} - \lim\limits_{x\to\infty} \frac{1}{x} + \lim\limits_{x\to\infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim\limits_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$$

- $= \lim\limits_{x\to 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} = \lim\limits_{x\to 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)} = \lim\limits_{x\to 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)}$

$$= -\lim_{x\to 1} \frac{(x+2)(x-1)}{(x-1)(1+x+x^2)} = -\lim_{x\to 1} \frac{x+2}{1+x+x^2} = -\frac{1+2}{1+1+1}$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

- $= \lim\limits_{x\to 0} \left(\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x} \right) = \lim\limits_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$

- $= \lim\limits_{x\to 0} \frac{\frac{1}{x}\cdot(3x^2+2)}{\frac{1}{x}\cdot(x^2+4)} = \lim\limits_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right) = \frac{2}{3}$$

- $= \lim\limits_{x\to\infty} \frac{1}{x^2} - \lim\limits_{x\to\infty} \frac{1}{x} + \lim\limits_{x\to\infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim\limits_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$$

- $= \lim\limits_{x\to 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} = \lim\limits_{x\to 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)} = \lim\limits_{x\to 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)}$

  $= -\lim\limits_{x\to 1} \frac{(x+2)(x-1)}{(x-1)(1+x+x^2)} = -\lim\limits_{x\to 1} \frac{x+2}{1+x+x^2} = -\frac{1+2}{1+1+1} = -\frac{3}{3}$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

- $= \lim_{x\to 0} \left(\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x} \right) = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$

- $= \lim_{x\to 0} \frac{\frac{1}{x}\cdot(3x^2+2)}{\frac{1}{x}\cdot(x^2+4)} = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right) = \frac{2}{3}$$

- $= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right) = -1$$

- $= \lim_{x\to 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} = \lim_{x\to 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)} = \lim_{x\to 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)}$

$$= -\lim_{x\to 1} \frac{(x+2)(x-1)}{(x-1)(1+x+x^2)} = -\lim_{x\to 1} \frac{x+2}{1+x+x^2} = -\frac{1+2}{1+1+1} = -\frac{3}{3} = -1.$$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$$

- $= \lim_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}}$

- $= \left[\begin{array}{r|l|l} \text{Subst. } z = 3x+1 & x\to\infty & 3x = z-1 \\ x = \frac{z-1}{3} & z\to\infty & 3x-2 = z-3 \end{array}\right]$

Riešené limity – 11

$\lim_{x\to\infty} \left(\frac{3x-2}{3x+1}\right)^{x}$

- $= \lim_{x\to\infty} \left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim_{x\to\infty} \left(1 + \frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}}$

- $= \left[\begin{array}{r|l|l} \text{Subst. } z = 3x+1 & x\to\infty & 3x = z-1 \\ x = \frac{z-1}{3} & z\to\infty & 3x-2 = z-3 \end{array} \right] = \lim_{z\to\infty} \left(\frac{z-3}{z}\right)^{\frac{z-1}{3}}$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$$

- $= \lim_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim_{x\to\infty}\left(1+\frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}} = \left[\text{Subst. } z=3x+1 \;\middle|\; \begin{matrix} x\to\infty \\ z\to\infty \end{matrix}\right]$

- $= \left[\begin{matrix}\text{Subst. } z=3x+1 \\ x=\frac{z-1}{3}\end{matrix} \;\middle|\; \begin{matrix} x\to\infty \\ z\to\infty \end{matrix} \;\middle|\; \begin{matrix} 3x=z-1 \\ 3x-2=z-3 \end{matrix}\right] = \lim_{z\to\infty}\left(\frac{z-3}{z}\right)^{\frac{z-1}{3}} = \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}-\frac{1}{3}}$

  $= \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}\right] = \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$$

- $= \lim_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim_{x\to\infty}\left(1+\frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}} = \left[\text{Subst. } z=3x+1 \,\middle|\, \begin{matrix} x\to\infty \\ z\to\infty \end{matrix}\right]$

  $= \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z-1}{3}}$

- $= \left[\begin{matrix}\text{Subst. } z=3x+1 \\ x=\frac{z-1}{3}\end{matrix} \,\middle|\, \begin{matrix} x\to\infty \\ z\to\infty \end{matrix} \,\middle|\, \begin{matrix} 3x=z-1 \\ 3x-2=z-3 \end{matrix}\right] = \lim_{z\to\infty}\left(\frac{z-3}{z}\right)^{\frac{z-1}{3}} = \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}-\frac{1}{3}}$

  $= \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}\right] = \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}$

  $= \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{1}{3}}\cdot\lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$$

- $= \lim\limits_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim\limits_{x\to\infty}\left(1+\frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}} = \left[\text{Subst. } z=3x+1 \,\middle|\, \begin{matrix} x\to\infty \\ z\to\infty \end{matrix}\right]$

  $= \lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z-1}{3}} = \lim\limits_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{z-1}{3\cdot z}}$

- $= \left[\begin{matrix}\text{Subst. } z=3x+1 \\ x=\frac{z-1}{3}\end{matrix} \,\middle|\, \begin{matrix} x\to\infty \\ z\to\infty \end{matrix} \,\middle|\, \begin{matrix} 3x=z-1 \\ 3x-2=z-3 \end{matrix}\right] = \lim\limits_{z\to\infty}\left(\frac{z-3}{z}\right)^{\frac{z-1}{3}} = \lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}-\frac{1}{3}}$

  $= \lim\limits_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}\right] = \lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}$

  $= \lim\limits_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{1}{3}}\cdot\lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}} = \left(e^{-3}\right)^{\frac{1}{3}}\cdot\left(1+\frac{-3}{\infty}\right)^{-\frac{1}{3}}$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$$

- $= \lim\limits_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim\limits_{x\to\infty}\left(1+\frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}} = \left[\text{Subst. } z=3x+1 \,\middle|\, \begin{matrix} x\to\infty \\ z\to\infty \end{matrix}\right]$

  $= \lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z-1}{3}} = \lim\limits_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{z-1}{3\cdot z}} = \left[\lim\limits_{z\to\infty}\frac{z-1}{3z} = \lim\limits_{z\to\infty}\left(\frac{z}{3z}-\frac{1}{3z}\right) = \lim\limits_{z\to\infty}\left(\frac{1}{3}-\frac{1}{3z}\right) = 0-\frac{1}{3}=\frac{1}{3}\right]$

- $= \left[\text{Subst. } z=3x+1,\ x=\frac{z-1}{3} \,\middle|\, \begin{matrix} x\to\infty \\ z\to\infty \end{matrix} \,\middle|\, \begin{matrix} 3x=z-1 \\ 3x-2=z-3 \end{matrix}\right] = \lim\limits_{z\to\infty}\left(\frac{z-3}{z}\right)^{\frac{z-1}{3}} = \lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}-\frac{1}{3}}$

  $= \lim\limits_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}\right] = \lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}$

  $= \lim\limits_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{1}{3}}\cdot\lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}} = \left(\mathrm{e}^{-3}\right)^{\frac{1}{3}}\cdot\left(1+\frac{-3}{\infty}\right)^{-\frac{1}{3}}$

  $= \left(\mathrm{e}^{-3}\right)^{\frac{1}{3}}\cdot(1+0)^{-\frac{1}{3}}$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$$

- $$= \lim_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim_{x\to\infty}\left(1+\frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}} = \left[\text{Subst. } z=3x+1 \;\middle|\; \begin{matrix} x\to\infty \\ z\to\infty \end{matrix}\right]$$

  $$= \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z-1}{3}} = \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{z-1}{3\cdot z}} = \left[\lim_{z\to\infty}\frac{z-1}{3z} = \lim_{z\to\infty}\left(\frac{z}{3z}-\frac{1}{3z}\right) = \lim_{z\to\infty}\left(\frac{1}{3}-\frac{1}{3z}\right) = 0-\frac{1}{3}=\frac{1}{3}\right]$$

  $$= \left(e^{-3}\right)^{\frac{1}{3}}$$

- $$= \left[\begin{matrix}\text{Subst. } z=3x+1 \\ x=\frac{z-1}{3}\end{matrix} \;\middle|\; \begin{matrix} x\to\infty \\ z\to\infty \end{matrix} \;\middle|\; \begin{matrix} 3x=z-1 \\ 3x-2=z-3 \end{matrix}\right] = \lim_{z\to\infty}\left(\frac{z-3}{z}\right)^{\frac{z-1}{3}} = \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}-\frac{1}{3}}$$

  $$= \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}\right] = \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}$$

  $$= \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{1}{3}}\cdot\lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}} = \left(e^{-3}\right)^{\frac{1}{3}}\cdot\left(1+\frac{-3}{\infty}\right)^{-\frac{1}{3}}$$

  $$= \left(e^{-3}\right)^{\frac{1}{3}}\cdot(1+0)^{-\frac{1}{3}} = e^{-1}\cdot 1^{-\frac{1}{3}}$$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x} = \mathrm{e}^{-1} = \frac{1}{\mathrm{e}}$$

- $= \lim\limits_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim\limits_{x\to\infty}\left(1+\frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}} = \left[\text{Subst. } z=3x+1 \,\middle|\, \begin{matrix} x\to\infty \\ z\to\infty \end{matrix}\right]$

  $= \lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z-1}{3}} = \lim\limits_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{z-1}{3\cdot z}} = \left[\lim\limits_{z\to\infty}\frac{z-1}{3z} = \lim\limits_{z\to\infty}\left(\frac{z}{3z}-\frac{1}{3z}\right) = \lim\limits_{z\to\infty}\left(\frac{1}{3}-\frac{1}{3z}\right) = 0-\frac{1}{3} = \frac{1}{3}\right]$

  $= \left(\mathrm{e}^{-3}\right)^{\frac{1}{3}} = \mathrm{e}^{-1} = \frac{1}{\mathrm{e}}.$

- $= \left[\begin{matrix}\text{Subst. } z=3x+1 \\ x=\frac{z-1}{3}\end{matrix} \,\middle|\, \begin{matrix} x\to\infty \\ z\to\infty \end{matrix} \,\middle|\, \begin{matrix} 3x=z-1 \\ 3x-2=z-3 \end{matrix}\right] = \lim\limits_{z\to\infty}\left(\frac{z-3}{z}\right)^{\frac{z-1}{3}} = \lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}-\frac{1}{3}}$

  $= \lim\limits_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}\right] = \lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}$

  $= \lim\limits_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{1}{3}}\cdot\lim\limits_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}} = \left(\mathrm{e}^{-3}\right)^{\frac{1}{3}}\cdot\left(1+\frac{-3}{\infty}\right)^{-\frac{1}{3}}$

  $= \left(\mathrm{e}^{-3}\right)^{\frac{1}{3}}\cdot(1+0)^{-\frac{1}{3}} = \mathrm{e}^{-1}\cdot 1^{-\frac{1}{3}} = \mathrm{e}^{-1} = \frac{1}{\mathrm{e}}.$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}$$

$$\lim_{x\to 0}\left(1+3\operatorname{tg}^2 x\right)^{\operatorname{cotg}^2 x}$$

$$\lim_{x\to 0}\left(1+3\operatorname{tg}^2 x\right)^{\operatorname{tg}^2 x}$$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^x$$

- $= \lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^x$

$$\lim_{x\to0}(1+3\operatorname{tg}^2 x)^{\operatorname{cotg}^2 x}$$

- $= \lim_{x\to0}(1+3\operatorname{tg}^2 x)^{\frac{1}{\operatorname{tg}^2 x}}$

$$\lim_{x\to0}(1+3\operatorname{tg}^2 x)^{\operatorname{tg}^2 x}$$

- $= (1+3\cdot 0)^0$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}$$

- $= \lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x} = \left[\text{Subst. } x=-z \,\middle|\, \begin{matrix} x\to-\infty \\ z\to\infty \end{matrix}\right]$

$$\lim_{x\to 0}(1+3\operatorname{tg}^2 x)^{\operatorname{cotg}^2 x}$$

- $= \lim_{x\to 0}(1+3\operatorname{tg}^2 x)^{\frac{1}{\operatorname{tg}^2 x}} = \left[\text{Subst. } z=\operatorname{tg}^2 x \,\middle|\, \begin{matrix} x\to 0 \\ z\to 0 \end{matrix}\right]$

$$\lim_{x\to 0}(1+3\operatorname{tg}^2 x)^{\operatorname{tg}^2 x}$$

- $= (1+3\cdot 0)^0 = 1^0$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}$$

- $= \lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x} = \left[\text{Subst. } x=-z \;\middle|\; \begin{array}{l} x\to-\infty \\ z\to\infty \end{array}\right] = \lim_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$

$$\lim_{x\to 0}\left(1+3\operatorname{tg}^2 x\right)^{\operatorname{cotg}^2 x}$$

- $= \lim_{x\to 0}\left(1+3\operatorname{tg}^2 x\right)^{\frac{1}{\operatorname{tg}^2 x}} = \left[\text{Subst. } z=\operatorname{tg}^2 x \;\middle|\; \begin{array}{l} x\to 0 \\ z\to 0 \end{array}\right] = \lim_{z\to 0}(1+3z)^{\frac{1}{z}}$

$$\lim_{x\to 0}\left(1+3\operatorname{tg}^2 x\right)^{\operatorname{tg}^2 x} = 1$$

- $= (1+3\cdot 0)^0 = 1^0 = 1.$

Riešené limity – 12, 13, 14

$\lim_{x\to-\infty} \left(\frac{x+1}{x}\right)^x$

- $= \lim_{x\to-\infty} \left(1+\frac{1}{x}\right)^x = \left[\text{Subst. } x=-z \;\middle|\; \begin{matrix} x\to-\infty \\ z\to\infty \end{matrix}\right] = \lim_{z\to\infty} \left(1+\frac{1}{-z}\right)^{-z}$

$$= \lim_{z\to\infty} \left(1+\frac{-1}{z}\right)^{-z}$$

$\lim_{x\to0} (1+3\operatorname{tg}^2 x)^{\operatorname{cotg}^2 x} = e^3$

- $= \lim_{x\to0} (1+3\operatorname{tg}^2 x)^{\frac{1}{\operatorname{tg}^2 x}} = \left[\text{Subst. } z=\operatorname{tg}^2 x \;\middle|\; \begin{matrix} x\to0 \\ z\to0 \end{matrix}\right] = \lim_{z\to0} (1+3z)^{\frac{1}{z}} = e^3.$

$\lim_{x\to0} (1+3\operatorname{tg}^2 x)^{\operatorname{tg}^2 x} = 1$

- $= (1+3\cdot0)^0 = 1^0 = 1.$

Riešené limity – 12, 13, 14

$\lim\limits_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}$

- $= \lim\limits_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x} = \left[\text{Subst. } x=-z \;\middle|\; \begin{array}{l} x\to-\infty \\ z\to\infty \end{array}\right] = \lim\limits_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$

$$= \lim\limits_{z\to\infty}\left(1+\frac{-1}{z}\right)^{-z} = \lim\limits_{z\to\infty}\left[\left(1+\frac{-1}{z}\right)^{z}\right]^{-1}$$

$\lim\limits_{x\to 0}(1+3\operatorname{tg}^2 x)^{\operatorname{cotg}^2 x} = \mathrm{e}^3$

- $= \lim\limits_{x\to 0}(1+3\operatorname{tg}^2 x)^{\frac{1}{\operatorname{tg}^2 x}} = \left[\text{Subst. } z=\operatorname{tg}^2 x \;\middle|\; \begin{array}{l} x\to 0 \\ z\to 0 \end{array}\right] = \lim\limits_{z\to 0}(1+3z)^{\frac{1}{z}} = \mathrm{e}^3.$

$\lim\limits_{x\to 0}(1+3\operatorname{tg}^2 x)^{\operatorname{tg}^2 x} = 1$

- $= (1+3\cdot 0)^0 = 1^0 = 1.$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}$$

- $= \lim\limits_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x} = \left[\text{Subst. } x=-z \,\middle|\, \begin{matrix} x\to-\infty \\ z\to\infty \end{matrix}\right] = \lim\limits_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$
  $= \lim\limits_{z\to\infty}\left(1+\frac{-1}{z}\right)^{-z} = \lim\limits_{z\to\infty}\left[\left(1+\frac{-1}{z}\right)^{z}\right]^{-1} = \left(\mathrm{e}^{-1}\right)^{-1}$

$$\lim_{x\to0}\left(1+3\,\mathrm{tg}^2 x\right)^{\mathrm{cotg}^2 x} = \mathrm{e}^3$$

- $= \lim\limits_{x\to0}\left(1+3\,\mathrm{tg}^2 x\right)^{\frac{1}{\mathrm{tg}^2 x}} = \left[\text{Subst. } z=\mathrm{tg}^2 x \,\middle|\, \begin{matrix} x\to0 \\ z\to0 \end{matrix}\right] = \lim\limits_{z\to0}\left(1+3z\right)^{\frac{1}{z}} = \mathrm{e}^3.$

$$\lim_{x\to0}\left(1+3\,\mathrm{tg}^2 x\right)^{\mathrm{tg}^2 x} = 1$$

- $= (1+3\cdot0)^0 = 1^0 = 1.$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}=\mathrm{e}$$

- $=\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x}=\left[\text{Subst. } x=-z \,\middle|\, \begin{matrix} x\to-\infty \\ z\to\infty \end{matrix}\right]=\lim_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$

$$=\lim_{z\to\infty}\left(1+\frac{-1}{z}\right)^{-z}=\lim_{z\to\infty}\left[\left(1+\frac{-1}{z}\right)^{z}\right]^{-1}=\left(\mathrm{e}^{-1}\right)^{-1}=\mathrm{e}.$$

$$\lim_{x\to0}\left(1+3\,\mathrm{tg}^2 x\right)^{\mathrm{cotg}^2 x}=\mathrm{e}^3$$

- $=\lim_{x\to0}\left(1+3\,\mathrm{tg}^2 x\right)^{\frac{1}{\mathrm{tg}^2 x}}=\left[\text{Subst. } z=\mathrm{tg}^2 x \,\middle|\, \begin{matrix} x\to0 \\ z\to0 \end{matrix}\right]=\lim_{z\to0}\left(1+3z\right)^{\frac{1}{z}}=\mathrm{e}^3.$

$$\lim_{x\to0}\left(1+3\,\mathrm{tg}^2 x\right)^{\mathrm{tg}^2 x}=1$$

- $=(1+3\cdot0)^0=1^0=1.$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{x^2}$$

$$\lim_{x\to\infty} x\cdot\left[\ln(x+2)-\ln x\right]$$

$$\lim_{x\to 0}\frac{x}{\ln(1+tx)} \quad \text{pre } t\in R-\{0\}$$

Riešené limity – 15, 16, 17

$\lim\limits_{x\to\infty} \left(1-\frac{1}{x}\right)^{x^2}$

- $= \lim\limits_{x\to\infty} \left(1-\frac{1}{x}\right)^{x\cdot x}$

$\lim\limits_{x\to\infty} x\cdot\left[\ln(x+2)-\ln x\right]$

- $= \lim\limits_{x\to\infty} \left(x\cdot\ln\frac{x+2}{x}\right)$

$\lim\limits_{x\to 0} \frac{x}{\ln(1+tx)}$ pre $t\in R-\{0\}$

- $= \lim\limits_{x\to 0} \frac{1}{\frac{1}{x}\ln(1+tx)}$

- $=$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty} (1-\tfrac{1}{x})^{x^2}$$

- $= \lim_{x\to\infty} (1-\frac{1}{x})^{x\cdot x} = \lim_{x\to\infty} \left[(1+\frac{-1}{x})^{x}\right]^{x}$

$$\lim_{x\to\infty} x\cdot\left[\ln(x+2)-\ln x\right]$$

- $= \lim_{x\to\infty} \left(x\cdot\ln\frac{x+2}{x}\right) = \lim_{x\to\infty} \ln\left(\frac{x+2}{x}\right)^{x}$

$$\lim_{x\to 0} \frac{x}{\ln(1+tx)}$$ pre $t\in R-\{0\}$

- $= \lim_{x\to 0} \frac{1}{\frac{1}{x}\ln(1+tx)} = \lim_{x\to 0} \frac{1}{\frac{t}{tx}\ln(1+tx)}$

- $= \qquad = \lim_{z\to 0} \frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}}$

Riešené limity – 15, 16, 17

$\lim\limits_{x\to\infty} (1-\frac{1}{x})^{x^2}$

- $= \lim\limits_{x\to\infty} (1-\frac{1}{x})^{x\cdot x} = \lim\limits_{x\to\infty} \left[(1+\frac{-1}{x})^{x}\right]^{x} = \left[\mathrm{e}^{-1}\right]^{\infty}$

$\lim\limits_{x\to\infty} x\cdot\left[\ln(x+2)-\ln x\right]$

- $= \lim\limits_{x\to\infty} \left(x\cdot\ln\frac{x+2}{x}\right) = \lim\limits_{x\to\infty} \ln\left(\frac{x+2}{x}\right)^{x} = \lim\limits_{x\to\infty} \ln\left(1+\frac{2}{x}\right)^{x}$

$\lim\limits_{x\to 0} \frac{x}{\ln(1+tx)}$ pre $t\in R-\{0\}$

- $= \lim\limits_{x\to 0} \frac{1}{\frac{1}{x}\ln(1+tx)} = \lim\limits_{x\to 0} \frac{1}{\frac{t}{tx}\ln(1+tx)} = \lim\limits_{x\to 0} \frac{1}{t\cdot\ln(1+tx)^{\frac{1}{tx}}}$

- $= \qquad = \lim\limits_{z\to 0} \frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}} = \lim\limits_{z\to 0} \frac{1}{\frac{t}{z}\ln(1+z)^{\frac{1}{z}}}$

Riešené limity – 15, 16, 17

$\lim\limits_{x\to\infty} (1 - \frac{1}{x})^{x^2}$

- $= \lim\limits_{x\to\infty} (1 - \frac{1}{x})^{x\cdot x} = \lim\limits_{x\to\infty} \left[(1 + \frac{-1}{x})^{x}\right]^{x} = \left[\mathrm{e}^{-1}\right]^{\infty} = \frac{1}{\mathrm{e}^{\infty}}$

$\lim\limits_{x\to\infty} x \cdot \left[\ln(x+2) - \ln x\right]$

- $= \lim\limits_{x\to\infty} \left(x \cdot \ln \frac{x+2}{x}\right) = \lim\limits_{x\to\infty} \ln \left(\frac{x+2}{x}\right)^{x} = \lim\limits_{x\to\infty} \ln \left(1 + \frac{2}{x}\right)^{x} = \ln \mathrm{e}^2$

$\lim\limits_{x\to 0} \frac{x}{\ln(1+tx)}$ pre $t \in R - \{0\}$

- $= \lim\limits_{x\to 0} \frac{1}{\frac{1}{x}\ln(1+tx)} = \lim\limits_{x\to 0} \frac{1}{\frac{t}{tx}\ln(1+tx)} = \lim\limits_{x\to 0} \frac{1}{t\cdot\ln(1+tx)^{\frac{1}{tx}}}$

  $= \left[\text{Subst. } z = tx \;\middle|\; \begin{matrix} x \to 0 \\ z \to 0 \end{matrix}\right]$

- $= \left[\begin{matrix}\text{Subst. } z = tx \\ x = \frac{z}{t}\end{matrix} \;\middle|\; \begin{matrix} x \to 0 \\ z \to 0 \end{matrix}\right] = \lim\limits_{z\to 0} \frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}} = \lim\limits_{z\to 0} \frac{1}{\frac{t}{z}\ln(1+z)^{\frac{1}{z}}} = \lim\limits_{z\to 0} \frac{1}{t\cdot\ln(1+z)^{\frac{1}{z}}}$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty} (1-\tfrac{1}{x})^{x^2}$$

- $= \lim_{x\to\infty} (1-\frac{1}{x})^{x\cdot x} = \lim_{x\to\infty} \left[(1+\frac{-1}{x})^x\right]^x = \left[\mathrm{e}^{-1}\right]^\infty = \frac{1}{\mathrm{e}^\infty} = \frac{1}{\infty}$

$$\lim_{x\to\infty} x\cdot\left[\ln(x+2)-\ln x\right] = 2$$

- $= \lim_{x\to\infty} \left(x\cdot\ln\frac{x+2}{x}\right) = \lim_{x\to\infty} \ln\left(\frac{x+2}{x}\right)^x = \lim_{x\to\infty} \ln\left(1+\frac{2}{x}\right)^x = \ln \mathrm{e}^2 = 2.$

$$\lim_{x\to 0} \frac{x}{\ln(1+tx)} \qquad \text{pre } t\in R-\{0\}$$

- $= \lim_{x\to 0} \frac{1}{\frac{1}{x}\ln(1+tx)} = \lim_{x\to 0} \frac{1}{\frac{t}{tx}\ln(1+tx)} = \lim_{x\to 0} \frac{1}{t\cdot\ln(1+tx)^{\frac{1}{tx}}}$
  $= \left[\text{Subst. } z=tx \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix}\right] = \lim_{z\to 0} \frac{1}{t\cdot\ln(1+z)^{\frac{1}{z}}}$

- $= \left[\begin{matrix}\text{Subst. } z=tx \\ x=\frac{z}{t}\end{matrix} \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix}\right] = \lim_{z\to 0} \frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}} = \lim_{z\to 0} \frac{1}{\frac{t}{z}\ln(1+z)^{\frac{1}{z}}} = \lim_{z\to 0} \frac{1}{t\cdot\ln(1+z)^{\frac{1}{z}}}$
  $= \frac{1}{t\cdot\ln \mathrm{e}}$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{x^2}=0$$

- $= \lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{x\cdot x} = \lim_{x\to\infty}\left[\left(1+\frac{-1}{x}\right)^{x}\right]^{x} = \left[\mathrm{e}^{-1}\right]^{\infty} = \frac{1}{\mathrm{e}^{\infty}} = \frac{1}{\infty} = 0.$

$$\lim_{x\to\infty} x\cdot\left[\ln(x+2)-\ln x\right]=2$$

- $= \lim_{x\to\infty}\left(x\cdot\ln\frac{x+2}{x}\right) = \lim_{x\to\infty}\ln\left(\frac{x+2}{x}\right)^{x} = \lim_{x\to\infty}\ln\left(1+\frac{2}{x}\right)^{x} = \ln\mathrm{e}^2 = 2.$

$$\lim_{x\to 0}\frac{x}{\ln(1+tx)} \quad \text{pre } t\in R-\{0\}$$

- $= \lim_{x\to 0}\frac{1}{\frac{1}{x}\ln(1+tx)} = \lim_{x\to 0}\frac{1}{\frac{t}{tx}\ln(1+tx)} = \lim_{x\to 0}\frac{1}{t\cdot\ln(1+tx)^{\frac{1}{tx}}}$

  $= \left[\text{Subst. } z=tx \;\middle|\; \begin{matrix} x\to 0\\ z\to 0\end{matrix}\right] = \lim_{z\to 0}\frac{1}{t\cdot\ln(1+z)^{\frac{1}{z}}} = \frac{1}{t\cdot\ln\mathrm{e}}$

- $= \left[\begin{matrix}\text{Subst. } z=tx\\ x=\frac{z}{t}\end{matrix} \;\middle|\; \begin{matrix} x\to 0\\ z\to 0\end{matrix}\right] = \lim_{z\to 0}\frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}} = \lim_{z\to 0}\frac{1}{\frac{t}{z}\ln(1+z)^{\frac{1}{z}}} = \lim_{z\to 0}\frac{1}{t\cdot\ln(1+z)^{\frac{1}{z}}}$

  $= \frac{1}{t\cdot\ln\mathrm{e}} = \frac{1}{t\cdot 1}$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty}\left(1-\tfrac{1}{x}\right)^{x^2}=0$$

- $= \lim\limits_{x\to\infty}\left(1-\frac{1}{x}\right)^{x\cdot x} = \lim\limits_{x\to\infty}\left[\left(1+\frac{-1}{x}\right)^{x}\right]^{x} = \left[\mathrm{e}^{-1}\right]^{\infty} = \frac{1}{\mathrm{e}^{\infty}} = \frac{1}{\infty} = 0.$

$$\lim_{x\to\infty} x\cdot\left[\ln(x+2)-\ln x\right]=2$$

- $= \lim\limits_{x\to\infty}\left(x\cdot\ln\frac{x+2}{x}\right) = \lim\limits_{x\to\infty}\ln\left(\frac{x+2}{x}\right)^{x} = \lim\limits_{x\to\infty}\ln\left(1+\frac{2}{x}\right)^{x} = \ln \mathrm{e}^2 = 2.$

$$\lim_{x\to 0}\frac{x}{\ln(1+tx)}=\frac{1}{t} \text{ pre } t\in R-\{0\}$$

- $= \lim\limits_{x\to 0}\frac{1}{\frac{1}{x}\ln(1+tx)} = \lim\limits_{x\to 0}\frac{1}{\frac{t}{tx}\ln(1+tx)} = \lim\limits_{x\to 0}\frac{1}{t\cdot\ln(1+tx)^{\frac{1}{tx}}}$
  $= \left[\text{Subst. } z=tx \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0\end{matrix}\right] = \lim\limits_{z\to 0}\frac{1}{t\cdot\ln(1+z)^{\frac{1}{z}}} = \frac{1}{t\cdot\ln \mathrm{e}} = \frac{1}{t\cdot 1}$

- $= \left[\begin{matrix}\text{Subst. } z=tx \\ x=\frac{z}{t}\end{matrix} \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0\end{matrix}\right] = \lim\limits_{z\to 0}\frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}} = \lim\limits_{z\to 0}\frac{1}{\frac{t}{z}\ln(1+z)^{\frac{1}{z}}} = \lim\limits_{z\to 0}\frac{1}{t\cdot\ln(1+z)^{\frac{1}{z}}}$
  $= \frac{1}{t\cdot\ln \mathrm{e}} = \frac{1}{t\cdot 1} = \frac{1}{t}.$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{x^2}=0$$

- $= \lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{x\cdot x} = \lim_{x\to\infty}\left[\left(1+\frac{-1}{x}\right)^{x}\right]^{x} = \left[\mathrm{e}^{-1}\right]^{\infty} = \frac{1}{\mathrm{e}^{\infty}} = \frac{1}{\infty} = 0.$

$$\lim_{x\to\infty} x\cdot\left[\ln(x+2)-\ln x\right]=2$$

- $= \lim_{x\to\infty}\left(x\cdot\ln\frac{x+2}{x}\right) = \lim_{x\to\infty}\ln\left(\frac{x+2}{x}\right)^{x} = \lim_{x\to\infty}\ln\left(1+\frac{2}{x}\right)^{x} = \ln\mathrm{e}^2 = 2.$

$$\lim_{x\to 0}\frac{x}{\ln(1+tx)}=\frac{1}{t} \text{ pre } t\in R-\{0\}$$

- $= \lim_{x\to 0}\frac{1}{\frac{1}{x}\ln(1+tx)} = \lim_{x\to 0}\frac{1}{\frac{t}{tx}\ln(1+tx)} = \lim_{x\to 0}\frac{1}{t\cdot\ln(1+tx)^{\frac{1}{tx}}}$
  $= \left[\text{Subst. } z=tx \,\middle|\, \begin{matrix} x\to 0 \\ z\to 0\end{matrix}\right] = \lim_{z\to 0}\frac{1}{t\cdot\ln(1+z)^{\frac{1}{z}}} = \frac{1}{t\cdot\ln\mathrm{e}} = \frac{1}{t\cdot 1} = \frac{1}{t}.$

- $= \left[\begin{matrix}\text{Subst. } z=tx \\ x=\frac{z}{t}\end{matrix} \,\middle|\, \begin{matrix} x\to 0 \\ z\to 0\end{matrix}\right] = \lim_{z\to 0}\frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}} = \lim_{z\to 0}\frac{1}{\frac{t}{z}\ln(1+z)^{\frac{1}{z}}} = \lim_{z\to 0}\frac{1}{t\cdot\ln(1+z)^{\frac{1}{z}}}$
  $= \frac{1}{t\cdot\ln\mathrm{e}} = \frac{1}{t\cdot 1} = \frac{1}{t}.$

Riešené limity – 18, 19

$$\lim_{x\to 0} \frac{1-\sqrt{1-x}}{x}$$

$$\lim_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$$

Riešené limity – 18, 19

$\lim\limits_{x\to 0} \frac{1-\sqrt{1-x}}{x}$

- $= \lim\limits_{x\to 0} \left(\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}} \right)$

- $= \left[\text{Subst. } \sqrt{1-x} = t \;\middle|\; \begin{matrix} x \to 0 \\ t \to 1 \end{matrix} \;\middle|\; \begin{matrix} 1-x = t^2 \\ x = 1-t^2 \end{matrix} \right]$

$\lim\limits_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$

- $= \lim\limits_{x\to 0} \left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}} \cdot \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \right)$

Riešené limity – 18, 19

$$\lim_{x\to 0}\frac{1-\sqrt{1-x}}{x}$$

- $= \lim\limits_{x\to 0}\left(\frac{1-\sqrt{1-x}}{x}\cdot\frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right) = \lim\limits_{x\to 0}\frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})}$

- $= \left[\text{Subst. } \sqrt{1-x}=t \;\middle|\; \begin{matrix} x\to 0 \\ t\to 1\end{matrix} \;\middle|\; \begin{matrix} 1-x=t^2 \\ x=1-t^2\end{matrix}\right] = \lim\limits_{t\to 1}\frac{1-t}{1-t^2}$

$$\lim_{x\to 0}\frac{x}{\sqrt{1+x}-\sqrt{1-x}}$$

- $= \lim\limits_{x\to 0}\left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}}\cdot\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \lim\limits_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2}$

Riešené limity – 18, 19

$\lim\limits_{x\to 0} \frac{1-\sqrt{1-x}}{x}$

- $= \lim\limits_{x\to 0} \left(\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}} \right) = \lim\limits_{x\to 0} \frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})} = \lim\limits_{x\to 0} \frac{1-(1-x)}{x(1+\sqrt{1-x})}$

- $= \left[\text{Subst. } \sqrt{1-x} = t \;\middle|\; \begin{matrix} x\to 0 \\ t \to 1 \end{matrix} \;\middle|\; \begin{matrix} 1-x = t^2 \\ x = 1-t^2 \end{matrix} \right] = \lim\limits_{t\to 1} \frac{1-t}{1-t^2} = \lim\limits_{t\to 1} \frac{1-t}{(1-t)\cdot(1+t)}$

$\lim\limits_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$

- $= \lim\limits_{x\to 0} \left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}} \cdot \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \right) = \lim\limits_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2} = \lim\limits_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)}$

Riešené limity – 18, 19

$\lim\limits_{x\to 0} \frac{1-\sqrt{1-x}}{x}$

- $= \lim\limits_{x\to 0} \left(\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}} \right) = \lim\limits_{x\to 0} \frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})} = \lim\limits_{x\to 0} \frac{1-(1-x)}{x(1+\sqrt{1-x})}$

  $= \lim\limits_{x\to 0} \frac{x}{x(1+\sqrt{1-x})}$

- $= \left[\text{Subst. } \sqrt{1-x} = t \;\middle|\; \begin{matrix} x \to 0 \\ t \to 1 \end{matrix} \;\middle|\; \begin{matrix} 1-x = t^2 \\ x = 1-t^2 \end{matrix} \right] = \lim\limits_{t\to 1} \frac{1-t}{1-t^2} = \lim\limits_{t\to 1} \frac{1-t}{(1-t)\cdot(1+t)} = \lim\limits_{t\to 1} \frac{1}{1+t}$

$\lim\limits_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$

- $= \lim\limits_{x\to 0} \left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}} \cdot \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \right) = \lim\limits_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2} = \lim\limits_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)}$

  $= \lim\limits_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{2x}$

Riešené limity – 18, 19

$$\lim_{x\to 0} \frac{1-\sqrt{1-x}}{x}$$

- $= \lim\limits_{x\to 0} \left(\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}} \right) = \lim\limits_{x\to 0} \frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})} = \lim\limits_{x\to 0} \frac{1-(1-x)}{x(1+\sqrt{1-x})}$
  $= \lim\limits_{x\to 0} \frac{x}{x(1+\sqrt{1-x})} = \lim\limits_{x\to 0} \frac{1}{1+\sqrt{1-x}}$

- $= \left[\text{Subst. } \sqrt{1-x} = t \;\middle|\; \begin{matrix} x \to 0 \\ t \to 1 \end{matrix} \;\middle|\; \begin{matrix} 1-x = t^2 \\ x = 1-t^2 \end{matrix} \right] = \lim\limits_{t\to 1} \frac{1-t}{1-t^2} = \lim\limits_{t\to 1} \frac{1-t}{(1-t)\cdot(1+t)} = \lim\limits_{t\to 1} \frac{1}{1+t} = \frac{1}{1+1}$

$$\lim_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$$

- $= \lim\limits_{x\to 0} \left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}} \cdot \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \right) = \lim\limits_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2} = \lim\limits_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)}$
  $= \lim\limits_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{2x} = \lim\limits_{x\to 0} \frac{\sqrt{1+x}+\sqrt{1-x}}{2}$

Riešené limity – 18, 19

$\lim\limits_{x\to 0} \frac{1-\sqrt{1-x}}{x} = \frac{1}{2}$

- $= \lim\limits_{x\to 0} \left(\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}} \right) = \lim\limits_{x\to 0} \frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})} = \lim\limits_{x\to 0} \frac{1-(1-x)}{x(1+\sqrt{1-x})}$
  $= \lim\limits_{x\to 0} \frac{x}{x(1+\sqrt{1-x})} = \lim\limits_{x\to 0} \frac{1}{1+\sqrt{1-x}} = \frac{1}{1+\sqrt{1-0}}$

- $= \left[\text{Subst. } \sqrt{1-x} = t \;\middle|\; \begin{array}{c} x\to 0 \\ t\to 1 \end{array} \;\middle|\; \begin{array}{c} 1-x = t^2 \\ x = 1-t^2 \end{array} \right] = \lim\limits_{t\to 1} \frac{1-t}{1-t^2} = \lim\limits_{t\to 1} \frac{1-t}{(1-t)\cdot(1+t)} = \lim\limits_{t\to 1} \frac{1}{1+t} = \frac{1}{1+1} = \frac{1}{2}.$

$\lim\limits_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$

- $= \lim\limits_{x\to 0} \left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}} \cdot \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \right) = \lim\limits_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2} = \lim\limits_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)}$
  $= \lim\limits_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{2x} = \lim\limits_{x\to 0} \frac{\sqrt{1+x}+\sqrt{1-x}}{2} = \frac{\sqrt{1+0}+\sqrt{1-0}}{2}$

Riešené limity – 18, 19

$$\lim_{x\to 0}\frac{1-\sqrt{1-x}}{x}=\frac{1}{2}$$

- $= \lim_{x\to 0}\left(\frac{1-\sqrt{1-x}}{x}\cdot\frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right)= \lim_{x\to 0}\frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})}= \lim_{x\to 0}\frac{1-(1-x)}{x(1+\sqrt{1-x})}$
  $= \lim_{x\to 0}\frac{x}{x(1+\sqrt{1-x})}= \lim_{x\to 0}\frac{1}{1+\sqrt{1-x}}=\frac{1}{1+\sqrt{1-0}}=\frac{1}{1+1}$

- $= \left[\text{Subst. } \sqrt{1-x}=t \;\middle|\; \begin{matrix} x\to 0 \\ t\to 1\end{matrix} \;\middle|\; \begin{matrix} 1-x=t^2 \\ x=1-t^2\end{matrix}\right] = \lim_{t\to 1}\frac{1-t}{1-t^2}= \lim_{t\to 1}\frac{1-t}{(1-t)\cdot(1+t)}= \lim_{t\to 1}\frac{1}{1+t}=\frac{1}{1+1}=\frac{1}{2}.$

$$\lim_{x\to 0}\frac{x}{\sqrt{1+x}-\sqrt{1-x}}$$

- $= \lim_{x\to 0}\left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}}\cdot\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)= \lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2}= \lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)}$
  $= \lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{2x}= \lim_{x\to 0}\frac{\sqrt{1+x}+\sqrt{1-x}}{2}=\frac{\sqrt{1+0}+\sqrt{1-0}}{2}=\frac{1+1}{2}$

Riešené limity – 18, 19

$$\lim_{x\to 0}\frac{1-\sqrt{1-x}}{x}=\frac{1}{2}$$

- $= \lim_{x\to 0}\left(\frac{1-\sqrt{1-x}}{x}\cdot\frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right) = \lim_{x\to 0}\frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})} = \lim_{x\to 0}\frac{1-(1-x)}{x(1+\sqrt{1-x})}$
  $= \lim_{x\to 0}\frac{x}{x(1+\sqrt{1-x})} = \lim_{x\to 0}\frac{1}{1+\sqrt{1-x}} = \frac{1}{1+\sqrt{1-0}} = \frac{1}{1+1} = \frac{1}{2}.$

- $= \left[\text{Subst. } \sqrt{1-x}=t \;\middle|\; \begin{matrix} x\to 0 \\ t\to 1\end{matrix} \;\middle|\; \begin{matrix} 1-x=t^2 \\ x=1-t^2\end{matrix}\right] = \lim_{t\to 1}\frac{1-t}{1-t^2} = \lim_{t\to 1}\frac{1-t}{(1-t)\cdot(1+t)} = \lim_{t\to 1}\frac{1}{1+t} = \frac{1}{1+1} = \frac{1}{2}.$

$$\lim_{x\to 0}\frac{x}{\sqrt{1+x}-\sqrt{1-x}}=1$$

- $= \lim_{x\to 0}\left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}}\cdot\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2} = \lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)}$
  $= \lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{2x} = \lim_{x\to 0}\frac{\sqrt{1+x}+\sqrt{1-x}}{2} = \frac{\sqrt{1+0}+\sqrt{1-0}}{2} = \frac{1+1}{2} = 1.$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$$

$$\lim_{x\to\infty} \left(x - \sqrt{x^2-1}\right)$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$$

- $= \lim\limits_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right)$

$$\lim_{x\to\infty} \left(x - \sqrt{x^2-1}\right)$$

- $= \lim\limits_{x\to\infty} \left(\left(x - \sqrt{x^2-1}\right) \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} \right)$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$$

- $= \lim\limits_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{array}{r|l} x \to \infty & \\ x > 0 & \sqrt{x^2} = |x| = x \end{array} \right]$

$$\lim_{x\to\infty} (x - \sqrt{x^2-1})$$

- $= \lim\limits_{x\to\infty} \left((x - \sqrt{x^2-1}) \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} \right) = \lim\limits_{x\to\infty} \frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$$

- $= \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{matrix} x\to\infty \\ x>0 \end{matrix} \middle| \sqrt{x^2} = |x| = x \right] = \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right)$

$$\lim_{x\to\infty} \left(x - \sqrt{x^2-1}\right)$$

- $= \lim_{x\to\infty} \left(\left(x - \sqrt{x^2-1}\right) \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} \right) = \lim_{x\to\infty} \frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$

  $= \lim_{x\to\infty} \frac{x^2-(\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}}$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$$

- $= \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x}\right) = \left[\begin{array}{c|c} x\to\infty & \\ x>0 & \sqrt{x^2}=|x|=x \end{array}\right] = \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$

  $= \lim_{x\to\infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}}\right)$

$$\lim_{x\to\infty} \left(x - \sqrt{x^2-1}\right)$$

- $= \lim_{x\to\infty} \left(\left(x - \sqrt{x^2-1}\right) \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right) = \lim_{x\to\infty} \frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$

  $= \lim_{x\to\infty} \frac{x^2-(\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}}$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$$

- $= \lim\limits_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x}\right) = \left[\begin{array}{c|c} x\to\infty & \\ x>0 & \sqrt{x^2}=|x|=x \end{array}\right] = \lim\limits_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$

  $= \lim\limits_{x\to\infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}}\right) = \lim\limits_{x\to\infty} \left(\sqrt{1-\frac{1}{x^2}} + \sqrt{1+\frac{1}{x^2}}\right)$

$$\lim_{x\to\infty} \left(x-\sqrt{x^2-1}\right)$$

- $= \lim\limits_{x\to\infty} \left(\left(x-\sqrt{x^2-1}\right)\cdot\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right) = \lim\limits_{x\to\infty} \frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$

  $= \lim\limits_{x\to\infty} \frac{x^2-(\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}} = \lim\limits_{x\to\infty} \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}} = \lim\limits_{x\to\infty} \frac{1}{x+\sqrt{x^2-1}}$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$$

- $= \lim\limits_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x}\right) = \left[\begin{array}{c|c} x\to\infty & \\ x>0 & \sqrt{x^2}=|x|=x \end{array}\right] = \lim\limits_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$

  $= \lim\limits_{x\to\infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}}\right) = \lim\limits_{x\to\infty} \left(\sqrt{1-\frac{1}{x^2}} + \sqrt{1+\frac{1}{x^2}}\right)$

  $= \sqrt{1-\frac{1}{\infty}} + \sqrt{1+\frac{1}{\infty}}$

$$\lim_{x\to\infty} \left(x - \sqrt{x^2-1}\right)$$

- $= \lim\limits_{x\to\infty} \left(\left(x-\sqrt{x^2-1}\right) \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right) = \lim\limits_{x\to\infty} \frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$

  $= \lim\limits_{x\to\infty} \frac{x^2-(\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}} = \lim\limits_{x\to\infty} \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}} = \lim\limits_{x\to\infty} \frac{1}{x+\sqrt{x^2-1}}$

  $= \frac{1}{\infty+\sqrt{\infty^2-1}}$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$$

- $= \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x}\right) = \left[\begin{array}{c|c} x\to\infty & \\ x>0 & \sqrt{x^2}=|x|=x \end{array}\right] = \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$

  $= \lim_{x\to\infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}}\right) = \lim_{x\to\infty} \left(\sqrt{1-\frac{1}{x^2}} + \sqrt{1+\frac{1}{x^2}}\right)$

  $= \sqrt{1-\frac{1}{\infty}} + \sqrt{1+\frac{1}{\infty}} = \sqrt{1-0} + \sqrt{1+0}$

$$\lim_{x\to\infty} \left(x - \sqrt{x^2-1}\right)$$

- $= \lim_{x\to\infty} \left(\left(x-\sqrt{x^2-1}\right) \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right) = \lim_{x\to\infty} \frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$

  $= \lim_{x\to\infty} \frac{x^2-(\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{1}{x+\sqrt{x^2-1}}$

  $= \frac{1}{\infty+\sqrt{\infty^2-1}} = \frac{1}{\infty+\infty}$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x} = 2$$

- $= \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x}\right) = \left[\begin{array}{c|c} x\to\infty & \\ x>0 & \sqrt{x^2}=|x|=x \end{array}\right] = \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$

$= \lim_{x\to\infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}}\right) = \lim_{x\to\infty} \left(\sqrt{1-\frac{1}{x^2}} + \sqrt{1+\frac{1}{x^2}}\right)$

$= \sqrt{1-\frac{1}{\infty}} + \sqrt{1+\frac{1}{\infty}} = \sqrt{1-0} + \sqrt{1+0} = 2.$

$$\lim_{x\to\infty} \left(x - \sqrt{x^2-1}\right)$$

- $= \lim_{x\to\infty} \left(\left(x - \sqrt{x^2-1}\right) \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right) = \lim_{x\to\infty} \frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$

$= \lim_{x\to\infty} \frac{x^2-(\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{1}{x+\sqrt{x^2-1}}$

$= \frac{1}{\infty+\sqrt{\infty^2-1}} = \frac{1}{\infty+\infty} = \frac{1}{\infty}$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x} = 2$$

- $= \lim\limits_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x}\right) = \left[\begin{array}{c|c} x\to\infty & \\ x>0 & \sqrt{x^2}=|x|=x \end{array}\right] = \lim\limits_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$

  $= \lim\limits_{x\to\infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}}\right) = \lim\limits_{x\to\infty} \left(\sqrt{1-\frac{1}{x^2}} + \sqrt{1+\frac{1}{x^2}}\right)$

  $= \sqrt{1-\frac{1}{\infty}} + \sqrt{1+\frac{1}{\infty}} = \sqrt{1-0} + \sqrt{1+0} = 2.$

$$\lim_{x\to\infty} \left(x - \sqrt{x^2-1}\right) = 0$$

- $= \lim\limits_{x\to\infty} \left(\left(x - \sqrt{x^2-1}\right) \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right) = \lim\limits_{x\to\infty} \frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$

  $= \lim\limits_{x\to\infty} \frac{x^2-(\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}} = \lim\limits_{x\to\infty} \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}} = \lim\limits_{x\to\infty} \frac{1}{x+\sqrt{x^2-1}}$

  $= \frac{1}{\infty+\sqrt{\infty^2-1}} = \frac{1}{\infty+\infty} = \frac{1}{\infty} = 0.$

Riešené limity – 22

$$\lim_{x \to a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} \quad \text{pre } a > 0$$

Riešené limity – 22

$$\lim_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}$$ pre $a>0$

- $= \lim_{x\to a}\left(\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\frac{\sqrt{ax}+a}{\sqrt{ax}+a}\right)$

- $= \left[\text{Subst. } z=\sqrt{ax} \;\middle|\; \begin{matrix} x\to a \\ z\to a\end{matrix} \;\middle|\; \begin{matrix} z^2=ax \\ x^2=\frac{z^4}{a^2}\end{matrix}\right]$

- $= \left[\begin{matrix}\text{Subst. } & z=\sqrt{x} \\ \text{Označme } & \sqrt{a}=b\end{matrix} \;\middle|\; \begin{matrix} x\to a \\ z\to\sqrt{a}=b\end{matrix} \;\middle|\; \begin{matrix} x=z^2 \\ a=b^2\end{matrix}\right]$

Riešené limity – 22

$\lim\limits_{x\to a} \dfrac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}$ pre $a>0$

- $= \lim\limits_{x\to a} \left(\dfrac{x^2-a\sqrt{ax}}{\sqrt{ax}-a} \cdot \dfrac{\sqrt{ax}+a}{\sqrt{ax}+a} \right) = \lim\limits_{x\to a} \dfrac{x^2\sqrt{ax}-a\cdot(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$

- $= \left[\text{Subst. } z=\sqrt{ax} \,\middle|\, \begin{matrix} x\to a \\ z\to a \end{matrix} \,\middle|\, \begin{matrix} z^2=ax \\ x^2=\frac{z^4}{a^2} \end{matrix} \right] = \lim\limits_{z\to a} \dfrac{\frac{z^4}{a^2}-a\cdot z}{z-a}$

- $= \left[\begin{matrix} \text{Subst. } z=\sqrt{x} \\ \text{Označme } \sqrt{a}=b \end{matrix} \,\middle|\, \begin{matrix} x\to a \\ z\to\sqrt{a}=b \end{matrix} \,\middle|\, \begin{matrix} x=z^2 \\ a=b^2 \end{matrix} \right] = \lim\limits_{z\to b} \dfrac{z^4-b^2\cdot bz}{bz-b^2}$

Riešené limity – 22

$$\lim_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}$$ pre $a>0$

- $= \lim_{x\to a}\left(\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\frac{\sqrt{ax}+a}{\sqrt{ax}+a}\right) = \lim_{x\to a}\frac{x^2\sqrt{ax}-a\cdot(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$

  $= \lim_{x\to a}\frac{\sqrt{ax}\cdot(x^2-a^2)+ax^2-a^2x}{ax-a^2}$

- $= \left[\text{Subst. } z=\sqrt{ax} \;\middle|\; \begin{matrix} x\to a \\ z\to a\end{matrix} \;\middle|\; \begin{matrix} z^2=ax \\ x^2=\frac{z^4}{a^2}\end{matrix}\right] = \lim_{z\to a}\frac{\frac{z^4}{a^2}-a\cdot z}{z-a} = \lim_{z\to a}\frac{\frac{z^4-a^3z}{a^2}}{z-a}$

- $= \left[\begin{matrix}\text{Subst. } z=\sqrt{x} \\ \text{Označme } \sqrt{a}=b\end{matrix} \;\middle|\; \begin{matrix} x\to a \\ z\to\sqrt{a}=b\end{matrix} \;\middle|\; \begin{matrix} x=z^2 \\ a=b^2\end{matrix}\right] = \lim_{z\to b}\frac{z^4-b^2\cdot bz}{bz-b^2} = \lim_{z\to b}\frac{z^4-b^3z}{bz-b^2}$

Riešené limity – 22

$$\lim_{x\to a} \frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a} \quad \text{pre } a>0$$

- $$= \lim_{x\to a}\left(\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\frac{\sqrt{ax}+a}{\sqrt{ax}+a}\right) = \lim_{x\to a}\frac{x^2\sqrt{ax}-a\cdot(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$$
  $$= \lim_{x\to a}\frac{\sqrt{ax}\cdot(x^2-a^2)+ax^2-a^2x}{ax-a^2} = \lim_{x\to a}\frac{\sqrt{ax}\cdot(x-a)\cdot(x+a)+ax\cdot(x-a)}{a\cdot(x-a)}$$

- $$= \left[\text{Subst. } z=\sqrt{ax} \;\middle|\; \begin{matrix} x\to a \\ z\to a\end{matrix} \;\middle|\; \begin{matrix} z^2=ax \\ x^2=\frac{z^4}{a^2}\end{matrix}\right] = \lim_{z\to a}\frac{\frac{z^4}{a^2}-a\cdot z}{z-a} = \lim_{z\to a}\frac{\frac{z^4-a^3z}{a^2}}{z-a} = \lim_{z\to a}\frac{z(z^3-a^3)}{a^2(z-a)}$$

- $$= \left[\begin{matrix}\text{Subst. } z=\sqrt{x} \\ \text{Označme } \sqrt{a}=b\end{matrix} \;\middle|\; \begin{matrix} x\to a \\ z\to\sqrt{a}=b\end{matrix} \;\middle|\; \begin{matrix} x=z^2 \\ a=b^2\end{matrix}\right] = \lim_{z\to b}\frac{z^4-b^2\cdot bz}{bz-b^2} = \lim_{z\to b}\frac{z^4-b^3z}{bz-b^2} = \lim_{z\to b}\frac{z(z^3-b^3)}{b(z-b)}$$

Riešené limity – 22

$\lim\limits_{x\to a} \dfrac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}$ pre $a>0$

- $= \lim\limits_{x\to a} \left(\dfrac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\dfrac{\sqrt{ax}+a}{\sqrt{ax}+a}\right) = \lim\limits_{x\to a} \dfrac{x^2\sqrt{ax}-a\cdot(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$

  $= \lim\limits_{x\to a} \dfrac{\sqrt{ax}\cdot(x^2-a^2)+ax^2-a^2x}{ax-a^2} = \lim\limits_{x\to a} \dfrac{\sqrt{ax}\cdot(x-a)\cdot(x+a)+ax\cdot(x-a)}{a\cdot(x-a)} = \lim\limits_{x\to a} \dfrac{\sqrt{ax}\cdot(x+a)+ax}{a}$

- $= \left[\text{Subst. } z=\sqrt{ax} \;\middle|\; \begin{matrix} x\to a \\ z\to a \end{matrix} \;\middle|\; \begin{matrix} z^2=ax \\ x^2=\frac{z^4}{a^2} \end{matrix}\right] = \lim\limits_{z\to a} \dfrac{\frac{z^4}{a^2}-a\cdot z}{z-a} = \lim\limits_{z\to a} \dfrac{\frac{z^4-a^3z}{a^2}}{z-a} = \lim\limits_{z\to a} \dfrac{z(z^3-a^3)}{a^2(z-a)}$

  $= \lim\limits_{z\to a} \dfrac{z(z-a)(z^2+za+a^2)}{a^2(z-a)}$

- $= \left[\begin{matrix} \text{Subst. } z=\sqrt{x} \\ \text{Označme } \sqrt{a}=b \end{matrix} \;\middle|\; \begin{matrix} x\to a \\ z\to\sqrt{a}=b \end{matrix} \;\middle|\; \begin{matrix} x=z^2 \\ a=b^2 \end{matrix}\right] = \lim\limits_{z\to b} \dfrac{z^4-b^2\cdot bz}{bz-b^2} = \lim\limits_{z\to b} \dfrac{z^4-b^3z}{bz-b^2} = \lim\limits_{z\to b} \dfrac{z(z^3-b^3)}{b(z-b)}$

  $= \lim\limits_{z\to b} \dfrac{z(z-b)(z^2+zb+b^2)}{b(z-b)}$

Riešené limity – 22

$$\lim_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a} \quad \text{pre } a>0$$

- $$= \lim_{x\to a}\left(\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\frac{\sqrt{ax}+a}{\sqrt{ax}+a}\right) = \lim_{x\to a}\frac{x^2\sqrt{ax}-a\cdot(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$$
$$= \lim_{x\to a}\frac{\sqrt{ax}\cdot(x^2-a^2)+ax^2-a^2x}{ax-a^2} = \lim_{x\to a}\frac{\sqrt{ax}\cdot(x-a)\cdot(x+a)+ax\cdot(x-a)}{a\cdot(x-a)} = \lim_{x\to a}\frac{\sqrt{ax}\cdot(x+a)+ax}{a}$$
$$= \frac{\sqrt{a\cdot a}\cdot(a+a)+a\cdot a}{a}$$

- $$= \left[\text{Subst. } z=\sqrt{ax} \;\middle|\; \begin{matrix} x\to a \\ z\to a\end{matrix} \;\middle|\; \begin{matrix} z^2=ax \\ x^2=\frac{z^4}{a^2}\end{matrix}\right] = \lim_{z\to a}\frac{\frac{z^4}{a^2}-a\cdot z}{z-a} = \lim_{z\to a}\frac{\frac{z^4-a^3z}{a^2}}{z-a} = \lim_{z\to a}\frac{z(z^3-a^3)}{a^2(z-a)}$$
$$= \lim_{z\to a}\frac{z(z-a)(z^2+za+a^2)}{a^2(z-a)} = \lim_{z\to a}\frac{z(z^2+za+a^2)}{a^2}$$

- $$= \left[\begin{matrix}\text{Subst. } z=\sqrt{x} \\ \text{Označme } \sqrt{a}=b\end{matrix} \;\middle|\; \begin{matrix} x\to a \\ z\to\sqrt{a}=b\end{matrix} \;\middle|\; \begin{matrix} x=z^2 \\ a=b^2\end{matrix}\right] = \lim_{z\to b}\frac{z^4-b^2\cdot bz}{bz-b^2} = \lim_{z\to b}\frac{z^4-b^3z}{bz-b^2} = \lim_{z\to b}\frac{z(z^3-b^3)}{b(z-b)}$$
$$= \lim_{z\to b}\frac{z(z-b)(z^2+zb+b^2)}{b(z-b)} = \lim_{z\to b}\frac{z(z^2+zb+b^2)}{b}$$

Riešené limity – 22

$$\lim_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a} \quad \text{pre } a>0$$

- $= \lim_{x\to a}\left(\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\frac{\sqrt{ax}+a}{\sqrt{ax}+a}\right) = \lim_{x\to a}\frac{x^2\sqrt{ax}-a\cdot(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$

  $= \lim_{x\to a}\frac{\sqrt{ax}\cdot(x^2-a^2)+ax^2-a^2x}{ax-a^2} = \lim_{x\to a}\frac{\sqrt{ax}\cdot(x-a)\cdot(x+a)+ax\cdot(x-a)}{a\cdot(x-a)} = \lim_{x\to a}\frac{\sqrt{ax}\cdot(x+a)+ax}{a}$

  $= \frac{\sqrt{a\cdot a}\cdot(a+a)+a\cdot a}{a} = \frac{a\cdot 2a+a^2}{a}$

- $= \left[\text{Subst. } z=\sqrt{ax} \;\middle|\; \begin{matrix} x\to a \\ z\to a\end{matrix} \;\middle|\; \begin{matrix} z^2=ax \\ x^2=\frac{z^4}{a^2}\end{matrix}\right] = \lim_{z\to a}\frac{\frac{z^4}{a^2}-a\cdot z}{z-a} = \lim_{z\to a}\frac{\frac{z^4-a^3z}{a^2}}{z-a} = \lim_{z\to a}\frac{z(z^3-a^3)}{a^2(z-a)}$

  $= \lim_{z\to a}\frac{z(z-a)(z^2+za+a^2)}{a^2(z-a)} = \lim_{z\to a}\frac{z(z^2+za+a^2)}{a^2} = \frac{a(a^2+a\cdot a+a^2)}{a^2}$

- $= \left[\begin{matrix}\text{Subst. } z=\sqrt{x} \\ \text{Označme } \sqrt{a}=b\end{matrix} \;\middle|\; \begin{matrix} x\to a \\ z\to\sqrt{a}=b\end{matrix} \;\middle|\; \begin{matrix} x=z^2 \\ a=b^2\end{matrix}\right] = \lim_{z\to b}\frac{z^4-b^2\cdot bz}{bz-b^2} = \lim_{z\to b}\frac{z^4-b^3z}{bz-b^2} = \lim_{z\to b}\frac{z(z^3-b^3)}{b(z-b)}$

  $= \lim_{z\to b}\frac{z(z-b)(z^2+zb+b^2)}{b(z-b)} = \lim_{z\to b}\frac{z(z^2+zb+b^2)}{b} = \frac{b(b^2+b\cdot b+b^2)}{b}$

Riešené limity – 22

$$\lim_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}$$ pre $a>0$

- $$= \lim_{x\to a}\left(\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\frac{\sqrt{ax}+a}{\sqrt{ax}+a}\right) = \lim_{x\to a}\frac{x^2\sqrt{ax}-a\cdot(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$$
$$= \lim_{x\to a}\frac{\sqrt{ax}\cdot(x^2-a^2)+ax^2-a^2x}{ax-a^2} = \lim_{x\to a}\frac{\sqrt{ax}\cdot(x-a)\cdot(x+a)+ax\cdot(x-a)}{a\cdot(x-a)} = \lim_{x\to a}\frac{\sqrt{ax}\cdot(x+a)+ax}{a}$$
$$= \frac{\sqrt{a\cdot a}\cdot(a+a)+a\cdot a}{a} = \frac{a\cdot 2a+a^2}{a} = \frac{3a^2}{a}$$

- $$= \left[\text{Subst. } z=\sqrt{ax} \;\middle|\; \begin{matrix} x\to a \\ z\to a\end{matrix} \;\middle|\; \begin{matrix} z^2=ax \\ x^2=\frac{z^4}{a^2}\end{matrix}\right] = \lim_{z\to a}\frac{\frac{z^4}{a^2}-a\cdot z}{z-a} = \lim_{z\to a}\frac{\frac{z^4-a^3z}{a^2}}{z-a} = \lim_{z\to a}\frac{z(z^3-a^3)}{a^2(z-a)}$$
$$= \lim_{z\to a}\frac{z(z-a)(z^2+za+a^2)}{a^2(z-a)} = \lim_{z\to a}\frac{z(z^2+za+a^2)}{a^2} = \frac{a(a^2+a\cdot a+a^2)}{a^2} = \frac{a\cdot 3a^2}{a^2}$$

- $$= \left[\begin{matrix}\text{Subst. } z=\sqrt{x} \\ \text{Označme } \sqrt{a}=b\end{matrix} \;\middle|\; \begin{matrix} x\to a \\ z\to\sqrt{a}=b\end{matrix} \;\middle|\; \begin{matrix} x=z^2 \\ a=b^2\end{matrix}\right] = \lim_{z\to b}\frac{z^4-b^2\cdot bz}{bz-b^2} = \lim_{z\to b}\frac{z^4-b^3z}{bz-b^2} = \lim_{z\to b}\frac{z(z^3-b^3)}{b(z-b)}$$
$$= \lim_{z\to b}\frac{z(z-b)(z^2+zb+b^2)}{b(z-b)} = \lim_{z\to b}\frac{z(z^2+zb+b^2)}{b} = \frac{b(b^2+b\cdot b+b^2)}{b} = \frac{b\cdot 3b^2}{b} = 3b^2$$

Riešené limity – 22

$\lim\limits_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}=3a$ pre $a>0$

- $$= \lim_{x\to a}\left(\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\frac{\sqrt{ax}+a}{\sqrt{ax}+a}\right)=\lim_{x\to a}\frac{x^2\sqrt{ax}-a\cdot(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$$
  $$=\lim_{x\to a}\frac{\sqrt{ax}\cdot(x^2-a^2)+ax^2-a^2x}{ax-a^2}=\lim_{x\to a}\frac{\sqrt{ax}\cdot(x-a)\cdot(x+a)+ax\cdot(x-a)}{a\cdot(x-a)}=\lim_{x\to a}\frac{\sqrt{ax}\cdot(x+a)+ax}{a}$$
  $$=\frac{\sqrt{a\cdot a}\cdot(a+a)+a\cdot a}{a}=\frac{a\cdot 2a+a^2}{a}=\frac{3a^2}{a}=3a.$$

- $$=\left[\text{Subst. } z=\sqrt{ax}\;\middle|\; \begin{matrix}x\to a\\ z\to a\end{matrix}\;\middle|\;\begin{matrix}z^2=ax\\ x^2=\frac{z^4}{a^2}\end{matrix}\right]=\lim_{z\to a}\frac{\frac{z^4}{a^2}-a\cdot z}{z-a}=\lim_{z\to a}\frac{\frac{z^4-a^3z}{a^2}}{z-a}=\lim_{z\to a}\frac{z(z^3-a^3)}{a^2(z-a)}$$
  $$=\lim_{z\to a}\frac{z(z-a)(z^2+za+a^2)}{a^2(z-a)}=\lim_{z\to a}\frac{z(z^2+za+a^2)}{a^2}=\frac{a(a^2+a\cdot a+a^2)}{a^2}=\frac{a\cdot 3a^2}{a^2}=3a.$$

- $$=\left[\begin{matrix}\text{Subst. } z=\sqrt{x}\\ \text{Označme } \sqrt{a}=b\end{matrix}\;\middle|\;\begin{matrix}x\to a\\ z\to\sqrt{a}=b\end{matrix}\;\middle|\;\begin{matrix}x=z^2\\ a=b^2\end{matrix}\right]=\lim_{z\to b}\frac{z^4-b^2\cdot bz}{bz-b^2}=\lim_{z\to b}\frac{z^4-b^3z}{bz-b^2}=\lim_{z\to b}\frac{z(z^3-b^3)}{b(z-b)}$$
  $$=\lim_{z\to b}\frac{z(z-b)(z^2+zb+b^2)}{b(z-b)}=\lim_{z\to b}\frac{z(z^2+zb+b^2)}{b}=\frac{b(b^2+b\cdot b+b^2)}{b}=\frac{b\cdot 3b^2}{b}=3b^2=3a.$$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}}$ pre $a \geq 0$

$\lim\limits_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1}$

$\lim\limits_{x\to 1} \frac{x^m-1}{x^n-1}$ pre $m, n \in N$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}}$ pre $a \geq 0$

- $= \lim\limits_{x\to a} \frac{(\sqrt{a}-\sqrt{x})\cdot(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}}$

- $= \lim\limits_{x\to a} \left(\frac{a-x}{\sqrt{a}-\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}}\right)$

$\lim\limits_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1}$

- $= \lim\limits_{x\to\infty} \left(\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right)$

$\lim\limits_{x\to 1} \frac{x^m-1}{x^n-1}$ pre $m,n \in N$

- $= \lim\limits_{x\to 1} \frac{(x-1)\cdot(x^{m-1}+x^{m-2}+\cdots+x+1)}{(x-1)\cdot(x^{n-1}+x^{n-2}+\cdots+x+1)}$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}}$ pre $a \geq 0$

- $= \lim\limits_{x\to a} \frac{(\sqrt{a}-\sqrt{x})\cdot(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim\limits_{x\to a} (\sqrt{a}+\sqrt{x})$

- $= \lim\limits_{x\to a} \left(\frac{a-x}{\sqrt{a}-\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}}\right) = \lim\limits_{x\to a} \frac{(a-x)\cdot(\sqrt{a}+\sqrt{x})}{a-x}$

$\lim\limits_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1}$

- $= \lim\limits_{x\to\infty} \left(\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right) = \lim\limits_{x\to\infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}}$

$\lim\limits_{x\to 1} \frac{x^m-1}{x^n-1}$ pre $m,n \in N$

- $= \lim\limits_{x\to 1} \frac{(x-1)\cdot(x^{m-1}+x^{m-2}+\cdots+x+1)}{(x-1)\cdot(x^{n-1}+x^{n-2}+\cdots+x+1)} = \lim\limits_{x\to 1} \frac{x^{m-1}+x^{m-2}+\cdots+x+1}{x^{n-1}+x^{n-2}+\cdots+x+1}$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}}$ pre $a \geq 0$

- $= \lim\limits_{x\to a} \frac{(\sqrt{a}-\sqrt{x})\cdot(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim\limits_{x\to a} (\sqrt{a}+\sqrt{x}) = \sqrt{a}+\sqrt{a}$

- $= \lim\limits_{x\to a} \left(\frac{a-x}{\sqrt{a}-\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}}\right) = \lim\limits_{x\to a} \frac{(a-x)\cdot(\sqrt{a}+\sqrt{x})}{a-x} = \lim\limits_{x\to a} (\sqrt{a}+\sqrt{x})$

$\lim\limits_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1}$

- $= \lim\limits_{x\to\infty} \left(\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right) = \lim\limits_{x\to\infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}}$

$\lim\limits_{x\to 1} \frac{x^m-1}{x^n-1}$ pre $m,n \in N$

- $= \lim\limits_{x\to 1} \frac{(x-1)\cdot(x^{m-1}+x^{m-2}+\cdots+x+1)}{(x-1)\cdot(x^{n-1}+x^{n-2}+\cdots+x+1)} = \lim\limits_{x\to 1} \frac{x^{m-1}+x^{m-2}+\cdots+x+1}{x^{n-1}+x^{n-2}+\cdots+x+1}$

$$= \frac{1^{m-1}+1^{m-2}+\cdots+1+1}{1^{n-1}+1^{n-2}+\cdots+1+1}$$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a}$ pre $a \geq 0$

- $= \lim\limits_{x\to a} \frac{(\sqrt{a}-\sqrt{x})\cdot(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim\limits_{x\to a} (\sqrt{a}+\sqrt{x}) = \sqrt{a}+\sqrt{a} = 2\sqrt{a}.$

- $= \lim\limits_{x\to a} \left(\frac{a-x}{\sqrt{a}-\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}}\right) = \lim\limits_{x\to a} \frac{(a-x)\cdot(\sqrt{a}+\sqrt{x})}{a-x} = \lim\limits_{x\to a} (\sqrt{a}+\sqrt{x}) = \sqrt{a}+\sqrt{a} = 2\sqrt{a}.$

$\lim\limits_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1}$

- $= \lim\limits_{x\to\infty} \left(\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right) = \lim\limits_{x\to\infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}} = \frac{0-6}{3+0}$

$\lim\limits_{x\to 1} \frac{x^m-1}{x^n-1}$ pre $m,n \in N$

- $= \lim\limits_{x\to 1} \frac{(x-1)\cdot(x^{m-1}+x^{m-2}+\cdots+x+1)}{(x-1)\cdot(x^{n-1}+x^{n-2}+\cdots+x+1)} = \lim\limits_{x\to 1} \frac{x^{m-1}+x^{m-2}+\cdots+x+1}{x^{n-1}+x^{n-2}+\cdots+x+1}$

$$= \frac{1^{m-1}+1^{m-2}+\cdots+1+1}{1^{n-1}+1^{n-2}+\cdots+1+1} = \frac{1^{m-1}+1^{m-2}+\cdots+1^1+1^0}{1^{n-1}+1^{n-2}+\cdots+1^1+1^0}$$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a}$ pre $a \geq 0$

- $= \lim\limits_{x\to a} \frac{(\sqrt{a}-\sqrt{x})\cdot(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim\limits_{x\to a} (\sqrt{a}+\sqrt{x}) = \sqrt{a}+\sqrt{a} = 2\sqrt{a}.$

- $= \lim\limits_{x\to a} \left(\frac{a-x}{\sqrt{a}-\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}}\right) = \lim\limits_{x\to a} \frac{(a-x)\cdot(\sqrt{a}+\sqrt{x})}{a-x} = \lim\limits_{x\to a} (\sqrt{a}+\sqrt{x}) = \sqrt{a}+\sqrt{a} = 2\sqrt{a}.$

$\lim\limits_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1} = -2$

- $= \lim\limits_{x\to\infty} \left(\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right) = \lim\limits_{x\to\infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}} = \frac{0-6}{3+0} = -2.$

$\lim\limits_{x\to 1} \frac{x^m-1}{x^n-1}$ pre $m,n \in N$

- $= \lim\limits_{x\to 1} \frac{(x-1)\cdot(x^{m-1}+x^{m-2}+\cdots+x+1)}{(x-1)\cdot(x^{n-1}+x^{n-2}+\cdots+x+1)} = \lim\limits_{x\to 1} \frac{x^{m-1}+x^{m-2}+\cdots+x+1}{x^{n-1}+x^{n-2}+\cdots+x+1}$

$$= \frac{1^{m-1}+1^{m-2}+\cdots+1+1}{1^{n-1}+1^{n-2}+\cdots+1+1} = \frac{1^{m-1}+1^{m-2}+\cdots+1^1+1^0}{1^{n-1}+1^{n-2}+\cdots+1^1+1^0} = \frac{m\cdot 1}{n\cdot 1}$$

Riešené limity – 23, 24, 25

$$\lim_{x\to a}\frac{a-x}{\sqrt{a}-\sqrt{x}}=2\sqrt{a} \text{ pre } a\geq 0$$

- $=\lim_{x\to a}\frac{(\sqrt{a}-\sqrt{x})\cdot(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}}=\lim_{x\to a}(\sqrt{a}+\sqrt{x})=\sqrt{a}+\sqrt{a}=2\sqrt{a}.$

- $=\lim_{x\to a}\left(\frac{a-x}{\sqrt{a}-\sqrt{x}}\cdot\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}}\right)=\lim_{x\to a}\frac{(a-x)\cdot(\sqrt{a}+\sqrt{x})}{a-x}=\lim_{x\to a}(\sqrt{a}+\sqrt{x})=\sqrt{a}+\sqrt{a}=2\sqrt{a}.$

$$\lim_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}=-2$$

- $=\lim_{x\to\infty}\left(\frac{\sqrt{x}-6x}{3x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}\right)=\lim_{x\to\infty}\frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}}=\frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}}=\frac{0-6}{3+0}=-2.$

$$\lim_{x\to 1}\frac{x^m-1}{x^n-1}=\frac{m}{n} \text{ pre } m,n\in N$$

- $=\lim_{x\to 1}\frac{(x-1)\cdot(x^{m-1}+x^{m-2}+\cdots+x+1)}{(x-1)\cdot(x^{n-1}+x^{n-2}+\cdots+x+1)}=\lim_{x\to 1}\frac{x^{m-1}+x^{m-2}+\cdots+x+1}{x^{n-1}+x^{n-2}+\cdots+x+1}$

$$=\frac{1^{m-1}+1^{m-2}+\cdots+1+1}{1^{n-1}+1^{n-2}+\cdots+1+1}=\frac{1^{m-1}+1^{m-2}+\cdots+1^1+1^0}{1^{n-1}+1^{n-2}+\cdots+1^1+1^0}=\frac{m\cdot 1}{n\cdot 1}=\frac{m}{n}.$$

Riešené limity – 26

$\lim_{x\to 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$ pre $m, n \in N$

Riešené limity – 26

$$\lim_{x\to 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \quad \text{pre } m,n \in N$$

- $= \lim\limits_{x\to 1} \left(\dfrac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \dfrac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1} \cdot \dfrac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1} \right)$

- $= \left[\text{Subst. } x = z^{mn} \,\middle|\, \begin{matrix} x \to 1 \\ z \to 1 \end{matrix} \,\middle|\, \begin{matrix} \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \\ \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \end{matrix} \right]$

Riešené limity – 26

$\lim\limits_{x\to 1} \dfrac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$ pre $m,n \in N$

- $= \lim\limits_{x\to 1} \left(\dfrac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \dfrac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1} \cdot \dfrac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1} \right)$

  $= \lim\limits_{x\to 1} \left(\dfrac{(\sqrt[m]{x}-1)\cdot\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1\right]}{(\sqrt[n]{x}-1)\cdot\left[(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1\right]} \cdot \dfrac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1} \right)$

- $= \left[\text{Subst. } x = z^{mn} \;\middle|\; \begin{array}{c} x\to 1 \\ z \to 1 \end{array} \;\middle|\; \begin{array}{l} \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \\ \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \end{array} \right]$

  $= \lim\limits_{z\to 1} \dfrac{z^n-1}{z^m-1}$

Riešené limity – 26

$\lim\limits_{x\to 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$ pre $m,n \in N$

- $= \lim\limits_{x\to 1} \left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \frac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\dots+\sqrt[m]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\dots+\sqrt[m]{x}+1} \cdot \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\dots+\sqrt[n]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\dots+\sqrt[n]{x}+1} \right)$

  $= \lim\limits_{x\to 1} \left(\frac{(\sqrt[m]{x}-1)\cdot\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\dots+\sqrt[m]{x}+1\right]}{(\sqrt[n]{x}-1)\cdot\left[(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\dots+\sqrt[n]{x}+1\right]} \cdot \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\dots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\dots+\sqrt[m]{x}+1} \right)$

  $= \lim\limits_{x\to 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\dots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\dots+\sqrt[m]{x}+1} \right)$

- $= \left[\text{Subst. } x = z^{mn} \;\middle|\; \begin{matrix} x \to 1 \\ z \to 1 \end{matrix} \;\middle|\; \begin{matrix} \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \\ \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \end{matrix} \right]$

  $= \lim\limits_{z\to 1} \frac{z^n-1}{z^m-1} = \lim\limits_{z\to 1} \frac{(z-1)\cdot(z^{n-1}+z^{n-2}+\dots+z+1)}{(z-1)\cdot(z^{m-1}+z^{m-2}+\dots+z+1)}$

Riešené limity – 26

$\lim\limits_{x\to 1} \dfrac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$ pre $m,n \in N$

- $= \lim\limits_{x\to 1} \left(\dfrac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \dfrac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1} \cdot \dfrac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1} \right)$

$$= \lim_{x\to 1} \left(\frac{(\sqrt[m]{x}-1)\cdot\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1\right]}{(\sqrt[n]{x}-1)\cdot\left[(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1\right]} \cdot \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1} \right)$$

$$= \lim_{x\to 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1} \right) = \lim_{x\to 1} \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}$$

- $= \left[\text{Subst. } x = z^{mn} \;\middle|\; \begin{matrix} x\to 1 \\ z\to 1 \end{matrix} \;\middle|\; \begin{matrix} \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \\ \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \end{matrix} \right]$

$$= \lim_{z\to 1} \frac{z^n-1}{z^m-1} = \lim_{z\to 1} \frac{(z-1)\cdot(z^{n-1}+z^{n-2}+\cdots+z+1)}{(z-1)\cdot(z^{m-1}+z^{m-2}+\cdots+z+1)} = \lim_{z\to 1} \frac{z^{n-1}+z^{n-2}+\cdots+z+1}{z^{m-1}+z^{m-2}+\cdots+z+1}$$

Riešené limity – 26

$$\lim_{x\to 1}\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}=\frac{n}{m}$$ pre $m,n\in N$

- $$=\lim_{x\to 1}\left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}\cdot\frac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}\cdot\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}\right)$$

$$=\lim_{x\to 1}\left(\frac{(\sqrt[m]{x}-1)\cdot\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1\right]}{(\sqrt[n]{x}-1)\cdot\left[(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1\right]}\cdot\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}\right)$$

$$=\lim_{x\to 1}\left(\frac{x-1}{x-1}\cdot\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}\right)=\lim_{x\to 1}\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}$$

$$=\frac{1^{n-1}+1^{n-2}+\cdots+1+1}{1^{m-1}+1^{m-2}+\cdots+1+1}$$

- $$=\left[\text{Subst. } x=z^{mn}\ \middle|\ \begin{matrix} x\to 1\\ z\to 1\end{matrix}\ \middle|\ \begin{matrix}\sqrt[m]{x}=\sqrt[m]{z^{mn}}=z^{\frac{mn}{m}}=z^n\\ \sqrt[n]{x}=\sqrt[n]{z^{mn}}=z^{\frac{mn}{n}}=z^m\end{matrix}\right]$$

$$=\lim_{z\to 1}\frac{z^n-1}{z^m-1}=\lim_{z\to 1}\frac{(z-1)\cdot(z^{n-1}+z^{n-2}+\cdots+z+1)}{(z-1)\cdot(z^{m-1}+z^{m-2}+\cdots+z+1)}=\lim_{z\to 1}\frac{z^{n-1}+z^{n-2}+\cdots+z+1}{z^{m-1}+z^{m-2}+\cdots+z+1}$$

$$=\frac{1^{n-1}+1^{n-2}+\cdots+1+1}{1^{m-1}+1^{m-2}+\cdots+1+1}$$

Riešené limity – 26

$$\lim_{x\to 1}\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}=\frac{n}{m} \text{ pre } m,n\in N$$

- $$= \lim_{x\to 1}\left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}\cdot\frac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}\cdot\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}\right)$$

$$= \lim_{x\to 1}\left(\frac{(\sqrt[m]{x}-1)\cdot\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1\right]}{(\sqrt[n]{x}-1)\cdot\left[(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1\right]}\cdot\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}\right)$$

$$= \lim_{x\to 1}\left(\frac{x-1}{x-1}\cdot\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}\right)=\lim_{x\to 1}\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}$$

$$= \frac{1^{n-1}+1^{n-2}+\cdots+1+1}{1^{m-1}+1^{m-2}+\cdots+1+1}=\frac{1^{n-1}+1^{n-2}+\cdots+1^1+1^0}{1^{m-1}+1^{m-2}+\cdots+1^1+1^0}$$

- $$= \left[\text{Subst. } x=z^{mn} \;\middle|\; \begin{array}{c} x\to 1 \\ z\to 1\end{array} \;\middle|\; \begin{array}{c}\sqrt[m]{x}=\sqrt[m]{z^{mn}}=z^{\frac{mn}{m}}=z^n \\ \sqrt[n]{x}=\sqrt[n]{z^{mn}}=z^{\frac{mn}{n}}=z^m\end{array}\right]$$

$$= \lim_{z\to 1}\frac{z^n-1}{z^m-1}=\lim_{z\to 1}\frac{(z-1)\cdot(z^{n-1}+z^{n-2}+\cdots+z+1)}{(z-1)\cdot(z^{m-1}+z^{m-2}+\cdots+z+1)}=\lim_{z\to 1}\frac{z^{n-1}+z^{n-2}+\cdots+z+1}{z^{m-1}+z^{m-2}+\cdots+z+1}$$

$$= \frac{1^{n-1}+1^{n-2}+\cdots+1+1}{1^{m-1}+1^{m-2}+\cdots+1+1}=\frac{1^{n-1}+1^{n-2}+\cdots+1^1+1^0}{1^{m-1}+1^{m-2}+\cdots+1^1+1^0}$$

Riešené limity – 26

$\lim\limits_{x\to 1}\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}=\frac{n}{m}$ pre $m,n\in N$

- $=\lim\limits_{x\to 1}\left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}\cdot\frac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}\cdot\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}\right)$

$$=\lim_{x\to 1}\left(\frac{(\sqrt[m]{x}-1)\cdot\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1\right]}{(\sqrt[n]{x}-1)\cdot\left[(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1\right]}\cdot\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}\right)$$

$$=\lim_{x\to 1}\left(\frac{x-1}{x-1}\cdot\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}\right)=\lim_{x\to 1}\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}$$

$$=\frac{1^{n-1}+1^{n-2}+\cdots+1+1}{1^{m-1}+1^{m-2}+\cdots+1+1}=\frac{1^{n-1}+1^{n-2}+\cdots+1^1+1^0}{1^{m-1}+1^{m-2}+\cdots+1^1+1^0}=\frac{n\cdot 1}{m\cdot 1}$$

- $=\left[\text{Subst. } x=z^{mn}\;\middle|\;\begin{matrix}x\to 1\\ z\to 1\end{matrix}\;\middle|\;\begin{matrix}\sqrt[m]{x}=\sqrt[m]{z^{mn}}=z^{\frac{mn}{m}}=z^n\\ \sqrt[n]{x}=\sqrt[n]{z^{mn}}=z^{\frac{mn}{n}}=z^m\end{matrix}\right]$

$$=\lim_{z\to 1}\frac{z^n-1}{z^m-1}=\lim_{z\to 1}\frac{(z-1)\cdot(z^{n-1}+z^{n-2}+\cdots+z+1)}{(z-1)\cdot(z^{m-1}+z^{m-2}+\cdots+z+1)}=\lim_{z\to 1}\frac{z^{n-1}+z^{n-2}+\cdots+z+1}{z^{m-1}+z^{m-2}+\cdots+z+1}$$

$$=\frac{1^{n-1}+1^{n-2}+\cdots+1+1}{1^{m-1}+1^{m-2}+\cdots+1+1}=\frac{1^{n-1}+1^{n-2}+\cdots+1^1+1^0}{1^{m-1}+1^{m-2}+\cdots+1^1+1^0}=\frac{n\cdot 1}{m\cdot 1}$$

Riešené limity – 26

$\lim\limits_{x\to 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m}$ pre $m,n \in N$

- $$= \lim_{x\to 1} \left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \frac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1} \cdot \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1} \right)$$

$$= \lim_{x\to 1} \left(\frac{(\sqrt[m]{x}-1)\cdot\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1\right]}{(\sqrt[n]{x}-1)\cdot\left[(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1\right]} \cdot \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1} \right)$$

$$= \lim_{x\to 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1} \right) = \lim_{x\to 1} \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}$$

$$= \frac{1^{n-1}+1^{n-2}+\cdots+1+1}{1^{m-1}+1^{m-2}+\cdots+1+1} = \frac{1^{n-1}+1^{n-2}+\cdots+1^1+1^0}{1^{m-1}+1^{m-2}+\cdots+1^1+1^0} = \frac{n\cdot 1}{m\cdot 1} = \frac{n}{m}.$$

- $$= \left[\text{Subst. } x = z^{mn} \;\middle|\; \begin{matrix} x\to 1 \\ z\to 1 \end{matrix} \;\middle|\; \begin{matrix} \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \\ \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \end{matrix} \right]$$

$$= \lim_{z\to 1} \frac{z^n-1}{z^m-1} = \lim_{z\to 1} \frac{(z-1)\cdot(z^{n-1}+z^{n-2}+\cdots+z+1)}{(z-1)\cdot(z^{m-1}+z^{m-2}+\cdots+z+1)} = \lim_{z\to 1} \frac{z^{n-1}+z^{n-2}+\cdots+z+1}{z^{m-1}+z^{m-2}+\cdots+z+1}$$

$$= \frac{1^{n-1}+1^{n-2}+\cdots+1+1}{1^{m-1}+1^{m-2}+\cdots+1+1} = \frac{1^{n-1}+1^{n-2}+\cdots+1^1+1^0}{1^{m-1}+1^{m-2}+\cdots+1^1+1^0} = \frac{n\cdot 1}{m\cdot 1} = \frac{n}{m}.$$

Riešené limity – 27

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

Riešené limity – 27

$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$

- $=$ [Príklad 26 pre $m=2$, $n=3$]

- $= \lim_{x\to 1} \left(\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} \right)$

- $= \left[\text{Subst. } x=z^6 \;\middle|\; \begin{matrix} x\to 1 \\ z\to 1 \end{matrix} \;\middle|\; \begin{matrix} \sqrt{x}=\sqrt{z^6}=z^{\frac{6}{2}}=z^3 \\ \sqrt[3]{x}=\sqrt[3]{z^6}=z^{\frac{6}{3}}=z^2 \end{matrix} \right]$

- $= \left[\begin{matrix} x\to 1,\ x>0 \\ x=(\sqrt[6]{x})^6 \end{matrix} \;\middle|\; \begin{matrix} \sqrt{x}=(\sqrt[6]{x})^{\frac{6}{2}}=(\sqrt[6]{x})^3 \\ \sqrt[3]{x}=(\sqrt[6]{x})^{\frac{6}{3}}=(\sqrt[6]{x})^2 \end{matrix} \right]$

Riešené limity – 27

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

- $= \left[\text{Príklad 26 pre } m=2,\ n=3 \right] = \lim_{x\to 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$

- $= \lim_{x\to 1} \left(\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} \right) = \lim_{x\to 1} \left(\frac{(\sqrt{x}-1)\cdot(\sqrt{x}+1)}{(\sqrt[3]{x}-1)\cdot\left[(\sqrt[3]{x})^2+\sqrt[3]{x}+1\right]} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right)$

- $= \left[\begin{array}{l|l|l} \text{Subst. } x=z^6 & x\to 1 & \sqrt{x}=\sqrt{z^6}=z^{\frac{6}{2}}=z^3 \\ & z\to 1 & \sqrt[3]{x}=\sqrt[3]{z^6}=z^{\frac{6}{3}}=z^2 \end{array} \right] = \lim_{z\to 1} \frac{z^3-1}{z^2-1}$

- $= \left[\begin{array}{r|l} x\to 1,\ x>0 & \sqrt{x}=(\sqrt[6]{x})^{\frac{6}{2}}=(\sqrt[6]{x})^3 \\ x=(\sqrt[6]{x})^6 & \sqrt[3]{x}=(\sqrt[6]{x})^{\frac{6}{3}}=(\sqrt[6]{x})^2 \end{array} \right] = \lim_{x\to 1} \frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1}$

Riešené limity – 27

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

- $= \left[\text{Príklad 26 pre } m=2,\ n=3 \right] = \lim_{x\to 1}\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m}$

- $= \lim_{x\to 1}\left(\frac{\sqrt{x}-1}{\sqrt[3]{x}-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}\cdot\frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}\right) = \lim_{x\to 1}\left(\frac{(\sqrt{x}-1)\cdot(\sqrt{x}+1)}{(\sqrt[3]{x}-1)\cdot\left[(\sqrt[3]{x})^2+\sqrt[3]{x}+1\right]}\cdot\frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1}\right)$

  $= \lim_{x\to 1}\left(\frac{x-1}{x-1}\cdot\frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1}\right)$

- $= \left[\text{Subst. } x=z^6 \;\middle|\; \begin{matrix} x\to 1 \\ z\to 1\end{matrix} \;\middle|\; \begin{matrix}\sqrt{x}=\sqrt{z^6}=z^{\frac{6}{2}}=z^3 \\ \sqrt[3]{x}=\sqrt[3]{z^6}=z^{\frac{6}{3}}=z^2\end{matrix}\right] = \lim_{z\to 1}\frac{z^3-1}{z^2-1} = \lim_{z\to 1}\frac{(z-1)(z^2+z+1)}{(z-1)(z+1)}$

- $= \left[\begin{matrix} x\to 1,\ x>0 \\ x=(\sqrt[6]{x})^6\end{matrix} \;\middle|\; \begin{matrix}\sqrt{x}=(\sqrt[6]{x})^{\frac{6}{2}}=(\sqrt[6]{x})^3 \\ \sqrt[3]{x}=(\sqrt[6]{x})^{\frac{6}{3}}=(\sqrt[6]{x})^2\end{matrix}\right] = \lim_{x\to 1}\frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1} = \lim_{x\to 1}\frac{(\sqrt[6]{x}-1)\cdot[(\sqrt[6]{x})^2+\sqrt[6]{x}+1]}{(\sqrt[6]{x}-1)\cdot(\sqrt[6]{x}+1)}$

Riešené limity – 27

$\lim\limits_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$

- $= \left[\text{Príklad 26 pre } m=2,\ n=3 \right] = \lim\limits_{x\to 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m}$

- $= \lim\limits_{x\to 1} \left(\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} \right) = \lim\limits_{x\to 1} \left(\frac{(\sqrt{x}-1)\cdot(\sqrt{x}+1)}{(\sqrt[3]{x}-1)\cdot\left[(\sqrt[3]{x})^2+\sqrt[3]{x}+1\right]} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right)$

  $= \lim\limits_{x\to 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right) = \lim\limits_{x\to 1} \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1}$

- $= \left[\begin{array}{l|l|l} \text{Subst. } x=z^6 & x\to 1 & \sqrt{x}=\sqrt{z^6}=z^{\frac{6}{2}}=z^3 \\ & z\to 1 & \sqrt[3]{x}=\sqrt[3]{z^6}=z^{\frac{6}{3}}=z^2 \end{array} \right] = \lim\limits_{z\to 1} \frac{z^3-1}{z^2-1} = \lim\limits_{z\to 1} \frac{(z-1)(z^2+z+1)}{(z-1)(z+1)} = \lim\limits_{z\to 1} \frac{z^2+z+1}{z+1}$

- $= \left[\begin{array}{r|l} x\to 1,\ x>0 & \sqrt{x}=(\sqrt[6]{x})^{\frac{6}{2}}=(\sqrt[6]{x})^3 \\ x=(\sqrt[6]{x})^6 & \sqrt[3]{x}=(\sqrt[6]{x})^{\frac{6}{3}}=(\sqrt[6]{x})^2 \end{array} \right] = \lim\limits_{x\to 1} \frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1} = \lim\limits_{x\to 1} \frac{(\sqrt[6]{x}-1)\cdot[(\sqrt[6]{x})^2+\sqrt[6]{x}+1]}{(\sqrt[6]{x}-1)\cdot(\sqrt[6]{x}+1)}$

  $= \lim\limits_{x\to 1} \frac{(\sqrt[6]{x})^2+\sqrt[6]{x}+1}{\sqrt[6]{x}+1}$

Riešené limity – 27

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

- $= \left[\text{Príklad 26 pre } m=2,\ n=3\right] = \lim_{x\to 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m}$

- $= \lim_{x\to 1} \left(\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} \right) = \lim_{x\to 1} \left(\frac{(\sqrt{x}-1)\cdot(\sqrt{x}+1)}{(\sqrt[3]{x}-1)\cdot\left[(\sqrt[3]{x})^2+\sqrt[3]{x}+1\right]} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right)$

  $= \lim_{x\to 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right) = \lim_{x\to 1} \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} = \frac{1+1+1}{1+1}$

- $= \left[\text{Subst. } x=z^6 \;\middle|\; \begin{matrix} x\to 1 \\ z\to 1 \end{matrix} \;\middle|\; \begin{matrix} \sqrt{x}=\sqrt{z^6}=z^{\frac{6}{2}}=z^3 \\ \sqrt[3]{x}=\sqrt[3]{z^6}=z^{\frac{6}{3}}=z^2 \end{matrix} \right] = \lim_{z\to 1} \frac{z^3-1}{z^2-1} = \lim_{z\to 1} \frac{(z-1)(z^2+z+1)}{(z-1)(z+1)} = \lim_{z\to 1} \frac{z^2+z+1}{z+1}$

  $= \frac{1+1+1}{1+1}$

- $= \left[\begin{matrix} x\to 1,\ x>0 \\ x=(\sqrt[6]{x})^6 \end{matrix} \;\middle|\; \begin{matrix} \sqrt{x}=(\sqrt[6]{x})^{\frac{6}{2}}=(\sqrt[6]{x})^3 \\ \sqrt[3]{x}=(\sqrt[6]{x})^{\frac{6}{3}}=(\sqrt[6]{x})^2 \end{matrix} \right] = \lim_{x\to 1} \frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1} = \lim_{x\to 1} \frac{(\sqrt[6]{x}-1)\cdot[(\sqrt[6]{x})^2+\sqrt[6]{x}+1]}{(\sqrt[6]{x}-1)\cdot(\sqrt[6]{x}+1)}$

  $= \lim_{x\to 1} \frac{(\sqrt[6]{x})^2+\sqrt[6]{x}+1}{\sqrt[6]{x}+1} = \frac{1+1+1}{1+1}$

Riešené limity – 27

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1} = \frac{3}{2}$$

- $= \Big[\text{Príklad 26 pre } m=2,\ n=3\Big] = \lim_{x\to 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m} = \frac{3}{2}.$

- $= \lim_{x\to 1} \left(\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} \right) = \lim_{x\to 1} \left(\frac{(\sqrt{x}-1)\cdot(\sqrt{x}+1)}{(\sqrt[3]{x}-1)\cdot\big[(\sqrt[3]{x})^2+\sqrt[3]{x}+1\big]} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right)$

  $= \lim_{x\to 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right) = \lim_{x\to 1} \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} = \frac{1+1+1}{1+1} = \frac{3}{2}.$

- $= \left[\text{Subst. } x = z^6 \;\middle|\; \begin{matrix} x\to 1 \\ z\to 1 \end{matrix} \;\middle|\; \begin{matrix} \sqrt{x} = \sqrt{z^6} = z^{\frac{6}{2}} = z^3 \\ \sqrt[3]{x} = \sqrt[3]{z^6} = z^{\frac{6}{3}} = z^2 \end{matrix} \right] = \lim_{z\to 1} \frac{z^3-1}{z^2-1} = \lim_{z\to 1} \frac{(z-1)(z^2+z+1)}{(z-1)(z+1)} = \lim_{z\to 1} \frac{z^2+z+1}{z+1}$

  $= \frac{1+1+1}{1+1} = \frac{3}{2}.$

- $= \left[\begin{matrix} x\to 1,\ x>0 \\ x = (\sqrt[6]{x})^6 \end{matrix} \;\middle|\; \begin{matrix} \sqrt{x} = (\sqrt[6]{x})^{\frac{6}{2}} = (\sqrt[6]{x})^3 \\ \sqrt[3]{x} = (\sqrt[6]{x})^{\frac{6}{3}} = (\sqrt[6]{x})^2 \end{matrix} \right] = \lim_{x\to 1} \frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1} = \lim_{x\to 1} \frac{(\sqrt[6]{x}-1)\cdot[(\sqrt[6]{x})^2+\sqrt[6]{x}+1]}{(\sqrt[6]{x}-1)\cdot(\sqrt[6]{x}+1)}$

  $= \lim_{x\to 1} \frac{(\sqrt[6]{x})^2+\sqrt[6]{x}+1}{\sqrt[6]{x}+1} = \frac{1+1+1}{1+1} = \frac{3}{2}.$

Riešené limity – 28, 29

$$\lim_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} \quad \text{pre } m \in R$$

$$\lim_{x\to 0} \frac{\sin 5x}{x}$$

Riešené limity – 28, 29

$\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x}$ pre $m \in R$

$m = 0. \Rightarrow$

$m \neq 0. \Rightarrow$

$\lim\limits_{x\to 0} \frac{\sin 5x}{x}$

- $= \left[\begin{array}{r|l} \text{Subst. } z = 5x & x \to 0 \\ x = \frac{z}{5} & z \to 0 \end{array} \right]$

- $= \lim\limits_{x\to 0} \frac{5 \cdot \sin 5x}{5x}$

Riešené limity – 28, 29

$\lim\limits_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}$ pre $m\in R$

$m=0. \Rightarrow$ • $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+0\cdot x}-1}{x}$

$m\neq 0. \Rightarrow$ • $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}$

$\lim\limits_{x\to 0}\frac{\sin 5x}{x}$

• $= \left[\begin{array}{r|l} \text{Subst. } z=5x & x\to 0 \\ x=\frac{z}{5} & z\to 0 \end{array}\right] = \lim\limits_{z\to 0}\frac{\sin z}{\frac{z}{5}}$

• $= \lim\limits_{x\to 0}\frac{5\cdot\sin 5x}{5x} = \left[\begin{array}{r|l} \text{Subst. } z=5x & x\to 0 \\ & z\to 0 \end{array}\right]$

Riešené limity – 28, 29

$$\lim_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} \quad \text{pre } m\in R$$

$m = 0. \Rightarrow$ • $\displaystyle\lim_{x\to 0} \frac{\sqrt[3]{1+0\cdot x}-1}{x} = \lim_{x\to 0} \frac{\sqrt[3]{1}-1}{x}$

$m \neq 0. \Rightarrow$ • $\displaystyle\lim_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} = \left[\text{Subst. } z^3 = 1+mx \;\middle|\; \begin{matrix} x\to 0 \\ z\to 1 \end{matrix} \;\middle|\; \begin{matrix} mx = z^3-1 \\ x = \frac{z^3-1}{m} \end{matrix} \right]$

$$\lim_{x\to 0} \frac{\sin 5x}{x}$$

• $\displaystyle = \left[\begin{matrix} \text{Subst. } z = 5x \\ x = \frac{z}{5} \end{matrix} \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix} \right] = \lim_{z\to 0} \frac{\sin z}{\frac{z}{5}} = 5\cdot \lim_{z\to 0} \frac{\sin z}{z}$

• $\displaystyle = \lim_{x\to 0} \frac{5\cdot \sin 5x}{5x} = \left[\text{Subst. } z = 5x \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix} \right] = 5\cdot \lim_{z\to 0} \frac{\sin z}{z}$

Riešené limity – 28, 29

$\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x}$ pre $m\in R$

$m=0. \Rightarrow$ • $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+0\cdot x}-1}{x} = \lim\limits_{x\to 0} \frac{\sqrt[3]{1}-1}{x} = \lim\limits_{x\to 0} 0$

$m\neq 0. \Rightarrow$ • $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} = \left[\text{Subst. } z^3=1+mx \;\middle|\; \begin{matrix} x\to 0 \\ z\to 1 \end{matrix} \;\middle|\; \begin{matrix} mx=z^3-1 \\ x=\frac{z^3-1}{m} \end{matrix} \right]$

$= \lim\limits_{z\to 1} \frac{z-1}{\frac{z^3-1}{m}}$

$\lim\limits_{x\to 0} \frac{\sin 5x}{x}$

• $= \left[\begin{matrix} \text{Subst. } z=5x \\ x=\frac{z}{5} \end{matrix} \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix} \right] = \lim\limits_{z\to 0} \frac{\sin z}{\frac{z}{5}} = 5\cdot \lim\limits_{z\to 0} \frac{\sin z}{z} = 5\cdot 1$

• $= \lim\limits_{x\to 0} \frac{5\cdot \sin 5x}{5x} = \left[\text{Subst. } z=5x \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix} \right] = 5\cdot \lim\limits_{z\to 0} \frac{\sin z}{z} = 5\cdot 1$

Riešené limity – 28, 29

$$\lim_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x} \quad \text{pre } m\in R$$

$m=0. \Rightarrow$ • $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+0\cdot x}-1}{x}=\lim\limits_{x\to 0}\frac{\sqrt[3]{1}-1}{x}=\lim\limits_{x\to 0}0=0$

$m\neq 0. \Rightarrow$ • $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}=\left[\text{Subst. } z^3=1+mx \;\middle|\; \begin{matrix} x\to 0 \\ z\to 1\end{matrix} \;\middle|\; \begin{matrix} mx=z^3-1 \\ x=\frac{z^3-1}{m}\end{matrix}\right]$

$$=\lim_{z\to 1}\frac{z-1}{\frac{z^3-1}{m}}=\lim_{z\to 1}\frac{m(z-1)}{(z-1)(z^2+z+1)}$$

$$\lim_{x\to 0}\frac{\sin 5x}{x}=5$$

• $=\left[\text{Subst. } z=5x, \; x=\frac{z}{5} \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0\end{matrix}\right]=\lim\limits_{z\to 0}\frac{\sin z}{\frac{z}{5}}=5\cdot\lim\limits_{z\to 0}\frac{\sin z}{z}=5\cdot 1=5.$

• $=\lim\limits_{x\to 0}\frac{5\cdot\sin 5x}{5x}=\left[\text{Subst. } z=5x \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0\end{matrix}\right]=5\cdot\lim\limits_{z\to 0}\frac{\sin z}{z}=5\cdot 1=5.$

Riešené limity – 28, 29

$\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x}$ pre $m \in R$

$m = 0. \Rightarrow$ • $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+0\cdot x}-1}{x} = \lim\limits_{x\to 0} \frac{\sqrt[3]{1}-1}{x} = \lim\limits_{x\to 0} 0 = 0$

$m \neq 0. \Rightarrow$ • $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} = \left[\text{Subst. } z^3 = 1+mx \;\middle|\; \begin{matrix} x\to 0 \\ z\to 1 \end{matrix} \;\middle|\; \begin{matrix} mx = z^3-1 \\ x = \frac{z^3-1}{m} \end{matrix} \right]$

$$= \lim_{z\to 1} \frac{z-1}{\frac{z^3-1}{m}} = \lim_{z\to 1} \frac{m(z-1)}{(z-1)(z^2+z+1)} = \lim_{z\to 1} \frac{m}{z^2+z+1}$$

$\lim\limits_{x\to 0} \frac{\sin 5x}{x} = 5$

• $= \left[\begin{matrix} \text{Subst. } z = 5x \\ x = \frac{z}{5} \end{matrix} \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix} \right] = \lim\limits_{z\to 0} \frac{\sin z}{\frac{z}{5}} = 5 \cdot \lim\limits_{z\to 0} \frac{\sin z}{z} = 5 \cdot 1 = 5.$

• $= \lim\limits_{x\to 0} \frac{5\cdot \sin 5x}{5x} = \left[\text{Subst. } z = 5x \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix} \right] = 5 \cdot \lim\limits_{z\to 0} \frac{\sin z}{z} = 5 \cdot 1 = 5.$

Riešené limity – 28, 29

$\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x}$ pre $m \in R$

$m = 0. \Rightarrow$ • $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+0\cdot x}-1}{x} = \lim\limits_{x\to 0} \frac{\sqrt[3]{1}-1}{x} = \lim\limits_{x\to 0} 0 = 0$

$m \neq 0. \Rightarrow$ • $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} = \left[\text{Subst. } z^3 = 1+mx \;\middle|\; \begin{matrix} x\to 0 \\ z\to 1 \end{matrix} \;\middle|\; \begin{matrix} mx = z^3-1 \\ x = \frac{z^3-1}{m} \end{matrix} \right]$

$= \lim\limits_{z\to 1} \frac{z-1}{\frac{z^3-1}{m}} = \lim\limits_{z\to 1} \frac{m(z-1)}{(z-1)(z^2+z+1)} = \lim\limits_{z\to 1} \frac{m}{z^2+z+1} = \frac{m}{1^2+1+1}$

$\lim\limits_{x\to 0} \frac{\sin 5x}{x} = 5$

• $= \left[\begin{matrix} \text{Subst. } z = 5x \\ x = \frac{z}{5} \end{matrix} \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix} \right] = \lim\limits_{z\to 0} \frac{\sin z}{\frac{z}{5}} = 5 \cdot \lim\limits_{z\to 0} \frac{\sin z}{z} = 5\cdot 1 = 5.$

• $= \lim\limits_{x\to 0} \frac{5\cdot \sin 5x}{5x} = \left[\text{Subst. } z = 5x \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix} \right] = 5 \cdot \lim\limits_{z\to 0} \frac{\sin z}{z} = 5\cdot 1 = 5.$

Riešené limity – 28, 29

$$\lim_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}=\frac{m}{3} \text{ pre } m\in R$$

$m=0. \Rightarrow$ • $\lim_{x\to 0}\frac{\sqrt[3]{1+0\cdot x}-1}{x}=\lim_{x\to 0}\frac{\sqrt[3]{1}-1}{x}=\lim_{x\to 0}0=0=\frac{0}{3}.$

$m\neq 0. \Rightarrow$ • $\lim_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}=\left[\text{Subst. } z^3=1+mx \;\middle|\; \begin{matrix} x\to 0 \\ z\to 1\end{matrix} \;\middle|\; \begin{matrix} mx=z^3-1 \\ x=\frac{z^3-1}{m}\end{matrix}\right]$

$$=\lim_{z\to 1}\frac{z-1}{\frac{z^3-1}{m}}=\lim_{z\to 1}\frac{m(z-1)}{(z-1)(z^2+z+1)}=\lim_{z\to 1}\frac{m}{z^2+z+1}=\frac{m}{1^2+1+1}=\frac{m}{3}.$$

$$\lim_{x\to 0}\frac{\sin 5x}{x}=5$$

• $=\left[\begin{matrix}\text{Subst. } z=5x \\ x=\frac{z}{5}\end{matrix} \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0\end{matrix}\right]=\lim_{z\to 0}\frac{\sin z}{\frac{z}{5}}=5\cdot\lim_{z\to 0}\frac{\sin z}{z}=5\cdot 1=5.$

• $=\lim_{x\to 0}\frac{5\cdot\sin 5x}{5x}=\left[\text{Subst. } z=5x \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0\end{matrix}\right]=5\cdot\lim_{z\to 0}\frac{\sin z}{z}=5\cdot 1=5.$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{x}$$

$$\lim_{x\to 0} \frac{\operatorname{arctg} x}{x}$$

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{\sin 2x}$$

$$\lim_{x\to \pi} \frac{\operatorname{tg} x}{\sin 2x}$$

Riešené limity – 30, 31, 32, 33

$\lim\limits_{x\to 0} \frac{\operatorname{tg} x}{x}$

- $= \lim\limits_{x\to 0} \frac{\frac{\sin x}{\cos x}}{x}$

$\lim\limits_{x\to 0} \frac{\operatorname{arctg} x}{x}$

- $= \left[\begin{array}{r|l} \text{Subst. } z = \operatorname{arctg} x & z \to 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} & x \to 0 \end{array} \right]$

$\lim\limits_{x\to 0} \frac{\operatorname{tg} x}{\sin 2x}$

- $= \lim\limits_{x\to 0} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cdot \cos x}$

$\lim\limits_{x\to \pi} \frac{\operatorname{tg} x}{\sin 2x}$

- $= \lim\limits_{x\to \pi} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cos x}$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x}$$

- $= \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right)$

$$\lim_{x\to 0}\frac{\operatorname{arctg} x}{x}$$

- $= \left[\begin{array}{c|c} \text{Subst. } z = \operatorname{arctg} x & z\to 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} & x\to 0 \end{array}\right] = \lim_{z\to 0}\frac{z}{\frac{\sin z}{\cos z}}$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{\sin 2x}$$

- $= \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{2\sin x\cdot\cos x} = \lim_{x\to 0}\frac{1}{2\cos^2 x}$

$$\lim_{x\to \pi}\frac{\operatorname{tg} x}{\sin 2x}$$

- $= \lim_{x\to \pi}\frac{\frac{\sin x}{\cos x}}{2\sin x\cos x} = \lim_{x\to \pi}\frac{1}{2\cos^2 x}$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{x} = 1$$

- $= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0} \left(\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right) = 1 \cdot \frac{1}{1} = 1.$

$$\lim_{x\to 0} \frac{\operatorname{arctg} x}{x}$$

- $= \left[\begin{array}{c|c} \text{Subst. } z = \operatorname{arctg} x & z \to 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} & x \to 0 \end{array} \right] = \lim_{z\to 0} \frac{z}{\frac{\sin z}{\cos z}} = \lim_{z\to 0} \left(\frac{z}{\sin z} \cdot \cos z \right)$

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{\sin 2x}$$

- $= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{2\sin x \cdot \cos x} = \lim_{x\to 0} \frac{1}{2\cos^2 x} = \frac{1}{2\cdot 1^2}$

$$\lim_{x\to \pi} \frac{\operatorname{tg} x}{\sin 2x}$$

- $= \lim_{x\to \pi} \frac{\frac{\sin x}{\cos x}}{2\sin x \cos x} = \lim_{x\to \pi} \frac{1}{2\cos^2 x} = \frac{1}{2\cdot(-1)^2}$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x} = 1$$

- $$= \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right) = 1\cdot\frac{1}{1} = 1.$$

$$\lim_{x\to 0}\frac{\operatorname{arctg} x}{x}$$

- $$= \left[\begin{array}{c|c} \text{Subst. } z = \operatorname{arctg} x & z\to 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} & x \to 0 \end{array}\right] = \lim_{z\to 0}\frac{z}{\frac{\sin z}{\cos z}} = \lim_{z\to 0}\left(\frac{z}{\sin z}\cdot\cos z\right) = 1\cdot 1$$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{\sin 2x} = \frac{1}{2}$$

- $$= \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{2\sin x\cdot\cos x} = \lim_{x\to 0}\frac{1}{2\cos^2 x} = \frac{1}{2\cdot 1^2} = \frac{1}{2}.$$

$$\lim_{x\to \pi}\frac{\operatorname{tg} x}{\sin 2x} = \frac{1}{2}$$

- $$= \lim_{x\to \pi}\frac{\frac{\sin x}{\cos x}}{2\sin x\cos x} = \lim_{x\to \pi}\frac{1}{2\cos^2 x} = \frac{1}{2\cdot(-1)^2} = \frac{1}{2}.$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{x} = 1$$

- $= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0} \left(\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right) = 1 \cdot \frac{1}{1} = 1.$

$$\lim_{x\to 0} \frac{\operatorname{arctg} x}{x} = 1$$

- $= \left[\begin{array}{r|l} \text{Subst. } z = \operatorname{arctg} x & z \to 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} & x \to 0 \end{array} \right] = \lim_{z\to 0} \frac{z}{\frac{\sin z}{\cos z}} = \lim_{z\to 0} \left(\frac{z}{\sin z} \cdot \cos z \right) = 1 \cdot 1 = 1.$

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{\sin 2x} = \frac{1}{2}$$

- $= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cdot \cos x} = \lim_{x\to 0} \frac{1}{2\cos^2 x} = \frac{1}{2 \cdot 1^2} = \frac{1}{2}.$

$$\lim_{x\to \pi} \frac{\operatorname{tg} x}{\sin 2x} = \frac{1}{2}$$

- $= \lim_{x\to \pi} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cos x} = \lim_{x\to \pi} \frac{1}{2\cos^2 x} = \frac{1}{2 \cdot (-1)^2} = \frac{1}{2}.$

Riešené limity – 34, 35

$\lim_{x \to 0} \frac{\sin mx}{\sin nx}$ pre $m, n \in Z - \{0\}$

$\lim_{x \to \pi} \frac{\sin mx}{\sin nx}$ pre $m, n \in Z - \{0\}$

Riešené limity – 34, 35

$\lim\limits_{x\to 0} \dfrac{\sin mx}{\sin nx}$ pre $m,n \in Z - \{0\}$

- $= \lim\limits_{x\to 0} \left(\dfrac{\sin mx}{mx} \cdot \dfrac{mx}{nx} \cdot \dfrac{nx}{\sin nx} \right)$

$\lim\limits_{x\to \pi} \dfrac{\sin mx}{\sin nx}$ pre $m,n \in Z - \{0\}$

- $= \left[\text{Subst. } \begin{array}{r|l} x = \pi + z & x \to \pi \\ z = \pi - x & z \to 0 \end{array} \right]$

Riešené limity – 34, 35

$\lim_{x\to 0} \frac{\sin mx}{\sin nx}$ pre $m,n \in Z - \{0\}$

- $= \lim_{x\to 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right) = \lim_{x\to 0} \frac{mx}{nx} \cdot \lim_{x\to 0} \frac{\sin mx}{mx} \cdot \lim_{x\to 0} \frac{nx}{\sin nx}$

$\lim_{x\to \pi} \frac{\sin mx}{\sin nx}$ pre $m,n \in Z - \{0\}$

- $= \left[\text{Subst. } \begin{array}{r|l} x = \pi + z & x \to \pi \\ z = \pi - x & z \to 0 \end{array} \right] = \lim_{z\to 0} \frac{\sin m(\pi+z)}{\sin n(\pi+z)}$

Riešené limity – 34, 35

$\lim\limits_{x\to 0} \dfrac{\sin mx}{\sin nx}$ pre $m,n \in Z - \{0\}$

- $= \lim\limits_{x\to 0} \left(\dfrac{\sin mx}{mx} \cdot \dfrac{mx}{nx} \cdot \dfrac{nx}{\sin nx} \right) = \lim\limits_{x\to 0} \dfrac{mx}{nx} \cdot \lim\limits_{x\to 0} \dfrac{\sin mx}{mx} \cdot \lim\limits_{x\to 0} \dfrac{nx}{\sin nx}$

  $= \left[\text{Subst. } u = mx \;\middle|\; \begin{matrix} x \to 0 \\ u \to 0 \end{matrix} \right] \left[\text{Subst. } v = nx \;\middle|\; \begin{matrix} x \to 0 \\ v \to 0 \end{matrix} \right]$

$\lim\limits_{x\to \pi} \dfrac{\sin mx}{\sin nx}$ pre $m,n \in Z - \{0\}$

- $= \left[\begin{matrix} \text{Subst. } x = \pi + z \\ z = \pi - x \end{matrix} \;\middle|\; \begin{matrix} x \to \pi \\ z \to 0 \end{matrix} \right] = \lim\limits_{z\to 0} \dfrac{\sin m(\pi+z)}{\sin n(\pi+z)} = \lim\limits_{z\to 0} \dfrac{\sin(m\pi+mz)}{\sin(n\pi+nz)}$

Riešené limity – 34, 35

$\lim\limits_{x\to 0} \frac{\sin mx}{\sin nx}$ pre $m,n \in Z - \{0\}$

- $= \lim\limits_{x\to 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right) = \lim\limits_{x\to 0} \frac{mx}{nx} \cdot \lim\limits_{x\to 0} \frac{\sin mx}{mx} \cdot \lim\limits_{x\to 0} \frac{nx}{\sin nx}$

$$= \left[\text{Subst. } u = mx \,\middle|\, \begin{matrix} x \to 0 \\ u \to 0 \end{matrix} \right] \left[\text{Subst. } v = nx \,\middle|\, \begin{matrix} x \to 0 \\ v \to 0 \end{matrix} \right] = \frac{m}{n} \cdot \lim_{u\to 0} \frac{\sin u}{u} \cdot \lim_{v\to 0} \frac{v}{\sin v}$$

$\lim\limits_{x\to \pi} \frac{\sin mx}{\sin nx}$ pre $m,n \in Z - \{0\}$

- $= \left[\begin{matrix} \text{Subst. } x = \pi + z \\ z = \pi - x \end{matrix} \,\middle|\, \begin{matrix} x \to \pi \\ z \to 0 \end{matrix} \right] = \lim\limits_{z\to 0} \frac{\sin m(\pi+z)}{\sin n(\pi+z)} = \lim\limits_{z\to 0} \frac{\sin(m\pi+mz)}{\sin(n\pi+nz)}$

$$= \lim_{z\to 0} \frac{\sin m\pi \cdot \cos mz + \cos m\pi \cdot \sin mz}{\sin n\pi \cdot \cos nz + \cos n\pi \cdot \sin nz}$$

Riešené limity – 34, 35

$$\lim_{x\to 0} \frac{\sin mx}{\sin nx} \quad \text{pre } m,n \in Z - \{0\}$$

- $= \lim_{x\to 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right) = \lim_{x\to 0} \frac{mx}{nx} \cdot \lim_{x\to 0} \frac{\sin mx}{mx} \cdot \lim_{x\to 0} \frac{nx}{\sin nx}$

$= \left[\text{Subst. } u = mx \;\middle|\; \begin{matrix} x \to 0 \\ u \to 0 \end{matrix} \right] \left[\text{Subst. } v = nx \;\middle|\; \begin{matrix} x \to 0 \\ v \to 0 \end{matrix} \right] = \frac{m}{n} \cdot \lim_{u\to 0} \frac{\sin u}{u} \cdot \lim_{v\to 0} \frac{v}{\sin v} = \frac{m}{n} \cdot 1 \cdot 1$

$$\lim_{x\to \pi} \frac{\sin mx}{\sin nx} \quad \text{pre } m,n \in Z - \{0\}$$

- $= \left[\begin{matrix} \text{Subst. } x = \pi + z \\ z = \pi - x \end{matrix} \;\middle|\; \begin{matrix} x \to \pi \\ z \to 0 \end{matrix} \right] = \lim_{z\to 0} \frac{\sin m(\pi+z)}{\sin n(\pi+z)} = \lim_{z\to 0} \frac{\sin(m\pi+mz)}{\sin(n\pi+nz)}$

$= \lim_{z\to 0} \frac{\sin m\pi \cdot \cos mz + \cos m\pi \cdot \sin mz}{\sin n\pi \cdot \cos nz + \cos n\pi \cdot \sin nz} = \lim_{z\to 0} \frac{0 \cdot \cos mz + (-1)^m \cdot \sin mz}{0 \cdot \cos nz + (-1)^n \cdot \sin nz}$

Riešené limity – 34, 35

$$\lim_{x\to 0} \frac{\sin mx}{\sin nx} = \frac{m}{n} \quad \text{pre} \quad m,n \in Z - \{0\}$$

- $= \lim_{x\to 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right) = \lim_{x\to 0} \frac{mx}{nx} \cdot \lim_{x\to 0} \frac{\sin mx}{mx} \cdot \lim_{x\to 0} \frac{nx}{\sin nx}$

$= \left[\text{Subst. } u = mx \;\middle|\; \begin{matrix} x \to 0 \\ u \to 0 \end{matrix} \right] \left[\text{Subst. } v = nx \;\middle|\; \begin{matrix} x \to 0 \\ v \to 0 \end{matrix} \right] = \frac{m}{n} \cdot \lim_{u\to 0} \frac{\sin u}{u} \cdot \lim_{v\to 0} \frac{v}{\sin v} = \frac{m}{n} \cdot 1 \cdot 1 = \frac{m}{n}.$

$$\lim_{x\to \pi} \frac{\sin mx}{\sin nx} \quad \text{pre} \quad m,n \in Z - \{0\}$$

- $= \left[\begin{matrix} \text{Subst. } x = \pi + z \\ z = \pi - x \end{matrix} \;\middle|\; \begin{matrix} x \to \pi \\ z \to 0 \end{matrix} \right] = \lim_{z\to 0} \frac{\sin m(\pi+z)}{\sin n(\pi+z)} = \lim_{z\to 0} \frac{\sin (m\pi+mz)}{\sin (n\pi+nz)}$

$= \lim_{z\to 0} \frac{\sin m\pi \cdot \cos mz + \cos m\pi \cdot \sin mz}{\sin n\pi \cdot \cos nz + \cos n\pi \cdot \sin nz} = \lim_{z\to 0} \frac{0 \cdot \cos mz + (-1)^m \cdot \sin mz}{0 \cdot \cos nz + (-1)^n \cdot \sin nz} = \lim_{z\to 0} \frac{(-1)^m \cdot \sin mz}{(-1)^n \cdot \sin nz}$

$= (-1)^{m-n} \cdot \lim_{z\to 0} \frac{\sin mz}{\sin nz}$

Riešené limity – 34, 35

$$\lim_{x\to 0} \frac{\sin mx}{\sin nx} = \frac{m}{n} \quad \text{pre } m,n \in Z - \{0\}$$

- $= \lim_{x\to 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right) = \lim_{x\to 0} \frac{mx}{nx} \cdot \lim_{x\to 0} \frac{\sin mx}{mx} \cdot \lim_{x\to 0} \frac{nx}{\sin nx}$

$= \left[\text{Subst. } u = mx \,\middle|\, \begin{matrix} x \to 0 \\ u \to 0 \end{matrix} \right] \left[\text{Subst. } v = nx \,\middle|\, \begin{matrix} x \to 0 \\ v \to 0 \end{matrix} \right] = \frac{m}{n} \cdot \lim_{u\to 0} \frac{\sin u}{u} \cdot \lim_{v\to 0} \frac{v}{\sin v} = \frac{m}{n} \cdot 1 \cdot 1 = \frac{m}{n}.$

$$\lim_{x\to \pi} \frac{\sin mx}{\sin nx} \quad \text{pre } m,n \in Z - \{0\}$$

- $= \left[\begin{matrix} \text{Subst. } x = \pi + z \\ z = \pi - x \end{matrix} \,\middle|\, \begin{matrix} x \to \pi \\ z \to 0 \end{matrix} \right] = \lim_{z\to 0} \frac{\sin m(\pi+z)}{\sin n(\pi+z)} = \lim_{z\to 0} \frac{\sin (m\pi+mz)}{\sin (n\pi+nz)}$

$= \lim_{z\to 0} \frac{\sin m\pi \cdot \cos mz + \cos m\pi \cdot \sin mz}{\sin n\pi \cdot \cos nz + \cos n\pi \cdot \sin nz} = \lim_{z\to 0} \frac{0 \cdot \cos mz + (-1)^m \cdot \sin mz}{0 \cdot \cos nz + (-1)^n \cdot \sin nz} = \lim_{z\to 0} \frac{(-1)^m \cdot \sin mz}{(-1)^n \cdot \sin nz}$

$= (-1)^{m-n} \cdot \lim_{z\to 0} \frac{\sin mz}{\sin nz} = \left[\text{Predchádzajúci príklad 34: } \lim_{z\to 0} \frac{\sin mz}{\sin nz} = \lim_{z\to 0} \left(\frac{\sin mz}{mz} \cdot \frac{mz}{nz} \cdot \frac{nz}{\sin nz} \right) = 1 \cdot \frac{m}{n} \cdot 1 = \frac{m}{n} \right]$

Riešené limity – 34, 35

$$\lim_{x\to 0}\frac{\sin mx}{\sin nx}=\frac{m}{n}\ \text{pre}\ m,n\in Z-\{0\}$$

- $= \lim_{x\to 0}\left(\frac{\sin mx}{mx}\cdot\frac{mx}{nx}\cdot\frac{nx}{\sin nx}\right)=\lim_{x\to 0}\frac{mx}{nx}\cdot\lim_{x\to 0}\frac{\sin mx}{mx}\cdot\lim_{x\to 0}\frac{nx}{\sin nx}$

$$=\left[\text{Subst. } u=mx \,\middle|\, \begin{matrix}x\to 0\\ u\to 0\end{matrix}\right]\left[\text{Subst. } v=nx \,\middle|\, \begin{matrix}x\to 0\\ v\to 0\end{matrix}\right]=\frac{m}{n}\cdot\lim_{u\to 0}\frac{\sin u}{u}\cdot\lim_{v\to 0}\frac{v}{\sin v}=\frac{m}{n}\cdot 1\cdot 1=\frac{m}{n}.$$

$$\lim_{x\to \pi}\frac{\sin mx}{\sin nx}=(-1)^{m-n}\cdot\frac{m}{n}\ \text{pre}\ m,n\in Z-\{0\}$$

- $=\left[\text{Subst. } \begin{matrix}x=\pi+z\\ z=\pi-x\end{matrix} \,\middle|\, \begin{matrix}x\to \pi\\ z\to 0\end{matrix}\right]=\lim_{z\to 0}\frac{\sin m(\pi+z)}{\sin n(\pi+z)}=\lim_{z\to 0}\frac{\sin(m\pi+mz)}{\sin(n\pi+nz)}$

$$=\lim_{z\to 0}\frac{\sin m\pi\cdot\cos mz+\cos m\pi\cdot\sin mz}{\sin n\pi\cdot\cos nz+\cos n\pi\cdot\sin nz}=\lim_{z\to 0}\frac{0\cdot\cos mz+(-1)^m\cdot\sin mz}{0\cdot\cos nz+(-1)^n\cdot\sin nz}=\lim_{z\to 0}\frac{(-1)^m\cdot\sin mz}{(-1)^n\cdot\sin nz}$$

$$=(-1)^{m-n}\cdot\lim_{z\to 0}\frac{\sin mz}{\sin nz}=\left[\text{Predchádzajúci príklad 34: } \lim_{z\to 0}\frac{\sin mz}{\sin nz}=\lim_{z\to 0}\left(\frac{\sin mz}{mz}\cdot\frac{mz}{nz}\cdot\frac{nz}{\sin nz}\right)=1\cdot\frac{m}{n}\cdot 1=\frac{m}{n}\right]$$

$$=(-1)^{m-n}\cdot\frac{m}{n}.$$

Riešené limity – 36, 37

$\lim\limits_{x\to 2\pi} \dfrac{\sin mx}{\sin nx}$ pre $m, n \in Z - \{0\}$

$\lim\limits_{x\to 0} \dfrac{1-\cos 2x}{x^2}$

Riešené limity – 36, 37

$$\lim_{x\to 2\pi} \frac{\sin mx}{\sin nx} \quad \text{pre } m,n \in Z - \{0\}$$

- $= \left[\begin{array}{r|l} \text{Subst. } x = 2\pi + z & x \to 2\pi \\ z = \pi - x & z \to 0 \end{array} \right]$

$$\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$$

- $= \lim_{x\to 0} \frac{1-(\cos^2 x - \sin^2 x)}{x^2}$

- $= \lim_{x\to 0} \left(\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x} \right)$

- $= \left[\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow 1 - \cos 2x = 2\sin^2 x \right]$

Riešené limity – 36, 37

$$\lim_{x\to 2\pi} \frac{\sin mx}{\sin nx} \quad \text{pre } m,n \in Z - \{0\}$$

- $= \left[\begin{array}{r|l} \text{Subst. } x = 2\pi + z & x \to 2\pi \\ z = \pi - x & z \to 0 \end{array} \right] = \lim\limits_{z\to 0} \frac{\sin m(2\pi+z)}{\sin n(2\pi+z)}$

$$\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$$

- $= \lim\limits_{x\to 0} \frac{1-(\cos^2 x - \sin^2 x)}{x^2} = \lim\limits_{x\to 0} \frac{\sin^2 x + \sin^2 x}{x^2}$

- $= \lim\limits_{x\to 0} \left(\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x} \right) = \lim\limits_{x\to 0} \frac{1-\cos^2 2x}{x^2(1+\cos 2x)}$

- $= \left[\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow 1 - \cos 2x = 2\sin^2 x \right] = \lim\limits_{x\to 0} \frac{2\sin^2 x}{x^2}$

Riešené limity – 36, 37

$\lim\limits_{x\to 2\pi} \frac{\sin mx}{\sin nx}$ pre $m, n \in Z - \{0\}$

- $= \left[\begin{array}{r|l} \text{Subst. } x = 2\pi + z & x \to 2\pi \\ z = \pi - x & z \to 0 \end{array} \right] = \lim\limits_{z\to 0} \frac{\sin m(2\pi + z)}{\sin n(2\pi + z)} = \lim\limits_{z\to 0} \frac{\sin(2m\pi + mz)}{\sin(2n\pi + nz)}$

$\lim\limits_{x\to 0} \frac{1-\cos 2x}{x^2}$

- $= \lim\limits_{x\to 0} \frac{1-(\cos^2 x - \sin^2 x)}{x^2} = \lim\limits_{x\to 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim\limits_{x\to 0} \frac{2\sin^2 x}{x^2}$

- $= \lim\limits_{x\to 0} \left(\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x} \right) = \lim\limits_{x\to 0} \frac{1-\cos^2 2x}{x^2(1+\cos 2x)} = \lim\limits_{x\to 0} \frac{\sin^2 2x}{x^2(1+\cos 2x)}$

- $= \left[\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow 1 - \cos 2x = 2\sin^2 x \right] = \lim\limits_{x\to 0} \frac{2\sin^2 x}{x^2} = 2 \lim\limits_{x\to 0} \left(\frac{\sin x}{x}\right)^2$

Riešené limity – 36, 37

$$\lim_{x\to 2\pi} \frac{\sin mx}{\sin nx} \quad \text{pre } m, n \in Z - \{0\}$$

- $= \left[\begin{array}{r|r} \text{Subst. } x = 2\pi + z & x \to 2\pi \\ z = \pi - x & z \to 0 \end{array} \right] = \lim_{z\to 0} \frac{\sin m(2\pi+z)}{\sin n(2\pi+z)} = \lim_{z\to 0} \frac{\sin(2m\pi+mz)}{\sin(2n\pi+nz)} =$

$$\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$$

- $= \lim_{x\to 0} \frac{1-(\cos^2 x-\sin^2 x)}{x^2} = \lim_{x\to 0} \frac{\sin^2 x+\sin^2 x}{x^2} = \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2$

- $= \lim_{x\to 0} \left(\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x}\right) = \lim_{x\to 0} \frac{1-\cos^2 2x}{x^2(1+\cos 2x)} = \lim_{x\to 0} \frac{\sin^2 2x}{x^2(1+\cos 2x)} = \lim_{x\to 0} \frac{4\sin^2 2x}{4x^2(1+\cos 2x)}$

- $= \left[\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow 1-\cos 2x = 2\sin^2 x \right] = \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = 2 \cdot 1^2$

Riešené limity – 36, 37

$\lim_{x\to 2\pi} \frac{\sin mx}{\sin nx}$ pre $m,n \in Z - \{0\}$

- $= \left[\begin{array}{r|l} \text{Subst. } x = 2\pi + z & x \to 2\pi \\ z = \pi - x & z \to 0 \end{array} \right] = \lim_{z\to 0} \frac{\sin m(2\pi+z)}{\sin n(2\pi+z)} = \lim_{z\to 0} \frac{\sin(2m\pi+mz)}{\sin(2n\pi+nz)} = \left[\begin{array}{c} \text{Funkcia sínus je periodická} \\ \text{s periódami } 2\pi,\ 2m\pi,\ 2n\pi \end{array} \right]$
  $= \lim_{z\to 0} \frac{\sin mz}{\sin nz}$

$\lim_{x\to 0} \frac{1-\cos 2x}{x^2} = 2$

- $= \lim_{x\to 0} \frac{1-(\cos^2 x-\sin^2 x)}{x^2} = \lim_{x\to 0} \frac{\sin^2 x+\sin^2 x}{x^2} = \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = 2\cdot 1^2$

- $= \lim_{x\to 0} \left(\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x}\right) = \lim_{x\to 0} \frac{1-\cos^2 2x}{x^2(1+\cos 2x)} = \lim_{x\to 0} \frac{\sin^2 2x}{x^2(1+\cos 2x)} = \lim_{x\to 0} \frac{4\sin^2 2x}{4x^2(1+\cos 2x)}$
  $= \lim_{x\to 0} \frac{4}{1+\cos 2x} \cdot \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right)^2$

- $= \left[\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow 1-\cos 2x = 2\sin^2 x \right] = \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = 2\cdot 1^2 = 2.$

Riešené limity – 36, 37

$\lim\limits_{x\to 2\pi} \frac{\sin mx}{\sin nx}$ pre $m, n \in Z - \{0\}$

- $= \left[\begin{array}{r|l} \text{Subst. } x = 2\pi + z & x \to 2\pi \\ z = \pi - x & z \to 0 \end{array} \right] = \lim\limits_{z\to 0} \frac{\sin m(2\pi + z)}{\sin n(2\pi + z)} = \lim\limits_{z\to 0} \frac{\sin(2m\pi + mz)}{\sin(2n\pi + nz)} = \left[\begin{array}{c} \text{Funkcia sínus je periodická} \\ \text{s periódami } 2\pi,\ 2m\pi,\ 2n\pi \end{array} \right]$

$= \lim\limits_{z\to 0} \frac{\sin mz}{\sin nz} = \left[\text{Predchádzajúci príklad 34: } \lim\limits_{z\to 0} \frac{\sin mz}{\sin nz} = \lim\limits_{z\to 0} \left(\frac{\sin mz}{mz} \cdot \frac{mz}{nz} \cdot \frac{nz}{\sin nz} \right) = 1 \cdot \frac{m}{n} \cdot 1 = \frac{m}{n} \right]$

$\lim\limits_{x\to 0} \frac{1 - \cos 2x}{x^2} = 2$

- $= \lim\limits_{x\to 0} \frac{1 - (\cos^2 x - \sin^2 x)}{x^2} = \lim\limits_{x\to 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim\limits_{x\to 0} \frac{2 \sin^2 x}{x^2} = 2 \lim\limits_{x\to 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2 = 2.$

- $= \lim\limits_{x\to 0} \left(\frac{1 - \cos 2x}{x^2} \cdot \frac{1 + \cos 2x}{1 + \cos 2x} \right) = \lim\limits_{x\to 0} \frac{1 - \cos^2 2x}{x^2 (1 + \cos 2x)} = \lim\limits_{x\to 0} \frac{\sin^2 2x}{x^2 (1 + \cos 2x)} = \lim\limits_{x\to 0} \frac{4 \sin^2 2x}{4x^2 (1 + \cos 2x)}$

$= \lim\limits_{x\to 0} \frac{4}{1 + \cos 2x} \cdot \lim\limits_{x\to 0} \left(\frac{\sin 2x}{2x} \right)^2 = \left[\begin{array}{r|l} \text{Subst. } z = 2x & x \to 0 \\ & z \to 0 \end{array} \right]$

- $= \left[\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow 1 - \cos 2x = 2 \sin^2 x \right] = \lim\limits_{x\to 0} \frac{2 \sin^2 x}{x^2} = 2 \lim\limits_{x\to 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2 = 2.$

Riešené limity – 36, 37

$$\lim_{x\to 2\pi} \frac{\sin mx}{\sin nx} = \frac{m}{n} \text{ pre } m,n \in Z - \{0\}$$

- $= \left[\begin{array}{r|l} \text{Subst. } x = 2\pi + z & x \to 2\pi \\ z = \pi - x & z \to 0 \end{array} \right] = \lim\limits_{z\to 0} \frac{\sin m(2\pi+z)}{\sin n(2\pi+z)} = \lim\limits_{z\to 0} \frac{\sin(2m\pi+mz)}{\sin(2n\pi+nz)} = \left[\begin{array}{c} \text{Funkcia sínus je periodická} \\ \text{s periódami } 2\pi,\ 2m\pi,\ 2n\pi \end{array} \right]$

$= \lim\limits_{z\to 0} \frac{\sin mz}{\sin nz} = \left[\text{Predchádzajúci príklad 34: } \lim\limits_{z\to 0} \frac{\sin mz}{\sin nz} = \lim\limits_{z\to 0} \left(\frac{\sin mz}{mz} \cdot \frac{mz}{nz} \cdot \frac{nz}{\sin nz} \right) = 1 \cdot \frac{m}{n} \cdot 1 = \frac{m}{n} \right] = \frac{m}{n}.$

$$\lim_{x\to 0} \frac{1-\cos 2x}{x^2} = 2$$

- $= \lim\limits_{x\to 0} \frac{1-(\cos^2 x - \sin^2 x)}{x^2} = \lim\limits_{x\to 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim\limits_{x\to 0} \frac{2\sin^2 x}{x^2} = 2 \lim\limits_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = 2 \cdot 1^2 = 2.$

- $= \lim\limits_{x\to 0} \left(\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x} \right) = \lim\limits_{x\to 0} \frac{1-\cos^2 2x}{x^2(1+\cos 2x)} = \lim\limits_{x\to 0} \frac{\sin^2 2x}{x^2(1+\cos 2x)} = \lim\limits_{x\to 0} \frac{4\sin^2 2x}{4x^2(1+\cos 2x)}$

$= \lim\limits_{x\to 0} \frac{4}{1+\cos 2x} \cdot \lim\limits_{x\to 0} \left(\frac{\sin 2x}{2x}\right)^2 = \left[\begin{array}{r|l} \text{Subst. } z = 2x & x \to 0 \\ & z \to 0 \end{array} \right] = \frac{4}{1+1} \cdot \lim\limits_{z\to 0} \left(\frac{\sin z}{z}\right)^2$

- $= \left[\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow 1 - \cos 2x = 2\sin^2 x \right] = \lim\limits_{x\to 0} \frac{2\sin^2 x}{x^2} = 2 \lim\limits_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = 2 \cdot 1^2 = 2.$

Riešené limity – 36, 37

$$\lim_{x\to 2\pi}\frac{\sin mx}{\sin nx}=\frac{m}{n}\ \text{pre}\ m,n\in Z-\{0\}$$

- $= \left[\begin{array}{r|l}\text{Subst. } x=2\pi+z & x\to 2\pi\\ z=\pi-x & z\to 0\end{array}\right] = \lim\limits_{z\to 0}\frac{\sin m(2\pi+z)}{\sin n(2\pi+z)} = \lim\limits_{z\to 0}\frac{\sin(2m\pi+mz)}{\sin(2n\pi+nz)} = \left[\begin{array}{c}\text{Funkcia sínus je periodická}\\ \text{s periódami } 2\pi,\ 2m\pi,\ 2n\pi\end{array}\right]$
  $= \lim\limits_{z\to 0}\frac{\sin mz}{\sin nz} = \left[\text{Predchádzajúci príklad 34: } \lim\limits_{z\to 0}\frac{\sin mz}{\sin nz}=\lim\limits_{z\to 0}\left(\frac{\sin mz}{mz}\cdot\frac{mz}{nz}\cdot\frac{nz}{\sin nz}\right)=1\cdot\frac{m}{n}\cdot 1=\frac{m}{n}\right] = \frac{m}{n}.$

$$\lim_{x\to 0}\frac{1-\cos 2x}{x^2}=2$$

- $= \lim\limits_{x\to 0}\frac{1-(\cos^2 x-\sin^2 x)}{x^2} = \lim\limits_{x\to 0}\frac{\sin^2 x+\sin^2 x}{x^2} = \lim\limits_{x\to 0}\frac{2\sin^2 x}{x^2} = 2\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^2 = 2\cdot 1^2 = 2.$

- $= \lim\limits_{x\to 0}\left(\frac{1-\cos 2x}{x^2}\cdot\frac{1+\cos 2x}{1+\cos 2x}\right) = \lim\limits_{x\to 0}\frac{1-\cos^2 2x}{x^2(1+\cos 2x)} = \lim\limits_{x\to 0}\frac{\sin^2 2x}{x^2(1+\cos 2x)} = \lim\limits_{x\to 0}\frac{4\sin^2 2x}{4x^2(1+\cos 2x)}$
  $= \lim\limits_{x\to 0}\frac{4}{1+\cos 2x}\cdot\lim\limits_{x\to 0}\left(\frac{\sin 2x}{2x}\right)^2 = \left[\begin{array}{l|l}\text{Subst. } z=2x & x\to 0\\ & z\to 0\end{array}\right] = \frac{4}{1+1}\cdot\lim\limits_{z\to 0}\left(\frac{\sin z}{z}\right)^2 = 2\cdot 1^2$

- $= \left[\sin^2 x=\frac{1-\cos 2x}{2}\Rightarrow 1-\cos 2x=2\sin^2 x\right] = \lim\limits_{x\to 0}\frac{2\sin^2 x}{x^2} = 2\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^2 = 2\cdot 1^2 = 2.$

Riešené limity – 36, 37

$$\lim_{x\to 2\pi}\frac{\sin mx}{\sin nx}=\frac{m}{n}\ \text{pre}\ m,n\in Z-\{0\}$$

- $= \left[\begin{array}{r|l}\text{Subst. } x=2\pi+z & x\to 2\pi\\ z=\pi-x & z\to 0\end{array}\right] = \lim\limits_{z\to 0}\dfrac{\sin m(2\pi+z)}{\sin n(2\pi+z)} = \lim\limits_{z\to 0}\dfrac{\sin(2m\pi+mz)}{\sin(2n\pi+nz)} = \left[\begin{array}{c}\text{Funkcia sínus je periodická}\\ \text{s periódami } 2\pi,\ 2m\pi,\ 2n\pi\end{array}\right]$

  $= \lim\limits_{z\to 0}\dfrac{\sin mz}{\sin nz} = \left[\text{Predchádzajúci príklad 34: } \lim\limits_{z\to 0}\frac{\sin mz}{\sin nz} = \lim\limits_{z\to 0}\left(\frac{\sin mz}{mz}\cdot\frac{mz}{nz}\cdot\frac{nz}{\sin nz}\right) = 1\cdot\frac{m}{n}\cdot 1 = \frac{m}{n}\right] = \dfrac{m}{n}.$

$$\lim_{x\to 0}\frac{1-\cos 2x}{x^2}=2$$

- $= \lim\limits_{x\to 0}\dfrac{1-(\cos^2 x-\sin^2 x)}{x^2} = \lim\limits_{x\to 0}\dfrac{\sin^2 x+\sin^2 x}{x^2} = \lim\limits_{x\to 0}\dfrac{2\sin^2 x}{x^2} = 2\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^2 = 2\cdot 1^2 = 2.$

- $= \lim\limits_{x\to 0}\left(\dfrac{1-\cos 2x}{x^2}\cdot\dfrac{1+\cos 2x}{1+\cos 2x}\right) = \lim\limits_{x\to 0}\dfrac{1-\cos^2 2x}{x^2(1+\cos 2x)} = \lim\limits_{x\to 0}\dfrac{\sin^2 2x}{x^2(1+\cos 2x)} = \lim\limits_{x\to 0}\dfrac{4\sin^2 2x}{4x^2(1+\cos 2x)}$

  $= \lim\limits_{x\to 0}\dfrac{4}{1+\cos 2x}\cdot\lim\limits_{x\to 0}\left(\dfrac{\sin 2x}{2x}\right)^2 = \left[\begin{array}{r|l}\text{Subst. } z=2x & x\to 0\\ & z\to 0\end{array}\right] = \dfrac{4}{1+1}\cdot\lim\limits_{z\to 0}\left(\dfrac{\sin z}{z}\right)^2 = 2\cdot 1^2 = 2.$

- $= \left[\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow 1-\cos 2x = 2\sin^2 x\right] = \lim\limits_{x\to 0}\dfrac{2\sin^2 x}{x^2} = 2\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^2 = 2\cdot 1^2 = 2.$

Riešené limity – 38, 39

$$\lim_{x \to \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

$$\lim_{x \to 0} \frac{\sin 4x}{\sqrt{x+1} - 1}$$

Riešené limity – 38, 39

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

- $= \lim\limits_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

- $= \lim\limits_{x\to 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right)$

- $= \lim\limits_{x\to 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1} \right)$

Riešené limity – 38, 39

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x-\sin x}$$

- $= \lim\limits_{x\to\frac{\pi}{4}} \frac{\cos^2 x-\sin^2 x}{\cos x-\sin x} = \lim\limits_{x\to\frac{\pi}{4}} \frac{(\cos x-\sin x)\cdot(\cos x+\sin x)}{\cos x-\sin x}$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

- $= \lim\limits_{x\to 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right) = \lim\limits_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1}$

- $= \lim\limits_{x\to 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1}\right) = \left[\begin{array}{c|c} \text{Subst. } z=4x & x\to 0 \\ & z\to 0 \end{array}\right]\left[\begin{array}{c|c} \text{Subst. } t=\sqrt{x+1} & x\to 0 \\ x+1=t^2 & t\to 1 \end{array}\right]$

Riešené limity – 38, 39

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

- $= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)\cdot(\cos x + \sin x)}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x)$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

- $= \lim_{x\to 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right) = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1} = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(x+1)-1}$

- $= \lim_{x\to 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1}\right) = \left[\begin{array}{l|l} \text{Subst. } z = 4x & x\to 0 \\ & z\to 0 \end{array}\right]\left[\begin{array}{r|l} \text{Subst. } t = \sqrt{x+1} & x\to 0 \\ x+1 = t^2 & t\to 1 \end{array}\right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} \frac{4(t^2-1)}{t-1}$

Riešené limity – 38, 39

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

- $= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)\cdot(\cos x + \sin x)}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x)$
  $= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

- $= \lim_{x\to 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right) = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1} = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(x+1)-1}$
  $= \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{x}$

- $= \lim_{x\to 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1}\right) = \left[\begin{array}{l|l} \text{Subst. } z = 4x & x\to 0 \\ & z \to 0 \end{array}\right]\left[\begin{array}{l|l} \text{Subst. } t = \sqrt{x+1} & x\to 0 \\ x+1 = t^2 & t\to 1 \end{array}\right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} \frac{4(t^2-1)}{t-1}$
  $= 4 \cdot \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} \frac{(t-1)\cdot(t+1)}{t-1}$

Riešené limity – 38, 39

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

- $= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)\cdot(\cos x + \sin x)}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x)$

  $= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

- $= \lim_{x\to 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right) = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1} = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(x+1)-1}$

  $= \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{x} = \lim_{x\to 0} \left(4 \cdot \frac{\sin 4x}{4x} \cdot (\sqrt{x+1}+1)\right)$

- $= \lim_{x\to 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1}\right) = \left[\text{Subst. } z = 4x \,\middle|\, \begin{array}{l} x \to 0 \\ z \to 0 \end{array}\right]\left[\begin{array}{r} \text{Subst. } t = \sqrt{x+1} \\ x+1 = t^2 \end{array} \,\middle|\, \begin{array}{l} x \to 0 \\ t \to 1 \end{array}\right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} \frac{4(t^2-1)}{t-1}$

  $= 4 \cdot \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} \frac{(t-1)\cdot(t+1)}{t-1} = 4 \cdot \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} (t+1)$

Riešené limity – 38, 39

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

- $= \lim\limits_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim\limits_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)\cdot(\cos x + \sin x)}{\cos x - \sin x} = \lim\limits_{x\to\frac{\pi}{4}} (\cos x + \sin x)$
  $= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

- $= \lim\limits_{x\to 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right) = \lim\limits_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1} = \lim\limits_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(x+1)-1}$
  $= \lim\limits_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{x} = \lim\limits_{x\to 0} \left(4 \cdot \frac{\sin 4x}{4x} \cdot \left(\sqrt{x+1}+1\right)\right) = \left[\text{Subst. } z = 4x \;\middle|\; \begin{matrix} x\to 0 \\ z \to 0 \end{matrix} \right]$

- $= \lim\limits_{x\to 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1} \right) = \left[\text{Subst. } z = 4x \;\middle|\; \begin{matrix} x\to 0 \\ z \to 0 \end{matrix} \right] \left[\begin{matrix} \text{Subst. } t = \sqrt{x+1} \\ x+1 = t^2 \end{matrix} \;\middle|\; \begin{matrix} x\to 0 \\ t \to 1 \end{matrix} \right] = \lim\limits_{z\to 0} \frac{\sin z}{z} \cdot \lim\limits_{t\to 1} \frac{4(t^2-1)}{t-1}$
  $= 4 \cdot \lim\limits_{z\to 0} \frac{\sin z}{z} \cdot \lim\limits_{t\to 1} \frac{(t-1)\cdot(t+1)}{t-1} = 4 \cdot \lim\limits_{z\to 0} \frac{\sin z}{z} \cdot \lim\limits_{t\to 1} (t+1) = 4 \cdot 1 \cdot (1+1)$

Riešené limity – 38, 39

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

- $= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)\cdot(\cos x + \sin x)}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x)$
  $= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1} = 8$$

- $= \lim_{x\to 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right) = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1} = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(x+1)-1}$
  $= \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{x} = \lim_{x\to 0} \left(4 \cdot \frac{\sin 4x}{4x} \cdot (\sqrt{x+1}+1)\right) = \left[\text{Subst. } z = 4x \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix}\right]$
  $= 4 \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to 0} (\sqrt{x+1}+1)$

- $= \lim_{x\to 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1}\right) = \left[\text{Subst. } z = 4x \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix}\right] \left[\text{Subst. } t = \sqrt{x+1},\ x+1 = t^2 \;\middle|\; \begin{matrix} x\to 0 \\ t\to 1 \end{matrix}\right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} \frac{4(t^2-1)}{t-1}$
  $= 4 \cdot \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} \frac{(t-1)\cdot(t+1)}{t-1} = 4 \cdot \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} (t+1) = 4\cdot 1\cdot(1+1) = 8.$

Riešené limity – 38, 39

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

- $= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)\cdot(\cos x + \sin x)}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x)$
  $= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1} = 8$$

- $= \lim_{x\to 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right) = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1} = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(x+1)-1}$
  $= \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{x} = \lim_{x\to 0} \left(4 \cdot \frac{\sin 4x}{4x} \cdot (\sqrt{x+1}+1)\right) = \left[\text{Subst. } z = 4x \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix}\right]$
  $= 4 \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to 0} (\sqrt{x+1}+1) = 4\cdot 1\cdot(\sqrt{0+1}+1)$

- $= \lim_{x\to 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1}\right) = \left[\text{Subst. } z = 4x \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix}\right] \left[\begin{matrix}\text{Subst. } t = \sqrt{x+1} \\ x+1 = t^2\end{matrix} \;\middle|\; \begin{matrix} x\to 0 \\ t\to 1 \end{matrix}\right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} \frac{4(t^2-1)}{t-1}$
  $= 4 \cdot \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} \frac{(t-1)\cdot(t+1)}{t-1} = 4 \cdot \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} (t+1) = 4\cdot 1\cdot(1+1) = 8.$

Riešené limity – 38, 39

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x-\sin x} = \sqrt{2}$$

- $= \lim\limits_{x\to\frac{\pi}{4}} \frac{\cos^2 x-\sin^2 x}{\cos x-\sin x} = \lim\limits_{x\to\frac{\pi}{4}} \frac{(\cos x-\sin x)\cdot(\cos x+\sin x)}{\cos x-\sin x} = \lim\limits_{x\to\frac{\pi}{4}} (\cos x + \sin x)$
  $= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1} = 8$$

- $= \lim\limits_{x\to 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right) = \lim\limits_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1} = \lim\limits_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(x+1)-1}$
  $= \lim\limits_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{x} = \lim\limits_{x\to 0} \left(4 \cdot \frac{\sin 4x}{4x} \cdot (\sqrt{x+1} + 1)\right) = \left[\text{Subst. } z = 4x \;\middle|\; \begin{matrix} x\to 0 \\ z \to 0 \end{matrix}\right]$
  $= 4 \lim\limits_{z\to 0} \frac{\sin z}{z} \cdot \lim\limits_{x\to 0} (\sqrt{x+1} + 1) = 4 \cdot 1 \cdot (\sqrt{0+1} + 1) = 4 \cdot 1 \cdot 2$

- $= \lim\limits_{x\to 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1}\right) = \left[\text{Subst. } z = 4x \;\middle|\; \begin{matrix} x\to 0 \\ z \to 0 \end{matrix}\right]\left[\begin{matrix}\text{Subst. } t = \sqrt{x+1} \\ x+1 = t^2\end{matrix} \;\middle|\; \begin{matrix} x\to 0 \\ t \to 1 \end{matrix}\right] = \lim\limits_{z\to 0} \frac{\sin z}{z} \cdot \lim\limits_{t\to 1} \frac{4(t^2-1)}{t-1}$
  $= 4 \cdot \lim\limits_{z\to 0} \frac{\sin z}{z} \cdot \lim\limits_{t\to 1} \frac{(t-1)\cdot(t+1)}{t-1} = 4 \cdot \lim\limits_{z\to 0} \frac{\sin z}{z} \cdot \lim\limits_{t\to 1} (t+1) = 4 \cdot 1 \cdot (1+1) = 8.$

Riešené limity – 38, 39

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

- $$= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)\cdot(\cos x + \sin x)}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x)$$
$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1} = 8$$

- $$= \lim_{x\to 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right) = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1} = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(x+1)-1}$$
$$= \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{x} = \lim_{x\to 0} \left(4 \cdot \frac{\sin 4x}{4x} \cdot (\sqrt{x+1}+1)\right) = \left[\text{Subst. } z = 4x \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix}\right]$$
$$= 4 \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to 0} (\sqrt{x+1}+1) = 4\cdot 1\cdot(\sqrt{0+1}+1) = 4\cdot 1\cdot 2 = 8.$$

- $$= \lim_{x\to 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1}\right) = \left[\text{Subst. } z = 4x \;\middle|\; \begin{matrix} x\to 0 \\ z\to 0 \end{matrix}\right]\left[\begin{matrix}\text{Subst. } t = \sqrt{x+1} \\ x+1 = t^2\end{matrix} \;\middle|\; \begin{matrix} x\to 0 \\ t\to 1 \end{matrix}\right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} \frac{4(t^2-1)}{t-1}$$
$$= 4 \cdot \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} \frac{(t-1)\cdot(t+1)}{t-1} = 4 \cdot \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{t\to 1} (t+1) = 4\cdot 1\cdot(1+1) = 8.$$

Riešené limity – 40, 41, 42

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x}$$

$$\lim_{x\to\infty} e^x(2+\cos x)$$

$$\lim_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

Riešené limity – 40, 41, 42

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x}$$

- $= \lim_{x\to 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2} \right)$

$$\lim_{x\to\infty} e^x (2+\cos x)$$

- $x \in R.$

$$\lim_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

$\left[x \in (-\pi;\pi) . \quad \middle| \begin{array}{l} x \in (-\pi;0) \\ x \in (0;\pi) \end{array} \right]$

Riešené limity – 40, 41, 42

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x}$$

- $= \lim_{x\to 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2} \right) = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z \to 0 \\ x-2 = \sin z & x \to 2 \end{array} \right]$

$$\lim_{x\to\infty} e^x (2 + \cos x)$$

- $x \in R. \Rightarrow e^x > 0, \ -1 \le \cos x \le 1.$

$$\lim_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

$\left[\begin{array}{l|l} x \in (-\pi;\pi). \Rightarrow -1 < \cos x < 1. & x \in (-\pi;0) \\ & x \in (0;\pi) \end{array} \right]$

Riešené limity – 40, 41, 42

$\lim\limits_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x}$

- $= \lim\limits_{x\to 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2}\right) = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z \to 0 \\ x-2 = \sin z & x \to 2 \end{array}\right] = \lim\limits_{x\to 2} \frac{1}{x} \cdot \lim\limits_{z\to 0} \frac{z}{\sin z}$

$\lim\limits_{x\to\infty} e^x(2+\cos x)$

- $x \in R. \Rightarrow e^x > 0,\ -1 \le \cos x \le 1. \Rightarrow 1 \le 2 + \cos x \le 3.$

$\lim\limits_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$

$\left[\begin{array}{l|l} x \in (-\pi;\pi). \Rightarrow -1 < \cos x < 1. \Rightarrow 0 < 1-\cos x,\ 0 < 1+\cos x. & x \in (-\pi;0) \\ & x \in (0;\pi) \end{array}\right]$

Riešené limity – 40, 41, 42

$\lim\limits_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x}$

- $= \lim\limits_{x\to 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2}\right) = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z\to 0 \\ x-2=\sin z & x\to 2 \end{array}\right] = \lim\limits_{x\to 2} \frac{1}{x} \cdot \lim\limits_{z\to 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1$

$\lim\limits_{x\to\infty} \mathrm{e}^x(2+\cos x)$

- $x \in R. \Rightarrow \mathrm{e}^x > 0,\ -1 \le \cos x \le 1. \Rightarrow 1 \le 2+\cos x \le 3. \Rightarrow \mathrm{e}^x \le \mathrm{e}^x(2+\cos x) \le 3\mathrm{e}^x.$

$\lim\limits_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$

$\left[\begin{array}{l|l} x\in(-\pi;\pi).\Rightarrow -1<\cos x<1. \Rightarrow 0<1-\cos x,\ 0<1+\cos x. & x\in(-\pi;0) \\ \Rightarrow \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x}. & x\in(0;\pi) \end{array}\right]$

Riešené limity – 40, 41, 42

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

- $= \lim_{x\to 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2}\right) = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z \to 0 \\ x-2 = \sin z & x \to 2 \end{array}\right] = \lim_{x\to 2} \frac{1}{x} \cdot \lim_{z\to 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$

$$\lim_{x\to\infty} e^x(2+\cos x)$$

- $\lim_{x\to\infty} e^x \leq \lim_{x\to\infty} e^x(2+\cos x) \leq \lim_{x\to\infty} 3e^x$

- $x \in R. \Rightarrow e^x > 0,\ -1 \leq \cos x \leq 1. \Rightarrow 1 \leq 2+\cos x \leq 3. \Rightarrow e^x \leq e^x(2+\cos x) \leq 3e^x.$

$$\lim_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

$$\left[\begin{array}{l|l} x\in(-\pi;\pi). \Rightarrow -1 < \cos x < 1. \Rightarrow 0 < 1-\cos x,\ 0 < 1+\cos x. & x\in(-\pi;0),\ x\to 0^-. \\ \Rightarrow \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x}. & x\in(0;\pi),\ x\to 0^+. \end{array}\right]$$

- $\lim_{x\to 0^-} \frac{\sin x}{\sqrt{1-\cos x}}$
- $\lim_{x\to 0^+} \frac{\sin x}{\sqrt{1-\cos x}}$

Riešené limity – 40, 41, 42

$$\lim_{x\to 2}\frac{\arcsin(x-2)}{x^2-2x}=\frac{1}{2}$$

- $= \lim_{x\to 2}\left(\frac{1}{x}\cdot\frac{\arcsin(x-2)}{x-2}\right) = \left[\begin{array}{r|l} \text{Subst. } z=\arcsin(x-2) & z\to 0 \\ x-2=\sin z & x\to 2 \end{array}\right] = \lim_{x\to 2}\frac{1}{x}\cdot\lim_{z\to 0}\frac{z}{\sin z} = \frac{1}{2}\cdot 1 = \frac{1}{2}.$

$$\lim_{x\to\infty} e^x(2+\cos x)$$

- $\infty = \lim_{x\to\infty} e^x \leq \lim_{x\to\infty} e^x(2+\cos x) \leq \lim_{x\to\infty} 3e^x = \infty.$

- $x\in R. \Rightarrow e^x > 0,\ -1\leq \cos x\leq 1. \Rightarrow 1\leq 2+\cos x\leq 3. \Rightarrow e^x \leq e^x(2+\cos x)\leq 3e^x.$

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{1-\cos x}}$$

$$\left[\begin{array}{l|l} x\in(-\pi;\pi).\Rightarrow -1<\cos x<1. \Rightarrow 0<1-\cos x,\ 0<1+\cos x. & x\in(-\pi;0),\ x\to 0^-. \Rightarrow\ |\sin x| = -\sin x. \\ \Rightarrow \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x}. & x\in(0;\pi),\ x\to 0^+. \end{array}\right]$$

- $\lim_{x\to 0^-}\frac{\sin x}{\sqrt{1-\cos x}} = -\lim_{x\to 0^-}\sqrt{1+\cos x}$
- $\lim_{x\to 0^+}\frac{\sin x}{\sqrt{1-\cos x}}$

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$$\lim_{x\to\infty}\mathrm{e}^x(2+\cos x)=\infty$$

- $\infty=\lim_{x\to\infty}\mathrm{e}^x\leq\lim_{x\to\infty}\mathrm{e}^x(2+\cos x)\leq\lim_{x\to\infty}3\mathrm{e}^x=\infty.\ \Rightarrow$ • $\lim_{x\to\infty}\mathrm{e}^x(2+\cos x)=\infty.$

- $x\in R.\ \Rightarrow\ \mathrm{e}^x>0,\ -1\leq\cos x\leq 1.\ \Rightarrow\ 1\leq 2+\cos x\leq 3.\ \Rightarrow\ \mathrm{e}^x\leq\mathrm{e}^x(2+\cos x)\leq 3\mathrm{e}^x.$

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$\left[\begin{array}{l|l} x\in(-\pi;\pi).\Rightarrow -1<\cos x<1.\Rightarrow 0<1-\cos x,\ 0<1+\cos x. & x\in(-\pi;0),\ x\to 0^-.\ \Rightarrow\ |\sin x|=-\sin x.\\ \Rightarrow\frac{|\sin x|}{\sqrt{1-\cos x}}=\frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}}=\frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}}=\frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}}=\frac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}}=\sqrt{1+\cos x}. & x\in(0;\pi),\ x\to 0^+.\ \Rightarrow\ |\sin x|=\sin x.\end{array}\right]$

- $\lim_{x\to 0^-}\frac{\sin x}{\sqrt{1-\cos x}}=-\lim_{x\to 0^-}\sqrt{1+\cos x}$
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- $=\lim_{x\to 2}\left(\frac{1}{x}\cdot\frac{\arcsin(x-2)}{x-2}\right)=\left[\begin{array}{r|l}\text{Subst. } z=\arcsin(x-2) & z\to 0\\ x-2=\sin z & x\to 2\end{array}\right]=\lim_{x\to 2}\frac{1}{x}\cdot\lim_{z\to 0}\frac{z}{\sin z}=\frac{1}{2}\cdot 1=\frac{1}{2}.$

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  - $x\in R.\Rightarrow e^x>0,\ -1\le\cos x\le 1.\Rightarrow 1\le 2+\cos x\le 3.\Rightarrow e^x\le e^x(2+\cos x)\le 3e^x.$

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$$\lim_{x\to 0}\frac{\sin x}{\sqrt{1-\cos x}}$$

$$\left[\begin{array}{l|l} x\in(-\pi;\pi).\Rightarrow -1<\cos x<1.\Rightarrow 0<1-\cos x,\ 0<1+\cos x. & x\in(-\pi;0),\ x\to 0^-.\Rightarrow\ |\sin x|=-\sin x.\\ \Rightarrow\frac{|\sin x|}{\sqrt{1-\cos x}}=\frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}}=\frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}}=\frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}}=\frac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}}=\sqrt{1+\cos x}. & x\in(0;\pi),\ x\to 0^+.\ \Rightarrow\ |\sin x|=\sin x.\end{array}\right]$$

- $\lim_{x\to 0^-}\frac{\sin x}{\sqrt{1-\cos x}}=-\lim_{x\to 0^-}\sqrt{1+\cos x}=-\sqrt{1+1}=-\sqrt{2}.$
- $\lim_{x\to 0^+}\frac{\sin x}{\sqrt{1-\cos x}}=\lim_{x\to 0^+}\sqrt{1+\cos x}=\sqrt{1+1}=\sqrt{2}.$

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- $\infty = \lim_{x\to\infty} e^x \le \lim_{x\to\infty} e^x(2+\cos x) \le \lim_{x\to\infty} 3e^x = \infty. \Rightarrow$ • $\lim_{x\to\infty} e^x(2+\cos x)=\infty.$

- $x\in R. \Rightarrow e^x>0,\ -1\le\cos x\le 1. \Rightarrow 1\le 2+\cos x\le 3. \Rightarrow e^x\le e^x(2+\cos x)\le 3e^x.$

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{1-\cos x}} \text{ neexistuje.}$$

$$\left[\begin{array}{l|l} x\in(-\pi;\pi). \Rightarrow -1<\cos x<1. \Rightarrow 0<1-\cos x,\ 0<1+\cos x. & x\in(-\pi;0),\ x\to 0^-. \Rightarrow |\sin x|=-\sin x.\\ \Rightarrow \frac{|\sin x|}{\sqrt{1-\cos x}}=\frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}}=\frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}}=\frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}}=\frac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}}=\sqrt{1+\cos x}. & x\in(0;\pi),\ x\to 0^+. \Rightarrow |\sin x|=\sin x.\end{array}\right]$$

$$\left.\begin{array}{l} \bullet\ \lim_{x\to 0^-}\frac{\sin x}{\sqrt{1-\cos x}} = -\lim_{x\to 0^-}\sqrt{1+\cos x} = -\sqrt{1+1} = -\sqrt{2}.\\ \bullet\ \lim_{x\to 0^+}\frac{\sin x}{\sqrt{1-\cos x}} = \lim_{x\to 0^+}\sqrt{1+\cos x} = \sqrt{1+1} = \sqrt{2}.\end{array}\right\} \Rightarrow \bullet\ \lim_{x\to 0}\frac{\sin x}{\sqrt{1-\cos x}}\ \nexists.$$

Riešené limity – 43, 44, 45

$\lim\limits_{x\to 0} \frac{x^n-\sin^n x}{x-\sin x}$ pre $n\in N,\ n\neq 1$

$\lim\limits_{x\to a} \frac{\sin x-\sin a}{x-a}$ pre $a\in R$

$\lim\limits_{x\to a} \frac{\cos x-\cos a}{x-a}$ pre $a\in R$

Riešené limity – 43, 44, 45

$\lim\limits_{x\to 0} \frac{x^n-\sin^n x}{x-\sin x}$ pre $n\in N,\ n\neq 1$

- $= \lim\limits_{x\to 0} \frac{(x-\sin x)\cdot(x^{n-1}+x^{n-2}\cdot\sin x+\cdots+x\cdot\sin^{n-2} x+\sin^{n-1} x)}{x-\sin x}$

$\lim\limits_{x\to a} \frac{\sin x-\sin a}{x-a}$ pre $a\in R$

- $= \lim\limits_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}}$

$\lim\limits_{x\to a} \frac{\cos x-\cos a}{x-a}$ pre $a\in R$

- $= \lim\limits_{x\to a} \frac{-2\sin\frac{x-a}{2}\sin\frac{x+a}{2}}{2\cdot\frac{x-a}{2}}$

Riešené limity – 43, 44, 45

$\lim\limits_{x\to 0} \frac{x^n - \sin^n x}{x - \sin x}$ pre $n \in N,\ n \neq 1$

- $= \lim\limits_{x\to 0} \frac{(x-\sin x)\cdot(x^{n-1}+x^{n-2}\cdot\sin x+\cdots+x\cdot\sin^{n-2}x+\sin^{n-1}x)}{x-\sin x}$
  $= \lim\limits_{x\to 0} (x^{n-1} + x^{n-2}\cdot\sin x + \cdots + \sin^{n-1} x)$

$\lim\limits_{x\to a} \frac{\sin x - \sin a}{x - a}$ pre $a \in R$

- $= \lim\limits_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim\limits_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim\limits_{x\to a} \cos\frac{x+a}{2}$

$\lim\limits_{x\to a} \frac{\cos x - \cos a}{x - a}$ pre $a \in R$

- $= \lim\limits_{x\to a} \frac{-2\sin\frac{x-a}{2}\sin\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = -\lim\limits_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim\limits_{x\to a} \sin\frac{x+a}{2}$

Riešené limity – 43, 44, 45

$\lim\limits_{x\to 0} \frac{x^n - \sin^n x}{x - \sin x}$ pre $n \in N,\ n \neq 1$

- $= \lim\limits_{x\to 0} \frac{(x-\sin x)\cdot(x^{n-1}+x^{n-2}\cdot\sin x+\cdots+x\cdot\sin^{n-2} x+\sin^{n-1} x)}{x-\sin x}$
  $= \lim\limits_{x\to 0} (x^{n-1} + x^{n-2}\cdot\sin x + \cdots + \sin^{n-1} x) = 0^{n-1} + 0^{n-2}\cdot 0 + \cdots + 0^{n-1}$

$\lim\limits_{x\to a} \frac{\sin x - \sin a}{x - a}$ pre $a \in R$

- $= \lim\limits_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim\limits_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim\limits_{x\to a} \cos\frac{x+a}{2} = \left[\text{Subst. } z = \frac{x-a}{2} \;\middle|\; \begin{matrix} x\to a \\ z\to 0 \end{matrix}\right]$

$\lim\limits_{x\to a} \frac{\cos x - \cos a}{x - a}$ pre $a \in R$

- $= \lim\limits_{x\to a} \frac{-2\sin\frac{x-a}{2}\sin\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = -\lim\limits_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim\limits_{x\to a} \sin\frac{x+a}{2} = \left[\text{Subst. } z = \frac{x-a}{2} \;\middle|\; \begin{matrix} x\to a \\ z\to 0 \end{matrix}\right]$

Riešené limity – 43, 44, 45

$\lim\limits_{x\to 0} \frac{x^n-\sin^n x}{x-\sin x} = 0$ pre $n\in N,\ n\neq 1$

- $= \lim\limits_{x\to 0} \frac{(x-\sin x)\cdot(x^{n-1}+x^{n-2}\cdot\sin x+\cdots+x\cdot\sin^{n-2}x+\sin^{n-1}x)}{x-\sin x}$

$$= \lim_{x\to 0}(x^{n-1}+x^{n-2}\cdot\sin x+\cdots+\sin^{n-1}x) = 0^{n-1}+0^{n-2}\cdot 0+\cdots+0^{n-1} = 0.$$

$\lim\limits_{x\to a} \frac{\sin x-\sin a}{x-a}$ pre $a\in R$

- $= \lim\limits_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim\limits_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim\limits_{x\to a}\cos\frac{x+a}{2} = \left[\text{Subst. } z=\frac{x-a}{2} \,\middle|\, \begin{matrix} x\to a \\ z\to 0\end{matrix}\right]$

$$= \lim_{z\to 0}\frac{\sin z}{z}\cdot\lim_{x\to a}\cos\frac{x+a}{2}$$

$\lim\limits_{x\to a} \frac{\cos x-\cos a}{x-a}$ pre $a\in R$

- $= \lim\limits_{x\to a} \frac{-2\sin\frac{x-a}{2}\sin\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = -\lim\limits_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim\limits_{x\to a}\sin\frac{x+a}{2} = \left[\text{Subst. } z=\frac{x-a}{2} \,\middle|\, \begin{matrix} x\to a \\ z\to 0\end{matrix}\right]$

$$= -\lim_{z\to 0}\frac{\sin z}{z}\cdot\lim_{x\to a}\sin\frac{x+a}{2}$$

Riešené limity – 43, 44, 45

$\lim\limits_{x\to 0} \frac{x^n-\sin^n x}{x-\sin x} = 0$ pre $n\in N,\ n\neq 1$

- $= \lim\limits_{x\to 0} \frac{(x-\sin x)\cdot(x^{n-1}+x^{n-2}\cdot\sin x+\cdots+x\cdot\sin^{n-2} x+\sin^{n-1} x)}{x-\sin x}$

$$= \lim_{x\to 0} (x^{n-1} + x^{n-2}\cdot\sin x + \cdots + \sin^{n-1} x) = 0^{n-1} + 0^{n-2}\cdot 0 + \cdots + 0^{n-1} = 0.$$

$\lim\limits_{x\to a} \frac{\sin x-\sin a}{x-a}$ pre $a\in R$

- $= \lim\limits_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim\limits_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim\limits_{x\to a} \cos\frac{x+a}{2} = \left[\text{Subst. } z=\frac{x-a}{2} \,\middle|\, \begin{matrix} x\to a \\ z\to 0\end{matrix}\right]$

$$= \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \cos\frac{x+a}{2} = 1\cdot\cos\frac{a+a}{2}$$

$\lim\limits_{x\to a} \frac{\cos x-\cos a}{x-a}$ pre $a\in R$

- $= \lim\limits_{x\to a} \frac{-2\sin\frac{x-a}{2}\sin\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = -\lim\limits_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim\limits_{x\to a} \sin\frac{x+a}{2} = \left[\text{Subst. } z=\frac{x-a}{2} \,\middle|\, \begin{matrix} x\to a \\ z\to 0\end{matrix}\right]$

$$= -\lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \sin\frac{x+a}{2} = -1\cdot\sin\frac{a+a}{2}$$

Riešené limity – 43, 44, 45

$\lim\limits_{x\to 0} \frac{x^n-\sin^n x}{x-\sin x} = 0$ pre $n \in N,\ n \neq 1$

- $= \lim\limits_{x\to 0} \frac{(x-\sin x)\cdot(x^{n-1}+x^{n-2}\cdot\sin x+\cdots+x\cdot\sin^{n-2} x+\sin^{n-1} x)}{x-\sin x}$

$$= \lim_{x\to 0} (x^{n-1} + x^{n-2}\cdot\sin x + \cdots + \sin^{n-1} x) = 0^{n-1} + 0^{n-2}\cdot 0 + \cdots + 0^{n-1} = 0.$$

$\lim\limits_{x\to a} \frac{\sin x-\sin a}{x-a}$ pre $a \in R$

- $= \lim\limits_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim\limits_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim\limits_{x\to a} \cos\frac{x+a}{2} = \left[\text{Subst. } z=\frac{x-a}{2} \;\middle|\; \begin{matrix} x\to a \\ z\to 0 \end{matrix}\right]$

$$= \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \cos\frac{x+a}{2} = 1\cdot\cos\frac{a+a}{2} = 1\cdot\cos a$$

$\lim\limits_{x\to a} \frac{\cos x-\cos a}{x-a}$ pre $a \in R$

- $= \lim\limits_{x\to a} \frac{-2\sin\frac{x-a}{2}\sin\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = -\lim\limits_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim\limits_{x\to a} \sin\frac{x+a}{2} = \left[\text{Subst. } z=\frac{x-a}{2} \;\middle|\; \begin{matrix} x\to a \\ z\to 0 \end{matrix}\right]$

$$= -\lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \sin\frac{x+a}{2} = -1\cdot\sin\frac{a+a}{2} = -1\cdot\sin a$$

Riešené limity – 43, 44, 45

$\lim\limits_{x\to 0} \frac{x^n-\sin^n x}{x-\sin x} = 0$ pre $n\in N,\ n\neq 1$

- $= \lim\limits_{x\to 0} \frac{(x-\sin x)\cdot(x^{n-1}+x^{n-2}\cdot\sin x+\cdots+x\cdot\sin^{n-2} x+\sin^{n-1} x)}{x-\sin x}$

$$= \lim_{x\to 0} (x^{n-1}+x^{n-2}\cdot\sin x+\cdots+\sin^{n-1} x) = 0^{n-1}+0^{n-2}\cdot 0+\cdots+0^{n-1} = 0.$$

$\lim\limits_{x\to a} \frac{\sin x-\sin a}{x-a} = \cos a$ pre $a\in R$

- $= \lim\limits_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim\limits_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim\limits_{x\to a} \cos\frac{x+a}{2} = \left[\text{Subst. } z=\frac{x-a}{2} \,\middle|\, \begin{matrix} x\to a \\ z\to 0 \end{matrix} \right]$

$$= \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \cos\frac{x+a}{2} = 1\cdot\cos\frac{a+a}{2} = 1\cdot\cos a = \cos a.$$

$\lim\limits_{x\to a} \frac{\cos x-\cos a}{x-a} = -\sin a$ pre $a\in R$

- $= \lim\limits_{x\to a} \frac{-2\sin\frac{x-a}{2}\sin\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = -\lim\limits_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim\limits_{x\to a} \sin\frac{x+a}{2} = \left[\text{Subst. } z=\frac{x-a}{2} \,\middle|\, \begin{matrix} x\to a \\ z\to 0 \end{matrix} \right]$

$$= -\lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \sin\frac{x+a}{2} = -1\cdot\sin\frac{a+a}{2} = -1\cdot\sin a = -\sin a.$$

Koniec 6. časti (príklady)

Ďakujem za pozornosť.