New models for the return bus scheduling problem

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We studed a MILP models for real-world situations which can occur in multi-depots bus scheduling problems. The discussion is focused one specific return bus scheduling problem.

- We show how this problem can be solved via two index MILP model.
- Our approach is based on an original constraint for assigning a timetable to the home depot.
- Computational experiments with the instance of public bus service for slovakian city Martin are presented.

Basic bus scheduling problem

consists of assigning buses to given set of (regular) trips in running board such that:

- each trip is performed exactly once,
- each bus must start and end its work day at the same depot,
- the number of buses is as low as possible minimum,
- the operate cost is minimum.

This problem has several variations with practical restriction (on number and types of depots, meal breaks, buses types, length of the running boards, flexible trips) studied in Slovakia by Palúch and his colleagues.

Given set of regular trips $\mathcal{S} = \{S_1, S_2, \dots, S_n\}$, each trip $S_i, i \in N = \{1, 2, \dots, n\}$ is represented by an arbitrary ordered four $S_i = (p_i^d, t_i^d, p_i^a, t_i^a)$, where

- p_i^d – the departure place,
- t_i^d – the departure time,
- p_i^a – the arrival place,
- t_i^a – the arrival time.

Let be $\tau(p_i^a, p_j^d)$ the times of the idle trip from place p_i^a to place p_j^d. Similarly we note the times of the idle trip from and to home depots d as $\tau(d, p_j^d)$ and $\tau(p_i^a, d)$.
The running board of the bus $\mathcal{T}_i(d)$ from home depot d with m trips from set $\mathcal{S}$ is sequence

$$S_{i_1} \prec S_{i_2} \prec \cdots \prec S_{i_m},$$

where for all $k \in \{1, 2, \dots, m-1\}$ is

$$t_{i_k} + \tau(p_{i_k}^a, p_{i_{k+1}}^d) < t_{i_{k+1}}.$$

We will consider the RBSP when the set of trips is given for one type of bus and no practical restrictions are presumed:

- the set of trips is given for one type of bus,
- one central depot with given number of buses,
- several single depots with one bus.

Consider a simple transport network with 4 stops, with place 1 where is located central depot and two busies and place 4 where is located single depot.

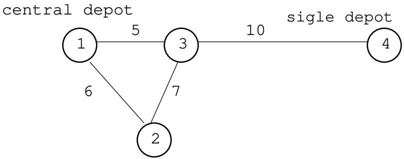

Figure 1: Simple transportation network.

Illustrative example

We have a set $\mathcal{S} = \{S_1, S_2, \ldots, S_9\}$ of nine regular trips given in table 1.

i	p_i^d	t_i^d	p_i^a	t_i^a
1	1	60	2	70
2	1	60	3	75
3	3	85	2	100
4	1	85	4	100
5	4	85	2	110
6	2	120	1	130
7	3	128	1	150
8	3	128	4	145
9	2	140	1	160

Table 1: Instance of trips.

Two solution of instance

The value in the columns bus I and bus II calculated bus's number. The home depot for busies 1 and 2 is central depot 1 and for bus 3 is single depot 4.

i	p_i^d	t_i^d	p_i^a	t_i^a	bus I.	bus II.
1	1	60	2	70	1	1
2	1	60	3	75	2	2
3	3	85	2	100	1	1
4	1	85	4	100	2	2
5	4	85	2	110	3	3
6	2	120	1	130	3	2
7	3	128	1	150	2	1
8	3	128	4	145	1	3
9	2	140	1	160	3	2

Table 2: Running boards for I. and II. solutions.

We can see that in the solution I. the bus 1 starts at depot 1 but ends at depot 4 and vehicle 3 opposite. In the solution II. busies start end end at its home depots.

Three-index model

Let $K = \{0, 1, \ldots, h\}$ be a set of indexes of depots and q is number of buses in central depot.

We can define vehicle scheduling network $\vec{G}_k = (V, A_k), k \in K$ corresponding to depot d_k which is a directed graph described above set of vertices $V = \{-h, \ldots, -1, 0, 1, \ldots, n\}$ and arcs $A_k = \{(i,j) \in N \times N : S_i \prec S_j\} \cup \{(-k, i), (i, -k) : i \in N\}$.

Let c_{ij} be the vehicle cost, time of empty trips of arc $(i,j) \in A_k$, which represents idle time activities for vehicles from home depot d_k.

Decision three-index variable x_{ij}^k indicates whether an arc (i,j) is used an assigned to the depot d_k or not. N

3RBSP model

$$\sum_{k\in K}\sum_{(i,j)\in A_k} c_{ij}x_{ij}^k \to \min \tag{1}$$

s.t.

$$\sum_{j:(i,j)\in A_k} x_{ij}^k - \sum_{j:(j,i)\in A_k} x_{ji}^k = 0 \qquad \forall k \in K, \forall i \in V, \tag{2}$$

$$\sum_{(-k,j)\in A_k} x_{-kj}^k = 1, \qquad \forall k \in K - \{0\}, \tag{3}$$

$$\sum_{(0,j)\in A_0} x_{0j}^0 = q, \tag{4}$$

$$\sum_{k\in K}\sum_{j:(i,j)\in A_k} x_{ij}^k = 1, \qquad \forall i \in V - K \tag{5}$$

$$x_{ij}^k \in \{0,1\} \qquad \forall k \in K, \forall (i,j) \in A_k. \tag{6}$$

Reduction of decision variables

In the 3RBSP model we have a decision three-index variable x_{ij}^k indicates whether an arc (i, j) will be selected and will be assigned to the object k, which in our case is one of depots. If $i \in I, j \in J$ and $k \in K$ than we need in the extreme case $|I| \cdot |J| \cdot |K|$ of 01 variables.

Using other variables $y_i \in R, i \in I \cup J$ and decision variables $z_{ij}, i \in I, j \in J$ we can replace the purpose of the variable x_{ij}^k via following inequalities:

$$M(z_{ij} - 1) \leq y_i - y_j \leq M(1 - z_{ij}), \tag{7}$$

where is M enough large positive number.

If $z_{ij} = 1$ than $0 \leq y_i - y_j \leq 0$ and so $y_i = y_j$. If $z_{ij} = 0$ than $-M \leq y_i - y_j \leq M$ and there are no restrictions on the variables y_i and y_j.

Two-index model

We can define basic vehicle scheduling network as a directed graf $\vec{G} = (V, A)$, where $V = \{-h, \ldots, -1, 0, 1, 2, \ldots, n\}$ and set of arc

$$A = \{(i,j) : S_i \prec S_j\} \cup \bigcup_{i \in N, k \in K} \{(-k, i), (i, -k)\}.$$

Vehicle cost of arc $(i,j) \in A$ we note c_{ij} again.

We will use decision two-index variable z_{ij} for indication whether an arc (i,j) is used or not.

For the assignment trip to home depot we use variable $y_i \in K$, $i \in V$. If $y_i = -k$ than trip $S_i \in \mathcal{S}$ is assigned to depot d_k and we can apply inequalities (7).

2RBSP model

$$\sum_{(i,j)\in A} c_{ij} z_{ij} \to \min \tag{8}$$

s.t.

$$\sum_{j:(i,j)\in A} z_{ij} - \sum_{j:(j,i)\in A} z_{ji} = 0 \qquad \forall i \in V, \tag{9}$$

$$\sum_{(-k,j)\in A} z_{-kj} = 1, \qquad \forall k \in K - \{0\}, \tag{10}$$

$$\sum_{(0,j)\in A} z_{0j} = q, \tag{11}$$

$$\sum_{j:(i,j)\in A} z_{ij} = 1, \qquad \forall i \in V - K \tag{12}$$

$$h(z_{ij} - 1) \le y_i - y_j \le h(1 - z_{ij}) \qquad \forall (i,j) \in A, \tag{13}$$

$$y_{-k} = k \qquad \forall k \in K, \tag{14}$$

$$z_{ij} \in \{0, 1\} \qquad \forall (i,j) \in A, \tag{15}$$

$$y_k \ge 0, \qquad \forall k \in V. \tag{16}$$

Mathematical model 3RBSP and 2RBSP was solved for instances from real study for bus company Martin in Slovakia containing maximum 726 trips, for bus network see figure 2.

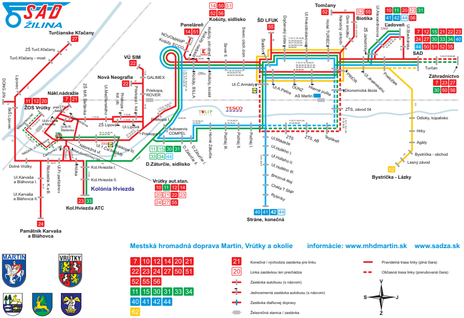

Figure 2: Bus network of city Martin.

Our experiments were conducted on HP XW6600 Workstation (8-core Xeon 3GHz, RAM 16GB) with OS Linux (Debian/jessie).

Trips	Depots	Operate cost [min.]	Time [sec.]
666	24; 1	1 303	914.0
666	24; 1, 17	1 269	877.33
666	24; 1, 17, 15	1 232	865.13
666	24; 1, 17, 15, 4	1 228	843.34
180	24; 1	1 209	1.80
180	24; 1, 17	1 084	1.78
180	24; 1, 17, 15	1 054	2.92
180	24; 1, 17, 15, 4	1 050	4.74

Table 3: Computational characteristics for the 3RBSP.

First experiments with tree-index model showed that reduction of decision 01 variables significantly speeds up calculations. Therefore, we did not even continue to increase the number of connections for the 3RBSP.

We continued with only model 2RBSP, where we required a minimum possible number of buses in the central depot 24.

Trips	Depots	Operate cost [min.]	Time [sec.]
726	24; 1	1 356	65.15
726	24; 1, 17	1 319	84.97
726	24; 1, 17, 15	1 299	76.33
726	24; 1, 17, 15, 4	1 295	85.52
720	24; 1	1 925	60.29
720	24; 1, 17	1 304	65.86
720	24; 1, 17, 15	1 299	76.94
720	24; 1, 17, 15, 4	1 295	74.57
666	24; 1	1 303	64.29
666	24; 1, 17	1 269	60.81
666	24; 1, 17, 15	1 232	74.44
666	24; 1, 17, 15, 4	1 228	60.61
180	24; 1	1 209	2.86
180	24; 1, 17	1 084	2.14
180	24; 1, 17, 15	1 054	1.93
180	24; 1, 17, 15, 4	1 050	2.36

Table 4: Computational characteristics for the 2RBSP.

Although it is not surprising that a suitable reduction of conversational variables can have such a significant effect on the computation time, the results surprised us.

Computer experiments have shown that the presented approach gives hope for incorporation into 2RBSP model a real operational limiting requirements:

- different type of vehicles,
- alternation of drivers in the depot,
- safety breaks.

Thank you for your attention...