

Matematická analýza 1

2025/2026

10. Neurčitý integrál Riešené príklady II

Pre správne zobrazenie, fungovanie tooltipov, 2D a 3D animácií je nevyhnutné súbor otvoriť pomocou programu Adobe Reader (zásuvný modul Adobe PDF Plug-In webového prehliadača nestačí).
Kliknutím na text pred ikonou  získate nápomoc.

Obsah – príklady 101–200

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Zoznam integrálov – príklady 101–200

101. $\int \frac{dx}{x^2+4x+5}$ 102. $\int \frac{dx}{x^2+4x+3}$ 103. $\int \frac{dx}{x^2-4x+6}$ 104. $\int \frac{dx}{x^2-4x+2}$ 105. $\int \frac{dx}{x^2+4x+4}$ 106. $\int \frac{dx}{\sqrt{x^2+4x+4}}$ 107. $\int \frac{dx}{\sqrt{x^2+4x+5}}$ 108. $\int \frac{dx}{\sqrt{x^2+4x+3}}$
 109. $\int \frac{dx}{\sqrt{x^2-4x+6}}$ 110. $\int \frac{dx}{\sqrt{x^2-4x+2}}$ 111. $\int \frac{dx}{\sqrt{-x^2+4x-5}}$ 112. $\int \frac{dx}{\sqrt{-x^2+4x-3}}$ 113. $\int \frac{dx}{\sqrt{x(x-1)}}$ 114. $\int \frac{dx}{\sqrt{x(2-x)}}$ 115. $\int \sqrt{x^2+4x+5} dx$
 116. $\int \sqrt{x^2+4x+3} dx$ 117. $\int \sqrt{x^2-4x+6} dx$ 118. $\int \sqrt{x^2-4x+2} dx$ 119. $\int \sqrt{-x^2+4x-3} dx$ 120. $\int \sqrt{x(2-x)} dx$ 121. $\int \frac{dx}{(x^2+a^2)^n}$ 122. $\int \frac{dx}{(x^2+a^2)^2}$
 123. $\int \frac{dx}{(x^2+a^2)^3}$ 124. $\int \frac{dx}{(x^2+a^2)^4}$ 125. $\int \frac{dx}{(x^2-a^2)^n}$ 126. $\int \frac{dx}{(x^2-a^2)^2}$ 127. $\int \frac{dx}{(x^2-a^2)^3}$ 128. $\int \frac{dx}{(x^2-a^2)^4}$ 129. $\int \frac{dx}{(x^2+4x+5)^2}$ 130. $\int \frac{dx}{(x^2+4x+3)^2}$ 131. $\int \frac{x^2 dx}{(x^2+a^2)^2}$
 132. $\int \frac{x^2 dx}{(x^2-a^2)^2}$ 133. $\int \frac{x dx}{x^2+a^2}$ 134. $\int \frac{x dx}{(x^2+a^2)^n}$ 135. $\int \frac{x dx}{x^2-a^2}$ 136. $\int \frac{x dx}{(x^2-a^2)^n}$ 137. $\int \frac{x^n dx}{x-1}$ 138. $\int \frac{x dx}{x-1}$ 139. $\int \frac{x^2 dx}{x-1}$ 140. $\int \frac{x^9 dx}{x-1}$ 141. $\int \frac{dx}{(1-x)x^2}$
 142. $\int \frac{dx}{x^6(1+x^2)}$ 143. $\int \frac{(x-2)^4 dx}{(x-1)^2}$ 144. $\int \frac{(x-1)^4 dx}{(x-2)^2}$ 145. $\int \frac{dx}{x^3-7x-6}$ 146. $\int \frac{dx}{x^3-2x^2-x+2}$ 147. $\int \frac{dx}{x^3-3x-2}$ 148. $\int \frac{dx}{x^3+x^2-x-1}$ 149. $\int \frac{dx}{x^3-3x^2+4x-2}$
 150. $\int \frac{dx}{x^3-3x^2+3x-1}$ 151. $\int \frac{dx}{x^3-2x-4}$ 152. $\int \frac{dx}{x^6+1}$ 153. $\int \frac{x dx}{x^6+1}$ 154. $\int \frac{x^2 dx}{x^6+1}$ 155. $\int \frac{x^3 dx}{x^6+1}$ 156. $\int \frac{x^4 dx}{x^6+1}$ 157. $\int \frac{x^5 dx}{x^6+1}$ 158. $\int \frac{x^6 dx}{x^6+1}$ 159. $\int \frac{dx}{2x+1}$ 160. $\int \frac{dx}{\sqrt{2x+1}}$
 161. $\int \frac{(1+x) dx}{\sqrt{1-x^2}}$ 162. $\int \sqrt{\frac{1+x}{1-x}} dx$ 163. $\int \sqrt{\frac{1-x}{1+x}} dx$ 164. $\int \sqrt{\frac{x+1}{x-1}} dx$ 165. $\int \sqrt{\frac{x-1}{x+1}} dx$ 166. $\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx$ 167. $\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx$
 168. $\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$ 169. $\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$ 170. $\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$ 171. $\int \frac{dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$ 172. $\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$ 173. $\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$
 174. $\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$ 175. $\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$ 176. $\int \frac{dx}{\sqrt{x-3}+\sqrt{x-5}}$ 177. $\int \frac{dx}{\sqrt{x-3}-\sqrt{x-5}}$ 178. $\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$ 179. $\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} dx$
 180. $\int \sqrt{\frac{x}{1-x\sqrt{x}}} dx$ 181. $\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$ 182. $\int \frac{dx}{\sqrt[5]{x}+\sqrt[4]{x}}$ 183. $\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}}$ 184. $\int \frac{dx}{\sqrt[3]{x}+1}$ 185. $\int \left[\sqrt{x^3} - \frac{1}{\sqrt{x}} \right] dx$ 186. $\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x} dx$ 187. $\int \frac{dx}{(x-1)\sqrt{x-2}}$
 188. $\int \frac{dx}{(x+1)\sqrt{1-x}}$ 189. $\int (x-1)\sqrt{x-2} dx$ 190. $\int \frac{\sqrt{1-x}}{x+1} dx$ 191. $\int \frac{dx}{(x-\sqrt{x^2-1})^2}$ 192. $\int \frac{x^5 dx}{\sqrt{x^2+1}}$ 193. $\int \frac{x^5 dx}{\sqrt{x^3+1}}$ 194. $\int \frac{x^5 dx}{\sqrt{x^2-1}}$ 195. $\int \frac{x^5 dx}{\sqrt{x^3-1}}$
 196. $\int \frac{x^5 dx}{\sqrt{1-x^2}}$ 197. $\int \frac{x^5 dx}{\sqrt{1-x^3}}$ 198. $\int \frac{\sqrt{1-x^2}}{x^2} dx$ 199. $\int \frac{dx}{x\sqrt{x^4+x^2+1}}$ 200. $\int \frac{dx}{x\sqrt{x^6+x^3+1}}$

Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5}$$

$$\int \frac{dx}{x^2+4x+3}$$

Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5}$$

$$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1}$$

$$\int \frac{dx}{x^2+4x+3}$$

$$= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1}$$

Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5}$$

$$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} \\ dt=dx \mid t \in \mathbb{R} \end{array} \right]$$

$$\int \frac{dx}{x^2+4x+3}$$

$$= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-1, -3\} \\ dt=dx \mid t \in \mathbb{R} - \{\pm 1\} \end{array} \right]$$

Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5}$$

$$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} \\ dt=dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dt}{t^2+1}$$

$$\int \frac{dx}{x^2+4x+3}$$

$$= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-1, -3\} \\ dt=dx \mid t \in \mathbb{R} - \{\pm 1\} \end{array} \right]$$

$$= \int \frac{dt}{t^2-1}$$

Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5}$$

$$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} \\ dt=dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dt}{t^2+1} = \operatorname{arctg} t + c$$

$$\int \frac{dx}{x^2+4x+3}$$

$$= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-1, -3\} \\ dt=dx \mid t \in \mathbb{R} - \{\pm 1\} \end{array} \right]$$

$$= \int \frac{dt}{t^2-1} = \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5} = \operatorname{arctg}(x+2) + c$$

$$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} \\ dt=dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dt}{t^2+1} = \operatorname{arctg} t + c = \operatorname{arctg}(x+2) + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$\int \frac{dx}{x^2+4x+3}$$

$$= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-1, -3\} \\ dt=dx \mid t \in \mathbb{R} - \{\pm 1\} \end{array} \right]$$

$$= \int \frac{dt}{t^2-1} = \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c = \frac{1}{2} \ln \left| \frac{x+2-1}{x+2+1} \right| + c$$

Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5} = \operatorname{arctg}(x+2) + c$$

$$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in R \\ dt=dx \mid t \in R \end{array} \right]$$

$$= \int \frac{dt}{t^2+1} = \operatorname{arctg} t + c = \operatorname{arctg}(x+2) + c, \quad x \in R, \quad c \in R.$$

$$\int \frac{dx}{x^2+4x+3} = \frac{1}{2} \ln \left| \frac{x+1}{x+3} \right| + c$$

$$= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in R - \{-1, -3\} \\ dt=dx \mid t \in R - \{\pm 1\} \end{array} \right]$$

$$= \int \frac{dt}{t^2-1} = \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c = \frac{1}{2} \ln \left| \frac{x+2-1}{x+2+1} \right| + c = \frac{1}{2} \ln \left| \frac{x+1}{x+3} \right| + c,$$

$$x \in R - \{-1, -3\}, \quad c \in R.$$

Riešené príklady – 103, 104

$$\int \frac{dx}{x^2 - 4x + 6}$$

$$\int \frac{dx}{x^2 - 4x + 2}$$

Riešené príklady – 103, 104

$$\int \frac{dx}{x^2 - 4x + 6}$$

$$= \int \frac{dx}{x^2 - 4x + 4 + 2} = \int \frac{dx}{(x-2)^2 + (\sqrt{2})^2}$$

$$\int \frac{dx}{x^2 - 4x + 2}$$

$$= \int \frac{dx}{x^2 - 4x + 4 - 2} = \int \frac{dx}{(x-2)^2 - (\sqrt{2})^2}$$

Riešené príklady – 103, 104

$$\int \frac{dx}{x^2 - 4x + 6}$$

$$= \int \frac{dx}{x^2 - 4x + 4 + 2} = \int \frac{dx}{(x-2)^2 + (\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } t = x-2 \mid x \in \mathbb{R} \\ dt = dx \mid t \in \mathbb{R} \end{array} \right]$$

$$\int \frac{dx}{x^2 - 4x + 2}$$

$$= \int \frac{dx}{x^2 - 4x + 4 - 2} = \int \frac{dx}{(x-2)^2 - (\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } t = x-2 \mid x \in \mathbb{R} - \{2 \pm \sqrt{2}\} \\ dt = dx \mid t \in \mathbb{R} - \{\pm \sqrt{2}\} \end{array} \right]$$

Riešené príklady – 103, 104

$$\int \frac{dx}{x^2 - 4x + 6}$$

$$= \int \frac{dx}{x^2 - 4x + 4 + 2} = \int \frac{dx}{(x-2)^2 + (\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x \in \mathbb{R} \\ dt = dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dt}{t^2 + (\sqrt{2})^2}$$

$$\int \frac{dx}{x^2 - 4x + 2}$$

$$= \int \frac{dx}{x^2 - 4x + 4 - 2} = \int \frac{dx}{(x-2)^2 - (\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x \in \mathbb{R} - \{2 \pm \sqrt{2}\} \\ dt = dx \mid t \in \mathbb{R} - \{\pm \sqrt{2}\} \end{array} \right]$$

$$= \int \frac{dt}{t^2 - (\sqrt{2})^2}$$

Riešené príklady – 103, 104

$$\int \frac{dx}{x^2 - 4x + 6}$$

$$= \int \frac{dx}{x^2 - 4x + 4 + 2} = \int \frac{dx}{(x-2)^2 + (\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x \in \mathbb{R} \\ dt = dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dt}{t^2 + (\sqrt{2})^2} = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + c$$

$$\int \frac{dx}{x^2 - 4x + 2}$$

$$= \int \frac{dx}{x^2 - 4x + 4 - 2} = \int \frac{dx}{(x-2)^2 - (\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x \in \mathbb{R} - \{2 \pm \sqrt{2}\} \\ dt = dx \mid t \in \mathbb{R} - \{\pm \sqrt{2}\} \end{array} \right]$$

$$= \int \frac{dt}{t^2 - (\sqrt{2})^2} = \frac{1}{2\sqrt{2}} \ln \left| \frac{t - \sqrt{2}}{t + \sqrt{2}} \right| + c$$

Riešené príklady – 103, 104

$$\int \frac{dx}{x^2-4x+6} = \arctg(x+2) + c$$

$$= \int \frac{dx}{x^2-4x+4+2} = \int \frac{dx}{(x-2)^2+(\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in R \\ dt=dx \mid t \in R \end{array} \right]$$

$$= \int \frac{dt}{t^2+(\sqrt{2})^2} = \frac{1}{\sqrt{2}} \arctg \frac{t}{\sqrt{2}} + c = \frac{1}{\sqrt{2}} \arctg \frac{x-2}{\sqrt{2}} + c, \quad x \in R, \quad c \in R.$$

$$\int \frac{dx}{x^2-4x+2} = \frac{1}{2} \ln \left| \frac{x-2-\sqrt{2}}{x-2+\sqrt{2}} \right| + c$$

$$= \int \frac{dx}{x^2-4x+4-2} = \int \frac{dx}{(x-2)^2-(\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in R - \{2 \pm \sqrt{2}\} \\ dt=dx \mid t \in R - \{\pm \sqrt{2}\} \end{array} \right]$$

$$= \int \frac{dt}{t^2-(\sqrt{2})^2} = \frac{1}{2\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c = \frac{1}{2\sqrt{2}} \ln \left| \frac{x-2-\sqrt{2}}{x-2+\sqrt{2}} \right| + c,$$

$$x \in R - \{2 \pm \sqrt{2}\}, \quad c \in R.$$

Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4}$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}}$$

Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4}$$

$$= \int \frac{dx}{(x+2)^2}$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}}$$

$$= \int \frac{dx}{\sqrt{(x+2)^2}} = \int \frac{dx}{|x+2|}$$

Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4}$$

$$= \int \frac{dx}{(x+2)^2} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right]$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}}$$

$$= \int \frac{dx}{\sqrt{(x+2)^2}} = \int \frac{dx}{|x+2|} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right]$$

Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4}$$

$$= \int \frac{dx}{(x+2)^2} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right]$$

$$= \int \frac{dt}{t^2} = \int t^{-2} dt$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}}$$

$$= \int \frac{dx}{\sqrt{(x+2)^2}} = \int \frac{dx}{|x+2|} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{dt}{|t|}$$

Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4}$$

$$= \int \frac{dx}{(x+2)^2} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right]$$

$$= \int \frac{dt}{t^2} = \int t^{-2} dt = \frac{t^{-2+1}}{-2+1} + c$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}}$$

$$= \int \frac{dx}{\sqrt{(x+2)^2}} = \int \frac{dx}{|x+2|} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{dt}{|t|}$$

$$\boxed{t > 0} = \int \frac{dt}{t}$$

$$\boxed{t < 0} = \int \frac{dt}{-t} = - \int \frac{dt}{t}$$

Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4}$$

$$= \int \frac{dx}{(x+2)^2} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right]$$

$$= \int \frac{dt}{t^2} = \int t^{-2} dt = \frac{t^{-2+1}}{-2+1} + c = -\frac{1}{t} + c$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}}$$

$$= \int \frac{dx}{\sqrt{(x+2)^2}} = \int \frac{dx}{|x+2|} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{dt}{|t|}$$

$$\boxed{t > 0} = \int \frac{dt}{t} = \ln |t| + c$$

$$\boxed{t < 0} = \int \frac{dt}{-t} = -\int \frac{dt}{t} = -\ln |t| + c$$

Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4} = -\frac{1}{x+2} + c$$

$$= \int \frac{dx}{(x+2)^2} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right]$$

$$= \int \frac{dt}{t^2} = \int t^{-2} dt = \frac{t^{-2+1}}{-2+1} + c = -\frac{1}{t} + c = -\frac{1}{x+2} + c, \quad x \in \mathbb{R} - \{-2\}, \quad c \in \mathbb{R}.$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}} = \operatorname{sgn}(x+2) \cdot \ln |x+2| + c$$

$$= \int \frac{dx}{\sqrt{(x+2)^2}} = \int \frac{dx}{|x+2|} = \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} - \{-2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{dt}{|t|}$$

$$\boxed{t > 0} = \int \frac{dt}{t} = \ln |t| + c = \ln |x+2| + c, \quad x \in (-2; \infty), \quad c \in \mathbb{R},$$

$$\boxed{t < 0} = \int \frac{dt}{-t} = -\int \frac{dt}{t} = -\ln |t| + c = -\ln |x+2| + c, \quad x \in (-\infty; -2), \quad c \in \mathbb{R}.$$

Riešené príklady – 107, 108

$$\int \frac{dx}{\sqrt{x^2+4x+5}}$$

$$\int \frac{dx}{\sqrt{x^2+4x+3}}$$

Riešené príklady – 107, 108

$$\int \frac{dx}{\sqrt{x^2+4x+5}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4+1}} = \int \frac{dx}{\sqrt{(x+2)^2+1}}$$

$$\int \frac{dx}{\sqrt{x^2+4x+3}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4-1}} = \int \frac{dx}{\sqrt{(x+2)^2-1}}$$

Riešené príklady – 107, 108

$$\int \frac{dx}{\sqrt{x^2+4x+5}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4+1}} = \int \frac{dx}{\sqrt{(x+2)^2+1}} \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} \\ dt=dx \mid t \in \mathbb{R} \end{array} \right]$$

$$\int \frac{dx}{\sqrt{x^2+4x+3}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4-1}} = \int \frac{dx}{\sqrt{(x+2)^2-1}} \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in (-\infty; -3), t \in (-\infty; -1) \\ dt=dx \mid x \in (-1; \infty), t \in (1; \infty) \end{array} \right]$$

Riešené príklady – 107, 108

$$\int \frac{dx}{\sqrt{x^2+4x+5}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4+1}} = \int \frac{dx}{\sqrt{(x+2)^2+1}} \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} \\ dt=dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2+1}}$$

$$\int \frac{dx}{\sqrt{x^2+4x+3}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4-1}} = \int \frac{dx}{\sqrt{(x+2)^2-1}} \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in (-\infty; -3), t \in (-\infty; -1) \\ dt=dx \mid x \in (-1; \infty), t \in (1; \infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2-1}}$$

Riešené príklady – 107, 108

$$\int \frac{dx}{\sqrt{x^2+4x+5}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4+1}} = \int \frac{dx}{\sqrt{(x+2)^2+1}} \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} \\ dt=dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2+1}} = \ln(t + \sqrt{t^2+1}) + c$$

$$\int \frac{dx}{\sqrt{x^2+4x+3}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4-1}} = \int \frac{dx}{\sqrt{(x+2)^2-1}} \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in (-\infty; -3), t \in (-\infty; -1) \\ dt=dx \mid x \in (-1; \infty), t \in (1; \infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2-1}} = \ln|t + \sqrt{t^2-1}| + c$$

Riešené príklady – 107, 108

$$\int \frac{dx}{\sqrt{x^2+4x+5}}$$

$$\begin{aligned}
 &= \int \frac{dx}{\sqrt{x^2+4x+4+1}} = \int \frac{dx}{\sqrt{(x+2)^2+1}} \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} \\ dt=dx \mid t \in \mathbb{R} \end{array} \right] \\
 &= \int \frac{dt}{\sqrt{t^2+1}} = \ln(t + \sqrt{t^2+1}) + c = \ln(x+2 + \sqrt{(x+2)^2+1}) + c
 \end{aligned}$$

$$\int \frac{dx}{\sqrt{x^2+4x+3}}$$

$$\begin{aligned}
 &= \int \frac{dx}{\sqrt{x^2+4x+4-1}} = \int \frac{dx}{\sqrt{(x+2)^2-1}} \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in (-\infty; -3), t \in (-\infty; -1) \\ dt=dx \mid x \in (-1; \infty), t \in (1; \infty) \end{array} \right] \\
 &= \int \frac{dt}{\sqrt{t^2-1}} = \ln|t + \sqrt{t^2-1}| + c = \ln|x+2 + \sqrt{(x+2)^2-1}| + c
 \end{aligned}$$

Riešené príklady – 107, 108

$$\int \frac{dx}{\sqrt{x^2+4x+5}} = \ln(x+2+\sqrt{x^2+4x+5})+c$$

$$\begin{aligned} &= \int \frac{dx}{\sqrt{x^2+4x+4+1}} = \int \frac{dx}{\sqrt{(x+2)^2+1}} \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in \mathbb{R} \\ dt=dx \mid t \in \mathbb{R} \end{array} \right] \\ &= \int \frac{dt}{\sqrt{t^2+1}} = \ln(t+\sqrt{t^2+1})+c = \ln(x+2+\sqrt{(x+2)^2+1})+c \\ &= \ln(x+2+\sqrt{x^2+4x+5})+c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}. \end{aligned}$$

$$\int \frac{dx}{\sqrt{x^2+4x+3}} = \ln|x+2+\sqrt{x^2+4x+3}|+c$$

$$\begin{aligned} &= \int \frac{dx}{\sqrt{x^2+4x+4-1}} = \int \frac{dx}{\sqrt{(x+2)^2-1}} \left[\begin{array}{l} \text{Subst. } t=x+2 \mid x \in (-\infty; -3), t \in (-\infty; -1) \\ dt=dx \mid x \in (-1; \infty), t \in (1; \infty) \end{array} \right] \\ &= \int \frac{dt}{\sqrt{t^2-1}} = \ln|t+\sqrt{t^2-1}|+c = \ln|x+2+\sqrt{(x+2)^2-1}|+c \\ &= \ln|x+2+\sqrt{x^2+4x+3}|+c, \quad x \in (-\infty; -3) \cup (-1; \infty), \quad c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2 - 4x + 6}}$$

$$\int \frac{dx}{\sqrt{x^2 - 4x + 2}}$$

Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2 - 4x + 6}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 4x + 4 + 2}} = \int \frac{dx}{\sqrt{(x-2)^2 + (\sqrt{2})^2}}$$

$$\int \frac{dx}{\sqrt{x^2 - 4x + 2}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 4x + 4 - 2}} = \int \frac{dx}{\sqrt{(x-2)^2 - (\sqrt{2})^2}}$$

Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2 - 4x + 6}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 4x + 4 + 2}} = \int \frac{dx}{\sqrt{(x-2)^2 + (\sqrt{2})^2}} = \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x \in \mathbb{R} \\ dt = dx \mid t \in \mathbb{R} \end{array} \right]$$

$$\int \frac{dx}{\sqrt{x^2 - 4x + 2}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 4x + 4 - 2}} = \int \frac{dx}{\sqrt{(x-2)^2 - (\sqrt{2})^2}} = \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x \in (-\infty; 2 - \sqrt{2}), t \in (-\infty; -\sqrt{2}) \\ dt = dx \mid x \in (2 + \sqrt{2}; \infty), t \in (\sqrt{2}; \infty) \end{array} \right]$$

Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2 - 4x + 6}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 4x + 4 + 2}} = \int \frac{dx}{\sqrt{(x-2)^2 + (\sqrt{2})^2}} \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x \in \mathbb{R} \\ dt = dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2 + (\sqrt{2})^2}}$$

$$\int \frac{dx}{\sqrt{x^2 - 4x + 2}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 4x + 4 - 2}} = \int \frac{dx}{\sqrt{(x-2)^2 - (\sqrt{2})^2}} \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x \in (-\infty; 2 - \sqrt{2}), t \in (-\infty; -\sqrt{2}) \\ dt = dx \mid x \in (2 + \sqrt{2}; \infty), t \in (\sqrt{2}; \infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2 - (\sqrt{2})^2}}$$

Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2 - 4x + 6}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 4x + 4 + 2}} = \int \frac{dx}{\sqrt{(x-2)^2 + (\sqrt{2})^2}} \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x \in \mathbb{R} \\ dt = dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2 + (\sqrt{2})^2}} = \ln(t + \sqrt{t^2 + (\sqrt{2})^2}) + c$$

$$\int \frac{dx}{\sqrt{x^2 - 4x + 2}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 4x + 4 - 2}} = \int \frac{dx}{\sqrt{(x-2)^2 - (\sqrt{2})^2}} \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x \in (-\infty; 2 - \sqrt{2}), t \in (-\infty; -\sqrt{2}) \\ dt = dx \mid x \in (2 + \sqrt{2}; \infty), t \in (\sqrt{2}; \infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2 - (\sqrt{2})^2}} = \ln |t + \sqrt{t^2 - (\sqrt{2})^2}| + c$$

Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2 - 4x + 6}}$$

$$\begin{aligned}
 &= \int \frac{dx}{\sqrt{x^2 - 4x + 4 + 2}} = \int \frac{dx}{\sqrt{(x-2)^2 + (\sqrt{2})^2}} \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x \in \mathbb{R} \\ dt = dx \mid t \in \mathbb{R} \end{array} \right] \\
 &= \int \frac{dt}{\sqrt{t^2 + (\sqrt{2})^2}} = \ln(t + \sqrt{t^2 + (\sqrt{2})^2}) + c = \ln(x + 2 + \sqrt{(x-2)^2 + 4}) + c
 \end{aligned}$$

$$\int \frac{dx}{\sqrt{x^2 - 4x + 2}}$$

$$\begin{aligned}
 &= \int \frac{dx}{\sqrt{x^2 - 4x + 4 - 2}} = \int \frac{dx}{\sqrt{(x-2)^2 - (\sqrt{2})^2}} \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x \in (-\infty; 2 - \sqrt{2}), t \in (-\infty; -\sqrt{2}) \\ dt = dx \mid x \in (2 + \sqrt{2}; \infty), t \in (\sqrt{2}; \infty) \end{array} \right] \\
 &= \int \frac{dt}{\sqrt{t^2 - (\sqrt{2})^2}} = \ln|t + \sqrt{t^2 - (\sqrt{2})^2}| + c = \ln|x - 2 + \sqrt{(x-2)^2 - 2}| + c
 \end{aligned}$$

Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2-4x+6}} = \ln(x-2+\sqrt{x^2-4x+6})+c$$

$$= \int \frac{dx}{\sqrt{x^2-4x+4+2}} = \int \frac{dx}{\sqrt{(x-2)^2+(\sqrt{2})^2}} \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in \mathbb{R} \\ dt=dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2+(\sqrt{2})^2}} = \ln(t+\sqrt{t^2+(\sqrt{2})^2})+c = \ln(x+2+\sqrt{(x-2)^2+4})+c$$

$$= \ln(x-2+\sqrt{x^2-4x+6})+c, x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \frac{dx}{\sqrt{x^2-4x+2}} = \ln|x-2+\sqrt{x^2-4x+2}|+c$$

$$= \int \frac{dx}{\sqrt{x^2-4x+4-2}} = \int \frac{dx}{\sqrt{(x-2)^2-(\sqrt{2})^2}} \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in (-\infty; 2-\sqrt{2}), t \in (-\infty; -\sqrt{2}) \\ dt=dx \mid x \in (2+\sqrt{2}; \infty), t \in (\sqrt{2}; \infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2-(\sqrt{2})^2}} = \ln|t+\sqrt{t^2-(\sqrt{2})^2}|+c = \ln|x-2+\sqrt{(x+2)^2-2}|+c$$

$$= \ln|x-2+\sqrt{x^2-4x+2}|+c, x \in (-\infty; 2-\sqrt{2}) \cup (2+\sqrt{2}; \infty), c \in \mathbb{R}.$$

Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}}$$

$$\int \frac{dx}{\sqrt{-x^2+4x-3}}$$

Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+5)}}$$

$$\int \frac{dx}{\sqrt{-x^2+4x-3}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+3)}}$$

Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+5)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4+1)}}$$

$$\int \frac{dx}{\sqrt{-x^2+4x-3}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+3)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4-1)}}$$

Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+5)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4+1)}} = \int \frac{dx}{\sqrt{-(x-2)^2-1}}$$

$$\int \frac{dx}{\sqrt{-x^2+4x-3}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+3)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4-1)}} = \int \frac{dx}{\sqrt{-(x-2)^2+1}} = \int \frac{dx}{\sqrt{1-(x-2)^2}}$$

Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+5)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4+1)}} = \int \frac{dx}{\sqrt{-(x-2)^2-1}} \left[\begin{array}{l} -(x-2)^2-1 \geq 1 > 0 \\ \text{pre všetky } x \in \mathbb{R} \end{array} \right]$$

$$\int \frac{dx}{\sqrt{-x^2+4x-3}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+3)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4-1)}} = \int \frac{dx}{\sqrt{-(x-2)^2+1}} = \int \frac{dx}{\sqrt{1-(x-2)^2}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in (1;3) \\ dt=dx \mid t \in (-1;1) \end{array} \right]$$

Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}} \text{ nemá riešenie}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+5)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4+1)}} = \int \frac{dx}{\sqrt{-(x-2)^2-1}} \left[\begin{array}{l} -(x-2)^2-1 \geq 1 > 0 \\ \text{pre všetky } x \in \mathbb{R} \end{array} \right].$$

Neexistuje riešenie pre žiadne $x \in \mathbb{R}$.

$$\int \frac{dx}{\sqrt{-x^2+4x-3}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+3)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4-1)}} = \int \frac{dx}{\sqrt{-(x-2)^2+1}} = \int \frac{dx}{\sqrt{1-(x-2)^2}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in (1;3) \\ dt=dx \mid t \in (-1;1) \end{array} \right] = \int \frac{dt}{\sqrt{1-t^2}}$$

Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}} \text{ nemá riešenie}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+5)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4+1)}} = \int \frac{dx}{\sqrt{-(x-2)^2-1}} \left[\begin{array}{l} -(x-2)^2-1 \geq 1 > 0 \\ \text{pre všetky } x \in \mathbb{R} \end{array} \right].$$

Neexistuje riešenie pre žiadne $x \in \mathbb{R}$.

$$\int \frac{dx}{\sqrt{-x^2+4x-3}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+3)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4-1)}} = \int \frac{dx}{\sqrt{-(x-2)^2+1}} = \int \frac{dx}{\sqrt{1-(x-2)^2}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in (1;3) \\ dt=dx \mid t \in (-1;1) \end{array} \right] = \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \arcsin t + c_1 = -\arccos t + c_2$$

Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}} \text{ nemá riešenie}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+5)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4+1)}} = \int \frac{dx}{\sqrt{-(x-2)^2-1}} \left[\begin{array}{l} -(x-2)^2-1 \geq 1 > 0 \\ \text{pre všetky } x \in \mathbb{R} \end{array} \right].$$

Neexistuje riešenie pre žiadne $x \in \mathbb{R}$.

$$\int \frac{dx}{\sqrt{-x^2+4x-3}} = \arcsin(x-2) + c_1 = -\arccos(x-2) + c_2$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+3)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4-1)}} = \int \frac{dx}{\sqrt{-(x-2)^2+1}} = \int \frac{dx}{\sqrt{1-(x-2)^2}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in (1;3) \\ dt=dx \mid t \in (-1;1) \end{array} \right] = \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \arcsin t + c_1 = -\arccos t + c_2$$

$$= \arcsin(x-2) + c_1 = -\arccos(x-2) + c_2, \quad x \in (1;3), \quad c_1, c_2 \in \mathbb{R}.$$

Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}}$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}}$$

Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}}$$

$$= \int \frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}} = \int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}}$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}}$$

Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}}$$

$$= \int \frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}} = \int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}} = \left[\begin{array}{l} \text{Subst. } t=x-\frac{1}{2} \mid x \in (-\infty; 0), t \in (-\infty; -\frac{1}{2}) \\ dt=dx \mid x \in (1; \infty), t \in (\frac{1}{2}; \infty) \end{array} \right]$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}}$$

Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}}$$

$$= \int \frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}} = \int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}} = \left[\begin{array}{l} \text{Subst. } t=x-\frac{1}{2} \mid x \in (-\infty; 0), t \in (-\infty; -\frac{1}{2}) \\ dt=dx \mid x \in (1; \infty), t \in (\frac{1}{2}; \infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2-(\frac{1}{2})^2}}$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} = \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in (0; 2) \\ dt=dx \mid t \in (-1; 1) \end{array} \right]$$

Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 2\frac{x}{2} + \frac{1}{4} - \frac{1}{4}}} = \int \frac{dx}{\sqrt{(x-\frac{1}{2})^2 - (\frac{1}{2})^2}} \left[\begin{array}{l} \text{Subst. } t = x - \frac{1}{2} \mid x \in (-\infty; 0), t \in (-\infty; -\frac{1}{2}) \\ dt = dx \mid x \in (1; \infty), t \in (\frac{1}{2}; \infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2 - (\frac{1}{2})^2}} = \ln \left| t + \sqrt{t^2 - (\frac{1}{2})^2} \right| + c_1$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} \left[\begin{array}{l} \text{Subst. } t = x - 1 \mid x \in (0; 2) \\ dt = dx \mid t \in (-1; 1) \end{array} \right] = \int \frac{dt}{\sqrt{1-t^2}}$$

Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}}$$

$$= \int \frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}} = \int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}} \left[\begin{array}{l} \text{Subst. } t=x-\frac{1}{2} \mid x \in (-\infty; 0), t \in (-\infty; -\frac{1}{2}) \\ dt=dx \mid x \in (1; \infty), t \in (\frac{1}{2}; \infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2-(\frac{1}{2})^2}} = \ln \left| t + \sqrt{t^2-(\frac{1}{2})^2} \right| + c_1 = \ln \left| x - \frac{1}{2} + \sqrt{x^2-x} \right| + c_1$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in (0; 2) \\ dt=dx \mid t \in (-1; 1) \end{array} \right] = \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \arcsin t + c_1 = -\arccos t + c_2$$

Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}} = \ln \left| x - \frac{1}{2} + \sqrt{x^2-x} \right| + c_1$$

$$= \int \frac{dx}{\sqrt{x^2 - 2\frac{x}{2} + \frac{1}{4} - \frac{1}{4}}} = \int \frac{dx}{\sqrt{(x - \frac{1}{2})^2 - (\frac{1}{2})^2}} \left[\begin{array}{l} \text{Subst. } t = x - \frac{1}{2} \mid x \in (-\infty; 0), t \in (-\infty; -\frac{1}{2}) \\ dt = dx \mid x \in (1; \infty), t \in (\frac{1}{2}; \infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2 - (\frac{1}{2})^2}} = \ln \left| t + \sqrt{t^2 - (\frac{1}{2})^2} \right| + c_1 = \ln \left| x - \frac{1}{2} + \sqrt{x^2-x} \right| + c_1$$

$$= \ln \left| \frac{2x-1 + \sqrt{x^2-x}}{2} \right| + c_1$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}} = \arcsin(x-1) + c_1 = -\arccos(x-1) + c_2$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} \left[\begin{array}{l} \text{Subst. } t = x-1 \mid x \in (0; 2) \\ dt = dx \mid t \in (-1; 1) \end{array} \right] = \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \arcsin t + c_1 = -\arccos t + c_2 = \arcsin(x-1) + c_1 = -\arccos(x-1) + c_2,$$

$$x \in (0; 2), c_1, c_2 \in \mathbb{R}.$$

Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}} = \ln \left| x - \frac{1}{2} + \sqrt{x^2-x} \right| + c_1$$

$$= \int \frac{dx}{\sqrt{x^2 - 2\frac{x}{2} + \frac{1}{4} - \frac{1}{4}}} = \int \frac{dx}{\sqrt{(x - \frac{1}{2})^2 - (\frac{1}{2})^2}} \left[\begin{array}{l} \text{Subst. } t = x - \frac{1}{2} \mid x \in (-\infty; 0), t \in (-\infty; -\frac{1}{2}) \\ dt = dx \mid x \in (1; \infty), t \in (\frac{1}{2}; \infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2 - (\frac{1}{2})^2}} = \ln \left| t + \sqrt{t^2 - (\frac{1}{2})^2} \right| + c_1 = \ln \left| x - \frac{1}{2} + \sqrt{x^2-x} \right| + c_1$$

$$= \ln \left| \frac{2x-1 + \sqrt{x^2-x}}{2} \right| + c_1 = \ln |2x-1 + \sqrt{x^2-x}| - \ln 2 + c_1$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}} = \arcsin(x-1) + c_1 = -\arccos(x-1) + c_2$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} \left[\begin{array}{l} \text{Subst. } t = x-1 \mid x \in (0; 2) \\ dt = dx \mid t \in (-1; 1) \end{array} \right] = \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \arcsin t + c_1 = -\arccos t + c_2 = \arcsin(x-1) + c_1 = -\arccos(x-1) + c_2,$$

$$x \in (0; 2), c_1, c_2 \in \mathbb{R}.$$

Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}} = \ln \left| x - \frac{1}{2} + \sqrt{x^2-x} \right| + c_1$$

$$= \int \frac{dx}{\sqrt{x^2 - 2\frac{x}{2} + \frac{1}{4} - \frac{1}{4}}} = \int \frac{dx}{\sqrt{(x - \frac{1}{2})^2 - (\frac{1}{2})^2}} \left[\begin{array}{l} \text{Subst. } t = x - \frac{1}{2} \mid x \in (-\infty; 0), t \in (-\infty; -\frac{1}{2}) \\ dt = dx \mid x \in (1; \infty), t \in (\frac{1}{2}; \infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2 - (\frac{1}{2})^2}} = \ln \left| t + \sqrt{t^2 - (\frac{1}{2})^2} \right| + c_1 = \ln \left| x - \frac{1}{2} + \sqrt{x^2-x} \right| + c_1$$

$$= \ln \left| \frac{2x-1+\sqrt{x^2-x}}{2} \right| + c_1 = \ln |2x-1+\sqrt{x^2-x}| - \ln 2 + c_1 = \left[\begin{array}{l} c_2 = c_1 - \ln 2 \\ c_1 \in R, c_2 \in R \end{array} \right]$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}} = \arcsin(x-1) + c_1 = -\arccos(x-1) + c_2$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} \left[\begin{array}{l} \text{Subst. } t = x-1 \mid x \in (0; 2) \\ dt = dx \mid t \in (-1; 1) \end{array} \right] = \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \arcsin t + c_1 = -\arccos t + c_2 = \arcsin(x-1) + c_1 = -\arccos(x-1) + c_2, \\ x \in (0; 2), c_1, c_2 \in R.$$

Riešené príklady – 113, 114

$$\begin{aligned}
 \int \frac{dx}{\sqrt{x(x-1)}} &= \int \frac{dx}{\sqrt{x^2-x}} = \ln \left| x - \frac{1}{2} + \sqrt{x^2-x} \right| + c_1 = \ln \left| 2x-1 + \sqrt{x^2-x} \right| + c_2 \\
 &= \int \frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}} = \int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}} \left[\begin{array}{l} \text{Subst. } t=x-\frac{1}{2} \mid x \in (-\infty; 0), t \in (-\infty; -\frac{1}{2}) \\ dt=dx \mid x \in (1; \infty), t \in (\frac{1}{2}; \infty) \end{array} \right] \\
 &= \int \frac{dt}{\sqrt{t^2-(\frac{1}{2})^2}} = \ln \left| t + \sqrt{t^2-(\frac{1}{2})^2} \right| + c_1 = \ln \left| x - \frac{1}{2} + \sqrt{x^2-x} \right| + c_1 \\
 &= \ln \left| \frac{2x-1+\sqrt{x^2-x}}{2} \right| + c_1 = \ln \left| 2x-1 + \sqrt{x^2-x} \right| - \ln 2 + c_1 = \left[\begin{array}{l} c_2 = c_1 - \ln 2 \\ c_1 \in R, c_2 \in R \end{array} \right] \\
 &= \ln \left| 2x-1 + \sqrt{x^2-x} \right| + c_2, \quad x \in (-\infty; 0) \cup (1; \infty), \quad c \in R.
 \end{aligned}$$

$$\begin{aligned}
 \int \frac{dx}{\sqrt{x(2-x)}} &= \int \frac{dx}{\sqrt{-x^2+2x}} = \arcsin(x-1) + c_1 = -\arccos(x-1) + c_2 \\
 &= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in (0; 2) \\ dt=dx \mid t \in (-1; 1) \end{array} \right] = \int \frac{dt}{\sqrt{1-t^2}} \\
 &= \arcsin t + c_1 = -\arccos t + c_2 = \arcsin(x-1) + c_1 = -\arccos(x-1) + c_2, \\
 &\quad x \in (0; 2), \quad c_1, c_2 \in R.
 \end{aligned}$$

Riešené príklady – 115, 116

$$\int \sqrt{x^2 + 4x + 5} \, dx$$

$$\int \sqrt{x^2 + 4x + 3} \, dx$$

Riešené príklady – 115, 116

$$\int \sqrt{x^2 + 4x + 5} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + 2 \\ dt = dx \end{array} \middle| \begin{array}{l} x^2 + 4x + 5 = x^2 + 4x + 4 + 1 \\ = (x + 2)^2 + 1 = t^2 + 1 \end{array} \right| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right]$$

$$\int \sqrt{x^2 + 4x + 3} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + 2 \\ dt = dx \end{array} \middle| \begin{array}{l} x^2 + 4x + 3 = x^2 + 4x + 4 - 1 \\ = (x + 2)^2 - 1 = t^2 - 1 \end{array} \right| \begin{array}{l} x \in (-\infty; -3), t \in (-\infty; -1) \\ x \in (-1; \infty), t \in (1; \infty) \end{array} \right]$$

Riešené príklady – 115, 116

$$\int \sqrt{x^2 + 4x + 5} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + 2 \\ dt = dx \end{array} \left| \begin{array}{l} x^2 + 4x + 5 = x^2 + 4x + 4 + 1 \\ = (x + 2)^2 + 1 = t^2 + 1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \sqrt{t^2 + 1} \, dt$$

$$\int \sqrt{x^2 + 4x + 3} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + 2 \\ dt = dx \end{array} \left| \begin{array}{l} x^2 + 4x + 3 = x^2 + 4x + 4 - 1 \\ = (x + 2)^2 - 1 = t^2 - 1 \end{array} \right. \begin{array}{l} x \in (-\infty; -3), t \in (-\infty; -1) \\ x \in (-1; \infty), t \in (1; \infty) \end{array} \right] = \int \sqrt{t^2 - 1} \, dt$$

Riešené príklady – 115, 116

$$\int \sqrt{x^2 + 4x + 5} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + 2 \\ dt = dx \end{array} \left| \begin{array}{l} x^2 + 4x + 5 = x^2 + 4x + 4 + 1 \\ = (x + 2)^2 + 1 = t^2 + 1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \sqrt{t^2 + 1} \, dt = \frac{t\sqrt{t^2 + 1}}{2} + \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + 1}}$$

$$\int \sqrt{x^2 + 4x + 3} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + 2 \\ dt = dx \end{array} \left| \begin{array}{l} x^2 + 4x + 3 = x^2 + 4x + 4 - 1 \\ = (x + 2)^2 - 1 = t^2 - 1 \end{array} \right. \begin{array}{l} x \in (-\infty; -3), t \in (-\infty; -1) \\ x \in (-1; \infty), t \in (1; \infty) \end{array} \right] = \int \sqrt{t^2 - 1} \, dt$$

$$= \frac{t\sqrt{t^2 - 1}}{2} - \frac{1}{2} \int \frac{dt}{\sqrt{t^2 - 1}}$$

Riešené príklady – 115, 116

$$\int \sqrt{x^2 + 4x + 5} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + 2 \\ dt = dx \end{array} \left| \begin{array}{l} x^2 + 4x + 5 = x^2 + 4x + 4 + 1 \\ = (x + 2)^2 + 1 = t^2 + 1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \sqrt{t^2 + 1} \, dt = \frac{t\sqrt{t^2 + 1}}{2} + \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + 1}}$$

$$= \frac{t\sqrt{t^2 + 1}}{2} + \frac{\ln(t + \sqrt{t^2 + 1})}{2} + C$$

$$\int \sqrt{x^2 + 4x + 3} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + 2 \\ dt = dx \end{array} \left| \begin{array}{l} x^2 + 4x + 3 = x^2 + 4x + 4 - 1 \\ = (x + 2)^2 - 1 = t^2 - 1 \end{array} \right. \begin{array}{l} x \in (-\infty; -3), t \in (-\infty; -1) \\ x \in (-1; \infty), t \in (1; \infty) \end{array} \right] = \int \sqrt{t^2 - 1} \, dt$$

$$= \frac{t\sqrt{t^2 - 1}}{2} - \frac{1}{2} \int \frac{dt}{\sqrt{t^2 - 1}} = \frac{t\sqrt{t^2 - 1}}{2} - \frac{\ln|t + \sqrt{t^2 - 1}|}{2} + C$$

Riešené príklady – 115, 116

$$\int \sqrt{x^2+4x+5} dx = \frac{(x+2)\sqrt{x^2+4x+5}}{2} + \frac{\ln(x+2+\sqrt{x^2+4x+5})}{2} + c$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \sqrt{t^2+1} dt = \frac{t\sqrt{t^2+1}}{2} + \frac{1}{2} \int \frac{dt}{\sqrt{t^2+1}} \\ &= \frac{t\sqrt{t^2+1}}{2} + \frac{\ln(t+\sqrt{t^2+1})}{2} + c = \frac{(x+2)\sqrt{(x+2)^2+1}}{2} + \frac{\ln(x+2+\sqrt{(x+2)^2+1})}{2} + c \\ &= \frac{(x+2)\sqrt{x^2+4x+5}}{2} + \frac{\ln(x+2+\sqrt{x^2+4x+5})}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}. \end{aligned}$$

$$\int \sqrt{x^2+4x+3} dx = \frac{(x+2)\sqrt{x^2+4x+3}}{2} - \frac{\ln|x+2+\sqrt{x^2+4x+3}|}{2} + c$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+3=x^2+4x+4-1 \\ = (x+2)^2-1=t^2-1 \end{array} \right| \begin{array}{l} x \in (-\infty; -3), \quad t \in (-\infty; -1) \\ x \in (-1; \infty), \quad t \in (1; \infty) \end{array} \right] = \int \sqrt{t^2-1} dt \\ &= \frac{t\sqrt{t^2-1}}{2} - \frac{1}{2} \int \frac{dt}{\sqrt{t^2-1}} = \frac{t\sqrt{t^2-1}}{2} - \frac{\ln|t+\sqrt{t^2-1}|}{2} + c \\ &= \frac{(x+2)\sqrt{x^2+4x+3}}{2} - \frac{\ln|x+2+\sqrt{x^2+4x+3}|}{2} + c, \quad x \in (-\infty; -3) \cup (-1; \infty), \quad c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 117, 118

$$\int \sqrt{x^2 - 4x + 6} \, dx$$

$$\int \sqrt{x^2 - 4x + 2} \, dx$$

Riešené príklady – 117, 118

$$\int \sqrt{x^2 - 4x + 6} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - 2 \\ dt = dx \end{array} \left| \begin{array}{l} x^2 - 4x + 6 = x^2 - 4x + 4 + 2 \\ = (x - 2)^2 + 2 = t^2 + (\sqrt{2})^2 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right]$$

$$\int \sqrt{x^2 - 4x + 2} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - 2 \\ dt = dx \end{array} \left| \begin{array}{l} x^2 - 4x + 2 = x^2 - 4x + 4 - 2 \\ = (x - 2)^2 - 2 = t^2 - (\sqrt{2})^2 \end{array} \right. \begin{array}{l} x \in (-\infty; -3), t \in (-\infty; -1) \\ x \in (-1; \infty), t \in (1; \infty) \end{array} \right]$$

Riešené príklady – 117, 118

$$\int \sqrt{x^2 - 4x + 6} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x^2 - 4x + 6 = x^2 - 4x + 4 + 2 \\ dt = dx \mid = (x - 2)^2 + 2 = t^2 + (\sqrt{2})^2 \end{array} \mid \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \sqrt{t^2 + (\sqrt{2})^2} \, dt$$

$$\int \sqrt{x^2 - 4x + 2} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - 2 \mid x^2 - 4x + 2 = x^2 - 4x + 4 - 2 \\ dt = dx \mid = (x - 2)^2 - 2 = t^2 - (\sqrt{2})^2 \end{array} \mid \begin{array}{l} x \in (-\infty; -3), t \in (-\infty; -1) \\ x \in (-1; \infty), t \in (1; \infty) \end{array} \right] = \int \sqrt{t^2 - (\sqrt{2})^2} \, dt$$

Riešené príklady – 117, 118

$$\int \sqrt{x^2 - 4x + 6} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x^2-4x+6=x^2+4x+4+2 \mid x \in \mathbb{R} \\ dt=dx \mid \quad \quad \quad = (x-2)^2+2=t^2+(\sqrt{2})^2 \mid t \in \mathbb{R} \end{array} \right] = \int \sqrt{t^2 + (\sqrt{2})^2} \, dt$$

$$= \frac{t\sqrt{t^2 + (\sqrt{2})^2}}{2} + \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (\sqrt{2})^2}}$$

$$\int \sqrt{x^2 - 4x + 2} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x^2-4x+2=x^2-4x+4-2 \mid x \in (-\infty; -3), t \in (-\infty; -1) \\ dt=dx \mid \quad \quad \quad = (x-2)^2-2=t^2-(\sqrt{2})^2 \mid x \in (-1; \infty), t \in (1; \infty) \end{array} \right] = \int \sqrt{t^2 - (\sqrt{2})^2} \, dt$$

$$= \frac{t\sqrt{t^2 - (\sqrt{2})^2}}{2} - \frac{1}{2} \int \frac{dt}{\sqrt{t^2 - (\sqrt{2})^2}}$$

Riešené príklady – 117, 118

$$\int \sqrt{x^2 - 4x + 6} \, dx$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t=x-2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2-4x+6=x^2-4x+4+2 \\ =(x-2)^2+2=t^2+(\sqrt{2})^2 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \sqrt{t^2 + (\sqrt{2})^2} \, dt \\
 &= \frac{t\sqrt{t^2 + (\sqrt{2})^2}}{2} + \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (\sqrt{2})^2}} = \frac{t\sqrt{t^2 + (\sqrt{2})^2}}{2} + \frac{\ln(t + \sqrt{t^2 + (\sqrt{2})^2})}{2} + c
 \end{aligned}$$

$$\int \sqrt{x^2 - 4x + 2} \, dx$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t=x-2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2-4x+2=x^2-4x+4-2 \\ =(x-2)^2-2=t^2-(\sqrt{2})^2 \end{array} \right. \begin{array}{l} x \in (-\infty; -3), t \in (-\infty; -1) \\ x \in (-1; \infty), t \in (1; \infty) \end{array} \right] = \int \sqrt{t^2 - (\sqrt{2})^2} \, dt \\
 &= \frac{t\sqrt{t^2 - (\sqrt{2})^2}}{2} - \frac{1}{2} \int \frac{dt}{\sqrt{t^2 - (\sqrt{2})^2}} = \frac{t\sqrt{t^2 - (\sqrt{2})^2}}{2} - \frac{\ln|t + \sqrt{t^2 - (\sqrt{2})^2}|}{2} + c
 \end{aligned}$$

Riešené príklady – 117, 118

$$\int \sqrt{x^2 - 4x + 6} \, dx = \frac{(x-2)\sqrt{x^2 - 4x + 6}}{2} + \frac{\ln(x-2 + \sqrt{x^2 - 4x + 6})}{2} + c$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x^2-4x+6=x^2-4x+4+2 \\ dt=dx \mid = (x-2)^2+2=t^2+(\sqrt{2})^2 \end{array} \mid \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \int \sqrt{t^2 + (\sqrt{2})^2} \, dt \\
 &= \frac{t\sqrt{t^2 + (\sqrt{2})^2}}{2} + \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (\sqrt{2})^2}} = \frac{t\sqrt{t^2 + (\sqrt{2})^2}}{2} + \frac{\ln(t + \sqrt{t^2 + (\sqrt{2})^2})}{2} + c \\
 &= \frac{(x-2)\sqrt{x^2 - 4x + 6}}{2} + \frac{\ln(x-2 + \sqrt{x^2 - 4x + 6})}{2} + c, \quad x \in R, \quad c \in R.
 \end{aligned}$$

$$\int \sqrt{x^2 - 4x + 2} \, dx = \frac{(x-2)\sqrt{x^2 - 4x + 2}}{2} - \frac{\ln|x-2 + \sqrt{x^2 - 4x + 2}|}{2} + c$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x^2-4x+2=x^2-4x+4-2 \\ dt=dx \mid = (x-2)^2-2=t^2-(\sqrt{2})^2 \end{array} \mid \begin{array}{l} x \in (-\infty; -3), \quad t \in (-\infty; -1) \\ x \in (-1; \infty), \quad t \in (1; \infty) \end{array} \right] = \int \sqrt{t^2 - (\sqrt{2})^2} \, dt \\
 &= \frac{t\sqrt{t^2 - (\sqrt{2})^2}}{2} - \frac{1}{2} \int \frac{dt}{\sqrt{t^2 - (\sqrt{2})^2}} = \frac{t\sqrt{t^2 - (\sqrt{2})^2}}{2} - \frac{\ln|t + \sqrt{t^2 - (\sqrt{2})^2}|}{2} + c \\
 &= \frac{(x-2)\sqrt{x^2 - 4x + 2}}{2} - \frac{\ln|x-2 + \sqrt{x^2 - 4x + 2}|}{2} + c, \quad x \in R - \langle 2 - \sqrt{2}; 2 + \sqrt{2} \rangle, \quad c \in R.
 \end{aligned}$$

Riešené príklady – 119, 120

$$\int \sqrt{-x^2 + 4x - 3} \, dx$$

$$\int \sqrt{x(2-x)} \, dx = \int \sqrt{-x^2 + 2x} \, dx$$

Riešené príklady – 119, 120

$$\int \sqrt{-x^2 + 4x - 3} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - 2 \\ dt = dx \end{array} \left| \begin{array}{l} -x^2 + 4x - 3 = -(x^2 - 4x + 4 - 1) \\ = -(x - 2)^2 + 1 = 1 - t^2 \end{array} \right| \begin{array}{l} x \in (1; 3) \\ t \in (-1; 1) \end{array} \right]$$

$$\int \sqrt{x(2-x)} \, dx = \int \sqrt{-x^2 + 2x} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - 1 \\ dt = dx \end{array} \left| \begin{array}{l} -x^2 + 2x = -(x^2 - 2x + 1 - 1) \\ = -(x - 1)^2 + 1 = 1 - t^2 \end{array} \right| \begin{array}{l} x \in (0; 2) \\ t \in (-1; 1) \end{array} \right]$$

Riešené príklady – 119, 120

$$\int \sqrt{-x^2 + 4x - 3} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - 2 \\ dt = dx \end{array} \left| \begin{array}{l} -x^2 + 4x - 3 = -(x^2 - 4x + 4 - 1) \\ = -(x - 2)^2 + 1 = 1 - t^2 \end{array} \right. \begin{array}{l} x \in (1; 3) \\ t \in (-1; 1) \end{array} \right] = \int \sqrt{1 - t^2} \, dt$$

$$\int \sqrt{x(2-x)} \, dx = \int \sqrt{-x^2 + 2x} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - 1 \\ dt = dx \end{array} \left| \begin{array}{l} -x^2 + 2x = -(x^2 - 2x + 1 - 1) \\ = -(x - 1)^2 + 1 = 1 - t^2 \end{array} \right. \begin{array}{l} x \in (0; 2) \\ t \in (-1; 1) \end{array} \right] = \int \sqrt{1 - t^2} \, dt$$

Riešené príklady – 119, 120

$$\int \sqrt{-x^2 + 4x - 3} \, dx$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t = x - 2 \\ dt = dx \end{array} \left| \begin{array}{l} -x^2 + 4x - 3 = -(x^2 - 4x + 4 - 1) \\ = -(x - 2)^2 + 1 = 1 - t^2 \end{array} \right. \begin{array}{l} x \in (1; 3) \\ t \in (-1; 1) \end{array} \right] = \int \sqrt{1 - t^2} \, dt \\
 &= \frac{t\sqrt{1-t^2}}{2} + \frac{1}{2} \int \frac{dt}{\sqrt{1-t^2}}
 \end{aligned}$$

$$\int \sqrt{x(2-x)} \, dx = \int \sqrt{-x^2 + 2x} \, dx$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t = x - 1 \\ dt = dx \end{array} \left| \begin{array}{l} -x^2 + 2x = -(x^2 - 2x + 1 - 1) \\ = -(x - 1)^2 + 1 = 1 - t^2 \end{array} \right. \begin{array}{l} x \in (0; 2) \\ t \in (-1; 1) \end{array} \right] = \int \sqrt{1 - t^2} \, dt = \frac{t\sqrt{1-t^2}}{2} + \frac{1}{2} \int \frac{dt}{\sqrt{1-t^2}}
 \end{aligned}$$

Riešené príklady – 119, 120

$$\int \sqrt{-x^2 + 4x - 3} \, dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = x - 2 \left| \begin{array}{l} -x^2 + 4x - 3 = -(x^2 - 4x + 4 - 1) \\ dt = dx \end{array} \right. \begin{array}{l} x \in (1; 3) \\ t \in (-1; 1) \end{array} \right] = \int \sqrt{1 - t^2} \, dt \\
 &= \frac{t\sqrt{1-t^2}}{2} + \frac{1}{2} \int \frac{dt}{\sqrt{1-t^2}} = \frac{t\sqrt{1-t^2}}{2} + \frac{\arcsin t}{2} + C_1 = \frac{t\sqrt{1-t^2}}{2} - \frac{\arccos t}{2} + C_2
 \end{aligned}$$

$$\int \sqrt{x(2-x)} \, dx = \int \sqrt{-x^2 + 2x} \, dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = x - 1 \left| \begin{array}{l} -x^2 + 2x = -(x^2 - 2x + 1 - 1) \\ dt = dx \end{array} \right. \begin{array}{l} x \in (0; 2) \\ t \in (-1; 1) \end{array} \right] = \int \sqrt{1 - t^2} \, dt = \frac{t\sqrt{1-t^2}}{2} + \frac{1}{2} \int \frac{dt}{\sqrt{1-t^2}} \\
 &= \frac{t\sqrt{1-t^2}}{2} + \frac{\arcsin t}{2} + C_1 = \frac{t\sqrt{1-t^2}}{2} - \frac{\arccos t}{2} + C_2
 \end{aligned}$$

Riešené príklady – 119, 120

$$\int \sqrt{-x^2+4x-3} dx = \frac{(x-2)\sqrt{-x^2+4x-3}}{2} + \frac{\arcsin(x-2)}{2} + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-2 \\ dt=dx \end{array} \left| \begin{array}{l} -x^2+4x-3=-(x^2-4x+4-1) \\ =-(x-2)^2+1=1-t^2 \end{array} \right. \begin{array}{l} x \in (1;3) \\ t \in (-1;1) \end{array} \right] = \int \sqrt{1-t^2} dt$$

$$= \frac{t\sqrt{1-t^2}}{2} + \frac{1}{2} \int \frac{dt}{\sqrt{1-t^2}} = \frac{t\sqrt{1-t^2}}{2} + \frac{\arcsin t}{2} + c_1 = \frac{t\sqrt{1-t^2}}{2} - \frac{\arccos t}{2} + c_2$$

$$= \frac{(x-2)\sqrt{-x^2+4x-3}}{2} + \frac{\arcsin(x-2)}{2} + c_1 = \frac{(x-2)\sqrt{-x^2+4x-3}}{2} - \frac{\arccos(x-2)}{2} + c_2, \\ x \in (1;3), c_1, c_2 \in \mathbb{R}.$$

$$\int \sqrt{x(2-x)} dx = \int \sqrt{-x^2+2x} dx = \frac{(x-1)\sqrt{-x^2+2x}}{2} + \frac{\arcsin(x-1)}{2} + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \left| \begin{array}{l} -x^2+2x=-(x^2-2x+1-1) \\ =-(x-1)^2+1=1-t^2 \end{array} \right. \begin{array}{l} x \in (0;2) \\ t \in (-1;1) \end{array} \right] = \int \sqrt{1-t^2} dt = \frac{t\sqrt{1-t^2}}{2} + \frac{1}{2} \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \frac{t\sqrt{1-t^2}}{2} + \frac{\arcsin t}{2} + c_1 = \frac{t\sqrt{1-t^2}}{2} - \frac{\arccos t}{2} + c_2 = \frac{(x-1)\sqrt{-x^2+2x}}{2} + \frac{\arcsin(x-1)}{2} + c_1$$

$$= \frac{(x-1)\sqrt{-x^2+2x}}{2} - \frac{\arccos(x-1)}{2} + c_2, x \in (0;2), c_1, c_2 \in \mathbb{R}.$$

Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$



Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^n}$$



Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2 - x^2) dx}{(x^2 + a^2)^n}$$



Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2 - x^2) dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2) dx}{(x^2 + a^2)^n} - \frac{1}{a^2} \int \frac{x^2 dx}{(x^2 + a^2)^n}$$



Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2 - x^2) dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2) dx}{(x^2 + a^2)^n} - \frac{1}{a^2} \int \frac{x^2 dx}{(x^2 + a^2)^n}$$

$$= \frac{1}{a^2} \int \frac{dx}{(x^2 + a^2)^{n-1}} - \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 + a^2)^n}$$

Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2 - x^2) dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2) dx}{(x^2 + a^2)^n} - \frac{1}{a^2} \int \frac{x^2 dx}{(x^2 + a^2)^n}$$

$$= \frac{1}{a^2} \int \frac{dx}{(x^2 + a^2)^{n-1}} - \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 + a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 + a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2 + a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2 + a^2)^{n-1}} \end{array} \right] \quad ?$$

Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2 - x^2) dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2) dx}{(x^2 + a^2)^n} - \frac{1}{a^2} \int \frac{x^2 dx}{(x^2 + a^2)^n}$$

$$= \frac{1}{a^2} \int \frac{dx}{(x^2 + a^2)^{n-1}} - \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 + a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 + a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2 + a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2 + a^2)^{n-1}} \end{array} \right] \quad ?$$

$$= \frac{1}{a^2} I_{n-1} - \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2 + a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2 + a^2)^{n-1}} \right]$$

Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2 - x^2) dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2) dx}{(x^2 + a^2)^n} - \frac{1}{a^2} \int \frac{x^2 dx}{(x^2 + a^2)^n}$$

$$= \frac{1}{a^2} \int \frac{dx}{(x^2 + a^2)^{n-1}} - \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 + a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 + a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2 + a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2 + a^2)^{n-1}} \end{array} \right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2 + a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2 + a^2)^{n-1}} \right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{x}{2a^2(1-n)(x^2 + a^2)^{n-1}} + \frac{1}{2a^2(1-n)} I_{n-1}$$

Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2 - x^2) dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2) dx}{(x^2 + a^2)^n} - \frac{1}{a^2} \int \frac{x^2 dx}{(x^2 + a^2)^n}$$

$$= \frac{1}{a^2} \int \frac{dx}{(x^2 + a^2)^{n-1}} - \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 + a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 + a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2 + a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2 + a^2)^{n-1}} \end{array} \right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2 + a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2 + a^2)^{n-1}} \right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{x}{2a^2(1-n)(x^2 + a^2)^{n-1}} + \frac{1}{2a^2(1-n)} I_{n-1}$$

$$= \frac{1}{2a^2} \left(2 + \frac{1}{1-n} \right) I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}}$$

Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2 - x^2) dx}{(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{(x^2 + a^2) dx}{(x^2 + a^2)^n} - \frac{1}{a^2} \int \frac{x^2 dx}{(x^2 + a^2)^n}$$

$$= \frac{1}{a^2} \int \frac{dx}{(x^2 + a^2)^{n-1}} - \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 + a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 + a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2 + a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2 + a^2)^{n-1}} \end{array} \right] \quad ?$$

$$= \frac{1}{a^2} I_{n-1} - \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2 + a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2 + a^2)^{n-1}} \right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{x}{2a^2(1-n)(x^2 + a^2)^{n-1}} + \frac{1}{2a^2(1-n)} I_{n-1}$$

$$= \frac{1}{2a^2} \left(2 + \frac{1}{1-n} \right) I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} = \frac{3-2n}{2a^2(1-n)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}}$$

Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}, \quad a>0, n \in \mathbb{N}$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2+a^2)^n} = \frac{1}{a^2} \int \frac{(x^2+a^2-x^2) dx}{(x^2+a^2)^n} = \frac{1}{a^2} \int \frac{(x^2+a^2) dx}{(x^2+a^2)^n} - \frac{1}{a^2} \int \frac{x^2 dx}{(x^2+a^2)^n}$$

$$= \frac{1}{a^2} \int \frac{dx}{(x^2+a^2)^{n-1}} - \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2+a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2+a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2+a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2+a^2)^{n-1}} \end{array} \right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2+a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2+a^2)^{n-1}} \right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{x}{2a^2(1-n)(x^2+a^2)^{n-1}} + \frac{1}{2a^2(1-n)} I_{n-1}$$

$$= \frac{1}{2a^2} \left(2 + \frac{1}{1-n} \right) I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} = \frac{3-2n}{2a^2(1-n)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$$

$$= \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}, \quad x \in \mathbb{R}, c \in \mathbb{R}, n=2,3,4,\dots$$

Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}, \quad I_1 = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c, \quad a > 0, n \in \mathbb{N}$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2+a^2)^n} = \frac{1}{a^2} \int \frac{(x^2+a^2-x^2) dx}{(x^2+a^2)^n} = \frac{1}{a^2} \int \frac{(x^2+a^2) dx}{(x^2+a^2)^n} - \frac{1}{a^2} \int \frac{x^2 dx}{(x^2+a^2)^n}$$

$$= \frac{1}{a^2} \int \frac{dx}{(x^2+a^2)^{n-1}} - \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2+a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2+a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2+a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2+a^2)^{n-1}} \end{array} \right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2+a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2+a^2)^{n-1}} \right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{x}{2a^2(1-n)(x^2+a^2)^{n-1}} + \frac{1}{2a^2(1-n)} I_{n-1}$$

$$= \frac{1}{2a^2} \left(2 + \frac{1}{1-n} \right) I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} = \frac{3-2n}{2a^2(1-n)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$$

$$= \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}, \quad x \in \mathbb{R}, c \in \mathbb{R}, \quad n = 2, 3, 4, \dots$$

$$I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}, \quad n = 1.$$

Riešené príklady – 122

$$I_2 = \int \frac{dx}{(x^2 + a^2)^2}$$

$$a > 0$$



Riešené príklady – 122

$$I_2 = \int \frac{dx}{(x^2 + a^2)^2}$$

$$a > 0$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^2}$$



$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right]$$

Riešené príklady – 122

$$I_2 = \int \frac{dx}{(x^2 + a^2)^2}$$

$$a > 0$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^2} = \frac{1}{a^2} \int \frac{(x^2 + a^2 - x^2) dx}{(x^2 + a^2)^2}$$



$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2 + a^2)}$$

Riešené príklady – 122

$$I_2 = \int \frac{dx}{(x^2 + a^2)^2}$$

$$a > 0$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^2} = \frac{1}{a^2} \int \frac{(x^2 + a^2 - x^2) dx}{(x^2 + a^2)^2} = \frac{1}{a^2} \int \frac{dx}{x^2 + a^2} - \frac{1}{2a^2} \int \frac{2x^2 dx}{(x^2 + a^2)^2}$$



$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2 + a^2)}$$

$$= \frac{1}{2a^2} \int \frac{dx}{x^2 + a^2} + \frac{x}{2a^2(x^2 + a^2)}$$

Riešené príklady – 122

$$I_2 = \int \frac{dx}{(x^2 + a^2)^2}$$

$$a > 0$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2 + a^2)^2} = \frac{1}{a^2} \int \frac{(x^2 + a^2 - x^2) dx}{(x^2 + a^2)^2} = \frac{1}{a^2} \int \frac{dx}{x^2 + a^2} - \frac{1}{2a^2} \int \frac{2x^2 dx}{(x^2 + a^2)^2}$$

$$= \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 + a^2)^2} \end{array} \right] \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2 + a^2} \end{array}$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2 + a^2)}$$

$$= \frac{1}{2a^2} \int \frac{dx}{x^2 + a^2} + \frac{x}{2a^2(x^2 + a^2)} = \frac{1}{2a^2} \cdot \frac{1}{a} \arctg \frac{x}{a} + \frac{x}{2a^2(x^2 + a^2)} + c$$

Riešené príklady – 122

$$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c \quad a > 0$$

$$\begin{aligned} &= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2+a^2)^2} = \frac{1}{a^2} \int \frac{(x^2+a^2-x^2) dx}{(x^2+a^2)^2} = \frac{1}{a^2} \int \frac{dx}{x^2+a^2} - \frac{1}{2a^2} \int \frac{2x^2 dx}{(x^2+a^2)^2} \\ &= \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2+a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2+a^2} \end{array} \right] = \frac{1}{a^2} \int \frac{dx}{x^2+a^2} - \frac{1}{2a^2} \left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2} \right] \end{aligned}$$

$$\begin{aligned} &= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)} \\ &= \frac{1}{2a^2} \int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} = \frac{1}{2a^2} \cdot \frac{1}{a} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c \\ &= \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 122

$$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c \quad a > 0$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2+a^2)^2} = \frac{1}{a^2} \int \frac{(x^2+a^2-x^2) dx}{(x^2+a^2)^2} = \frac{1}{a^2} \int \frac{dx}{x^2+a^2} - \frac{1}{2a^2} \int \frac{2x^2 dx}{(x^2+a^2)^2}$$

$$= \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2+a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2+a^2} \end{array} \right] = \frac{1}{a^2} \int \frac{dx}{x^2+a^2} - \frac{1}{2a^2} \left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2} \right]$$

$$= \frac{1}{a^2} \int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} - \frac{1}{2a^2} \int \frac{dx}{x^2+a^2}$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)}$$

$$= \frac{1}{2a^2} \int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} = \frac{1}{2a^2} \cdot \frac{1}{a} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c$$

$$= \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

Riešené príklady – 122

$$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c \quad a > 0$$

$$\begin{aligned} &= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2+a^2)^2} = \frac{1}{a^2} \int \frac{(x^2+a^2-x^2) dx}{(x^2+a^2)^2} = \frac{1}{a^2} \int \frac{dx}{x^2+a^2} - \frac{1}{2a^2} \int \frac{2x^2 dx}{(x^2+a^2)^2} \\ &= \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2+a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2+a^2} \end{array} \right] = \frac{1}{a^2} \int \frac{dx}{x^2+a^2} - \frac{1}{2a^2} \left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2} \right] \\ &= \frac{1}{a^2} \int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} - \frac{1}{2a^2} \int \frac{dx}{x^2+a^2} = \frac{1}{2a^2} \int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)} \\ &= \frac{1}{2a^2} \int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} = \frac{1}{2a^2} \cdot \frac{1}{a} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c \\ &= \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 122

$$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c \quad a > 0$$

$$\begin{aligned} &= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2+a^2)^2} = \frac{1}{a^2} \int \frac{(x^2+a^2-x^2) dx}{(x^2+a^2)^2} = \frac{1}{a^2} \int \frac{dx}{x^2+a^2} - \frac{1}{2a^2} \int \frac{2x^2 dx}{(x^2+a^2)^2} \\ &= \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2+a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2+a^2} \end{array} \right] = \frac{1}{a^2} \int \frac{dx}{x^2+a^2} - \frac{1}{2a^2} \left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2} \right] \\ &= \frac{1}{a^2} \int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} - \frac{1}{2a^2} \int \frac{dx}{x^2+a^2} = \frac{1}{2a^2} \int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} \\ &= \frac{1}{2a^2} \cdot \frac{1}{a} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c \end{aligned}$$

$$\begin{aligned} &= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)} \\ &= \frac{1}{2a^2} \int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} = \frac{1}{2a^2} \cdot \frac{1}{a} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c \\ &= \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 122

$$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c \quad a > 0$$

$$= \frac{1}{a^2} \int \frac{a^2 dx}{(x^2+a^2)^2} = \frac{1}{a^2} \int \frac{(x^2+a^2-x^2) dx}{(x^2+a^2)^2} = \frac{1}{a^2} \int \frac{dx}{x^2+a^2} - \frac{1}{2a^2} \int \frac{2x^2 dx}{(x^2+a^2)^2}$$

$$= \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2+a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2+a^2} \end{array} \right] = \frac{1}{a^2} \int \frac{dx}{x^2+a^2} - \frac{1}{2a^2} \left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2} \right]$$

$$= \frac{1}{a^2} \int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} - \frac{1}{2a^2} \int \frac{dx}{x^2+a^2} = \frac{1}{2a^2} \int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)}$$

$$= \frac{1}{2a^2} \cdot \frac{1}{a} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c = \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c, \quad x \in R, \quad c \in R.$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)}$$

$$= \frac{1}{2a^2} \int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} = \frac{1}{2a^2} \cdot \frac{1}{a} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c$$

$$= \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c, \quad x \in R, \quad c \in R.$$

Riešené príklady – 123, 124

$$I_3 = \int \frac{dx}{(x^2 + a^2)^3}$$

$$a > 0$$

$$I_4 = \int \frac{dx}{(x^2 + a^2)^4}$$

$$a > 0$$

Riešené príklady – 123, 124

$$I_3 = \int \frac{dx}{(x^2 + a^2)^3}$$

$$a > 0$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right]$$

$$I_4 = \int \frac{dx}{(x^2 + a^2)^4}$$

$$a > 0$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right]$$

Riešené príklady – 123, 124

$$I_3 = \int \frac{dx}{(x^2 + a^2)^3}$$

$$a > 0$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{2 \cdot 3 - 3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2 + a^2)^2}$$

$$I_4 = \int \frac{dx}{(x^2 + a^2)^4}$$

$$a > 0$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{2 \cdot 4 - 3}{2a^2(4-1)} I_3 + \frac{x}{2a^2(4-1)(x^2 + a^2)^3}$$

Riešené príklady – 123, 124

$$I_3 = \int \frac{dx}{(x^2 + a^2)^3}$$

$$a > 0$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{2 \cdot 3 - 3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2 + a^2)^2} = \left[\text{Pr. 121} \right]$$

$$I_4 = \int \frac{dx}{(x^2 + a^2)^4}$$

$$a > 0$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{2 \cdot 4 - 3}{2a^2(4-1)} I_3 + \frac{x}{2a^2(4-1)(x^2 + a^2)^3} = \left[\text{Pr. 121} \right]$$

Riešené príklady – 123, 124

$$I_3 = \int \frac{dx}{(x^2 + a^2)^3} \quad a > 0$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{2 \cdot 3 - 3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2 + a^2)^2} = \left[\text{Pr. 121} \right]$$

$$= \frac{3}{4a^2} \left[\frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2 + a^2)} \right] + \frac{x}{4a^2(x^2 + a^2)^2}$$

$$I_4 = \int \frac{dx}{(x^2 + a^2)^4} \quad a > 0$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{2 \cdot 4 - 3}{2a^2(4-1)} I_3 + \frac{x}{2a^2(4-1)(x^2 + a^2)^3} = \left[\text{Pr. 121} \right]$$

$$= \frac{5}{6a^2} \left[\frac{2 \cdot 3 - 3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2 + a^2)^2} \right] + \frac{x}{6a^2(x^2 + a^2)^3}$$

Riešené príklady – 123, 124

$$I_3 = \int \frac{dx}{(x^2 + a^2)^3}$$

$$a > 0$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{2 \cdot 3 - 3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2 + a^2)^2} = \left[\text{Pr. 121} \right]$$

$$= \frac{3}{4a^2} \left[\frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2 + a^2)} \right] + \frac{x}{4a^2(x^2 + a^2)^2} = \left[\text{Pr. 121: } I_1 = \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctg \frac{x}{a} \right]$$

$$I_4 = \int \frac{dx}{(x^2 + a^2)^4}$$

$$a > 0$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{2 \cdot 4 - 3}{2a^2(4-1)} I_3 + \frac{x}{2a^2(4-1)(x^2 + a^2)^3} = \left[\text{Pr. 121} \right]$$

$$= \frac{5}{6a^2} \left[\frac{2 \cdot 3 - 3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2 + a^2)^2} \right] + \frac{x}{6a^2(x^2 + a^2)^3} = \left[\text{Pr. 121} \right]$$

Riešené príklady – 123, 124

$$I_3 = \int \frac{dx}{(x^2+a^2)^3} = \frac{3}{8a^5} \arctg \frac{x}{a} + \frac{3x}{8a^4(x^2+a^2)} + \frac{x}{4a^2(x^2+a^2)^2} + c \quad a > 0$$

$$\begin{aligned} &= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2 \cdot 3 - 3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^2} = \left[\text{Pr. 121} \right] \\ &= \frac{3}{4a^2} \left[\frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)} \right] + \frac{x}{4a^2(x^2+a^2)^2} = \left[\text{Pr. 121: } I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctg \frac{x}{a} \right] \\ &= \frac{3}{8a^5} \arctg \frac{x}{a} + \frac{3x}{8a^4(x^2+a^2)} + \frac{x}{4a^2(x^2+a^2)^2} + c, \quad x \in R, \quad c \in R. \end{aligned}$$

$$I_4 = \int \frac{dx}{(x^2+a^2)^4} \quad a > 0$$

$$\begin{aligned} &= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2 \cdot 4 - 3}{2a^2(4-1)} I_3 + \frac{x}{2a^2(4-1)(x^2+a^2)^3} = \left[\text{Pr. 121} \right] \\ &= \frac{5}{6a^2} \left[\frac{2 \cdot 3 - 3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^2} \right] + \frac{x}{6a^2(x^2+a^2)^3} = \left[\text{Pr. 121} \right] \\ &= \frac{15}{24a^4} \left[\frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)} \right] + \frac{5x}{24a^4(x^2+a^2)^2} + \frac{x}{6a^2(x^2+a^2)^3} \end{aligned}$$

Riešené príklady – 123, 124

$$I_3 = \int \frac{dx}{(x^2+a^2)^3} = \frac{3}{8a^5} \arctg \frac{x}{a} + \frac{3x}{8a^4(x^2+a^2)} + \frac{x}{4a^2(x^2+a^2)^2} + c \quad a > 0$$

$$\begin{aligned} &= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2 \cdot 3 - 3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^2} = \left[\text{Pr. 121} \right] \\ &= \frac{3}{4a^2} \left[\frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)} \right] + \frac{x}{4a^2(x^2+a^2)^2} = \left[\text{Pr. 121: } I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctg \frac{x}{a} \right] \\ &= \frac{3}{8a^5} \arctg \frac{x}{a} + \frac{3x}{8a^4(x^2+a^2)} + \frac{x}{4a^2(x^2+a^2)^2} + c, \quad x \in R, \quad c \in R. \end{aligned}$$

$$I_4 = \int \frac{dx}{(x^2+a^2)^4} \quad a > 0$$

$$\begin{aligned} &= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2 \cdot 4 - 3}{2a^2(4-1)} I_3 + \frac{x}{2a^2(4-1)(x^2+a^2)^3} = \left[\text{Pr. 121} \right] \\ &= \frac{5}{6a^2} \left[\frac{2 \cdot 3 - 3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^2} \right] + \frac{x}{6a^2(x^2+a^2)^3} = \left[\text{Pr. 121} \right] \\ &= \frac{15}{24a^4} \left[\frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)} \right] + \frac{5x}{24a^4(x^2+a^2)^2} + \frac{x}{6a^2(x^2+a^2)^3} = \left[\text{Pr. 121: } I_1 = \frac{1}{a} \arctg \frac{x}{a} \right] \end{aligned}$$

Riešené príklady – 123, 124

$$I_3 = \int \frac{dx}{(x^2+a^2)^3} = \frac{3}{8a^5} \arctg \frac{x}{a} + \frac{3x}{8a^4(x^2+a^2)} + \frac{x}{4a^2(x^2+a^2)^2} + c \quad a > 0$$

$$\begin{aligned} &= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2 \cdot 3 - 3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^2} = \left[\text{Pr. 121} \right] \\ &= \frac{3}{4a^2} \left[\frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)} \right] + \frac{x}{4a^2(x^2+a^2)^2} = \left[\text{Pr. 121: } I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctg \frac{x}{a} \right] \\ &= \frac{3}{8a^5} \arctg \frac{x}{a} + \frac{3x}{8a^4(x^2+a^2)} + \frac{x}{4a^2(x^2+a^2)^2} + c, \quad x \in R, \quad c \in R. \end{aligned}$$

$$I_4 = \int \frac{dx}{(x^2+a^2)^4} = \frac{5}{16a^7} \arctg \frac{x}{a} + \frac{5x}{16a^2(x^2+a^2)} + \frac{5x}{24a^4(x^2+a^2)^2} + \frac{x}{6a^2(x^2+a^2)^3} + c \quad a > 0$$

$$\begin{aligned} &= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2 \cdot 4 - 3}{2a^2(4-1)} I_3 + \frac{x}{2a^2(4-1)(x^2+a^2)^3} = \left[\text{Pr. 121} \right] \\ &= \frac{5}{6a^2} \left[\frac{2 \cdot 3 - 3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^2} \right] + \frac{x}{6a^2(x^2+a^2)^3} = \left[\text{Pr. 121} \right] \\ &= \frac{15}{24a^4} \left[\frac{2 \cdot 2 - 3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)} \right] + \frac{5x}{24a^4(x^2+a^2)^2} + \frac{x}{6a^2(x^2+a^2)^3} = \left[\text{Pr. 121: } I_1 = \frac{1}{a} \arctg \frac{x}{a} \right] \\ &= \frac{5}{16a^7} \arctg \frac{x}{a} + \frac{5x}{16a^2(x^2+a^2)} + \frac{5x}{24a^4(x^2+a^2)^2} + \frac{x}{6a^2(x^2+a^2)^3} + c, \quad x \in R, \quad c \in R. \end{aligned}$$

Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$



Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^n}$$



Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^n}$$



Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2) dx}{(x^2 - a^2)^n} - \frac{1}{-a^2} \int \frac{x^2 dx}{(x^2 - a^2)^n}$$



Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2) dx}{(x^2 - a^2)^n} - \frac{1}{-a^2} \int \frac{x^2 dx}{(x^2 - a^2)^n}$$

$$= -\frac{1}{a^2} \int \frac{dx}{(x^2 - a^2)^{n-1}} + \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 - a^2)^n}$$

Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2) dx}{(x^2 - a^2)^n} - \frac{1}{-a^2} \int \frac{x^2 dx}{(x^2 - a^2)^n}$$

$$= -\frac{1}{a^2} \int \frac{dx}{(x^2 - a^2)^{n-1}} + \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 - a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 - a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2 - a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2 - a^2)^{n-1}} \end{array} \right]$$

Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2) dx}{(x^2 - a^2)^n} - \frac{1}{-a^2} \int \frac{x^2 dx}{(x^2 - a^2)^n}$$

$$= -\frac{1}{a^2} \int \frac{dx}{(x^2 - a^2)^{n-1}} + \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 - a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 - a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2 - a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2 - a^2)^{n-1}} \end{array} \right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2 - a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2 - a^2)^{n-1}} \right]$$

Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2) dx}{(x^2 - a^2)^n} - \frac{1}{-a^2} \int \frac{x^2 dx}{(x^2 - a^2)^n}$$

$$= -\frac{1}{a^2} \int \frac{dx}{(x^2 - a^2)^{n-1}} + \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 - a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 - a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2 - a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2 - a^2)^{n-1}} \end{array} \right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2 - a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2 - a^2)^{n-1}} \right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{x}{2a^2(1-n)(x^2 - a^2)^{n-1}} - \frac{1}{2a^2(1-n)} I_{n-1}$$

Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2) dx}{(x^2 - a^2)^n} - \frac{1}{-a^2} \int \frac{x^2 dx}{(x^2 - a^2)^n}$$

$$= -\frac{1}{a^2} \int \frac{dx}{(x^2 - a^2)^{n-1}} + \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 - a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 - a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2 - a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2 - a^2)^{n-1}} \end{array} \right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2 - a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2 - a^2)^{n-1}} \right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{x}{2a^2(1-n)(x^2 - a^2)^{n-1}} - \frac{1}{2a^2(1-n)} I_{n-1}$$

$$= -\frac{1}{2a^2} \left(2 + \frac{1}{1-n} \right) I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}}$$

Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2) dx}{(x^2 - a^2)^n} - \frac{1}{-a^2} \int \frac{x^2 dx}{(x^2 - a^2)^n}$$

$$= -\frac{1}{a^2} \int \frac{dx}{(x^2 - a^2)^{n-1}} + \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 - a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 - a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2 - a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2 - a^2)^{n-1}} \end{array} \right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2 - a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2 - a^2)^{n-1}} \right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{x}{2a^2(1-n)(x^2 - a^2)^{n-1}} - \frac{1}{2a^2(1-n)} I_{n-1}$$

$$= -\frac{1}{2a^2} \left(2 + \frac{1}{1-n} \right) I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} = -\frac{3-2n}{2a^2(1-n)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}}$$

Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}}, \quad a > 0, n \in \mathbb{N}$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2) dx}{(x^2 - a^2)^n} - \frac{1}{-a^2} \int \frac{x^2 dx}{(x^2 - a^2)^n}$$

$$= -\frac{1}{a^2} \int \frac{dx}{(x^2 - a^2)^{n-1}} + \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 - a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 - a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2 - a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2 - a^2)^{n-1}} \end{array} \right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2 - a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2 - a^2)^{n-1}} \right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{x}{2a^2(1-n)(x^2 - a^2)^{n-1}} - \frac{1}{2a^2(1-n)} I_{n-1}$$

$$= -\frac{1}{2a^2} \left(2 + \frac{1}{1-n} \right) I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} = -\frac{3-2n}{2a^2(1-n)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}}$$

$$= \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}}, \quad x \in \mathbb{R} - \{\pm a\}, \quad c \in \mathbb{R}, \quad n = 2, 3, 4, \dots$$

Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}}, \quad I_1 = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c, \quad a > 0, \quad n \in \mathbb{N}$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2) dx}{(x^2 - a^2)^n} - \frac{1}{-a^2} \int \frac{x^2 dx}{(x^2 - a^2)^n}$$

$$= -\frac{1}{a^2} \int \frac{dx}{(x^2 - a^2)^{n-1}} + \frac{1}{2a^2} \int \frac{x \cdot 2x dx}{(x^2 - a^2)^n} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 - a^2)^n} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{(x^2 - a^2)^{1-n}}{1-n} = \frac{1}{(1-n)(x^2 - a^2)^{n-1}} \end{array} \right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2 - a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2 - a^2)^{n-1}} \right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{x}{2a^2(1-n)(x^2 - a^2)^{n-1}} - \frac{1}{2a^2(1-n)} I_{n-1}$$

$$= -\frac{1}{2a^2} \left(2 + \frac{1}{1-n} \right) I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} = -\frac{3-2n}{2a^2(1-n)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}}$$

$$= \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}}, \quad x \in \mathbb{R} - \{\pm a\}, \quad c \in \mathbb{R}, \quad n = 2, 3, 4, \dots$$

$$I_1 = \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c, \quad x \in \mathbb{R} - \{\pm a\}, \quad c \in \mathbb{R}, \quad n = 1.$$

Riešené príklady – 126

$$I_2 = \int \frac{dx}{(x^2 - a^2)^2}$$

$$a > 0$$



Riešené príklady – 126

$$I_2 = \int \frac{dx}{(x^2 - a^2)^2}$$

$$a > 0$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^2}$$



$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right]$$

Riešené príklady – 126

$$I_2 = \int \frac{dx}{(x^2 - a^2)^2}$$

$$a > 0$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^2}$$



$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)}$$

Riešené príklady – 126

$$I_2 = \int \frac{dx}{(x^2 - a^2)^2}$$

$$a > 0$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{dx}{x^2 - a^2} - \frac{1}{-2a^2} \int \frac{2x^2 dx}{(x^2 - a^2)^2}$$



$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)}$$

$$= -\frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)}$$

Riešené príklady – 126

$$I_2 = \int \frac{dx}{(x^2 - a^2)^2}$$

$$a > 0$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{dx}{x^2 - a^2} - \frac{1}{-2a^2} \int \frac{2x^2 dx}{(x^2 - a^2)^2}$$

$$= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \frac{2x}{(x^2 - a^2)^2} & v = -\frac{1}{x^2 - a^2} \end{array} \right]$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)}$$

$$= -\frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)} = -\frac{1}{2a^2} \cdot \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c$$

Riešené príklady – 126

$$I_2 = \int \frac{dx}{(x^2 - a^2)^2} = -\frac{1}{4a^3} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c \quad a > 0$$

$$\begin{aligned} &= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{dx}{x^2 - a^2} - \frac{1}{-2a^2} \int \frac{2x^2 dx}{(x^2 - a^2)^2} \\ &= \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 - a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2 - a^2} \end{array} \right] = -\frac{1}{a^2} \int \frac{dx}{x^2 - a^2} + \frac{1}{2a^2} \left[-\frac{x}{x^2 - a^2} + \int \frac{dx}{x^2 - a^2} \right] \end{aligned}$$

$$\begin{aligned} &= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)} \\ &= -\frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)} = -\frac{1}{2a^2} \cdot \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c \\ &= -\frac{1}{4a^3} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c, \quad x \in \mathbb{R} - \{\pm a\}, \quad c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 126

$$I_2 = \int \frac{dx}{(x^2 - a^2)^2} = -\frac{1}{4a^3} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c \quad a > 0$$

$$\begin{aligned} &= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{dx}{x^2 - a^2} - \frac{1}{-2a^2} \int \frac{2x^2 dx}{(x^2 - a^2)^2} \\ &= \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 - a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2 - a^2} \end{array} \right] = -\frac{1}{a^2} \int \frac{dx}{x^2 - a^2} + \frac{1}{2a^2} \left[-\frac{x}{x^2 - a^2} + \int \frac{dx}{x^2 - a^2} \right] \\ &= -\frac{1}{a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)} + \frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)} \\ &= -\frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)} = -\frac{1}{2a^2} \cdot \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c \\ &= -\frac{1}{4a^3} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c, \quad x \in \mathbb{R} - \{\pm a\}, \quad c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 126

$$I_2 = \int \frac{dx}{(x^2 - a^2)^2} = -\frac{1}{4a^3} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c \quad a > 0$$

$$\begin{aligned} &= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{dx}{x^2 - a^2} - \frac{1}{-2a^2} \int \frac{2x^2 dx}{(x^2 - a^2)^2} \\ &= \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 - a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2 - a^2} \end{array} \right] = -\frac{1}{a^2} \int \frac{dx}{x^2 - a^2} + \frac{1}{2a^2} \left[-\frac{x}{x^2 - a^2} + \int \frac{dx}{x^2 - a^2} \right] \\ &= -\frac{1}{a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)} + \frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} = -\frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)} \\ &= -\frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)} = -\frac{1}{2a^2} \cdot \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c \\ &= -\frac{1}{4a^3} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c, \quad x \in \mathbb{R} - \{\pm a\}, \quad c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 126

$$I_2 = \int \frac{dx}{(x^2 - a^2)^2} = -\frac{1}{4a^3} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c \quad a > 0$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{dx}{x^2 - a^2} - \frac{1}{-2a^2} \int \frac{2x^2 dx}{(x^2 - a^2)^2}$$

$$= \left[\begin{matrix} u = x \\ v' = \frac{2x}{(x^2 - a^2)^2} \end{matrix} \middle| \begin{matrix} u' = 1 \\ v = -\frac{1}{x^2 - a^2} \end{matrix} \right] = -\frac{1}{a^2} \int \frac{dx}{x^2 - a^2} + \frac{1}{2a^2} \left[-\frac{x}{x^2 - a^2} + \int \frac{dx}{x^2 - a^2} \right]$$

$$= -\frac{1}{a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)} + \frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} = -\frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)}$$

$$= -\frac{1}{2a^2} \cdot \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)}$$

$$= -\frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)} = -\frac{1}{2a^2} \cdot \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c$$

$$= -\frac{1}{4a^3} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c, \quad x \in \mathbb{R} - \{\pm a\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 126

$$I_2 = \int \frac{dx}{(x^2 - a^2)^2} = -\frac{1}{4a^3} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c \quad a > 0$$

$$= \frac{1}{-a^2} \int \frac{-a^2 dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2) dx}{(x^2 - a^2)^2} = \frac{1}{-a^2} \int \frac{dx}{x^2 - a^2} - \frac{1}{-2a^2} \int \frac{2x^2 dx}{(x^2 - a^2)^2}$$

$$= \left[\begin{matrix} u = x \\ v' = \frac{2x}{(x^2 - a^2)^2} \end{matrix} \middle| \begin{matrix} u' = 1 \\ v = -\frac{1}{x^2 - a^2} \end{matrix} \right] = -\frac{1}{a^2} \int \frac{dx}{x^2 - a^2} + \frac{1}{2a^2} \left[-\frac{x}{x^2 - a^2} + \int \frac{dx}{x^2 - a^2} \right]$$

$$= -\frac{1}{a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)} + \frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} = -\frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)}$$

$$= -\frac{1}{2a^2} \cdot \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c = -\frac{1}{4a^3} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c, \\ x \in R - \{\pm a\}, c \in R.$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2 + a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2 + a^2)^{n-1}} \right] = \frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)}$$

$$= -\frac{1}{2a^2} \int \frac{dx}{x^2 - a^2} - \frac{x}{2a^2(x^2 - a^2)} = -\frac{1}{2a^2} \cdot \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c$$

$$= -\frac{1}{4a^3} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2a^2(x^2 - a^2)} + c, x \in R - \{\pm a\}, c \in R.$$

Riešené príklady – 127, 128

$$I_3 = \int \frac{dx}{(x^2 - a^2)^3}$$

$$a > 0$$

$$I_4 = \int \frac{dx}{(x^2 - a^2)^4}$$

$$a > 0$$

Riešené príklady – 127, 128

$$I_3 = \int \frac{dx}{(x^2 - a^2)^3}$$

$$a > 0$$

$$= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right]$$

$$I_4 = \int \frac{dx}{(x^2 - a^2)^4}$$

$$a > 0$$

$$= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right]$$

Riešené príklady – 127, 128

$$I_3 = \int \frac{dx}{(x^2 - a^2)^3}$$

$$a > 0$$

$$= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 3 - 3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2 - a^2)^2}$$

$$I_4 = \int \frac{dx}{(x^2 - a^2)^4}$$

$$a > 0$$

$$= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 4 - 3)}{2a^2(4-1)} I_3 - \frac{x}{2a^2(4-1)(x^2 - a^2)^3}$$

Riešené príklady – 127, 128

$$I_3 = \int \frac{dx}{(x^2 - a^2)^3} \quad a > 0$$

$$= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 3 - 3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2 - a^2)^2} = \left[\text{Pr. 125} \right]$$

$$I_4 = \int \frac{dx}{(x^2 - a^2)^4} \quad a > 0$$

$$= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 4 - 3)}{2a^2(4-1)} I_3 - \frac{x}{2a^2(4-1)(x^2 - a^2)^3} = \left[\text{Pr. 125} \right]$$

Riešené príklady – 127, 128

$$I_3 = \int \frac{dx}{(x^2 - a^2)^3} \quad a > 0$$

$$\begin{aligned}
 &= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 3 - 3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2 - a^2)^2} = \left[\text{Pr. 125} \right] \\
 &= -\frac{3}{4a^2} \left[\frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)} \right] - \frac{x}{4a^2(x^2 - a^2)^2}
 \end{aligned}$$

$$I_4 = \int \frac{dx}{(x^2 - a^2)^4} \quad a > 0$$

$$\begin{aligned}
 &= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 4 - 3)}{2a^2(4-1)} I_3 - \frac{x}{2a^2(4-1)(x^2 - a^2)^3} = \left[\text{Pr. 125} \right] \\
 &= -\frac{5}{6a^2} \left[\frac{-(2 \cdot 3 - 3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2 - a^2)^2} \right] - \frac{x}{6a^2(x^2 - a^2)^3}
 \end{aligned}$$

Riešené príklady – 127, 128

$$I_3 = \int \frac{dx}{(x^2 - a^2)^3} \quad a > 0$$

$$\begin{aligned}
 &= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 3 - 3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2 - a^2)^2} = \left[\text{Pr. 125} \right] \\
 &= -\frac{3}{4a^2} \left[\frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)} \right] - \frac{x}{4a^2(x^2 - a^2)^2} = \left[\text{Pr. 125: } I_1 = \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| \right]
 \end{aligned}$$

$$I_4 = \int \frac{dx}{(x^2 - a^2)^4} \quad a > 0$$

$$\begin{aligned}
 &= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 4 - 3)}{2a^2(4-1)} I_3 - \frac{x}{2a^2(4-1)(x^2 - a^2)^3} = \left[\text{Pr. 125} \right] \\
 &= -\frac{5}{6a^2} \left[\frac{-(2 \cdot 3 - 3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2 - a^2)^2} \right] - \frac{x}{6a^2(x^2 - a^2)^3} = \left[\text{Pr. 125} \right]
 \end{aligned}$$

Riešené príklady – 127, 128

$$I_3 = \int \frac{dx}{(x^2 - a^2)^3} = \frac{3}{16a^5} \ln \left| \frac{x-a}{x+a} \right| + \frac{3x}{8a^4(x^2 - a^2)} - \frac{x}{4a^2(x^2 - a^2)^2} + C \quad a > 0$$

$$\begin{aligned} &= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 3 - 3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2 - a^2)^2} = \left[\text{Pr. 125} \right] \\ &= -\frac{3}{4a^2} \left[\frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)} \right] - \frac{x}{4a^2(x^2 - a^2)^2} = \left[\text{Pr. 125: } I_1 = \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| \right] \\ &= \frac{3}{16a^5} \ln \left| \frac{x-a}{x+a} \right| + \frac{3x}{8a^4(x^2 - a^2)} - \frac{x}{4a^2(x^2 - a^2)^2} + C, \quad x \in \mathbb{R} - \{\pm a\}, \quad C \in \mathbb{R}. \end{aligned}$$

$$I_4 = \int \frac{dx}{(x^2 - a^2)^4} \quad a > 0$$

$$\begin{aligned} &= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 4 - 3)}{2a^2(4-1)} I_3 - \frac{x}{2a^2(4-1)(x^2 - a^2)^3} = \left[\text{Pr. 125} \right] \\ &= -\frac{5}{6a^2} \left[\frac{-(2 \cdot 3 - 3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2 - a^2)^2} \right] - \frac{x}{6a^2(x^2 - a^2)^3} = \left[\text{Pr. 125} \right] \\ &= \frac{15}{24a^4} \left[\frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)} \right] + \frac{5x}{24a^4(x^2 - a^2)^2} - \frac{x}{6a^2(x^2 - a^2)^3} \end{aligned}$$

Riešené príklady – 127, 128

$$I_3 = \int \frac{dx}{(x^2 - a^2)^3} = \frac{3}{16a^5} \ln \left| \frac{x-a}{x+a} \right| + \frac{3x}{8a^4(x^2 - a^2)} - \frac{x}{4a^2(x^2 - a^2)^2} + C \quad a > 0$$

$$\begin{aligned} &= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 3 - 3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2 - a^2)^2} = \left[\text{Pr. 125} \right] \\ &= -\frac{3}{4a^2} \left[\frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)} \right] - \frac{x}{4a^2(x^2 - a^2)^2} = \left[\text{Pr. 125: } I_1 = \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| \right] \\ &= \frac{3}{16a^5} \ln \left| \frac{x-a}{x+a} \right| + \frac{3x}{8a^4(x^2 - a^2)} - \frac{x}{4a^2(x^2 - a^2)^2} + C, \quad x \in \mathbb{R} - \{\pm a\}, \quad C \in \mathbb{R}. \end{aligned}$$

$$I_4 = \int \frac{dx}{(x^2 - a^2)^4} \quad a > 0$$

$$\begin{aligned} &= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 4 - 3)}{2a^2(4-1)} I_3 - \frac{x}{2a^2(4-1)(x^2 - a^2)^3} = \left[\text{Pr. 125} \right] \\ &= -\frac{5}{6a^2} \left[\frac{-(2 \cdot 3 - 3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2 - a^2)^2} \right] - \frac{x}{6a^2(x^2 - a^2)^3} = \left[\text{Pr. 125} \right] \\ &= \frac{15}{24a^4} \left[\frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)} \right] + \frac{5x}{24a^4(x^2 - a^2)^2} - \frac{x}{6a^2(x^2 - a^2)^3} = \left[\text{Pr. 125: } I_1 = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| \right] \end{aligned}$$

Riešené príklady – 127, 128

$$I_3 = \int \frac{dx}{(x^2 - a^2)^3} = \frac{3}{16a^5} \ln \left| \frac{x-a}{x+a} \right| + \frac{3x}{8a^4(x^2 - a^2)} - \frac{x}{4a^2(x^2 - a^2)^2} + C \quad a > 0$$

$$\begin{aligned} &= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 3 - 3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2 - a^2)^2} = \left[\text{Pr. 125} \right] \\ &= -\frac{3}{4a^2} \left[\frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)} \right] - \frac{x}{4a^2(x^2 - a^2)^2} = \left[\text{Pr. 125: } I_1 = \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| \right] \\ &= \frac{3}{16a^5} \ln \left| \frac{x-a}{x+a} \right| + \frac{3x}{8a^4(x^2 - a^2)} - \frac{x}{4a^2(x^2 - a^2)^2} + C, \quad x \in R - \{\pm a\}, \quad C \in R. \end{aligned}$$

$$I_4 = \int \frac{dx}{(x^2 - a^2)^4} = -\frac{5}{32a^7} \ln \left| \frac{x-a}{x+a} \right| - \frac{5x}{16a^6(x^2 - a^2)} + \frac{5x}{24a^4(x^2 - a^2)^2} - \frac{x}{6a^2(x^2 - a^2)^3} + C \quad a > 0$$

$$\begin{aligned} &= \left[\text{Pr. 125: } I_n = \int \frac{dx}{(x^2 - a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} \right] = \frac{-(2 \cdot 4 - 3)}{2a^2(4-1)} I_3 - \frac{x}{2a^2(4-1)(x^2 - a^2)^3} = \left[\text{Pr. 125} \right] \\ &= -\frac{5}{6a^2} \left[\frac{-(2 \cdot 3 - 3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2 - a^2)^2} \right] - \frac{x}{6a^2(x^2 - a^2)^3} = \left[\text{Pr. 125} \right] \\ &= \frac{15}{24a^4} \left[\frac{-(2 \cdot 2 - 3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2 - a^2)} \right] + \frac{5x}{24a^4(x^2 - a^2)^2} - \frac{x}{6a^2(x^2 - a^2)^3} = \left[\text{Pr. 125: } I_1 = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| \right] \\ &= -\frac{5}{32a^7} \ln \left| \frac{x-a}{x+a} \right| - \frac{5x}{16a^6(x^2 - a^2)} + \frac{5x}{24a^4(x^2 - a^2)^2} - \frac{x}{6a^2(x^2 - a^2)^3} + C, \\ &\quad x \in R - \{\pm a\}, \quad C \in R. \end{aligned}$$

Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$



Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in R \\ t \in R \end{array} \right]$$



$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in R \\ t \in R \end{array} \right]$$

Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ =(x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \frac{1 \cdot dt}{(t^2+1)^2}$$



$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ =(x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \frac{dt}{(t^2+1)^2}$$

Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \frac{1 \cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2) dt}{(t^2+1)^2}$$



$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \frac{dt}{(t^2+1)^2} = \left[\begin{array}{l} \text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} \\ I_n = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array} \right]$$

Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \frac{1 \cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2) dt}{(t^2+1)^2}$$

$$= \int \frac{dt}{t^2+1} - \frac{1}{2} \int \frac{2t^2 dt}{(t^2+1)^2} \quad \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2+1)^2} \end{array} \left| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2+1} \end{array} \right. \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \frac{dt}{(t^2+1)^2} = \left[\begin{array}{l} \text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} \\ I_n = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array} \right]$$

$$= \frac{2 \cdot 2 - 3}{2(2-1)} I_1 + \frac{t}{2(2-1)(t^2+1)}$$

Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \frac{1 \cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2) dt}{(t^2+1)^2}$$

$$= \int \frac{dt}{t^2+1} - \frac{1}{2} \int \frac{2t^2 dt}{(t^2+1)^2} = \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2+1)^2} \end{array} \left| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2+1} \end{array} \right. \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \frac{dt}{(t^2+1)^2} = \left[\begin{array}{l} \text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} \\ I_n = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array} \right]$$

$$= \frac{2 \cdot 2 - 3}{2(2-1)} I_1 + \frac{t}{2(2-1)(t^2+1)} = \frac{1}{2} \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)}$$

Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \int \frac{1 \cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2) dx}{(t^2+1)^2}$$

$$= \int \frac{dt}{t^2+1} - \frac{1}{2} \int \frac{2t^2 dt}{(t^2+1)^2} = \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2+1)^2} \end{array} \left| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2+1} \end{array} \right. \right] = \int \frac{dt}{t^2+1} - \frac{1}{2} \left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2} = \left[\begin{array}{l} \text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} \\ I_n = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array} \right]$$

$$= \frac{2 \cdot 2 - 3}{2(2-1)} I_1 + \frac{t}{2(2-1)(t^2+1)} = \frac{1}{2} \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2} \arctg t + \frac{t}{2(t^2+1)} + c$$

Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2} = \frac{1}{2} \operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \middle| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right| \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \int \frac{1 \cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2) dt}{(t^2+1)^2} \\ &= \int \frac{dt}{t^2+1} - \frac{1}{2} \int \frac{2t^2 dt}{(t^2+1)^2} = \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2+1)^2} \end{array} \middle| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2+1} \end{array} \right] = \int \frac{dt}{t^2+1} - \frac{1}{2} \left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right] \\ &= \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} - \frac{1}{2} \int \frac{dt}{t^2+1} \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \middle| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right| \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2} = \left[\begin{array}{l} \text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} \\ I_n = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array} \right] \\ &= \frac{2 \cdot 2 - 3}{2(2-1)} I_1 + \frac{t}{2(2-1)(t^2+1)} = \frac{1}{2} \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2} \operatorname{arctg} t + \frac{t}{2(t^2+1)} + c \\ &= \frac{1}{2} \operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c, \quad x \in R, \quad c \in R. \end{aligned}$$

Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2} = \frac{1}{2} \operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \middle| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right| \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \int \frac{1 \cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2) dt}{(t^2+1)^2} \\ &= \int \frac{dt}{t^2+1} - \frac{1}{2} \int \frac{2t^2 dt}{(t^2+1)^2} = \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2+1)^2} \end{array} \middle| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2+1} \end{array} \right] = \int \frac{dt}{t^2+1} - \frac{1}{2} \left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right] \\ &= \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} - \frac{1}{2} \int \frac{dt}{t^2+1} = \frac{1}{2} \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \middle| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right| \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2} = \left[\begin{array}{l} \text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} \\ I_n = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array} \right] \\ &= \frac{2 \cdot 2 - 3}{2(2-1)} I_1 + \frac{t}{2(2-1)(t^2+1)} = \frac{1}{2} \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2} \operatorname{arctg} t + \frac{t}{2(t^2+1)} + c \\ &= \frac{1}{2} \operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c, \quad x \in R, \quad c \in R. \end{aligned}$$

Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2} = \frac{1}{2} \operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \middle| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right| \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \int \frac{1 \cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2) dt}{(t^2+1)^2} \\ &= \int \frac{dt}{t^2+1} - \frac{1}{2} \int \frac{2t^2 dt}{(t^2+1)^2} = \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2+1)^2} \end{array} \middle| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2+1} \end{array} \right] = \int \frac{dt}{t^2+1} - \frac{1}{2} \left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right] \\ &= \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} - \frac{1}{2} \int \frac{dt}{t^2+1} = \frac{1}{2} \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2} \operatorname{arctg} t + \frac{t}{2(t^2+1)} + c \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \middle| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right| \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2} = \left[\begin{array}{l} \text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} \\ I_n = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array} \right] \\ &= \frac{2 \cdot 2 - 3}{2(2-1)} I_1 + \frac{t}{2(2-1)(t^2+1)} = \frac{1}{2} \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2} \operatorname{arctg} t + \frac{t}{2(t^2+1)} + c \\ &= \frac{1}{2} \operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c, \quad x \in R, \quad c \in R. \end{aligned}$$

Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2} = \frac{1}{2} \operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \int \frac{1 \cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2) dt}{(t^2+1)^2} \\ &= \int \frac{dt}{t^2+1} - \frac{1}{2} \int \frac{2t^2 dt}{(t^2+1)^2} = \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2+1)^2} \end{array} \left| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2+1} \end{array} \right. \right] = \int \frac{dt}{t^2+1} - \frac{1}{2} \left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right] \\ &= \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} - \frac{1}{2} \int \frac{dt}{t^2+1} = \frac{1}{2} \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2} \operatorname{arctg} t + \frac{t}{2(t^2+1)} + c \\ &= \frac{1}{2} \operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c, \quad x \in R, \quad c \in R. \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+5=x^2+4x+4+1 \\ = (x+2)^2+1=t^2+1 \end{array} \right. \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2} = \left[\begin{array}{l} \text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} \\ I_n = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array} \right] \\ &= \frac{2 \cdot 2 - 3}{2(2-1)} I_1 + \frac{t}{2(2-1)(t^2+1)} = \frac{1}{2} \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2} \operatorname{arctg} t + \frac{t}{2(t^2+1)} + c \\ &= \frac{1}{2} \operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c, \quad x \in R, \quad c \in R. \end{aligned}$$

Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$



Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \middle| \begin{array}{l} x^2+4x+3=x^2+4x+4-1 \\ = (x+2)^2-1=t^2-1 \end{array} \middle| \begin{array}{l} x \in \mathbb{R} - \{-3, -1\} \\ t \in \mathbb{R} - \{\pm 1\} \end{array} \right]$$



Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \middle| \begin{array}{l} x^2+4x+3=x^2+4x+4-1 \\ = (x+2)^2-1=t^2-1 \end{array} \middle| \begin{array}{l} x \in \mathbb{R} - \{-3, -1\} \\ t \in \mathbb{R} - \{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = - \int \frac{-1 \cdot dt}{(t^2-1)^2}$$



Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+3=x^2+4x+4-1 \\ = (x+2)^2-1=t^2-1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} - \{-3, -1\} \\ t \in \mathbb{R} - \{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = - \int \frac{-1 \cdot dt}{(t^2-1)^2}$$

$$= - \int \frac{(t^2-1-t^2) dt}{(t^2-1)^2}$$



Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[\begin{array}{c} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{c} x^2+4x+3=x^2+4x+4-1 \\ = (x+2)^2-1=t^2-1 \end{array} \right. \begin{array}{c} x \in \mathbb{R} - \{-3, -1\} \\ t \in \mathbb{R} - \{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = - \int \frac{-1 \cdot dt}{(t^2-1)^2}$$

$$= - \int \frac{(t^2-1-t^2) dt}{(t^2-1)^2} = - \int \frac{dt}{t^2-1} + \frac{1}{2} \int \frac{2t^2 dt}{(t^2-1)^2}$$



Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+3=x^2+4x+4-1 \\ = (x+2)^2-1=t^2-1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} - \{-3, -1\} \\ t \in \mathbb{R} - \{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = - \int \frac{1 \cdot dt}{(t^2-1)^2}$$

$$= - \int \frac{(t^2-1-t^2) dt}{(t^2-1)^2} = - \int \frac{dt}{t^2-1} + \frac{1}{2} \int \frac{2t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2-1)^2} \end{array} \left| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2-1} \end{array} \right. \right] \quad \text{☞}$$

Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+3=x^2+4x+4-1 \\ =(x+2)^2-1=t^2-1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} - \{-3, -1\} \\ t \in \mathbb{R} - \{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = - \int \frac{-1 \cdot dt}{(t^2-1)^2}$$

$$= - \int \frac{(t^2-1-t^2) dt}{(t^2-1)^2} = - \int \frac{dt}{t^2-1} + \frac{1}{2} \int \frac{2t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2-1)^2} \end{array} \left| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2-1} \end{array} \right. \right] = - \int \frac{dt}{t^2-1} + \frac{1}{2} \left[- \frac{t}{t^2-1} + \int \frac{dt}{t^2-1} \right]$$

Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+3=x^2+4x+4-1 \\ =(x+2)^2-1=t^2-1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} - \{-3, -1\} \\ t \in \mathbb{R} - \{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = - \int \frac{-1 \cdot dt}{(t^2-1)^2}$$

$$= - \int \frac{(t^2-1-t^2) dt}{(t^2-1)^2} = - \int \frac{dt}{t^2-1} + \frac{1}{2} \int \frac{2t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2-1)^2} \end{array} \left| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2-1} \end{array} \right. \right] = - \int \frac{dt}{t^2-1} + \frac{1}{2} \left[-\frac{t}{t^2-1} + \int \frac{dt}{t^2-1} \right]$$

$$= - \int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} + \frac{1}{2} \int \frac{dt}{t^2-1}$$

Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+3=x^2+4x+4-1 \\ = (x+2)^2-1=t^2-1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} - \{-3, -1\} \\ t \in \mathbb{R} - \{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = - \int \frac{-1 \cdot dt}{(t^2-1)^2}$$

$$= - \int \frac{(t^2-1-t^2) dt}{(t^2-1)^2} = - \int \frac{dt}{t^2-1} + \frac{1}{2} \int \frac{2t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2-1)^2} \end{array} \left| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2-1} \end{array} \right. \right] = - \int \frac{dt}{t^2-1} + \frac{1}{2} \left[-\frac{t}{t^2-1} + \int \frac{dt}{t^2-1} \right]$$

$$= - \int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} + \frac{1}{2} \int \frac{dt}{t^2-1} = -\frac{1}{2} \int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)}$$

Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+3=x^2+4x+4-1 \\ =(x+2)^2-1=t^2-1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} - \{-3, -1\} \\ t \in \mathbb{R} - \{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = - \int \frac{-1 \cdot dt}{(t^2-1)^2}$$

$$= - \int \frac{(t^2-1-t^2)dt}{(t^2-1)^2} = - \int \frac{dt}{t^2-1} + \frac{1}{2} \int \frac{2t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2-1)^2} \end{array} \left| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2-1} \end{array} \right. \right] = - \int \frac{dt}{t^2-1} + \frac{1}{2} \left[-\frac{t}{t^2-1} + \int \frac{dt}{t^2-1} \right]$$

$$= - \int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} + \frac{1}{2} \int \frac{dt}{t^2-1} = -\frac{1}{2} \int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)}$$

$$= -\frac{1}{2} \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| - \frac{t}{2(t^2-1)} + C$$

Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+3=x^2+4x+4-1 \\ =(x+2)^2-1=t^2-1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} - \{-3, -1\} \\ t \in \mathbb{R} - \{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = - \int \frac{-1 \cdot dt}{(t^2-1)^2}$$

$$= - \int \frac{(t^2-1-t^2) dt}{(t^2-1)^2} = - \int \frac{dt}{t^2-1} + \frac{1}{2} \int \frac{2t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2-1)^2} \end{array} \left| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2-1} \end{array} \right. \right] = - \int \frac{dt}{t^2-1} + \frac{1}{2} \left[-\frac{t}{t^2-1} + \int \frac{dt}{t^2-1} \right]$$

$$= - \int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} + \frac{1}{2} \int \frac{dt}{t^2-1} = -\frac{1}{2} \int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)}$$

$$= -\frac{1}{2} \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| - \frac{t}{2(t^2-1)} + C = -\frac{1}{4} \ln \left| \frac{x+2-1}{x+2+1} \right| - \frac{x+2}{2(x^2+4x+3)} + C$$

Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2} = -\frac{1}{4} \ln \left| \frac{x+1}{x+3} \right| - \frac{x+2}{2(x^2+4x+3)} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+2 \\ dt=dx \end{array} \left| \begin{array}{l} x^2+4x+3=x^2+4x+4-1 \\ =(x+2)^2-1=t^2-1 \end{array} \right. \begin{array}{l} x \in \mathbb{R} - \{-3, -1\} \\ t \in \mathbb{R} - \{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = - \int \frac{-1 \cdot dt}{(t^2-1)^2}$$

$$= - \int \frac{(t^2-1-t^2) dt}{(t^2-1)^2} = - \int \frac{dt}{t^2-1} + \frac{1}{2} \int \frac{2t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u=t \\ v'=\frac{2t}{(t^2-1)^2} \end{array} \left| \begin{array}{l} u'=1 \\ v=-\frac{1}{t^2-1} \end{array} \right. \right] = - \int \frac{dt}{t^2-1} + \frac{1}{2} \left[- \frac{t}{t^2-1} + \int \frac{dt}{t^2-1} \right]$$

$$= - \int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} + \frac{1}{2} \int \frac{dt}{t^2-1} = -\frac{1}{2} \int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)}$$

$$= -\frac{1}{2} \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| - \frac{t}{2(t^2-1)} + c = -\frac{1}{4} \ln \left| \frac{x+2-1}{x+2+1} \right| - \frac{x+2}{2(x^2+4x+3)} + c$$

$$= -\frac{1}{4} \ln \left| \frac{x+1}{x+3} \right| - \frac{x+2}{2(x^2+4x+3)} + c$$

$$= \frac{1}{4} \ln |x+3| - \frac{1}{4} \ln |x+1| - \frac{x+2}{2(x^2+4x+3)} + c, \quad x \in \mathbb{R} - \{-3, -1\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 131, 132

$$\int \frac{x^2 dx}{(x^2 + a^2)^2}$$

$$a > 0$$



$$\int \frac{x^2 dx}{(x^2 - a^2)^2}$$

$$a > 0$$



Riešené príklady – 131, 132

$$\int \frac{x^2 dx}{(x^2 + a^2)^2}$$

$$a > 0$$

$$= \frac{1}{2} \int \frac{2x^2 dx}{(x^2 + a^2)^2}$$



$$\int \frac{x^2 dx}{(x^2 - a^2)^2}$$

$$a > 0$$

$$= \frac{1}{2} \int \frac{2x^2 dx}{(x^2 - a^2)^2}$$



Riešené príklady – 131, 132

$$\int \frac{x^2 dx}{(x^2 + a^2)^2}$$

$$a > 0$$

$$= \frac{1}{2} \int \frac{2x^2 dx}{(x^2 + a^2)^2} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 + a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2 + a^2} \end{array} \right] \text{ ☞ }$$

$$\int \frac{x^2 dx}{(x^2 - a^2)^2}$$

$$a > 0$$

$$= \frac{1}{2} \int \frac{2x^2 dx}{(x^2 - a^2)^2} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2 - a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2 - a^2} \end{array} \right] \text{ ☞ }$$

Riešené príklady – 131, 132

$$\int \frac{x^2 dx}{(x^2+a^2)^2}$$

$$a > 0$$

$$= \frac{1}{2} \int \frac{2x^2 dx}{(x^2+a^2)^2} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2+a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2+a^2} \end{array} \right] \stackrel{?}{=} \frac{1}{2} \left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2} \right]$$

$$\int \frac{x^2 dx}{(x^2-a^2)^2}$$

$$a > 0$$

$$= \frac{1}{2} \int \frac{2x^2 dx}{(x^2-a^2)^2} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2-a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2-a^2} \end{array} \right] \stackrel{?}{=} \frac{1}{2} \left[-\frac{x}{x^2-a^2} + \int \frac{dx}{x^2-a^2} \right]$$

Riešené príklady – 131, 132

$$\int \frac{x^2 dx}{(x^2+a^2)^2}$$

 $a > 0$

$$= \frac{1}{2} \int \frac{2x^2 dx}{(x^2+a^2)^2} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2+a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2+a^2} \end{array} \right] = \frac{1}{2} \left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2} \right]$$

$$= -\frac{x}{2(x^2+a^2)} + \frac{1}{2} \cdot \frac{1}{a} \arctg \frac{x}{a} + c$$

$$\int \frac{x^2 dx}{(x^2-a^2)^2}$$

 $a > 0$

$$= \frac{1}{2} \int \frac{2x^2 dx}{(x^2-a^2)^2} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2-a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2-a^2} \end{array} \right] = \frac{1}{2} \left[-\frac{x}{x^2-a^2} + \int \frac{dx}{x^2-a^2} \right]$$

$$= -\frac{x}{2(x^2-a^2)} + \frac{1}{2} \cdot \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$$

Riešené príklady – 131, 132

$$\int \frac{x^2 dx}{(x^2+a^2)^2} = \frac{1}{2a} \operatorname{arctg} \frac{x}{a} - \frac{x}{2(x^2+a^2)} + c \quad a > 0$$

$$= \frac{1}{2} \int \frac{2x^2 dx}{(x^2+a^2)^2} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2+a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2+a^2} \end{array} \right] = \frac{1}{2} \left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2} \right]$$

$$= -\frac{x}{2(x^2+a^2)} + \frac{1}{2} \cdot \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c = \frac{1}{2a} \operatorname{arctg} \frac{x}{a} - \frac{x}{2(x^2+a^2)} + c, x \in R, c \in R.$$

$$\int \frac{x^2 dx}{(x^2-a^2)^2} = \frac{1}{4a} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2(x^2-a^2)} + c \quad a > 0$$

$$= \frac{1}{2} \int \frac{2x^2 dx}{(x^2-a^2)^2} = \left[\begin{array}{l} u = x \\ v' = \frac{2x}{(x^2-a^2)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{x^2-a^2} \end{array} \right] = \frac{1}{2} \left[-\frac{x}{x^2-a^2} + \int \frac{dx}{x^2-a^2} \right]$$

$$= -\frac{x}{2(x^2-a^2)} + \frac{1}{2} \cdot \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c = \frac{1}{4a} \ln \left| \frac{x-a}{x+a} \right| - \frac{x}{2(x^2-a^2)} + c$$

$$= \frac{1}{4a} \ln |x-a| - \frac{1}{4a} \ln |x+a| - \frac{x}{2(x^2-a^2)} + c, x \in R - \{\pm a\}, c \in R.$$

Riešené príklady – 133, 134

$$\int \frac{x \, dx}{x^2 + a^2}$$

$$a > 0$$

$$\int \frac{x \, dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}, n \neq 1$$

Riešené príklady – 133, 134

$$\int \frac{x \, dx}{x^2 + a^2}$$

$$a > 0$$

$$= \frac{1}{2} \int \frac{2x \, dx}{x^2 + a^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 + a^2 \mid x \in (-\infty; 0), t \in \langle a^2; \infty \rangle \\ \quad \quad \quad dt = 2x \, dx \mid x \in \langle 0; \infty \rangle, t \in \langle a^2; \infty \rangle \end{array} \right]$$

$$\int \frac{x \, dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}, n \neq 1$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 + a^2 \mid x \in (-\infty; 0), t \in \langle a^2; \infty \rangle \\ \quad \quad \quad dt = 2x \, dx \mid x \in \langle 0; \infty \rangle, t \in \langle a^2; \infty \rangle \end{array} \right]$$

Riešené príklady – 133, 134

$$\int \frac{x \, dx}{x^2 + a^2}$$

$$a > 0$$

$$= \frac{1}{2} \int \frac{2x \, dx}{x^2 + a^2} = \frac{1}{2} \int \frac{(x^2 + a^2)' \, dx}{x^2 + a^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 + a^2 \mid x \in (-\infty; 0), t \in \langle a^2; \infty \rangle \\ \quad \quad \quad dt = 2x \, dx \mid x \in \langle 0; \infty \rangle, t \in \langle a^2; \infty \rangle \end{array} \right] = \frac{1}{2} \int \frac{dt}{t}$$

$$\int \frac{x \, dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}, n \neq 1$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 + a^2 \mid x \in (-\infty; 0), t \in \langle a^2; \infty \rangle \\ \quad \quad \quad dt = 2x \, dx \mid x \in \langle 0; \infty \rangle, t \in \langle a^2; \infty \rangle \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^n}$$

Riešené príklady – 133, 134

$$\int \frac{x \, dx}{x^2 + a^2}$$

$$a > 0$$

$$= \frac{1}{2} \int \frac{2x \, dx}{x^2 + a^2} = \frac{1}{2} \int \frac{(x^2 + a^2)' \, dx}{x^2 + a^2} = \frac{1}{2} \ln |x^2 + a^2| + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 + a^2 \mid x \in (-\infty; 0), t \in (a^2; \infty) \\ \quad \quad \quad dt = 2x \, dx \mid x \in (0; \infty), t \in (a^2; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln t + c$$

$$\int \frac{x \, dx}{(x^2 + a^2)^n}$$

$$a > 0, n \in \mathbb{N}, n \neq 1$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 + a^2 \mid x \in (-\infty; 0), t \in (a^2; \infty) \\ \quad \quad \quad dt = 2x \, dx \mid x \in (0; \infty), t \in (a^2; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^n} = \frac{1}{2} \int t^{-n} \, dt$$

Riešené príklady – 133, 134

$$\int \frac{x \, dx}{x^2 + a^2} = \frac{1}{2} \ln(x^2 + a^2) + c \quad a > 0$$

$$= \frac{1}{2} \int \frac{2x \, dx}{x^2 + a^2} = \frac{1}{2} \int \frac{(x^2 + a^2)' \, dx}{x^2 + a^2} = \frac{1}{2} \ln |x^2 + a^2| + c = \frac{1}{2} \ln(x^2 + a^2) + c, \\ x \in \mathbb{R}, c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 + a^2 \mid x \in (-\infty; 0), t \in (a^2; \infty) \\ \quad \quad \quad dt = 2x \, dx \mid x \in (0; \infty), t \in (a^2; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln t + c = \frac{1}{2} \ln(x^2 + a^2) + c, \\ x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \frac{x \, dx}{(x^2 + a^2)^n} \quad a > 0, n \in \mathbb{N}, n \neq 1$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 + a^2 \mid x \in (-\infty; 0), t \in (a^2; \infty) \\ \quad \quad \quad dt = 2x \, dx \mid x \in (0; \infty), t \in (a^2; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^n} = \frac{1}{2} \int t^{-n} \, dt = \frac{t^{1-n}}{2(1-n)} + c$$

Riešené príklady – 133, 134

$$\int \frac{x \, dx}{x^2 + a^2} = \frac{1}{2} \ln(x^2 + a^2) + c \quad a > 0$$

$$= \frac{1}{2} \int \frac{2x \, dx}{x^2 + a^2} = \frac{1}{2} \int \frac{(x^2 + a^2)' \, dx}{x^2 + a^2} = \frac{1}{2} \ln |x^2 + a^2| + c = \frac{1}{2} \ln(x^2 + a^2) + c, \\ x \in R, c \in R.$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 + a^2 \mid x \in (-\infty; 0), t \in (a^2; \infty) \\ dt = 2x \, dx \mid x \in (0; \infty), t \in (a^2; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln t + c = \frac{1}{2} \ln(x^2 + a^2) + c, \\ x \in R, c \in R.$$

$$\int \frac{x \, dx}{(x^2 + a^2)^n} = -\frac{1}{2(n-1)(x^2 + a^2)^{n-1}} + c \quad a > 0, n \in N, n \neq 1$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 + a^2 \mid x \in (-\infty; 0), t \in (a^2; \infty) \\ dt = 2x \, dx \mid x \in (0; \infty), t \in (a^2; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^n} = \frac{1}{2} \int t^{-n} \, dt = \frac{t^{1-n}}{2(1-n)} + c \\ = \frac{(x^2 + a^2)^{1-n}}{2(1-n)} + c = -\frac{1}{2(n-1)(x^2 + a^2)^{n-1}} + c, x \in R, c \in R.$$

Riešené príklady – 135, 136

$$\int \frac{x \, dx}{x^2 - a^2}$$

$$a > 0$$

$$\int \frac{x \, dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}, n \neq 1$$

Riešené príklady – 135, 136

$$\int \frac{x \, dx}{x^2 - a^2}$$

$$a > 0$$

$$= \frac{1}{2} \int \frac{2x \, dx}{x^2 - a^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - a^2 \mid x \in (-\infty; 0) - \{-a\}, t \in \langle -a^2; \infty \rangle - \{0\} \\ \quad \quad \quad dt = 2x \, dx \mid x \in \langle 0; \infty \rangle - \{a\}, t \in \langle -a^2; \infty \rangle - \{0\} \end{array} \right]$$

$$\int \frac{x \, dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}, n \neq 1$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - a^2 \mid x \in (-\infty; 0) - \{-a\}, t \in \langle -a^2; \infty \rangle - \{0\} \\ \quad \quad \quad dt = 2x \, dx \mid x \in \langle 0; \infty \rangle - \{a\}, t \in \langle -a^2; \infty \rangle - \{0\} \end{array} \right]$$

Riešené príklady – 135, 136

$$\int \frac{x \, dx}{x^2 - a^2}$$

$$a > 0$$

$$= \frac{1}{2} \int \frac{2x \, dx}{x^2 - a^2} = \frac{1}{2} \int \frac{(x^2 - a^2)' \, dx}{x^2 - a^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - a^2 \mid x \in (-\infty; 0) - \{-a\}, t \in \langle -a^2; \infty \rangle - \{0\} \\ \quad \quad \quad dt = 2x \, dx \mid x \in \langle 0; \infty \rangle - \{a\}, t \in \langle -a^2; \infty \rangle - \{0\} \end{array} \right] = \frac{1}{2} \int \frac{dt}{t}$$

$$\int \frac{x \, dx}{(x^2 - a^2)^n}$$

$$a > 0, n \in \mathbb{N}, n \neq 1$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - a^2 \mid x \in (-\infty; 0) - \{-a\}, t \in \langle -a^2; \infty \rangle - \{0\} \\ \quad \quad \quad dt = 2x \, dx \mid x \in \langle 0; \infty \rangle - \{a\}, t \in \langle -a^2; \infty \rangle - \{0\} \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^n}$$

Riešené príklady – 135, 136

$$\int \frac{x dx}{x^2 - a^2} = \frac{1}{2} \ln |x^2 - a^2| + c \quad a > 0$$

$$= \frac{1}{2} \int \frac{2x dx}{x^2 - a^2} = \frac{1}{2} \int \frac{(x^2 - a^2)' dx}{x^2 - a^2} = \frac{1}{2} \ln |x^2 - a^2| + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - a^2 \mid x \in (-\infty; 0) - \{-a\}, t \in (-a^2; \infty) - \{0\} \\ \quad \quad \quad dt = 2x dx \mid x \in (0; \infty) - \{a\}, t \in (-a^2; \infty) - \{0\} \end{array} \right] = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln |t| + c$$

$$\int \frac{x dx}{(x^2 - a^2)^n} \quad a > 0, n \in \mathbb{N}, n \neq 1$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - a^2 \mid x \in (-\infty; 0) - \{-a\}, t \in (-a^2; \infty) - \{0\} \\ \quad \quad \quad dt = 2x dx \mid x \in (0; \infty) - \{a\}, t \in (-a^2; \infty) - \{0\} \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^n} = \frac{1}{2} \int t^{-n} dt$$

Riešené príklady – 135, 136

$$\int \frac{x dx}{x^2 - a^2} = \frac{1}{2} \ln |x^2 - a^2| + c \quad a > 0$$

$$= \frac{1}{2} \int \frac{2x dx}{x^2 - a^2} = \frac{1}{2} \int \frac{(x^2 - a^2)' dx}{x^2 - a^2} = \frac{1}{2} \ln |x^2 - a^2| + c, \quad x \in \mathbb{R} - \{\pm a\}, \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - a^2 \mid x \in (-\infty; 0) - \{-a\}, t \in (-a^2; \infty) - \{0\} \\ dt = 2x dx \mid x \in (0; \infty) - \{a\}, t \in (-a^2; \infty) - \{0\} \end{array} \right] = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln |t| + c$$

$$= \frac{1}{2} \ln |x^2 - a^2| + c, \quad x \in \mathbb{R} - \{\pm a\}, \quad c \in \mathbb{R}.$$

$$\int \frac{x dx}{(x^2 - a^2)^n} \quad a > 0, n \in \mathbb{N}, n \neq 1$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - a^2 \mid x \in (-\infty; 0) - \{-a\}, t \in (-a^2; \infty) - \{0\} \\ dt = 2x dx \mid x \in (0; \infty) - \{a\}, t \in (-a^2; \infty) - \{0\} \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^n} = \frac{1}{2} \int t^{-n} dt = \frac{t^{1-n}}{2(1-n)} + c$$

Riešené príklady – 135, 136

$$\int \frac{x dx}{x^2 - a^2} = \frac{1}{2} \ln |x^2 - a^2| + c \quad a > 0$$

$$= \frac{1}{2} \int \frac{2x dx}{x^2 - a^2} = \frac{1}{2} \int \frac{(x^2 - a^2)' dx}{x^2 - a^2} = \frac{1}{2} \ln |x^2 - a^2| + c, \quad x \in \mathbb{R} - \{\pm a\}, \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - a^2 \mid x \in (-\infty; 0) - \{-a\}, t \in (-a^2; \infty) - \{0\} \\ dt = 2x dx \mid x \in (0; \infty) - \{a\}, t \in (-a^2; \infty) - \{0\} \end{array} \right] = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln |t| + c$$

$$= \frac{1}{2} \ln |x^2 - a^2| + c, \quad x \in \mathbb{R} - \{\pm a\}, \quad c \in \mathbb{R}.$$

$$\int \frac{x dx}{(x^2 - a^2)^n} = -\frac{1}{2(n-1)(x^2 - a^2)^{n-1}} + c \quad a > 0, n \in \mathbb{N}, n \neq 1$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - a^2 \mid x \in (-\infty; 0) - \{-a\}, t \in (-a^2; \infty) - \{0\} \\ dt = 2x dx \mid x \in (0; \infty) - \{a\}, t \in (-a^2; \infty) - \{0\} \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^n} = \frac{1}{2} \int t^{-n} dt = \frac{t^{1-n}}{2(1-n)} + c$$

$$= \frac{(x^2 - a^2)^{1-n}}{2(1-n)} + c = -\frac{1}{2(n-1)(x^2 - a^2)^{n-1}} + c, \quad x \in \mathbb{R} - \{\pm a\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 137, 138

$$\int \frac{x^n dx}{x-1}$$

$$n \in \mathbb{N}$$

$$\int \frac{x dx}{x-1}$$

Riešené príklady – 137, 138

$$\int \frac{x^n dx}{x-1}$$

 $n \in \mathbb{N}$

$$= \int \frac{x^n - 1 + 1}{x-1} dx$$

$$\int \frac{x dx}{x-1}$$

$$= \int \frac{x-1+1}{x-1} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{t+1}{t} dt$$

Riešené príklady – 137, 138

$$\int \frac{x^n dx}{x-1}$$

 $n \in \mathbb{N}$

$$= \int \frac{x^n - 1 + 1}{x-1} dx = \int \left[\frac{x^n - 1}{x-1} + \frac{1}{x-1} \right] dx$$

$$\int \frac{x dx}{x-1}$$

$$= \int \frac{x-1+1}{x-1} dx = \int \left[1 + \frac{1}{x-1} \right] dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{t+1}{t} dt = \int \left[1 + \frac{1}{t} \right] dt$$

Riešené príklady – 137, 138

$$\int \frac{x^n dx}{x-1}$$

 $n \in \mathbb{N}$

$$= \int \frac{x^n - 1 + 1}{x-1} dx = \int \left[\frac{x^n - 1}{x-1} + \frac{1}{x-1} \right] dx = \int \left[x^{n-1} + \dots + x + 1 + \frac{1}{x-1} \right] dx$$

$$\int \frac{x dx}{x-1} = x + \ln |x-1| + c$$

$$= \int \frac{x-1+1}{x-1} dx = \int \left[1 + \frac{1}{x-1} \right] dx = x + \ln |x-1| + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{t+1}{t} dt = \int \left[1 + \frac{1}{t} \right] dt = t + \ln |t| + c_1$$

Riešené príklady – 137, 138

$$\int \frac{x^n dx}{x-1} = \frac{x^n}{n} + \dots + \frac{x^2}{2} + x + \ln|x-1| + c = \ln|x-1| + \sum_{i=1}^n \frac{x^i}{i} + c \quad n \in \mathbb{N}$$

$$\begin{aligned} &= \int \frac{x^n - 1 + 1}{x-1} dx = \int \left[\frac{x^n - 1}{x-1} + \frac{1}{x-1} \right] dx = \int \left[x^{n-1} + \dots + x + 1 + \frac{1}{x-1} \right] dx \\ &= \frac{x^n}{n} + \dots + \frac{x^2}{2} + x + \ln|x-1| + c = \ln|x-1| + \sum_{i=1}^n \frac{x^i}{i} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}. \end{aligned}$$

$$\int \frac{x dx}{x-1} = x + \ln|x-1| + c$$

$$= \int \frac{x-1+1}{x-1} dx = \int \left[1 + \frac{1}{x-1} \right] dx = x + \ln|x-1| + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{t+1}{t} dt = \int \left[1 + \frac{1}{t} \right] dt = t + \ln|t| + c_1 \\ &= x - 1 + \ln|x-1| + c_1 \end{aligned}$$

Riešené príklady – 137, 138

$$\int \frac{x^n dx}{x-1} = \frac{x^n}{n} + \dots + \frac{x^2}{2} + x + \ln|x-1| + c = \ln|x-1| + \sum_{i=1}^n \frac{x^i}{i} + c \quad n \in \mathbb{N}$$

$$\begin{aligned} &= \int \frac{x^n - 1 + 1}{x-1} dx = \int \left[\frac{x^n - 1}{x-1} + \frac{1}{x-1} \right] dx = \int \left[x^{n-1} + \dots + x + 1 + \frac{1}{x-1} \right] dx \\ &= \frac{x^n}{n} + \dots + \frac{x^2}{2} + x + \ln|x-1| + c = \ln|x-1| + \sum_{i=1}^n \frac{x^i}{i} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}. \end{aligned}$$

$$\int \frac{x dx}{x-1} = x + \ln|x-1| + c$$

$$= \int \frac{x-1+1}{x-1} dx = \int \left[1 + \frac{1}{x-1} \right] dx = x + \ln|x-1| + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{t+1}{t} dt = \int \left[1 + \frac{1}{t} \right] dt = t + \ln|t| + c_1$$

$$= x - 1 + \ln|x-1| + c_1 = \left[\begin{array}{l} c = c_1 - 1 \\ c \in \mathbb{R}, \quad c_1 \in \mathbb{R} \end{array} \right]$$

Riešené príklady – 137, 138

$$\int \frac{x^n dx}{x-1} = \frac{x^n}{n} + \dots + \frac{x^2}{2} + x + \ln|x-1| + c = \ln|x-1| + \sum_{i=1}^n \frac{x^i}{i} + c \quad n \in \mathbb{N}$$

$$\begin{aligned} &= \int \frac{x^n - 1 + 1}{x-1} dx = \int \left[\frac{x^n - 1}{x-1} + \frac{1}{x-1} \right] dx = \int \left[x^{n-1} + \dots + x + 1 + \frac{1}{x-1} \right] dx \\ &= \frac{x^n}{n} + \dots + \frac{x^2}{2} + x + \ln|x-1| + c = \ln|x-1| + \sum_{i=1}^n \frac{x^i}{i} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}. \end{aligned}$$

$$\int \frac{x dx}{x-1} = x + \ln|x-1| + c$$

$$= \int \frac{x-1+1}{x-1} dx = \int \left[1 + \frac{1}{x-1} \right] dx = x + \ln|x-1| + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{t+1}{t} dt = \int \left[1 + \frac{1}{t} \right] dt = t + \ln|t| + c_1$$

$$= x - 1 + \ln|x-1| + c_1 = \left[\begin{array}{l} c = c_1 - 1 \\ c \in \mathbb{R}, c_1 \in \mathbb{R} \end{array} \right] = x + \ln|x-1| + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 139, 140

$$\int \frac{x^2 dx}{x-1}$$

$$\int \frac{x^9 dx}{x-1}$$

Riešené príklady – 139, 140

$$\int \frac{x^2 dx}{x-1}$$

$$= \int \frac{x^2-1+1}{x-1} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right]$$

$$\int \frac{x^9 dx}{x-1}$$

$$= \int \frac{x^9-1+1}{x-1} dx$$

Riešené príklady – 139, 140

$$\int \frac{x^2 dx}{x-1}$$

$$= \int \frac{x^2-1+1}{x-1} dx = \int \left[x+1 + \frac{1}{x-1} \right] dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t+1)^2 dt}{t}$$

$$\int \frac{x^9 dx}{x-1}$$

$$= \int \frac{x^9-1+1}{x-1} dx = \int \left[x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 + \frac{1}{x-1} \right] dx$$

Riešené príklady – 139, 140

$$\int \frac{x^2 dx}{x-1} = \frac{x^2}{2} + x + \ln|x-1| + c$$

$$= \int \frac{x^2-1+1}{x-1} dx = \int \left[x+1 + \frac{1}{x-1} \right] dx = \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \middle| \begin{array}{l} x \in \mathbb{R} - \{1\} \\ t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t+1)^2 dt}{t} = \int \frac{(t^2+2t+1) dt}{t}$$

$$\int \frac{x^9 dx}{x-1} =$$

$$= \int \frac{x^9-1+1}{x-1} dx = \int \left[x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 + \frac{1}{x-1} \right] dx$$

$$= \frac{x^9}{9} + \frac{x^8}{8} + \frac{x^7}{7} + \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}.$$

Riešené príklady – 139, 140

$$\int \frac{x^2 dx}{x-1} = \frac{x^2}{2} + x + \ln|x-1| + c$$

$$= \int \frac{x^2-1+1}{x-1} dx = \int \left[x+1 + \frac{1}{x-1} \right] dx = \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \middle| \begin{array}{l} x \in \mathbb{R} - \{1\} \\ t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t+1)^2 dt}{t} = \int \frac{(t^2+2t+1) dt}{t} = \int \left[t+2+\frac{1}{t} \right] dt$$

$$\int \frac{x^9 dx}{x-1} =$$

$$= \int \frac{x^9-1+1}{x-1} dx = \int \left[x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 + \frac{1}{x-1} \right] dx$$

$$= \frac{x^9}{9} + \frac{x^8}{8} + \frac{x^7}{7} + \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}.$$

Riešené príklady – 139, 140

$$\int \frac{x^2 dx}{x-1} = \frac{x^2}{2} + x + \ln|x-1| + c$$

$$= \int \frac{x^2-1+1}{x-1} dx = \int \left[x+1 + \frac{1}{x-1} \right] dx = \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t+1)^2 dt}{t} = \int \frac{(t^2+2t+1) dt}{t} = \int \left[t+2+\frac{1}{t} \right] dt$$

$$= \frac{t^2}{2} + 2t + \ln|t| + c_1$$

$$\int \frac{x^9 dx}{x-1} =$$

$$= \int \frac{x^9-1+1}{x-1} dx = \int \left[x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 + \frac{1}{x-1} \right] dx$$

$$= \frac{x^9}{9} + \frac{x^8}{8} + \frac{x^7}{7} + \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}.$$

Riešené príklady – 139, 140

$$\int \frac{x^2 dx}{x-1} = \frac{x^2}{2} + x + \ln|x-1| + c$$

$$= \int \frac{x^2-1+1}{x-1} dx = \int \left[x+1 + \frac{1}{x-1} \right] dx = \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t+1)^2 dt}{t} = \int \frac{(t^2+2t+1) dt}{t} = \int \left[t+2+\frac{1}{t} \right] dt$$

$$= \frac{t^2}{2} + 2t + \ln|t| + c_1 = \frac{(x-1)^2}{2} + 2(x-1) + \ln|x-1| + c_1$$

$$\int \frac{x^9 dx}{x-1} =$$

$$= \int \frac{x^9-1+1}{x-1} dx = \int \left[x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 + \frac{1}{x-1} \right] dx$$

$$= \frac{x^9}{9} + \frac{x^8}{8} + \frac{x^7}{7} + \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}.$$

Riešené príklady – 139, 140

$$\int \frac{x^2 dx}{x-1} = \frac{x^2}{2} + x + \ln|x-1| + c$$

$$= \int \frac{x^2-1+1}{x-1} dx = \int \left[x+1 + \frac{1}{x-1} \right] dx = \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t+1)^2 dt}{t} = \int \frac{(t^2+2t+1) dt}{t} = \int \left[t+2+\frac{1}{t} \right] dt$$

$$= \frac{t^2}{2} + 2t + \ln|t| + c_1 = \frac{(x-1)^2}{2} + 2(x-1) + \ln|x-1| + c_1$$

$$= \left[\begin{array}{l} \frac{x^2-2x+1}{2} + 2(x-1) \mid c = c_1 - \frac{3}{2} \\ = \frac{x^2}{2} - x + \frac{1}{2} + 2x - 2 \mid c \in \mathbb{R}, c_1 \in \mathbb{R} \end{array} \right]$$

$$\int \frac{x^9 dx}{x-1} =$$

$$= \int \frac{x^9-1+1}{x-1} dx = \int \left[x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 + \frac{1}{x-1} \right] dx$$

$$= \frac{x^9}{9} + \frac{x^8}{8} + \frac{x^7}{7} + \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}.$$

Riešené príklady – 139, 140

$$\int \frac{x^2 dx}{x-1} = \frac{x^2}{2} + x + \ln|x-1| + c$$

$$= \int \frac{x^2-1+1}{x-1} dx = \int \left[x+1 + \frac{1}{x-1} \right] dx = \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}.$$

$$\begin{aligned} &= \left[\text{Subst. } t=x-1 \begin{array}{l} x \in \mathbb{R} - \{1\} \\ dt = dx \end{array} \middle| \begin{array}{l} t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t+1)^2 dt}{t} = \int \frac{(t^2+2t+1) dt}{t} = \int \left[t+2+\frac{1}{t} \right] dt \\ &= \frac{t^2}{2} + 2t + \ln|t| + c_1 = \frac{(x-1)^2}{2} + 2(x-1) + \ln|x-1| + c_1 \\ &= \left[\begin{array}{l} \frac{x^2-2x+1}{2} + 2(x-1) \\ = \frac{x^2}{2} - x + \frac{1}{2} + 2x - 2 \end{array} \middle| \begin{array}{l} c = c_1 - \frac{3}{2} \\ c \in \mathbb{R}, c_1 \in \mathbb{R} \end{array} \right] = \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}. \end{aligned}$$

$$\int \frac{x^9 dx}{x-1} =$$

$$\begin{aligned} &= \int \frac{x^9-1+1}{x-1} dx = \int \left[x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 + \frac{1}{x-1} \right] dx \\ &= \frac{x^9}{9} + \frac{x^8}{8} + \frac{x^7}{7} + \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + c, x \in \mathbb{R} - \{1\}, c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2}$$

$$\int \frac{dx}{x^6(1+x^2)}$$

Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{(1-x)x^2} = \frac{\alpha}{x-1} + \frac{\beta}{x} + \frac{\gamma}{x^2} \\ 1 = \alpha x^2 + \beta x(x-1) + \gamma(x-1) = -\gamma + (-\beta+\gamma)x + (\alpha+\beta)x^2 \end{array} \right]$$

$$\int \frac{dx}{x^6(1+x^2)}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{x^6(1+x^2)} = \frac{1}{t^3(1+t)} = \frac{\alpha}{t+1} + \frac{\beta}{t} + \frac{\gamma}{t^2} + \frac{\delta}{t^3} \\ 1 = \alpha t^3 + \beta t^2(t+1) + \gamma t(t+1) + \delta(t+1) = \delta + (\gamma+\delta)t + (\beta+\gamma)t^2 + (\alpha+\beta)t^3 \end{array} \right]$$

Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(1-x)x^2} = \frac{\alpha}{x-1} + \frac{\beta}{x} + \frac{\gamma}{x^2} \right. \\ \left. 1 = \alpha x^2 + \beta x(x-1) + \gamma(x-1) = -\gamma + (-\beta+\gamma)x + (\alpha+\beta)x^2 \Rightarrow 1 = -\gamma, 0 = -\beta+\gamma, 0 = \alpha+\beta \right]$$

$$\int \frac{dx}{x^6(1+x^2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{x^6(1+x^2)} = \frac{1}{t^3(1+t)} = \frac{\alpha}{t+1} + \frac{\beta}{t} + \frac{\gamma}{t^2} + \frac{\delta}{t^3} \right. \\ \left. 1 = \alpha t^3 + \beta t^2(t+1) + \gamma t(t+1) + \delta(t+1) = \delta + (\gamma+\delta)t + (\beta+\gamma)t^2 + (\alpha+\beta)t^3 \Rightarrow 1 = \delta, 0 = \gamma+\delta, 0 = \beta+\gamma, 0 = \alpha+\beta \right]$$

Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(1-x)x^2} = \frac{\alpha}{x-1} + \frac{\beta}{x} + \frac{\gamma}{x^2} = \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}, \quad \gamma = -1, \quad \beta = -1, \quad \alpha = 1 \right]$$

$$1 = \alpha x^2 + \beta x(x-1) + \gamma(x-1) = -\gamma + (-\beta + \gamma)x + (\alpha + \beta)x^2 \Rightarrow 1 = -\gamma, \quad 0 = -\beta + \gamma, \quad 0 = \alpha + \beta$$

$$\int \frac{dx}{x^6(1+x^2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{x^6(1+x^2)} = \frac{1}{t^3(1+t)} = \frac{\alpha}{t+1} + \frac{\beta}{t} + \frac{\gamma}{t^2} + \frac{\delta}{t^3} = -\frac{1}{x^2+1} + \frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6}, \quad \delta = 1, \quad \gamma = -1, \quad \beta = 1, \quad \alpha = -1 \right]$$

$$1 = \alpha t^3 + \beta t^2(t+1) + \gamma t(t+1) + \delta(t+1) = \delta + (\gamma + \delta)t + (\beta + \gamma)t^2 + (\alpha + \beta)t^3 \Rightarrow 1 = \delta, \quad 0 = \gamma + \delta, \quad 0 = \beta + \gamma, \quad 0 = \alpha + \beta$$

Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(1-x)x^2} = \frac{\alpha}{x-1} + \frac{\beta}{x} + \frac{\gamma}{x^2} = \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}, \quad \gamma = -1, \quad \beta = -1, \quad \alpha = 1 \right]$$

$$1 = \alpha x^2 + \beta x(x-1) + \gamma(x-1) = -\gamma + (-\beta + \gamma)x + (\alpha + \beta)x^2 \Rightarrow 1 = -\gamma, \quad 0 = -\beta + \gamma, \quad 0 = \alpha + \beta$$

$$= \int \left[\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} \right] dx = \int \left[\frac{1}{x-1} - \frac{1}{x} - x^{-2} \right] dx$$

$$\int \frac{dx}{x^6(1+x^2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{x^6(1+x^2)} = \frac{1}{t^3(1+t)} = \frac{\alpha}{t+1} + \frac{\beta}{t} + \frac{\gamma}{t^2} + \frac{\delta}{t^3} = -\frac{1}{x^2+1} + \frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6}, \quad \delta = 1, \quad \gamma = -1, \quad \beta = 1, \quad \alpha = -1 \right]$$

$$1 = \alpha t^3 + \beta t^2(t+1) + \gamma t(t+1) + \delta(t+1) = \delta + (\gamma + \delta)t + (\beta + \gamma)t^2 + (\alpha + \beta)t^3 \Rightarrow 1 = \delta, \quad 0 = \gamma + \delta, \quad 0 = \beta + \gamma, \quad 0 = \alpha + \beta$$

$$= \int \left[\frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6} - \frac{1}{x^2+1} \right] dx = \int \left[x^{-2} - x^{-4} + x^{-6} - \frac{1}{x^2+1} \right] dx$$

Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(1-x)x^2} = \frac{\alpha}{x-1} + \frac{\beta}{x} + \frac{\gamma}{x^2} = \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}, \quad \gamma = -1, \quad \beta = -1, \quad \alpha = 1 \right]$$

$$1 = \alpha x^2 + \beta x(x-1) + \gamma(x-1) = -\gamma + (-\beta + \gamma)x + (\alpha + \beta)x^2 \Rightarrow 1 = -\gamma, \quad 0 = -\beta + \gamma, \quad 0 = \alpha + \beta$$

$$= \int \left[\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} \right] dx = \int \left[\frac{1}{x-1} - \frac{1}{x} - x^{-2} \right] dx = \ln|x-1| - \ln|x| - \frac{x^{-1}}{-1} + c$$

$$\int \frac{dx}{x^6(1+x^2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{x^6(1+x^2)} = \frac{1}{t^3(1+t)} = \frac{\alpha}{t+1} + \frac{\beta}{t} + \frac{\gamma}{t^2} + \frac{\delta}{t^3} = -\frac{1}{x^2+1} + \frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6}, \quad \delta = 1, \quad \gamma = -1, \quad \beta = 1, \quad \alpha = -1 \right]$$

$$1 = \alpha t^3 + \beta t^2(t+1) + \gamma t(t+1) + \delta(t+1) = \delta + (\gamma + \delta)t + (\beta + \gamma)t^2 + (\alpha + \beta)t^3 \Rightarrow 1 = \delta, \quad 0 = \gamma + \delta, \quad 0 = \beta + \gamma, \quad 0 = \alpha + \beta$$

$$= \int \left[\frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6} - \frac{1}{x^2+1} \right] dx = \int \left[x^{-2} - x^{-4} + x^{-6} - \frac{1}{x^2+1} \right] dx$$

$$= \frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + \frac{x^{-5}}{-5} - \arctg x + c$$

Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2} = \ln \left| \frac{x-1}{x} \right| + \frac{1}{x} + c$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(1-x)x^2} = \frac{\alpha}{x-1} + \frac{\beta}{x} + \frac{\gamma}{x^2} = \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}, \quad \gamma = -1, \quad \beta = -1, \quad \alpha = 1 \right]$$

$$1 = \alpha x^2 + \beta x(x-1) + \gamma(x-1) = -\gamma + (-\beta + \gamma)x + (\alpha + \beta)x^2 \Rightarrow 1 = -\gamma, \quad 0 = -\beta + \gamma, \quad 0 = \alpha + \beta$$

$$= \int \left[\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} \right] dx = \int \left[\frac{1}{x-1} - \frac{1}{x} - x^{-2} \right] dx = \ln |x-1| - \ln |x| - \frac{x^{-1}}{-1} + c$$

$$= \ln \left| \frac{x-1}{x} \right| + \frac{1}{x} + c, \quad x \in R - \{0, 1\}, \quad c \in R.$$

$$\int \frac{dx}{x^6(1+x^2)} = -\frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} - \arctg x + c$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{x^6(1+x^2)} = \frac{1}{t^3(1+t)} = \frac{\alpha}{t+1} + \frac{\beta}{t} + \frac{\gamma}{t^2} + \frac{\delta}{t^3} = -\frac{1}{x^2+1} + \frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6}, \quad \delta = 1, \quad \gamma = -1, \quad \beta = 1, \quad \alpha = -1 \right]$$

$$1 = \alpha t^3 + \beta t^2(t+1) + \gamma t(t+1) + \delta(t+1) = \delta + (\gamma + \delta)t + (\beta + \gamma)t^2 + (\alpha + \beta)t^3 \Rightarrow 1 = \delta, \quad 0 = \gamma + \delta, \quad 0 = \beta + \gamma, \quad 0 = \alpha + \beta$$

$$= \int \left[\frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6} - \frac{1}{x^2+1} \right] dx = \int \left[x^{-2} - x^{-4} + x^{-6} - \frac{1}{x^2+1} \right] dx$$

$$= \frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + \frac{x^{-5}}{-5} - \arctg x + c = -\frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} - \arctg x + c,$$

$$x \in R - \{0\}, \quad c \in R.$$

Riešené príklady – 143, 144

$$\int \frac{(x-2)^4 dx}{(x-1)^2}$$

$$\int \frac{(x-1)^4 dx}{(x-2)^2}$$

Riešené príklady – 143, 144

$$\int \frac{(x-2)^4 dx}{(x-1)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right]$$

$$\int \frac{(x-1)^4 dx}{(x-2)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in \mathbb{R} - \{2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right]$$

Riešené príklady – 143, 144

$$\int \frac{(x-2)^4 dx}{(x-1)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t-1)^4 dt}{t^2}$$

$$\int \frac{(x-1)^4 dx}{(x-2)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in \mathbb{R} - \{2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t+1)^4 dt}{t^2}$$

Riešené príklady – 143, 144

$$\int \frac{(x-2)^4 dx}{(x-1)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \left| \begin{array}{l} x \in \mathbb{R} - \{1\} \\ t \in \mathbb{R} - \{0\} \end{array} \right. \right] = \int \frac{(t-1)^4 dt}{t^2} = \int \frac{(t^4 - 4t^3 + 6t^2 - 4t + 1) dt}{t^2}$$

$$\int \frac{(x-1)^4 dx}{(x-2)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-2 \\ dt=dx \end{array} \left| \begin{array}{l} x \in \mathbb{R} - \{2\} \\ t \in \mathbb{R} - \{0\} \end{array} \right. \right] = \int \frac{(t+1)^4 dt}{t^2} = \int \frac{(t^4 + 4t^3 + 6t^2 + 4t + 1) dt}{t^2}$$

Riešené príklady – 143, 144

$$\int \frac{(x-2)^4 dx}{(x-1)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t-1)^4 dt}{t^2} = \int \frac{(t^4 - 4t^3 + 6t^2 - 4t + 1) dt}{t^2}$$

$$= \int \left[t^2 - 4t + 6 - \frac{4}{t} + t^{-2} \right] dt$$

$$\int \frac{(x-1)^4 dx}{(x-2)^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in \mathbb{R} - \{2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t+1)^4 dt}{t^2} = \int \frac{(t^4 + 4t^3 + 6t^2 + 4t + 1) dt}{t^2}$$

$$= \int \left[t^2 + 4t + 6 + \frac{4}{t} + t^{-2} \right] dt$$

Riešené príklady – 143, 144

$$\int \frac{(x-2)^4 dx}{(x-1)^2}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t-1)^4 dt}{t^2} = \int \frac{(t^4 - 4t^3 + 6t^2 - 4t + 1) dt}{t^2} \\
 &= \int \left[t^2 - 4t + 6 - \frac{4}{t} + t^{-2} \right] dt = \frac{t^3}{3} - \frac{4t^2}{2} + 6t - 4 \ln |t| + \frac{t^{-1}}{-1} + c
 \end{aligned}$$

$$\int \frac{(x-1)^4 dx}{(x-2)^2}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in \mathbb{R} - \{2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t+1)^4 dt}{t^2} = \int \frac{(t^4 + 4t^3 + 6t^2 + 4t + 1) dt}{t^2} \\
 &= \int \left[t^2 + 4t + 6 + \frac{4}{t} + t^{-2} \right] dt = \frac{t^3}{3} + \frac{4t^2}{2} + 6t + 4 \ln |t| + \frac{t^{-1}}{-1} + c
 \end{aligned}$$

Riešené príklady – 143, 144

$$\int \frac{(x-2)^4 dx}{(x-1)^2} = \frac{(x-1)^3}{3} - 2(x-1)^2 + 6(x-1) - 4 \ln |x-1| - \frac{1}{x-1} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x \in \mathbb{R} - \{1\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t-1)^4 dt}{t^2} = \int \frac{(t^4 - 4t^3 + 6t^2 - 4t + 1) dt}{t^2}$$

$$= \int \left[t^2 - 4t + 6 - \frac{4}{t} + t^{-2} \right] dt = \frac{t^3}{3} - \frac{4t^2}{2} + 6t - 4 \ln |t| + \frac{t^{-1}}{-1} + c$$

$$= \frac{(x-1)^3}{3} - 2(x-1)^2 + 6(x-1) - 4 \ln |x-1| - \frac{1}{x-1} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

$$\int \frac{(x-1)^4 dx}{(x-2)^2} = \frac{(x-2)^3}{3} + 2(x-2)^2 + 6(x-2) + 4 \ln |x-2| - \frac{1}{x-2} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-2 \mid x \in \mathbb{R} - \{2\} \\ dt=dx \mid t \in \mathbb{R} - \{0\} \end{array} \right] = \int \frac{(t+1)^4 dt}{t^2} = \int \frac{(t^4 + 4t^3 + 6t^2 + 4t + 1) dt}{t^2}$$

$$= \int \left[t^2 + 4t + 6 + \frac{4}{t} + t^{-2} \right] dt = \frac{t^3}{3} + \frac{4t^2}{2} + 6t + 4 \ln |t| + \frac{t^{-1}}{-1} + c$$

$$= \frac{(x-2)^3}{3} + 2(x-2)^2 + 6(x-2) + 4 \ln |x-2| - \frac{1}{x-2} + c, \quad x \in \mathbb{R} - \{2\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 145, 146

$$\int \frac{dx}{x^3 - 7x - 6}$$



$$\int \frac{dx}{x^3 - 2x^2 - x + 2}$$



Riešené príklady – 145, 146

$$\int \frac{dx}{x^3-7x-6} = \int \frac{dx}{(x-1)(x-2)(x+3)}$$



$$\int \frac{dx}{x^3-2x^2-x+2} = \int \frac{dx}{(x+1)(x-1)(x-2)}$$



Riešené príklady – 145, 146

$$\int \frac{dx}{x^3-7x-6} = \int \frac{dx}{(x-1)(x-2)(x+3)}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x-2)(x+3)} = \frac{\alpha}{x-1} + \frac{\beta}{x-2} + \frac{\gamma}{x+3} \\ 1 = \alpha(x-2)(x+3) + \beta(x-1)(x+3) + \gamma(x-1)(x-2) = (-6\alpha-3\beta+2\gamma) + (\alpha+2\beta-3\gamma)x + (\alpha+\beta+\gamma)x^2 \end{array} \right] \text{!}$$

$$\int \frac{dx}{x^3-2x^2-x+2} = \int \frac{dx}{(x+1)(x-1)(x-2)}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{(x+1)(x-1)(x-2)} = \frac{\alpha}{x+1} + \frac{\beta}{x-1} + \frac{\gamma}{x-2} \\ 1 = \alpha(x-1)(x-2) + \beta(x+1)(x-2) + \gamma(x+1)(x-1) = (2\alpha-2\beta-\gamma) + (-3\alpha-\beta)x + (\alpha+\beta+\gamma)x^2 \end{array} \right] \text{!}$$

Riešené príklady – 145, 146

$$\int \frac{dx}{x^3-7x-6} = \int \frac{dx}{(x-1)(x-2)(x+3)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x-2)(x+3)} = \frac{\alpha}{x-1} + \frac{\beta}{x-2} + \frac{\gamma}{x+3} = \frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{5}}{x-2} + \frac{\frac{1}{20}}{x+3}, \quad \alpha = -\frac{1}{4}, \quad \beta = \frac{1}{5}, \quad \gamma = \frac{1}{20} \right]$$

$$1 = \alpha(x-2)(x+3) + \beta(x-1)(x+3) + \gamma(x-1)(x-2) = (-6\alpha-3\beta+2\gamma) + (\alpha+2\beta-3\gamma)x + (\alpha+\beta+\gamma)x^2$$

$$\int \frac{dx}{x^3-2x^2-x+2} = \int \frac{dx}{(x+1)(x-1)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x+1)(x-1)(x-2)} = \frac{\alpha}{x+1} + \frac{\beta}{x-1} + \frac{\gamma}{x-2} = \frac{\frac{1}{6}}{x+1} + \frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{3}}{x-2}, \quad \alpha = \frac{1}{6}, \quad \beta = -\frac{1}{2}, \quad \gamma = \frac{1}{3} \right]$$

$$1 = \alpha(x-1)(x-2) + \beta(x+1)(x-2) + \gamma(x+1)(x-1) = (2\alpha-2\beta-\gamma) + (-3\alpha-\beta)x + (\alpha+\beta+\gamma)x^2$$

Riešené príklady – 145, 146

$$\int \frac{dx}{x^3-7x-6} = \int \frac{dx}{(x-1)(x-2)(x+3)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x-2)(x+3)} = \frac{\alpha}{x-1} + \frac{\beta}{x-2} + \frac{\gamma}{x+3} = \frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{5}}{x-2} + \frac{\frac{1}{20}}{x+3}, \quad \alpha = -\frac{1}{4}, \quad \beta = \frac{1}{5}, \quad \gamma = \frac{1}{20} \right]$$

$$1 = \alpha(x-2)(x+3) + \beta(x-1)(x+3) + \gamma(x-1)(x-2) = (-6\alpha-3\beta+2\gamma) + (\alpha+2\beta-3\gamma)x + (\alpha+\beta+\gamma)x^2$$

$$= \int \left[\frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{5}}{x-2} + \frac{\frac{1}{20}}{x+3} \right] dx$$

$$\int \frac{dx}{x^3-2x^2-x+2} = \int \frac{dx}{(x+1)(x-1)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x+1)(x-1)(x-2)} = \frac{\alpha}{x+1} + \frac{\beta}{x-1} + \frac{\gamma}{x-2} = \frac{\frac{1}{6}}{x+1} + \frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{3}}{x-2}, \quad \alpha = \frac{1}{6}, \quad \beta = -\frac{1}{2}, \quad \gamma = \frac{1}{3} \right]$$

$$1 = \alpha(x-1)(x-2) + \beta(x+1)(x-2) + \gamma(x+1)(x-1) = (2\alpha-2\beta-\gamma) + (-3\alpha-\beta)x + (\alpha+\beta+\gamma)x^2$$

$$= \int \left[\frac{\frac{1}{6}}{x+1} + \frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{3}}{x-2} \right] dx$$

Riešené príklady – 145, 146

$$\int \frac{dx}{x^3-7x-6} = \int \frac{dx}{(x-1)(x-2)(x+3)} = -\frac{1}{4} \ln|x-1| + \frac{1}{5} \ln|x-2| + \frac{1}{20} \ln|x+3| + c$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x-2)(x+3)} = \frac{\alpha}{x-1} + \frac{\beta}{x-2} + \frac{\gamma}{x+3} = \frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{5}}{x-2} + \frac{\frac{1}{20}}{x+3}, \quad \alpha = -\frac{1}{4}, \quad \beta = \frac{1}{5}, \quad \gamma = \frac{1}{20} \right]$$

$$1 = \alpha(x-2)(x+3) + \beta(x-1)(x+3) + \gamma(x-1)(x-2) = (-6\alpha-3\beta+2\gamma) + (\alpha+2\beta-3\gamma)x + (\alpha+\beta+\gamma)x^2$$

$$= \int \left[\frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{5}}{x-2} + \frac{\frac{1}{20}}{x+3} \right] dx = -\frac{1}{4} \ln|x-1| + \frac{1}{5} \ln|x-2| + \frac{1}{20} \ln|x+3| + c,$$

$$x \in \mathbb{R} - \{1, 2, -3\}, c \in \mathbb{R}.$$

$$\int \frac{dx}{x^3-2x^2-x+2} = \int \frac{dx}{(x+1)(x-1)(x-2)} = \frac{1}{6} \ln|x+1| - \frac{1}{2} \ln|x-1| + \frac{1}{3} \ln|x-2| + c$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x+1)(x-1)(x-2)} = \frac{\alpha}{x+1} + \frac{\beta}{x-1} + \frac{\gamma}{x-2} = \frac{\frac{1}{6}}{x+1} + \frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{3}}{x-2}, \quad \alpha = \frac{1}{6}, \quad \beta = -\frac{1}{2}, \quad \gamma = \frac{1}{3} \right]$$

$$1 = \alpha(x-1)(x-2) + \beta(x+1)(x-2) + \gamma(x+1)(x-1) = (2\alpha-2\beta-\gamma) + (-3\alpha-\beta)x + (\alpha+\beta+\gamma)x^2$$

$$= \int \left[\frac{\frac{1}{6}}{x+1} + \frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{3}}{x-2} \right] dx = \frac{1}{6} \ln|x+1| - \frac{1}{2} \ln|x-1| + \frac{1}{3} \ln|x-2| + c,$$

$$x \in \mathbb{R} - \{\pm 1, 2\}, c \in \mathbb{R}.$$

Riešené príklady – 147, 148

$$\int \frac{dx}{x^3 - 3x - 2}$$



$$\int \frac{dx}{x^3 + x^2 - x - 1}$$



Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x-2)(x+1)^2}$$



$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x-1)(x+1)^2}$$



Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x-2)(x+1)^2}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{(x-2)(x+1)^2} = \frac{\alpha}{x-2} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} \\ 1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-2\beta-2\gamma) + (2\alpha-\beta+\gamma)x + (\alpha+\beta)x^2 \end{array} \right] \text{!}$$

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x-1)(x+1)^2}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x+1)^2} = \frac{\alpha}{x-1} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} \\ 1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-\beta-\gamma) + (2\alpha+\gamma)x + (\alpha+\beta)x^2 \end{array} \right] \text{!}$$

Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x-2)(x+1)^2}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x-2)(x+1)^2} = \frac{\alpha}{x-2} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2}, \quad \alpha = \frac{1}{9}, \quad \beta = -\frac{1}{9}, \quad \gamma = -\frac{1}{3} \right] \quad \text{☞}$$

$$1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-2\beta-2\gamma) + (2\alpha-\beta+\gamma)x + (\alpha+\beta)x^2$$

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x-1)(x+1)^2}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x+1)^2} = \frac{\alpha}{x-1} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2}, \quad \alpha = \frac{1}{4}, \quad \beta = -\frac{1}{4}, \quad \gamma = -\frac{1}{2} \right] \quad \text{☞}$$

$$1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-\beta-\gamma) + (2\alpha+\gamma)x + (\alpha+\beta)x^2$$

Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x-2)(x+1)^2}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x-2)(x+1)^2} = \frac{\alpha}{x-2} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2}, \quad \alpha = \frac{1}{9}, \quad \beta = -\frac{1}{9}, \quad \gamma = -\frac{1}{3} \right] \Rightarrow$$

$$1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-2\beta-2\gamma) + (2\alpha-\beta+\gamma)x + (\alpha+\beta)x^2$$

$$= \int \left[\frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2} \right] dx$$

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x-1)(x+1)^2}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x+1)^2} = \frac{\alpha}{x-1} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2}, \quad \alpha = \frac{1}{4}, \quad \beta = -\frac{1}{4}, \quad \gamma = -\frac{1}{2} \right] \Rightarrow$$

$$1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-\beta-\gamma) + (2\alpha+\gamma)x + (\alpha+\beta)x^2$$

$$= \int \left[\frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2} \right] dx$$

Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x-2)(x+1)^2}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x-2)(x+1)^2} = \frac{\alpha}{x-2} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2}, \quad \alpha = \frac{1}{9}, \quad \beta = -\frac{1}{9}, \quad \gamma = -\frac{1}{3} \right] \Rightarrow$$

$$1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-2\beta-2\gamma) + (2\alpha-\beta+\gamma)x + (\alpha+\beta)x^2$$

$$= \int \left[\frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2} \right] dx = \frac{1}{9} \ln |x-2| - \frac{1}{9} \ln |x+1| - \frac{(x+1)^{-1}}{3 \cdot (-1)} + c$$

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x-1)(x+1)^2}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x+1)^2} = \frac{\alpha}{x-1} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2}, \quad \alpha = \frac{1}{4}, \quad \beta = -\frac{1}{4}, \quad \gamma = -\frac{1}{2} \right] \Rightarrow$$

$$1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-\beta-\gamma) + (2\alpha+\gamma)x + (\alpha+\beta)x^2$$

$$= \int \left[\frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2} \right] dx = \frac{1}{4} \ln |x-1| - \frac{1}{4} \ln |x+1| - \frac{(x+1)^{-1}}{2 \cdot (-1)} + c$$

Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x-2)(x+1)^2} = \frac{1}{9} \ln |x-2| - \frac{1}{9} \ln |x+1| + \frac{1}{3(x+1)} + c$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x-2)(x+1)^2} = \frac{\alpha}{x-2} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2}, \quad \alpha = \frac{1}{9}, \quad \beta = -\frac{1}{9}, \quad \gamma = -\frac{1}{3} \right] \Rightarrow$$

$$1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-2\beta-2\gamma) + (2\alpha-\beta+\gamma)x + (\alpha+\beta)x^2$$

$$= \int \left[\frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2} \right] dx = \frac{1}{9} \ln |x-2| - \frac{1}{9} \ln |x+1| - \frac{(x+1)^{-1}}{3 \cdot (-1)} + c$$

$$= \frac{1}{9} \ln |x-2| - \frac{1}{9} \ln |x+1| + \frac{1}{3(x+1)} + c, \quad x \in \mathbb{R} - \{-1, 2\}, \quad c \in \mathbb{R}.$$

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x-1)(x+1)^2} = \frac{1}{4} \ln |x-1| - \frac{1}{4} \ln |x+1| + \frac{1}{2(x+1)} + c$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x+1)^2} = \frac{\alpha}{x-1} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2}, \quad \alpha = \frac{1}{4}, \quad \beta = -\frac{1}{4}, \quad \gamma = -\frac{1}{2} \right] \Rightarrow$$

$$1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-\beta-\gamma) + (2\alpha+\gamma)x + (\alpha+\beta)x^2$$

$$= \int \left[\frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2} \right] dx = \frac{1}{4} \ln |x-1| - \frac{1}{4} \ln |x+1| - \frac{(x+1)^{-1}}{2 \cdot (-1)} + c$$

$$= \frac{1}{4} \ln |x-1| - \frac{1}{4} \ln |x+1| + \frac{1}{2(x+1)} + c, \quad x \in \mathbb{R} - \{\pm 1\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 149, 150

$$\int \frac{dx}{x^3 - 3x^2 + 4x - 2}$$



$$\int \frac{dx}{x^3 - 3x^2 + 3x - 1}$$

Riešené príklady – 149, 150

$$\int \frac{dx}{x^3 - 3x^2 + 4x - 2} = \int \frac{dx}{(x^2 - 2x + 2)(x - 1)}$$

$$\int \frac{dx}{x^3 - 3x^2 + 3x - 1} = \int \frac{dx}{(x - 1)^3}$$

Riešené príklady – 149, 150

$$\int \frac{dx}{x^3 - 3x^2 + 4x - 2} = \int \frac{dx}{(x^2 - 2x + 2)(x - 1)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2 + 2x + 2)(x - 1)} = \frac{\alpha x + \beta}{x^2 - 2x + 2} + \frac{\gamma}{x - 1} \right]$$
$$1 = (\alpha x + \beta)(x - 1) + \gamma(x^2 - 2x + 2) = (-\beta + 2\gamma) + (-\alpha + \beta - 2\gamma)x + (\alpha + \gamma)x^2$$

$$\int \frac{dx}{x^3 - 3x^2 + 3x - 1} = \int \frac{dx}{(x - 1)^3}$$

$$= \int (x - 1)^{-3} dx$$

Riešené príklady – 149, 150

$$\int \frac{dx}{x^3 - 3x^2 + 4x - 2} = \int \frac{dx}{(x^2 - 2x + 2)(x - 1)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2 + 2x + 2)(x - 1)} = \frac{\alpha x + \beta}{x^2 - 2x + 2} + \frac{\gamma}{x - 1} = \frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1}, \quad \alpha = -1, \quad \beta = 1, \quad \gamma = 1 \right]$$

$$1 = (\alpha x + \beta)(x - 1) + \gamma(x^2 - 2x + 2) = (-\beta + 2\gamma) + (-\alpha + \beta - 2\gamma)x + (\alpha + \gamma)x^2$$

$$\int \frac{dx}{x^3 - 3x^2 + 3x - 1} = \int \frac{dx}{(x - 1)^3}$$

$$= \int (x - 1)^{-3} dx = \frac{(x - 1)^{-2}}{-2} + c$$

Riešené príklady – 149, 150

$$\int \frac{dx}{x^3 - 3x^2 + 4x - 2} = \int \frac{dx}{(x^2 - 2x + 2)(x - 1)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2 + 2x + 2)(x - 1)} = \frac{\alpha x + \beta}{x^2 - 2x + 2} + \frac{\gamma}{x - 1} = \frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1}, \quad \alpha = -1, \quad \beta = 1, \quad \gamma = 1 \right]$$

$$1 = (\alpha x + \beta)(x - 1) + \gamma(x^2 - 2x + 2) = (-\beta + 2\gamma) + (-\alpha + \beta - 2\gamma)x + (\alpha + \gamma)x^2$$

$$= \int \left[\frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1} \right] dx$$

$$\int \frac{dx}{x^3 - 3x^2 + 3x - 1} = \int \frac{dx}{(x - 1)^3} = -\frac{1}{2(x - 1)^2} + c$$

$$= \int (x - 1)^{-3} dx = \frac{(x - 1)^{-2}}{-2} + c = -\frac{1}{2(x - 1)^2} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 149, 150

$$\int \frac{dx}{x^3 - 3x^2 + 4x - 2} = \int \frac{dx}{(x^2 - 2x + 2)(x - 1)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2 + 2x + 2)(x - 1)} = \frac{\alpha x + \beta}{x^2 - 2x + 2} + \frac{\gamma}{x - 1} = \frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1}, \quad \alpha = -1, \quad \beta = 1, \quad \gamma = 1 \right]$$

$$1 = (\alpha x + \beta)(x - 1) + \gamma(x^2 - 2x + 2) = (-\beta + 2\gamma) + (-\alpha + \beta - 2\gamma)x + (\alpha + \gamma)x^2$$

$$= \int \left[\frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1} \right] dx = - \int \frac{(x - 1) dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1}$$

$$\int \frac{dx}{x^3 - 3x^2 + 3x - 1} = \int \frac{dx}{(x - 1)^3} = -\frac{1}{2(x - 1)^2} + c$$

$$= \int (x - 1)^{-3} dx = \frac{(x - 1)^{-2}}{-2} + c = -\frac{1}{2(x - 1)^2} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 149, 150

$$\int \frac{dx}{x^3 - 3x^2 + 4x - 2} = \int \frac{dx}{(x^2 - 2x + 2)(x - 1)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2 + 2x + 2)(x - 1)} = \frac{\alpha x + \beta}{x^2 - 2x + 2} + \frac{\gamma}{x - 1} = \frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1}, \quad \alpha = -1, \quad \beta = 1, \quad \gamma = 1 \right]$$

$$1 = (\alpha x + \beta)(x - 1) + \gamma(x^2 - 2x + 2) = (-\beta + 2\gamma) + (-\alpha + \beta - 2\gamma)x + (\alpha + \gamma)x^2$$

$$= \int \left[\frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1} \right] dx = - \int \frac{(x - 1) dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1} = - \frac{1}{2} \int \frac{(2x - 2) dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1}$$

$$\int \frac{dx}{x^3 - 3x^2 + 3x - 1} = \int \frac{dx}{(x - 1)^3} = - \frac{1}{2(x - 1)^2} + c$$

$$= \int (x - 1)^{-3} dx = \frac{(x - 1)^{-2}}{-2} + c = - \frac{1}{2(x - 1)^2} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 149, 150

$$\int \frac{dx}{x^3 - 3x^2 + 4x - 2} = \int \frac{dx}{(x^2 - 2x + 2)(x - 1)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2 + 2x + 2)(x - 1)} = \frac{\alpha x + \beta}{x^2 - 2x + 2} + \frac{\gamma}{x - 1} = \frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1}, \quad \alpha = -1, \quad \beta = 1, \quad \gamma = 1 \right]$$

$$1 = (\alpha x + \beta)(x - 1) + \gamma(x^2 - 2x + 2) = (-\beta + 2\gamma) + (-\alpha + \beta - 2\gamma)x + (\alpha + \gamma)x^2$$

$$= \int \left[\frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1} \right] dx = - \int \frac{(x - 1) dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1} = - \frac{1}{2} \int \frac{(2x - 2) dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1}$$

$$= - \frac{1}{2} \int \frac{(x^2 - 2x + 2)' dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1}$$

$$\int \frac{dx}{x^3 - 3x^2 + 3x - 1} = \int \frac{dx}{(x - 1)^3} = - \frac{1}{2(x - 1)^2} + c$$

$$= \int (x - 1)^{-3} dx = \frac{(x - 1)^{-2}}{-2} + c = - \frac{1}{2(x - 1)^2} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 149, 150

$$\int \frac{dx}{x^3 - 3x^2 + 4x - 2} = \int \frac{dx}{(x^2 - 2x + 2)(x - 1)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2 - 2x + 2)(x - 1)} = \frac{\alpha x + \beta}{x^2 - 2x + 2} + \frac{\gamma}{x - 1} = \frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1}, \quad \alpha = -1, \quad \beta = 1, \quad \gamma = 1 \right]$$

$$1 = (\alpha x + \beta)(x - 1) + \gamma(x^2 - 2x + 2) = (-\beta + 2\gamma) + (-\alpha + \beta - 2\gamma)x + (\alpha + \gamma)x^2$$

$$= \int \left[\frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1} \right] dx = - \int \frac{(x - 1) dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1} = - \frac{1}{2} \int \frac{(2x - 2) dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1}$$

$$= - \frac{1}{2} \int \frac{(x^2 - 2x + 2)' dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1} = - \frac{1}{2} \ln |x^2 - 2x + 2| + \ln |x - 1| + c$$

$$\int \frac{dx}{x^3 - 3x^2 + 3x - 1} = \int \frac{dx}{(x - 1)^3} = - \frac{1}{2(x - 1)^2} + c$$

$$= \int (x - 1)^{-3} dx = \frac{(x - 1)^{-2}}{-2} + c = - \frac{1}{2(x - 1)^2} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 149, 150

$$\int \frac{dx}{x^3 - 3x^2 + 4x - 2} = \int \frac{dx}{(x^2 - 2x + 2)(x - 1)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2 - 2x + 2)(x - 1)} = \frac{\alpha x + \beta}{x^2 - 2x + 2} + \frac{\gamma}{x - 1} = \frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1}, \quad \alpha = -1, \quad \beta = 1, \quad \gamma = 1 \right]$$

$$1 = (\alpha x + \beta)(x - 1) + \gamma(x^2 - 2x + 2) = (-\beta + 2\gamma) + (-\alpha + \beta - 2\gamma)x + (\alpha + \gamma)x^2$$

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$$= - \frac{1}{2} \int \frac{(x^2 - 2x + 2)' dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1} = - \frac{1}{2} \ln |x^2 - 2x + 2| + \ln |x - 1| + c$$

$$= \left[\begin{array}{l} x^2 - 2x + 2 \\ = (x - 1)^2 + 1 > 0 \end{array} \right]$$

$$\int \frac{dx}{x^3 - 3x^2 + 3x - 1} = \int \frac{dx}{(x - 1)^3} = - \frac{1}{2(x - 1)^2} + c$$

$$= \int (x - 1)^{-3} dx = \frac{(x - 1)^{-2}}{-2} + c = - \frac{1}{2(x - 1)^2} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 149, 150

$$\int \frac{dx}{x^3 - 3x^2 + 4x - 2} = \int \frac{dx}{(x^2 - 2x + 2)(x - 1)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2 + 2x + 2)(x - 1)} = \frac{\alpha x + \beta}{x^2 - 2x + 2} + \frac{\gamma}{x - 1} = \frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1}, \quad \alpha = -1, \quad \beta = 1, \quad \gamma = 1 \right]$$

$$1 = (\alpha x + \beta)(x - 1) + \gamma(x^2 - 2x + 2) = (-\beta + 2\gamma) + (-\alpha + \beta - 2\gamma)x + (\alpha + \gamma)x^2$$

$$= \int \left[\frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1} \right] dx = - \int \frac{(x - 1) dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1} = - \frac{1}{2} \int \frac{(2x - 2) dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1}$$

$$= - \frac{1}{2} \int \frac{(x^2 - 2x + 2)' dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1} = - \frac{1}{2} \ln |x^2 - 2x + 2| + \ln |x - 1| + c$$

$$= \left[\begin{array}{l} x^2 - 2x + 2 \\ = (x - 1)^2 + 1 > 0 \end{array} \right] = - \frac{1}{2} \ln (x^2 - 2x + 2) + \frac{1}{2} \ln (x - 1)^2 + c$$

$$\int \frac{dx}{x^3 - 3x^2 + 3x - 1} = \int \frac{dx}{(x - 1)^3} = - \frac{1}{2(x - 1)^2} + c$$

$$= \int (x - 1)^{-3} dx = \frac{(x - 1)^{-2}}{-2} + c = - \frac{1}{2(x - 1)^2} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 149, 150

$$\int \frac{dx}{x^3 - 3x^2 + 4x - 2} = \int \frac{dx}{(x^2 - 2x + 2)(x - 1)} = \frac{1}{2} \ln \frac{(x-1)^2}{x^2 - 2x + 2} + c$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2 + 2x + 2)(x - 1)} = \frac{\alpha x + \beta}{x^2 - 2x + 2} + \frac{\gamma}{x - 1} = \frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1}, \quad \alpha = -1, \quad \beta = 1, \quad \gamma = 1 \right]$$

$$1 = (\alpha x + \beta)(x - 1) + \gamma(x^2 - 2x + 2) = (-\beta + 2\gamma) + (-\alpha + \beta - 2\gamma)x + (\alpha + \gamma)x^2$$

$$= \int \left[\frac{-x + 1}{x^2 - 2x + 2} + \frac{1}{x - 1} \right] dx = - \int \frac{(x - 1) dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1} = - \frac{1}{2} \int \frac{(2x - 2) dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1}$$

$$= - \frac{1}{2} \int \frac{(x^2 - 2x + 2)' dx}{x^2 - 2x + 2} + \int \frac{dx}{x - 1} = - \frac{1}{2} \ln |x^2 - 2x + 2| + \ln |x - 1| + c$$

$$= \left[\begin{array}{l} x^2 - 2x + 2 \\ = (x - 1)^2 + 1 > 0 \end{array} \right] = - \frac{1}{2} \ln (x^2 - 2x + 2) + \frac{1}{2} \ln (x - 1)^2 + c$$

$$= \frac{1}{2} \ln \frac{(x - 1)^2}{x^2 - 2x + 2} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

$$\int \frac{dx}{x^3 - 3x^2 + 3x - 1} = \int \frac{dx}{(x - 1)^3} = - \frac{1}{2(x - 1)^2} + c$$

$$= \int (x - 1)^{-3} dx = \frac{(x - 1)^{-2}}{-2} + c = - \frac{1}{2(x - 1)^2} + c, \quad x \in \mathbb{R} - \{1\}, \quad c \in \mathbb{R}.$$

Koniec 10. časti príklady b

Ďakujem za pozornosť.

Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3 - 2x - 4}$$



Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$



Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x + \beta}{x^2+2x+2} + \frac{\gamma}{x-2} \right] \text{ ☞}$$

$$1 = (\alpha x + \beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x + \beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{4}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha = -\frac{1}{10}, \quad \beta = -\frac{4}{10}, \quad \gamma = \frac{1}{10} \right] \quad \text{☞}$$

$$1 = (\alpha x + \beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x + \beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha = -\frac{1}{10}, \quad \beta = -\frac{4}{10}, \quad \gamma = \frac{1}{10} \right] \quad \text{☞}$$

$$1 = (\alpha x + \beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

$$= \int \left[\frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx$$

Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x + \beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha = -\frac{1}{10}, \quad \beta = -\frac{4}{10}, \quad \gamma = \frac{1}{10} \right] \quad \text{☞}$$

$$1 = (\alpha x + \beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

$$= \int \left[\frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10} \int \frac{(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x + \beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha = -\frac{1}{10}, \quad \beta = -\frac{4}{10}, \quad \gamma = \frac{1}{10} \right] \quad \text{☞}$$

$$1 = (\alpha x + \beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

$$= \int \left[\frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10} \int \frac{(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{2(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x + \beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha = -\frac{1}{10}, \quad \beta = -\frac{4}{10}, \quad \gamma = \frac{1}{10} \right] \quad \text{☞}$$

$$1 = (\alpha x + \beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

$$= \int \left[\frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10} \int \frac{(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{2(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= -\frac{1}{20} \int \frac{(2x+2+6) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x + \beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{6}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha = -\frac{1}{10}, \quad \beta = -\frac{4}{10}, \quad \gamma = \frac{1}{10} \right] \quad \text{☞}$$

$$1 = (\alpha x + \beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

$$= \int \left[\frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10} \int \frac{(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{2(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= -\frac{1}{20} \int \frac{(2x+2+6) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{(2x+2) dx}{x^2+2x+2} - \frac{6}{20} \int \frac{dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x + \beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{6}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha = -\frac{1}{10}, \quad \beta = -\frac{4}{10}, \quad \gamma = \frac{1}{10} \right] \quad \text{☞}$$

$$1 = (\alpha x + \beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

$$= \int \left[\frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10} \int \frac{(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{2(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= -\frac{1}{20} \int \frac{(2x+2+6) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{(2x+2) dx}{x^2+2x+2} - \frac{6}{20} \int \frac{dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x+1 \\ dt=dx \end{array} \middle| \begin{array}{l} x^2+2x+2 = x^2+2x+1+1 \\ = (x+1)^2+1 = t^2+1 \end{array} \right| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array}$$

Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x + \beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{6}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha = -\frac{1}{10}, \quad \beta = -\frac{4}{10}, \quad \gamma = \frac{1}{10} \right] \quad \text{☞}$$

$$1 = (\alpha x + \beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

$$= \int \left[\frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10} \int \frac{(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{2(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= -\frac{1}{20} \int \frac{(2x+2+6) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{(2x+2) dx}{x^2+2x+2} - \frac{6}{20} \int \frac{dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= \left[\text{Subst. } \begin{matrix} t=x+1 \\ dt=dx \end{matrix} \left| \begin{matrix} x^2+2x+2 = x^2+2x+1+1 \\ = (x+1)^2+1 = t^2+1 \end{matrix} \right| \begin{matrix} x \in \mathbb{R} \\ t \in \mathbb{R} \end{matrix} \right] = -\frac{1}{20} \int \frac{(x^2+2x+2)' dx}{x^2+2x+2} - \frac{6}{20} \int \frac{dt}{t^2+1} + \frac{1}{10} \int \frac{dx}{x-2}$$

Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x + \beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{6}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha = -\frac{1}{10}, \quad \beta = -\frac{4}{10}, \quad \gamma = \frac{1}{10} \right] \quad \text{☞}$$

$$1 = (\alpha x + \beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

$$= \int \left[\frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10} \int \frac{(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{2(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= -\frac{1}{20} \int \frac{(2x+2+6) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{(2x+2) dx}{x^2+2x+2} - \frac{6}{20} \int \frac{dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= \left[\text{Subst. } t=x+1 \begin{array}{l} x^2+2x+2 = x^2+2x+1+1 \\ dt=dx \end{array} \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = -\frac{1}{20} \int \frac{(x^2+2x+2)' dx}{x^2+2x+2} - \frac{6}{20} \int \frac{dt}{t^2+1} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= -\frac{1}{20} \ln |x^2+2x+2| - \frac{6}{20} \arctg t + \frac{1}{10} \ln |x-2| + c$$

Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x + \beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{6}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha = -\frac{1}{10}, \quad \beta = -\frac{4}{10}, \quad \gamma = \frac{1}{10} \right] \Rightarrow$$

$$1 = (\alpha x + \beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

$$= \int \left[\frac{-\frac{x}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10} \int \frac{(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{2(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= -\frac{1}{20} \int \frac{(2x+2+6) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{(2x+2) dx}{x^2+2x+2} - \frac{6}{20} \int \frac{dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= \left[\text{Subst. } t=x+1 \begin{array}{l} x^2+2x+2 = x^2+2x+1+1 \\ dt=dx \end{array} \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = -\frac{1}{20} \int \frac{(x^2+2x+2)' dx}{x^2+2x+2} - \frac{6}{20} \int \frac{dt}{t^2+1} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= -\frac{1}{20} \ln |x^2+2x+2| - \frac{6}{20} \arctg t + \frac{1}{10} \ln |x-2| + c$$

$$= \frac{1}{20} \ln (x-2)^2 - \frac{1}{20} \ln (x^2+2x+2) - \frac{3}{10} \arctg (x+1) + c$$

Koniec 10. časti príklady b – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)} = \frac{1}{20} \ln \frac{(x-2)^2}{x^2+2x+2} - \frac{3}{10} \operatorname{arctg}(x+1) + c$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x + \beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{3}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha = -\frac{1}{10}, \quad \beta = -\frac{4}{10}, \quad \gamma = \frac{1}{10} \right] \Rightarrow$$

$$1 = (\alpha x + \beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

$$= \int \left[\frac{-\frac{3}{10} - \frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10} \int \frac{(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{2(x+4) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= -\frac{1}{20} \int \frac{(2x+2+6) dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{(2x+2) dx}{x^2+2x+2} - \frac{6}{20} \int \frac{dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= \left[\text{Subst. } t=x+1 \begin{cases} x^2+2x+2 = x^2+2x+1+1 \\ dt=dx \end{cases} \begin{matrix} x \in R \\ t \in R \end{matrix} \right] = -\frac{1}{20} \int \frac{(x^2+2x+2)' dx}{x^2+2x+2} - \frac{6}{20} \int \frac{dt}{t^2+1} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= -\frac{1}{20} \ln |x^2+2x+2| - \frac{6}{20} \operatorname{arctg} t + \frac{1}{10} \ln |x-2| + c$$

$$= \frac{1}{20} \ln (x-2)^2 - \frac{1}{20} \ln (x^2+2x+2) - \frac{3}{10} \operatorname{arctg}(x+1) + c$$

$$= \frac{1}{20} \ln \frac{(x-2)^2}{x^2+2x+2} - \frac{3}{10} \operatorname{arctg}(x+1) + c, \quad x \in R - \{2\}, \quad c \in R.$$

Koniec 10. časti príklady b – 152

$$\int \frac{dx}{x^6+1}$$



Koniec 10. časti príklady b – 152

$$\int \frac{dx}{x^6+1}$$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}$$



Koniec 10. časti príklady b – 152

$$\int \frac{dx}{x^6+1}$$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

Koniec 10. časti príklady b – 152

$$\int \frac{dx}{x^6+1}$$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\frac{4\sqrt{3}}{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

Koniec 10. časti príklady b – 152

$$\int \frac{dx}{x^6+1}$$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

Koniec 10. časti príklady b – 152

$$\int \frac{dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx
 \end{aligned}$$

Koniec 10. časti príklady b – 152

$$\int \frac{dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \quad du=dx \quad \left| \begin{array}{l} x \in R, \quad u \in R \\ x^2 \pm \sqrt{3}x+1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} \end{array} \right| \frac{1}{4\sqrt{3}} \left(\frac{4\sqrt{3}}{3} - \sqrt{3} \right) \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \quad dv=dx \quad \left| \begin{array}{l} x \in R, \quad v \in R \\ = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \end{array} \right| = \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 152

$$\int \frac{dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{-x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \left| \begin{array}{l} du=dx \\ x \in R, u \in R \end{array} \right. \left| \begin{array}{l} x^2 \pm \sqrt{3}x+1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} \\ = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \end{array} \right. \left| \begin{array}{l} \frac{1}{4\sqrt{3}} (\frac{4\sqrt{3}}{3} - \sqrt{3}) \\ = \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right. \end{array} \right] \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12} \int \frac{dv}{v^2+\frac{1}{4}}
 \end{aligned}$$

Koniec 10. časti príklady b – 152

$$\int \frac{dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{-x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \left| \begin{array}{l} du=dx \\ x \in \mathbb{R}, u \in \mathbb{R} \end{array} \right. \left| \begin{array}{l} x^2 \pm \sqrt{3}x+1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} \\ = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \end{array} \right. \left| \begin{array}{l} \frac{1}{4\sqrt{3}}(\frac{4\sqrt{3}}{3}-\sqrt{3}) \\ = \frac{1}{3}-\frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right. \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \left| \begin{array}{l} dv=dx \\ x \in \mathbb{R}, v \in \mathbb{R} \end{array} \right. \end{array} \right. \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\
 &= \frac{1}{3} \arctg x + \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{u}{\frac{1}{2}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{v}{\frac{1}{2}} + c
 \end{aligned}$$

Koniec 10. časti príklady b – 152

$$\int \frac{dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{-x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \left| \begin{array}{l} du=dx \\ x \in \mathbb{R}, u \in \mathbb{R} \end{array} \right. \left| \begin{array}{l} x^2 \pm \sqrt{3}x+1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} \\ = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \end{array} \right. \left| \begin{array}{l} \frac{1}{4\sqrt{3}}(\frac{4\sqrt{3}}{3}-\sqrt{3}) \\ = \frac{1}{3}-\frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right. \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \left| \begin{array}{l} dv=dx \\ x \in \mathbb{R}, v \in \mathbb{R} \end{array} \right. \end{array} \right. \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\
 &= \frac{1}{3} \arctg x + \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{u}{\frac{1}{2}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{v}{\frac{1}{2}} + c \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{1}{6} \arctg 2u + \frac{1}{6} \arctg 2v + c
 \end{aligned}$$

Koniec 10. časti príklady b – 152

$$\int \frac{dx}{x^6+1} = \frac{1}{3} \operatorname{arctg} x + \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{\operatorname{arctg} (2x+\sqrt{3})}{6} - \frac{\operatorname{arctg} (2x-\sqrt{3})}{6} + c$$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{-x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3} \operatorname{arctg} x + \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \mid du=dx \mid x \in R, u \in R \mid x^2 \pm \sqrt{3}x+1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \mid dv=dx \mid x \in R, v \in R \mid = (x \pm \frac{\sqrt{3}}{2})^2 + 1 + \frac{3}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \end{array} \right] \left[\frac{1}{4\sqrt{3}} \left(\frac{4\sqrt{3}}{3} - \sqrt{3} \right) \right] = \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$$

$$= \frac{1}{3} \operatorname{arctg} x + \frac{1}{4\sqrt{3}} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12} \int \frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{1}{3} \operatorname{arctg} x + \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{u}{\frac{1}{2}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{v}{\frac{1}{2}} + c$$

$$= \frac{1}{3} \operatorname{arctg} x + \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{1}{6} \operatorname{arctg} 2u + \frac{1}{6} \operatorname{arctg} 2v + c$$

$$= \frac{1}{3} \operatorname{arctg} x + \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{\operatorname{arctg} (2x+\sqrt{3})}{6} + \frac{\operatorname{arctg} (2x-\sqrt{3})}{6} + c, x \in R, c \in R.$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$



Koniec 10. časti príklady b – 153

$$\int \frac{x \, dx}{x^6 + 1}$$

$$= \int \frac{x \, dx}{(x^2 + 1)(x^2 - \sqrt{3}x + 1)(x^2 + \sqrt{3}x + 1)}$$



Koniec 10. časti príklady b – 153

$$\int \frac{x \, dx}{x^6 + 1}$$

$$= \int \frac{x \, dx}{(x^2 + 1)(x^2 - \sqrt{3}x + 1)(x^2 + \sqrt{3}x + 1)} = \int \left[\frac{\frac{x}{3}}{x^2 + 1} + \frac{-\frac{x}{6} - \frac{\sqrt{3}}{6}}{x^2 + \sqrt{3}x + 1} + \frac{-\frac{x}{6} + \frac{\sqrt{3}}{6}}{x^2 - \sqrt{3}x + 1} \right] dx$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$= \int \frac{x dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6} \int \frac{2x dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{6} \int \frac{2x dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx
 \end{aligned}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{6} \int \frac{2x dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \mid du=dx \mid x \in R, u \in R \mid x^2 \pm \sqrt{3}x+1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \mid dv=dx \mid x \in R, v \in R \mid (x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1) = (x^2+1)^2 - (\sqrt{3}x)^2 = x^4 - x^2 + 1 > 0 \end{array} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{6} \int \frac{2x dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \mid du=dx \mid x \in \mathbb{R}, u \in \mathbb{R} \mid x^2 \pm \sqrt{3}x+1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \mid dv=dx \mid x \in \mathbb{R}, v \in \mathbb{R} \mid (x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1) = (x^2+1)^2 - (\sqrt{3}x)^2 = x^4 - x^2 + 1 > 0 \end{array} \right] \\
 &= \frac{1}{6} \ln(x^2+1) - \frac{1}{12} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{\sqrt{3}}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12} \int \frac{dv}{v^2+\frac{1}{4}}
 \end{aligned}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{6} \int \frac{2x dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \mid du = dx \mid x \in \mathbb{R}, u \in \mathbb{R} \mid x^2 \pm \sqrt{3}x + 1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \\ \text{Subst. } v = x - \frac{\sqrt{3}}{2} \mid dv = dx \mid x \in \mathbb{R}, v \in \mathbb{R} \mid (x^2 + \sqrt{3}x + 1)(x^2 - \sqrt{3}x + 1) = (x^2 + 1)^2 - (\sqrt{3}x)^2 = x^4 - x^2 + 1 > 0 \end{array} \right] \\
 &= \frac{1}{6} \ln(x^2+1) - \frac{1}{12} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{\sqrt{3}}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\
 &= \frac{\ln(x^2+1)^2}{12} - \frac{\ln(x^2+\sqrt{3}x+1) + \ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{u}{\frac{1}{2}} + \frac{\sqrt{3}}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{v}{\frac{1}{2}} + c_1
 \end{aligned}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{6} \int \frac{2x dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \mid du = dx \mid x \in \mathbb{R}, u \in \mathbb{R} \mid x^2 \pm \sqrt{3}x + 1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \\ \text{Subst. } v = x - \frac{\sqrt{3}}{2} \mid dv = dx \mid x \in \mathbb{R}, v \in \mathbb{R} \mid (x^2 + \sqrt{3}x + 1)(x^2 - \sqrt{3}x + 1) = (x^2 + 1)^2 - (\sqrt{3}x)^2 = x^4 - x^2 + 1 > 0 \end{array} \right] \\
 &= \frac{1}{6} \ln(x^2+1) - \frac{1}{12} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{\sqrt{3}}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\
 &= \frac{\ln(x^2+1)^2}{12} - \frac{\ln(x^2+\sqrt{3}x+1) + \ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{u}{\frac{1}{2}} + \frac{\sqrt{3}}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{v}{\frac{1}{2}} + c_1 \\
 &= \frac{1}{12} \ln \frac{(x^2+1)^2}{(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)} - \frac{\sqrt{3}}{6} \arctg 2u + \frac{\sqrt{3}}{6} \arctg 2v + c_1
 \end{aligned}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1} = \frac{1}{12} \ln \frac{(x^2+1)^2}{x^4-x^2+1} - \frac{\sqrt{3} \operatorname{arctg} (2x+\sqrt{3})}{6} + \frac{\sqrt{3} \operatorname{arctg} (2x-\sqrt{3})}{6} + c_1$$

$$\begin{aligned} &= \int \frac{x dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{6} \int \frac{2x dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} - \frac{1}{12} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \mid du = dx \mid x \in R, u \in R \mid x^2 \pm \sqrt{3}x + 1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \\ \text{Subst. } v = x - \frac{\sqrt{3}}{2} \mid dv = dx \mid x \in R, v \in R \mid (x^2 + \sqrt{3}x + 1)(x^2 - \sqrt{3}x + 1) = (x^2 + 1)^2 - (\sqrt{3}x)^2 = x^4 - x^2 + 1 > 0 \end{array} \right] \\ &= \frac{1}{6} \ln(x^2+1) - \frac{1}{12} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{\sqrt{3}}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\ &= \frac{\ln(x^2+1)^2}{12} - \frac{\ln(x^2+\sqrt{3}x+1) + \ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{u}{\frac{1}{2}} + \frac{\sqrt{3}}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{v}{\frac{1}{2}} + c_1 \\ &= \frac{1}{12} \ln \frac{(x^2+1)^2}{(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)} - \frac{\sqrt{3}}{6} \operatorname{arctg} 2u + \frac{\sqrt{3}}{6} \operatorname{arctg} 2v + c_1 \\ &= \frac{1}{12} \ln \frac{(x^2+1)^2}{x^4-x^2+1} - \frac{\sqrt{3} \operatorname{arctg} (2x+\sqrt{3})}{6} + \frac{\sqrt{3} \operatorname{arctg} (2x-\sqrt{3})}{6} + c_1, x \in R, c_1 \in R. \end{aligned}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$



Koniec 10. časti príklady b – 153

$$\int \frac{x \, dx}{x^6 + 1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^2 \\ dt=2x \, dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; \infty) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right]$$



Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^2 \\ dt=2x dx \end{array} \middle| \begin{array}{l} x \in (-\infty; 0), t \in (0; \infty) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^3+1}$$



Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^2 \\ dt=2x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; \infty) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \frac{1}{2} \int \frac{dt}{t^3+1} = \frac{1}{2} \int \frac{dt}{(t+1)(t^2-t+1)}$$



Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^2 \\ dt=2x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; \infty) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \frac{1}{2} \int \frac{dt}{t^3+1} = \frac{1}{2} \int \frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2} \int \left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3} + \frac{2}{3}}{t^2-t+1} \right] dt$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^2 \\ dt=2x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; \infty) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \frac{1}{2} \int \frac{dt}{t^3+1} = \frac{1}{2} \int \frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2} \int \left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3} + \frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-4}{t^2-t+1} dt$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t=x^2 \\ dt=2x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; \infty) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \frac{1}{2} \int \frac{dt}{t^3+1} = \frac{1}{2} \int \frac{dt}{(t+1)(t^2-t+1)} \\
 &= \frac{1}{2} \int \left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3} + \frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-4}{t^2-t+1} dt \\
 &= \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-1-3}{t^2-t+1} dt
 \end{aligned}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^2 \\ dt=2x dx \end{array} \middle| \begin{array}{l} x \in (-\infty; 0), t \in (0; \infty) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^3+1} = \frac{1}{2} \int \frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2} \int \left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3} + \frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-4}{t^2-t+1} dt$$

$$= \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-1-3}{t^2-t+1} dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{(2t-1) dt}{t^2-t+1} + \frac{1}{4} \int \frac{dt}{t^2-t+1}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \\ dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^3+1} = \frac{1}{2} \int \frac{dt}{(t+1)(t^2-t+1)} \\
 &= \frac{1}{2} \int \left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3} + \frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-4}{t^2-t+1} dt \\
 &= \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-1-3}{t^2-t+1} dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{(2t-1)dt}{t^2-t+1} + \frac{1}{4} \int \frac{dt}{t^2-t+1} \\
 &= \left[\begin{array}{l} \text{Subst. } u=t-\frac{1}{2} \mid t \in (0; \infty) \\ du=dt \mid u \in (-\frac{1}{2}; \infty) \end{array} \right] \left| t^2-t+1 = (t-\frac{1}{2})^2 + 1 - \frac{1}{4} = (t-\frac{1}{2})^2 + \frac{3}{4} = u^2 + (\frac{\sqrt{3}}{2})^2 > 0 \right|
 \end{aligned}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \\ dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^3+1} = \frac{1}{2} \int \frac{dt}{(t+1)(t^2-t+1)} \\
 &= \frac{1}{2} \int \left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3} + \frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-4}{t^2-t+1} dt \\
 &= \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-1-3}{t^2-t+1} dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{(2t-1)dt}{t^2-t+1} + \frac{1}{4} \int \frac{dt}{t^2-t+1} \\
 &= \left[\begin{array}{l} \text{Subst. } u=t-\frac{1}{2} \mid t \in (0; \infty) \\ du=dt \mid u \in (-\frac{1}{2}; \infty) \end{array} \right] \left[t^2-t+1 = (t-\frac{1}{2})^2 + 1 - \frac{1}{4} = (t-\frac{1}{2})^2 + \frac{3}{4} = u^2 + (\frac{\sqrt{3}}{2})^2 > 0 \right] \\
 &= \frac{1}{6} \ln(t+1) - \frac{1}{12} \int \frac{(t^2-t+1)' dt}{t^2-t+1} + \frac{1}{4} \int \frac{du}{u^2 + \frac{3}{4}}
 \end{aligned}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \\ dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^3+1} = \frac{1}{2} \int \frac{dt}{(t+1)(t^2-t+1)} \\
 &= \frac{1}{2} \int \left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3} + \frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-4}{t^2-t+1} dt \\
 &= \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-1-3}{t^2-t+1} dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{(2t-1)dt}{t^2-t+1} + \frac{1}{4} \int \frac{dt}{t^2-t+1} \\
 &= \left[\begin{array}{l} \text{Subst. } u=t-\frac{1}{2} \mid t \in (0; \infty) \\ du=dt \mid u \in (-\frac{1}{2}; \infty) \end{array} \right] \left[t^2-t+1 = (t-\frac{1}{2})^2 + 1 - \frac{1}{4} = (t-\frac{1}{2})^2 + \frac{3}{4} = u^2 + (\frac{\sqrt{3}}{2})^2 > 0 \right] \\
 &= \frac{1}{6} \ln(t+1) - \frac{1}{12} \int \frac{(t^2-t+1)' dt}{t^2-t+1} + \frac{1}{4} \int \frac{du}{u^2 + \frac{3}{4}} \\
 &= \frac{1}{12} \ln(t+1)^2 - \frac{1}{12} \ln(t^2-t+1) + \frac{1}{4} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arctg} \frac{u}{\frac{\sqrt{3}}{2}} + c_2
 \end{aligned}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \\ dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^3+1} = \frac{1}{2} \int \frac{dt}{(t+1)(t^2-t+1)} \\
 &= \frac{1}{2} \int \left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3} + \frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-4}{t^2-t+1} dt \\
 &= \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-1-3}{t^2-t+1} dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{(2t-1)dt}{t^2-t+1} + \frac{1}{4} \int \frac{dt}{t^2-t+1} \\
 &= \left[\begin{array}{l} \text{Subst. } u=t-\frac{1}{2} \mid t \in (0; \infty) \\ du=dt \mid u \in (-\frac{1}{2}; \infty) \end{array} \right] \left[t^2-t+1 = (t-\frac{1}{2})^2 + 1 - \frac{1}{4} = (t-\frac{1}{2})^2 + \frac{3}{4} = u^2 + (\frac{\sqrt{3}}{2})^2 > 0 \right] \\
 &= \frac{1}{6} \ln(t+1) - \frac{1}{12} \int \frac{(t^2-t+1)' dt}{t^2-t+1} + \frac{1}{4} \int \frac{du}{u^2 + \frac{3}{4}} \\
 &= \frac{1}{12} \ln(t+1)^2 - \frac{1}{12} \ln(t^2-t+1) + \frac{1}{4} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \arctg \frac{u}{\frac{\sqrt{3}}{2}} + c_2 \\
 &= \frac{1}{12} \ln \frac{(t+1)^2}{t^2-t+1} + \frac{1}{2\sqrt{3}} \arctg \frac{2u}{\sqrt{3}} + c_2
 \end{aligned}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \\ dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^3+1} = \frac{1}{2} \int \frac{dt}{(t+1)(t^2-t+1)} \\
 &= \frac{1}{2} \int \left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3} + \frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-4}{t^2-t+1} dt \\
 &= \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-1-3}{t^2-t+1} dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{(2t-1)dt}{t^2-t+1} + \frac{1}{4} \int \frac{dt}{t^2-t+1} \\
 &= \left[\begin{array}{l} \text{Subst. } u=t-\frac{1}{2} \mid t \in (0; \infty) \\ du=dt \mid u \in (-\frac{1}{2}; \infty) \end{array} \right] \left[t^2-t+1 = (t-\frac{1}{2})^2 + 1 - \frac{1}{4} = (t-\frac{1}{2})^2 + \frac{3}{4} = u^2 + (\frac{\sqrt{3}}{2})^2 > 0 \right] \\
 &= \frac{1}{6} \ln(t+1) - \frac{1}{12} \int \frac{(t^2-t+1)' dt}{t^2-t+1} + \frac{1}{4} \int \frac{du}{u^2 + \frac{3}{4}} \\
 &= \frac{1}{12} \ln(t+1)^2 - \frac{1}{12} \ln(t^2-t+1) + \frac{1}{4} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arctg} \frac{u}{\frac{\sqrt{3}}{2}} + c_2 \\
 &= \frac{1}{12} \ln \frac{(t+1)^2}{t^2-t+1} + \frac{1}{2\sqrt{3}} \operatorname{arctg} \frac{2u}{\sqrt{3}} + c_2 = \frac{1}{12} \ln \frac{(t+1)^2}{t^2-t+1} + \frac{1}{2\sqrt{3}} \operatorname{arctg} \frac{2t-1}{\sqrt{3}} + c_2
 \end{aligned}$$

Koniec 10. časti príklady b – 153

$$\int \frac{x dx}{x^6+1} = \frac{1}{12} \ln \frac{(x^2+1)^2}{x^4-x^2+1} + \frac{1}{2\sqrt{3}} \operatorname{arctg} \frac{2x^2-1}{\sqrt{3}} + c_2$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \\ dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^3+1} = \frac{1}{2} \int \frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2} \int \left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3} + \frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-4}{t^2-t+1} dt$$

$$= \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{2t-1-3}{t^2-t+1} dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{12} \int \frac{(2t-1)dt}{t^2-t+1} + \frac{1}{4} \int \frac{dt}{t^2-t+1}$$

$$= \left[\begin{array}{l} \text{Subst. } u=t-\frac{1}{2} \mid t \in (0; \infty) \\ du=dt \mid u \in (-\frac{1}{2}; \infty) \end{array} \right] \left| t^2-t+1 = (t-\frac{1}{2})^2 + 1 - \frac{1}{4} = (t-\frac{1}{2})^2 + \frac{3}{4} = u^2 + (\frac{\sqrt{3}}{2})^2 > 0 \right|$$

$$= \frac{1}{6} \ln(t+1) - \frac{1}{12} \int \frac{(t^2-t+1)' dt}{t^2-t+1} + \frac{1}{4} \int \frac{du}{u^2 + \frac{3}{4}}$$

$$= \frac{1}{12} \ln(t+1)^2 - \frac{1}{12} \ln(t^2-t+1) + \frac{1}{4} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arctg} \frac{u}{\frac{\sqrt{3}}{2}} + c_2$$

$$= \frac{1}{12} \ln \frac{(t+1)^2}{t^2-t+1} + \frac{1}{2\sqrt{3}} \operatorname{arctg} \frac{2u}{\sqrt{3}} + c_2 = \frac{1}{12} \ln \frac{(t+1)^2}{t^2-t+1} + \frac{1}{2\sqrt{3}} \operatorname{arctg} \frac{2t-1}{\sqrt{3}} + c_2$$

$$= \frac{1}{12} \ln \frac{(x^2+1)^2}{x^4-x^2+1} + \frac{1}{2\sqrt{3}} \operatorname{arctg} \frac{2x^2-1}{\sqrt{3}} + c_2, x \in \mathbb{R}, c_2 \in \mathbb{R}.$$

Koniec 10. časti príklady b – 154

$$\int \frac{x^2 dx}{x^6+1}$$



Koniec 10. časti príklady b – 154

$$\int \frac{x^2 dx}{x^6+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \in \mathbb{R} \\ dt=3x^2 dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{x^2 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}$$



Koniec 10. časti príklady b – 154

$$\int \frac{x^2 dx}{x^6+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \in \mathbb{R} \\ dt=3x^2 dx \mid t \in \mathbb{R} \end{array} \right] = \frac{1}{3} \int \frac{dt}{t^2+1}$$

$$= \int \frac{x^2 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

Koniec 10. časti príklady b – 154

$$\int \frac{x^2 dx}{x^6+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \\ dt=3x^2 dx \end{array} \middle| \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \frac{1}{3} \int \frac{dt}{t^2+1} = \frac{1}{3} \arctg t + c_1$$

$$= \int \frac{x^2 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \end{array} \middle| \begin{array}{l} du=dx \\ dv=dx \end{array} \middle| \begin{array}{l} x \in R, u \in R \\ x \in R, v \in R \end{array} \right] x^2 \pm \sqrt{3}x + 1 = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \frac{1}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 > 0$$

Koniec 10. časti príklady b – 154

$$\int \frac{x^2 dx}{x^6+1} = \frac{1}{3} \operatorname{arctg} x^3 + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \\ dt=3x^2 dx \end{array} \middle| \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \frac{1}{3} \int \frac{dt}{t^2+1} = \frac{1}{3} \operatorname{arctg} t + c_1 = \frac{1}{3} \operatorname{arctg} x^3 + c_1, \quad x \in R, \quad c \in R.$$

$$= \int \frac{x^2 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \end{array} \middle| \begin{array}{l} du=dx \\ dv=dx \end{array} \middle| \begin{array}{l} x \in R, \quad u \in R \\ x \in R, \quad v \in R \end{array} \right] x^2 \pm \sqrt{3}x + 1 = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \frac{1}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 > 0$$

Koniec 10. časti príklady b – 154

$$\int \frac{x^2 dx}{x^6+1} = \frac{1}{3} \operatorname{arctg} x^3 + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \\ dt=3x^2 dx \end{array} \middle| \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \frac{1}{3} \int \frac{dt}{t^2+1} = \frac{1}{3} \operatorname{arctg} t + c_1 = \frac{1}{3} \operatorname{arctg} x^3 + c_1, \quad x \in R, \quad c \in R.$$

$$= \int \frac{x^2 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \end{array} \middle| \begin{array}{l} du=dx \\ dv=dx \end{array} \middle| \begin{array}{l} x \in R, \quad u \in R \\ x \in R, \quad v \in R \end{array} \right] x^2 \pm \sqrt{3}x + 1 = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \frac{1}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 > 0$$

$$= -\frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{6} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{6} \int \frac{dv}{v^2+\frac{1}{4}}$$

Koniec 10. časti príklady b – 154

$$\int \frac{x^2 dx}{x^6+1} = \frac{1}{3} \operatorname{arctg} x^3 + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \in R \\ dt=3x^2 dx \mid t \in R \end{array} \right] = \frac{1}{3} \int \frac{dt}{t^2+1} = \frac{1}{3} \operatorname{arctg} t + c_1 = \frac{1}{3} \operatorname{arctg} x^3 + c_1, \quad x \in R, \quad c \in R.$$

$$= \int \frac{x^2 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \mid du=dx \mid x \in R, u \in R \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \mid dv=dx \mid x \in R, v \in R \end{array} \right] x^2 \pm \sqrt{3}x + 1 = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \frac{1}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 > 0$$

$$= -\frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{6} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{6} \int \frac{dv}{v^2+\frac{1}{4}}$$

$$= -\frac{1}{3} \operatorname{arctg} x + \frac{1}{6} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{u}{\frac{1}{2}} + \frac{1}{6} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{v}{\frac{1}{2}} + c_2$$

Koniec 10. časti príklady b – 154

$$\int \frac{x^2 dx}{x^6+1} = \frac{1}{3} \operatorname{arctg} x^3 + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \\ dt=3x^2 dx \end{array} \middle| \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \frac{1}{3} \int \frac{dt}{t^2+1} = \frac{1}{3} \operatorname{arctg} t + c_1 = \frac{1}{3} \operatorname{arctg} x^3 + c_1, \quad x \in R, \quad c \in R.$$

$$= \int \frac{x^2 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \end{array} \middle| \begin{array}{l} du=dx \\ dv=dx \end{array} \middle| \begin{array}{l} x \in R, u \in R \\ x \in R, v \in R \end{array} \right] \left[x^2 \pm \sqrt{3}x + 1 = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \frac{1}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 > 0 \right]$$

$$= -\frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{6} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{6} \int \frac{dv}{v^2+\frac{1}{4}}$$

$$= -\frac{1}{3} \operatorname{arctg} x + \frac{1}{6} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{u}{\frac{1}{2}} + \frac{1}{6} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{v}{\frac{1}{2}} + c_2$$

$$= -\frac{1}{3} \operatorname{arctg} x + \frac{1}{3} \operatorname{arctg} 2u + \frac{1}{3} \operatorname{arctg} 2v + c_2$$

Koniec 10. časti príklady b – 154

$$\int \frac{x^2 dx}{x^6+1} = \frac{1}{3} \operatorname{arctg} x^3 + c_1 = -\frac{1}{3} \operatorname{arctg} x + \frac{1}{3} \operatorname{arctg} (2x + \sqrt{3}) + \frac{1}{3} \operatorname{arctg} (2x - \sqrt{3}) + c_2$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \\ dt=3x^2 dx \end{array} \middle| \begin{array}{l} x \in R \\ t \in R \end{array} \right] = \frac{1}{3} \int \frac{dt}{t^2+1} = \frac{1}{3} \operatorname{arctg} t + c_1 = \frac{1}{3} \operatorname{arctg} x^3 + c_1, \quad x \in R, \quad c \in R.$$

$$= \int \frac{x^2 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \end{array} \middle| \begin{array}{l} du=dx \\ dv=dx \end{array} \middle| \begin{array}{l} x \in R, u \in R \\ x \in R, v \in R \end{array} \right] x^2 \pm \sqrt{3}x + 1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0$$

$$= -\frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{6} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{6} \int \frac{dv}{v^2+\frac{1}{4}}$$

$$= -\frac{1}{3} \operatorname{arctg} x + \frac{1}{6} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{u}{\frac{1}{2}} + \frac{1}{6} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{v}{\frac{1}{2}} + c_2$$

$$= -\frac{1}{3} \operatorname{arctg} x + \frac{1}{3} \operatorname{arctg} 2u + \frac{1}{3} \operatorname{arctg} 2v + c_2$$

$$= -\frac{1}{3} \operatorname{arctg} x + \frac{1}{3} \operatorname{arctg} (2x + \sqrt{3}) + \frac{1}{3} \operatorname{arctg} (2x - \sqrt{3}) + c_2, \quad x \in R, \quad c_2 \in R.$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6 + 1}$$



Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$= \int \frac{x^3 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}$$



Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$= \int \frac{x^3 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$= \int \frac{x^3 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= -\frac{1}{6} \int \frac{2x dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1} \right] dx$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x^3 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= -\frac{1}{6} \int \frac{2x dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1} \right] dx \\
 &= -\frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx
 \end{aligned}$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x^3 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= -\frac{1}{6} \int \frac{2x dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1} \right] dx \\
 &= -\frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \mid du = dx \mid x \in R, u \in R \mid x^2 \pm \sqrt{3}x + 1 = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \frac{1}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 > 0 \\ \text{Subst. } v = x - \frac{\sqrt{3}}{2} \mid dv = dx \mid x \in R, v \in R \mid (x^2 + \sqrt{3}x + 1)(x^2 - \sqrt{3}x + 1) = (x^2 + 1)^2 - (\sqrt{3}x)^2 = x^4 - x^2 + 1 > 0 \end{array} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x^3 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= -\frac{1}{6} \int \frac{2x dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1} \right] dx \\
 &= -\frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \mid du = dx \mid x \in \mathbb{R}, u \in \mathbb{R} \mid x^2 \pm \sqrt{3}x + 1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \\ \text{Subst. } v = x - \frac{\sqrt{3}}{2} \mid dv = dx \mid x \in \mathbb{R}, v \in \mathbb{R} \mid (x^2 + \sqrt{3}x + 1)(x^2 - \sqrt{3}x + 1) = (x^2 + 1)^2 - (\sqrt{3}x)^2 = x^4 - x^2 + 1 > 0 \end{array} \right] \\
 &= -\frac{1}{6} \ln(x^2+1) + \frac{1}{12} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{\sqrt{3}}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12} \int \frac{dv}{v^2+\frac{1}{4}}
 \end{aligned}$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x^3 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= -\frac{1}{6} \int \frac{2x dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1} \right] dx \\
 &= -\frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \mid du = dx \mid x \in \mathbb{R}, u \in \mathbb{R} \mid x^2 \pm \sqrt{3}x + 1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \\ \text{Subst. } v = x - \frac{\sqrt{3}}{2} \mid dv = dx \mid x \in \mathbb{R}, v \in \mathbb{R} \mid (x^2 + \sqrt{3}x + 1)(x^2 - \sqrt{3}x + 1) = (x^2 + 1)^2 - (\sqrt{3}x)^2 = x^4 - x^2 + 1 > 0 \end{array} \right] \\
 &= -\frac{1}{6} \ln(x^2+1) + \frac{1}{12} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{\sqrt{3}}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\
 &= -\frac{\ln(x^2+1)^2}{12} + \frac{\ln(x^2+\sqrt{3}x+1) + \ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{u}{\frac{1}{2}} + \frac{\sqrt{3}}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{v}{\frac{1}{2}} + c_1
 \end{aligned}$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x^3 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= -\frac{1}{6} \int \frac{2x dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1} \right] dx \\
 &= -\frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \mid du = dx \mid x \in \mathbb{R}, u \in \mathbb{R} \mid x^2 \pm \sqrt{3}x + 1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \\ \text{Subst. } v = x - \frac{\sqrt{3}}{2} \mid dv = dx \mid x \in \mathbb{R}, v \in \mathbb{R} \mid (x^2 + \sqrt{3}x + 1)(x^2 - \sqrt{3}x + 1) = (x^2 + 1)^2 - (\sqrt{3}x)^2 = x^4 - x^2 + 1 > 0 \end{array} \right] \\
 &= -\frac{1}{6} \ln(x^2+1) + \frac{1}{12} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{\sqrt{3}}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\
 &= -\frac{\ln(x^2+1)^2}{12} + \frac{\ln(x^2+\sqrt{3}x+1) + \ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{u}{\frac{1}{2}} + \frac{\sqrt{3}}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{v}{\frac{1}{2}} + c_1 \\
 &= \frac{1}{12} \ln \frac{(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)}{(x^2+1)^2} - \frac{\sqrt{3}}{6} \arctg 2u + \frac{\sqrt{3}}{6} \arctg 2v + c_1
 \end{aligned}$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1} = \frac{1}{12} \ln \frac{x^4-x^2+1}{(x^2+1)^2} - \frac{\sqrt{3} \operatorname{arctg} (2x+\sqrt{3})}{6} + \frac{\sqrt{3} \operatorname{arctg} (2x-\sqrt{3})}{6} + c_1$$

$$\begin{aligned} &= \int \frac{x^3 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx \\ &= -\frac{1}{6} \int \frac{2x dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1} \right] dx \\ &= -\frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} + \frac{1}{12} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \mid du = dx \mid x \in \mathbb{R}, u \in \mathbb{R} \mid x^2 \pm \sqrt{3}x + 1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \\ \text{Subst. } v = x - \frac{\sqrt{3}}{2} \mid dv = dx \mid x \in \mathbb{R}, v \in \mathbb{R} \mid (x^2 + \sqrt{3}x + 1)(x^2 - \sqrt{3}x + 1) = (x^2 + 1)^2 - (\sqrt{3}x)^2 = x^4 - x^2 + 1 > 0 \end{array} \right] \\ &= -\frac{1}{6} \ln(x^2+1) + \frac{1}{12} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{\sqrt{3}}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\ &= -\frac{\ln(x^2+1)^2}{12} + \frac{\ln(x^2+\sqrt{3}x+1) + \ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{u}{\frac{1}{2}} + \frac{\sqrt{3}}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{v}{\frac{1}{2}} + c_1 \\ &= \frac{1}{12} \ln \frac{(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)}{(x^2+1)^2} - \frac{\sqrt{3}}{6} \operatorname{arctg} 2u + \frac{\sqrt{3}}{6} \operatorname{arctg} 2v + c_1 \\ &= \frac{1}{12} \ln \frac{x^4-x^2+1}{(x^2+1)^2} - \frac{\sqrt{3} \operatorname{arctg} (2x+\sqrt{3})}{6} + \frac{\sqrt{3} \operatorname{arctg} (2x-\sqrt{3})}{6} + c_1, x \in \mathbb{R}, c_1 \in \mathbb{R}. \end{aligned}$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6 + 1}$$



Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$= \int \frac{x \cdot x^2 dx}{x^6+1}$$



Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$= \int \frac{x \cdot x^2 dx}{x^6+1} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \\ dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \end{array} \right]$$



Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$= \int \frac{x \cdot x^2 dx}{x^6+1} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \\ \quad \quad \quad dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{t dt}{t^3+1}$$



Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$= \int \frac{x \cdot x^2 dx}{x^6+1} = \left[\begin{array}{l} \text{Subst. } t=x^2 \\ dt=2x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; \infty) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \frac{1}{2} \int \frac{t dt}{t^3+1} = \frac{1}{2} \int \frac{t dt}{(t+1)(t^2-t+1)}$$



Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$= \int \frac{x \cdot x^2 dx}{x^6+1} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \\ \quad \quad \quad dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{t dt}{t^3+1} = \frac{1}{2} \int \frac{t dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2} \int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3} + \frac{1}{3}}{t^2-t+1} \right] dt$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$= \int \frac{x \cdot x^2 dx}{x^6+1} = \left[\begin{array}{l} \text{Subst. } t=x^2 \\ dt=2x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; \infty) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \frac{1}{2} \int \frac{t dt}{t^3+1} = \frac{1}{2} \int \frac{t dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2} \int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3} + \frac{1}{3}}{t^2-t+1} \right] dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t+2}{t^2-t+1} dt$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x \cdot x^2 dx}{x^6+1} = \left[\begin{array}{l} \text{Subst. } t=x^2 \\ dt=2x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; \infty) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \frac{1}{2} \int \frac{t dt}{t^3+1} = \frac{1}{2} \int \frac{t dt}{(t+1)(t^2-t+1)} \\
 &= \frac{1}{2} \int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3} + \frac{1}{3}}{t^2-t+1} \right] dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t+2}{t^2-t+1} dt \\
 &= -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t-1+3}{t^2-t+1} dt
 \end{aligned}$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6 + 1}$$

$$\begin{aligned}
 &= \int \frac{x \cdot x^2 dx}{x^6 + 1} = \left[\begin{array}{l} \text{Subst. } t = x^2 \\ dt = 2x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in \langle 0; \infty \rangle \\ x \in \langle 0; \infty \rangle, t \in \langle 0; \infty \rangle \end{array} \right. \right] = \frac{1}{2} \int \frac{t dt}{t^3 + 1} = \frac{1}{2} \int \frac{t dt}{(t+1)(t^2 - t + 1)} \\
 &= \frac{1}{2} \int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3} + \frac{1}{3}}{t^2 - t + 1} \right] dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t+2}{t^2 - t + 1} dt \\
 &= -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t-1+3}{t^2 - t + 1} dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{(2t-1) dt}{t^2 - t + 1} + \frac{1}{4} \int \frac{dt}{t^2 - t + 1}
 \end{aligned}$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x \cdot x^2 dx}{x^6+1} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in \langle 0; \infty \rangle \\ \quad \quad \quad dt=2x dx \mid x \in \langle 0; \infty \rangle, t \in \langle 0; \infty \rangle \end{array} \right] = \frac{1}{2} \int \frac{t dt}{t^3+1} = \frac{1}{2} \int \frac{t dt}{(t+1)(t^2-t+1)} \\
 &= \frac{1}{2} \int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3} + \frac{1}{3}}{t^2-t+1} \right] dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t+2}{t^2-t+1} dt \\
 &= -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t-1+3}{t^2-t+1} dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{(2t-1) dt}{t^2-t+1} + \frac{1}{4} \int \frac{dt}{t^2-t+1} \\
 &= \left[\begin{array}{l} \text{Subst. } u=t-\frac{1}{2} \mid t \in \langle 0; \infty \rangle \\ \quad \quad \quad du=dt \mid u \in \langle -\frac{1}{2}; \infty \rangle \end{array} \right] \left[t^2-t+1 = (t-\frac{1}{2})^2 + 1 - \frac{1}{4} = (t-\frac{1}{2})^2 + \frac{3}{4} = u^2 + (\frac{\sqrt{3}}{2})^2 > 0 \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 155

$$\int \frac{x^3 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x \cdot x^2 dx}{x^6+1} = \left[\begin{array}{l} \text{Subst. } t=x^2 \left| \begin{array}{l} x \in (-\infty; 0), t \in \langle 0; \infty \rangle \\ dt=2x dx \left| \begin{array}{l} x \in \langle 0; \infty \rangle, t \in \langle 0; \infty \rangle \end{array} \right. \end{array} \right] = \frac{1}{2} \int \frac{t dt}{t^3+1} = \frac{1}{2} \int \frac{t dt}{(t+1)(t^2-t+1)} \\
 &= \frac{1}{2} \int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3} + \frac{1}{3}}{t^2-t+1} \right] dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t+2}{t^2-t+1} dt \\
 &= -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t-1+3}{t^2-t+1} dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{(2t-1) dt}{t^2-t+1} + \frac{1}{4} \int \frac{dt}{t^2-t+1} \\
 &= \left[\begin{array}{l} \text{Subst. } u=t-\frac{1}{2} \left| \begin{array}{l} t \in \langle 0; \infty \rangle \\ du=dt \left| \begin{array}{l} u \in \langle -\frac{1}{2}; \infty \rangle \end{array} \right. \end{array} \right] \left[t^2-t+1 = (t-\frac{1}{2})^2 + 1 - \frac{1}{4} = (t-\frac{1}{2})^2 + \frac{3}{4} = u^2 + (\frac{\sqrt{3}}{2})^2 > 0 \right] \\
 &= -\frac{1}{6} \ln(t+1) + \frac{1}{12} \int \frac{(t^2-t+1)' dt}{t^2-t+1} + \frac{1}{4} \int \frac{du}{u^2 + \frac{3}{4}}
 \end{aligned}$$

Koniec 10. časti príklady b – 156

$$\int \frac{x^4 dx}{x^6 + 1}$$



Koniec 10. časti príklady b – 156

$$\int \frac{x^4 dx}{x^6+1}$$

$$= \int \frac{x^4 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}$$



Koniec 10. časti príklady b – 156

$$\int \frac{x^4 dx}{x^6+1}$$

$$= \int \frac{x^4 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

Koniec 10. časti príklady b – 156

$$\int \frac{x^4 dx}{x^6+1}$$

$$= \int \frac{x^4 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\frac{4\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{2x-\frac{4\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

Koniec 10. časti príklady b – 156

$$\int \frac{x^4 dx}{x^6+1}$$

$$= \int \frac{x^4 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x + \frac{4\sqrt{3}}{6} + \sqrt{3} - \sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x - \frac{4\sqrt{3}}{6} - \sqrt{3} + \sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

Koniec 10. časti príklady b – 156

$$\int \frac{x^4 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x^4 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx
 \end{aligned}$$

Koniec 10. časti príklady b – 156

$$\int \frac{x^4 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x^4 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \mid du=dx \mid x \in R, u \in R \mid x^2 \pm \sqrt{3}x+1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \mid dv=dx \mid x \in R, v \in R \mid = (x \pm \frac{\sqrt{3}}{2})^2 + 1 + \frac{3}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \end{array} \right] \left[\frac{1}{4\sqrt{3}} \left(\frac{4\sqrt{3}}{6} - \sqrt{3} \right) \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 156

$$\int \frac{x^4 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x^4 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \left| \begin{array}{l} du=dx \\ x \in R, u \in R \end{array} \right. \left| \begin{array}{l} x^2 \pm \sqrt{3}x+1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} \\ = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \end{array} \right. \left| \begin{array}{l} \frac{1}{4\sqrt{3}} (\frac{4\sqrt{3}}{6} - \sqrt{3}) \\ = \frac{1}{6} - \frac{1}{4} = \frac{2-3}{12} = -\frac{1}{12} \end{array} \right. \end{array} \right] \\
 &= \frac{1}{3} \arctg x - \frac{1}{4\sqrt{3}} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12} \int \frac{dv}{v^2+\frac{1}{4}}
 \end{aligned}$$

Koniec 10. časti príklady b – 156

$$\int \frac{x^4 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x^4 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \left| \frac{du}{dx}=1 \right| x \in \mathbb{R}, u \in \mathbb{R} \left| x^2 \pm \sqrt{3}x+1 = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} \right. \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \left| \frac{dv}{dx}=1 \right| x \in \mathbb{R}, v \in \mathbb{R} \left| = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 > 0 \right. \end{array} \right] \left[\frac{1}{4\sqrt{3}} \left(\frac{4\sqrt{3}}{6} - \sqrt{3} \right) \right] \\
 &= \frac{1}{3} \arctg x - \frac{1}{4\sqrt{3}} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\
 &= \frac{1}{3} \arctg x - \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{u}{\frac{1}{2}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{v}{\frac{1}{2}} + c
 \end{aligned}$$

Koniec 10. časti príklady b – 156

$$\int \frac{x^4 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{x^4 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \frac{1}{3} \arctg x + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \left| \begin{array}{l} du=dx \\ x \in R, u \in R \end{array} \right. \left| \begin{array}{l} x^2 \pm \sqrt{3}x+1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} \\ = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \end{array} \right. \left| \begin{array}{l} \frac{1}{4\sqrt{3}}(\frac{4\sqrt{3}}{6}-\sqrt{3}) \\ = \frac{1}{6} - \frac{1}{4} = \frac{2-3}{12} = -\frac{1}{12} \end{array} \right. \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \left| \begin{array}{l} dv=dx \\ x \in R, v \in R \end{array} \right. \end{array} \right] \\
 &= \frac{1}{3} \arctg x - \frac{1}{4\sqrt{3}} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\
 &= \frac{1}{3} \arctg x - \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{u}{\frac{1}{2}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{v}{\frac{1}{2}} + c \\
 &= \frac{1}{3} \arctg x - \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{1}{6} \arctg 2u + \frac{1}{6} \arctg 2v + c
 \end{aligned}$$

Koniec 10. časti príklady b – 156

$$\int \frac{x^4 dx}{x^6+1} = \frac{1}{3} \operatorname{arctg} x - \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{\operatorname{arctg}(2x+\sqrt{3})}{6} + \frac{\operatorname{arctg}(2x-\sqrt{3})}{6} + c$$

$$\begin{aligned} &= \int \frac{x^4 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}} - \frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{3} \int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{3} \operatorname{arctg} x + \frac{1}{4\sqrt{3}} \int \left[-\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \left[\begin{array}{l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} \left| \frac{du}{dx}=1 \right| x \in \mathbb{R}, u \in \mathbb{R} \left| x^2 \pm \sqrt{3}x+1 = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} \right. \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} \left| \frac{dv}{dx}=1 \right| x \in \mathbb{R}, v \in \mathbb{R} \left| = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 + \frac{3}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 > 0 \right. \end{array} \right] \left[\frac{1}{4\sqrt{3}} \left(\frac{4\sqrt{3}}{6} - \sqrt{3} \right) \right] \\ &= \frac{1}{3} \operatorname{arctg} x - \frac{1}{4\sqrt{3}} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\ &= \frac{1}{3} \operatorname{arctg} x - \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{u}{\frac{1}{2}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{v}{\frac{1}{2}} + c \\ &= \frac{1}{3} \operatorname{arctg} x - \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{1}{6} \operatorname{arctg} 2u + \frac{1}{6} \operatorname{arctg} 2v + c \\ &= \frac{1}{3} \operatorname{arctg} x - \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{\operatorname{arctg}(2x+\sqrt{3})}{6} + \frac{\operatorname{arctg}(2x-\sqrt{3})}{6} + c, x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

Koniec 10. časti príklady b – 157

$$\int \frac{x^5 dx}{x^6 + 1}$$



Koniec 10. časti príklady b – 157

$$\int \frac{x^5 dx}{x^6+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^6+1 \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt=6x^5 dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right]$$

$$= \int \frac{x^5 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}$$



Koniec 10. časti príklady b – 157

$$\int \frac{x^5 dx}{x^6+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^6+1 \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt=6x^5 dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \frac{1}{6} \int \frac{dt}{t}$$

$$= \int \frac{x^5 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3} + \frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3} - \frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

Koniec 10. časti príklady b – 157

$$\int \frac{x^5 dx}{x^6+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^6+1 \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt=6x^5 dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \frac{1}{6} \int \frac{dt}{t} = \frac{1}{6} \ln t + c$$

$$\begin{aligned} &= \int \frac{x^5 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3} + \frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3} - \frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{6} \int \frac{2x dx}{x^2+1} + \frac{1}{6} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \end{aligned}$$

Koniec 10. časti príklady b – 157

$$\int \frac{x^5 dx}{x^6+1} = \frac{1}{6} \ln(x^6+1) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^6+1 \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt=6x^5 dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \frac{1}{6} \int \frac{dt}{t} = \frac{1}{6} \ln t + c = \frac{1}{6} \ln(x^6+1) + c,$$

$x \in R, c \in R.$

$$\begin{aligned} &= \int \frac{x^5 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3} + \frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3} - \frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{6} \int \frac{2x dx}{x^2+1} + \frac{1}{6} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} + \frac{1}{6} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx \end{aligned}$$

Koniec 10. časti príklady b – 157

$$\int \frac{x^5 dx}{x^6+1} = \frac{1}{6} \ln(x^6+1) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^6+1 \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt=6x^5 dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \frac{1}{6} \int \frac{dt}{t} = \frac{1}{6} \ln t + c = \frac{1}{6} \ln(x^6+1) + c,$$

$x \in R, c \in R.$

$$\begin{aligned} &= \int \frac{x^5 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3} + \frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3} - \frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{6} \int \frac{2x dx}{x^2+1} + \frac{1}{6} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} + \frac{1}{6} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{6} \ln(x^2+1) + \frac{1}{6} \ln(x^2+\sqrt{3}x+1) + \frac{1}{6} \ln(x^2-\sqrt{3}x+1) + c \end{aligned}$$

Koniec 10. časti príklady b – 157

$$\int \frac{x^5 dx}{x^6+1} = \frac{1}{6} \ln(x^6+1) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^6+1 \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt=6x^5 dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \frac{1}{6} \int \frac{dt}{t} = \frac{1}{6} \ln t + c = \frac{1}{6} \ln(x^6+1) + c,$$

$x \in R, c \in R.$

$$\begin{aligned} &= \int \frac{x^5 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3} + \frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3} - \frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{6} \int \frac{2x dx}{x^2+1} + \frac{1}{6} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} + \frac{1}{6} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx \\ &= \frac{1}{6} \ln(x^2+1) + \frac{1}{6} \ln(x^2+\sqrt{3}x+1) + \frac{1}{6} \ln(x^2-\sqrt{3}x+1) + c \\ &= \frac{1}{6} \ln[(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)] + c \end{aligned}$$

Koniec 10. časti príklady b – 157

$$\int \frac{x^5 dx}{x^6+1} = \frac{1}{6} \ln(x^6+1) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^6+1 \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt=6x^5 dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \frac{1}{6} \int \frac{dt}{t} = \frac{1}{6} \ln t + c = \frac{1}{6} \ln(x^6+1) + c,$$

$$x \in R, c \in R.$$

$$= \int \frac{x^5 dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3} + \frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3} - \frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6} \int \frac{2x dx}{x^2+1} + \frac{1}{6} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6} \int \frac{(x^2+1)' dx}{x^2+1} + \frac{1}{6} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6} \ln(x^2+1) + \frac{1}{6} \ln(x^2+\sqrt{3}x+1) + \frac{1}{6} \ln(x^2-\sqrt{3}x+1) + c$$

$$= \frac{1}{6} \ln[(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)] + c = \frac{1}{6} \ln(x^6+1) + c,$$

$$x \in R, c \in R.$$

Koniec 10. časti príklady b – 158

$$\int \frac{x^6 dx}{x^6+1}$$



Koniec 10. časti príklady b – 158

$$\int \frac{x^6 dx}{x^6+1}$$

$$= \int \frac{(x^6+1-1) dx}{x^6+1}$$

Koniec 10. časti príklady b – 158

$$\int \frac{x^6 dx}{x^6+1}$$

$$= \int \frac{(x^6+1-1) dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1}$$



Koniec 10. časti príklady b – 158

$$\int \frac{x^6 dx}{x^6+1}$$

$$= \int \frac{(x^6+1-1) dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2 + \sqrt{3}x + 1} + \frac{-\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2 - \sqrt{3}x + 1} \right] dx$$

Koniec 10. časti príklady b – 158

$$\int \frac{x^6 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{(x^6+1-1) dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2 + \sqrt{3}x + 1} + \frac{-\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2 - \sqrt{3}x + 1} \right] dx \\
 &= x - \frac{1}{3} \int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x + \frac{4\sqrt{3}}{3}}{x^2 + \sqrt{3}x + 1} - \frac{2x - \frac{4\sqrt{3}}{3}}{x^2 - \sqrt{3}x + 1} \right] dx
 \end{aligned}$$

Koniec 10. časti príklady b – 158

$$\int \frac{x^6 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{(x^6+1-1) dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= x - \frac{1}{3} \int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x + \frac{4\sqrt{3}}{3} + \sqrt{3} - \sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x - \frac{4\sqrt{3}}{3} - \sqrt{3} + \sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx
 \end{aligned}$$

Koniec 10. časti príklady b – 158

$$\int \frac{x^6 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{(x^6+1-1) dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= x - \frac{1}{3} \int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x + \frac{4\sqrt{3}}{3} + \sqrt{3} - \sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x - \frac{4\sqrt{3}}{3} - \sqrt{3} + \sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= x - \frac{\arctg x}{3} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx
 \end{aligned}$$

Koniec 10. časti príklady b – 158

$$\int \frac{x^6 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{(x^6+1-1) dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= x - \frac{1}{3} \int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x + \frac{4\sqrt{3}}{3} + \sqrt{3} - \sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x - \frac{4\sqrt{3}}{3} - \sqrt{3} + \sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= x - \frac{\arctg x}{3} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \mid du = dx \mid x \in R, u \in R \mid x^2 \pm \sqrt{3}x + 1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} \\ \text{Subst. } v = x - \frac{\sqrt{3}}{2} \mid dv = dx \mid x \in R, v \in R \mid = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \end{array} \right] \left[\frac{1}{4\sqrt{3}} \left(\frac{4\sqrt{3}}{3} - \sqrt{3} \right) \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 158

$$\int \frac{x^6 dx}{x^6+1}$$

$$= \int \frac{(x^6+1-1) dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{1}{3} \int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x + \frac{4\sqrt{3}}{3} + \sqrt{3} - \sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x - \frac{4\sqrt{3}}{3} - \sqrt{3} + \sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{\arctg x}{3} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \left| \frac{du}{dx} = 1 \right| x \in \mathbb{R}, u \in \mathbb{R} \left| x^2 \pm \sqrt{3}x + 1 = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} \right. \\ \text{Subst. } v = x - \frac{\sqrt{3}}{2} \left| \frac{dv}{dx} = 1 \right| x \in \mathbb{R}, v \in \mathbb{R} \left| = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \frac{1}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 > 0 \right. \end{array} \left| \frac{1}{4\sqrt{3}} \left(\frac{4\sqrt{3}}{3} - \sqrt{3} \right) \right. \right]$$

$$= x - \frac{\arctg x}{3} - \frac{1}{4\sqrt{3}} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{1}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12} \int \frac{dv}{v^2+\frac{1}{4}}$$

Koniec 10. časti príklady b – 158

$$\int \frac{x^6 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{(x^6+1-1) dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{-x}{2\sqrt{3}} + \frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= x - \frac{1}{3} \int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x + \frac{4\sqrt{3}}{3} + \sqrt{3} - \sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x - \frac{4\sqrt{3}}{3} - \sqrt{3} + \sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \left| \begin{array}{l} du = dx \\ x \in \mathbb{R}, u \in \mathbb{R} \end{array} \right. \left| \begin{array}{l} x^2 \pm \sqrt{3}x + 1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} \\ = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \end{array} \right. \left| \begin{array}{l} \frac{1}{4\sqrt{3}} (\frac{4\sqrt{3}}{3} - \sqrt{3}) \\ = \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right. \end{array} \right] \\
 &= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{1}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\
 &= x - \frac{\operatorname{arctg} x}{3} - \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} - \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{u}{\frac{1}{2}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{v}{\frac{1}{2}} + c
 \end{aligned}$$

Koniec 10. časti príklady b – 158

$$\int \frac{x^6 dx}{x^6+1}$$

$$\begin{aligned}
 &= \int \frac{(x^6+1-1) dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= x - \frac{1}{3} \int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x + \frac{4\sqrt{3}}{3} + \sqrt{3} - \sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x - \frac{4\sqrt{3}}{3} - \sqrt{3} + \sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= x - \frac{\arctg x}{3} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx \\
 &= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \left| \begin{array}{l} du = dx \\ x \in \mathbb{R}, u \in \mathbb{R} \end{array} \right. \left| \begin{array}{l} x^2 \pm \sqrt{3}x + 1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} \\ = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \end{array} \right. \left| \begin{array}{l} \frac{1}{4\sqrt{3}} (\frac{4\sqrt{3}}{3} - \sqrt{3}) \\ = \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right. \\ \text{Subst. } v = x - \frac{\sqrt{3}}{2} \left| \begin{array}{l} dv = dx \\ x \in \mathbb{R}, v \in \mathbb{R} \end{array} \right. \end{array} \right] \\
 &= x - \frac{\arctg x}{3} - \frac{1}{4\sqrt{3}} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{1}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12} \int \frac{dv}{v^2+\frac{1}{4}} \\
 &= x - \frac{\arctg x}{3} - \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} - \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{u}{\frac{1}{2}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \arctg \frac{v}{\frac{1}{2}} + c \\
 &= x - \frac{\arctg x}{3} - \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} - \frac{1}{6} \arctg 2u + \frac{1}{6} \arctg 2v + c
 \end{aligned}$$

Koniec 10. časti príklady b – 158

$$\int \frac{x^6 dx}{x^6+1} = x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} - \frac{\operatorname{arctg} (2x+\sqrt{3})}{6} + \frac{\operatorname{arctg} (2x-\sqrt{3})}{6} + c$$

$$= \int \frac{(x^6+1-1) dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}} + \frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{1}{3} \int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x + \frac{4\sqrt{3}}{3} + \sqrt{3} - \sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x - \frac{4\sqrt{3}}{3} - \sqrt{3} + \sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l} \text{Subst. } u = x + \frac{\sqrt{3}}{2} \quad du = dx \quad \left| \begin{array}{l} x \in R, u \in R \\ x^2 \pm \sqrt{3}x + 1 = (x \pm \frac{\sqrt{3}}{2})^2 + 1 - \frac{3}{4} \end{array} \right. \\ \text{Subst. } v = x - \frac{\sqrt{3}}{2} \quad dv = dx \quad \left| \begin{array}{l} x \in R, v \in R \\ = (x \pm \frac{\sqrt{3}}{2})^2 + \frac{1}{4} = (x \pm \frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 > 0 \end{array} \right. \end{array} \right] \frac{1}{4\sqrt{3}} \left(\frac{4\sqrt{3}}{3} - \sqrt{3} \right) = \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{1}{12} \int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12} \int \frac{dv}{v^2+\frac{1}{4}}$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} - \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{u}{\frac{1}{2}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{v}{\frac{1}{2}} + c$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} - \frac{1}{6} \operatorname{arctg} 2u + \frac{1}{6} \operatorname{arctg} 2v + c$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} - \frac{\operatorname{arctg} (2x+\sqrt{3})}{6} + \frac{\operatorname{arctg} (2x-\sqrt{3})}{6} + c, \quad x \in R, \quad c \in R.$$

Koniec 10. časti príklady b – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$\int \frac{dx}{\sqrt{2^x+1}}$$

Koniec 10. časti príklady b – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=2^x+1, 2^x=t-1 \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} \end{array} \middle| x \in R, t \in (1; \infty) \right] dx = \frac{dt}{(t-1)\ln 2}$$

$$\int \frac{dx}{\sqrt{2^x+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=\sqrt{2^x+1}, 2^x=t^2-1 \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} \end{array} \middle| x \in R, t \in (1; \infty) \right] dx = \frac{2t dt}{(t^2-1)\ln 2}$$

Koniec 10. časti príklady b – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=2^x+1, 2^x=t-1 \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} \end{array} \middle| \begin{array}{l} x \in R, t \in (1; \infty) \\ dx = \frac{dt}{(t-1)\ln 2} \end{array} \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \int \frac{dt}{(t-1)t \ln 2}$$

$$\int \frac{dx}{\sqrt{2^x+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=\sqrt{2^x+1}, 2^x=t^2-1 \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} \end{array} \middle| \begin{array}{l} x \in R, t \in (1; \infty) \\ dx = \frac{2t dt}{(t^2-1)\ln 2} \end{array} \right] = \int \frac{\frac{2t dt}{(t^2-1)\ln 2}}{t} = \int \frac{2 dt}{(t^2-1)\ln 2}$$

Koniec 10. časti príklady b – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=2^x+1, 2^x=t-1 \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} \end{array} \middle| \begin{array}{l} x \in R, t \in (1; \infty) \\ dx = \frac{dt}{(t-1)\ln 2} \end{array} \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \frac{1}{\ln 2} \int \frac{dt}{t(t-1)}$$

$$\int \frac{dx}{\sqrt{2^x+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=\sqrt{2^x+1}, 2^x=t^2-1 \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} \end{array} \middle| \begin{array}{l} x \in R, t \in (1; \infty) \\ dx = \frac{2t dt}{(t^2-1)\ln 2} \end{array} \right] = \int \frac{\frac{2t dt}{(t^2-1)\ln 2}}{t} = \frac{2}{\ln 2} \int \frac{dt}{t^2-1}$$

Koniec 10. časti príklady b – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=2^x+1, 2^x=t-1 \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} \end{array} \middle| \begin{array}{l} x \in R, t \in (1; \infty) \\ dx = \frac{dt}{(t-1)\ln 2} \end{array} \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \frac{1}{\ln 2} \int \frac{dt}{t(t-1)} = \frac{1}{\ln 2} \int \frac{(t-t+1) dt}{t(t-1)}$$

$$\int \frac{dx}{\sqrt{2^x+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=\sqrt{2^x+1}, 2^x=t^2-1 \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} \end{array} \middle| \begin{array}{l} x \in R, t \in (1; \infty) \\ dx = \frac{2t dt}{(t^2-1)\ln 2} \end{array} \right] = \int \frac{\frac{2t dt}{(t^2-1)\ln 2}}{t} = \frac{2}{\ln 2} \int \frac{dt}{t^2-1} = \frac{2}{2\ln 2} \ln \frac{t-1}{t+1} + c$$

Koniec 10. časti príklady b – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=2^x+1, 2^x=t-1 \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} \end{array} \middle| \begin{array}{l} x \in R, t \in (1; \infty) \\ dx = \frac{dt}{(t-1)\ln 2} \end{array} \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \frac{1}{\ln 2} \int \frac{dt}{t(t-1)} = \frac{1}{\ln 2} \int \frac{(t-t+1) dt}{t(t-1)}$$

$$= \frac{1}{\ln 2} \int \left[\frac{1}{t-1} - \frac{1}{t} \right] dt$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=\sqrt{2^x+1}, 2^x=t^2-1 \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} \end{array} \middle| \begin{array}{l} x \in R, t \in (1; \infty) \\ dx = \frac{2t dt}{(t^2-1)\ln 2} \end{array} \right] = \int \frac{\frac{2t dt}{(t^2-1)\ln 2}}{t} = \frac{2}{\ln 2} \int \frac{dt}{t^2-1} = \frac{2}{2\ln 2} \ln \frac{t-1}{t+1} + c$$

$$= \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c$$

Koniec 10. časti príklady b – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=2^x+1, 2^x=t-1 \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} \end{array} \middle| x \in \mathbb{R}, t \in (1; \infty) \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \frac{1}{\ln 2} \int \frac{dt}{t(t-1)} = \frac{1}{\ln 2} \int \frac{(t-t+1) dt}{t(t-1)}$$

$$= \frac{1}{\ln 2} \int \left[\frac{1}{t-1} - \frac{1}{t} \right] dt = \frac{1}{\ln 2} \left[\ln(t-1) - \ln t \right] + c$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=\sqrt{2^x+1}, 2^x=t^2-1 \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} \end{array} \middle| x \in \mathbb{R}, t \in (1; \infty) \right] = \int \frac{\frac{2t dt}{(t^2-1)\ln 2}}{t} = \frac{2}{\ln 2} \int \frac{dt}{t^2-1} = \frac{2}{2\ln 2} \ln \frac{t-1}{t+1} + c$$

$$= \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c = \frac{1}{\ln 2} \ln \left[\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}-1} \right] + c$$

Koniec 10. časti príklady b – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$= \left[\begin{array}{l} \text{Subst. } t=2^x+1, 2^x=t-1 \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} \end{array} \middle| x \in \mathbb{R}, t \in (1; \infty) \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \frac{1}{\ln 2} \int \frac{dt}{t(t-1)} = \frac{1}{\ln 2} \int \frac{(t-t+1) dt}{t(t-1)}$$

$$= \frac{1}{\ln 2} \int \left[\frac{1}{t-1} - \frac{1}{t} \right] dt = \frac{1}{\ln 2} \left[\ln(t-1) - \ln t \right] + C = \frac{1}{\ln 2} \ln \frac{t-1}{t} + C$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + C$$

$$= \left[\begin{array}{l} \text{Subst. } t=\sqrt{2^x+1}, 2^x=t^2-1 \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} \end{array} \middle| x \in \mathbb{R}, t \in (1; \infty) \right] = \int \frac{\frac{2t dt}{(t^2-1)\ln 2}}{t} = \frac{2}{\ln 2} \int \frac{dt}{t^2-1} = \frac{2}{2\ln 2} \ln \frac{t-1}{t+1} + C$$

$$= \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + C = \frac{1}{\ln 2} \ln \left[\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}-1} \right] + C = \frac{1}{\ln 2} \ln \frac{2^x+1-2\sqrt{2^x+1}+1}{2^x+1-1} + C$$

Koniec 10. časti príklady b – 159, 160

$$\int \frac{dx}{2^x+1} = \frac{1}{\ln 2} \ln \frac{2^x}{2^x+1} + c$$

$$\begin{aligned}
 &= \left[\text{Subst. } t=2^x+1, 2^x=t-1 \mid x \in \mathbb{R}, t \in (1; \infty) \right] = \int \frac{\frac{dt}{(t-1) \ln 2}}{t} = \frac{1}{\ln 2} \int \frac{dt}{t(t-1)} = \frac{1}{\ln 2} \int \frac{(t-t+1) dt}{t(t-1)} \\
 &= \frac{1}{\ln 2} \int \left[\frac{1}{t-1} - \frac{1}{t} \right] dt = \frac{1}{\ln 2} \left[\ln(t-1) - \ln t \right] + c = \frac{1}{\ln 2} \ln \frac{t-1}{t} + c = \frac{1}{\ln 2} \ln \frac{2^x}{2^x+1} + c
 \end{aligned}$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c$$

$$\begin{aligned}
 &= \left[\text{Subst. } t=\sqrt{2^x+1}, 2^x=t^2-1 \mid x \in \mathbb{R}, t \in (1; \infty) \right] = \int \frac{\frac{2t dt}{(t^2-1) \ln 2}}{t} = \frac{2}{\ln 2} \int \frac{dt}{t^2-1} = \frac{2}{2 \ln 2} \ln \frac{t-1}{t+1} + c \\
 &= \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c = \frac{1}{\ln 2} \ln \left[\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}-1} \right] + c = \frac{1}{\ln 2} \ln \frac{2^x+1-2\sqrt{2^x+1}+1}{2^x+1-1} + c \\
 &= \frac{\ln(2^x+2-2\sqrt{2^x+1}) - \ln 2^x}{\ln 2} + c
 \end{aligned}$$

Koniec 10. časti príklady b – 159, 160

$$\int \frac{dx}{2^x+1} = \frac{1}{\ln 2} \ln \frac{2^x}{2^x+1} + c$$

$$\begin{aligned}
 &= \left[\text{Subst. } t=2^x+1, 2^x=t-1 \mid x \in \mathbb{R}, t \in (1; \infty) \right] = \int \frac{\frac{dt}{(t-1) \ln 2}}{t} = \frac{1}{\ln 2} \int \frac{dt}{t(t-1)} = \frac{1}{\ln 2} \int \frac{(t-t+1) dt}{t(t-1)} \\
 &= \frac{1}{\ln 2} \int \left[\frac{1}{t-1} - \frac{1}{t} \right] dt = \frac{1}{\ln 2} \left[\ln(t-1) - \ln t \right] + c = \frac{1}{\ln 2} \ln \frac{t-1}{t} + c = \frac{1}{\ln 2} \ln \frac{2^x}{2^x+1} + c \\
 &= \frac{\ln 2^x - \ln(2^x+1)}{\ln 2} + c
 \end{aligned}$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c$$

$$\begin{aligned}
 &= \left[\text{Subst. } t=\sqrt{2^x+1}, 2^x=t^2-1 \mid x \in \mathbb{R}, t \in (1; \infty) \right] = \int \frac{\frac{2t dt}{(t^2-1) \ln 2}}{t} = \frac{2}{\ln 2} \int \frac{dt}{t^2-1} = \frac{2}{2 \ln 2} \ln \frac{t-1}{t+1} + c \\
 &= \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c = \frac{1}{\ln 2} \ln \left[\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}-1} \right] + c = \frac{1}{\ln 2} \ln \frac{2^x+1-2\sqrt{2^x+1}+1}{2^x+1-1} + c \\
 &= \frac{\ln(2^x+2-2\sqrt{2^x+1}) - \ln 2^x}{\ln 2} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1}) - x \ln 2}{\ln 2} + c
 \end{aligned}$$

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$$\int \frac{dx}{2^x+1} = \frac{1}{\ln 2} \ln \frac{2^x}{2^x+1} + c$$

$$\begin{aligned}
 &= \left[\text{Subst. } t=2^x+1, 2^x=t-1 \mid x \in \mathbb{R}, t \in (1; \infty) \right] = \int \frac{\frac{dt}{(t-1) \ln 2}}{t} = \frac{1}{\ln 2} \int \frac{dt}{t(t-1)} = \frac{1}{\ln 2} \int \frac{(t-t+1) dt}{t(t-1)} \\
 &= \frac{1}{\ln 2} \int \left[\frac{1}{t-1} - \frac{1}{t} \right] dt = \frac{1}{\ln 2} [\ln(t-1) - \ln t] + c = \frac{1}{\ln 2} \ln \frac{t-1}{t} + c = \frac{1}{\ln 2} \ln \frac{2^x}{2^x+1} + c \\
 &= \frac{\ln 2^x - \ln(2^x+1)}{\ln 2} + c = \frac{x \ln 2 - \ln(2^x+1)}{\ln 2} + c
 \end{aligned}$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1})}{\ln 2} - x + c$$

$$\begin{aligned}
 &= \left[\text{Subst. } t=\sqrt{2^x+1}, 2^x=t^2-1 \mid x \in \mathbb{R}, t \in (1; \infty) \right] = \int \frac{\frac{2t dt}{(t^2-1) \ln 2}}{t} = \frac{2}{\ln 2} \int \frac{dt}{t^2-1} = \frac{2}{2 \ln 2} \ln \frac{t-1}{t+1} + c \\
 &= \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c = \frac{1}{\ln 2} \ln \left[\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}-1} \right] + c = \frac{1}{\ln 2} \ln \frac{2^x+1-2\sqrt{2^x+1}+1}{2^x+1-1} + c \\
 &= \frac{\ln(2^x+2-2\sqrt{2^x+1}) - \ln 2^x}{\ln 2} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1}) - x \ln 2}{\ln 2} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1})}{\ln 2} - x + c, \\
 &\quad x \in \mathbb{R}, c \in \mathbb{R}.
 \end{aligned}$$

Koniec 10. časti príklady b – 159, 160

$$\int \frac{dx}{2^x+1} = \frac{1}{\ln 2} \ln \frac{2^x}{2^x+1} + c = x - \frac{\ln(2^x+1)}{\ln 2} + c$$

$$\begin{aligned} &= \left[\text{Subst. } t=2^x+1, 2^x=t-1 \mid x \in R, t \in (1; \infty) \right] = \int \frac{\frac{dt}{(t-1) \ln 2}}{t} = \frac{1}{\ln 2} \int \frac{dt}{t(t-1)} = \frac{1}{\ln 2} \int \frac{(t-t+1) dt}{t(t-1)} \\ &= \frac{1}{\ln 2} \int \left[\frac{1}{t-1} - \frac{1}{t} \right] dt = \frac{1}{\ln 2} [\ln(t-1) - \ln t] + c = \frac{1}{\ln 2} \ln \frac{t-1}{t} + c = \frac{1}{\ln 2} \ln \frac{2^x}{2^x+1} + c \\ &= \frac{\ln 2^x - \ln(2^x+1)}{\ln 2} + c = \frac{x \ln 2 - \ln(2^x+1)}{\ln 2} + c = x - \frac{\ln(2^x+1)}{\ln 2} + c, x \in R, c \in R. \end{aligned}$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1})}{\ln 2} - x + c$$

$$\begin{aligned} &= \left[\text{Subst. } t=\sqrt{2^x+1}, 2^x=t^2-1 \mid x \in R, t \in (1; \infty) \right] = \int \frac{\frac{2t dt}{(t^2-1) \ln 2}}{t} = \frac{2}{\ln 2} \int \frac{dt}{t^2-1} = \frac{2}{2 \ln 2} \ln \frac{t-1}{t+1} + c \\ &= \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c = \frac{1}{\ln 2} \ln \left[\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}-1} \right] + c = \frac{1}{\ln 2} \ln \frac{2^x+1-2\sqrt{2^x+1}+1}{2^x+1-1} + c \\ &= \frac{\ln(2^x+2-2\sqrt{2^x+1}) - \ln 2^x}{\ln 2} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1}) - x \ln 2}{\ln 2} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1})}{\ln 2} - x + c, \\ &\quad x \in R, c \in R. \end{aligned}$$

Koniec 10. časti príklady b – 161

$$\int \frac{(1+x) dx}{\sqrt{1-x^2}}$$

Koniec 10. časti príklady b – 161

$$\int \frac{(1+x) dx}{\sqrt{1-x^2}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}}$$

$$= \left[\text{Subst. } t = \frac{1+x}{\sqrt{1-x^2}} = \frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x} - 1} \mid x \in (-1; 1) \right. \\ \left. t^2 = \frac{1+x}{1-x}, t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right]$$

Koniec 10. časti príklady b – 161

$$\int \frac{(1+x) dx}{\sqrt{1-x^2}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}} = \left[\begin{array}{l} \text{Subst. } t=1-x^2 \mid x \in (-1; 0), t \in (0; 1) \\ dt = -2x dx \mid x \in (0; 1), t \in (0; 1) \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1+x}{\sqrt{1-x^2}} = \frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}-1} \mid x \in (-1; 1) \\ t^2 = \frac{1+x}{1-x}, t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \end{array} \right] = \int \frac{4t^2 dt}{(t^2+1)^2}$$

Koniec 10. časti príklady b – 161

$$\int \frac{(1+x) dx}{\sqrt{1-x^2}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}} \left[\begin{array}{l} \text{Subst. } t=1-x^2 \quad x \in (-1; 0), t \in (0; 1) \\ \quad \quad \quad dt = -2x dx \quad x \in (0; 1), t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int \frac{dt}{\sqrt{t}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int t^{-\frac{1}{2}} dt$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1+x}{\sqrt{1-x^2}} = \frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}-1} \quad x \in (-1; 1) \\ t^2 = \frac{1+x}{1-x}, t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \quad t \in (0; \infty) \end{array} \right] = \int \frac{4t^2 dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = 2t \quad | \quad u' = 2 \\ v' = \frac{2t}{(t^2+1)^2} \quad | \quad v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right]$$

Koniec 10. časti príklady b – 161

$$\int \frac{(1+x) dx}{\sqrt{1-x^2}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}} = \left[\begin{array}{l} \text{Subst. } t=1-x^2 \quad x \in (-1; 0), t \in (0; 1) \\ dt = -2x dx \quad x \in (0; 1), t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int \frac{dt}{\sqrt{t}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int t^{-\frac{1}{2}} dt = \arcsin x - \frac{1}{2} t^{\frac{1}{2}} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1+x}{1-x} = \frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}-1} \quad x \in (-1; 1) \\ t^2 = \frac{1+x}{1-x}, t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \quad t \in (0; \infty) \end{array} \right] = \int \frac{4t^2 dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = 2t \quad u' = 2 \\ v' = \frac{2t}{(t^2+1)^2} \quad v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2 dt}{t^2+1}$$

Koniec 10. časti príklady b – 161

$$\int \frac{(1+x) dx}{\sqrt{1-x^2}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}} = \left[\text{Subst. } t=1-x^2 \begin{array}{l} x \in (-1; 0), t \in (0; 1) \\ dt = -2x dx \quad x \in (0; 1), t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int \frac{dt}{\sqrt{t}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int t^{-\frac{1}{2}} dt = \arcsin x - \frac{1}{2} \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = \arcsin x - \sqrt{t} + c$$

$$= \left[\text{Subst. } t = \frac{1+x}{\sqrt{1-x^2}} = \frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}-1} \begin{array}{l} x \in (-1; 1) \\ t^2 = \frac{1+x}{1-x}, t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, dx = \frac{2t(t^2+1)-(t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \quad t \in (0; \infty) \end{array} \right] = \int \frac{4t^2 dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = 2t \\ v' = \frac{2t}{(t^2+1)^2} \end{array} \begin{array}{l} u' = 2 \\ v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2 dt}{t^2+1} = -\frac{2t}{t^2+1} + 2 \operatorname{arctg} t + c$$

Koniec 10. časti príklady b – 161

$$\int \frac{(1+x) dx}{\sqrt{1-x^2}} = \arcsin x - \sqrt{1-x^2} + c$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}} = \left[\text{Subst. } t=1-x^2 \begin{array}{l} x \in (-1; 0), t \in (0; 1) \\ dt = -2x dx \quad x \in (0; 1), t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int \frac{dt}{\sqrt{t}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int t^{-\frac{1}{2}} dt = \arcsin x - \frac{1}{2} \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = \arcsin x - \sqrt{t} + c$$

$$= \arcsin x - \sqrt{1-x^2} + c, x \in (-1; 1), c \in R.$$

$$= \left[\text{Subst. } t = \frac{1+x}{\sqrt{1-x^2}} = \frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}-1} \begin{array}{l} x \in (-1; 1) \\ t^2 = \frac{1+x}{1-x}, t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, dx = \frac{2t(t^2+1)-(t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \quad t \in (0; \infty) \end{array} \right] = \int \frac{4t^2 dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = 2t \\ v' = \frac{2t}{(t^2+1)^2} \end{array} \begin{array}{l} u' = 2 \\ v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2 dt}{t^2+1} = -\frac{2t}{t^2+1} + 2 \arctg t + c$$

$$= -\frac{2 \frac{1+x}{\sqrt{1-x^2}}}{\frac{1+x}{1-x}+1} + 2 \arctg \frac{1+x}{\sqrt{1-x^2}} + c$$

Koniec 10. časti príklady b – 161

$$\int \frac{(1+x) dx}{\sqrt{1-x^2}} = \arcsin x - \sqrt{1-x^2} + c$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}} = \left[\begin{array}{l} \text{Subst. } t=1-x^2 \mid x \in (-1; 0), t \in (0; 1) \\ dt = -2x dx \mid x \in (0; 1), t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int \frac{dt}{\sqrt{t}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int t^{-\frac{1}{2}} dt = \arcsin x - \frac{1}{2} \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = \arcsin x - \sqrt{t} + c$$

$$= \arcsin x - \sqrt{1-x^2} + c, x \in (-1; 1), c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1+x}{\sqrt{1-x^2}} = \frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}-1} \mid x \in (-1; 1) \\ t^2 = \frac{1+x}{1-x}, t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, dx = \frac{2t(t^2+1)-(t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \end{array} \right] = \int \frac{4t^2 dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = 2t \mid u' = 2 \\ v' = \frac{2t}{(t^2+1)^2} \mid v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2 dt}{t^2+1} = -\frac{2t}{t^2+1} + 2 \arctg t + c$$

$$= -\frac{2 \frac{1+x}{\sqrt{1-x^2}}}{\frac{1+x}{1-x} + 1} + 2 \arctg \frac{1+x}{\sqrt{1-x^2}} + c = \left[\frac{2 \frac{1+x}{\sqrt{1-x^2}}}{\frac{1+x}{1-x} + 1} = \frac{\frac{2(1+x)}{\sqrt{1-x^2}}}{\frac{1+x+1-x}{1-x}} = \frac{\frac{2(1+x)}{\sqrt{1-x^2}}}{\frac{2}{1-x}} = \frac{(1+x)(1-x)}{\sqrt{1-x^2}} = \frac{1-x^2}{\sqrt{1-x^2}} = \sqrt{1-x^2} \right]$$

Koniec 10. časti príklady b – 161

$$\int \frac{(1+x) dx}{\sqrt{1-x^2}} = \arcsin x - \sqrt{1-x^2} + c = 2 \operatorname{arctg} \frac{1+x}{\sqrt{1-x^2}} - \sqrt{1-x^2} + c$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}} = \left[\begin{array}{l} \text{Subst. } t=1-x^2 \mid x \in (-1; 0), t \in (0; 1) \\ dt = -2x dx \mid x \in (0; 1), t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int \frac{dt}{\sqrt{t}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int t^{-\frac{1}{2}} dt = \arcsin x - \frac{1}{2} \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = \arcsin x - \sqrt{t} + c$$

$$= \arcsin x - \sqrt{1-x^2} + c, x \in (-1; 1), c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1+x}{\sqrt{1-x^2}} = \frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}-1} \mid x \in (-1; 1) \\ t^2 = \frac{1+x}{1-x}, t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, dx = \frac{2t(t^2+1)-(t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \end{array} \right] = \int \frac{4t^2 dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = 2t \mid u' = 2 \\ v' = \frac{2t}{(t^2+1)^2} \mid v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2 dt}{t^2+1} = -\frac{2t}{t^2+1} + 2 \operatorname{arctg} t + c$$

$$= -\frac{2 \frac{1+x}{\sqrt{1-x^2}}}{\frac{1+x}{1-x} + 1} + 2 \operatorname{arctg} \frac{1+x}{\sqrt{1-x^2}} + c = \left[\frac{2 \frac{1+x}{\sqrt{1-x^2}}}{\frac{1+x}{1-x} + 1} = \frac{\frac{2(1+x)}{\sqrt{1-x^2}}}{\frac{1+x+1-x}{1-x}} = \frac{\frac{2(1+x)}{\sqrt{1-x^2}}}{\frac{2}{1-x}} = \frac{(1+x)(1-x)}{\sqrt{1-x^2}} = \frac{1-x^2}{\sqrt{1-x^2}} = \sqrt{1-x^2} \right]$$

$$= 2 \operatorname{arctg} \frac{1+x}{\sqrt{1-x^2}} - \sqrt{1-x^2} + c, x \in (-1; 1), c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x} - 1}, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right. \\ \left. t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right]$$

$$= \left[\sqrt{\frac{1+x}{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \frac{1+x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right]$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right. \\ \left. t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{4t^2 dt}{(t^2+1)^2}$$

$$= \left[\sqrt{\frac{1+x}{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \frac{1+x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1+x) dx}{\sqrt{1-x^2}}$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x} - 1}, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right] = \int \frac{4t^2 dt}{(t^2+1)^2}$$

$$t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty)$$

$$= \left[\begin{array}{l} u = 2t \\ v' = \frac{2t}{(t^2+1)^2} \end{array} \mid \begin{array}{l} u' = 2 \\ v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right]$$

$$= \left[\sqrt{\frac{1+x}{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \frac{1+x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1+x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}}$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \\ t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \end{array} \right] = \int \frac{4t^2 dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = 2t \mid u' = 2 \\ v' = \frac{2t}{(t^2+1)^2} \mid v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2 dt}{t^2+1}$$

$$= \left[\sqrt{\frac{1+x}{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \frac{1+x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1+x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1-x^2 \mid x \in (-1; 0), t \in (0; 1) \\ dt = -2x dx \mid x \in (0; 1), t \in (0; 1) \end{array} \right]$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \\ t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \end{array} \right] = \int \frac{4t^2 dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = 2t \mid u' = 2 \\ v' = \frac{2t}{(t^2+1)^2} \mid v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2 dt}{t^2+1} = -\frac{2t}{t^2+1} + 2 \operatorname{arctg} t + c_1$$

$$= \left[\sqrt{\frac{1+x}{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \frac{1+x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1+x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1-x^2 \mid x \in (-1; 0), t \in (0; 1) \\ dt = -2x dx \mid x \in (0; 1), t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int \frac{dt}{\sqrt{t}}$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right] \\
 &\quad \left[t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{4t^2 dt}{(t^2+1)^2} \\
 &= \left[\begin{array}{l} u = 2t \\ v' = \frac{2t}{(t^2+1)^2} \end{array} \mid \begin{array}{l} u' = 2 \\ v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2 dt}{t^2+1} = -\frac{2t}{t^2+1} + 2 \operatorname{arctg} t + c_1 \\
 &= \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x} + 1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x+1-x}{1-x}} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{2}{1-x}} = \sqrt{\frac{1+x}{1-x}} \cdot (1-x) = \sqrt{(1+x)(1-x)} = \sqrt{1-x^2} \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \left[\sqrt{\frac{1+x}{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \frac{1+x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1+x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}} \\
 &= \left[\text{Subst. } t = 1-x^2 \mid x \in (-1; 0), t \in (0; 1) \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int t^{-\frac{1}{2}} dt
 \end{aligned}$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx = 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1$$

$$\begin{aligned} &= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right] = \int \frac{4t^2 dt}{(t^2+1)^2} \\ &\quad t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \\ &= \left[\begin{array}{l} u = 2t \\ v' = \frac{2t}{(t^2+1)^2} \end{array} \mid \begin{array}{l} u' = 2 \\ v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2 dt}{t^2+1} = -\frac{2t}{t^2+1} + 2 \operatorname{arctg} t + c_1 \\ &= \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x} + 1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x+1-x}{1-x}} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{2}{1-x}} = \sqrt{\frac{1+x}{1-x}} \cdot (1-x) = \sqrt{(1+x)(1-x)} = \sqrt{1-x^2} \right] \\ &= 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1, \quad x \in (-1; 1), \quad c_1 \in \mathbb{R}. \end{aligned}$$

$$\begin{aligned} &= \left[\sqrt{\frac{1+x}{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \frac{1+x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1+x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}} \\ &= \left[\begin{array}{l} \text{Subst. } t = 1-x^2 \mid x \in (-1; 0), t \in (0; 1) \\ dt = -2x dx \mid x \in (0; 1), t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int t^{-\frac{1}{2}} dt \\ &= \arcsin x - \frac{1}{2} t^{\frac{1}{2}} + c_2 \end{aligned}$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx = 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1$$

$$\begin{aligned} &= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right] = \int \frac{4t^2 dt}{(t^2+1)^2} \\ &\quad t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \\ &= \left[\begin{array}{l} u = 2t \\ v' = \frac{2t}{(t^2+1)^2} \end{array} \mid \begin{array}{l} u' = 2 \\ v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2 dt}{t^2+1} = -\frac{2t}{t^2+1} + 2 \operatorname{arctg} t + c_1 \\ &= \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x} + 1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x+1-x}{1-x}} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{2}{1-x}} = \sqrt{\frac{1+x}{1-x}} \cdot (1-x) = \sqrt{(1+x)(1-x)} = \sqrt{1-x^2} \right] \\ &= 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1, \quad x \in (-1; 1), \quad c_1 \in \mathbb{R}. \end{aligned}$$

$$\begin{aligned} &= \left[\sqrt{\frac{1+x}{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \frac{1+x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1+x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}} \\ &= \left[\text{Subst. } t = 1-x^2 \mid x \in (-1; 0), t \in (0; 1) \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int t^{-\frac{1}{2}} dt \\ &\quad dt = -2x dx \mid x \in (0; 1), t \in (0; 1) \\ &= \arcsin x - \frac{1}{2} t^{\frac{1}{2}} + c_2 = \arcsin x - \sqrt{t} + c_2 \end{aligned}$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx = 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1 = \arcsin x - \sqrt{1-x^2} + c_2$$

$$\begin{aligned} &= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right] = \int \frac{4t^2 dt}{(t^2+1)^2} \\ & \quad t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \\ &= \left[\begin{array}{l} u = 2t \\ v' = \frac{2t}{(t^2+1)^2} \end{array} \mid \begin{array}{l} u' = 2 \\ v = -\frac{1}{t^2+1} = -(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2 dt}{t^2+1} = -\frac{2t}{t^2+1} + 2 \operatorname{arctg} t + c_1 \\ &= \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x} + 1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x+1-x}{1-x}} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{2}{1-x}} = \sqrt{\frac{1+x}{1-x}} \cdot (1-x) = \sqrt{(1+x)(1-x)} = \sqrt{1-x^2} \right] \\ &= 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1, \quad x \in (-1; 1), \quad c_1 \in \mathbb{R}. \end{aligned}$$

$$\begin{aligned} &= \left[\sqrt{\frac{1+x}{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \frac{1+x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1+x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}} \\ &= \left[\text{Subst. } t = 1-x^2 \mid x \in (-1; 0), t \in (0; 1) \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int t^{-\frac{1}{2}} dt \\ & \quad dt = -2x dx \mid x \in (0; 1), t \in (0; 1) \\ &= \arcsin x - \frac{1}{2} t^{\frac{1}{2}} + c_2 = \arcsin x - \sqrt{t} + c_2 = \arcsin x - \sqrt{1-x^2} + c_2, \\ & \quad x \in (-1; 1), \quad c_2 \in \mathbb{R}. \end{aligned}$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=-x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=-x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right]$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=-x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1-t}{1+t}} dt$$

$$= \left[\begin{array}{l} \text{Subst. } t=-x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1-t}{1+t}} dt$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1-t}{1+t}} dt$$

$$= \left[\text{Pr. 163: } \int \sqrt{\frac{1-x}{1+x}} dx = \sqrt{1-x^2} - 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + c_1, x \in (-1; 1) \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1-t}{1+t}} dt$$

$$= \left[\text{Pr. 163: } \int \sqrt{\frac{1-x}{1+x}} dx = \arcsin x + \sqrt{1-x^2} + c_2, x \in (-1; 1) \right]$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1-t}{1+t}} dt$$

$$= \left[\text{Pr. 163: } \int \sqrt{\frac{1-x}{1+x}} dx = \sqrt{1-x^2} - 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + c_1, x \in (-1; 1) \right]$$

$$= - \left[\sqrt{1-t^2} - 2 \operatorname{arctg} \sqrt{\frac{1-t}{1+t}} \right] + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1-t}{1+t}} dt$$

$$= \left[\text{Pr. 163: } \int \sqrt{\frac{1-x}{1+x}} dx = \arcsin x + \sqrt{1-x^2} + c_2, x \in (-1; 1) \right] = - \left[\arcsin t + \sqrt{1-t^2} \right] + c_2$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1-t}{1+t}} dt$$

$$= \left[\text{Pr. 163: } \int \sqrt{\frac{1-x}{1+x}} dx = \sqrt{1-x^2} - 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + c_1, x \in (-1; 1) \right]$$

$$= - \left[\sqrt{1-t^2} - 2 \operatorname{arctg} \sqrt{\frac{1-t}{1+t}} \right] + c_1 = 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-(-x)^2} + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1-t}{1+t}} dt$$

$$= \left[\text{Pr. 163: } \int \sqrt{\frac{1-x}{1+x}} dx = \arcsin x + \sqrt{1-x^2} + c_2, x \in (-1; 1) \right] = - \left[\arcsin t + \sqrt{1-t^2} \right] + c_2$$

$$= - \arcsin(-x) - \sqrt{1-(-x)^2} + c_2$$

Koniec 10. časti príklady b – 162

$$\int \sqrt{\frac{1+x}{1-x}} dx = 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} - \sqrt{1-x^2} + c_1 = \arcsin x - \sqrt{1-x^2} + c_2$$

$$= \left[\begin{array}{l} \text{Subst. } t=-x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1-t}{1+t}} dt$$

$$= \left[\text{Pr. 163: } \int \sqrt{\frac{1-x}{1+x}} dx = \sqrt{1-x^2} - 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + c_1, x \in (-1; 1) \right]$$

$$= - \left[\sqrt{1-t^2} - 2 \operatorname{arctg} \sqrt{\frac{1-t}{1+t}} \right] + c_1 = 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-(-x)^2} + c_1$$

$$= 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1, x \in (-1; 1), c_1 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=-x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1-t}{1+t}} dt$$

$$= \left[\text{Pr. 163: } \int \sqrt{\frac{1-x}{1+x}} dx = \arcsin x + \sqrt{1-x^2} + c_2, x \in (-1; 1) \right] = - \left[\arcsin t + \sqrt{1-t^2} \right] + c_2$$

$$= - \arcsin(-x) - \sqrt{1-(-x)^2} + c_2 = \arcsin x - \sqrt{1-x^2} + c_2,$$

$$x \in (-1; 1), c_2 \in \mathbb{R}.$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \quad \left| x \in (-1; 1) \right. \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \quad \left| t \in (0; \infty) \right. \end{array} \right]$$

$$= \left[\sqrt{\frac{1-x}{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} = \frac{1-x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right]$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \mid x \in (-1; 1) \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \end{array} \right] = \int \frac{-4t^2 dt}{(t^2+1)^2}$$

$$= \left[\sqrt{\frac{1-x}{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} = \frac{1-x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1-x) dx}{\sqrt{1-x^2}}$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \mid x \in (-1; 1) \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \end{array} \right] = \int \frac{-4t^2 dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = 2t \\ v' = -\frac{2t}{(t^2+1)^2} \mid u' = 2 \\ v = \frac{1}{t^2+1} = (t^2+1)^{-1} \end{array} \right]$$

$$= \left[\sqrt{\frac{1-x}{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} = \frac{1-x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1-x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x dx}{\sqrt{1-x^2}}$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \mid x \in (-1; 1) \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \end{array} \right] = \int \frac{-4t^2 dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = 2t \\ v' = -\frac{2t}{(t^2+1)^2} \mid u' = 2 \\ v = \frac{1}{t^2+1} = (t^2+1)^{-1} \end{array} \right] = \frac{2t}{t^2+1} - \int \frac{2 dt}{t^2+1}$$

$$= \left[\sqrt{\frac{1-x}{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} = \frac{1-x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1-x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x dx}{\sqrt{1-x^2}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1-x^2 \mid x \in (-1; 0), t \in (0; 1) \\ dt = -2x dx \mid x \in (0; 1), t \in (0; 1) \end{array} \right]$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \mid x \in (-1; 1) \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \end{array} \right] = \int \frac{-4t^2 dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = 2t \\ v' = -\frac{2t}{(t^2+1)^2} \mid u' = 2 \\ v = \frac{1}{t^2+1} = (t^2+1)^{-1} \end{array} \right] = \frac{2t}{t^2+1} - \int \frac{2 dt}{t^2+1} = \frac{2t}{t^2+1} - 2 \operatorname{arctg} t + c_1$$

$$= \left[\sqrt{\frac{1-x}{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} = \frac{1-x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1-x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x dx}{\sqrt{1-x^2}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1-x^2 \mid x \in (-1; 0), t \in (0; 1) \\ dt = -2x dx \mid x \in (0; 1), t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} + \frac{1}{2} \int \frac{dt}{\sqrt{t}}$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \mid x \in (-1; 1) \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \end{array} \right] = \int \frac{-4t^2 dt}{(t^2+1)^2} \\
 &= \left[\begin{array}{l} u = 2t \\ v' = -\frac{2t}{(t^2+1)^2} \mid u' = 2 \\ v = \frac{1}{t^2+1} = (t^2+1)^{-1} \end{array} \right] = \frac{2t}{t^2+1} - \int \frac{2 dt}{t^2+1} = \frac{2t}{t^2+1} - 2 \operatorname{arctg} t + c_1 \\
 &= \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x}{1+x} + 1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x+1+x}{1+x}} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{2}{1+x}} = \sqrt{\frac{1-x}{1+x}} \cdot (1+x) = \sqrt{(1-x)(1+x)} = \sqrt{1-x^2} \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \left[\sqrt{\frac{1-x}{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} = \frac{1-x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1-x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x dx}{\sqrt{1-x^2}} \\
 &= \left[\begin{array}{l} \text{Subst. } t = 1-x^2 \mid x \in (-1; 0), t \in (0; 1) \\ dt = -2x dx \mid x \in (0; 1), t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} + \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \int \frac{dx}{\sqrt{1-x^2}} + \frac{1}{2} \int t^{-\frac{1}{2}} dt
 \end{aligned}$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx = \sqrt{1-x^2} - 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + c_1$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x}-1}, \quad t^2 = \frac{1-x}{1+x} \quad x \in (-1; 1) \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \quad t \in (0; \infty) \end{array} \right] = \int \frac{-4t^2 dt}{(t^2+1)^2} \\ &= \left[\begin{array}{l} u = 2t \\ v' = -\frac{2t}{(t^2+1)^2} \quad u' = 2 \\ v = \frac{1}{t^2+1} = (t^2+1)^{-1} \end{array} \right] = \frac{2t}{t^2+1} - \int \frac{2 dt}{t^2+1} = \frac{2t}{t^2+1} - 2 \operatorname{arctg} t + c_1 \\ &= \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x}{1+x}+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x+1+x}{1+x}} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{2}{1+x}} = \sqrt{\frac{1-x}{1+x}} \cdot (1+x) = \sqrt{(1-x)(1+x)} = \sqrt{1-x^2} \right] \\ &= \sqrt{1-x^2} - 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + c_1, \quad x \in (-1; 1), \quad c_1 \in \mathbb{R}. \end{aligned}$$

$$\begin{aligned} &= \left[\sqrt{\frac{1-x}{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} = \frac{1-x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1-x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x dx}{\sqrt{1-x^2}} \\ &= \left[\begin{array}{l} \text{Subst. } t = 1-x^2 \quad x \in (-1; 0), \quad t \in (0; 1) \\ dt = -2x dx \quad x \in (0; 1), \quad t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} + \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \int \frac{dx}{\sqrt{1-x^2}} + \frac{1}{2} \int t^{-\frac{1}{2}} dt \\ &= \arcsin x + \frac{1}{2} t^{\frac{1}{2}} + c_2 \end{aligned}$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx = \sqrt{1-x^2} - 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + c_1$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x}-1}, \quad t^2 = \frac{1-x}{1+x} \quad x \in (-1; 1) \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1)-(1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \quad t \in (0; \infty) \end{array} \right] = \int \frac{-4t^2 dt}{(t^2+1)^2} \\ &= \left[\begin{array}{l} u = 2t \\ v' = -\frac{2t}{(t^2+1)^2} \quad u' = 2 \\ v = \frac{1}{t^2+1} = (t^2+1)^{-1} \end{array} \right] = \frac{2t}{t^2+1} - \int \frac{2 dt}{t^2+1} = \frac{2t}{t^2+1} - 2 \operatorname{arctg} t + c_1 \\ &= \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x}{1+x}+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x+1+x}{1+x}} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{2}{1+x}} = \sqrt{\frac{1-x}{1+x}} \cdot (1+x) = \sqrt{(1-x)(1+x)} = \sqrt{1-x^2} \right] \\ &= \sqrt{1-x^2} - 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + c_1, \quad x \in (-1; 1), \quad c_1 \in \mathbb{R}. \end{aligned}$$

$$\begin{aligned} &= \left[\sqrt{\frac{1-x}{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} = \frac{1-x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1-x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x dx}{\sqrt{1-x^2}} \\ &= \left[\begin{array}{l} \text{Subst. } t = 1-x^2 \quad x \in (-1; 0), \quad t \in (0; 1) \\ dt = -2x dx \quad x \in (0; 1), \quad t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} + \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \int \frac{dx}{\sqrt{1-x^2}} + \frac{1}{2} \int t^{-\frac{1}{2}} dt \\ &= \arcsin x + \frac{1}{2} t^{\frac{1}{2}} + c_2 = \arcsin x + \sqrt{t} + c_2 \end{aligned}$$

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$$\int \sqrt{\frac{1-x}{1+x}} dx = \sqrt{1-x^2} - 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + c_1 = \arcsin x + \sqrt{1-x^2} + c_2$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x}-1}, \quad t^2 = \frac{1-x}{1+x} \quad x \in (-1; 1) \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \quad t \in (0; \infty) \end{array} \right] = \int \frac{-4t^2 dt}{(t^2+1)^2} \\ &= \left[\begin{array}{l} u = 2t \\ v' = -\frac{2t}{(t^2+1)^2} \end{array} \right] \begin{array}{l} u' = 2 \\ v = \frac{1}{t^2+1} = (t^2+1)^{-1} \end{array} \right] = \frac{2t}{t^2+1} - \int \frac{2 dt}{t^2+1} = \frac{2t}{t^2+1} - 2 \operatorname{arctg} t + c_1 \\ &= \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x}{1+x}+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x+1+x}{1+x}} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{2}{1+x}} = \sqrt{\frac{1-x}{1+x}} \cdot (1+x) = \sqrt{(1-x)(1+x)} = \sqrt{1-x^2} \right] \\ &= \sqrt{1-x^2} - 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + c_1, \quad x \in (-1; 1), \quad c_1 \in \mathbb{R}. \end{aligned}$$

$$\begin{aligned} &= \left[\sqrt{\frac{1-x}{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} = \frac{1-x}{\sqrt{1-x^2}}, \quad x \in (-1; 1) \right] = \int \frac{(1-x) dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x dx}{\sqrt{1-x^2}} \\ &= \left[\begin{array}{l} \text{Subst. } t = 1-x^2 \quad x \in (-1; 0), \quad t \in (0; 1) \\ dt = -2x dx \quad x \in (0; 1), \quad t \in (0; 1) \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} + \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \int \frac{dx}{\sqrt{1-x^2}} + \frac{1}{2} \int t^{-\frac{1}{2}} dt \\ &= \arcsin x + \frac{1}{2} t^{\frac{1}{2}} + c_2 = \arcsin x + \sqrt{t} + c_2 = \arcsin x + \sqrt{1-x^2} + c_2, \\ &\quad x \in (-1; 1), \quad c_2 \in \mathbb{R}. \end{aligned}$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=-x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=-x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right]$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=-x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1+t}{1-t}} dt$$

$$= \left[\begin{array}{l} \text{Subst. } t=-x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1+t}{1-t}} dt$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1+t}{1-t}} dt$$

$$= \left[\text{Pr. 162: } \int \sqrt{\frac{1+x}{1-x}} dx = 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1, x \in (-1; 1) \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1+t}{1-t}} dt$$

$$= \left[\text{Pr. 162: } \int \sqrt{\frac{1+x}{1-x}} dx = \arcsin x - \sqrt{1-x^2} + c_2, x \in (-1; 1) \right]$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1+t}{1-t}} dt$$

$$= \left[\text{Pr. 162: } \int \sqrt{\frac{1+x}{1-x}} dx = 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1, x \in (-1; 1) \right]$$

$$= - \left[2 \operatorname{arctg} \sqrt{\frac{1+t}{1-t}} - \sqrt{1-t^2} \right] + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1+t}{1-t}} dt$$

$$= \left[\text{Pr. 162: } \int \sqrt{\frac{1+x}{1-x}} dx = \arcsin x - \sqrt{1-x^2} + c_2, x \in (-1; 1) \right] = - \left[\arcsin t - \sqrt{1-t^2} \right] + c_2$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1+t}{1-t}} dt$$

$$= \left[\text{Pr. 162: } \int \sqrt{\frac{1+x}{1-x}} dx = 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1, x \in (-1; 1) \right]$$

$$= - \left[2 \operatorname{arctg} \sqrt{\frac{1+t}{1-t}} - \sqrt{1-t^2} \right] + c_1 = -2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + \sqrt{1-(-x)^2} + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1+t}{1-t}} dt$$

$$= \left[\text{Pr. 162: } \int \sqrt{\frac{1+x}{1-x}} dx = \arcsin x - \sqrt{1-x^2} + c_2, x \in (-1; 1) \right] = - \left[\arcsin t - \sqrt{1-t^2} \right] + c_2$$

$$= - \arcsin(-x) + \sqrt{1-(-x)^2} + c_2$$

Koniec 10. časti príklady b – 163

$$\int \sqrt{\frac{1-x}{1+x}} dx = -2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + \sqrt{1-x^2} + c_1 = \arcsin x + \sqrt{1-x^2} + c_2$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1+t}{1-t}} dt$$

$$= \left[\text{Pr. 162: } \int \sqrt{\frac{1+x}{1-x}} dx = 2 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1, x \in (-1; 1) \right]$$

$$= - \left[2 \operatorname{arctg} \sqrt{\frac{1+t}{1-t}} - \sqrt{1-t^2} \right] + c_1 = -2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + \sqrt{1-(-x)^2} + c_1$$

$$= -2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + \sqrt{1-x^2} + c_1, x \in (-1; 1), c_1 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = -x \mid x \in (-1; 1) \\ dt = -dx \mid t \in (-1; 1) \end{array} \right] = - \int \sqrt{\frac{1+t}{1-t}} dt$$

$$= \left[\text{Pr. 162: } \int \sqrt{\frac{1+x}{1-x}} dx = \arcsin x - \sqrt{1-x^2} + c_2, x \in (-1; 1) \right] = - \left[\arcsin t - \sqrt{1-t^2} \right] + c_2$$

$$= - \arcsin(-x) + \sqrt{1-(-x)^2} + c_2 = \arcsin x + \sqrt{1-x^2} + c_2,$$

$$x \in (-1; 1), c_2 \in \mathbb{R}.$$

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$$\int \sqrt{\frac{x+1}{x-1}} dx$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ t^2(x-1) = x+1, x = \frac{t^2+1}{t^2-1}, dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (1; \infty) \end{array} \right. \right. \right]$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ t^2(x-1) = x+1, x = \frac{t^2+1}{t^2-1}, dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (1; \infty) \end{array} \right. \right] = \int \frac{-4t^2 dt}{(t^2-1)^2}$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ t^2(x-1) = x+1, x = \frac{t^2+1}{t^2-1}, dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (1; \infty) \end{array} \right. \right] = \int \frac{-4t^2 dt}{(t^2-1)^2} \right.$$

$$= \left[\begin{array}{l} u = 2t \\ v' = \frac{-2t}{(t^2-1)^2} \end{array} \quad \left| \begin{array}{l} u' = 2 \\ v = \frac{1}{t^2-1} = (t^2-1)^{-1} \end{array} \right. \right]$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ t^2(x-1) = x+1, x = \frac{t^2+1}{t^2-1}, dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (1; \infty) \end{array} \right. \right] = \int \frac{-4t^2 dt}{(t^2-1)^2} \right.$$

$$= \left[\begin{array}{l} u = 2t \\ v' = \frac{-2t}{(t^2-1)^2} \end{array} \quad \left| \begin{array}{l} u' = 2 \\ v = \frac{1}{t^2-1} = (t^2-1)^{-1} \end{array} \right. \right] = \frac{2t}{t^2-1} - \int \frac{2 dt}{t^2-1}$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ t^2(x-1) = x+1, x = \frac{t^2+1}{t^2-1}, dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (1; \infty) \end{array} \right. \right] = \int \frac{-4t^2 dt}{(t^2-1)^2} \right.$$

$$= \left[\begin{array}{l} u = 2t \\ v' = \frac{-2t}{(t^2-1)^2} \end{array} \quad \left| \quad \begin{array}{l} u' = 2 \\ v = \frac{1}{t^2-1} = (t^2-1)^{-1} \end{array} \right. \right] = \frac{2t}{t^2-1} - \int \frac{2 dt}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \mid x \in (-\infty; -1), t \in (0; 1) \right. \\
 &\quad \left. t^2(x-1) = x+1, \quad x = \frac{t^2+1}{t^2-1}, \quad dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (1; \infty) \right] = \int \frac{-4t^2 dt}{(t^2-1)^2} \\
 &= \left[u = 2t \mid u' = 2 \right. \\
 &\quad \left. v' = \frac{-2t}{(t^2-1)^2} \mid v = \frac{1}{t^2-1} = (t^2-1)^{-1} \right] = \frac{2t}{t^2-1} - \int \frac{2 dt}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c \\
 &= \frac{2t}{t^2-1} + \ln |t+1| - \ln |t-1| + c
 \end{aligned}$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \mid x \in (-\infty; -1), t \in (0; 1) \right] = \int \frac{-4t^2 dt}{(t^2-1)^2} \\
 &= \left[\begin{array}{l} u = 2t \mid u' = 2 \\ v' = \frac{-2t}{(t^2-1)^2} \mid v = \frac{1}{t^2-1} = (t^2-1)^{-1} \end{array} \right] = \frac{2t}{t^2-1} - \int \frac{2 dt}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c \\
 &= \frac{2t}{t^2-1} + \ln |t+1| - \ln |t-1| + c = \left[\frac{2t}{t^2-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}} = (x-1)\sqrt{\frac{x+1}{x-1}} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx = (x-1) \sqrt{\frac{x+1}{x-1}} + \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] - \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + c$$

$$= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ t^2(x-1) = x+1, x = \frac{t^2+1}{t^2-1}, dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (1; \infty) \end{array} \right. \right] = \int \frac{-4t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u = 2t \\ v' = \frac{-2t}{(t^2-1)^2} \end{array} \quad \left| \begin{array}{l} u' = 2 \\ v = \frac{1}{t^2-1} = (t^2-1)^{-1} \end{array} \right. \right] = \frac{2t}{t^2-1} - \int \frac{2 dt}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

$$= \frac{2t}{t^2-1} + \ln |t+1| - \ln |t-1| + c = \left[\frac{2t}{t^2-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1} - 1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}} = (x-1) \sqrt{\frac{x+1}{x-1}} \right]$$

$$= (x-1) \sqrt{\frac{x+1}{x-1}} + \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] - \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx = (x-1)\sqrt{\frac{x+1}{x-1}} + \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] - \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + c$$

$$= \left[\text{Subst. } t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \mid x \in (-\infty; -1), t \in (0; 1) \right] = \int \frac{-4t^2 dt}{(t^2-1)^2}$$

$$\left[t^2(x-1) = x+1, \quad x = \frac{t^2+1}{t^2-1}, \quad dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (1; \infty) \right]$$

$$= \left[u = 2t \mid u' = 2 \right. \\ \left. v' = \frac{-2t}{(t^2-1)^2} \mid v = \frac{1}{t^2-1} = (t^2-1)^{-1} \right] = \frac{2t}{t^2-1} - \int \frac{2 dt}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

$$= \frac{2t}{t^2-1} + \ln |t+1| - \ln |t-1| + c = \left[\frac{2t}{t^2-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1} - 1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x-1}{x-1}} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{-2}{x-1}} = (x-1)\sqrt{\frac{x+1}{x-1}} \right]$$

$$= (x-1)\sqrt{\frac{x+1}{x-1}} + \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] - \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

$$= (x-1) \frac{\sqrt{x+1}}{\sqrt{x-1}} + \ln \frac{\frac{\sqrt{x+1}}{\sqrt{x-1}} + 1}{\frac{\sqrt{x+1}}{\sqrt{x-1}} - 1} + c$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx = (x-1)\sqrt{\frac{x+1}{x-1}} + \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] - \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + c$$

$$= \left[\text{Subst. } t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \mid x \in (-\infty; -1), t \in (0; 1) \right] = \int \frac{-4t^2 dt}{(t^2-1)^2}$$

$$\left[t^2(x-1) = x+1, \quad x = \frac{t^2+1}{t^2-1}, \quad dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (1; \infty) \right]$$

$$= \left[u = 2t \mid u' = 2 \right. \\ \left. v' = \frac{-2t}{(t^2-1)^2} \mid v = \frac{1}{t^2-1} = (t^2-1)^{-1} \right] = \frac{2t}{t^2-1} - \int \frac{2 dt}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

$$= \frac{2t}{t^2-1} + \ln |t+1| - \ln |t-1| + c = \left[\frac{2t}{t^2-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1} - 1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}} = (x-1)\sqrt{\frac{x+1}{x-1}} \right]$$

$$= (x-1)\sqrt{\frac{x+1}{x-1}} + \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] - \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

$$= (x-1) \frac{\sqrt{x+1}}{\sqrt{x-1}} + \ln \frac{\frac{\sqrt{x+1}}{\sqrt{x-1}} + 1}{\frac{\sqrt{x+1}}{\sqrt{x-1}} - 1} + c = \sqrt{x^2-1} + \ln \left[\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} \cdot \frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} \right] + c$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx = (x-1)\sqrt{\frac{x+1}{x-1}} + \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] - \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + c$$

$$= \left[\text{Subst. } t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \mid x \in (-\infty; -1), t \in (0; 1) \right] = \int \frac{-4t^2 dt}{(t^2-1)^2}$$

$$= \left[t^2(x-1) = x+1, \quad x = \frac{t^2+1}{t^2-1}, \quad dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (1; \infty) \right]$$

$$= \left[u = 2t \mid u' = 2 \right. \\ \left. v' = \frac{-2t}{(t^2-1)^2} \mid v = \frac{1}{t^2-1} = (t^2-1)^{-1} \right] = \frac{2t}{t^2-1} - \int \frac{2 dt}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

$$= \frac{2t}{t^2-1} + \ln |t+1| - \ln |t-1| + c = \left[\frac{2t}{t^2-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}} = (x-1)\sqrt{\frac{x+1}{x-1}} \right]$$

$$= (x-1)\sqrt{\frac{x+1}{x-1}} + \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] - \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

$$= (x-1) \frac{\sqrt{\frac{x+1}{x-1}}}{\frac{\sqrt{\frac{x+1}{x-1}}}{\frac{\sqrt{\frac{x+1}{x-1}}}{\sqrt{x-1}} - 1}} + 1 + c = \sqrt{x^2-1} + \ln \left[\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} \cdot \frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} \right] + c$$

$$= \sqrt{x^2-1} + \ln \frac{x+1+2\sqrt{x^2-1}+x-1}{x+1-(x-1)} + c$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx = (x-1) \sqrt{\frac{x+1}{x-1}} + \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] - \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + c$$

$$= \left[\text{Subst. } t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \mid x \in (-\infty; -1), t \in (0; 1) \right] = \int \frac{-4t^2 dt}{(t^2-1)^2}$$

$$= \left[t^2(x-1) = x+1, \quad x = \frac{t^2+1}{t^2-1}, \quad dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (1; \infty) \right]$$

$$= \left[u = 2t \mid u' = 2 \right. \\ \left. v' = \frac{-2t}{(t^2-1)^2} \mid v = \frac{1}{t^2-1} = (t^2-1)^{-1} \right] = \frac{2t}{t^2-1} - \int \frac{2 dt}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

$$= \frac{2t}{t^2-1} + \ln |t+1| - \ln |t-1| + c = \left[\frac{2t}{t^2-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}} = (x-1) \sqrt{\frac{x+1}{x-1}} \right]$$

$$= (x-1) \sqrt{\frac{x+1}{x-1}} + \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] - \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

$$= (x-1) \frac{\sqrt{\frac{x+1}{x-1}}}{\frac{\sqrt{\frac{x+1}{x-1}}}{\frac{\sqrt{\frac{x+1}{x-1}}}{\sqrt{x-1}}}-1} + c = \sqrt{x^2-1} + \ln \left[\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} \cdot \frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}} \right] + c$$

$$= \sqrt{x^2-1} + \ln \frac{x+1+2\sqrt{x^2-1}+x-1}{x+1-(x-1)} + c = \sqrt{x^2-1} + \ln \frac{2x+2\sqrt{x^2-1}}{2} + c$$

Koniec 10. časti príklady b – 164

$$\int \sqrt{\frac{x+1}{x-1}} dx = (x-1)\sqrt{\frac{x+1}{x-1}} + \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] - \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + c$$

$$= \left[\text{Subst. } t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \mid x \in (-\infty; -1), t \in (0; 1) \right] = \int \frac{-4t^2 dt}{(t^2-1)^2}$$

$$= \left[t^2(x-1) = x+1, \quad x = \frac{t^2+1}{t^2-1}, \quad dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (1; \infty) \right]$$

$$= \left[u = 2t \mid u' = 2 \right. \\ \left. v' = \frac{-2t}{(t^2-1)^2} \mid v = \frac{1}{t^2-1} = (t^2-1)^{-1} \right] = \frac{2t}{t^2-1} - \int \frac{2 dt}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

$$= \frac{2t}{t^2-1} + \ln |t+1| - \ln |t-1| + c = \left[\frac{2t}{t^2-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}} = (x-1)\sqrt{\frac{x+1}{x-1}} \right]$$

$$= (x-1)\sqrt{\frac{x+1}{x-1}} + \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] - \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

$$= (x-1) \frac{\sqrt{\frac{x+1}{x-1}}}{\frac{\sqrt{\frac{x+1}{x-1}}}{\frac{\sqrt{x+1}}{\sqrt{x-1}}}-1} + c = \sqrt{x^2-1} + \ln \left[\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} \cdot \frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}} \right] + c$$

$$= \sqrt{x^2-1} + \ln \frac{x+1+2\sqrt{x^2-1}+x-1}{x+1-(x-1)} + c = \sqrt{x^2-1} + \ln \frac{2x+2\sqrt{x^2-1}}{2} + c$$

$$= \sqrt{x^2-1} + \ln [x + \sqrt{x^2-1}] + c, \quad x \in (1; \infty), c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \quad \left| \begin{array}{l} x \in (-\infty; -1), \quad t \in (1; \infty) \\ t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), \quad t \in (0; 1) \end{array} \right. \end{array} \right. \end{array} \right]$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \quad \left| \begin{array}{l} x \in (-\infty; -1), \quad t \in (1; \infty) \\ t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), \quad t \in (0; 1) \end{array} \right. \end{array} \right] = \int \frac{4t^2 dt}{(t^2-1)^2}$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (1; \infty) \\ t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (0; 1) \end{array} \right. \right] = \int \frac{4t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u = 2t \\ v' = \frac{2t}{(t^2-1)^2} \end{array} \quad \left| \quad \begin{array}{l} u' = 2 \\ v = -\frac{1}{t^2-1} = -(t^2-1)^{-1} \end{array} \right. \right]$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (1; \infty) \\ t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1) \cdot (-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (0; 1) \end{array} \right. \end{array} \right] = \int \frac{4t^2 dt}{(t^2-1)^2} \\ = \left[\begin{array}{l} u = 2t \quad \left| \quad u' = 2 \\ v' = \frac{2t}{(t^2-1)^2} \quad \left| \quad v = -\frac{1}{t^2-1} = -(t^2-1)^{-1} \end{array} \right] = -\frac{2t}{t^2-1} + \int \frac{2 dt}{t^2-1} \right]$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right] = \int \frac{4t^2 dt}{(t^2-1)^2} \\
 &\quad \left[t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right] \\
 &= \left[u = 2t \mid u' = 2 \right. \\
 &\quad \left. v' = \frac{2t}{(t^2-1)^2} \mid v = -\frac{1}{t^2-1} = -(t^2-1)^{-1} \right] = -\frac{2t}{t^2-1} + \int \frac{2 dt}{t^2-1} = -\frac{2t}{t^2-1} + \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c
 \end{aligned}$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right] = \int \frac{4t^2 dt}{(t^2-1)^2} \\
 &\quad \left[t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right] \\
 &= \left[u = 2t \mid u' = 2 \right. \\
 &\quad \left. v' = \frac{2t}{(t^2-1)^2} \mid v = -\frac{1}{t^2-1} = -(t^2-1)^{-1} \right] = -\frac{2t}{t^2-1} + \int \frac{2 dt}{t^2-1} = -\frac{2t}{t^2-1} + \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c \\
 &= -\frac{2t}{t^2-1} + \ln |t-1| - \ln |t+1| + c
 \end{aligned}$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right] = \int \frac{4t^2 dt}{(t^2-1)^2} \\
 &\quad \left[t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right] \\
 &= \left[u = 2t \mid u' = 2 \right. \\
 &\quad \left. v' = \frac{2t}{(t^2-1)^2} \mid v = -\frac{1}{t^2-1} = -(t^2-1)^{-1} \right] = -\frac{2t}{t^2-1} + \int \frac{2 dt}{t^2-1} = -\frac{2t}{t^2-1} + \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c \\
 &= -\frac{2t}{t^2-1} + \ln |t-1| - \ln |t+1| + c = \left[-\frac{2t}{t^2-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x+1}{x+1}} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}} = (x+1)\sqrt{\frac{x-1}{x+1}} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx = (x+1) \sqrt{\frac{x-1}{x+1}} + \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c$$

$$= \left[\text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right] = \int \frac{4t^2 dt}{(t^2-1)^2}$$

$$\left[t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right]$$

$$= \left[u = 2t \mid u' = 2 \right. \\ \left. v' = \frac{2t}{(t^2-1)^2} \mid v = -\frac{1}{t^2-1} = -(t^2-1)^{-1} \right] = -\frac{2t}{t^2-1} + \int \frac{2 dt}{t^2-1} = -\frac{2t}{t^2-1} + \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

$$= -\frac{2t}{t^2-1} + \ln |t-1| - \ln |t+1| + c = \left[-\frac{2t}{t^2-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x-1}{x+1}} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}} = (x+1) \sqrt{\frac{x-1}{x+1}} \right]$$

$$= (x+1) \sqrt{\frac{x-1}{x+1}} + \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx = (x+1)\sqrt{\frac{x-1}{x+1}} + \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (1; \infty) \\ t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (0; 1) \end{array} \right. \end{array} \right] = \int \frac{4t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u = 2t \quad \left| \quad u' = 2 \right. \\ v' = \frac{2t}{(t^2-1)^2} \quad \left| \quad v = -\frac{1}{t^2-1} = -(t^2-1)^{-1} \right. \end{array} \right] = -\frac{2t}{t^2-1} + \int \frac{2 dt}{t^2-1} = -\frac{2t}{t^2-1} + \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

$$= -\frac{2t}{t^2-1} + \ln |t-1| - \ln |t+1| + c = \left[-\frac{2t}{t^2-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x+1}{x+1}} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}} = (x+1)\sqrt{\frac{x-1}{x+1}} \right]$$

$$= (x+1)\sqrt{\frac{x-1}{x+1}} + \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

$$= (x+1) \frac{\sqrt{x-1}}{\sqrt{x+1}} + \ln \frac{1 - \frac{\sqrt{x-1}}{\sqrt{x+1}}}{1 + \frac{\sqrt{x-1}}{\sqrt{x+1}}} + c$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx = (x+1) \sqrt{\frac{x-1}{x+1}} + \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \quad x \in (-\infty; -1), t \in (1; \infty) \\ t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \quad x \in (1; \infty), t \in (0; 1) \end{array} \right] = \int \frac{4t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u = 2t \quad u' = 2 \\ v' = \frac{2t}{(t^2-1)^2} \quad v = -\frac{1}{t^2-1} = -(t^2-1)^{-1} \end{array} \right] = -\frac{2t}{t^2-1} + \int \frac{2 dt}{t^2-1} = -\frac{2t}{t^2-1} + \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

$$= -\frac{2t}{t^2-1} + \ln |t-1| - \ln |t+1| + c = \left[-\frac{2t}{t^2-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x+1}{x+1}} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}} = (x+1) \sqrt{\frac{x-1}{x+1}} \right]$$

$$= (x+1) \sqrt{\frac{x-1}{x+1}} + \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

$$= (x+1) \frac{\sqrt{x-1}}{\sqrt{x+1}} + \ln \frac{1 - \frac{\sqrt{x-1}}{\sqrt{x+1}}}{1 + \frac{\sqrt{x-1}}{\sqrt{x+1}}} + c = \sqrt{x^2-1} + \ln \left[\frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} \cdot \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} \right] + c$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx = (x+1) \sqrt{\frac{x-1}{x+1}} + \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c$$

$$= \left[\text{Subst. } t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right] = \int \frac{4t^2 dt}{(t^2-1)^2}$$

$$= \left[t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right]$$

$$= \left[u = 2t \mid u' = 2 \right. \\ \left. v' = \frac{2t}{(t^2-1)^2} \mid v = -\frac{1}{t^2-1} = -(t^2-1)^{-1} \right] = -\frac{2t}{t^2-1} + \int \frac{2 dt}{t^2-1} = -\frac{2t}{t^2-1} + \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

$$= -\frac{2t}{t^2-1} + \ln |t-1| - \ln |t+1| + c = \left[-\frac{2t}{t^2-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x+1}{x+1}} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}} = (x+1)\sqrt{\frac{x-1}{x+1}} \right]$$

$$= (x+1) \sqrt{\frac{x-1}{x+1}} + \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

$$= (x+1) \frac{\sqrt{x-1}}{\sqrt{x+1}} + \ln \frac{1 - \frac{\sqrt{x-1}}{\sqrt{x+1}}}{1 + \frac{\sqrt{x-1}}{\sqrt{x+1}}} + c = \sqrt{x^2-1} + \ln \left[\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}} \cdot \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} \right] + c$$

$$= \sqrt{x^2-1} + \ln \frac{x+1-2\sqrt{x^2-1}+x-1}{x+1-(x-1)} + c$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx = (x+1) \sqrt{\frac{x-1}{x+1}} + \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \\ t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \end{array} \right] = \int \frac{4t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u = 2t \mid u' = 2 \\ v' = \frac{2t}{(t^2-1)^2} \mid v = -\frac{1}{t^2-1} = -(t^2-1)^{-1} \end{array} \right] = -\frac{2t}{t^2-1} + \int \frac{2 dt}{t^2-1} = -\frac{2t}{t^2-1} + \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

$$= -\frac{2t}{t^2-1} + \ln |t-1| - \ln |t+1| + c = \left[-\frac{2t}{t^2-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x+1}{x+1}} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}} = (x+1) \sqrt{\frac{x-1}{x+1}} \right]$$

$$= (x+1) \sqrt{\frac{x-1}{x+1}} + \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

$$= (x+1) \frac{\sqrt{x-1}}{\sqrt{x+1}} + \ln \frac{1 - \frac{\sqrt{x-1}}{\sqrt{x+1}}}{1 + \frac{\sqrt{x-1}}{\sqrt{x+1}}} + c = \sqrt{x^2-1} + \ln \left[\frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} \cdot \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} \right] + c$$

$$= \sqrt{x^2-1} + \ln \frac{x+1-2\sqrt{x^2-1}+x-1}{x+1-(x-1)} + c = \sqrt{x^2-1} + \ln \frac{2x-2\sqrt{x^2-1}}{2} + c$$

Koniec 10. časti príklady b – 165

$$\int \sqrt{\frac{x-1}{x+1}} dx = (x+1)\sqrt{\frac{x-1}{x+1}} + \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (1; \infty) \\ t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (0; 1) \end{array} \right. \end{array} \right] = \int \frac{4t^2 dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l} u = 2t \quad \left| \quad u' = 2 \right. \\ v' = \frac{2t}{(t^2-1)^2} \quad \left| \quad v = -\frac{1}{t^2-1} = -(t^2-1)^{-1} \right. \end{array} \right] = -\frac{2t}{t^2-1} + \int \frac{2 dt}{t^2-1} = -\frac{2t}{t^2-1} + \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

$$= -\frac{2t}{t^2-1} + \ln |t-1| - \ln |t+1| + c = \left[-\frac{2t}{t^2-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x+1}{x+1}} = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}} = (x+1)\sqrt{\frac{x-1}{x+1}} \right]$$

$$= (x+1)\sqrt{\frac{x-1}{x+1}} + \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

$$= (x+1) \frac{\sqrt{x-1}}{\sqrt{x+1}} + \ln \frac{1 - \frac{\sqrt{x-1}}{\sqrt{x+1}}}{1 + \frac{\sqrt{x-1}}{\sqrt{x+1}}} + c = \sqrt{x^2-1} + \ln \left[\frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} \cdot \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} \right] + c$$

$$= \sqrt{x^2-1} + \ln \frac{x+1-2\sqrt{x^2-1}+x-1}{x+1-(x-1)} + c = \sqrt{x^2-1} + \ln \frac{2x-2\sqrt{x^2-1}}{2} + c$$

$$= \sqrt{x^2-1} + \ln [x - \sqrt{x^2-1}] + c, x \in (1; \infty), c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx$$

Koniec 10. časti príklady b – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right. \\ \left. t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right]$$

Koniec 10. časti príklady b – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right. \\ \left. t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2+1)^2}$$

Koniec 10. časti príklady b – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right. \\ \left. t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2+1)^2}$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4 + 2t^2 + 1 - 2t^2 - 2 + 1}{(t^2+1)^2} = \frac{(t^2+1)^2 - 2(t^2+1) + 1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right]$$

Koniec 10. časti príklady b – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x} - 1}, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right. \\ \left. t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2+1)^2}$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4 + 2t^2 + 1 - 2t^2 - 2 + 1}{(t^2+1)^2} = \frac{(t^2+1)^2 - 2(t^2+1) + 1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = 4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \int \frac{dt}{(t^2+1)^2}$$

Koniec 10. časti príklady b – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right. \\ \left. t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2+1)^2}$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4 + 2t^2 + 1 - 2t^2 - 2 + 1}{(t^2+1)^2} = \frac{(t^2+1)^2 - 2(t^2+1) + 1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = 4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right]$$

Koniec 10. časti príklady b – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right. \\ \left. t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2+1)^2}$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4 + 2t^2 + 1 - 2t^2 - 2 + 1}{(t^2+1)^2} = \frac{(t^2+1)^2 - 2(t^2+1) + 1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = 4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right]$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \left[\frac{2 \cdot 2 - 3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right]$$

Koniec 10. časti príklady b – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right. \\ \left. t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2+1)^2}$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = 4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right]$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \left[\frac{2 \cdot 2 - 3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right]$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 2 \int \frac{dt}{t^2+1} + \frac{2t}{t^2+1}$$

Koniec 10. časti príklady b – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right. \\ \left. t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2+1)^2}$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = 4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right]$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \left[\frac{2 \cdot 2 - 3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right]$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 2 \int \frac{dt}{t^2+1} + \frac{2t}{t^2+1} = 4 \int dt - 6 \int \frac{dt}{t^2+1} + \frac{2t}{t^2+1}$$

Koniec 10. časti príklady b – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right. \\ \left. t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2+1)^2}$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = 4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right]$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \left[\frac{2 \cdot 2 - 3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right]$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 2 \int \frac{dt}{t^2+1} + \frac{2t}{t^2+1} = 4 \int dt - 6 \int \frac{dt}{t^2+1} + \frac{2t}{t^2+1}$$

$$= 4t - 6 \operatorname{arctg} t + \frac{2t}{t^2+1} + C$$

Koniec 10. časti príklady b – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right. \\
 &\quad \left. t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2+1)^2} \\
 &= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4 + 2t^2 + 1 - 2t^2 - 2 + 1}{(t^2+1)^2} = \frac{(t^2+1)^2 - 2(t^2+1) + 1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = 4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt \\
 &= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right] \\
 &= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \left[\frac{2 \cdot 2 - 3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right] \\
 &= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 2 \int \frac{dt}{t^2+1} + \frac{2t}{t^2+1} = 4 \int dt - 6 \int \frac{dt}{t^2+1} + \frac{2t}{t^2+1} \\
 &= 4t - 6 \operatorname{arctg} t + \frac{2t}{t^2+1} + C = \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x} + 1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x+1-x}{1-x}} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{2}{1-x}} = \sqrt{\frac{1+x}{1-x}} \cdot (1-x) = \sqrt{1-x^2} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3} dx = 4\sqrt{\frac{1+x}{1-x}} - 6 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} + \sqrt{1-x^2} + c$$

$$= \left[\text{Subst. } t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}} - 1, \quad t^2 = \frac{1+x}{1-x} \mid x \in (-1; 1) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2+1)^2}$$

$$t^2(1-x) = 1+x \Rightarrow x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1) - (t^2-1) \cdot 2t}{(t^2+1)^2} dt = \frac{4t dt}{(t^2+1)^2} \mid t \in (0; \infty)$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4 + 2t^2 + 1 - 2t^2 - 2 + 1}{(t^2+1)^2} = \frac{(t^2+1)^2 - 2(t^2+1) + 1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = 4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{1}{2a^2(n-1)} \frac{t}{(t^2+a^2)^{n-1}} \right]$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 4 \left[\frac{2 \cdot 2 - 3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right]$$

$$= 4 \int dt - 8 \int \frac{dt}{t^2+1} + 2 \int \frac{dt}{t^2+1} + \frac{2t}{t^2+1} = 4 \int dt - 6 \int \frac{dt}{t^2+1} + \frac{2t}{t^2+1}$$

$$= 4t - 6 \operatorname{arctg} t + \frac{2t}{t^2+1} + c = \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x} + 1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x+1-x}{1-x}} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{2}{1-x}} = \sqrt{\frac{1+x}{1-x}} \cdot (1-x) = \sqrt{1-x^2} \right]$$

$$= 4\sqrt{\frac{1+x}{1-x}} - 6 \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} + \sqrt{1-x^2} + c, \quad x \in (-1; 1), \quad c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx$$

Koniec 10. časti príklady b – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \quad \left| x \in (-1; 1) \right. \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \quad \left| t \in (0; \infty) \right. \end{array} \right]$$

Koniec 10. časti príklady b – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \quad \left| x \in (-1; 1) \right. \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \quad \left| t \in (0; \infty) \right. \end{array} \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2+1)^4}$$

Koniec 10. časti príklady b – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \quad \left| x \in (-1; 1) \right. \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \quad \left| t \in (0; \infty) \right. \end{array} \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2+1)^4}$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4 + 2t^2 + 1 - 2t^2 - 2 + 1}{(t^2+1)^2} = \frac{(t^2+1)^2 - 2(t^2+1) + 1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right]$$

Koniec 10. časti príklady b – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst.} \quad t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \quad \left| x \in (-1; 1) \right. \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \quad \left| t \in (0; \infty) \right. \end{array} \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2+1)^4} \\
 &= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4 + 2t^2 + 1 - 2t^2 - 2 + 1}{(t^2+1)^2} = \frac{(t^2+1)^2 - 2(t^2+1) + 1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = -4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \int \frac{dt}{(t^2+1)^2}
 \end{aligned}$$

Koniec 10. časti príklady b – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst.} \quad t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \quad \left| x \in (-1; 1) \right. \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \quad \left| t \in (0; \infty) \right. \end{array} \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2+1)^4} \\
 &= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4 + 2t^2 + 1 - 2t^2 - 2 + 1}{(t^2+1)^2} = \frac{(t^2+1)^2 - 2(t^2+1) + 1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = -4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \mid x \in (-1; 1) \right. \\
 &\quad \left. t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2+1)^4} \\
 &= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = -4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right] \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \left[\frac{2 \cdot 2 - 3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst.} \quad t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \quad \left| x \in (-1; 1) \right. \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \quad \left| t \in (0; \infty) \right. \end{array} \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2+1)^4} \\
 &= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4 + 2t^2 + 1 - 2t^2 - 2 + 1}{(t^2+1)^2} = \frac{(t^2+1)^2 - 2(t^2+1) + 1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = -4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right] \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \left[\frac{2 \cdot 2 - 3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right] \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 2 \int \frac{dt}{t^2+1} - \frac{2t}{t^2+1}
 \end{aligned}$$

Koniec 10. časti príklady b – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst.} \quad t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \quad x \in (-1; 1) \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \quad t \in (0; \infty) \end{array} \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2+1)^4} \\
 &= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = -4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right] \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \left[\frac{2 \cdot 2 - 3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right] \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 2 \int \frac{dt}{t^2+1} - \frac{2t}{t^2+1} = -4 \int dt + 6 \int \frac{dt}{t^2+1} - \frac{2t}{t^2+1}
 \end{aligned}$$

Koniec 10. časti príklady b – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} \mid x \in (-1; 1) \right. \\
 &\quad \left. t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2+1)^4} \\
 &= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4 + 2t^2 + 1 - 2t^2 - 2 + 1}{(t^2+1)^2} = \frac{(t^2+1)^2 - 2(t^2+1) + 1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = -4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right] \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \left[\frac{2 \cdot 2 - 3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right] \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 2 \int \frac{dt}{t^2+1} - \frac{2t}{t^2+1} = -4 \int dt + 6 \int \frac{dt}{t^2+1} - \frac{2t}{t^2+1} \\
 &= -4t + 6 \operatorname{arctg} t - \frac{2t}{t^2+1} + C
 \end{aligned}$$

Koniec 10. časti príklady b – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x}-1}, \quad t^2 = \frac{1-x}{1+x} \mid x \in (-1; 1) \right. \\
 &\quad \left. t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1) - (1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \mid t \in (0; \infty) \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2+1)^4} \\
 &= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = -4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right] \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \left[\frac{2 \cdot 2 - 3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right] \\
 &= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 2 \int \frac{dt}{t^2+1} - \frac{2t}{t^2+1} = -4 \int dt + 6 \int \frac{dt}{t^2+1} - \frac{2t}{t^2+1} \\
 &= -4t + 6 \operatorname{arctg} t - \frac{2t}{t^2+1} + C = \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x}{1+x}+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x+1+x}{1+x}} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{2}{1+x}} = \sqrt{\frac{1-x}{1+x}} \cdot (1+x) = \sqrt{1-x^2} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3} dx = -4\sqrt{\frac{1-x}{1+x}} + 6 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} - \sqrt{1-x^2} + c$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x}-1}, \quad t^2 = \frac{1-x}{1+x} \quad | \quad x \in (-1; 1) \\ t^2(1+x) = 1-x \Rightarrow x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1)-(1-t^2) \cdot 2t}{(t^2+1)^2} dt = \frac{-4t dt}{(t^2+1)^2} \quad | \quad t \in (0; \infty) \end{array} \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2+1)^4}$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = -4 \int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt$$

$$= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right]$$

$$= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 4 \left[\frac{2 \cdot 2 - 3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right]$$

$$= -4 \int dt + 8 \int \frac{dt}{t^2+1} - 2 \int \frac{dt}{t^2+1} - \frac{2t}{t^2+1} = -4 \int dt + 6 \int \frac{dt}{t^2+1} - \frac{2t}{t^2+1}$$

$$= -4t + 6 \operatorname{arctg} t - \frac{2t}{t^2+1} + c = \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x}{1+x}+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x+1+x}{1+x}} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{2}{1+x}} = \sqrt{\frac{1-x}{1+x} \cdot (1+x)^2} = \sqrt{1-x^2} \right]$$

$$= -4\sqrt{\frac{1-x}{1+x}} + 6 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} - \sqrt{1-x^2} + c, \quad x \in (-1; 1), \quad c \in R.$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ t^2(x-1) = x+1, \quad x = \frac{t^2+1}{t^2-1}, \quad dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (1; \infty) \end{array} \right. \end{array} \right. \end{array} \right]$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$$

$$= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ t^2(x-1) = x+1, x = \frac{t^2+1}{t^2-1}, dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (1; \infty) \end{array} \right. \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2-1)^2}$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$$

$$= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ t^2(x-1) = x+1, x = \frac{t^2+1}{t^2-1}, dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (1; \infty) \end{array} \right. \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2-1)^2}$$

$$= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right]$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ x \in (1; \infty), t \in (1; \infty) \end{array} \right. \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = -4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \int \frac{dt}{(t^2-1)^2}
 \end{aligned}$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ x \in (1; \infty), t \in (1; \infty) \end{array} \right. \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = -4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ x \in (1; \infty), t \in (1; \infty) \end{array} \right. \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = -4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \left| \begin{array}{l} x \in (-\infty; -1), t \in (0; 1) \\ x \in (1; \infty), t \in (1; \infty) \end{array} \right. \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = -4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} + 2 \int \frac{dt}{t^2-1} + \frac{2t}{t^2-1}
 \end{aligned}$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \mid x \in (-\infty; -1), t \in (0; 1) \right. \\
 &\quad \left. t^2(x-1) = x+1, \quad x = \frac{t^2+1}{t^2-1}, \quad dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (1; \infty) \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = -4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} + 2 \int \frac{dt}{t^2-1} + \frac{2t}{t^2-1} = -4 \int dt - 6 \int \frac{dt}{t^2-1} + \frac{2t}{t^2-1}
 \end{aligned}$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \mid x \in (-\infty; -1), t \in (0; 1) \right. \\
 &\quad \left. t^2(x-1) = x+1, \quad x = \frac{t^2+1}{t^2-1}, \quad dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (1; \infty) \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = -4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} + 2 \int \frac{dt}{t^2-1} + \frac{2t}{t^2-1} = -4 \int dt - 6 \int \frac{dt}{t^2-1} + \frac{2t}{t^2-1} \\
 &= -4t - 6 \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + \frac{2t}{t^2-1} + c
 \end{aligned}$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \mid x \in (-\infty; -1), t \in (0; 1) \right. \\
 &\quad \left. t^2(x-1) = x+1, \quad x = \frac{t^2+1}{t^2-1}, \quad dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (1; \infty) \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = -4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} + 2 \int \frac{dt}{t^2-1} + \frac{2t}{t^2-1} = -4 \int dt - 6 \int \frac{dt}{t^2-1} + \frac{2t}{t^2-1} \\
 &= -4t - 6 \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + \frac{2t}{t^2-1} + C = \frac{2t}{t^2-1} - 4t - 3 \ln |t-1| + 3 \ln |t+1| + C
 \end{aligned}$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \mid x \in (-\infty; -1), t \in (0; 1) \right. \\
 &\quad \left. t^2(x-1) = x+1, \quad x = \frac{t^2+1}{t^2-1}, \quad dx = \frac{2t(t^2-1) - (t^2+1) \cdot 2t}{(t^2-1)^2} dt = \frac{-4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (1; \infty) \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = -4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} + 2 \int \frac{dt}{t^2-1} + \frac{2t}{t^2-1} = -4 \int dt - 6 \int \frac{dt}{t^2-1} + \frac{2t}{t^2-1} \\
 &= -4t - 6 \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + \frac{2t}{t^2-1} + C = \frac{2t}{t^2-1} - 4t - 3 \ln |t-1| + 3 \ln |t+1| + C \\
 &= \left[\frac{2t}{t^2-1} - 4t = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1} - 1} - 4\sqrt{\frac{x+1}{x-1}} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-(x-1)}{x-1}} - 4\sqrt{\frac{x+1}{x-1}} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}} - 4\sqrt{\frac{x+1}{x-1}} = (x-1)\sqrt{\frac{x+1}{x-1}} - 4\sqrt{\frac{x+1}{x-1}} = (x-5)\sqrt{\frac{x+1}{x-1}} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3} dx = (x-5)\sqrt{\frac{x+1}{x-1}} - 3 \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + 3 \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] + c$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1 + \frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} \mid x \in (-\infty; -1), t \in (0; 1) \right] = \int \frac{t^3 \cdot (-4t) dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2} = \frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = -4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} - 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right] \\
 &= -4 \int dt - 8 \int \frac{dt}{t^2-1} + 2 \int \frac{dt}{t^2-1} + \frac{2t}{t^2-1} = -4 \int dt - 6 \int \frac{dt}{t^2-1} + \frac{2t}{t^2-1} \\
 &= -4t - 6 \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + \frac{2t}{t^2-1} + c = \frac{2t}{t^2-1} - 4t - 3 \ln |t-1| + 3 \ln |t+1| + c \\
 &= \left[\frac{2t}{t^2-1} - 4t = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1} - 4\sqrt{\frac{x+1}{x-1}} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-(x-1)}{x-1}} - 4\sqrt{\frac{x+1}{x-1}} = \frac{2\sqrt{\frac{x+1}{x-1}}}{2} - 4\sqrt{\frac{x+1}{x-1}} = (x-1)\sqrt{\frac{x+1}{x-1}} - 4\sqrt{\frac{x+1}{x-1}} = (x-5)\sqrt{\frac{x+1}{x-1}} \right] \\
 &= (x-5)\sqrt{\frac{x+1}{x-1}} - 3 \ln \left| \sqrt{\frac{x+1}{x-1}} - 1 \right| + 3 \ln \left[\sqrt{\frac{x+1}{x-1}} + 1 \right] + c, \\
 &\quad x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.
 \end{aligned}$$

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$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$$

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$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \quad \left| \begin{array}{l} x \in (-\infty; -1), \quad t \in (1; \infty) \\ t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), \quad t \in (0; 1) \end{array} \right. \end{array} \right] \end{array} \right.$$

Koniec 10. časti príklady b – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \quad \left| \begin{array}{l} x \in (-\infty; -1), \quad t \in (1; \infty) \\ t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1) \cdot (-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), \quad t \in (0; 1) \end{array} \right. \end{array} \right] = \int \frac{t^3 \cdot 4t dt}{(t^2-1)^2}$$

Koniec 10. časti príklady b – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \\ t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \end{array} \right] = \int \frac{t^3 \cdot 4t dt}{(t^2-1)^2}$$

$$= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right]$$

Koniec 10. časti príklady b – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \quad \left| \begin{array}{l} x \in (-\infty; -1), t \in (1; \infty) \\ t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \quad \left| \begin{array}{l} x \in (1; \infty), t \in (0; 1) \end{array} \right. \right] = \int \frac{t^3 \cdot 4t dt}{(t^2-1)^2} \right. \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = 4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \int \frac{dt}{(t^2-1)^2}
 \end{aligned}$$

Koniec 10. časti príklady b – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2-1)^2} \\
 &\quad \left[t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right] \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = 4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2-1)^2} \\
 &= \left[t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = 4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right. \\
 &\quad \left. t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = 4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right] \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} - 2 \int \frac{dt}{t^2-1} - \frac{2t}{t^2-1}
 \end{aligned}$$

Koniec 10. časti príklady b – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst.} \quad t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right. \\
 &\quad \left. t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = 4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right] \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} - 2 \int \frac{dt}{t^2-1} - \frac{2t}{t^2-1} = 4 \int dt + 6 \int \frac{dt}{t^2-1} - \frac{2t}{t^2-1}
 \end{aligned}$$

Koniec 10. časti príklady b – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2-1)^2} \\
 &\quad \left[t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1) \cdot (-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right] \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = 4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right] \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} - 2 \int \frac{dt}{t^2-1} - \frac{2t}{t^2-1} = 4 \int dt + 6 \int \frac{dt}{t^2-1} - \frac{2t}{t^2-1} \\
 &= 4t + 6 \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| - \frac{2t}{t^2-1} + c
 \end{aligned}$$

Koniec 10. časti príklady b – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right. \\
 &\quad \left. t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1) \cdot (-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = 4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right] \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} - 2 \int \frac{dt}{t^2-1} - \frac{2t}{t^2-1} = 4 \int dt + 6 \int \frac{dt}{t^2-1} - \frac{2t}{t^2-1} \\
 &= 4t + 6 \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| - \frac{2t}{t^2-1} + c = 4t + 3 \ln |t-1| - 3 \ln |t+1| - \frac{2t}{t^2-1} + c
 \end{aligned}$$

Koniec 10. časti príklady b – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right. \\
 &\quad \left. t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1) \cdot (-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2-1)^2} \\
 &= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = 4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right] \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right] \\
 &= 4 \int dt + 8 \int \frac{dt}{t^2-1} - 2 \int \frac{dt}{t^2-1} - \frac{2t}{t^2-1} = 4 \int dt + 6 \int \frac{dt}{t^2-1} - \frac{2t}{t^2-1} \\
 &= 4t + 6 \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| - \frac{2t}{t^2-1} + C = 4t + 3 \ln |t-1| - 3 \ln |t+1| - \frac{2t}{t^2-1} + C \\
 &= \left[4t - \frac{2t}{t^2-1} = 4\sqrt{\frac{x-1}{x+1}} - \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1} - 1} = 4\sqrt{\frac{x-1}{x+1}} - \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-(x+1)}{x+1}} = 4\sqrt{\frac{x-1}{x+1}} - \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}} = 4\sqrt{\frac{x-1}{x+1}} + (x+1)\sqrt{\frac{x-1}{x+1}} = (x+5)\sqrt{\frac{x-1}{x+1}} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3} dx = (x+5)\sqrt{\frac{x-1}{x+1}} + 3 \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - 3 \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c$$

$$= \left[\text{Subst. } t = \sqrt{\frac{x-1}{x+1}} = \sqrt{\frac{(x+1)-2}{x+1}} = \sqrt{1 - \frac{2}{x+1}}, \quad t^2 = \frac{x-1}{x+1} \mid x \in (-\infty; -1), t \in (1; \infty) \right] = \int \frac{t^3 \cdot 4t dt}{(t^2-1)^2}$$

$$\left[t^2(x+1) = x-1, \quad x = \frac{t^2+1}{1-t^2}, \quad dx = \frac{2t(1-t^2) - (t^2+1)(-2t)}{(1-t^2)^2} dt = \frac{4t dt}{(t^2-1)^2} \mid x \in (1; \infty), t \in (0; 1) \right]$$

$$= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4 - 2t^2 + 1 + 2t^2 - 2 + 1}{(t^2-1)^2} = \frac{(t^2-1)^2 + 2(t^2-1) + 1}{(t^2-1)^2} = 1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] = 4 \int \left[1 + \frac{2}{t^2-1} + \frac{1}{(t^2-1)^2} \right] dt$$

$$= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right]$$

$$= 4 \int dt + 8 \int \frac{dt}{t^2-1} + 4 \left[\frac{-(2 \cdot 2 - 3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right]$$

$$= 4 \int dt + 8 \int \frac{dt}{t^2-1} - 2 \int \frac{dt}{t^2-1} - \frac{2t}{t^2-1} = 4 \int dt + 6 \int \frac{dt}{t^2-1} - \frac{2t}{t^2-1}$$

$$= 4t + 6 \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| - \frac{2t}{t^2-1} + c = 4t + 3 \ln |t-1| - 3 \ln |t+1| - \frac{2t}{t^2-1} + c$$

$$= \left[4t - \frac{2t}{t^2-1} = 4\sqrt{\frac{x-1}{x+1}} - \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1} - 1} = 4\sqrt{\frac{x-1}{x+1}} - \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-(x+1)}{x+1}} = 4\sqrt{\frac{x-1}{x+1}} - \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}} = 4\sqrt{\frac{x-1}{x+1}} + (x+1)\sqrt{\frac{x-1}{x+1}} = (x+5)\sqrt{\frac{x-1}{x+1}} \right]$$

$$= (x+5)\sqrt{\frac{x-1}{x+1}} + 3 \ln \left| \sqrt{\frac{x-1}{x+1}} - 1 \right| - 3 \ln \left[\sqrt{\frac{x-1}{x+1}} + 1 \right] + c,$$

$$x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$



Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx$$



Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx = \left[\begin{array}{l} \sqrt{x} \in R: x \in \langle 0; \infty \rangle \\ 1-\sqrt{x} > 0: x \in \langle 0; 1 \rangle \end{array} \middle| x \in \langle 0; 1 \rangle \right]$$



Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx = \left[\begin{array}{l} \sqrt{x} \in \mathbb{R}: x \in \langle 0; \infty \rangle \\ 1-\sqrt{x} > 0: x \in \langle 0; 1 \rangle \end{array} \middle| x \in \langle 0; 1 \rangle \right] = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-(\sqrt{x})^2}}$$



Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx = \left[\sqrt{x} \in \mathbb{R}: x \in \langle 0; \infty \rangle \mid 1-\sqrt{x} > 0: x \in \langle 0; 1 \rangle \right] = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-x}}$$



Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$\begin{aligned}
 &= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx = \left[\sqrt{x} \in \mathbb{R}: x \in \langle 0; \infty \rangle \mid 1-\sqrt{x} > 0: x \in \langle 0; 1 \rangle \right] = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-x}} \\
 &= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x} dx}{\sqrt{1-x}}
 \end{aligned}$$



Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$\begin{aligned}
 &= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx = \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid \begin{matrix} 1-\sqrt{x} > 0: x \in \langle 0; 1 \rangle \\ x \in \langle 0; 1 \rangle \end{matrix} \right] = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-x}} \\
 &= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \int (1-x)^{-\frac{1}{2}} dx - \int \frac{\sqrt{x} dx}{\sqrt{1-x}}
 \end{aligned}$$



Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx = \left[\begin{array}{l} \sqrt{x} \in R: x \in \langle 0; \infty \rangle \\ 1-\sqrt{x} > 0: x \in \langle 0; 1 \rangle \end{array} \middle| x \in \langle 0; 1 \rangle \right] = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-x}}$$

$$= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \int (1-x)^{-\frac{1}{2}} dx - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \left[\begin{array}{l} \text{Subst. } z=1-x \mid x \in \langle 0; 1 \rangle \\ dz = -dx \mid z \in \langle 0; 1 \rangle \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}} = \sqrt{\frac{1}{1-x} - 1} \mid t^2(1-x) = x \mid x \in \langle 0; 1 \rangle \\ dx = \frac{2t(t^2+1) - t^2 \cdot 2t}{(t^2+1)^2} dt = \frac{2t dt}{(t^2+1)^2} \mid x = \frac{t^2}{t^2+1} \mid t \in \langle 0; \infty \rangle \end{array} \right]$$



Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx = \left[\begin{array}{l} \sqrt{x} \in R: x \in (0; \infty) \\ 1-\sqrt{x} > 0: x \in (0; 1) \end{array} \middle| x \in (0; 1) \right] = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-x}}$$

$$= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \int (1-x)^{-\frac{1}{2}} dx - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \left[\begin{array}{l} \text{Subst. } z=1-x \mid x \in (0; 1) \\ dz = -dx \mid z \in (0; 1) \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}} = \sqrt{\frac{1}{1-x} - 1} \mid t^2(1-x) = x \mid x \in (0; 1) \\ dx = \frac{2t(t^2+1) - t^2 \cdot 2t}{(t^2+1)^2} dt = \frac{2t dt}{(t^2+1)^2} \mid x = \frac{t^2}{t^2+1} \mid t \in (0; \infty) \end{array} \right] = - \int z^{-\frac{1}{2}} dz - \int \frac{t \cdot 2t dt}{(t^2+1)^2}$$



Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx = \left[\begin{array}{l} \sqrt{x} \in R: x \in (0; \infty) \\ 1-\sqrt{x} > 0: x \in (0; 1) \end{array} \middle| x \in (0; 1) \right] = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-x}}$$

$$= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \int (1-x)^{-\frac{1}{2}} dx - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \left[\begin{array}{l} \text{Subst. } z=1-x \mid x \in (0; 1) \\ dz = -dx \mid z \in (0; 1) \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}} = \sqrt{\frac{1}{1-x} - 1} \mid t^2(1-x) = x \mid x \in (0; 1) \\ dx = \frac{2t(t^2+1) - t^2 \cdot 2t}{(t^2+1)^2} dt = \frac{2t dt}{(t^2+1)^2} \mid x = \frac{t^2}{t^2+1} \mid t \in (0; \infty) \end{array} \right] = - \int z^{-\frac{1}{2}} dz - \int \frac{t \cdot 2t dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = t \mid u' = 1 \\ v' = \frac{2t}{(t^2+1)^2} \mid v = -\frac{1}{t^2+1} \end{array} \right]$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx = \left[\begin{array}{l} \sqrt{x} \in \mathbb{R}: x \in \langle 0; \infty \rangle \\ 1-\sqrt{x} > 0: x \in \langle 0; 1 \rangle \end{array} \middle| x \in \langle 0; 1 \rangle \right] = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-x}}$$

$$= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \int (1-x)^{-\frac{1}{2}} dx - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \left[\begin{array}{l} \text{Subst. } z=1-x \mid x \in \langle 0; 1 \rangle \\ dz = -dx \mid z \in \langle 0; 1 \rangle \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}} = \sqrt{\frac{1}{1-x} - 1} \mid t^2(1-x) = x \mid x \in \langle 0; 1 \rangle \\ dx = \frac{2t(t^2+1)-t^2 \cdot 2t}{(t^2+1)^2} dt = \frac{2t dt}{(t^2+1)^2} \mid x = \frac{t^2}{t^2+1} \mid t \in \langle 0; \infty \rangle \end{array} \right] = -\int z^{-\frac{1}{2}} dz - \int \frac{t \cdot 2t dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = t \\ v' = \frac{2t}{(t^2+1)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{t^2+1} \end{array} \right] = -\frac{z^{\frac{1}{2}}}{\frac{1}{2}} - \left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right] = -2z^{\frac{1}{2}} + \frac{t}{t^2+1} - \int \frac{dt}{t^2+1}$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx = \left[\begin{array}{l} \sqrt{x} \in \mathbb{R}: x \in \langle 0; \infty \rangle \\ 1-\sqrt{x} > 0: x \in \langle 0; 1 \rangle \end{array} \middle| x \in \langle 0; 1 \rangle \right] = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-x}}$$

$$= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \int (1-x)^{-\frac{1}{2}} dx - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \left[\begin{array}{l} \text{Subst. } z=1-x \mid x \in \langle 0; 1 \rangle \\ dz = -dx \mid z \in \langle 0; 1 \rangle \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}} = \sqrt{\frac{1}{1-x} - 1} \mid t^2(1-x) = x \mid x \in \langle 0; 1 \rangle \\ dx = \frac{2t(t^2+1) - t^2 \cdot 2t}{(t^2+1)^2} dt = \frac{2t dt}{(t^2+1)^2} \mid x = \frac{t^2}{t^2+1} \mid t \in \langle 0; \infty \rangle \end{array} \right] = -\int z^{-\frac{1}{2}} dz - \int \frac{t \cdot 2t dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = t \mid u' = 1 \\ v' = \frac{2t}{(t^2+1)^2} \mid v = -\frac{1}{t^2+1} \end{array} \right] = -\frac{z^{\frac{1}{2}}}{\frac{1}{2}} - \left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right] = -2z^{\frac{1}{2}} + \frac{t}{t^2+1} - \int \frac{dt}{t^2+1}$$

$$= -2\sqrt{z} + \frac{t}{t^2+1} - \arctg t + c_1$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx = \left[\begin{array}{l} \sqrt{x} \in \mathbb{R}: x \in \langle 0; \infty \rangle \\ 1-\sqrt{x} > 0: x \in \langle 0; 1 \rangle \end{array} \middle| x \in \langle 0; 1 \rangle \right] = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-x}}$$

$$= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \int (1-x)^{-\frac{1}{2}} dx - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \left[\begin{array}{l} \text{Subst. } z=1-x \mid x \in \langle 0; 1 \rangle \\ dz = -dx \mid z \in \langle 0; 1 \rangle \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}} = \sqrt{\frac{1}{1-x} - 1} \mid t^2(1-x) = x \mid x \in \langle 0; 1 \rangle \\ dx = \frac{2t(t^2+1) - t^2 \cdot 2t}{(t^2+1)^2} dt = \frac{2t dt}{(t^2+1)^2} \mid x = \frac{t^2}{t^2+1} \mid t \in \langle 0; \infty \rangle \end{array} \right] = -\int z^{-\frac{1}{2}} dz - \int \frac{t \cdot 2t dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = t \mid u' = 1 \\ v' = \frac{2t}{(t^2+1)^2} \mid v = -\frac{1}{t^2+1} \end{array} \right] = -\frac{z^{\frac{1}{2}}}{\frac{1}{2}} - \left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right] = -2z^{\frac{1}{2}} + \frac{t}{t^2+1} - \int \frac{dt}{t^2+1}$$

$$= -2\sqrt{z} + \frac{t}{t^2+1} - \arctg t + c_1 = \left[\frac{t}{t^2+1} = \frac{\frac{\sqrt{x}}{\sqrt{1-x}}}{\frac{x}{1-x} + 1} = \frac{\frac{\sqrt{x}}{\sqrt{1-x}}}{\frac{x+1-x}{1-x}} = \frac{(1-x)\sqrt{x}}{\sqrt{1-x}} = \sqrt{x(1-x)} = \sqrt{x-x^2} \right]$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx = -2\sqrt{1-x} + \sqrt{x-x^2} - \operatorname{arctg} \frac{\sqrt{x}}{\sqrt{1-x}} + c_1$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}} dx = \left[\begin{array}{l} \sqrt{x} \in R: x \in \langle 0; \infty \rangle \\ 1-\sqrt{x} > 0: x \in \langle 0; 1 \rangle \end{array} \middle| x \in \langle 0; 1 \rangle \right] = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x}) dx}{\sqrt{1-x}}$$

$$= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} = \int (1-x)^{-\frac{1}{2}} dx - \int \frac{\sqrt{x} dx}{\sqrt{1-x}} \left[\begin{array}{l} \text{Subst. } z=1-x \mid x \in \langle 0; 1 \rangle \\ dz = -dx \mid z \in \langle 0; 1 \rangle \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}} = \sqrt{\frac{1}{1-x} - 1} \mid t^2(1-x) = x \mid x \in \langle 0; 1 \rangle \\ dx = \frac{2t(t^2+1) - t^2 \cdot 2t}{(t^2+1)^2} dt = \frac{2t dt}{(t^2+1)^2} \mid x = \frac{t^2}{t^2+1} \mid t \in \langle 0; \infty \rangle \end{array} \right] = -\int z^{-\frac{1}{2}} dz - \int \frac{t \cdot 2t dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l} u = t \\ v' = \frac{2t}{(t^2+1)^2} \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{1}{t^2+1} \end{array} \right] = -\frac{z^{\frac{1}{2}}}{\frac{1}{2}} - \left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right] = -2z^{\frac{1}{2}} + \frac{t}{t^2+1} - \int \frac{dt}{t^2+1}$$

$$= -2\sqrt{z} + \frac{t}{t^2+1} - \operatorname{arctg} t + c_1 = \left[\frac{t}{t^2+1} = \frac{\frac{\sqrt{x}}{\sqrt{1-x}}}{\frac{x}{1-x} + 1} = \frac{\frac{\sqrt{x}}{\sqrt{1-x}}}{\frac{x+1-x}{1-x}} = \frac{(1-x)\sqrt{x}}{\sqrt{1-x}} = \sqrt{x(1-x)} = \sqrt{x-x^2} \right]$$

$$= -2\sqrt{1-x} + \sqrt{x-x^2} - \operatorname{arctg} \frac{\sqrt{x}}{\sqrt{1-x}} + c_1, x \in \langle 0; 1 \rangle, c_1 \in R.$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\begin{array}{l} \sqrt{x} \in R: x \in \langle 0; \infty \rangle \\ 1 - \sqrt{x} \geq 0: x \in \langle 0; 1 \rangle \end{array} \middle| x \in \langle 0; 1 \rangle \right]$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\begin{array}{l} \sqrt{x} \in \mathbb{R}: x \in \langle 0; \infty \rangle \\ 1 - \sqrt{x} \geq 0: x \in \langle 0; 1 \rangle \end{array} \middle| x \in \langle 0; 1 \rangle \right] = \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \\ x = t^2, \quad dx = 2t \, dt \mid t \in \langle 0; 1 \rangle \end{array} \right]$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in \mathbb{R} : x \in \langle 0; \infty \rangle \mid 1 - \sqrt{x} \geq 0 : x \in \langle 0; 1 \rangle \right] \left[x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } \begin{array}{l} t = \sqrt{x} \\ x = t^2, dx = 2t dt \end{array} \mid \begin{array}{l} x \in \langle 0; 1 \rangle \\ t \in \langle 0; 1 \rangle \end{array} \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t}-1} \mid z^2(1+t)=1-t \right]$$

$$\left[t = \frac{1-z^2}{z^2+1}, dt = \frac{-2z(z^2+1) - (1-z^2) \cdot 2z}{(z^2+1)^2} dz = \frac{-4z dz}{(z^2+1)^2} \mid t \in \langle 0; 1 \rangle, z \in \langle 0; 1 \rangle \right]$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t}-1} \mid z^2(1+t)=1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid z^2(1+t) = 1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3}$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid z^2(1+t) = 1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3} = \left[\text{Rozklad na parciálne zlomky: } \frac{z^4-z^2}{(z^2+1)^3} = \frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right]$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid z^2(1+t) = 1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3} = \left[\text{Rozklad na parciálne zlomky: } \frac{z^4-z^2}{(z^2+1)^3} = \frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] = 8 \int \left[\frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] dz$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t}-1} \mid z^2(1+t)=1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3} = \left[\text{Rozklad na parciálne zlomky: } \frac{z^4-z^2}{(z^2+1)^3} = \frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] = 8 \int \left[\frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] dz$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dz}{(z^2+1)^n} \right] = \left[\frac{2n-3}{2n-2} I_{n-1} + \frac{z}{(2n-2)(z^2+1)^{n-1}} \right]$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t}-1} \mid z^2(1+t)=1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3} = \left[\text{Rozklad na parciálne zlomky: } \frac{z^4-z^2}{(z^2+1)^3} = \frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] = 8 \int \left[\frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] dz$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dz}{(z^2+1)^n} \right] = 16I_3 - 24I_2 + 8I_1$$

$$= \frac{2n-3}{2n-2} I_{n-1} + \frac{z}{(2n-2)(z^2+1)^{n-1}}$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t}-1} \mid z^2(1+t)=1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3} = \left[\text{Rozklad na parciálne zlomky: } \frac{z^4-z^2}{(z^2+1)^3} = \frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] = 8 \int \left[\frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] dz$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dz}{(z^2+1)^n} \right] = 16I_3 - 24I_2 + 8I_1 = 16 \left[\frac{3I_2}{4} + \frac{z}{4(z^2+1)^2} \right] - 24I_2 + 8I_1$$



Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid z^2(1+t) = 1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3} = \left[\text{Rozklad na parciálne zlomky: } \frac{z^4-z^2}{(z^2+1)^3} = \frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] = 8 \int \left[\frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] dz$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dz}{(z^2+1)^n} \right] = 16I_3 - 24I_2 + 8I_1 = 16 \left[\frac{3I_2}{4} + \frac{z}{4(z^2+1)^2} \right] - 24I_2 + 8I_1$$

$$= \frac{4z}{(z^2+1)^2} - 12I_2 + 8I_1$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid z^2(1+t) = 1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3} = \left[\text{Rozklad na parciálne zlomky: } \frac{z^4-z^2}{(z^2+1)^3} = \frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] = 8 \int \left[\frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] dz$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dz}{(z^2+1)^n} \right] = 16I_3 - 24I_2 + 8I_1 = 16 \left[\frac{3I_2}{4} + \frac{z}{4(z^2+1)^2} \right] - 24I_2 + 8I_1$$

$$= \frac{4z}{(z^2+1)^2} - 12I_2 + 8I_1 = \frac{4z}{(z^2+1)^2} - 12 \left[\frac{I_1}{2} + \frac{z}{2(z^2+1)} \right] + 8I_1$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid z^2(1+t) = 1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3} = \left[\text{Rozklad na parciálne zlomky: } \frac{z^4-z^2}{(z^2+1)^3} = \frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] = 8 \int \left[\frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] dz$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dz}{(z^2+1)^n} \right] = 16I_3 - 24I_2 + 8I_1 = 16 \left[\frac{3I_2}{4} + \frac{z}{4(z^2+1)^2} \right] - 24I_2 + 8I_1$$

$$= \frac{4z}{(z^2+1)^2} - 12I_2 + 8I_1 = \frac{4z}{(z^2+1)^2} - 12 \left[\frac{I_1}{2} + \frac{z}{2(z^2+1)} \right] + 8I_1 = \frac{4z}{(z^2+1)^2} - \frac{6z}{z^2+1} + 2I_1$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid z^2(1+t) = 1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3} = \left[\text{Rozklad na parciálne zlomky: } \frac{z^4-z^2}{(z^2+1)^3} = \frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] = 8 \int \left[\frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] dz$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dz}{(z^2+1)^n} \right] = 16I_3 - 24I_2 + 8I_1 = 16 \left[\frac{3I_2}{4} + \frac{z}{4(z^2+1)^2} \right] - 24I_2 + 8I_1$$

$$= \frac{4z}{(z^2+1)^2} - 12I_2 + 8I_1 = \frac{4z}{(z^2+1)^2} - 12 \left[\frac{I_1}{2} + \frac{z}{2(z^2+1)} \right] + 8I_1 = \frac{4z}{(z^2+1)^2} - \frac{6z}{z^2+1} + 2I_1$$

$$= \frac{4z}{(z^2+1)^2} - \frac{6z}{z^2+1} + 2 \operatorname{arctg} z + c$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid z^2(1+t) = 1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3} = \left[\text{Rozklad na parciálne zlomky: } \frac{z^4-z^2}{(z^2+1)^3} = \frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] = 8 \int \left[\frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] dz$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dz}{(z^2+1)^n} \right] = 16I_3 - 24I_2 + 8I_1 = 16 \left[\frac{3I_2}{4} + \frac{z}{4(z^2+1)^2} \right] - 24I_2 + 8I_1$$

$$= \frac{4z}{(z^2+1)^2} - 12I_2 + 8I_1 = \frac{4z}{(z^2+1)^2} - 12 \left[\frac{I_1}{2} + \frac{z}{2(z^2+1)} \right] + 8I_1 = \frac{4z}{(z^2+1)^2} - \frac{6z}{z^2+1} + 2I_1$$

$$= \frac{4z}{(z^2+1)^2} - \frac{6z}{z^2+1} + 2 \operatorname{arctg} z + c = \left[z = \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \mid \frac{4z}{(z^2+1)^2} = \sqrt{1-x} + \sqrt{x-x^2} \mid \frac{6z}{z^2+1} = \sqrt{x-x^2} - 2\sqrt{1-x} \right]$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid z^2(1+t) = 1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3} = \left[\text{Rozklad na parciálne zlomky: } \frac{z^4-z^2}{(z^2+1)^3} = \frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] = 8 \int \left[\frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] dz$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dz}{(z^2+1)^n} \right] = 16I_3 - 24I_2 + 8I_1 = 16 \left[\frac{3I_2}{4} + \frac{z}{4(z^2+1)^2} \right] - 24I_2 + 8I_1$$

$$= \frac{4z}{(z^2+1)^2} - 12I_2 + 8I_1 = \frac{4z}{(z^2+1)^2} - 12 \left[\frac{I_1}{2} + \frac{z}{2(z^2+1)} \right] + 8I_1 = \frac{4z}{(z^2+1)^2} - \frac{6z}{z^2+1} + 2I_1$$

$$= \frac{4z}{(z^2+1)^2} - \frac{6z}{z^2+1} + 2 \operatorname{arctg} z + c = \left[z = \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \mid \frac{4z}{(z^2+1)^2} = \sqrt{1-x} + \sqrt{x-x^2} \mid \frac{6z}{z^2+1} = \sqrt{x-x^2} - 2\sqrt{1-x} \right]$$

$$= \sqrt{1-x} + \sqrt{x-x^2} - 3\sqrt{1-x} + 2 \operatorname{arctg} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} + c$$

Koniec 10. časti príklady b – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} dx = -2\sqrt{1-x} + \sqrt{x-x^2} + 2 \operatorname{arctg} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} + c$$

$$= \left[\sqrt{x} \in R: x \in \langle 0; \infty \rangle \mid x \in \langle 0; 1 \rangle \right] = \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \cdot 2t dt$$

$$= \left[\text{Subst. } z = \frac{\sqrt{1-t}}{\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t}-1} \mid z^2(1+t)=1-t \right] = \int z \cdot \frac{2(1-z^2)}{z^2+1} \cdot \frac{-4z dz}{(z^2+1)^2}$$

$$= 8 \int \frac{z^2(z^2-1) dz}{(z^2+1)^3} = \left[\text{Rozklad na parciálne zlomky: } \frac{z^4-z^2}{(z^2+1)^3} = \frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] = 8 \int \left[\frac{2}{(z^2+1)^3} - \frac{3}{(z^2+1)^2} + \frac{1}{z^2+1} \right] dz$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dz}{(z^2+1)^n} \right] = 16I_3 - 24I_2 + 8I_1 = 16 \left[\frac{3I_2}{4} + \frac{z}{4(z^2+1)^2} \right] - 24I_2 + 8I_1$$

$$= \frac{4z}{(z^2+1)^2} - 12I_2 + 8I_1 = \frac{4z}{(z^2+1)^2} - 12 \left[\frac{I_1}{2} + \frac{z}{2(z^2+1)} \right] + 8I_1 = \frac{4z}{(z^2+1)^2} - \frac{6z}{z^2+1} + 2I_1$$

$$= \frac{4z}{(z^2+1)^2} - \frac{6z}{z^2+1} + 2 \operatorname{arctg} z + c = \left[z = \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \mid \frac{4z}{(z^2+1)^2} = \sqrt{1-x} + \sqrt{x-x^2} \mid \frac{6z}{z^2+1} = \sqrt{x-x^2} - 2\sqrt{1-x} \right]$$

$$= \sqrt{1-x} + \sqrt{x-x^2} - 3\sqrt{1-x} + 2 \operatorname{arctg} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} + c$$

$$= \sqrt{x-x^2} - 2\sqrt{1-x} + 2 \operatorname{arctg} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} + c, x \in \langle 0; 1 \rangle, c \in R.$$

Koniec 10. časti príklady b – 171

$$\int \frac{dx}{\sqrt{x+1} + \sqrt[3]{x+1}}$$



Koniec 10. časti príklady b – 171

$$\int \frac{dx}{\sqrt{x+1} + \sqrt[3]{x+1}}$$

$$= \left[\text{Subst.} \left| \begin{array}{l} t = \sqrt[6]{x+1} \\ t^6 = x+1 \end{array} \right. \begin{array}{l} \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 \\ \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 \end{array} \left| \begin{array}{l} x \in (-1; \infty) \\ t \in (0; \infty) \end{array} \right. \right]$$



Koniec 10. časti príklady b – 171

$$\int \frac{dx}{\sqrt{x+1} + \sqrt[3]{x+1}}$$

$$= \left[\text{Subst.} \left| \begin{array}{l} t = \sqrt[6]{x+1} \\ t^6 = x+1 \end{array} \right. \begin{array}{l} \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 \\ \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 \end{array} \left| \begin{array}{l} x \in (-1; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{6t^5 dt}{t^3 + t^2}$$



Koniec 10. časti príklady b – 171

$$\int \frac{dx}{\sqrt{x+1} + \sqrt[3]{x+1}}$$

$$= \left[\text{Subst.} \left| \begin{array}{l} t = \sqrt[6]{x+1} \\ t^6 = x+1 \end{array} \right. \begin{array}{l} \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 \\ \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 \end{array} \left| \begin{array}{l} x \in (-1; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{6t^5 dt}{t^3 + t^2} = 6 \int \frac{t^3 dt}{t+1}$$



Koniec 10. časti príklady b – 171

$$\int \frac{dx}{\sqrt{x+1} + \sqrt[3]{x+1}}$$

$$= \left[\text{Subst.} \quad \left. \begin{array}{l} t = \sqrt[6]{x+1} \\ t^6 = x+1 \end{array} \right| \begin{array}{l} \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 \\ \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 \end{array} \right| \begin{array}{l} x \in (-1; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{6t^5 dt}{t^3 + t^2} = 6 \int \frac{t^3 dt}{t+1}$$

$$= 6 \int \frac{t^3 + t^2 - t^2 - t + t + 1 - 1}{t+1} dt$$

$$= \left[\text{Subst.} \quad \left. \begin{array}{l} u = t+1 = \sqrt[6]{x+1} + 1 \\ t = u-1 \end{array} \right| \begin{array}{l} dt = du \\ u \in (1; \infty) \end{array} \right| \begin{array}{l} t \in (0; \infty) \\ u \in (1; \infty) \end{array} \right]$$



Koniec 10. časti príklady b – 171

$$\int \frac{dx}{\sqrt{x+1} + \sqrt[3]{x+1}}$$

$$= \left[\text{Subst.} \quad \left| \begin{array}{l} t = \sqrt[6]{x+1} \\ t^6 = x+1 \end{array} \right. \begin{array}{l} \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 \\ \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 \end{array} \quad \left| \begin{array}{l} x \in (-1; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{6t^5 dt}{t^3 + t^2} = 6 \int \frac{t^3 dt}{t+1}$$

$$= 6 \int \frac{t^3 + t^2 - t^2 - t + t + 1 - 1}{t+1} dt = 6 \int \frac{t^3 + 1 - 1}{t+1} dt$$

$$= \left[\text{Subst.} \quad \left| \begin{array}{l} u = t+1 = \sqrt[6]{x+1} + 1 \\ t = u-1 \end{array} \right. \quad \left| \begin{array}{l} t \in (0; \infty) \\ u \in (1; \infty) \end{array} \right. \right] = 6 \int \frac{(u-1)^3 du}{u}$$



Koniec 10. časti príklady b – 171

$$\int \frac{dx}{\sqrt{x+1} + \sqrt[3]{x+1}}$$

$$= \left[\text{Subst.} \quad \left| \begin{array}{l} t = \sqrt[6]{x+1} \\ t^6 = x+1 \\ 6t^5 dt = dx \end{array} \right. \quad \left| \begin{array}{l} \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 \\ \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 \end{array} \right. \quad \left| \begin{array}{l} x \in (-1; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{6t^5 dt}{t^3 + t^2} = 6 \int \frac{t^3 dt}{t+1}$$

$$= 6 \int \frac{t^3 + t^2 - t^2 - t + t + 1 - 1}{t+1} dt = 6 \int \frac{t^3 + 1 - 1}{t+1} dt = 6 \int \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt$$

$$= \left[\text{Subst.} \quad \left| \begin{array}{l} u = t+1 = \sqrt[6]{x+1} + 1 \\ t = u-1 \\ dt = du \end{array} \right. \quad \left| \begin{array}{l} t \in (0; \infty) \\ u \in (1; \infty) \end{array} \right. \right] = 6 \int \frac{(u-1)^3 du}{u} = 6 \int \frac{u^3 - 3u^2 + 3u - 1}{u} du$$



Koniec 10. časti príklady b – 171

$$\int \frac{dx}{\sqrt{x+1} + \sqrt[3]{x+1}}$$

$$= \left[\text{Subst.} \left| \begin{array}{l} t = \sqrt[6]{x+1} \\ t^6 = x+1 \end{array} \right. \begin{array}{l} \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 \\ \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 \end{array} \left| \begin{array}{l} x \in (-1; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{6t^5 dt}{t^3 + t^2} = 6 \int \frac{t^3 dt}{t+1}$$

$$= 6 \int \frac{t^3 + t^2 - t^2 - t + t + 1 - 1}{t+1} dt = 6 \int \frac{t^3 + 1 - 1}{t+1} dt = 6 \int \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt$$

$$= 6 \left(\frac{t^3}{3} - \frac{t^2}{2} + t - \ln |t+1| \right) + c$$

$$= \left[\text{Subst.} \left| \begin{array}{l} u = t+1 = \sqrt[6]{x+1} + 1 \\ t = u-1 \end{array} \right. \left| \begin{array}{l} t \in (0; \infty) \\ u \in (1; \infty) \end{array} \right. \right] = 6 \int \frac{(u-1)^3 du}{u} = 6 \int \frac{u^3 - 3u^2 + 3u - 1}{u} du$$

$$= 6 \int \left(u^2 - 3u + 3 - \frac{1}{u} \right) du$$



Koniec 10. časti príklady b – 171

$$\int \frac{dx}{\sqrt{x+1} + \sqrt[3]{x+1}}$$

$$= \left[\text{Subst.} \left| \begin{array}{l} t = \sqrt[6]{x+1} \\ t^6 = x+1 \end{array} \right. \begin{array}{l} \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 \\ \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 \end{array} \right| \begin{array}{l} x \in (-1; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{6t^5 dt}{t^3 + t^2} = 6 \int \frac{t^3 dt}{t+1}$$

$$\begin{aligned} &= 6 \int \frac{t^3 + t^2 - t^2 - t + t + 1 - 1}{t+1} dt = 6 \int \frac{t^3 + 1 - 1}{t+1} dt = 6 \int \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt \\ &= 6 \left(\frac{t^3}{3} - \frac{t^2}{2} + t - \ln |t+1| \right) + c = 2t^3 - 3t^2 + 6t - 6 \ln(t+1) + c \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst.} \left| \begin{array}{l} u = t+1 = \sqrt[6]{x+1} + 1 \\ t = u-1 \end{array} \right. \begin{array}{l} t \in (0; \infty) \\ u \in (1; \infty) \end{array} \right] = 6 \int \frac{(u-1)^3 du}{u} = 6 \int \frac{u^3 - 3u^2 + 3u - 1}{u} du \\ &= 6 \int \left(u^2 - 3u + 3 - \frac{1}{u} \right) du = 6 \left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln |u| \right) + c \end{aligned}$$



Koniec 10. časti príklady b – 171

$$\int \frac{dx}{\sqrt{x+1} + \sqrt[3]{x+1}} = 2\sqrt{x+1} - 3\sqrt[3]{x+1} + 6\sqrt[6]{x+1} - 6\ln(\sqrt[6]{x+1} + 1) + c$$

$$= \left[\text{Subst.} \left| \begin{array}{l} t = \sqrt[6]{x+1} \\ t^6 = x+1 \end{array} \right. \begin{array}{l} \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 \\ \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 \end{array} \right| \begin{array}{l} x \in (-1; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{6t^5 dt}{t^3 + t^2} = 6 \int \frac{t^3 dt}{t+1}$$

$$\begin{aligned} &= 6 \int \frac{t^3 + t^2 - t^2 - t + t + 1 - 1}{t+1} dt = 6 \int \frac{t^3 + 1 - 1}{t+1} dt = 6 \int \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt \\ &= 6 \left(\frac{t^3}{3} - \frac{t^2}{2} + t - \ln|t+1| \right) + c = 2t^3 - 3t^2 + 6t - 6\ln(t+1) + c \\ &= 2\sqrt{x+1} - 3\sqrt[3]{x+1} + 6\sqrt[6]{x+1} - 6\ln(\sqrt[6]{x+1} + 1) + c, \quad x \in (-1; \infty), \quad c \in \mathbb{R}. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst.} \left| \begin{array}{l} u = t+1 = \sqrt[6]{x+1} + 1 \\ t = u-1 \end{array} \right. \right| \begin{array}{l} t \in (0; \infty) \\ u \in (1; \infty) \end{array} \right] = 6 \int \frac{(u-1)^3 du}{u} = 6 \int \frac{u^3 - 3u^2 + 3u - 1}{u} du \\ &= 6 \int \left(u^2 - 3u + 3 - \frac{1}{u} \right) du = 6 \left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln|u| \right) + c \\ &= 2(\sqrt[6]{x+1} + 1)^3 - 9(\sqrt[6]{x+1} + 1)^2 + (\sqrt[6]{x+1} + 1) - 6\ln(\sqrt[6]{x+1} + 1) + c, \\ &\quad x \in (-1; \infty), \quad c \in \mathbb{R}. \end{aligned}$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{1+x}{1-x} \leq 1 & 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in \langle -1; 0 \rangle \\ x \neq 1 & 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \right| \begin{array}{l} x \in \langle -1; 0 \rangle \\ \text{t. j. } -1 \leq x \leq 0 \end{array} \right]$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in \langle -1; 0 \rangle \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in \langle -1; 0 \rangle \\ \text{t. j. } -1 \leq x \leq 0 \end{array} \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{1+x}{1-x}} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{1+x}{1-x}}} \cdot \frac{1}{2} \left(\frac{1+x}{1-x} \right)^{-\frac{1}{2}} \cdot \frac{(1-x) - (1+x) \cdot (-1)}{(1-x)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{1-x-(1+x)}{1-x}} \sqrt{\frac{1+x}{1-x}} \sqrt{(1-x)^4}} = \frac{1}{\sqrt{\frac{-2x}{1-x} \cdot \frac{1+x}{1-x} \cdot (1-x)^4}} \\ v = x \\ = \frac{1}{\sqrt{-2x(1+x) \cdot (1-x)^2}} = \frac{1}{\sqrt{2} |1-x| \sqrt{-x(1+x)}} = \frac{\sqrt{2}}{2(1-x) \sqrt{-x(x+1)}} = \frac{-\sqrt{2}}{2(x-1) \sqrt{-x^2-x}} \end{array} \right]$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in (-1; 0) \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in (-1; 0) \\ \text{t. j. } -1 \leq x \leq 0 \end{array} \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{1+x}{1-x}} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{1+x}{1-x}}} \cdot \frac{1}{2} \left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} \cdot \frac{(1-x) - (1+x) \cdot (-1)}{(1-x)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{1-x-(1+x)}{1-x}} \sqrt{\frac{1+x}{1-x}} \sqrt{(1-x)^4}} = \frac{1}{\sqrt{\frac{-2x}{1-x} \cdot \frac{1+x}{1-x} \cdot (1-x)^4}} \\ v = x \end{array} \right]$$

$$= \frac{1}{\sqrt{-2x(1+x) \cdot (1-x)^2}} = \frac{1}{\sqrt{2} |1-x| \sqrt{-x(1+x)}} = \frac{\sqrt{2}}{2(1-x) \sqrt{-x(x+1)}} = \frac{-\sqrt{2}}{2(x-1) \sqrt{-x^2-x}}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x-1) \sqrt{-x^2-x}} dx$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in (-1; 0) \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in (-1; 0) \\ \text{t. j. } -1 \leq x \leq 0 \end{array} \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{1+x}{1-x}} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{1+x}{1-x}}} \cdot \frac{1}{2} \left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} \cdot \frac{(1-x) - (1+x) \cdot (-1)}{(1-x)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{1-x-(1+x)}{1-x}} \sqrt{\frac{1+x}{1-x}} \sqrt{(1-x)^4}} = \frac{1}{\sqrt{\frac{-2x}{1-x} \cdot \frac{1+x}{1-x} \cdot (1-x)^4}} \\ v = x \end{array} \right]$$

$$= \frac{1}{\sqrt{-2x(1+x) \cdot (1-x)^2}} = \frac{1}{\sqrt{2} |1-x| \sqrt{-x(1+x)}} = \frac{\sqrt{2}}{2(1-x) \sqrt{-x(x+1)}} = \frac{-\sqrt{2}}{2(x-1) \sqrt{-x^2-x}}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x-1) \sqrt{-x^2-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x-1+1}{(x-1) \sqrt{-x^2-x}} dx$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in (-1; 0) \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in (-1; 0) \\ \text{t. j. } -1 \leq x \leq 0 \end{array} \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{1+x}{1-x}} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{1+x}{1-x}}} \cdot \frac{1}{2} \left(\frac{1+x}{1-x} \right)^{-\frac{1}{2}} \cdot \frac{(1-x) - (1+x) \cdot (-1)}{(1-x)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{1-x-(1+x)}{1-x}} \sqrt{\frac{1+x}{1-x}} \sqrt{(1-x)^4}} = \frac{1}{\sqrt{\frac{-2x}{1-x} \cdot \frac{1+x}{1-x} \cdot (1-x)^4}} \\ v = x \end{array} \right] = \frac{1}{\sqrt{-2x(1+x) \cdot (1-x)^2}} = \frac{1}{\sqrt{2} |1-x| \sqrt{-x(1+x)}} = \frac{\sqrt{2}}{2(1-x) \sqrt{-x(x+1)}} = \frac{-\sqrt{2}}{2(x-1) \sqrt{-x^2-x}}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x-1) \sqrt{-x^2-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x-1+1}{(x-1) \sqrt{-x^2-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{(x-1) \sqrt{-x^2-x}}$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in (-1; 0) \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in (-1; 0) \\ \text{t. j. } -1 \leq x \leq 0 \end{array} \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{1+x}{1-x}} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{1+x}{1-x}}} \cdot \frac{1}{2} \left(\frac{1+x}{1-x} \right)^{-\frac{1}{2}} \cdot \frac{(1-x) - (1+x) \cdot (-1)}{(1-x)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{1-x-(1+x)}{1-x}} \sqrt{\frac{1+x}{1-x}} \sqrt{(1-x)^4}} = \frac{1}{\sqrt{\frac{-2x}{1-x} \cdot \frac{1+x}{1-x} \cdot (1-x)^4}} \\ v = x \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x-1)\sqrt{-x^2-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x-1+1}{(x-1)\sqrt{-x^2-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{(x-1)\sqrt{-x^2-x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + \frac{1}{2} \\ \text{Subst. } z = -\frac{1}{x} \end{array} \middle| \begin{array}{l} dt = dx \\ x = -z^{-1} \end{array} \middle| \begin{array}{l} x \in (-1; 0) \\ x \in (1; \infty) \end{array} \middle| \begin{array}{l} t \in (-\frac{1}{2}; \frac{1}{2}) \\ z \in (1; \infty) \end{array} \right]$$

$$\left[\begin{array}{l} -x^2 - x = -(x^2 + x) = -(x^2 + 2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x + \frac{1}{2})^2 \\ x-1 = -\frac{1}{z} - 1 = -\frac{1+z}{z}, \quad \sqrt{-x^2-x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \right]$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{1+x}{1-x}} \\ v' = 1 \end{array} \right] \left[\begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{1+x}{1-x}}} \cdot \frac{1}{2} \left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} \cdot \frac{(1-x)-(1+x) \cdot (-1)}{(1-x)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{1-x-(1+x)}{1-x}} \sqrt{\frac{1+x}{1-x}} \sqrt{(1-x)^4}} = \frac{1}{\sqrt{\frac{-2x}{1-x} \cdot \frac{1+x}{1-x} \cdot (1-x)^4}} \\ v = x \end{array} \right] = \frac{1}{\sqrt{-2x(1+x) \cdot (1-x)^2}} = \frac{1}{\sqrt{2} |1-x| \sqrt{-x(1+x)}} = \frac{\sqrt{2}}{2(1-x) \sqrt{-x(x+1)}} = \frac{-\sqrt{2}}{2(x-1) \sqrt{-x^2-x}}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x-1) \sqrt{-x^2-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x-1+1}{(x-1) \sqrt{-x^2-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{(x-1) \sqrt{-x^2-x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + \frac{1}{2} \left| \begin{array}{l} dt = dx \\ x \in (-1; 0) \end{array} \right. \left| \begin{array}{l} t \in (-\frac{1}{2}; \frac{1}{2}) \\ -x^2 - x = -(x^2 + x) = -(x^2 + 2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x + \frac{1}{2})^2 \end{array} \right. \\ \text{Subst. } z = -\frac{1}{x} \left| \begin{array}{l} dx = -\frac{dz}{z^2} \\ x \in (-1; 0) \end{array} \right. \left| \begin{array}{l} z \in (1; \infty) \\ x-1 = -\frac{1}{z} - 1 = -\frac{1+z}{z}, \quad \sqrt{-x^2-x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \right. \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} + \frac{\sqrt{2}}{2} \int \frac{\frac{dz}{z^2}}{-\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

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$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x-1)\sqrt{-x^2-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x-1+1}{(x-1)\sqrt{-x^2-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{(x-1)\sqrt{-x^2-x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + \frac{1}{2} \mid dt = dx \\ \text{Subst. } z = -\frac{1}{x} \mid x = -z^{-1} \mid dx = \frac{dz}{z^2} \end{array} \left| \begin{array}{l} x \in (-1; 0) \mid t \in (-\frac{1}{2}; \frac{1}{2}) \\ z \in (1; \infty) \end{array} \right| \begin{array}{l} -x^2 - x = -(x^2 + x) = -(x^2 + 2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x + \frac{1}{2})^2 \\ x-1 = -\frac{1}{z} - 1 = -\frac{1+z}{z}, \quad \sqrt{-x^2-x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} + \frac{\sqrt{2}}{2} \int \frac{\frac{dz}{z^2}}{-\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} - \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}}$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x-1)\sqrt{-x^2-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x-1+1}{(x-1)\sqrt{-x^2-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{(x-1)\sqrt{-x^2-x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + \frac{1}{2} \mid dt = dx \\ \text{Subst. } z = -\frac{1}{x} \mid x = -z^{-1} \mid dx = \frac{dz}{z^2} \end{array} \right. \left. \begin{array}{l} x \in (-1; 0) \mid t \in (-\frac{1}{2}; \frac{1}{2}) \mid -x^2 - x = -(x^2 + x) = -(x^2 + 2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x + \frac{1}{2})^2 \\ z \in (1; \infty) \mid x - 1 = -\frac{1}{z} - 1 = -\frac{1+z}{z}, \quad \sqrt{-x^2-x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} + \frac{\sqrt{2}}{2} \int \frac{\frac{dz}{z^2}}{-\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} - \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \mid u^2 = z-1 \\ z = u^2 + 1 \mid dz = 2u du \\ z \in (1; \infty) \mid u \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{x-1+1}{(x-1)\sqrt{-x^2-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{(x-1)\sqrt{-x^2-x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + \frac{1}{2} \mid dt = dx \mid x \in (-1; 0) \mid t \in (-\frac{1}{2}; \frac{1}{2}) \mid -x^2 - x = -(x^2 + x) = -(x^2 + 2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x + \frac{1}{2})^2 \\ \text{Subst. } z = -\frac{1}{x} \mid x = -z^{-1} \mid dx = \frac{dz}{z^2} \mid z \in (1; \infty) \mid x - 1 = -\frac{1}{z} - 1 = -\frac{1+z}{z}, \quad \sqrt{-x^2-x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} + \frac{\sqrt{2}}{2} \int \frac{\frac{dz}{z^2}}{-\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} - \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \mid u^2 = z-1 \\ z = u^2 + 1 \mid dz = 2u du \\ z \in (1; \infty) \mid u \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \frac{\sqrt{2}}{2} \int \frac{2u du}{(u^2+2)u}$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2-x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{(x-1)\sqrt{-x^2-x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + \frac{1}{2} \mid dt = dx \mid x \in (-1; 0) \mid t \in (-\frac{1}{2}; \frac{1}{2}) \mid -x^2 - x = -(x^2 + x) = -(x^2 + 2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x + \frac{1}{2})^2 \\ \text{Subst. } z = -\frac{1}{x} \mid x = -z^{-1} \mid dx = \frac{dz}{z^2} \mid z \in (1; \infty) \mid x - 1 = -\frac{1}{z} - 1 = -\frac{1+z}{z}, \quad \sqrt{-x^2-x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} + \frac{\sqrt{2}}{2} \int \frac{\frac{dz}{z^2}}{-\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} - \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \mid u^2 = z-1 \\ z = u^2 + 1 \mid dz = 2u du \\ z \in (1; \infty) \mid u \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \frac{\sqrt{2}}{2} \int \frac{2u du}{(u^2+2)u}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \sqrt{2} \int \frac{du}{u^2+2}$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x + \frac{1}{2} \mid dt = dx \mid x \in (-1; 0) \mid t \in (-\frac{1}{2}; \frac{1}{2}) \mid -x^2 - x = -(x^2 + x) = -(x^2 + 2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x + \frac{1}{2})^2 \\ \text{Subst. } z = -\frac{1}{x} \mid x = -z^{-1} \mid dx = \frac{dz}{z^2} \mid z \in (1; \infty) \mid x - 1 = -\frac{1}{z} - 1 = -\frac{1+z}{z}, \quad \sqrt{-x^2 - x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} + \frac{\sqrt{2}}{2} \int \frac{\frac{dz}{z^2}}{-\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} - \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \mid u^2 = z-1 \\ z = u^2 + 1 \mid dz = 2u du \\ z \in (1; \infty) \mid u \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \frac{\sqrt{2}}{2} \int \frac{2u du}{(u^2+2)u}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \sqrt{2} \int \frac{du}{u^2+2}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \frac{\sqrt{2}}{\sqrt{2}} \operatorname{arctg} \frac{u}{\sqrt{2}} + c_1$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4}-t^2}} + \frac{\sqrt{2}}{2} \int \frac{\frac{dz}{z^2}}{-\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} - \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \mid u^2 = z-1 \\ z = u^2 + 1 \mid dz = 2u du \\ z \in (1; \infty) \mid u \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \frac{\sqrt{2}}{2} \int \frac{2u du}{(u^2+2)u}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \sqrt{2} \int \frac{du}{u^2+2}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \frac{\sqrt{2}}{2} \operatorname{arctg} \frac{u}{\sqrt{2}} + c_1$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin (2x+1) - \operatorname{arctg} \frac{\sqrt{z-1}}{\sqrt{2}} + c_1$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4}-t^2}} + \frac{\sqrt{2}}{2} \int \frac{\frac{dz}{z^2}}{-\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} - \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \mid u^2 = z-1 \\ z = u^2 + 1 \mid dz = 2u du \\ z \in (1; \infty) \mid u \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \frac{\sqrt{2}}{2} \int \frac{2u du}{(u^2+2)u}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \sqrt{2} \int \frac{du}{u^2+2}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \frac{\sqrt{2}}{2} \operatorname{arctg} \frac{u}{\sqrt{2}} + c_1$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin (2x+1) - \operatorname{arctg} \frac{\sqrt{z-1}}{\sqrt{2}} + c_1 = \left[\begin{array}{l} \frac{\sqrt{z-1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \sqrt{-\frac{1}{x}-1} \\ = \frac{1}{\sqrt{2}} \sqrt{-\frac{1-x}{x}} = \sqrt{\frac{-1-x}{2x}} \end{array} \right]$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx = x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin (2x+1) - \operatorname{arctg} \sqrt{\frac{-1-x}{2x}} + c_1$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} - \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \mid u^2 = z-1 \\ z = u^2 + 1 \mid dz = 2u du \\ z \in (1; \infty) \mid u \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \frac{\sqrt{2}}{2} \int \frac{2u du}{(u^2+2)u}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \sqrt{2} \int \frac{du}{u^2+2}$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin 2t - \frac{\sqrt{2}}{\sqrt{2}} \operatorname{arctg} \frac{u}{\sqrt{2}} + c_1$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin (2x+1) - \operatorname{arctg} \frac{\sqrt{z-1}}{\sqrt{2}} + c_1 = \left[\begin{array}{l} \frac{\sqrt{z-1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \sqrt{-\frac{1}{x} - 1} \\ = \frac{1}{\sqrt{2}} \sqrt{-\frac{1-x}{x}} = \sqrt{\frac{-1-x}{2x}} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin (2x+1) - \operatorname{arctg} \sqrt{-\frac{1+x}{2x}} + c_1, \quad x \in (-1; 0), \quad c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in \langle -1; 0 \rangle \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right. \left. \begin{array}{l} x \in \langle -1; 0 \rangle \\ \text{t. j. } -1 \leq x < 0 \end{array} \right]$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in \langle -1; 0 \rangle \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right. \left. \begin{array}{l} x \in \langle -1; 0 \rangle \\ \text{t. j. } -1 \leq x \leq 0 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1-x \\ dt = -dx \end{array} \left| \begin{array}{l} x \in \langle -1; 0 \rangle \\ t \in \langle 1; 2 \rangle \end{array} \right. \right]$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in \langle -1; 0 \rangle \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right| x \in \langle -1; 0 \rangle \quad \left. \begin{array}{l} \\ \text{t. j. } -1 \leq x \leq 0 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1-x \\ dt=-dx \end{array} \left| \begin{array}{l} x \in \langle -1; 0 \rangle \\ t \in \langle 1; 2 \rangle \end{array} \right. \right] = - \int \arcsin \sqrt{\frac{2-t}{t}} dt = - \int \arcsin \sqrt{\frac{2}{t}-1} dt$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in \langle -1; 0 \rangle \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in \langle -1; 0 \rangle \\ \text{t. j. } -1 \leq x \leq 0 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1-x \\ dt=-dx \end{array} \middle| \begin{array}{l} x \in \langle -1; 0 \rangle \\ t \in \langle 1; 2 \rangle \end{array} \right] = - \int \arcsin \sqrt{\frac{2-t}{t}} dt = - \int \arcsin \sqrt{\frac{2}{t}-1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t}-1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-(\frac{2}{t}-1)}} \cdot \frac{1}{2} \left(\frac{2}{t}-1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}} \cdot \sqrt{\frac{2-t}{t}} \cdot \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t^2}{t} \cdot \frac{2-t}{t}} \cdot t^2} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)} t^2} \\ v = t \end{array} \right]$$

$$= \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2+2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}}$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in \langle -1; 0 \rangle \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in \langle -1; 0 \rangle \\ \text{t. j. } -1 \leq x \leq 0 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1-x \\ dt=-dx \end{array} \middle| \begin{array}{l} x \in \langle -1; 0 \rangle \\ t \in \langle 1; 2 \rangle \end{array} \right] = - \int \arcsin \sqrt{\frac{2-t}{t}} dt = - \int \arcsin \sqrt{\frac{2}{t}-1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t}-1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-(\frac{2}{t}-1)}} \cdot \frac{1}{2} \left(\frac{2}{t}-1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}} \cdot \sqrt{\frac{2-t}{t}} \cdot \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t^2}{t} \cdot \frac{2-t}{t}} \cdot t^2} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)} t^2} \\ v = t \end{array} \right]$$

$$= \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2+2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}}$$

$$= - \left[t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}} \right]$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in \langle -1; 0 \rangle \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in \langle -1; 0 \rangle \\ \text{t. j. } -1 \leq x \leq 0 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1-x \\ dt = -dx \end{array} \middle| \begin{array}{l} x \in \langle -1; 0 \rangle \\ t \in \langle 1; 2 \rangle \end{array} \right] = - \int \arcsin \sqrt{\frac{2-t}{t}} dt = - \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t} - 1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (\frac{2}{t} - 1)}} \cdot \frac{1}{2} \left(\frac{2}{t} - 1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2 - \frac{2}{t}} \sqrt{\frac{2}{t} - 1} \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t^2}{t} \cdot \frac{2-t}{t} \cdot t^2}} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)t^2}} \\ v = t \end{array} \right]$$

$$= \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t - t^2 - 2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2 + 2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t - \frac{3}{2})^2}}$$

$$= - \left[t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t - \frac{3}{2})^2}} \right] = \left[\begin{array}{l} \text{Subst. } z = t - \frac{3}{2} \\ dz = dt \end{array} \middle| \begin{array}{l} t \in (1; 2) \\ z \in (-\frac{1}{2}; \frac{1}{2}) \end{array} \right]$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in (-1; 0) \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \right] \begin{array}{l} x \in (-1; 0) \\ \text{t. j. } -1 \leq x \leq 0 \end{array}$$

$$= \left[\begin{array}{l} \text{Subst. } t=1-x \\ dt=-dx \end{array} \middle| \begin{array}{l} x \in (-1; 0) \\ t \in (1; 2) \end{array} \right] = - \int \arcsin \sqrt{\frac{2-t}{t}} dt = - \int \arcsin \sqrt{\frac{2}{t}-1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t}-1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-(\frac{2}{t}-1)}} \cdot \frac{1}{2} \left(\frac{2}{t}-1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}} \cdot \sqrt{\frac{2}{t}} \cdot \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t}{t} \cdot \frac{2}{t} \cdot t^2}} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)t^2}} \\ v = t \end{array} \right] = \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2+2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}}$$

$$= - \left[t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}} \right] = \left[\begin{array}{l} \text{Subst. } z=t-\frac{3}{2} \\ dz=dt \end{array} \middle| \begin{array}{l} t \in (1; 2) \\ z \in (-\frac{1}{2}; \frac{1}{2}) \end{array} \right]$$

$$= -t \arcsin \sqrt{\frac{2-t}{t}} - \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{(\frac{1}{2})^2 - z^2}}$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in (-1; 0) \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \right| \begin{array}{l} x \in (-1; 0) \\ \text{t. j. } -1 \leq x \leq 0 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1-x \\ dt=-dx \end{array} \middle| \begin{array}{l} x \in (-1; 0) \\ t \in (1; 2) \end{array} \right] = - \int \arcsin \sqrt{\frac{2-t}{t}} dt = - \int \arcsin \sqrt{\frac{2}{t}-1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t}-1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-(\frac{2}{t}-1)}} \cdot \frac{1}{2} \left(\frac{2}{t}-1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}} \cdot \sqrt{\frac{2}{t}} \cdot \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t}{t} \cdot \frac{2-t}{t} \cdot t^2}} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)}} \\ v = t \end{array} \right] = \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2+2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}}$$

$$= - \left[t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}} \right] = \left[\begin{array}{l} \text{Subst. } z=t-\frac{3}{2} \\ dz=dt \end{array} \middle| \begin{array}{l} t \in (1; 2) \\ z \in (-\frac{1}{2}; \frac{1}{2}) \end{array} \right]$$

$$= -t \arcsin \sqrt{\frac{2-t}{t}} - \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{(\frac{1}{2})^2 - z^2}} = -t \arcsin \sqrt{\frac{2-t}{t}} - \frac{\sqrt{2}}{2} \arcsin \frac{z}{\frac{1}{2}} + c_2$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in (-1; 0) \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right] x \in (-1; 0)$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1-x \\ dt = -dx \end{array} \left| \begin{array}{l} x \in (-1; 0) \\ t \in (1; 2) \end{array} \right. \right] = - \int \arcsin \sqrt{\frac{2-t}{t}} dt = - \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t} - 1} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (\frac{2}{t} - 1)}} \cdot \frac{1}{2} \left(\frac{2}{t} - 1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2 - \frac{2}{t}} \sqrt{\frac{2-t}{t}} \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t^2}{t} \cdot \frac{2-t}{t} \cdot t^2}} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)^2}} \\ v = t \end{array} \right. \right] = \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t - t^2 - 2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2 + 2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t - \frac{3}{2})^2}}$$

$$= - \left[t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t - \frac{3}{2})^2}} \right] = \left[\begin{array}{l} \text{Subst. } z = t - \frac{3}{2} \\ dz = dt \end{array} \left| \begin{array}{l} t \in (1; 2) \\ z \in (-\frac{1}{2}; \frac{1}{2}) \end{array} \right. \right]$$

$$= -t \arcsin \sqrt{\frac{2-t}{t}} - \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{(\frac{1}{2})^2 - z^2}} = -t \arcsin \sqrt{\frac{2-t}{t}} - \frac{\sqrt{2}}{2} \arcsin \frac{z}{\frac{1}{2}} + c_2$$

$$= -(1-x) \arcsin \sqrt{\frac{1+x}{1-x}} - \frac{\sqrt{2}}{2} \arcsin 2z + c_2$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in (-1; 0) \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \right] \quad \begin{array}{l} x \in (-1; 0) \\ \text{t. j. } -1 \leq x \leq 0 \end{array}$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1-x \\ dt = -dx \end{array} \middle| \begin{array}{l} x \in (-1; 0) \\ t \in (1; 2) \end{array} \right] = - \int \arcsin \sqrt{\frac{2-t}{t}} dt = - \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t} - 1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (\frac{2}{t} - 1)}} \cdot \frac{1}{2} \left(\frac{2}{t} - 1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2 - \frac{2}{t}} \cdot \sqrt{\frac{2}{t}} \cdot \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t}{t} \cdot \frac{2}{t} \cdot t^2}} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)^2}} \\ v = t \end{array} \right] = \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t - t^2 - 2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2 + 2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t - \frac{3}{2})^2}}$$

$$= - \left[t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t - \frac{3}{2})^2}} \right] = \left[\begin{array}{l} \text{Subst. } z = t - \frac{3}{2} \\ dz = dt \end{array} \middle| \begin{array}{l} t \in (1; 2) \\ z \in (-\frac{1}{2}; \frac{1}{2}) \end{array} \right]$$

$$= -t \arcsin \sqrt{\frac{2-t}{t}} - \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{(\frac{1}{2})^2 - z^2}} = -t \arcsin \sqrt{\frac{2-t}{t}} - \frac{\sqrt{2}}{2} \arcsin \frac{z}{\frac{1}{2}} + c_2$$

$$= -(1-x) \arcsin \sqrt{\frac{1+x}{1-x}} - \frac{\sqrt{2}}{2} \arcsin 2z + c_2 = \left[\begin{array}{l} 2z = 2(t - \frac{3}{2}) = 2(-x - \frac{1}{2}) \\ = -2x - 1 = -(2x + 1) \end{array} \right]$$

Koniec 10. časti príklady b – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}} dx = (x-1) \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin (2x+1) + c_2$$

$$= \left[\begin{array}{l} 0 \leq \frac{1+x}{1-x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1-x > 0 \Leftrightarrow 1 > x: \quad 0 \leq 1+x \leq 1-x, \text{ t. j. } -1 \leq x, 2x \leq 0, \text{ t. j. } x \in (-1; 0) \\ 1-x < 0 \Leftrightarrow 1 < x: \quad 0 \geq 1+x \geq 1-x, \text{ t. j. } -1 \geq x, 2x \geq 0, \text{ t. j. } x \text{ neexistuje} \end{array} \right] \begin{array}{l} x \in (-1; 0) \\ \text{t. j. } -1 \leq x \leq 0 \end{array}$$

$$= \left[\begin{array}{l} \text{Subst. } t=1-x \\ dt = -dx \end{array} \middle| \begin{array}{l} x \in (-1; 0) \\ t \in (1; 2) \end{array} \right] = - \int \arcsin \sqrt{\frac{2-t}{t}} dt = - \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t} - 1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-(\frac{2}{t}-1)}} \cdot \frac{1}{2} \left(\frac{2}{t}-1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}} \cdot \frac{2}{t} \cdot \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t}{t} \cdot \frac{2-t}{t} \cdot t^2}} = \frac{-1}{\sqrt{2(t-1)(2-t)t^2}} \\ v = t \end{array} \right] = \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2+2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}}$$

$$= - \left[t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}} \right] = \left[\begin{array}{l} \text{Subst. } z = t - \frac{3}{2} \\ dz = dt \end{array} \middle| \begin{array}{l} t \in (1; 2) \\ z \in (-\frac{1}{2}; \frac{1}{2}) \end{array} \right]$$

$$= -t \arcsin \sqrt{\frac{2-t}{t}} - \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{(\frac{1}{2})^2 - z^2}} = -t \arcsin \sqrt{\frac{2-t}{t}} - \frac{\sqrt{2}}{2} \arcsin \frac{z}{\frac{1}{2}} + c_2$$

$$= -(1-x) \arcsin \sqrt{\frac{1+x}{1-x}} - \frac{\sqrt{2}}{2} \arcsin 2z + c_2 = \left[\begin{array}{l} 2z = 2(t - \frac{3}{2}) = 2(-x - \frac{1}{2}) \\ = -2x - 1 = -(2x+1) \end{array} \right]$$

$$= (x-1) \arcsin \sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2} \arcsin (2x+1) + c_2, \quad x \in (-1; 0), \quad c \in R.$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{1-x}{1+x} \leq 1 & 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in \langle 0; 1 \rangle \\ x \neq 1 & 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in \langle 0; 1 \rangle \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{1-x}{1+x} \leq 1 & 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in \langle 0; 1 \rangle \\ x \neq 1 & 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \left. \begin{array}{l} x \in \langle 0; 1 \rangle \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l|l} u = \arcsin \sqrt{\frac{1-x}{1+x}} & u' = \frac{1}{\sqrt{1-\frac{1-x}{1+x}}} \cdot \frac{1}{2} \left(\frac{1-x}{1+x} \right)^{-\frac{1}{2}} \cdot \frac{-(1+x) - (1-x)}{(1+x)^2} = \frac{-\frac{1}{2} \cdot 2}{\sqrt{\frac{1+x-(1-x)}{1+x}} \sqrt{\frac{1-x}{1+x}} \sqrt{(1+x)^4}} = \frac{-1}{\sqrt{\frac{2x}{1+x} \cdot \frac{1-x}{1+x}} \cdot (1+x)^2} \\ v' = 1 & v = x \end{array} \right. \left. \begin{array}{l} = \frac{-1}{\sqrt{2x(1-x)} \cdot (1+x)^2} = \frac{-1}{\sqrt{2} |1+x| \sqrt{x(1-x)}} = \frac{-\sqrt{2}}{2(1+x) \sqrt{x(1-x)}} = \frac{-\sqrt{2}}{2(x+1) \sqrt{-x^2+x}} \end{array} \right]$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{1-x}{1+x} \leq 1 & 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in (0; 1) \\ x \neq 1 & 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \left. \begin{array}{l} x \in (0; 1) \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l|l} u = \arcsin \sqrt{\frac{1-x}{1+x}} & u' = \frac{1}{\sqrt{1-\frac{1-x}{1+x}}} \cdot \frac{1}{2} \left(\frac{1-x}{1+x} \right)^{-\frac{1}{2}} \cdot \frac{-(1+x)-(1-x)}{(1+x)^2} = \frac{-\frac{1}{2} \cdot 2}{\sqrt{\frac{1+x-(1-x)}{1+x}} \sqrt{\frac{1-x}{1+x}} \sqrt{(1+x)^4}} = \frac{-1}{\sqrt{\frac{2x}{1+x} \cdot \frac{1-x}{1+x}} \cdot (1+x)^2} \\ v' = 1 & v = x \end{array} \right. \left. \begin{array}{l} = \frac{-1}{\sqrt{2x(1-x)} \cdot (1+x)^2} = \frac{-1}{\sqrt{2} |1+x| \sqrt{x(1-x)}} = \frac{-\sqrt{2}}{2(1+x) \sqrt{x(1-x)}} = \frac{-\sqrt{2}}{2(x+1) \sqrt{-x^2+x}} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x+1) \sqrt{-x^2+x}} dx$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{1-x}{1+x} \leq 1 & 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in \langle 0; 1 \rangle \\ x \neq 1 & 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \left. \begin{array}{l} x \in \langle 0; 1 \rangle \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l|l} u = \arcsin \sqrt{\frac{1-x}{1+x}} & u' = \frac{1}{\sqrt{1-\frac{1-x}{1+x}}} \cdot \frac{1}{2} \left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}} \cdot \frac{-(1+x)-(1-x)}{(1+x)^2} = \frac{-\frac{1}{2} \cdot 2}{\sqrt{\frac{1+x-(1-x)}{1+x}} \sqrt{\frac{1-x}{1+x}} \sqrt{(1+x)^4}} = \frac{-1}{\sqrt{\frac{2x}{1+x} \cdot \frac{1-x}{1+x}} \cdot (1+x)^2} \\ v' = 1 & v = x \end{array} \right. \left. \begin{array}{l} = \frac{-1}{\sqrt{2x(1-x)} \cdot (1+x)^2} = \frac{-1}{\sqrt{2} |1+x| \sqrt{x(1-x)}} = \frac{-\sqrt{2}}{2(1+x) \sqrt{x(1-x)}} = \frac{-\sqrt{2}}{2(x+1) \sqrt{-x^2+x}} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x+1) \sqrt{-x^2+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x+1-1}{(x+1) \sqrt{-x^2+x}} dx$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{1-x}{1+x} \leq 1 & 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in (0; 1) \\ x \neq 1 & 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \left. \begin{array}{l} x \in (0; 1) \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l|l} u = \arcsin \sqrt{\frac{1-x}{1+x}} & u' = \frac{1}{\sqrt{1-\frac{1-x}{1+x}}} \cdot \frac{1}{2} \left(\frac{1-x}{1+x} \right)^{-\frac{1}{2}} \cdot \frac{-(1+x)-(1-x)}{(1+x)^2} = \frac{-\frac{1}{2} \cdot 2}{\sqrt{\frac{1+x-(1-x)}{1+x}} \sqrt{\frac{1-x}{1+x}} \sqrt{(1+x)^4}} = \frac{-1}{\sqrt{\frac{2x}{1+x} \cdot \frac{1-x}{1+x}} \cdot (1+x)^2} \\ v' = 1 & v = x \end{array} \right. \left. \begin{array}{l} = \frac{-1}{\sqrt{2x(1-x)} \cdot (1+x)^2} = \frac{-1}{\sqrt{2} |1+x| \sqrt{x(1-x)}} = \frac{-\sqrt{2}}{2(1+x) \sqrt{x(1-x)}} = \frac{-\sqrt{2}}{2(x+1) \sqrt{-x^2+x}} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x+1) \sqrt{-x^2+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x+1-1}{(x+1) \sqrt{-x^2+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2+x}} - \frac{\sqrt{2}}{2} \int \frac{dx}{(x+1) \sqrt{-x^2+x}}$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{1-x}{1+x} \leq 1 & 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in (0; 1) \\ x \neq 1 & 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \left. \begin{array}{l} x \in (0; 1) \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l|l} u = \arcsin \sqrt{\frac{1-x}{1+x}} & u' = \frac{1}{\sqrt{1-\frac{1-x}{1+x}}} \cdot \frac{1}{2} \left(\frac{1-x}{1+x} \right)^{-\frac{1}{2}} \cdot \frac{-(1+x)-(1-x)}{(1+x)^2} = \frac{-\frac{1}{2} \cdot 2}{\sqrt{\frac{1+x-(1-x)}{1+x}} \sqrt{\frac{1-x}{1+x}} \sqrt{(1+x)^4}} = \frac{-1}{\sqrt{\frac{2x}{1+x} \cdot \frac{1-x}{1+x}} \cdot (1+x)^2} \\ v' = 1 & v = x \end{array} \right. \left. \begin{array}{l} = \frac{-1}{\sqrt{2x(1-x)} \cdot (1+x)^2} = \frac{-1}{\sqrt{2} |1+x| \sqrt{x(1-x)}} = \frac{-\sqrt{2}}{2(1+x) \sqrt{x(1-x)}} = \frac{-\sqrt{2}}{2(x+1) \sqrt{-x^2+x}} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x+1) \sqrt{-x^2+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x+1-1}{(x+1) \sqrt{-x^2+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2+x}} - \frac{\sqrt{2}}{2} \int \frac{dx}{(x+1) \sqrt{-x^2+x}}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } t = x - \frac{1}{2} & dt = dx & x \in (0; 1) & t \in (-\frac{1}{2}; \frac{1}{2}) \\ \text{Subst. } z = \frac{1}{x} & x = z^{-1} & dx = -\frac{dz}{z^2} & z \in (1; \infty) \end{array} \right. \left. \begin{array}{l} -x^2+x = -(x^2-x) = -(x^2-2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x-\frac{1}{2})^2 \\ x+1 = \frac{1}{z} + 1 = \frac{1+z}{z}, \quad \sqrt{-x^2+x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \right]$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{1-x}{1+x}} \\ v' = 1 \end{array} \right] \quad \left[\begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{1-x}{1+x}}} \cdot \frac{1}{2} \left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}} \cdot \frac{-(1+x)-(1-x)}{(1+x)^2} = \frac{-\frac{1}{2} \cdot 2}{\sqrt{\frac{1+x-(1-x)}{1+x}} \sqrt{\frac{1-x}{1+x}} \sqrt{(1+x)^4}} = \frac{-1}{\sqrt{\frac{2x}{1+x} \cdot \frac{1-x}{1+x}} \cdot (1+x)^4} \\ v = x \end{array} \right] = \frac{-1}{\sqrt{2x(1-x)} \cdot (1+x)^2} = \frac{-1}{\sqrt{2} |1+x| \sqrt{x(1-x)}} = \frac{-\sqrt{2}}{2(1+x)\sqrt{x(1-x)}} = \frac{-\sqrt{2}}{2(x+1)\sqrt{-x^2+x}}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x+1)\sqrt{-x^2+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x+1-1}{(x+1)\sqrt{-x^2+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2+x}} - \frac{\sqrt{2}}{2} \int \frac{dx}{(x+1)\sqrt{-x^2+x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - \frac{1}{2} \left| \frac{dt}{dx} = 1 \right| x \in (0; 1) \left| t \in \left(-\frac{1}{2}; \frac{1}{2}\right) \right| -x^2+x = -(x^2-x) = -(x^2-2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = \left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2 \\ \text{Subst. } z = \frac{1}{x} \left| \frac{dz}{dx} = -\frac{1}{x^2} \right| x \in (1; \infty) \left| x+1 = \frac{1}{z} + 1 = \frac{1+z}{z}, \sqrt{-x^2+x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \right| \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} - \frac{\sqrt{2}}{2} \int \frac{-\frac{dz}{z^2}}{\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x+1)\sqrt{-x^2+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x+1-1}{(x+1)\sqrt{-x^2+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2+x}} - \frac{\sqrt{2}}{2} \int \frac{dx}{(x+1)\sqrt{-x^2+x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - \frac{1}{2} \mid dt = dx \mid x \in (0; 1) \mid t \in (-\frac{1}{2}; \frac{1}{2}) \mid -x^2 + x = -(x^2 - x) = -(x^2 - 2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x - \frac{1}{2})^2 \\ \text{Subst. } z = \frac{1}{x} \mid x = z^{-1} \mid dx = -\frac{dz}{z^2} \mid z \in (1; \infty) \mid x+1 = \frac{1}{z} + 1 = \frac{1+z}{z}, \quad \sqrt{-x^2+x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} - \frac{\sqrt{2}}{2} \int \frac{-\frac{dz}{z^2}}{\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} + \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}}$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x}{(x+1)\sqrt{-x^2+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x+1-1}{(x+1)\sqrt{-x^2+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2+x}} - \frac{\sqrt{2}}{2} \int \frac{dx}{(x+1)\sqrt{-x^2+x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - \frac{1}{2} \mid dt = dx \mid x \in (0; 1) \mid t \in (-\frac{1}{2}; \frac{1}{2}) \mid -x^2 + x = -(x^2 - x) = -(x^2 - 2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x - \frac{1}{2})^2 \\ \text{Subst. } z = \frac{1}{x} \mid x = z^{-1} \mid dx = -\frac{dz}{z^2} \mid z \in (1; \infty) \mid x+1 = \frac{1}{z} + 1 = \frac{1+z}{z}, \sqrt{-x^2+x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} - \frac{\sqrt{2}}{2} \int \frac{-\frac{dz}{z^2}}{\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} + \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \mid u^2 = z-1 \\ z = u^2 + 1 \mid dz = 2u du \\ z \in (1; \infty) \mid u \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{x+1-1}{(x+1)\sqrt{-x^2+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2+x}} - \frac{\sqrt{2}}{2} \int \frac{dx}{(x+1)\sqrt{-x^2+x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - \frac{1}{2} \left| \begin{array}{l} dt = dx \\ x \in (0; 1) \end{array} \right. \left| \begin{array}{l} t \in (-\frac{1}{2}; \frac{1}{2}) \\ -x^2 + x = -(x^2 - x) = -(x^2 - 2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x - \frac{1}{2})^2 \end{array} \right. \\ \text{Subst. } z = \frac{1}{x} \left| \begin{array}{l} x = z^{-1} \\ dx = -\frac{dz}{z^2} \end{array} \right. \left| \begin{array}{l} z \in (1; \infty) \\ x+1 = \frac{1}{z} + 1 = \frac{1+z}{z}, \quad \sqrt{-x^2+x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \right. \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} - \frac{\sqrt{2}}{2} \int \frac{-\frac{dz}{z^2}}{\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} + \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \left| \begin{array}{l} u^2 = z-1 \\ z = u^2+1 \\ dz = 2u du \\ z \in (1; \infty) \end{array} \right. \left| \begin{array}{l} u \in (0; \infty) \end{array} \right. \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \frac{\sqrt{2}}{2} \int \frac{2u du}{(u^2+2)u}$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dx}{\sqrt{-x^2+x}} - \frac{\sqrt{2}}{2} \int \frac{dx}{(x+1)\sqrt{-x^2+x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - \frac{1}{2} \left| \begin{array}{l} dt = dx \\ x \in (0; 1) \end{array} \right| \begin{array}{l} t \in (-\frac{1}{2}; \frac{1}{2}) \\ -x^2 + x = -(x^2 - x) = -(x^2 - 2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x - \frac{1}{2})^2 \end{array} \\ \text{Subst. } z = \frac{1}{x} \left| \begin{array}{l} x = z^{-1} \\ dx = -\frac{dz}{z^2} \end{array} \right| \begin{array}{l} z \in (1; \infty) \\ x+1 = \frac{1}{z} + 1 = \frac{1+z}{z}, \quad \sqrt{-x^2+x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} - \frac{\sqrt{2}}{2} \int \frac{-\frac{dz}{z^2}}{\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} + \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \left| \begin{array}{l} u^2 = z-1 \\ z = u^2+1 \\ dz = 2u du \\ z \in (1; \infty) \end{array} \right| \begin{array}{l} u \in (0; \infty) \end{array} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \frac{\sqrt{2}}{2} \int \frac{2u du}{(u^2+2)u}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \sqrt{2} \int \frac{du}{u^2+2}$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x - \frac{1}{2} \mid dt = dx \mid x \in (0; 1) \mid t \in (-\frac{1}{2}; \frac{1}{2}) \mid -x^2 + x = -(x^2 - x) = -(x^2 - 2 \cdot \frac{x}{2} + \frac{1}{4}) + \frac{1}{4} = (\frac{1}{2})^2 - (x - \frac{1}{2})^2 \\ \text{Subst. } z = \frac{1}{x} \mid x = z^{-1} \mid dx = -\frac{dz}{z^2} \mid z \in (1; \infty) \mid x + 1 = \frac{1}{z} + 1 = \frac{1+z}{z}, \quad \sqrt{-x^2 + x} = \sqrt{-\frac{1}{z^2} + \frac{1}{z}} = \sqrt{\frac{-1+z}{z^2}} = \frac{\sqrt{z-1}}{z} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} - \frac{\sqrt{2}}{2} \int \frac{-\frac{dz}{z^2}}{\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} + \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \mid u^2 = z-1 \\ z = u^2 + 1 \mid dz = 2u du \\ z \in (1; \infty) \mid u \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \frac{\sqrt{2}}{2} \int \frac{2u du}{(u^2+2)u}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \sqrt{2} \int \frac{du}{u^2+2}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \frac{\sqrt{2}}{\sqrt{2}} \operatorname{arctg} \frac{u}{\sqrt{2}} + c_1$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4}-t^2}} - \frac{\sqrt{2}}{2} \int \frac{-\frac{dz}{z^2}}{\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} + \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \mid u^2 = z-1 \\ z = u^2 + 1 \mid dz = 2u du \\ z \in (1; \infty) \mid u \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \frac{\sqrt{2}}{2} \int \frac{2u du}{(u^2+2)u}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \sqrt{2} \int \frac{du}{u^2+2}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \frac{\sqrt{2}}{2} \operatorname{arctg} \frac{u}{\sqrt{2}} + c_1$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin (2x-1) + \operatorname{arctg} \frac{\sqrt{z-1}}{\sqrt{2}} + c_1$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sqrt{\frac{1}{4}-t^2}} - \frac{\sqrt{2}}{2} \int \frac{-\frac{dz}{z^2}}{\frac{z+1}{z} \cdot \frac{\sqrt{z-1}}{z}}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} + \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \mid u^2 = z-1 \\ z = u^2 + 1 \mid dz = 2u du \\ z \in (1; \infty) \mid u \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \frac{\sqrt{2}}{2} \int \frac{2u du}{(u^2+2)u}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \sqrt{2} \int \frac{du}{u^2+2}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \frac{\sqrt{2}}{2} \operatorname{arctg} \frac{u}{\sqrt{2}} + c_1$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin (2x-1) + \operatorname{arctg} \frac{\sqrt{z-1}}{\sqrt{2}} + c_1 = \left[\begin{array}{l} \frac{\sqrt{z-1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \sqrt{\frac{1}{x}-1} \\ = \frac{1}{\sqrt{2}} \sqrt{\frac{1-x}{x}} = \sqrt{\frac{1-x}{2x}} \end{array} \right]$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx = x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin (2x-1) + \operatorname{arctg} \sqrt{\frac{1-x}{2x}} + c_1$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} + \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{z-1} \mid u^2 = z-1 \\ z = u^2 + 1 \mid dz = 2u du \\ z \in (1; \infty) \mid u \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \frac{\sqrt{2}}{2} \int \frac{2u du}{(u^2+2)u}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \sqrt{2} \int \frac{du}{u^2+2}$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \frac{\sqrt{2}}{\sqrt{2}} \operatorname{arctg} \frac{u}{\sqrt{2}} + c_1$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin (2x-1) + \operatorname{arctg} \frac{\sqrt{z-1}}{\sqrt{2}} + c_1 = \left[\begin{array}{l} \frac{\sqrt{z-1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \sqrt{\frac{1}{x} - 1} \\ = \frac{1}{\sqrt{2}} \sqrt{\frac{1-x}{x}} = \sqrt{\frac{1-x}{2x}} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin (2x-1) + \operatorname{arctg} \sqrt{\frac{1-x}{2x}} + c_1, \quad x \in (0; 1), \quad c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{1-x}{1+x} \leq 1 & 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in \langle 0; 1 \rangle \\ x \neq 1 & 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \left| \begin{array}{l} x \in \langle 0; 1 \rangle \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right. \right]$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1-x}{1+x} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in \langle 0; 1 \rangle \\ 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \left. \begin{array}{l} x \in \langle 0; 1 \rangle \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1+x \\ dt=dx \end{array} \left| \begin{array}{l} x \in \langle 0; 1 \rangle \\ t \in \langle 1; 2 \rangle \end{array} \right. \right]$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{1-x}{1+x} \leq 1 & 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in \langle 0; 1 \rangle \\ x \neq 1 & 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in \langle 0; 1 \rangle \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=1+x & x \in \langle 0; 1 \rangle \\ dt=dx & t \in \langle 1; 2 \rangle \end{array} \right] = \int \arcsin \sqrt{\frac{2-t}{t}} dt = \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{1-x}{1+x} \leq 1 & 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in \langle 0; 1 \rangle \\ x \neq 1 & 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \right| x \in \langle 0; 1 \rangle$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=1+x & x \in \langle 0; 1 \rangle \\ dt=dx & t \in \langle 1; 2 \rangle \end{array} \right] = \int \arcsin \sqrt{\frac{2-t}{t}} dt = \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

$$= \left[\begin{array}{l|l} u = \arcsin \sqrt{\frac{2}{t} - 1} & u' = \frac{1}{\sqrt{1 - (\frac{2}{t} - 1)}} \cdot \frac{1}{2} \left(\frac{2}{t} - 1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2 - \frac{2}{t}} \sqrt{\frac{2-t}{t}} \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t^2}{t} \cdot \frac{2-t}{t}} \cdot t^2} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)} t^2} \\ v' = 1 & v = t \end{array} \right] = \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t - t^2 - 2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2 + 2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t - \frac{3}{2})^2}}$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1-x}{1+x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in \langle 0; 1 \rangle \\ 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in \langle 0; 1 \rangle \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1+x \\ dt=dx \end{array} \middle| \begin{array}{l} x \in \langle 0; 1 \rangle \\ t \in \langle 1; 2 \rangle \end{array} \right] = \int \arcsin \sqrt{\frac{2-t}{t}} dt = \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t} - 1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (\frac{2}{t} - 1)}} \cdot \frac{1}{2} \left(\frac{2}{t} - 1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2 - \frac{2}{t}} \cdot \sqrt{\frac{2-t}{t}} \cdot \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t^2}{t} \cdot \frac{2-t}{t}} \cdot t^2} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)} t^2} \\ v = t \end{array} \right]$$

$$= \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2+2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t - \frac{3}{2})^2}}$$

$$= t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t - \frac{3}{2})^2}}$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1-x}{1+x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in (0; 1) \\ 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in (0; 1) \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1+x \\ dt=dx \end{array} \middle| \begin{array}{l} x \in (0; 1) \\ t \in (1; 2) \end{array} \right] = \int \arcsin \sqrt{\frac{2-t}{t}} dt = \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t} - 1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (\frac{2}{t} - 1)}} \cdot \frac{1}{2} \left(\frac{2}{t} - 1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2 - \frac{2}{t}} \sqrt{\frac{2}{t} - 1} \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t}{t} \cdot \frac{2-t}{t} \cdot t^2}} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)t^2}} \\ v = t \end{array} \right]$$

$$= \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2+2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t - \frac{3}{2})^2}}$$

$$= t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t - \frac{3}{2})^2}} = \left[\begin{array}{l} \text{Subst. } z = t - \frac{3}{2} \\ dz = dt \end{array} \middle| \begin{array}{l} t \in (1; 2) \\ z \in (-\frac{1}{2}; \frac{1}{2}) \end{array} \right]$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1-x}{1+x} \leq 1 \\ x \neq 1 \end{array} \middle| \begin{array}{l} 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in \langle 0; 1 \rangle \\ 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in \langle 0; 1 \rangle \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1+x \\ dt=dx \end{array} \middle| \begin{array}{l} x \in \langle 0; 1 \rangle \\ t \in \langle 1; 2 \rangle \end{array} \right] = \int \arcsin \sqrt{\frac{2-t}{t}} dt = \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t} - 1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-(\frac{2}{t}-1)}} \cdot \frac{1}{2} \left(\frac{2}{t}-1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}} \cdot \sqrt{\frac{2}{t}} \cdot \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t}{t} \cdot \frac{2-t}{t} \cdot t^2}} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)t^2}} \\ v = t \end{array} \right]$$

$$= \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2+2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}}$$

$$= t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}} = \left[\begin{array}{l} \text{Subst. } z=t-\frac{3}{2} \\ dz=dt \end{array} \middle| \begin{array}{l} t \in (1; 2) \\ z \in (-\frac{1}{2}; \frac{1}{2}) \end{array} \right]$$

$$= t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{(\frac{1}{2})^2 - z^2}}$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1-x}{1+x} \leq 1 \\ x \neq -1 \end{array} \middle| \begin{array}{l} 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in (0; 1) \\ 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \right| \begin{array}{l} x \in (0; 1) \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1+x \\ dt=dx \end{array} \middle| \begin{array}{l} x \in (0; 1) \\ t \in (1; 2) \end{array} \right] = \int \arcsin \sqrt{\frac{2-t}{t}} dt = \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t} - 1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-(\frac{2}{t}-1)}} \cdot \frac{1}{2} \left(\frac{2}{t}-1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}} \sqrt{\frac{2}{t}} \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-2}{t} \cdot \frac{2-t}{t} \cdot t^2}} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)t^2}} \\ v = t \end{array} \right] = \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2+2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}}$$

$$= t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}} = \left[\begin{array}{l} \text{Subst. } z=t-\frac{3}{2} \\ dz=dt \end{array} \middle| \begin{array}{l} t \in (1; 2) \\ z \in (-\frac{1}{2}; \frac{1}{2}) \end{array} \right]$$

$$= t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{(\frac{1}{2})^2 - z^2}} = t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \arcsin \frac{z}{\frac{1}{2}} + c_2$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1-x}{1+x} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in (0; 1) \\ 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right. \left. \begin{array}{l} x \in (0; 1) \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1+x \\ dt=dx \end{array} \left| \begin{array}{l} x \in (0; 1) \\ t \in (1; 2) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{2-t}{t}} dt = \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t} - 1} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-(\frac{2}{t}-1)}} \cdot \frac{1}{2} \left(\frac{2}{t}-1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}} \sqrt{\frac{2}{t}} \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t^2}{t} \cdot \frac{2-t}{t} \cdot t^2}} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)}} \\ v = t \end{array} \right. \right] = \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2+2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}}$$

$$= t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}} = \left[\begin{array}{l} \text{Subst. } z=t-\frac{3}{2} \\ dz=dt \end{array} \left| \begin{array}{l} t \in (1; 2) \\ z \in (-\frac{1}{2}; \frac{1}{2}) \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{(\frac{1}{2})^2 - z^2}} = t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \arcsin \frac{z}{\frac{1}{2}} + c_2$$

$$= (1+x) \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2z + c_2$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{1-x}{1+x} \leq 1 \\ x \neq -1 \end{array} \middle| \begin{array}{l} 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in (0; 1) \\ 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \right| \begin{array}{l} x \in (0; 1) \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1+x \\ dt=dx \end{array} \middle| \begin{array}{l} x \in (0; 1) \\ t \in (1; 2) \end{array} \right] = \int \arcsin \sqrt{\frac{2-t}{t}} dt = \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t} - 1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-(\frac{2}{t}-1)}} \cdot \frac{1}{2} \left(\frac{2}{t}\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}} \sqrt{\frac{2}{t}} \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-t^2}{t} \cdot \frac{2-t}{t} \cdot t^2}} = \frac{-1}{\sqrt{2} \sqrt{(t-1)(2-t)t^2}} \\ v = t \end{array} \right] = \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2+2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}}$$

$$= t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}} = \left[\begin{array}{l} \text{Subst. } z=t-\frac{3}{2} \\ dz=dt \end{array} \middle| \begin{array}{l} t \in (1; 2) \\ z \in (-\frac{1}{2}; \frac{1}{2}) \end{array} \right]$$

$$= t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{(\frac{1}{2})^2 - z^2}} = t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \arcsin \frac{z}{\frac{1}{2}} + c_2$$

$$= (1+x) \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2z + c_2 = \left[2z = 2(t-\frac{3}{2}) = 2(x-\frac{1}{2}) = 2x-1 \right]$$

Koniec 10. časti príklady b – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}} dx = (x+1) \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin (2x-1) + c_2$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{1-x}{1+x} \leq 1 & 1+x > 0 \Leftrightarrow x > -1: \quad 0 \leq 1-x \leq 1+x, \text{ t. j. } x \leq 1, 0 \leq 2x, \text{ t. j. } x \in (0; 1) \\ x \neq 1 & 1+x < 0 \Leftrightarrow x < -1: \quad 0 \geq 1-x \geq 1+x, \text{ t. j. } x \geq 1, 0 \geq 2x, \text{ t. j. } x \text{ neexistuje} \end{array} \middle| \begin{array}{l} x \in (0; 1) \\ \text{t. j. } 0 \leq x \leq 1 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=1+x \\ dt=dx \end{array} \middle| \begin{array}{l} x \in (0; 1) \\ t \in (1; 2) \end{array} \right] = \int \arcsin \sqrt{\frac{2-t}{t}} dt = \int \arcsin \sqrt{\frac{2}{t} - 1} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{2}{t} - 1} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1-(\frac{2}{t}-1)}} \cdot \frac{1}{2} \left(\frac{2}{t}-1\right)^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}} \sqrt{\frac{2}{t}} \sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-2}{t} \cdot \frac{2-t}{t} \cdot t^2}} = \frac{-1}{\sqrt{2(t-1)(2-t)t^2}} \\ v = t \end{array} \right] = \frac{-\sqrt{2}}{2|t| \sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t \sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t \sqrt{-t^2+2 \cdot \frac{3t}{2} - \frac{9}{4} + \frac{9}{4} - 2}} = \frac{-\sqrt{2}}{2t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}}$$

$$= t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{\frac{1}{4} - (t-\frac{3}{2})^2}} = \left[\begin{array}{l} \text{Subst. } z=t-\frac{3}{2} \\ dz=dt \end{array} \middle| \begin{array}{l} t \in (1; 2) \\ z \in (-\frac{1}{2}; \frac{1}{2}) \end{array} \right]$$

$$= t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{(\frac{1}{2})^2 - z^2}} = t \arcsin \sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2} \arcsin \frac{z}{\frac{1}{2}} + c_2$$

$$= (1+x) \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2z + c_2 = \left[2z = 2(t-\frac{3}{2}) = 2(x-\frac{1}{2}) = 2x-1 \right]$$

$$= (x+1) \arcsin \sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin (2x-1) + c_2, \quad x \in (0; 1), \quad c \in R.$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x+1}{x-1} \leq 1 & x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x \neq 1 & x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right] \quad \begin{array}{l} x \in (-\infty; -1) \\ \text{t. j. } x \leq -1 \end{array}$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x+1}{x-1} \leq 1 & x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x \neq 1 & x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right] \quad \text{t. j. } x \leq -1$$

$$= \left[\begin{array}{l|l} u = \arcsin \sqrt{\frac{x+1}{x-1}} & u' = \frac{1}{\sqrt{1-\frac{x+1}{x-1}}} \cdot \frac{1}{2} \left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}} \cdot \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{\frac{1}{2} \cdot (-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}} \sqrt{\frac{x+1}{x-1}} \sqrt{(x-1)^4}} = \frac{-1}{\sqrt{\frac{-2}{x-1} \cdot \frac{x+1}{x-1}} \cdot (x-1)^4} \\ v' = 1 & v = x \end{array} \right] \\ = \frac{-1}{\sqrt{-2(x+1) \cdot (x-1)^2}} = \frac{-1}{\sqrt{2} |x-1| \sqrt{-(x+1)}} = \frac{-\sqrt{2}}{2(1-x) \sqrt{-(x+1)}} = \frac{\sqrt{2}}{2(x-1) \sqrt{-x-1}}$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x+1}{x-1}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{x+1}{x-1}}} \cdot \frac{1}{2} \left(\frac{x+1}{x-1} \right)^{-\frac{1}{2}} \cdot \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{\frac{1}{2} \cdot (-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}} \sqrt{\frac{x+1}{x-1}} \sqrt{(x-1)^4}} = \frac{-1}{\sqrt{\frac{-2}{x-1} \cdot \frac{x+1}{x-1} \cdot (x-1)^4}} \\ v = x \end{array} \right. \right]$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x-1)\sqrt{-x-1}}$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x+1}{x-1}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{x+1}{x-1}}} \cdot \frac{1}{2} \left(\frac{x+1}{x-1} \right)^{-\frac{1}{2}} \cdot \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{\frac{1}{2} \cdot (-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}} \sqrt{\frac{x+1}{x-1}} \sqrt{(x-1)^4}} = \frac{-1}{\sqrt{\frac{-2}{x-1} \cdot \frac{x+1}{x-1}} \cdot (x-1)^4} \\ v = x \end{array} \right. \right]$$

$$= \frac{-1}{\sqrt{-2(x+1) \cdot (x-1)^2}} = \frac{-1}{\sqrt{2} |x-1| \sqrt{-(x+1)}} = \frac{-\sqrt{2}}{2(1-x) \sqrt{-(x+1)}} = \frac{\sqrt{2}}{2(x-1) \sqrt{-x-1}}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x-1) \sqrt{-x-1}} = \left[\begin{array}{l} \text{Subst. } t = \sqrt{-x-1} \mid t^2 = -x-1 \mid x \in (-\infty; -1) \\ x = -t^2 - 1 \mid dx = -2t dt \mid t \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x+1}{x-1}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{x+1}{x-1}}} \cdot \frac{1}{2} \left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}} \cdot \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{\frac{1}{2} \cdot (-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}} \sqrt{\frac{x+1}{x-1}} \sqrt{(x-1)^4}} = \frac{-1}{\sqrt{\frac{-2}{x-1} \cdot \frac{x+1}{x-1}} \cdot (x-1)^4} \\ v = x \end{array} \right. \right]$$

$$= \frac{-1}{\sqrt{-2(x+1) \cdot (x-1)^2}} = \frac{-1}{\sqrt{2}|x-1| \sqrt{-(x+1)}} = \frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}} = \frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x-1)\sqrt{-x-1}} = \left[\begin{array}{l} \text{Subst. } t = \sqrt{-x-1} \mid t^2 = -x-1 \mid x \in (-\infty; -1) \\ x = -t^2-1 \mid dx = -2t dt \mid t \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{(-t^2-1)(-2t) dt}{(-t^2-2)t}$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x+1}{x-1}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{x+1}{x-1}}} \cdot \frac{1}{2} \left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}} \cdot \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{\frac{1}{2} \cdot (-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}} \sqrt{\frac{x+1}{x-1}} \sqrt{(x-1)^4}} = \frac{-1}{\sqrt{\frac{-2}{x-1} \cdot \frac{x+1}{x-1} \cdot (x-1)^4}} \\ v = x \end{array} \right. \right]$$

$$= \frac{-1}{\sqrt{-2(x+1) \cdot (x-1)^2}} = \frac{-1}{\sqrt{2}|x-1| \sqrt{-(x+1)}} = \frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}} = \frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x-1)\sqrt{-x-1}} = \left[\begin{array}{l} \text{Subst. } t = \sqrt{-x-1} \mid t^2 = -x-1 \mid x \in (-\infty; -1) \\ x = -t^2-1 \mid dx = -2t dt \mid t \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{(-t^2-1)(-2t) dt}{(-t^2-2)t} = x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} \int \frac{(t^2+1) dt}{t^2+2}$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x+1}{x-1}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{x+1}{x-1}}} \cdot \frac{1}{2} \left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}} \cdot \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{\frac{1}{2} \cdot (-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}} \sqrt{\frac{x+1}{x-1}} \sqrt{(x-1)^4}} = \frac{-1}{\sqrt{\frac{-2}{x-1} \cdot \frac{x+1}{x-1} \cdot (x-1)^4}} \\ v = x \end{array} \right. \right]$$

$$= \frac{-1}{\sqrt{-2(x+1) \cdot (x-1)^2}} = \frac{-1}{\sqrt{2}|x-1| \sqrt{-(x+1)}} = \frac{-\sqrt{2}}{2(1-x) \sqrt{-(x+1)}} = \frac{\sqrt{2}}{2(x-1) \sqrt{-x-1}}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x-1) \sqrt{-x-1}} = \left[\begin{array}{l} \text{Subst. } t = \sqrt{-x-1} \mid t^2 = -x-1 \mid x \in (-\infty; -1) \\ x = -t^2-1 \mid dx = -2t dt \mid t \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{(-t^2-1)(-2t) dt}{(-t^2-2)t} = x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} \int \frac{(t^2+1) dt}{t^2+2}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} \int \frac{(t^2+2-1) dt}{t^2+2}$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x+1}{x-1}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{x+1}{x-1}}} \cdot \frac{1}{2} \left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}} \cdot \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{\frac{1}{2} \cdot (-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}} \sqrt{\frac{x+1}{x-1}} \sqrt{(x-1)^4}} = \frac{-1}{\sqrt{\frac{-2}{x-1} \cdot \frac{x+1}{x-1} \cdot (x-1)^4}} \\ v = x \end{array} \right. \right]$$

$$= \frac{-1}{\sqrt{-2(x+1) \cdot (x-1)^2}} = \frac{-1}{\sqrt{2}|x-1| \sqrt{-(x+1)}} = \frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}} = \frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x-1)\sqrt{-x-1}} = \left[\begin{array}{l} \text{Subst. } t = \sqrt{-x-1} \quad t^2 = -x-1 \quad x \in (-\infty; -1) \\ x = -t^2-1 \quad dx = -2t dt \quad t \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{(-t^2-1)(-2t) dt}{(-t^2-2)t} = x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} \int \frac{(t^2+1) dt}{t^2+2}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} \int \frac{(t^2+2-1) dt}{t^2+2} = x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} \int dt - \sqrt{2} \int \frac{dt}{t^2+(\sqrt{2})^2}$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x+1}{x-1}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{x+1}{x-1}}} \cdot \frac{1}{2} \left(\frac{x+1}{x-1} \right)^{-\frac{1}{2}} \cdot \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{\frac{1}{2} \cdot (-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}} \sqrt{\frac{x+1}{x-1}} \sqrt{(x-1)^4}} = \frac{-1}{\sqrt{\frac{-2}{x-1} \cdot \frac{x+1}{x-1} \cdot (x-1)^4}} \\ v = x \end{array} \right. \right]$$

$$= \frac{-1}{\sqrt{-2(x+1) \cdot (x-1)^2}} = \frac{-1}{\sqrt{2}|x-1| \sqrt{-(x+1)}} = \frac{-\sqrt{2}}{2(1-x) \sqrt{-(x+1)}} = \frac{\sqrt{2}}{2(x-1) \sqrt{-x-1}}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x-1) \sqrt{-x-1}} = \left[\begin{array}{l} \text{Subst. } t = \sqrt{-x-1} \quad t^2 = -x-1 \quad x \in (-\infty; -1) \\ x = -t^2-1 \quad dx = -2t dt \quad t \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{(-t^2-1)(-2t) dt}{(-t^2-2)t} = x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} \int \frac{(t^2+1) dt}{t^2+2}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} \int \frac{(t^2+2-1) dt}{t^2+2} = x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} \int dt - \sqrt{2} \int \frac{dt}{t^2+(\sqrt{2})^2}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} t - \frac{\sqrt{2}}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + c_1$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx = x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2(-x-1)} - \operatorname{arctg} \sqrt{\frac{-x-1}{2}} + c_1$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x+1}{x-1}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{x+1}{x-1}}} \cdot \frac{1}{2} \left(\frac{x+1}{x-1} \right)^{-\frac{1}{2}} \cdot \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{\frac{1}{2} \cdot (-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}} \sqrt{\frac{x+1}{x-1}} \sqrt{(x-1)^4}} = \frac{-1}{\sqrt{\frac{-2}{x-1} \cdot \frac{x+1}{x-1} \cdot (x-1)^4}} \\ v = x \end{array} \right. \right]$$

$$= \frac{-1}{\sqrt{-2(x+1) \cdot (x-1)^2}} = \frac{-1}{\sqrt{2}|x-1| \sqrt{-(x+1)}} = \frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}} = \frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x-1)\sqrt{-x-1}} = \left[\begin{array}{l} \text{Subst. } t = \sqrt{-x-1} \quad t^2 = -x-1 \quad x \in (-\infty; -1) \\ x = -t^2 - 1 \quad dx = -2t dt \quad t \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2} \int \frac{(-t^2-1)(-2t) dt}{(-t^2-2)t} = x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} \int \frac{(t^2+1) dt}{t^2+2}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} \int \frac{(t^2+2-1) dt}{t^2+2} = x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} \int dt - \sqrt{2} \int \frac{dt}{t^2+(\sqrt{2})^2}$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2} t - \frac{\sqrt{2}}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + c_1$$

$$= x \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2(-x-1)} - \operatorname{arctg} \sqrt{\frac{-x-1}{2}} + c_1, \quad x \in (-\infty; -1), \quad c_1 \in \mathbb{R}.$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x+1}{x-1} \leq 1 & x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x \neq 1 & x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right] \quad \begin{array}{l} x \in (-\infty; -1) \\ \text{t. j. } x \leq -1 \end{array}$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (-\infty; -1) \\ t \in (-\infty; -2) \end{array} \right. \right]$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x+1}{x-1} \leq 1 & x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x \neq 1 & x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right] \quad \begin{array}{l} x \in (-\infty; -1) \\ \text{t. j. } x \leq -1 \end{array}$$

$$= \left[\begin{array}{l|l} \text{Subst. } t = x-1 & x \in (-\infty; -1) \\ dt = dx & t \in (-\infty; -2) \end{array} \right] = \int \arcsin \sqrt{\frac{t+2}{t}} dt = \int \arcsin \sqrt{1 + \frac{2}{t}} dt$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (-\infty; -1) \\ t \in (-\infty; -2) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{t+2}{t}} dt = \int \arcsin \sqrt{1 + \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 + \frac{2}{t}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 + \frac{2}{t})}} \cdot \frac{1}{2} (1 + \frac{2}{t})^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{-\frac{2}{t}} \sqrt{\frac{t+2}{t}} \sqrt{t^2}} = \frac{-1}{\sqrt{-2(t+2)t^2}} \\ v = t \end{array} \right. \right]$$

$$= \frac{-1}{\sqrt{2}|t| \sqrt{-(t+2)}} = \frac{-\sqrt{2}}{2(-t)\sqrt{-t-2}} = \frac{\sqrt{2}}{2t\sqrt{-t-2}}$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t\sqrt{-t-2}}$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (-\infty; -1) \\ t \in (-\infty; -2) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{t+2}{t}} dt = \int \arcsin \sqrt{1 + \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 + \frac{2}{t}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 + \frac{2}{t})}} \cdot \frac{1}{2} (1 + \frac{2}{t})^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{-\frac{2}{t}} \sqrt{\frac{t+2}{t}} \sqrt{t^2}} = \frac{-1}{\sqrt{-2(t+2)t^2}} \\ v = t \end{array} \right. \right]$$

$$= \frac{-1}{\sqrt{2}|t| \sqrt{-(t+2)}} = \frac{-\sqrt{2}}{2(-t)\sqrt{-t-2}} = \frac{\sqrt{2}}{2t\sqrt{-t-2}}$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t\sqrt{-t-2}} = \left[\begin{array}{l} \text{Subst. } z = -t-2 \\ dz = -dt \end{array} \left| \begin{array}{l} t \in (-\infty; -2) \\ z \in (0; \infty) \end{array} \right. \right]$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (-\infty; -1) \\ t \in (-\infty; -2) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{t+2}{t}} dt = \int \arcsin \sqrt{1 + \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 + \frac{2}{t}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 + \frac{2}{t})}} \cdot \frac{1}{2} (1 + \frac{2}{t})^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{-\frac{2}{t}} \sqrt{\frac{t+2}{t}} \sqrt{t^4}} = \frac{-1}{\sqrt{-2(t+2)t^2}} \\ v = t \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{-t-2}} = \left[\begin{array}{l} \text{Subst. } z = -t-2 \\ dz = -dt \end{array} \left| \begin{array}{l} t \in (-\infty; -2) \\ z \in (0; \infty) \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2} \int \frac{-dz}{\sqrt{z}}$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \\ \text{t. j. } x \leq -1 \end{array} \right. \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (-\infty; -1) \\ t \in (-\infty; -2) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{t+2}{t}} dt = \int \arcsin \sqrt{1 + \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 + \frac{2}{t}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 + \frac{2}{t})}} \cdot \frac{1}{2} (1 + \frac{2}{t})^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{-\frac{2}{t}} \sqrt{1 + \frac{2}{t}} \sqrt{t^4}} = \frac{-1}{\sqrt{-2(t+2)t^2}} \\ v = t \end{array} \right. \right]$$

$$= \frac{-1}{\sqrt{2}|t| \sqrt{-(t+2)}} = \frac{-\sqrt{2}}{2(-t)\sqrt{-t-2}} = \frac{\sqrt{2}}{2t\sqrt{-t-2}}$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t\sqrt{-t-2}} = \left[\begin{array}{l} \text{Subst. } z = -t-2 \\ dz = -dt \end{array} \left| \begin{array}{l} t \in (-\infty; -2) \\ z \in (0; \infty) \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2} \int \frac{-dz}{\sqrt{z}} = t \arcsin \sqrt{\frac{t+2}{t}} + \frac{\sqrt{2}}{2} \int z^{-\frac{1}{2}} dz$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \end{array} \right. \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (-\infty; -1) \\ t \in (-\infty; -2) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{t+2}{t}} dt = \int \arcsin \sqrt{1 + \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 + \frac{2}{t}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 + \frac{2}{t})}} \cdot \frac{1}{2} (1 + \frac{2}{t})^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{-\frac{2}{t}} \sqrt{\frac{t+2}{t}} \sqrt{t^2}} = \frac{-1}{\sqrt{-2(t+2)t^2}} \\ v = t \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t \sqrt{-t-2}} = \left[\begin{array}{l} \text{Subst. } z = -t-2 \\ dz = -dt \end{array} \left| \begin{array}{l} t \in (-\infty; -2) \\ z \in (0; \infty) \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2} \int \frac{-dz}{\sqrt{z}} = t \arcsin \sqrt{\frac{t+2}{t}} + \frac{\sqrt{2}}{2} \int z^{-\frac{1}{2}} dz$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} + \frac{\sqrt{2}}{2} \cdot \frac{z^{\frac{1}{2}}}{\frac{1}{2}} + c_2$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \\ \text{t. j. } x \leq -1 \end{array} \right. \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (-\infty; -1) \\ t \in (-\infty; -2) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{t+2}{t}} dt = \int \arcsin \sqrt{1 + \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 + \frac{2}{t}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 + \frac{2}{t})}} \cdot \frac{1}{2} (1 + \frac{2}{t})^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{-\frac{2}{t}} \sqrt{\frac{t+2}{t}} \sqrt{t^2}} = \frac{-1}{\sqrt{-2(t+2)t^2}} \\ v = t \end{array} \right. \right]$$

$$= \frac{-1}{\sqrt{2}|t| \sqrt{-(t+2)}} = \frac{-\sqrt{2}}{2(-t)\sqrt{-t-2}} = \frac{\sqrt{2}}{2t\sqrt{-t-2}}$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t\sqrt{-t-2}} = \left[\begin{array}{l} \text{Subst. } z = -t-2 \\ dz = -dt \end{array} \left| \begin{array}{l} t \in (-\infty; -2) \\ z \in (0; \infty) \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2} \int \frac{-dz}{\sqrt{z}} = t \arcsin \sqrt{\frac{t+2}{t}} + \frac{\sqrt{2}}{2} \int z^{-\frac{1}{2}} dz$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} + \frac{\sqrt{2}}{2} \cdot \frac{z^{\frac{1}{2}}}{\frac{1}{2}} + c_2 = t \arcsin \sqrt{\frac{t+2}{t}} + \sqrt{2z} + c_2$$

Koniec 10. časti príklady b – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}} dx = (x-1) \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2(-x-1)} + c_2$$

$$= \left[\begin{array}{l} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{array} \left| \begin{array}{l} x-1 > 0 \Leftrightarrow x > 1: \quad 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x, 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1 < 0 \Leftrightarrow x < 1: \quad 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x, 1 \geq -1, \text{ t. j. } x \in (-\infty; -1) \\ \text{t. j. } x \leq -1 \end{array} \right. \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (-\infty; -1) \\ t \in (-\infty; -2) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{t+2}{t}} dt = \int \arcsin \sqrt{1 + \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 + \frac{2}{t}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 + \frac{2}{t})}} \cdot \frac{1}{2} (1 + \frac{2}{t})^{-\frac{1}{2}} \cdot (-2t^{-2}) = \frac{-1}{\sqrt{-\frac{2}{t}} \sqrt{\frac{t+2}{t}} \sqrt{t}} = \frac{-1}{\sqrt{-2(t+2)t^2}} \\ v = t \end{array} \right. \right]$$

$$= \frac{-1}{\sqrt{2}|t| \sqrt{-(t+2)}} = \frac{-\sqrt{2}}{2(-t)\sqrt{-t-2}} = \frac{\sqrt{2}}{2t\sqrt{-t-2}}$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t\sqrt{-t-2}} = \left[\begin{array}{l} \text{Subst. } z = -t-2 = -x-1 \\ dz = -dt \end{array} \left| \begin{array}{l} t \in (-\infty; -2) \\ z \in (0; \infty) \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2} \int \frac{-dz}{\sqrt{z}} = t \arcsin \sqrt{\frac{t+2}{t}} + \frac{\sqrt{2}}{2} \int z^{-\frac{1}{2}} dz$$

$$= t \arcsin \sqrt{\frac{t+2}{t}} + \frac{\sqrt{2}}{2} \cdot \frac{z^{\frac{1}{2}}}{\frac{1}{2}} + c_2 = t \arcsin \sqrt{\frac{t+2}{t}} + \sqrt{2z} + c_2$$

$$= (x-1) \arcsin \sqrt{\frac{x+1}{x-1}} + \sqrt{2(-x-1)} + c_2, \quad x \in (-\infty; -1), \quad c_2 \in \mathbb{R}.$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x-1}{x+1} \leq 1 & x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in \langle 1; \infty \rangle \\ x \neq -1 & x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right| \begin{array}{l} x \in \langle 1; \infty \rangle \\ \text{t. j. } 1 \leq x \end{array} \right]$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x-1}{x+1} \leq 1 & x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in \langle 1; \infty \rangle \\ x \neq -1 & x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \left. \begin{array}{l} x \in \langle 1; \infty \rangle \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l|l} u = \arcsin \sqrt{\frac{x-1}{x+1}} & u' = \frac{1}{\sqrt{1-\frac{x-1}{x+1}}} \cdot \frac{1}{2} \left(\frac{x-1}{x+1} \right)^{-\frac{1}{2}} \cdot \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}} \sqrt{\frac{x-1}{x+1}} \sqrt{(x+1)^4}} = \frac{1}{\sqrt{\frac{2}{x+1} \cdot \frac{x-1}{x+1} \cdot (x+1)^4}} \\ v' = 1 & v = x \\ & = \frac{1}{\sqrt{2(x+1) \cdot (x+1)^2}} = \frac{1}{\sqrt{2} |x+1| \sqrt{x-1}} = \frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \end{array} \right]$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x-1}{x+1} \leq 1 & x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in \langle 1; \infty \rangle \\ x \neq -1 & x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right| x \in \langle 1; \infty \rangle$$

$$= \left[\begin{array}{l|l} u = \arcsin \sqrt{\frac{x-1}{x+1}} & u' = \frac{1}{\sqrt{1-\frac{x-1}{x+1}}} \cdot \frac{1}{2} \left(\frac{x-1}{x+1} \right)^{-\frac{1}{2}} \cdot \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}} \sqrt{\frac{x-1}{x+1}} \sqrt{(x+1)^4}} = \frac{1}{\sqrt{\frac{2}{x+1} \cdot \frac{x-1}{x+1} \cdot (x+1)^4}} \\ v' = 1 & v = x \end{array} \right| = \frac{1}{\sqrt{2(x+1) \cdot (x+1)^2}} = \frac{1}{\sqrt{2} |x+1| \sqrt{x-1}} = \frac{\sqrt{2}}{2(x+1)\sqrt{x-1}}$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x+1)\sqrt{x-1}}$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x-1}{x+1} \leq 1 \\ x \neq -1 \end{array} \left| \begin{array}{l} x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \\ x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right. \left. \begin{array}{l} x \in (1; \infty) \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x-1}{x+1}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{x-1}{x+1}}} \cdot \frac{1}{2} \left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}} \cdot \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}} \sqrt{\frac{x-1}{x+1}} \sqrt{(x+1)^4}} = \frac{1}{\sqrt{\frac{2}{x+1} \cdot \frac{x-1}{x+1}} \cdot (x+1)^4} \\ v = x \end{array} \right. \right. \left. \begin{array}{l} = \frac{1}{\sqrt{2(x+1) \cdot (x+1)^2}} = \frac{1}{\sqrt{2} |x+1| \sqrt{x-1}} = \frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x+1)\sqrt{x-1}} = \left[\begin{array}{l} \text{Subst. } t = \sqrt{x-1} \\ x = t^2 + 1 \end{array} \left| \begin{array}{l} t^2 = x-1 \\ dx = 2t dt \end{array} \right. \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x-1}{x+1} \leq 1 & x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \quad \left| \quad x \in (1; \infty) \right. \\ x \neq -1 & x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \quad \left| \quad \text{t. j. } 1 \leq x \right. \end{array} \right]$$

$$= \left[\begin{array}{l|l} u = \arcsin \sqrt{\frac{x-1}{x+1}} & u' = \frac{1}{\sqrt{1-\frac{x-1}{x+1}}} \cdot \frac{1}{2} \left(\frac{x-1}{x+1} \right)^{-\frac{1}{2}} \cdot \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}} \sqrt{\frac{x-1}{x+1}} \sqrt{(x+1)^4}} = \frac{1}{\sqrt{\frac{2}{x+1} \cdot \frac{x-1}{x+1} \cdot (x+1)^4}} \\ v' = 1 & v = x \end{array} \right]$$

$$= \frac{1}{\sqrt{2(x+1) \cdot (x+1)^2}} = \frac{1}{\sqrt{2} |x+1| \sqrt{x-1}} = \frac{\sqrt{2}}{2(x+1)\sqrt{x-1}}$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x+1)\sqrt{x-1}} = \left[\begin{array}{l|l} \text{Subst. } t = \sqrt{x-1} & t^2 = x-1 \quad \left| \quad x \in (1; \infty) \right. \\ x = t^2 + 1 & dx = 2t dt \quad \left| \quad t \in (0; \infty) \right. \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{(t^2+1) \cdot 2t dt}{(t^2+2)t}$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x-1}{x+1} \leq 1 \\ x \neq -1 \end{array} \left| \begin{array}{l} x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \\ x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right. \left. \begin{array}{l} x \in (1; \infty) \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x-1}{x+1}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{x-1}{x+1}}} \cdot \frac{1}{2} \left(\frac{x-1}{x+1} \right)^{-\frac{1}{2}} \cdot \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}} \sqrt{\frac{x-1}{x+1}} \sqrt{(x+1)^4}} = \frac{1}{\sqrt{\frac{2}{x+1} \cdot \frac{x-1}{x+1} \cdot (x+1)^4}} \\ v = x \end{array} \right. \right. \left. \begin{array}{l} = \frac{1}{\sqrt{2(x+1) \cdot (x+1)^2}} = \frac{1}{\sqrt{2} |x+1| \sqrt{x-1}} = \frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x+1)\sqrt{x-1}} = \left[\begin{array}{l} \text{Subst. } t = \sqrt{x-1} \mid t^2 = x-1 \\ x = t^2+1 \mid dx = 2t dt \end{array} \left| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right. \right]$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{(t^2+1) \cdot 2t dt}{(t^2+2)t} = x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} \int \frac{(t^2+1) dt}{t^2+2}$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x-1}{x+1} \leq 1 \\ x \neq -1 \end{array} \left| \begin{array}{l} x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \\ x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right. \left. \begin{array}{l} x \in (1; \infty) \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x-1}{x+1}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{x-1}{x+1}}} \cdot \frac{1}{2} \left(\frac{x-1}{x+1} \right)^{-\frac{1}{2}} \cdot \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}} \sqrt{\frac{x-1}{x+1}} \sqrt{(x+1)^4}} = \frac{1}{\sqrt{\frac{2}{x+1} \cdot \frac{x-1}{x+1} \cdot (x+1)^4}} \\ v = x \end{array} \right. \right. \left. \begin{array}{l} = \frac{1}{\sqrt{2(x+1) \cdot (x+1)^2}} = \frac{1}{\sqrt{2} |x+1| \sqrt{x-1}} = \frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x+1)\sqrt{x-1}} = \left[\begin{array}{l} \text{Subst. } t = \sqrt{x-1} \mid t^2 = x-1 \\ x = t^2+1 \mid dx = 2t dt \end{array} \left| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right. \right]$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{(t^2+1) \cdot 2t dt}{(t^2+2)t} = x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} \int \frac{(t^2+1) dt}{t^2+2}$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} \int \frac{(t^2+2-1) dt}{t^2+2}$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x-1}{x+1} \leq 1 \\ x \neq -1 \end{array} \left| \begin{array}{l} x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \\ x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right. \left. \begin{array}{l} x \in (1; \infty) \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x-1}{x+1}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1-\frac{x-1}{x+1}}} \cdot \frac{1}{2} \left(\frac{x-1}{x+1} \right)^{-\frac{1}{2}} \cdot \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}} \sqrt{\frac{x-1}{x+1}} \sqrt{(x+1)^4}} = \frac{1}{\sqrt{\frac{2}{x+1} \cdot \frac{x-1}{x+1} \cdot (x+1)^4}} \\ v = x \end{array} \right. \right. \left. \begin{array}{l} = \frac{1}{\sqrt{2(x+1) \cdot (x+1)^2}} = \frac{1}{\sqrt{2} |x+1| \sqrt{x-1}} = \frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x+1)\sqrt{x-1}} = \left[\begin{array}{l} \text{Subst. } t = \sqrt{x-1} \mid t^2 = x-1 \\ x = t^2+1 \mid dx = 2t dt \end{array} \left| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right. \right]$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{(t^2+1) \cdot 2t dt}{(t^2+2)t} = x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} \int \frac{(t^2+1) dt}{t^2+2}$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} \int \frac{(t^2+2-1) dt}{t^2+2} = x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} \int dt + \sqrt{2} \int \frac{dt}{t^2+(\sqrt{2})^2}$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x-1}{x+1} \leq 1 \mid x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \mid x \in (1; \infty) \\ x \neq -1 \mid x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \mid \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{\frac{x-1}{x+1}} \mid u' = \frac{1}{\sqrt{1-\frac{x-1}{x+1}}} \cdot \frac{1}{2} \left(\frac{x-1}{x+1} \right)^{-\frac{1}{2}} \cdot \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}} \sqrt{\frac{x-1}{x+1}} \sqrt{(x+1)^4}} = \frac{1}{\sqrt{\frac{2}{x+1} \cdot \frac{x-1}{x+1} \cdot (x+1)^4}} \\ v' = 1 \mid v = x \end{array} \right] \\ = \frac{1}{\sqrt{2(x+1) \cdot (x+1)^2}} = \frac{1}{\sqrt{2} |x+1| \sqrt{x-1}} = \frac{\sqrt{2}}{2(x+1)\sqrt{x-1}}$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x+1)\sqrt{x-1}} = \left[\begin{array}{l} \text{Subst. } t = \sqrt{x-1} \mid t^2 = x-1 \mid x \in (1; \infty) \\ x = t^2+1 \mid dx = 2t dt \mid t \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{(t^2+1) \cdot 2t dt}{(t^2+2)t} = x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} \int \frac{(t^2+1) dt}{t^2+2}$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} \int \frac{(t^2+2-1) dt}{t^2+2} = x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} \int dt + \sqrt{2} \int \frac{dt}{t^2+(\sqrt{2})^2}$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} t + \frac{\sqrt{2}}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + c_1$$

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$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx = x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2(x-1)} + \operatorname{arctg} \sqrt{\frac{x-1}{2}} + c_1$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x-1}{x+1} \leq 1 & x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \\ x \neq -1 & x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \left. \begin{array}{l} x \in (1; \infty) \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l|l} u = \arcsin \sqrt{\frac{x-1}{x+1}} & u' = \frac{1}{\sqrt{1-\frac{x-1}{x+1}}} \cdot \frac{1}{2} \left(\frac{x-1}{x+1} \right)^{-\frac{1}{2}} \cdot \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{\frac{1}{2} \cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}} \sqrt{\frac{x-1}{x+1}} \sqrt{(x+1)^4}} = \frac{1}{\sqrt{\frac{2}{x+1} \cdot \frac{x-1}{x+1} \cdot (x+1)^4}} \\ v' = 1 & v = x \end{array} \right. \left. \begin{array}{l} = \frac{1}{\sqrt{2(x+1) \cdot (x+1)^2}} = \frac{1}{\sqrt{2} |x+1| \sqrt{x-1}} = \frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{x dx}{(x+1)\sqrt{x-1}} = \left[\begin{array}{l|l} \text{Subst. } t = \sqrt{x-1} & t^2 = x-1 \\ x = t^2+1 & dx = 2t dt \end{array} \right. \left. \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right]$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2} \int \frac{(t^2+1) \cdot 2t dt}{(t^2+2)t} = x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} \int \frac{(t^2+1) dt}{t^2+2}$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} \int \frac{(t^2+2-1) dt}{t^2+2} = x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} \int dt + \sqrt{2} \int \frac{dt}{t^2+(\sqrt{2})^2}$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2} t + \frac{\sqrt{2}}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + c_1$$

$$= x \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2(x-1)} + \operatorname{arctg} \sqrt{\frac{x-1}{2}} + c_1, \quad x \in (-\infty; -1), \quad c_1 \in \mathbb{R}.$$

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$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

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$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x-1}{x+1} \leq 1 & x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in \langle 1; \infty \rangle \\ x \neq -1 & x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right| \begin{array}{l} x \in \langle 1; \infty \rangle \\ \text{t. j. } 1 \leq x \end{array} \right]$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x-1}{x+1} \leq 1 & x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in \langle 1; \infty \rangle \\ x \neq -1 & x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right| \begin{array}{l} x \in \langle 1; \infty \rangle \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=x+1 & x \in \langle 1; \infty \rangle \\ dt=dx & t \in \langle 2; \infty \rangle \end{array} \right]$$

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$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x-1}{x+1} \leq 1 & x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in \langle 1; \infty \rangle \quad \left| \begin{array}{l} x \in \langle 1; \infty \rangle \\ x \neq -1 \end{array} \right. \\ & x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \quad \left| \begin{array}{l} \\ \text{t. j. } 1 \leq x \end{array} \right. \end{array} \right]$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=x+1 & x \in \langle 1; \infty \rangle \\ dt=dx & t \in \langle 2; \infty \rangle \end{array} \right] = \int \arcsin \sqrt{\frac{t-2}{t}} dt = \int \arcsin \sqrt{1 - \frac{2}{t}} dt$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x-1}{x+1} \leq 1 \\ x \neq -1 \end{array} \left| \begin{array}{l} x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \\ x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right. \left. \begin{array}{l} x \in (1; \infty) \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x+1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (1; \infty) \\ t \in (2; \infty) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{t-2}{t}} dt = \int \arcsin \sqrt{1 - \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 - \frac{2}{t}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 - \frac{2}{t})}} \cdot \frac{1}{2} (1 - \frac{2}{t})^{-\frac{1}{2}} \cdot (-1) \cdot (-2t^{-2}) = \frac{1}{\sqrt{\frac{2}{t}} \sqrt{\frac{t-2}{t}} \sqrt{t^4}} = \frac{1}{\sqrt{2(t-2)t^2}} = \frac{1}{\sqrt{2}|t| \sqrt{t-2}} = \frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v = t \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t\sqrt{t-2}}$$

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$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x-1}{x+1} \leq 1 \\ x \neq -1 \end{array} \left| \begin{array}{l} x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \\ x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right. \left. \begin{array}{l} x \in (1; \infty) \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x+1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (1; \infty) \\ t \in (2; \infty) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{t-2}{t}} dt = \int \arcsin \sqrt{1 - \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 - \frac{2}{t}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 - \frac{2}{t})}} \cdot \frac{1}{2} (1 - \frac{2}{t})^{-\frac{1}{2}} \cdot (-1) \cdot (-2t^{-2}) = \frac{1}{\sqrt{\frac{2}{t}} \sqrt{\frac{t-2}{t}} \sqrt{t^4}} = \frac{1}{\sqrt{2(t-2)t^2}} = \frac{1}{\sqrt{2}|t| \sqrt{t-2}} = \frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v = t \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } z = t-2 \\ dz = dt \end{array} \left| \begin{array}{l} t \in (2; \infty) \\ z \in (0; \infty) \end{array} \right. \right]$$

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$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x-1}{x+1} \leq 1 \\ x \neq -1 \end{array} \left| \begin{array}{l} x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \\ x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right. \left. \begin{array}{l} x \in (1; \infty) \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x+1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (1; \infty) \\ t \in (2; \infty) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{t-2}{t}} dt = \int \arcsin \sqrt{1 - \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 - \frac{2}{t}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 - \frac{2}{t})}} \cdot \frac{1}{2} (1 - \frac{2}{t})^{-\frac{1}{2}} \cdot (-1) \cdot (-2t^{-2}) = \frac{1}{\sqrt{\frac{2}{t}} \sqrt{1 - \frac{2}{t}} \sqrt{t^4}} = \frac{1}{\sqrt{2(t-2)t^2}} = \frac{1}{\sqrt{2}|t| \sqrt{t-2}} = \frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v = t \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } z = t-2 \\ dz = dt \end{array} \left| \begin{array}{l} t \in (2; \infty) \\ z \in (0; \infty) \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{z}}$$

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$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l|l} 0 \leq \frac{x-1}{x+1} \leq 1 & x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in \langle 1; \infty \rangle \\ x \neq -1 & x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \left. \begin{array}{l} x \in \langle 1; \infty \rangle \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l|l} \text{Subst. } t = x+1 & x \in \langle 1; \infty \rangle \\ dt = dx & t \in \langle 2; \infty \rangle \end{array} \right] = \int \arcsin \sqrt{\frac{t-2}{t}} dt = \int \arcsin \sqrt{1 - \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 - \frac{2}{t}} \\ v' = 1 \end{array} \right. \left. \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 - \frac{2}{t})}} \cdot \frac{1}{2} (1 - \frac{2}{t})^{-\frac{1}{2}} \cdot (-1) \cdot (-2t^{-2}) = \frac{1}{\sqrt{\frac{2}{t}} \sqrt{1 - \frac{2}{t}} \sqrt{t^4}} = \frac{1}{\sqrt{2(t-2)t^2}} = \frac{1}{\sqrt{2}|t| \sqrt{t-2}} = \frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v = t \end{array} \right]$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t\sqrt{t-2}} = \left[\begin{array}{l|l} \text{Subst. } z = t-2 & t \in \langle 2; \infty \rangle \\ dz = dt & z \in \langle 0; \infty \rangle \end{array} \right]$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{z}} = t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int z^{-\frac{1}{2}} dz$$

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$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x-1}{x+1} \leq 1 \\ x \neq -1 \end{array} \left| \begin{array}{l} x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \\ x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right] \left. \begin{array}{l} x \in (1; \infty) \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x+1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (1; \infty) \\ t \in (2; \infty) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{t-2}{t}} dt = \int \arcsin \sqrt{1 - \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 - \frac{2}{t}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 - \frac{2}{t})}} \cdot \frac{1}{2} (1 - \frac{2}{t})^{-\frac{1}{2}} \cdot (-1) \cdot (-2t^{-2}) = \frac{1}{\sqrt{\frac{2}{t}} \sqrt{1 - \frac{2}{t}} \sqrt{t^4}} = \frac{1}{\sqrt{2(t-2)t^2}} = \frac{1}{\sqrt{2}|t| \sqrt{t-2}} = \frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v = t \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } z = t-2 \\ dz = dt \end{array} \left| \begin{array}{l} t \in (2; \infty) \\ z \in (0; \infty) \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{z}} = t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int z^{-\frac{1}{2}} dz$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \cdot \frac{z^{\frac{1}{2}}}{\frac{1}{2}} + c_2$$

Koniec 10. časti príklady b – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx$$

$$= \left[\begin{array}{l} 0 \leq \frac{x-1}{x+1} \leq 1 \\ x \neq -1 \end{array} \left| \begin{array}{l} x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \\ x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right. \right. \left. \begin{array}{l} x \in (1; \infty) \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x+1 \\ dt = dx \end{array} \left| \begin{array}{l} x \in (1; \infty) \\ t \in (2; \infty) \end{array} \right. \right] = \int \arcsin \sqrt{\frac{t-2}{t}} dt = \int \arcsin \sqrt{1 - \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 - \frac{2}{t}} \\ v' = 1 \end{array} \left| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 - \frac{2}{t})}} \cdot \frac{1}{2} (1 - \frac{2}{t})^{-\frac{1}{2}} \cdot (-1) \cdot (-2t^{-2}) = \frac{1}{\sqrt{\frac{2}{t}} \sqrt{1 - \frac{2}{t}} \sqrt{t^4}} = \frac{1}{\sqrt{2(t-2)t^2}} = \frac{1}{\sqrt{2}|t| \sqrt{t-2}} = \frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v = t \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } z = t-2 \\ dz = dt \end{array} \left| \begin{array}{l} t \in (2; \infty) \\ z \in (0; \infty) \end{array} \right. \right]$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{z}} = t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int z^{-\frac{1}{2}} dz$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \cdot \frac{z^{\frac{1}{2}}}{\frac{1}{2}} + c_2 = t \arcsin \sqrt{\frac{t-2}{t}} - \sqrt{2z} + c_2$$

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$$\int \arcsin \sqrt{\frac{x-1}{x+1}} dx = (x+1) \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2(x+1)} + c_2$$

$$= \left[\begin{array}{l} 0 \leq \frac{x-1}{x+1} \leq 1 \\ x \neq -1 \end{array} \middle| \begin{array}{l} x+1 > 0 \Leftrightarrow x > -1: \quad 0 \leq x-1 \leq x+1, \text{ t. j. } 1 \leq x, -1 \leq 1, \text{ t. j. } x \in (1; \infty) \\ x+1 < 0 \Leftrightarrow x < -1: \quad 0 \geq x-1 \geq x+1, \text{ t. j. } 1 \geq x, -1 \geq 1, \text{ t. j. } x \text{ neexistuje} \end{array} \right| \begin{array}{l} x \in (1; \infty) \\ \text{t. j. } 1 \leq x \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x+1 \\ dt = dx \end{array} \middle| \begin{array}{l} x \in (1; \infty) \\ t \in (2; \infty) \end{array} \right] = \int \arcsin \sqrt{\frac{t-2}{t}} dt = \int \arcsin \sqrt{1 - \frac{2}{t}} dt$$

$$= \left[\begin{array}{l} u = \arcsin \sqrt{1 - \frac{2}{t}} \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{\sqrt{1 - (1 - \frac{2}{t})}} \cdot \frac{1}{2} (1 - \frac{2}{t})^{-\frac{1}{2}} \cdot (-1) \cdot (-2t^{-2}) = \frac{1}{\sqrt{\frac{2}{t}} \sqrt{1 - \frac{2}{t}} \sqrt{t^4}} = \frac{1}{\sqrt{2(t-2)t^2}} = \frac{1}{\sqrt{2}|t| \sqrt{t-2}} = \frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v = t \end{array} \right]$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int \frac{t dt}{t\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } z = t-2 = x+1 \\ dz = dt \end{array} \middle| \begin{array}{l} t \in (2; \infty) \\ z \in (0; \infty) \end{array} \right]$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int \frac{dz}{\sqrt{z}} = t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \int z^{-\frac{1}{2}} dz$$

$$= t \arcsin \sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2} \cdot \frac{z^{\frac{1}{2}}}{\frac{1}{2}} + c_2 = t \arcsin \sqrt{\frac{t-2}{t}} - \sqrt{2z} + c_2$$

$$= (x+1) \arcsin \sqrt{\frac{x-1}{x+1}} - \sqrt{2(x+1)} + c_2, \quad x \in (1; \infty), \quad c_2 \in \mathbb{R}.$$

Koniec 10. časti príklady b – 176, 177

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{x-5}}$$

$$\int \frac{dx}{\sqrt{x-3} - \sqrt{x-5}}$$

Koniec 10. časti príklady b – 176, 177

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{x-5}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{x-5}} \cdot \frac{\sqrt{x-3} - \sqrt{x-5}}{\sqrt{x-3} - \sqrt{x-5}} dx$$

$$\int \frac{dx}{\sqrt{x-3} - \sqrt{x-5}}$$

$$= \int \frac{1}{\sqrt{x-3} - \sqrt{x-5}} \cdot \frac{\sqrt{x-3} + \sqrt{x-5}}{\sqrt{x-3} + \sqrt{x-5}} dx$$

Koniec 10. časti príklady b – 176, 177

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{x-5}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{x-5}} \cdot \frac{\sqrt{x-3} - \sqrt{x-5}}{\sqrt{x-3} - \sqrt{x-5}} dx = \int \frac{\sqrt{x-3} - \sqrt{x-5}}{x-3 - (x-5)} dx$$

$$\int \frac{dx}{\sqrt{x-3} - \sqrt{x-5}}$$

$$= \int \frac{1}{\sqrt{x-3} - \sqrt{x-5}} \cdot \frac{\sqrt{x-3} + \sqrt{x-5}}{\sqrt{x-3} + \sqrt{x-5}} dx = \int \frac{\sqrt{x-3} + \sqrt{x-5}}{x-3 - (x-5)} dx$$

Koniec 10. časti príklady b – 176, 177

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{x-5}}$$

$$\begin{aligned} &= \int \frac{1}{\sqrt{x-3} + \sqrt{x-5}} \cdot \frac{\sqrt{x-3} - \sqrt{x-5}}{\sqrt{x-3} - \sqrt{x-5}} dx = \int \frac{\sqrt{x-3} - \sqrt{x-5}}{x-3 - (x-5)} dx = \int \frac{\sqrt{x-3} - \sqrt{x-5}}{2} dx \\ &= \frac{1}{2} \int (x-3)^{\frac{1}{2}} dx - \frac{1}{2} \int (x-5)^{\frac{1}{2}} dx \end{aligned}$$

$$\int \frac{dx}{\sqrt{x-3} - \sqrt{x-5}}$$

$$\begin{aligned} &= \int \frac{1}{\sqrt{x-3} - \sqrt{x-5}} \cdot \frac{\sqrt{x-3} + \sqrt{x-5}}{\sqrt{x-3} + \sqrt{x-5}} dx = \int \frac{\sqrt{x-3} + \sqrt{x-5}}{x-3 - (x-5)} dx = \int \frac{\sqrt{x-3} + \sqrt{x-5}}{2} dx \\ &= \frac{1}{2} \int (x-3)^{\frac{1}{2}} dx + \frac{1}{2} \int (x-5)^{\frac{1}{2}} dx \end{aligned}$$

Koniec 10. časti príklady b – 176, 177

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{x-5}}$$

$$\begin{aligned} &= \int \frac{1}{\sqrt{x-3} + \sqrt{x-5}} \cdot \frac{\sqrt{x-3} - \sqrt{x-5}}{\sqrt{x-3} - \sqrt{x-5}} dx = \int \frac{\sqrt{x-3} - \sqrt{x-5}}{x-3 - (x-5)} dx = \int \frac{\sqrt{x-3} - \sqrt{x-5}}{2} dx \\ &= \frac{1}{2} \int (x-3)^{\frac{1}{2}} dx - \frac{1}{2} \int (x-5)^{\frac{1}{2}} dx = \frac{1}{2} \cdot \frac{(x-3)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2} \cdot \frac{(x-5)^{\frac{3}{2}}}{\frac{3}{2}} + c \end{aligned}$$

$$\int \frac{dx}{\sqrt{x-3} - \sqrt{x-5}}$$

$$\begin{aligned} &= \int \frac{1}{\sqrt{x-3} - \sqrt{x-5}} \cdot \frac{\sqrt{x-3} + \sqrt{x-5}}{\sqrt{x-3} + \sqrt{x-5}} dx = \int \frac{\sqrt{x-3} + \sqrt{x-5}}{x-3 - (x-5)} dx = \int \frac{\sqrt{x-3} + \sqrt{x-5}}{2} dx \\ &= \frac{1}{2} \int (x-3)^{\frac{1}{2}} dx + \frac{1}{2} \int (x-5)^{\frac{1}{2}} dx = \frac{1}{2} \cdot \frac{(x-3)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2} \cdot \frac{(x-5)^{\frac{3}{2}}}{\frac{3}{2}} + c \end{aligned}$$

Koniec 10. časti príklady b – 176, 177

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{x-5}} = \frac{\sqrt{(x-3)^3}}{3} - \frac{\sqrt{(x-5)^3}}{3} + c$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{x-5}} \cdot \frac{\sqrt{x-3}-\sqrt{x-5}}{\sqrt{x-3}-\sqrt{x-5}} dx = \int \frac{\sqrt{x-3}-\sqrt{x-5}}{x-3-(x-5)} dx = \int \frac{\sqrt{x-3}-\sqrt{x-5}}{2} dx$$

$$= \frac{1}{2} \int (x-3)^{\frac{1}{2}} dx - \frac{1}{2} \int (x-5)^{\frac{1}{2}} dx = \frac{1}{2} \cdot \frac{(x-3)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2} \cdot \frac{(x-5)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{\sqrt{(x-3)^3}}{3} - \frac{\sqrt{(x-5)^3}}{3} + c, x \in \langle 5; \infty \rangle, c \in \mathbb{R}.$$

$$\int \frac{dx}{\sqrt{x-3}-\sqrt{x-5}} = \frac{\sqrt{(x-3)^3}}{3} + \frac{\sqrt{(x-5)^3}}{3} + c$$

$$= \int \frac{1}{\sqrt{x-3}-\sqrt{x-5}} \cdot \frac{\sqrt{x-3}+\sqrt{x-5}}{\sqrt{x-3}+\sqrt{x-5}} dx = \int \frac{\sqrt{x-3}+\sqrt{x-5}}{x-3-(x-5)} dx = \int \frac{\sqrt{x-3}+\sqrt{x-5}}{2} dx$$

$$= \frac{1}{2} \int (x-3)^{\frac{1}{2}} dx + \frac{1}{2} \int (x-5)^{\frac{1}{2}} dx = \frac{1}{2} \cdot \frac{(x-3)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2} \cdot \frac{(x-5)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{\sqrt{(x-3)^3}}{3} + \frac{\sqrt{(x-5)^3}}{3} + c, x \in \langle 5; \infty \rangle, c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{5-x}}$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{5-x}} \cdot \frac{\sqrt{x-3} - \sqrt{5-x}}{\sqrt{x-3} - \sqrt{5-x}} dx$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{5-x}} \cdot \frac{\sqrt{x-3} - \sqrt{5-x}}{\sqrt{x-3} - \sqrt{5-x}} dx = \int \frac{\sqrt{x-3} - \sqrt{5-x}}{x-3 - (5-x)} dx$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{5-x}} \cdot \frac{\sqrt{x-3} - \sqrt{5-x}}{\sqrt{x-3} - \sqrt{5-x}} dx = \int \frac{\sqrt{x-3} - \sqrt{5-x}}{x-3-(5-x)} dx = \int \frac{\sqrt{x-3} dx}{2x-8} - \int \frac{\sqrt{5-x} dx}{2x-8}$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{5-x}} \cdot \frac{\sqrt{x-3} - \sqrt{5-x}}{\sqrt{x-3} - \sqrt{5-x}} dx = \int \frac{\sqrt{x-3} - \sqrt{5-x}}{x-3 - (5-x)} dx = \int \frac{\sqrt{x-3} dx}{2x-8} - \int \frac{\sqrt{5-x} dx}{2x-8}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sqrt{x-3} \mid u^2 = x-3 \mid x = u^2 + 3 \mid dx = 2u du \mid x \in (3; 4), u \in (0; 1), v \in (1; \sqrt{2}) \\ \text{Subst. } v = \sqrt{5-x} \mid v^2 = 5-x \mid x = 5 - v^2 \mid dx = -2v dv \mid x \in (4; 5), u \in (1; \sqrt{2}), v \in (0; 1) \end{array} \right]$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{5-x}} \cdot \frac{\sqrt{x-3} - \sqrt{5-x}}{\sqrt{x-3} - \sqrt{5-x}} dx = \int \frac{\sqrt{x-3} - \sqrt{5-x}}{x-3 - (5-x)} dx = \int \frac{\sqrt{x-3} dx}{2x-8} - \int \frac{\sqrt{5-x} dx}{2x-8}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sqrt{x-3} \mid u^2 = x-3 \mid x = u^2 + 3 \mid dx = 2u du \mid x \in (3; 4), u \in (0; 1), v \in (1; \sqrt{2}) \\ \text{Subst. } v = \sqrt{5-x} \mid v^2 = 5-x \mid x = 5 - v^2 \mid dx = -2v dv \mid x \in (4; 5), u \in (1; \sqrt{2}), v \in (0; 1) \end{array} \right]$$

$$= \int \frac{2u^2 du}{2u^2 + 6 - 8} + \int \frac{2v^2 dv}{-2v^2 + 10 - 8}$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{5-x}} \cdot \frac{\sqrt{x-3} - \sqrt{5-x}}{\sqrt{x-3} - \sqrt{5-x}} dx = \int \frac{\sqrt{x-3} - \sqrt{5-x}}{x-3 - (5-x)} dx = \int \frac{\sqrt{x-3} dx}{2x-8} - \int \frac{\sqrt{5-x} dx}{2x-8}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sqrt{x-3} \mid u^2 = x-3 \mid x = u^2 + 3 \mid dx = 2u du \mid x \in (3; 4), u \in (0; 1), v \in (1; \sqrt{2}) \\ \text{Subst. } v = \sqrt{5-x} \mid v^2 = 5-x \mid x = 5 - v^2 \mid dx = -2v dv \mid x \in (4; 5), u \in (1; \sqrt{2}), v \in (0; 1) \end{array} \right]$$

$$= \int \frac{2u^2 du}{2u^2 + 6 - 8} + \int \frac{2v^2 dv}{-2v^2 + 10 - 8} = \int \frac{u^2 du}{u^2 - 1} - \int \frac{v^2 dv}{v^2 - 1}$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{5-x}} \cdot \frac{\sqrt{x-3} - \sqrt{5-x}}{\sqrt{x-3} - \sqrt{5-x}} dx = \int \frac{\sqrt{x-3} - \sqrt{5-x}}{x-3 - (5-x)} dx = \int \frac{\sqrt{x-3} dx}{2x-8} - \int \frac{\sqrt{5-x} dx}{2x-8}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sqrt{x-3} \mid u^2 = x-3 \mid x = u^2 + 3 \mid dx = 2u du \mid x \in (3; 4), u \in (0; 1), v \in (1; \sqrt{2}) \\ \text{Subst. } v = \sqrt{5-x} \mid v^2 = 5-x \mid x = 5 - v^2 \mid dx = -2v dv \mid x \in (4; 5), u \in (1; \sqrt{2}), v \in (0; 1) \end{array} \right]$$

$$= \int \frac{2u^2 du}{2u^2 + 6 - 8} + \int \frac{2v^2 dv}{-2v^2 + 10 - 8} = \int \frac{u^2 du}{u^2 - 1} - \int \frac{v^2 dv}{v^2 - 1} = \int \frac{u^2 - 1 + 1}{u^2 - 1} du - \int \frac{v^2 - 1 + 1}{v^2 - 1} dv$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{5-x}} \cdot \frac{\sqrt{x-3} - \sqrt{5-x}}{\sqrt{x-3} - \sqrt{5-x}} dx = \int \frac{\sqrt{x-3} - \sqrt{5-x}}{x-3 - (5-x)} dx = \int \frac{\sqrt{x-3} dx}{2x-8} - \int \frac{\sqrt{5-x} dx}{2x-8}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sqrt{x-3} \mid u^2 = x-3 \mid x = u^2 + 3 \mid dx = 2u du \mid x \in (3; 4), u \in (0; 1), v \in (1; \sqrt{2}) \\ \text{Subst. } v = \sqrt{5-x} \mid v^2 = 5-x \mid x = 5 - v^2 \mid dx = -2v dv \mid x \in (4; 5), u \in (1; \sqrt{2}), v \in (0; 1) \end{array} \right]$$

$$= \int \frac{2u^2 du}{2u^2 + 6 - 8} + \int \frac{2v^2 dv}{-2v^2 + 10 - 8} = \int \frac{u^2 du}{u^2 - 1} - \int \frac{v^2 dv}{v^2 - 1} = \int \frac{u^2 - 1 + 1}{u^2 - 1} du - \int \frac{v^2 - 1 + 1}{v^2 - 1} dv$$

$$= \int du + \int \frac{du}{u^2 - 1} - \int dv - \int \frac{dv}{v^2 - 1}$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{5-x}} \cdot \frac{\sqrt{x-3} - \sqrt{5-x}}{\sqrt{x-3} - \sqrt{5-x}} dx = \int \frac{\sqrt{x-3} - \sqrt{5-x}}{x-3 - (5-x)} dx = \int \frac{\sqrt{x-3} dx}{2x-8} - \int \frac{\sqrt{5-x} dx}{2x-8}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sqrt{x-3} \mid u^2 = x-3 \mid x = u^2 + 3 \mid dx = 2u du \mid x \in (3; 4), u \in (0; 1), v \in (1; \sqrt{2}) \\ \text{Subst. } v = \sqrt{5-x} \mid v^2 = 5-x \mid x = 5 - v^2 \mid dx = -2v dv \mid x \in (4; 5), u \in (1; \sqrt{2}), v \in (0; 1) \end{array} \right]$$

$$= \int \frac{2u^2 du}{2u^2 + 6 - 8} + \int \frac{2v^2 dv}{-2v^2 + 10 - 8} = \int \frac{u^2 du}{u^2 - 1} - \int \frac{v^2 dv}{v^2 - 1} = \int \frac{u^2 - 1 + 1}{u^2 - 1} du - \int \frac{v^2 - 1 + 1}{v^2 - 1} dv$$

$$= \int du + \int \frac{du}{u^2 - 1} - \int dv - \int \frac{dv}{v^2 - 1} = u + \frac{1}{2} \ln \left| \frac{u-1}{u+1} \right| - v - \frac{1}{2} \ln \left| \frac{v-1}{v+1} \right| + c$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{5-x}} \cdot \frac{\sqrt{x-3} - \sqrt{5-x}}{\sqrt{x-3} - \sqrt{5-x}} dx = \int \frac{\sqrt{x-3} - \sqrt{5-x}}{x-3 - (5-x)} dx = \int \frac{\sqrt{x-3} dx}{2x-8} - \int \frac{\sqrt{5-x} dx}{2x-8}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sqrt{x-3} \mid u^2 = x-3 \mid x = u^2 + 3 \mid dx = 2u du \mid x \in (3; 4), u \in (0; 1), v \in (1; \sqrt{2}) \\ \text{Subst. } v = \sqrt{5-x} \mid v^2 = 5-x \mid x = 5 - v^2 \mid dx = -2v dv \mid x \in (4; 5), u \in (1; \sqrt{2}), v \in (0; 1) \end{array} \right]$$

$$= \int \frac{2u^2 du}{2u^2 + 6 - 8} + \int \frac{2v^2 dv}{-2v^2 + 10 - 8} = \int \frac{u^2 du}{u^2 - 1} - \int \frac{v^2 dv}{v^2 - 1} = \int \frac{u^2 - 1 + 1}{u^2 - 1} du - \int \frac{v^2 - 1 + 1}{v^2 - 1} dv$$

$$= \int du + \int \frac{du}{u^2 - 1} - \int dv - \int \frac{dv}{v^2 - 1} = u + \frac{1}{2} \ln \left| \frac{u-1}{u+1} \right| - v - \frac{1}{2} \ln \left| \frac{v-1}{v+1} \right| + c$$

$$= \sqrt{x-3} + \frac{1}{2} \ln \left| \frac{\sqrt{x-3}-1}{\sqrt{x-3}+1} \right| - \sqrt{5-x} - \frac{1}{2} \ln \left| \frac{\sqrt{5-x}-1}{\sqrt{5-x}+1} \right| + c$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3} + \sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3} + \sqrt{5-x}} \cdot \frac{\sqrt{x-3} - \sqrt{5-x}}{\sqrt{x-3} - \sqrt{5-x}} dx = \int \frac{\sqrt{x-3} - \sqrt{5-x}}{x-3 - (5-x)} dx = \int \frac{\sqrt{x-3} dx}{2x-8} - \int \frac{\sqrt{5-x} dx}{2x-8}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sqrt{x-3} \mid u^2 = x-3 \mid x = u^2 + 3 \mid dx = 2u du \mid x \in (3; 4), u \in (0; 1), v \in (1; \sqrt{2}) \\ \text{Subst. } v = \sqrt{5-x} \mid v^2 = 5-x \mid x = 5 - v^2 \mid dx = -2v dv \mid x \in (4; 5), u \in (1; \sqrt{2}), v \in (0; 1) \end{array} \right]$$

$$= \int \frac{2u^2 du}{2u^2 + 6 - 8} + \int \frac{2v^2 dv}{-2v^2 + 10 - 8} = \int \frac{u^2 du}{u^2 - 1} - \int \frac{v^2 dv}{v^2 - 1} = \int \frac{u^2 - 1 + 1}{u^2 - 1} du - \int \frac{v^2 - 1 + 1}{v^2 - 1} dv$$

$$= \int du + \int \frac{du}{u^2 - 1} - \int dv - \int \frac{dv}{v^2 - 1} = u + \frac{1}{2} \ln \left| \frac{u-1}{u+1} \right| - v - \frac{1}{2} \ln \left| \frac{v-1}{v+1} \right| + c$$

$$= \sqrt{x-3} + \frac{1}{2} \ln \left| \frac{\sqrt{x-3}-1}{\sqrt{x-3}+1} \right| - \sqrt{5-x} - \frac{1}{2} \ln \left| \frac{\sqrt{5-x}-1}{\sqrt{5-x}+1} \right| + c$$

$$= \left[\begin{array}{l} \ln \left| \frac{\sqrt{x-3}-1}{\sqrt{x-3}+1} \right| - \ln \left| \frac{\sqrt{5-x}-1}{\sqrt{5-x}+1} \right| = \ln \left| \frac{\sqrt{x-3}-1}{\sqrt{x-3}+1} \cdot \frac{\sqrt{x-3}-1}{\sqrt{x-3}-1} \right| - \ln \left| \frac{\sqrt{5-x}-1}{\sqrt{5-x}+1} \cdot \frac{\sqrt{5-x}-1}{\sqrt{5-x}-1} \right| = \ln \left| \frac{x-3-2\sqrt{x-3}+1}{x-3-1} \right| - \ln \left| \frac{5-x-2\sqrt{5-x}+1}{5-x-1} \right| \\ = \ln \left| \frac{x-2-2\sqrt{x-3}}{x-4} \right| - \ln \left| \frac{6-x-2\sqrt{5-x}+1}{4-x} \right| = \ln \left| \frac{x-2-2\sqrt{x-3}}{x-4} \cdot \frac{4-x}{6-x-2\sqrt{5-x}} \right| = \ln \left| \frac{x-2-2\sqrt{x-3}}{6-x-2\sqrt{5-x}} \right| \end{array} \right]$$

Koniec 10. časti príklady b – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}} = \sqrt{x-3} - \sqrt{5-x} + \frac{1}{2} \ln \left| \frac{x-2-2\sqrt{x-3}}{6-x-2\sqrt{5-x}} \right| + c$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}} dx = \int \frac{\sqrt{x-3}-\sqrt{5-x}}{x-3-(5-x)} dx = \int \frac{\sqrt{x-3} dx}{2x-8} - \int \frac{\sqrt{5-x} dx}{2x-8}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sqrt{x-3} \mid u^2 = x-3 \mid x = u^2+3 \mid dx = 2u du \mid x \in \langle 3; 4 \rangle, u \in (0; 1), v \in (1; \sqrt{2}) \\ \text{Subst. } v = \sqrt{5-x} \mid v^2 = 5-x \mid x = 5-v^2 \mid dx = -2v dv \mid x \in (4; 5), u \in (1; \sqrt{2}), v \in (0; 1) \end{array} \right]$$

$$= \int \frac{2u^2 du}{2u^2+6-8} + \int \frac{2v^2 dv}{-2v^2+10-8} = \int \frac{u^2 du}{u^2-1} - \int \frac{v^2 dv}{v^2-1} = \int \frac{u^2-1+1}{u^2-1} du - \int \frac{v^2-1+1}{v^2-1} dv$$

$$= \int du + \int \frac{du}{u^2-1} - \int dv - \int \frac{dv}{v^2-1} = u + \frac{1}{2} \ln \left| \frac{u-1}{u+1} \right| - v - \frac{1}{2} \ln \left| \frac{v-1}{v+1} \right| + c$$

$$= \sqrt{x-3} + \frac{1}{2} \ln \left| \frac{\sqrt{x-3}-1}{\sqrt{x-3}+1} \right| - \sqrt{5-x} - \frac{1}{2} \ln \left| \frac{\sqrt{5-x}-1}{\sqrt{5-x}+1} \right| + c$$

$$= \left[\begin{array}{l} \ln \left| \frac{\sqrt{x-3}-1}{\sqrt{x-3}+1} \right| - \ln \left| \frac{\sqrt{5-x}-1}{\sqrt{5-x}+1} \right| = \ln \left| \frac{\sqrt{x-3}-1}{\sqrt{x-3}+1} \cdot \frac{\sqrt{x-3}-1}{\sqrt{x-3}-1} \right| - \ln \left| \frac{\sqrt{5-x}-1}{\sqrt{5-x}+1} \cdot \frac{\sqrt{5-x}-1}{\sqrt{5-x}-1} \right| = \ln \left| \frac{x-3-2\sqrt{x-3}+1}{x-3-1} \right| - \ln \left| \frac{5-x-2\sqrt{5-x}+1}{5-x-1} \right| \\ = \ln \left| \frac{x-2-2\sqrt{x-3}}{x-4} \right| - \ln \left| \frac{6-x-2\sqrt{5-x}+1}{4-x} \right| = \ln \left| \frac{x-2-2\sqrt{x-3}}{x-4} \cdot \frac{4-x}{6-x-2\sqrt{5-x}} \right| = \ln \left| \frac{x-2-2\sqrt{x-3}}{6-x-2\sqrt{5-x}} \right| \end{array} \right]$$

$$= \sqrt{x-3} - \sqrt{5-x} + \frac{1}{2} \ln \left| \frac{x-2-2\sqrt{x-3}}{6-x-2\sqrt{5-x}} \right| + c, x \in \langle 3; 4 \rangle \cup (4; 5), c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} dx$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}} dx = \int \frac{\sqrt{x} dx}{\sqrt{1-x\sqrt{x}}}$$

Koniec 10. časti príklady b – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} dx$$

$$= \int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} dx$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}} dx = \int \frac{\sqrt{x} dx}{\sqrt{1-x\sqrt{x}}}$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{x} \mid x \in (0; 1) \\ x = t^2, \quad dx = 2t dt \mid t \in (0; 1) \end{array} \right]$$

Koniec 10. časti príklady b – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} dx$$

$$= \int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} dx = \int \frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)} dx$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}} dx = \int \frac{\sqrt{x} dx}{\sqrt{1-x\sqrt{x}}}$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt{x} \mid x \in (0; 1) \\ x = t^2, \quad dx = 2t dt \mid t \in (0; 1) \end{array} \right] = \int \frac{2t^2 dt}{\sqrt{1-t^3}}$$

Koniec 10. časti príklady b – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} dx$$

$$= \int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} dx = \int \frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)} dx = \int \frac{2-x^2+2\sqrt{1-x^2}}{x^2} dx$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}} dx = \int \frac{\sqrt{x} dx}{\sqrt{1-x\sqrt{x}}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{x} \mid x \in (0; 1) \\ x = t^2, dx = 2t dt \mid t \in (0; 1) \end{array} \right] = \int \frac{2t^2 dt}{\sqrt{1-t^3}} = \left[\begin{array}{l} \text{Subst. } u = 1-t^3 \mid t \in (0; 1) \\ du = -3t^2 dt \mid u \in (0; 1) \end{array} \right]$$

Koniec 10. časti príklady b – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} dx$$

$$\begin{aligned} &= \int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} dx = \int \frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)} dx = \int \frac{2-x^2+2\sqrt{1-x^2}}{x^2} dx \\ &= 2 \int x^{-2} dx - \int dx + 2 \int \frac{\sqrt{1-x^2}}{x^2} dx \end{aligned}$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}} dx = \int \frac{\sqrt{x} dx}{\sqrt{1-x\sqrt{x}}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{x} \mid x \in (0; 1) \\ x = t^2, dx = 2t dt \mid t \in (0; 1) \end{array} \right] = \int \frac{2t^2 dt}{\sqrt{1-t^3}} = \left[\begin{array}{l} \text{Subst. } u = 1-t^3 \mid t \in (0; 1) \\ du = -3t^2 dt \mid u \in (0; 1) \end{array} \right] = - \int \frac{2 du}{3\sqrt{u}} = -\frac{2}{3} \int u^{-\frac{1}{2}} du$$

Koniec 10. časti príklady b – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} dx$$

$$\begin{aligned} &= \int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} dx = \int \frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)} dx = \int \frac{2-x^2+2\sqrt{1-x^2}}{x^2} dx \\ &= 2 \int x^{-2} dx - \int dx + 2 \int \frac{\sqrt{1-x^2}}{x^2} dx = \left[u = \sqrt{1-x^2} \mid u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{1-x^2}} \right. \\ &\quad \left. v' = \frac{1}{x^2} = x^{-2} \mid v = \frac{x^{-1}}{-1} = -\frac{1}{x} \right] \end{aligned}$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}} dx = \int \frac{\sqrt{x} dx}{\sqrt{1-x\sqrt{x}}}$$

$$\begin{aligned} &= \left[\text{Subst. } t = \sqrt{x} \mid x \in (0; 1) \right] = \int \frac{2t^2 dt}{\sqrt{1-t^3}} = \left[\text{Subst. } u = 1-t^3 \mid t \in (0; 1) \right] = - \int \frac{2 du}{3\sqrt{u}} = -\frac{2}{3} \int u^{-\frac{1}{2}} du \\ &= -\frac{2}{3} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \end{aligned}$$

Koniec 10. časti príklady b – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} dx$$

$$\begin{aligned} &= \int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} dx = \int \frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)} dx = \int \frac{2-x^2+2\sqrt{1-x^2}}{x^2} dx \\ &= 2 \int x^{-2} dx - \int dx + 2 \int \frac{\sqrt{1-x^2}}{x^2} dx = \left[u = \sqrt{1-x^2} \mid u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{1-x^2}} \right. \\ &\quad \left. v' = \frac{1}{x^2} = x^{-2} \mid v = \frac{x^{-1}}{-1} = -\frac{1}{x} \right] \\ &= 2 \frac{x^{-1}}{-1} - x + 2 \left[-\frac{\sqrt{1-x^2}}{x} - \int \frac{dx}{\sqrt{1-x^2}} \right] \end{aligned}$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}} dx = \int \frac{\sqrt{x} dx}{\sqrt{1-x\sqrt{x}}}$$

$$\begin{aligned} &= \left[\text{Subst. } t = \sqrt{x} \mid x \in (0; 1) \right] = \int \frac{2t^2 dt}{\sqrt{1-t^3}} = \left[\text{Subst. } u = 1-t^3 \mid t \in (0; 1) \right] = -\int \frac{2 du}{3\sqrt{u}} = -\frac{2}{3} \int u^{-\frac{1}{2}} du \\ &= -\frac{2}{3} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = -\frac{4\sqrt{u}}{3} + C \end{aligned}$$

Koniec 10. časti príklady b – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} dx = -\frac{2}{x} - x - \frac{2\sqrt{1-x^2}}{x} - 2 \arcsin x + c$$

$$= \int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} dx = \int \frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)} dx = \int \frac{2-x^2+2\sqrt{1-x^2}}{x^2} dx$$

$$= 2 \int x^{-2} dx - \int dx + 2 \int \frac{\sqrt{1-x^2}}{x^2} dx = \left[u = \sqrt{1-x^2} \mid u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{1-x^2}} \right]$$

$$= 2 \frac{x^{-1}}{-1} - x + 2 \left[-\frac{\sqrt{1-x^2}}{x} - \int \frac{dx}{\sqrt{1-x^2}} \right] = -\frac{2}{x} - x - \frac{2\sqrt{1-x^2}}{x} - 2 \arcsin x + c,$$

$$x \in \langle -1; 0 \rangle \cup (0; 1), c \in \mathbb{R}.$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}} dx = \int \frac{\sqrt{x} dx}{\sqrt{1-x\sqrt{x}}}$$

$$= \left[\text{Subst. } t = \sqrt{x} \mid x \in \langle 0; 1 \rangle \right] = \int \frac{2t^2 dt}{\sqrt{1-t^3}} = \left[\text{Subst. } u = 1-t^3 \mid t \in \langle 0; 1 \rangle \right] = -\int \frac{2 du}{3\sqrt{u}} = -\frac{2}{3} \int u^{-\frac{1}{2}} du$$

$$= -\frac{2}{3} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = -\frac{4\sqrt{u}}{3} + c = -\frac{4\sqrt{1-t^3}}{3} + c$$

Koniec 10. časti príklady b – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} dx = -\frac{2}{x} - x - \frac{2\sqrt{1-x^2}}{x} - 2 \arcsin x + c$$

$$= \int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} dx = \int \frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)} dx = \int \frac{2-x^2+2\sqrt{1-x^2}}{x^2} dx$$

$$= 2 \int x^{-2} dx - \int dx + 2 \int \frac{\sqrt{1-x^2}}{x^2} dx = \left[\begin{array}{l} u = \sqrt{1-x^2} \quad \left| \begin{array}{l} u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{1-x^2}} \\ v' = \frac{1}{x^2} = x^{-2} \quad \left| \begin{array}{l} v = \frac{x^{-1}}{-1} = -\frac{1}{x} \end{array} \right. \end{array} \right.$$

$$= 2 \frac{x^{-1}}{-1} - x + 2 \left[-\frac{\sqrt{1-x^2}}{x} - \int \frac{dx}{\sqrt{1-x^2}} \right] = -\frac{2}{x} - x - \frac{2\sqrt{1-x^2}}{x} - 2 \arcsin x + c,$$

$$x \in \langle -1; 0 \rangle \cup (0; 1), c \in \mathbb{R}.$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}} dx = \int \frac{\sqrt{x} dx}{\sqrt{1-x\sqrt{x}}} = -\frac{4\sqrt{1-x\sqrt{x}}}{3} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{x} \quad \left| \begin{array}{l} x \in \langle 0; 1 \rangle \\ x = t^2, dx = 2t dt \quad \left| \begin{array}{l} t \in \langle 0; 1 \rangle \end{array} \right. \end{array} \right. \end{array} \right] = \int \frac{2t^2 dt}{\sqrt{1-t^3}} = \left[\begin{array}{l} \text{Subst. } u = 1-t^3 \quad \left| \begin{array}{l} t \in \langle 0; 1 \rangle \\ du = -3t^2 dt \quad \left| \begin{array}{l} u \in (0; 1) \end{array} \right. \end{array} \right. \end{array} \right] = -\int \frac{2 du}{3\sqrt{u}} = -\frac{2}{3} \int u^{-\frac{1}{2}} du$$

$$= -\frac{2}{3} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = -\frac{4\sqrt{u}}{3} + c = -\frac{4\sqrt{1-t^3}}{3} + c = -\frac{4\sqrt{1-x\sqrt{x}}}{3} + c, x \in \langle 0; 1 \rangle, c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 181

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[4]{x}}$$

Koniec 10. časti príklady b – 181

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[4]{x}}$$

$$= \left[\begin{array}{l} \text{Subst. } x=t^{12}, t=\sqrt[12]{x} \mid \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 \mid x \in (0; \infty) \\ dx=12t^{11} dt \mid \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 \mid t \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 181

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[4]{x}}$$

$$= \left[\text{Subst. } x = t^{12}, t = \sqrt[12]{x} \mid \begin{array}{l} \sqrt[3]{x} = \sqrt[3]{t^{12}} = t^4 \\ \sqrt[4]{x} = \sqrt[4]{t^{12}} = t^3 \end{array} \mid \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^4 + t^3}$$

Koniec 10. časti príklady b – 181

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[4]{x}}$$

$$= \left[\text{Subst. } x = t^{12}, t = \sqrt[12]{x} \mid \begin{array}{l} \sqrt[3]{x} = \sqrt[3]{t^{12}} = t^4 \\ \sqrt[4]{x} = \sqrt[4]{t^{12}} = t^3 \end{array} \mid \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^4 + t^3} = 12 \int \frac{t^8 dt}{t+1}$$

Koniec 10. časti príklady b – 181

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[4]{x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } x=t^{12}, t=\sqrt[12]{x} \left| \begin{array}{l} \sqrt[3]{x} = \sqrt[3]{t^{12}} = t^4 \\ \sqrt[4]{x} = \sqrt[4]{t^{12}} = t^3 \end{array} \right. \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^4 + t^3} = 12 \int \frac{t^8 dt}{t+1} \\
 &= 12 \int \frac{t^8 + (t^7 - t^7) - (t^6 - t^6) + (t^5 - t^5) - (t^4 - t^4) + (t^3 - t^3) - (t^2 - t^2) + (t - t) - (1 - 1)}{t+1} dt
 \end{aligned}$$

Koniec 10. časti príklady b – 181

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[4]{x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } x=t^{12}, t=\sqrt[12]{x} \mid \begin{array}{l} \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 \\ \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 \end{array} \mid \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^4+t^3} = 12 \int \frac{t^8 dt}{t+1} \\
 &= 12 \int \frac{t^8 + (t^7 - t^7) - (t^6 - t^6) + (t^5 - t^5) - (t^4 - t^4) + (t^3 - t^3) - (t^2 - t^2) + (t - t) - (1 - 1)}{t+1} dt \\
 &= 12 \int \frac{(t^8 + t^7) - (t^7 + t^6) + (t^6 + t^5) - (t^5 + t^4) + (t^4 + t^3) - (t^3 + t^2) + (t^2 + t) - (t + 1) + 1}{t+1} dt
 \end{aligned}$$

Koniec 10. časti príklady b – 181

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[4]{x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } x=t^{12}, t=\sqrt[12]{x} \mid \begin{array}{l} \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 \\ \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 \end{array} \mid \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^4+t^3} = 12 \int \frac{t^8 dt}{t+1} \\
 &= 12 \int \frac{t^8 + (t^7 - t^7) - (t^6 - t^6) + (t^5 - t^5) - (t^4 - t^4) + (t^3 - t^3) - (t^2 - t^2) + (t - t) - (1 - 1)}{t+1} dt \\
 &= 12 \int \frac{(t^8 + t^7) - (t^7 + t^6) + (t^6 + t^5) - (t^5 + t^4) + (t^4 + t^3) - (t^3 + t^2) + (t^2 + t) - (t + 1) + 1}{t+1} dt \\
 &= 12 \int \left[t^7 - t^6 + t^5 - t^4 + t^3 - t^2 + t - 1 + \frac{1}{t+1} \right] dt
 \end{aligned}$$

Koniec 10. časti príklady b – 181

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[4]{x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } x=t^{12}, t=\sqrt[12]{x} \mid \begin{array}{l} \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 \\ \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 \end{array} \mid \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^4+t^3} = 12 \int \frac{t^8 dt}{t+1} \\
 &= 12 \int \frac{t^8 + (t^7 - t^7) - (t^6 - t^6) + (t^5 - t^5) - (t^4 - t^4) + (t^3 - t^3) - (t^2 - t^2) + (t - t) - (1 - 1)}{t+1} dt \\
 &= 12 \int \frac{(t^8 + t^7) - (t^7 + t^6) + (t^6 + t^5) - (t^5 + t^4) + (t^4 + t^3) - (t^3 + t^2) + (t^2 + t) - (t+1) + 1}{t+1} dt \\
 &= 12 \int \left[t^7 - t^6 + t^5 - t^4 + t^3 - t^2 + t - 1 + \frac{1}{t+1} \right] dt \\
 &= 12 \left[\frac{t^8}{8} - \frac{t^7}{7} + \frac{t^6}{6} - \frac{t^5}{5} + \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} - t + \ln |t+1| \right] + c
 \end{aligned}$$

Koniec 10. časti príklady b – 181

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[4]{x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } x=t^{12}, t=\sqrt[12]{x} \mid \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 \mid x \in (0; \infty) \right. \\
 &\quad \left. dx=12t^{11} dt \mid \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 \mid t \in (0; \infty) \right] = \int \frac{12t^{11} dt}{t^4+t^3} = 12 \int \frac{t^8 dt}{t+1} \\
 &= 12 \int \frac{t^8 + (t^7 - t^7) - (t^6 - t^6) + (t^5 - t^5) - (t^4 - t^4) + (t^3 - t^3) - (t^2 - t^2) + (t - t) - (1 - 1)}{t+1} dt \\
 &= 12 \int \frac{(t^8 + t^7) - (t^7 + t^6) + (t^6 + t^5) - (t^5 + t^4) + (t^4 + t^3) - (t^3 + t^2) + (t^2 + t) - (t+1) + 1}{t+1} dt \\
 &= 12 \int \left[t^7 - t^6 + t^5 - t^4 + t^3 - t^2 + t - 1 + \frac{1}{t+1} \right] dt \\
 &= 12 \left[\frac{t^8}{8} - \frac{t^7}{7} + \frac{t^6}{6} - \frac{t^5}{5} + \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} - t + \ln |t+1| \right] + c \\
 &= 12 \left[\frac{\sqrt[12]{x^8}}{8} - \frac{\sqrt[12]{x^7}}{7} + \frac{\sqrt[12]{x^6}}{6} - \frac{\sqrt[12]{x^5}}{5} + \frac{\sqrt[12]{x^4}}{4} - \frac{\sqrt[12]{x^3}}{3} + \frac{\sqrt[12]{x^2}}{2} - \sqrt[12]{x} + \ln |\sqrt[12]{x} + 1| \right] + c
 \end{aligned}$$

Koniec 10. časti príklady b – 181

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[4]{x}} = \frac{3\sqrt[3]{x^2}}{2} - \frac{12\sqrt[12]{x^7}}{7} + 2\sqrt{x} - \frac{12\sqrt[12]{x^5}}{5} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12 \ln(\sqrt[12]{x} + 1) + c$$

$$\begin{aligned} &= \left[\text{Subst. } x=t^{12}, t=\sqrt[12]{x} \left| \begin{array}{l} \sqrt[3]{x} = \sqrt[3]{t^{12}} = t^4 \\ \sqrt[4]{x} = \sqrt[4]{t^{12}} = t^3 \end{array} \right. \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^4 + t^3} = 12 \int \frac{t^8 dt}{t+1} \\ &= 12 \int \frac{t^8 + (t^7 - t^7) - (t^6 - t^6) + (t^5 - t^5) - (t^4 - t^4) + (t^3 - t^3) - (t^2 - t^2) + (t - t) - (1 - 1)}{t+1} dt \\ &= 12 \int \frac{(t^8 + t^7) - (t^7 + t^6) + (t^6 + t^5) - (t^5 + t^4) + (t^4 + t^3) - (t^3 + t^2) + (t^2 + t) - (t+1) + 1}{t+1} dt \\ &= 12 \int \left[t^7 - t^6 + t^5 - t^4 + t^3 - t^2 + t - 1 + \frac{1}{t+1} \right] dt \\ &= 12 \left[\frac{t^8}{8} - \frac{t^7}{7} + \frac{t^6}{6} - \frac{t^5}{5} + \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} - t + \ln|t+1| \right] + c \\ &= 12 \left[\frac{\sqrt[12]{x^8}}{8} - \frac{\sqrt[12]{x^7}}{7} + \frac{\sqrt[12]{x^6}}{6} - \frac{\sqrt[12]{x^5}}{5} + \frac{\sqrt[12]{x^4}}{4} - \frac{\sqrt[12]{x^3}}{3} + \frac{\sqrt[12]{x^2}}{2} - \sqrt[12]{x} + \ln|\sqrt[12]{x} + 1| \right] + c \\ &= \frac{3\sqrt[3]{x^2}}{2} - \frac{12\sqrt[12]{x^7}}{7} + 2\sqrt{x} - \frac{12\sqrt[12]{x^5}}{5} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12 \ln(\sqrt[12]{x} + 1) + c, x \in (0; \infty), c \in \mathbb{R}. \end{aligned}$$

Koniec 10. časti príklady b – 182

$$\int \frac{dx}{\sqrt[6]{x} + \sqrt[4]{x}}$$

Koniec 10. časti príklady b – 182

$$\int \frac{dx}{\sqrt[6]{x} + \sqrt[4]{x}}$$

$$= \left[\begin{array}{l} \text{Subst. } x=t^{12}, t=\sqrt[12]{x} \mid \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 \mid x \in (0; \infty) \\ dx=12t^{11} dt \mid \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 \mid t \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 182

$$\int \frac{dx}{\sqrt[6]{x} + \sqrt[4]{x}}$$

$$= \left[\text{Subst. } x = t^{12}, t = \sqrt[12]{x} \mid \begin{array}{l} \sqrt[6]{x} = \sqrt[6]{t^{12}} = t^2 \\ \sqrt[4]{x} = \sqrt[4]{t^{12}} = t^3 \end{array} \mid \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^2 + t^3}$$

Koniec 10. časti príklady b – 182

$$\int \frac{dx}{\sqrt[6]{x} + \sqrt[4]{x}}$$

$$= \left[\text{Subst. } x = t^{12}, t = \sqrt[12]{x} \left| \begin{array}{l} \sqrt[6]{x} = \sqrt[6]{t^{12}} = t^2 \\ \sqrt[4]{x} = \sqrt[4]{t^{12}} = t^3 \end{array} \right. \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^2 + t^3} = 12 \int \frac{t^9 dt}{1+t}$$

Koniec 10. časti príklady b – 182

$$\int \frac{dx}{\sqrt[6]{x} + \sqrt[4]{x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } x=t^{12}, t=\sqrt[12]{x} \left| \begin{array}{l} \sqrt[6]{x} = \sqrt[6]{t^{12}} = t^2 \\ \sqrt[4]{x} = \sqrt[4]{t^{12}} = t^3 \end{array} \right. \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^2 + t^3} = 12 \int \frac{t^9 dt}{1+t} \\
 &= 12 \int \frac{t^9 + (t^8 - t^8) - (t^7 - t^7) + (t^6 - t^6) - (t^5 - t^5) + (t^4 - t^4) - (t^3 - t^3) + (t^2 - t^2) - (t - t) + (1 - 1)}{t+1} dt
 \end{aligned}$$

Koniec 10. časti príklady b – 182

$$\int \frac{dx}{\sqrt[6]{x} + \sqrt[4]{x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } x=t^{12}, t=\sqrt[12]{x} \mid \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 \mid x \in (0; \infty) \right. \\
 &\quad \left. dx=12t^{11} dt \mid \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 \mid t \in (0; \infty) \right] = \int \frac{12t^{11} dt}{t^2+t^3} = 12 \int \frac{t^9 dt}{1+t} \\
 &= 12 \int \frac{t^9 + (t^8 - t^8) - (t^7 - t^7) + (t^6 - t^6) - (t^5 - t^5) + (t^4 - t^4) - (t^3 - t^3) + (t^2 - t^2) - (t - t) + (1 - 1)}{t+1} dt \\
 &= 12 \int \frac{(t^9 + t^8) - (t^8 + t^7) + (t^7 + t^6) - (t^6 + t^5) + (t^5 + t^4) - (t^4 + t^3) + (t^3 + t^2) - (t^2 + t) + (t+1) - 1}{t+1} dt
 \end{aligned}$$

Koniec 10. časti príklady b – 182

$$\int \frac{dx}{\sqrt[6]{x} + \sqrt[4]{x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } x=t^{12}, t=\sqrt[12]{x} \left| \begin{array}{l} \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 \\ \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 \end{array} \right. \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^2+t^3} = 12 \int \frac{t^9 dt}{1+t} \\
 &= 12 \int \frac{t^9 + (t^8 - t^8) - (t^7 - t^7) + (t^6 - t^6) - (t^5 - t^5) + (t^4 - t^4) - (t^3 - t^3) + (t^2 - t^2) - (t - t) + (1 - 1)}{t+1} dt \\
 &= 12 \int \frac{(t^9 + t^8) - (t^8 + t^7) + (t^7 + t^6) - (t^6 + t^5) + (t^5 + t^4) - (t^4 + t^3) + (t^3 + t^2) - (t^2 + t) + (t+1) - 1}{t+1} dt \\
 &= 12 \int \left[t^8 - t^7 + t^6 - t^5 + t^4 - t^3 + t^2 - t + 1 - \frac{1}{t+1} \right] dt
 \end{aligned}$$

Koniec 10. časti príklady b – 182

$$\int \frac{dx}{\sqrt[6]{x} + \sqrt[4]{x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } x=t^{12}, t=\sqrt[12]{x} \left| \begin{array}{l} \sqrt[6]{x} = \sqrt[6]{t^{12}} = t^2 \\ \sqrt[4]{x} = \sqrt[4]{t^{12}} = t^3 \end{array} \right. \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^2 + t^3} = 12 \int \frac{t^9 dt}{1+t} \\
 &= 12 \int \frac{t^9 + (t^8 - t^8) - (t^7 - t^7) + (t^6 - t^6) - (t^5 - t^5) + (t^4 - t^4) - (t^3 - t^3) + (t^2 - t^2) - (t - t) + (1 - 1)}{t+1} dt \\
 &= 12 \int \frac{(t^9 + t^8) - (t^8 + t^7) + (t^7 + t^6) - (t^6 + t^5) + (t^5 + t^4) - (t^4 + t^3) + (t^3 + t^2) - (t^2 + t) + (t+1) - 1}{t+1} dt \\
 &= 12 \int \left[t^8 - t^7 + t^6 - t^5 + t^4 - t^3 + t^2 - t + 1 - \frac{1}{t+1} \right] dt \\
 &= 12 \left[\frac{t^9}{9} - \frac{t^8}{8} + \frac{t^7}{7} - \frac{t^6}{6} + \frac{t^5}{5} - \frac{t^4}{4} + \frac{t^3}{3} - \frac{t^2}{2} + t - \ln |t+1| \right] + c
 \end{aligned}$$

Koniec 10. časti príklady b – 182

$$\int \frac{dx}{\sqrt[6]{x} + \sqrt[4]{x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } x=t^{12}, t=\sqrt[12]{x} \left| \begin{array}{l} \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 \\ \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 \end{array} \right. \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^2+t^3} = 12 \int \frac{t^9 dt}{1+t} \\
 &= 12 \int \frac{t^9 + (t^8 - t^8) - (t^7 - t^7) + (t^6 - t^6) - (t^5 - t^5) + (t^4 - t^4) - (t^3 - t^3) + (t^2 - t^2) - (t - t) + (1 - 1)}{t+1} dt \\
 &= 12 \int \frac{(t^9 + t^8) - (t^8 + t^7) + (t^7 + t^6) - (t^6 + t^5) + (t^5 + t^4) - (t^4 + t^3) + (t^3 + t^2) - (t^2 + t) + (t+1) - 1}{t+1} dt \\
 &= 12 \int \left[t^8 - t^7 + t^6 - t^5 + t^4 - t^3 + t^2 - t + 1 - \frac{1}{t+1} \right] dt \\
 &= 12 \left[\frac{t^9}{9} - \frac{t^8}{8} + \frac{t^7}{7} - \frac{t^6}{6} + \frac{t^5}{5} - \frac{t^4}{4} + \frac{t^3}{3} - \frac{t^2}{2} + t - \ln |t+1| \right] + c \\
 &= 12 \left[\frac{\sqrt[12]{x^9}}{9} - \frac{\sqrt[12]{x^8}}{8} + \frac{\sqrt[12]{x^7}}{7} - \frac{\sqrt[12]{x^6}}{6} + \frac{\sqrt[12]{x^5}}{5} - \frac{\sqrt[12]{x^4}}{4} + \frac{\sqrt[12]{x^3}}{3} - \frac{\sqrt[12]{x^2}}{2} + \sqrt[12]{x} - \ln \left| \sqrt[12]{x} + 1 \right| \right] + c
 \end{aligned}$$

Koniec 10. časti príklady b – 182

$$\begin{aligned}
 \int \frac{dx}{\sqrt[6]{x} + \sqrt[4]{x}} &= \frac{4\sqrt[4]{x^3}}{3} - \frac{3\sqrt[3]{x^2}}{2} + \frac{12\sqrt[12]{x^7}}{7} - 2\sqrt{x} + \frac{12\sqrt[12]{x^5}}{5} - 3\sqrt[3]{x} + 4\sqrt[4]{x} \\
 &\quad - 6\sqrt[6]{x} + 12\sqrt[12]{x} - 12 \ln(\sqrt[12]{x} + 1) + c \\
 &= \left[\text{Subst. } x = t^{12}, t = \sqrt[12]{x} \left| \begin{array}{l} \sqrt[6]{x} = \sqrt[6]{t^{12}} = t^2 \\ \sqrt[4]{x} = \sqrt[4]{t^{12}} = t^3 \end{array} \right. \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{12t^{11} dt}{t^2 + t^3} = 12 \int \frac{t^9 dt}{1+t} \\
 &= 12 \int \frac{t^9 + (t^8 - t^8) - (t^7 - t^7) + (t^6 - t^6) - (t^5 - t^5) + (t^4 - t^4) - (t^3 - t^3) + (t^2 - t^2) - (t - t) + (1 - 1)}{t+1} dt \\
 &= 12 \int \frac{(t^9 + t^8) - (t^8 + t^7) + (t^7 + t^6) - (t^6 + t^5) + (t^5 + t^4) - (t^4 + t^3) + (t^3 + t^2) - (t^2 + t) + (t + 1) - 1}{t+1} dt \\
 &= 12 \int \left[t^8 - t^7 + t^6 - t^5 + t^4 - t^3 + t^2 - t + 1 - \frac{1}{t+1} \right] dt \\
 &= 12 \left[\frac{t^9}{9} - \frac{t^8}{8} + \frac{t^7}{7} - \frac{t^6}{6} + \frac{t^5}{5} - \frac{t^4}{4} + \frac{t^3}{3} - \frac{t^2}{2} + t - \ln|t+1| \right] + c \\
 &= 12 \left[\frac{\sqrt[12]{x^9}}{9} - \frac{\sqrt[12]{x^8}}{8} + \frac{\sqrt[12]{x^7}}{7} - \frac{\sqrt[12]{x^6}}{6} + \frac{\sqrt[12]{x^5}}{5} - \frac{\sqrt[12]{x^4}}{4} + \frac{\sqrt[12]{x^3}}{3} - \frac{\sqrt[12]{x^2}}{2} + \sqrt[12]{x} - \ln|\sqrt[12]{x} + 1| \right] + c \\
 &= \frac{4\sqrt[4]{x^3}}{3} - \frac{3\sqrt[3]{x^2}}{2} + \frac{12\sqrt[12]{x^7}}{7} - 2\sqrt{x} + \frac{12\sqrt[12]{x^5}}{5} - 3\sqrt[3]{x} + 4\sqrt[4]{x} - 6\sqrt[6]{x} + 12\sqrt[12]{x} \\
 &\quad - 12 \ln(\sqrt[12]{x} + 1) + c, x \in (0; \infty), c \in \mathbb{R}.
 \end{aligned}$$

Koniec 10. časti príklady b – 183

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[5]{x}}$$

Koniec 10. časti príklady b – 183

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[5]{x}}$$

$$= \left[\begin{array}{l} \text{Subst. } x = t^{15}, t = \sqrt[15]{x} \mid \sqrt[3]{x} = \sqrt[3]{t^{15}} = t^5 \mid x \in (0; \infty) \\ dx = 15t^{14} dt \mid \sqrt[5]{x} = \sqrt[5]{t^{15}} = t^3 \mid t \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 183

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[5]{x}}$$

$$= \left[\begin{array}{l} \text{Subst. } x = t^{15}, t = \sqrt[15]{x} \mid \sqrt[3]{x} = \sqrt[3]{t^{15}} = t^5 \mid x \in (0; \infty) \\ dx = 15t^{14} dt \mid \sqrt[5]{x} = \sqrt[5]{t^{15}} = t^3 \mid t \in (0; \infty) \end{array} \right] = \int \frac{15t^{14} dt}{t^5 + t^3}$$

Koniec 10. časti príklady b – 183

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[5]{x}}$$

$$= \left[\begin{array}{l} \text{Subst. } x = t^{15}, t = \sqrt[15]{x} \mid \sqrt[3]{x} = \sqrt[3]{t^{15}} = t^5 \mid x \in (0; \infty) \\ dx = 15t^{14} dt \mid \sqrt[5]{x} = \sqrt[5]{t^{15}} = t^3 \mid t \in (0; \infty) \end{array} \right] = \int \frac{15t^{14} dt}{t^5 + t^3} = 15 \int \frac{t^{11} dt}{t^2 + 1}$$

Koniec 10. časti príklady b – 183

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[5]{x}}$$

$$= \left[\begin{array}{l} \text{Subst. } x = t^{15}, t = \sqrt[15]{x} \mid \sqrt[3]{x} = \sqrt[3]{t^{15}} = t^5 \mid x \in (0; \infty) \\ dx = 15t^{14} dt \mid \sqrt[5]{x} = \sqrt[5]{t^{15}} = t^3 \mid t \in (0; \infty) \end{array} \right] = \int \frac{15t^{14} dt}{t^5 + t^3} = 15 \int \frac{t^{11} dt}{t^2 + 1}$$

$$= 15 \int \frac{t^{11} + (t^9 - t^9) - (t^7 - t^7) + (t^5 - t^5) - (t^3 - t^3) + (t - t)}{t^2 + 1} dt$$

Koniec 10. časti príklady b – 183

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[5]{x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } x = t^{15}, t = \sqrt[15]{x} \mid \begin{array}{l} \sqrt[3]{x} = \sqrt[3]{t^{15}} = t^5 \mid x \in (0; \infty) \\ dx = 15t^{14} dt \mid \sqrt[5]{x} = \sqrt[5]{t^{15}} = t^3 \mid t \in (0; \infty) \end{array} \right] = \int \frac{15t^{14} dt}{t^5 + t^3} = 15 \int \frac{t^{11} dt}{t^2 + 1} \\
 &= 15 \int \frac{t^{11} + (t^9 - t^9) - (t^7 - t^7) + (t^5 - t^5) - (t^3 - t^3) + (t - t)}{t^2 + 1} dt \\
 &= 15 \int \frac{(t^{11} + t^9) - (t^9 + t^7) + (t^7 + t^5) - (t^5 + t^3) + (t^3 + t) - t}{t^2 + 1} dt
 \end{aligned}$$

Koniec 10. časti príklady b – 183

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[5]{x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } x = t^{15}, t = \sqrt[15]{x} \mid \sqrt[3]{x} = \sqrt[3]{t^{15}} = t^5 \mid x \in (0; \infty) \right. \\
 &\quad \left. dx = 15t^{14} dt \mid \sqrt[5]{x} = \sqrt[5]{t^{15}} = t^3 \mid t \in (0; \infty) \right] = \int \frac{15t^{14} dt}{t^5 + t^3} = 15 \int \frac{t^{11} dt}{t^2 + 1} \\
 &= 15 \int \frac{t^{11} + (t^9 - t^9) - (t^7 - t^7) + (t^5 - t^5) - (t^3 - t^3) + (t - t)}{t^2 + 1} dt \\
 &= 15 \int \frac{(t^{11} + t^9) - (t^9 + t^7) + (t^7 + t^5) - (t^5 + t^3) + (t^3 + t) - t}{t^2 + 1} dt \\
 &= 15 \int \left[t^9 - t^7 + t^5 - t^3 + t - \frac{t}{t^2 + 1} \right] dt
 \end{aligned}$$

Koniec 10. časti príklady b – 183

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[5]{x}}$$

$$= \left[\begin{array}{l} \text{Subst. } x = t^{15}, t = \sqrt[15]{x} \mid \sqrt[3]{x} = \sqrt[3]{t^{15}} = t^5 \mid x \in (0; \infty) \\ dx = 15t^{14} dt \mid \sqrt[5]{x} = \sqrt[5]{t^{15}} = t^3 \mid t \in (0; \infty) \end{array} \right] = \int \frac{15t^{14} dt}{t^5 + t^3} = 15 \int \frac{t^{11} dt}{t^2 + 1}$$

$$= 15 \int \frac{t^{11} + (t^9 - t^9) - (t^7 - t^7) + (t^5 - t^5) - (t^3 - t^3) + (t - t)}{t^2 + 1} dt$$

$$= 15 \int \frac{(t^{11} + t^9) - (t^9 + t^7) + (t^7 + t^5) - (t^5 + t^3) + (t^3 + t) - t}{t^2 + 1} dt$$

$$= 15 \int \left[t^9 - t^7 + t^5 - t^3 + t - \frac{t}{t^2 + 1} \right] dt = 15 \int \left[t^9 - t^7 + t^5 - t^3 + t - \frac{1}{2} \cdot \frac{2t}{t^2 + 1} \right] dt$$

Koniec 10. časti príklady b – 183

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[5]{x}}$$

$$= \left[\begin{array}{l} \text{Subst. } x = t^{15}, \quad t = \sqrt[15]{x} \mid \sqrt[3]{x} = \sqrt[3]{t^{15}} = t^5 \mid x \in (0; \infty) \\ \quad \quad \quad dx = 15t^{14} dt \mid \sqrt[5]{x} = \sqrt[5]{t^{15}} = t^3 \mid t \in (0; \infty) \end{array} \right] = \int \frac{15t^{14} dt}{t^5 + t^3} = 15 \int \frac{t^{11} dt}{t^2 + 1}$$

$$= 15 \int \frac{t^{11} + (t^9 - t^9) - (t^7 - t^7) + (t^5 - t^5) - (t^3 - t^3) + (t - t)}{t^2 + 1} dt$$

$$= 15 \int \frac{(t^{11} + t^9) - (t^9 + t^7) + (t^7 + t^5) - (t^5 + t^3) + (t^3 + t) - t}{t^2 + 1} dt$$

$$= 15 \int \left[t^9 - t^7 + t^5 - t^3 + t - \frac{t}{t^2 + 1} \right] dt = 15 \int \left[t^9 - t^7 + t^5 - t^3 + t - \frac{1}{2} \cdot \frac{2t}{t^2 + 1} \right] dt$$

$$= 15 \left[\frac{t^{10}}{10} - \frac{t^8}{8} + \frac{t^6}{6} - \frac{t^4}{4} + \frac{t^2}{2} - \frac{1}{2} \ln |t^2 + 1| \right] + c$$

Koniec 10. časti príklady b – 183

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[5]{x}}$$

$$= \left[\begin{array}{l} \text{Subst. } x = t^{15}, t = \sqrt[15]{x} \mid \sqrt[3]{x} = \sqrt[3]{t^{15}} = t^5 \mid x \in (0; \infty) \\ dx = 15t^{14} dt \mid \sqrt[5]{x} = \sqrt[5]{t^{15}} = t^3 \mid t \in (0; \infty) \end{array} \right] = \int \frac{15t^{14} dt}{t^5 + t^3} = 15 \int \frac{t^{11} dt}{t^2 + 1}$$

$$= 15 \int \frac{t^{11} + (t^9 - t^9) - (t^7 - t^7) + (t^5 - t^5) - (t^3 - t^3) + (t - t)}{t^2 + 1} dt$$

$$= 15 \int \frac{(t^{11} + t^9) - (t^9 + t^7) + (t^7 + t^5) - (t^5 + t^3) + (t^3 + t) - t}{t^2 + 1} dt$$

$$= 15 \int \left[t^9 - t^7 + t^5 - t^3 + t - \frac{t}{t^2 + 1} \right] dt = 15 \int \left[t^9 - t^7 + t^5 - t^3 + t - \frac{1}{2} \cdot \frac{2t}{t^2 + 1} \right] dt$$

$$= 15 \left[\frac{t^{10}}{10} - \frac{t^8}{8} + \frac{t^6}{6} - \frac{t^4}{4} + \frac{t^2}{2} - \frac{1}{2} \ln |t^2 + 1| \right] + c$$

$$= 15 \left[\frac{\sqrt[15]{x^{10}}}{10} - \frac{\sqrt[15]{x^8}}{8} + \frac{\sqrt[15]{x^6}}{6} - \frac{\sqrt[15]{x^4}}{4} + \frac{\sqrt[15]{x^2}}{2} - \frac{1}{2} \ln |\sqrt[15]{x^2} + 1| \right] + c$$

Koniec 10. časti príklady b – 183

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt[5]{x}} = \frac{3\sqrt[3]{x^2}}{2} - \frac{15\sqrt[15]{x^8}}{8} + \frac{5\sqrt[5]{x^2}}{2} - \frac{15\sqrt[15]{x^4}}{4} + \frac{15\sqrt[15]{x^2}}{2} - \frac{15}{2} \ln(\sqrt[15]{x^2} + 1) + c$$

$$\begin{aligned} &= \left[\text{Subst. } x = t^{15}, t = \sqrt[15]{x} \mid \sqrt[3]{x} = \sqrt[3]{t^{15}} = t^5 \mid x \in (0; \infty) \right] = \int \frac{15t^{14} dt}{t^5 + t^3} = 15 \int \frac{t^{11} dt}{t^2 + 1} \\ &= 15 \int \frac{t^{11} + (t^9 - t^9) - (t^7 - t^7) + (t^5 - t^5) - (t^3 - t^3) + (t - t)}{t^2 + 1} dt \\ &= 15 \int \frac{(t^{11} + t^9) - (t^9 + t^7) + (t^7 + t^5) - (t^5 + t^3) + (t^3 + t) - t}{t^2 + 1} dt \\ &= 15 \int \left[t^9 - t^7 + t^5 - t^3 + t - \frac{t}{t^2 + 1} \right] dt = 15 \int \left[t^9 - t^7 + t^5 - t^3 + t - \frac{1}{2} \cdot \frac{2t}{t^2 + 1} \right] dt \\ &= 15 \left[\frac{t^{10}}{10} - \frac{t^8}{8} + \frac{t^6}{6} - \frac{t^4}{4} + \frac{t^2}{2} - \frac{1}{2} \ln |t^2 + 1| \right] + c \\ &= 15 \left[\frac{\sqrt[15]{x^{10}}}{10} - \frac{\sqrt[15]{x^8}}{8} + \frac{\sqrt[15]{x^6}}{6} - \frac{\sqrt[15]{x^4}}{4} + \frac{\sqrt[15]{x^2}}{2} - \frac{1}{2} \ln |\sqrt[15]{x^2} + 1| \right] + c \\ &= \frac{3\sqrt[3]{x^2}}{2} - \frac{15\sqrt[15]{x^8}}{8} + \frac{5\sqrt[5]{x^2}}{2} - \frac{15\sqrt[15]{x^4}}{4} + \frac{15\sqrt[15]{x^2}}{2} - \frac{15}{2} \ln(\sqrt[15]{x^2} + 1) + c, \\ &\quad x \in (0; \infty), c \in \mathbb{R}. \end{aligned}$$

Koniec 10. časti príklady b – 184, 185

$$\int \frac{dx}{\sqrt[3]{x+1}}$$

$$\int \left[\sqrt{x^3} - \frac{1}{\sqrt{x}} \right] dx$$

Koniec 10. časti príklady b – 184, 185

$$\int \frac{dx}{\sqrt[3]{x+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } x=t^3 \mid x \in (0; \infty) \\ dx = 3t^2 dx \mid t \in (0; \infty) \end{array} \right]$$

$$\int \left[\sqrt{x^3} - \frac{1}{\sqrt{x}} \right] dx$$

$$= \int \left[x^{\frac{3}{2}} - x^{-\frac{1}{2}} \right] dx$$

Koniec 10. časti príklady b – 184, 185

$$\int \frac{dx}{\sqrt[3]{x+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } x=t^3 \mid x \in (0; \infty) \\ dx=3t^2 dx \mid t \in (0; \infty) \end{array} \right] = \int \frac{3t^2 dt}{t+1}$$

$$\int \left[\sqrt{x^3} - \frac{1}{\sqrt{x}} \right] dx$$

$$= \int \left[x^{\frac{3}{2}} - x^{-\frac{1}{2}} \right] dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

Koniec 10. časti príklady b – 184, 185

$$\int \frac{dx}{\sqrt[3]{x+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } x=t^3 \mid x \in (0; \infty) \\ dx=3t^2 dx \mid t \in (0; \infty) \end{array} \right] = \int \frac{3t^2 dt}{t+1} = 3 \int \frac{t^2+t-t-1+1}{t+1} dt$$

$$\int \left[\sqrt{x^3} - \frac{1}{\sqrt{x}} \right] dx$$

$$= \int \left[x^{\frac{3}{2}} - x^{-\frac{1}{2}} \right] dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2}{5}x^{\frac{5}{2}} - 2x^{\frac{1}{2}} + c$$

Koniec 10. časti príklady b – 184, 185

$$\int \frac{dx}{\sqrt[3]{x+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } x=t^3 \\ dx=3t^2 dx \end{array} \left| \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{3t^2 dt}{t+1} = 3 \int \frac{t^2+t-t-1+1}{t+1} dt = 3 \int \left[t - 1 + \frac{1}{t+1} \right] dt$$

$$\int \left[\sqrt{x^3} - \frac{1}{\sqrt{x}} \right] dx = \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c$$

$$= \int \left[x^{\frac{3}{2}} - x^{-\frac{1}{2}} \right] dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2}{5}x^{\frac{5}{2}} - 2x^{\frac{1}{2}} + c$$

$$= \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c, x \in (0; \infty), c \in R.$$

Koniec 10. časti príklady b – 184, 185

$$\int \frac{dx}{\sqrt[3]{x+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } x=t^3 \mid x \in (0; \infty) \\ dx=3t^2 dx \mid t \in (0; \infty) \end{array} \right] = \int \frac{3t^2 dt}{t+1} = 3 \int \frac{t^2+t-t-1+1}{t+1} dt = 3 \int \left[t - 1 + \frac{1}{t+1} \right] dt$$

$$= \frac{3t^2}{2} - 3t + 3 \ln(t+1) + c$$

$$\int \left[\sqrt{x^3} - \frac{1}{\sqrt{x}} \right] dx = \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c$$

$$= \int \left[x^{\frac{3}{2}} - x^{-\frac{1}{2}} \right] dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2}{5}x^{\frac{5}{2}} - 2x^{\frac{1}{2}} + c$$

$$= \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c, x \in (0; \infty), c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 184, 185

$$\int \frac{dx}{\sqrt[3]{x+1}} = \frac{3\sqrt[3]{x^2}}{2} - 3\sqrt[3]{x} + 3\ln(\sqrt[3]{x}+1) + c$$

$$= \left[\begin{array}{l} \text{Subst. } x=t^3 \\ dx=3t^2 dx \end{array} \middle| \begin{array}{l} x \in \langle 0; \infty \rangle \\ t \in \langle 0; \infty \rangle \end{array} \right] = \int \frac{3t^2 dt}{t+1} = 3 \int \frac{t^2+t-t-1+1}{t+1} dt = 3 \int \left[t - 1 + \frac{1}{t+1} \right] dt$$

$$= \frac{3t^2}{2} - 3t + 3\ln(t+1) + c$$

$$= \frac{3\sqrt[3]{x^2}}{2} - 3\sqrt[3]{x} + 3\ln(\sqrt[3]{x}+1) + c, x \in \langle 0; \infty \rangle, c \in R.$$

$$\int \left[\sqrt{x^3} - \frac{1}{\sqrt{x}} \right] dx = \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c$$

$$= \int \left[x^{\frac{3}{2}} - x^{-\frac{1}{2}} \right] dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2}{5}x^{\frac{5}{2}} - 2x^{\frac{1}{2}} + c$$

$$= \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c, x \in (0; \infty), c \in R.$$

Koniec 10. časti príklady b – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x} dx$$

Koniec 10. časti príklady b – 186

$$\int \frac{x-1}{(\sqrt{x} + \sqrt[3]{x^2})x} dx$$

$$= \left[\begin{array}{l} \text{Subst.} \quad t = \sqrt[6]{x} \mid \sqrt{x} = \sqrt{t^6} = t^3 \\ x = t^6, \, dx = 6t^5 dt \mid \sqrt[3]{x^2} = \sqrt[3]{t^{12}} = t^4 \end{array} \right. \left. \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x} dx$$

$$= \left[\text{Subst.} \quad t = \sqrt[6]{x} \mid \begin{array}{l} \sqrt{x} = \sqrt{t^6} = t^3 \\ \sqrt[3]{x^2} = \sqrt[3]{t^{12}} = t^4 \end{array} \mid \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{(t^6-1)6t^5 dt}{(t^3+t^4)t^6}$$

Koniec 10. časti príklady b – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x} dx$$

$$= \left[\text{Subst.} \quad t = \sqrt[6]{x} \mid \begin{array}{l} \sqrt{x} = \sqrt{t^6} = t^3 \\ \sqrt[3]{x^2} = \sqrt[3]{t^{12}} = t^4 \end{array} \mid \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{(t^6-1)6t^5 dt}{(t^3+t^4)t^6} = 6 \int \frac{(t^6-1) dt}{t^4+t^5}$$

Koniec 10. časti príklady b – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt[6]{x} \mid \sqrt{x} = \sqrt{t^6} = t^3 \\ x = t^6, dx = 6t^5 dt \mid \sqrt[3]{x^2} = \sqrt[3]{t^{12}} = t^4 \end{array} \left| \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{(t^6-1)6t^5 dt}{(t^3+t^4)t^6} = 6 \int \frac{(t^6-1) dt}{t^4+t^5}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1 + \frac{t^4-1}{t^5+t^4} = t-1 + \frac{(t^2-1)(t^2+1)}{t^4(t+1)} \\ = t-1 + \frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1 + \frac{(t-1)(t^2+1)}{t^4} = t-1 + \frac{t^3-t^2+t-1}{t^4} = t-1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \end{array} \right]$$

Koniec 10. časti príklady b – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt[6]{x} \mid \sqrt{x} = \sqrt{t^6} = t^3 \\ x = t^6, dx = 6t^5 dt \mid \sqrt[3]{x^2} = \sqrt[3]{t^{12}} = t^4 \end{array} \left| \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{(t^6-1)6t^5 dt}{(t^3+t^4)t^6} = 6 \int \frac{(t^6-1) dt}{t^4+t^5}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1 + \frac{t^4-1}{t^5+t^4} = t-1 + \frac{(t^2-1)(t^2+1)}{t^4(t+1)} \right. \\ \left. = t-1 + \frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1 + \frac{(t-1)(t^2+1)}{t^4} = t-1 + \frac{t^3-t^2+t-1}{t^4} = t-1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \right]$$

$$= 6 \int \left(t - 1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \right) dt$$

Koniec 10. časti príklady b – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt[6]{x} \mid \sqrt{x} = \sqrt{t^6} = t^3 \\ x = t^6, dx = 6t^5 dt \mid \sqrt[3]{x^2} = \sqrt[3]{t^{12}} = t^4 \end{array} \left| \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{(t^6-1)6t^5 dt}{(t^3+t^4)t^6} = 6 \int \frac{(t^6-1) dt}{t^4+t^5}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1 + \frac{t^4-1}{t^5+t^4} = t-1 + \frac{(t^2-1)(t^2+1)}{t^4(t+1)} \right. \\ \left. = t-1 + \frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1 + \frac{(t-1)(t^2+1)}{t^4} = t-1 + \frac{t^3-t^2+t-1}{t^4} = t-1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \right]$$

$$= 6 \int (t-1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4}) dt = 6 \int (t-1 + \frac{1}{t} - t^{-2} + t^{-3} - t^{-4}) dt$$

Koniec 10. časti príklady b – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x} dx$$

$$= \left[\text{Subst.} \quad \begin{array}{l} t = \sqrt[6]{x} \mid \sqrt{x} = \sqrt{t^6} = t^3 \\ x = t^6, \, dx = 6t^5 dt \mid \sqrt[3]{x^2} = \sqrt[3]{t^{12}} = t^4 \end{array} \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{(t^6-1)6t^5 dt}{(t^3+t^4)t^6} = 6 \int \frac{(t^6-1) dt}{t^4+t^5}$$

$$= \left[\text{Rozklad na parciálne zlomky:} \quad \begin{aligned} \frac{t^6-1}{t^5+t^4} &= \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1 + \frac{t^4-1}{t^5+t^4} = t-1 + \frac{(t^2-1)(t^2+1)}{t^4(t+1)} \\ &= t-1 + \frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1 + \frac{(t-1)(t^2+1)}{t^4} = t-1 + \frac{t^3-t^2+t-1}{t^4} = t-1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \end{aligned} \right]$$

$$= 6 \int \left(t - 1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \right) dt = 6 \int \left(t - 1 + \frac{1}{t} - t^{-2} + t^{-3} - t^{-4} \right) dt$$

$$= 6 \left(\frac{t^2}{2} - t + \ln |t| - \frac{t^{-1}}{-1} + \frac{t^{-2}}{-2} + \frac{t^{-3}}{-3} \right) + c$$

Koniec 10. časti príklady b – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt[6]{x} \mid \sqrt{x} = \sqrt{t^6} = t^3 \\ x = t^6, dx = 6t^5 dt \mid \sqrt[3]{x^2} = \sqrt[3]{t^{12}} = t^4 \end{array} \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{(t^6-1)6t^5 dt}{(t^3+t^4)t^6} = 6 \int \frac{(t^6-1) dt}{t^4+t^5}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1 + \frac{t^4-1}{t^5+t^4} = t-1 + \frac{(t^2-1)(t^2+1)}{t^4(t+1)} \right. \\ \left. = t-1 + \frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1 + \frac{(t-1)(t^2+1)}{t^4} = t-1 + \frac{t^3-t^2+t-1}{t^4} = t-1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \right]$$

$$= 6 \int \left(t - 1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \right) dt = 6 \int \left(t - 1 + \frac{1}{t} - t^{-2} + t^{-3} - t^{-4} \right) dt$$

$$= 6 \left(\frac{t^2}{2} - t + \ln |t| - \frac{t^{-1}}{-1} + \frac{t^{-2}}{-2} + \frac{t^{-3}}{-3} \right) + c$$

$$= 3t^2 - 6t + 6 \ln t + \frac{6}{t} - \frac{12}{t^2} + \frac{18}{t^3} + c$$

Koniec 10. časti príklady b – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt[6]{x} \mid \sqrt{x} = \sqrt{t^6} = t^3 \\ x = t^6, dx = 6t^5 dt \mid \sqrt[3]{x^2} = \sqrt[3]{t^{12}} = t^4 \end{array} \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{(t^6-1)6t^5 dt}{(t^3+t^4)t^6} = 6 \int \frac{(t^6-1) dt}{t^4+t^5}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1 + \frac{t^4-1}{t^5+t^4} = t-1 + \frac{(t^2-1)(t^2+1)}{t^4(t+1)} \right. \\ \left. = t-1 + \frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1 + \frac{(t-1)(t^2+1)}{t^4} = t-1 + \frac{t^3-t^2+t-1}{t^4} = t-1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \right]$$

$$= 6 \int \left(t - 1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \right) dt = 6 \int \left(t - 1 + \frac{1}{t} - t^{-2} + t^{-3} - t^{-4} \right) dt$$

$$= 6 \left(\frac{t^2}{2} - t + \ln |t| - \frac{t^{-1}}{-1} + \frac{t^{-2}}{-2} + \frac{t^{-3}}{-3} \right) + c$$

$$= 3t^2 - 6t + 6 \ln t + \frac{6}{t} - \frac{12}{t^2} + \frac{18}{t^3} + c = 3t^2 - 6t + \ln t^6 + \frac{6}{t} - \frac{12}{t^2} + \frac{18}{t^3} + c$$

Koniec 10. časti príklady b – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt[6]{x} \mid \sqrt{x} = \sqrt{t^6} = t^3 \\ x = t^6, dx = 6t^5 dt \mid \sqrt[3]{x^2} = \sqrt[3]{t^{12}} = t^4 \end{array} \left| \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{(t^6-1)6t^5 dt}{(t^3+t^4)t^6} = 6 \int \frac{(t^6-1) dt}{t^4+t^5}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1 + \frac{t^4-1}{t^5+t^4} = t-1 + \frac{(t^2-1)(t^2+1)}{t^4(t+1)} \right. \\ \left. = t-1 + \frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1 + \frac{(t-1)(t^2+1)}{t^4} = t-1 + \frac{t^3-t^2+t-1}{t^4} = t-1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \right]$$

$$= 6 \int \left(t - 1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \right) dt = 6 \int \left(t - 1 + \frac{1}{t} - t^{-2} + t^{-3} - t^{-4} \right) dt$$

$$= 6 \left(\frac{t^2}{2} - t + \ln |t| - \frac{t^{-1}}{-1} + \frac{t^{-2}}{-2} + \frac{t^{-3}}{-3} \right) + c$$

$$= 3t^2 - 6t + 6 \ln t + \frac{6}{t} - \frac{12}{t^2} + \frac{18}{t^3} + c = 3t^2 - 6t + \ln t^6 + \frac{6}{t} - \frac{12}{t^2} + \frac{18}{t^3} + c$$

$$= 3\sqrt[6]{x^2} - 6\sqrt[6]{x} + \ln \sqrt[6]{x^6} + \frac{6}{\sqrt[6]{x}} - \frac{12}{\sqrt[6]{x^2}} + \frac{18}{\sqrt[6]{x^3}} + c$$

Koniec 10. časti príklady b – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x} dx = 3\sqrt[3]{x} - 6\sqrt[6]{x} + \ln x + \frac{6}{\sqrt[6]{x}} - \frac{12}{\sqrt[3]{x}} + \frac{18}{\sqrt{x}} + c$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt[6]{x} \mid \sqrt{x} = \sqrt{t^6} = t^3 \\ x = t^6, dx = 6t^5 dt \mid \sqrt[3]{x^2} = \sqrt[3]{t^{12}} = t^4 \end{array} \mid \begin{array}{l} x \in (0; \infty) \\ t \in (0; \infty) \end{array} \right] = \int \frac{(t^6-1)6t^5 dt}{(t^3+t^4)t^6} = 6 \int \frac{(t^6-1) dt}{t^4+t^5}$$

$$= \left[\text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1 + \frac{t^4-1}{t^5+t^4} = t-1 + \frac{(t^2-1)(t^2+1)}{t^4(t+1)} \right. \\ \left. = t-1 + \frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1 + \frac{(t-1)(t^2+1)}{t^4} = t-1 + \frac{t^3-t^2+t-1}{t^4} = t-1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \right]$$

$$= 6 \int \left(t - 1 + \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t^4} \right) dt = 6 \int \left(t - 1 + \frac{1}{t} - t^{-2} + t^{-3} - t^{-4} \right) dt$$

$$= 6 \left(\frac{t^2}{2} - t + \ln |t| - \frac{t^{-1}}{-1} + \frac{t^{-2}}{-2} + \frac{t^{-3}}{-3} \right) + c$$

$$= 3t^2 - 6t + 6 \ln t + \frac{6}{t} - \frac{12}{t^2} + \frac{18}{t^3} + c = 3t^2 - 6t + \ln t^6 + \frac{6}{t} - \frac{12}{t^2} + \frac{18}{t^3} + c$$

$$= 3\sqrt[6]{x^2} - 6\sqrt[6]{x} + \ln \sqrt[6]{x^6} + \frac{6}{\sqrt[6]{x}} - \frac{12}{\sqrt[6]{x^2}} + \frac{18}{\sqrt[6]{x^3}} + c$$

$$= 3\sqrt[3]{x} - 6\sqrt[6]{x} + \ln x + \frac{6}{\sqrt[6]{x}} - \frac{12}{\sqrt[3]{x}} + \frac{18}{\sqrt{x}} + c, x \in (0; \infty), c \in R.$$

Koniec 10. časti príklady b – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}}$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}}$$

Koniec 10. časti príklady b – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, dx = 2 dt \mid t \in (0; \infty) \end{array} \right]$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1 - t^2, dx = -2 dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right]$$

Koniec 10. časti príklady b – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, dx = 2t dt \mid t \in (0; \infty) \end{array} \right] = \int \frac{2t dt}{(t^2+1)t}$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1 - t^2, dx = -2t dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right] = \int \frac{-2t dt}{(2-t^2)t}$$

Koniec 10. časti príklady b – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, dx = 2t dt \mid t \in (0; \infty) \end{array} \right] = \int \frac{2t dt}{(t^2+1)t} = \int \frac{2 dt}{t^2+1}$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1 - t^2, dx = -2t dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right] = \int \frac{-2t dt}{(2-t^2)t} = 2 \int \frac{dt}{t^2-2}$$

Koniec 10. časti príklady b – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, dx = 2t dt \mid t \in (0; \infty) \end{array} \right] = \int \frac{2t dt}{(t^2+1)t} = \int \frac{2 dt}{t^2+1} = 2 \operatorname{arctg} t + c$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1 - t^2, dx = -2t dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right] = \int \frac{-2t dt}{(2-t^2)t} = 2 \int \frac{dt}{t^2-2} = \frac{2}{2\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c$$

Koniec 10. časti príklady b – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}} = 2 \operatorname{arctg} \sqrt{x-2} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, dx = 2t dt \mid t \in (0; \infty) \end{array} \right] = \int \frac{2t dt}{(t^2+1)t} = \int \frac{2 dt}{t^2+1} = 2 \operatorname{arctg} t + c$$

$$= 2 \operatorname{arctg} \sqrt{x-2} + c, x \in (2; \infty), c \in \mathbb{R}.$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}} = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1 - t^2, dx = -2t dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right] = \int \frac{-2t dt}{(2-t^2)t} = 2 \int \frac{dt}{t^2-2} = \frac{2}{2\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c$$

Koniec 10. časti príklady b – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}} = 2 \operatorname{arctg} \sqrt{x-2} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, dx = 2t dt \mid t \in (0; \infty) \end{array} \right] = \int \frac{2t dt}{(t^2+1)t} = \int \frac{2 dt}{t^2+1} = 2 \operatorname{arctg} t + c$$

$$= 2 \operatorname{arctg} \sqrt{x-2} + c, x \in (2; \infty), c \in \mathbb{R}.$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}} = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1 - t^2, dx = -2t dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right] = \int \frac{-2t dt}{(2-t^2)t} = 2 \int \frac{dt}{t^2-2} = \frac{2}{2\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \cdot \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}-\sqrt{2}} \right| + c$$

Koniec 10. časti príklady b – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}} = 2 \operatorname{arctg} \sqrt{x-2} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, dx = 2t dt \mid t \in (0; \infty) \end{array} \right] = \int \frac{2t dt}{(t^2+1)t} = \int \frac{2 dt}{t^2+1} = 2 \operatorname{arctg} t + c$$

$$= 2 \operatorname{arctg} \sqrt{x-2} + c, x \in (2; \infty), c \in \mathbb{R}.$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}} = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1 - t^2, dx = -2t dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right] = \int \frac{-2t dt}{(2-t^2)t} = 2 \int \frac{dt}{t^2-2} = \frac{2}{2\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \cdot \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}-\sqrt{2}} \right| + c$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{1-x-\sqrt{2} \cdot \sqrt{1-x}+2}{1-x-2} \right| + c$$

Koniec 10. časti príklady b – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}} = 2 \operatorname{arctg} \sqrt{x-2} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, dx = 2t dt \mid t \in (0; \infty) \end{array} \right] = \int \frac{2t dt}{(t^2+1)t} = \int \frac{2 dt}{t^2+1} = 2 \operatorname{arctg} t + c$$

$$= 2 \operatorname{arctg} \sqrt{x-2} + c, x \in (2; \infty), c \in \mathbb{R}.$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}} = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1 - t^2, dx = -2t dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right] = \int \frac{-2t dt}{(2-t^2)t} = 2 \int \frac{dt}{t^2-2} = \frac{2}{2\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \cdot \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}-\sqrt{2}} \right| + c$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{1-x-\sqrt{2} \cdot \sqrt{1-x}+2}{1-x-2} \right| + c = \frac{1}{\sqrt{2}} \ln \left| \frac{3-x-\sqrt{2(1-x)}}{-x-1} \right| + c$$

Koniec 10. časti príklady b – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}} = 2 \operatorname{arctg} \sqrt{x-2} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, dx = 2t dt \mid t \in (0; \infty) \end{array} \right] = \int \frac{2t dt}{(t^2+1)t} = \int \frac{2 dt}{t^2+1} = 2 \operatorname{arctg} t + c$$

$$= 2 \operatorname{arctg} \sqrt{x-2} + c, x \in (2; \infty), c \in R.$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}} = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1-t^2, dx = -2t dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right] = \int \frac{-2t dt}{(2-t^2)t} = 2 \int \frac{dt}{t^2-2} = \frac{2}{2\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \cdot \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}-\sqrt{2}} \right| + c$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{1-x-\sqrt{2} \cdot \sqrt{1-x}+2}{1-x-2} \right| + c = \frac{1}{\sqrt{2}} \ln \left| \frac{3-x-\sqrt{2(1-x)}}{-x-1} \right| + c$$

$$= \frac{1}{\sqrt{2}} \ln |3-x-\sqrt{2(1-x)}| - \frac{1}{\sqrt{2}} \ln |x+1| + c, x \in (-\infty; 1) - \{-1\}, c \in R.$$

Koniec 10. časti príklady b – 189, 190

$$\int (x-1)\sqrt{x-2} dx$$

$$\int \frac{\sqrt{1-x}}{x+1} dx$$

Koniec 10. časti príklady b – 189, 190

$$\int (x-1)\sqrt{x-2} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, dx = 2 dt \mid t \in (0; \infty) \end{array} \right]$$

$$\int \frac{\sqrt{1-x}}{x+1} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1 - t^2, dx = -2 dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right]$$

Koniec 10. časti príklady b – 189, 190

$$\int (x-1)\sqrt{x-2} \, dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, \, dx = 2t \, dt \mid t \in (0; \infty) \end{array} \right] = \int (t^2 + 1)t \cdot 2t \, dt$$

$$\int \frac{\sqrt{1-x}}{x+1} \, dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1 - t^2, \, dx = -2t \, dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right] = \int \frac{-2t^2 \, dt}{2 - t^2}$$

Koniec 10. časti príklady b – 189, 190

$$\int (x-1)\sqrt{x-2} \, dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{x-2} \\ x = t^2 + 2, \, dx = 2t \, dt \end{array} \left| \begin{array}{l} x \in (2; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = \int (t^2 + 1)t \cdot 2t \, dt = \int (2t^4 + 2t^2) \, dt$$

$$\int \frac{\sqrt{1-x}}{x+1} \, dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{1-x} \\ x = 1 - t^2, \, dx = -2t \, dt \end{array} \left| \begin{array}{l} x \in (-\infty; 1) - \{-1\} \\ t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right. \right] = \int \frac{-2t^2 \, dt}{2 - t^2} = 2 \int \frac{t^2 \, dt}{t^2 - 2}$$

Koniec 10. časti príklady b – 189, 190

$$\int (x-1)\sqrt{x-2} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{x-2} \\ x = t^2 + 2, dx = 2t dt \end{array} \left| \begin{array}{l} x \in (2; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = \int (t^2 + 1)t \cdot 2t dt = \int (2t^4 + 2t^2) dt$$

$$= \frac{2t^5}{5} + \frac{2t^3}{3} + c$$

$$\int \frac{\sqrt{1-x}}{x+1} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{1-x} \\ x = 1 - t^2, dx = -2t dt \end{array} \left| \begin{array}{l} x \in (-\infty; 1) - \{-1\} \\ t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right. \right] = \int \frac{-2t^2 dt}{2-t^2} = 2 \int \frac{t^2 dt}{t^2-2} = 2 \int \frac{t^2-2+2 dt}{t^2-2}$$

Koniec 10. časti príklady b – 189, 190

$$\int (x-1)\sqrt{x-2} dx = \frac{2\sqrt{(x-2)^5}}{5} + \frac{2\sqrt{(x-2)^3}}{3} + c$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, dx = 2t dt \mid t \in (0; \infty) \end{array} \right] = \int (t^2 + 1)t \cdot 2t dt = \int (2t^4 + 2t^2) dt$$

$$= \frac{2t^5}{5} + \frac{2t^3}{3} + c = \frac{2\sqrt{(x-2)^5}}{5} + \frac{2\sqrt{(x-2)^3}}{3} + c, x \in (2; \infty), c \in \mathbb{R}.$$

$$\int \frac{\sqrt{1-x}}{x+1} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1 - t^2, dx = -2t dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right] = \int \frac{-2t^2 dt}{2-t^2} = 2 \int \frac{t^2 dt}{t^2-2} = 2 \int \frac{t^2-2+2}{t^2-2} dt$$

$$= 2 \int dt + 4 \int \frac{dt}{t^2-2}$$

Koniec 10. časti príklady b – 189, 190

$$\int (x-1)\sqrt{x-2} dx = \frac{2\sqrt{(x-2)^5}}{5} + \frac{2\sqrt{(x-2)^3}}{3} + c$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{x-2} \mid x \in (2; \infty) \\ x = t^2 + 2, dx = 2t dt \mid t \in (0; \infty) \end{array} \right] = \int (t^2 + 1)t \cdot 2t dt = \int (2t^4 + 2t^2) dt$$

$$= \frac{2t^5}{5} + \frac{2t^3}{3} + c = \frac{2\sqrt{(x-2)^5}}{5} + \frac{2\sqrt{(x-2)^3}}{3} + c, x \in (2; \infty), c \in \mathbb{R}.$$

$$\int \frac{\sqrt{1-x}}{x+1} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{1-x} \mid x \in (-\infty; 1) - \{-1\} \\ x = 1 - t^2, dx = -2t dt \mid t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right] = \int \frac{-2t^2 dt}{2-t^2} = 2 \int \frac{t^2 dt}{t^2-2} = 2 \int \frac{t^2-2+2}{t^2-2} dt$$

$$= 2 \int dt + 4 \int \frac{dt}{t^2-2} = 2t + \frac{4}{2\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c$$

Koniec 10. časti príklady b – 189, 190

$$\int (x-1)\sqrt{x-2} dx = \frac{2\sqrt{(x-2)^5}}{5} + \frac{2\sqrt{(x-2)^3}}{3} + c$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{x-2} \\ x = t^2 + 2, dx = 2 dt \end{array} \middle| \begin{array}{l} x \in (2; \infty) \\ t \in (0; \infty) \end{array} \right] = \int (t^2 + 1)t \cdot 2t dt = \int (2t^4 + 2t^2) dt$$

$$= \frac{2t^5}{5} + \frac{2t^3}{3} + c = \frac{2\sqrt{(x-2)^5}}{5} + \frac{2\sqrt{(x-2)^3}}{3} + c, x \in (2; \infty), c \in \mathbb{R}.$$

$$\int \frac{\sqrt{1-x}}{x+1} dx = 2\sqrt{1-x} + \sqrt{2} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sqrt{1-x} \\ x = 1-t^2, dx = -2t dt \end{array} \middle| \begin{array}{l} x \in (-\infty; 1) - \{-1\} \\ t \in (0; \infty) - \{\sqrt{2}\} \end{array} \right] = \int \frac{-2t^2 dt}{2-t^2} = 2 \int \frac{t^2 dt}{t^2-2} = 2 \int \frac{t^2-2+2 dt}{t^2-2}$$

$$= 2 \int dt + 4 \int \frac{dt}{t^2-2} = 2t + \frac{4}{2\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c$$

$$= 2\sqrt{1-x} + \sqrt{2} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c, x \in (-\infty; 1) - \{-1\}, c \in \mathbb{R}.$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{(x - \sqrt{x^2 - 1})^2}$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{(x - \sqrt{x^2 - 1})^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{(x - \sqrt{x^2 - 1})^2 \cdot (x + \sqrt{x^2 - 1})^2}$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{(x - \sqrt{x^2 - 1})^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{(x - \sqrt{x^2 - 1})^2 \cdot (x + \sqrt{x^2 - 1})^2} = \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[(x - \sqrt{x^2 - 1}) \cdot (x + \sqrt{x^2 - 1})]^2}$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{(x - \sqrt{x^2 - 1})^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{(x - \sqrt{x^2 - 1})^2 \cdot (x + \sqrt{x^2 - 1})^2} = \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[(x - \sqrt{x^2 - 1}) \cdot (x + \sqrt{x^2 - 1})]^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[x^2 - (x^2 - 1)]^2}$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{\left(x - \sqrt{x^2 - 1}\right)^2}$$

$$= \int \frac{\left(x + \sqrt{x^2 - 1}\right)^2 dx}{\left(x - \sqrt{x^2 - 1}\right)^2 \cdot \left(x + \sqrt{x^2 - 1}\right)^2} = \int \frac{\left(x + \sqrt{x^2 - 1}\right)^2 dx}{\left[\left(x - \sqrt{x^2 - 1}\right) \cdot \left(x + \sqrt{x^2 - 1}\right)\right]^2}$$

$$= \int \frac{\left(x + \sqrt{x^2 - 1}\right)^2 dx}{\left[x^2 - (x^2 - 1)\right]^2} = \int \frac{\left(x^2 + 2x\sqrt{x^2 - 1} + x^2 - 1\right) dx}{1}$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{(x - \sqrt{x^2 - 1})^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{(x - \sqrt{x^2 - 1})^2 \cdot (x + \sqrt{x^2 - 1})^2} = \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[(x - \sqrt{x^2 - 1}) \cdot (x + \sqrt{x^2 - 1})]^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[x^2 - (x^2 - 1)]^2} = \int \frac{(x^2 + 2x\sqrt{x^2 - 1} + x^2 - 1) dx}{1}$$

$$= \int (2x^2 - 1 + 2x\sqrt{x^2 - 1}) dx$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{(x - \sqrt{x^2 - 1})^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{(x - \sqrt{x^2 - 1})^2 \cdot (x + \sqrt{x^2 - 1})^2} = \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[(x - \sqrt{x^2 - 1}) \cdot (x + \sqrt{x^2 - 1})]^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[x^2 - (x^2 - 1)]^2} = \int \frac{(x^2 + 2x\sqrt{x^2 - 1} + x^2 - 1) dx}{1}$$

$$= \int (2x^2 - 1 + 2x\sqrt{x^2 - 1}) dx = 2 \int x^2 dx - \int dx + \int 2x\sqrt{x^2 - 1} dx$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{(x - \sqrt{x^2 - 1})^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{(x - \sqrt{x^2 - 1})^2 \cdot (x + \sqrt{x^2 - 1})^2} = \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[(x - \sqrt{x^2 - 1}) \cdot (x + \sqrt{x^2 - 1})]^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[x^2 - (x^2 - 1)]^2} = \int \frac{(x^2 + 2x\sqrt{x^2 - 1} + x^2 - 1) dx}{1}$$

$$= \int (2x^2 - 1 + 2x\sqrt{x^2 - 1}) dx = 2 \int x^2 dx - \int dx + \int 2x\sqrt{x^2 - 1} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - 1 \mid x \in (-\infty; -1), t \in (0; \infty) \\ \quad \quad \quad dt = 2x dx \mid x \in (1; \infty), t \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{(x - \sqrt{x^2 - 1})^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{(x - \sqrt{x^2 - 1})^2 \cdot (x + \sqrt{x^2 - 1})^2} = \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[(x - \sqrt{x^2 - 1}) \cdot (x + \sqrt{x^2 - 1})]^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[x^2 - (x^2 - 1)]^2} = \int \frac{(x^2 + 2x\sqrt{x^2 - 1} + x^2 - 1) dx}{1}$$

$$= \int (2x^2 - 1 + 2x\sqrt{x^2 - 1}) dx = 2 \int x^2 dx - \int dx + \int 2x\sqrt{x^2 - 1} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - 1 \mid x \in (-\infty; -1), t \in (0; \infty) \\ \quad \quad \quad dt = 2x dx \mid x \in (1; \infty), t \in (0; \infty) \end{array} \right] = \frac{2x^3}{3} - x + \int \sqrt{t} dt$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{(x - \sqrt{x^2 - 1})^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{(x - \sqrt{x^2 - 1})^2 \cdot (x + \sqrt{x^2 - 1})^2} = \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[(x - \sqrt{x^2 - 1}) \cdot (x + \sqrt{x^2 - 1})]^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[x^2 - (x^2 - 1)]^2} = \int \frac{(x^2 + 2x\sqrt{x^2 - 1} + x^2 - 1) dx}{1}$$

$$= \int (2x^2 - 1 + 2x\sqrt{x^2 - 1}) dx = 2 \int x^2 dx - \int dx + \int 2x\sqrt{x^2 - 1} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - 1 \mid x \in (-\infty; -1), t \in (0; \infty) \\ \quad \quad \quad dt = 2x dx \mid x \in (1; \infty), t \in (0; \infty) \end{array} \right] = \frac{2x^3}{3} - x + \int \sqrt{t} dt$$

$$= \frac{2x^3}{3} - x + \int t^{\frac{1}{2}} dt$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{(x - \sqrt{x^2 - 1})^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{(x - \sqrt{x^2 - 1})^2 \cdot (x + \sqrt{x^2 - 1})^2} = \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[(x - \sqrt{x^2 - 1}) \cdot (x + \sqrt{x^2 - 1})]^2}$$

$$= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[x^2 - (x^2 - 1)]^2} = \int \frac{(x^2 + 2x\sqrt{x^2 - 1} + x^2 - 1) dx}{1}$$

$$= \int (2x^2 - 1 + 2x\sqrt{x^2 - 1}) dx = 2 \int x^2 dx - \int dx + \int 2x\sqrt{x^2 - 1} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - 1 \mid x \in (-\infty; -1), t \in (0; \infty) \\ \quad \quad \quad dt = 2x dx \mid x \in (1; \infty), t \in (0; \infty) \end{array} \right] = \frac{2x^3}{3} - x + \int \sqrt{t} dt$$

$$= \frac{2x^3}{3} - x + \int t^{\frac{1}{2}} dt = \frac{2x^3}{3} - x + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{\left(x - \sqrt{x^2 - 1}\right)^2}$$

$$= \int \frac{\left(x + \sqrt{x^2 - 1}\right)^2 dx}{\left(x - \sqrt{x^2 - 1}\right)^2 \cdot \left(x + \sqrt{x^2 - 1}\right)^2} = \int \frac{\left(x + \sqrt{x^2 - 1}\right)^2 dx}{\left[\left(x - \sqrt{x^2 - 1}\right) \cdot \left(x + \sqrt{x^2 - 1}\right)\right]^2}$$

$$= \int \frac{\left(x + \sqrt{x^2 - 1}\right)^2 dx}{\left[x^2 - (x^2 - 1)\right]^2} = \int \frac{\left(x^2 + 2x\sqrt{x^2 - 1} + x^2 - 1\right) dx}{1}$$

$$= \int (2x^2 - 1 + 2x\sqrt{x^2 - 1}) dx = 2 \int x^2 dx - \int dx + \int 2x\sqrt{x^2 - 1} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^2 - 1 \mid x \in (-\infty; -1), t \in (0; \infty) \\ \quad \quad \quad dt = 2x dx \mid x \in (1; \infty), t \in (0; \infty) \end{array} \right] = \frac{2x^3}{3} - x + \int \sqrt{t} dt$$

$$= \frac{2x^3}{3} - x + \int t^{\frac{1}{2}} dt = \frac{2x^3}{3} - x + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2x^3}{3} - x + \frac{2\sqrt{t^3}}{3} + c$$

Koniec 10. časti príklady b – 191

$$\int \frac{dx}{(x - \sqrt{x^2 - 1})^2} = \frac{2x^3}{3} - x + \frac{2\sqrt{(x^2 - 1)^3}}{3} + c$$

$$\begin{aligned} &= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{(x - \sqrt{x^2 - 1})^2 \cdot (x + \sqrt{x^2 - 1})^2} = \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[(x - \sqrt{x^2 - 1}) \cdot (x + \sqrt{x^2 - 1})]^2} \\ &= \int \frac{(x + \sqrt{x^2 - 1})^2 dx}{[x^2 - (x^2 - 1)]^2} = \int \frac{(x^2 + 2x\sqrt{x^2 - 1} + x^2 - 1) dx}{1} \\ &= \int (2x^2 - 1 + 2x\sqrt{x^2 - 1}) dx = 2 \int x^2 dx - \int dx + \int 2x\sqrt{x^2 - 1} dx \\ &= \left[\begin{array}{l} \text{Subst. } t = x^2 - 1 \mid x \in (-\infty; -1), t \in (0; \infty) \\ \quad \quad \quad dt = 2x dx \mid x \in (1; \infty), t \in (0; \infty) \end{array} \right] = \frac{2x^3}{3} - x + \int \sqrt{t} dt \\ &= \frac{2x^3}{3} - x + \int t^{\frac{1}{2}} dt = \frac{2x^3}{3} - x + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2x^3}{3} - x + \frac{2\sqrt{t^3}}{3} + c \\ &= \frac{2x^3}{3} - x + \frac{2\sqrt{(x^2 - 1)^3}}{3} + c, \quad x \in (-\infty; -1) \cup (1; \infty), \quad c \in \mathbb{R}. \end{aligned}$$

Koniec 10. časti príklady b – 192, 193

$$\int \frac{x^5 dx}{\sqrt{x^2+1}}$$

$$\int \frac{x^5 dx}{\sqrt{x^3+1}}$$

Koniec 10. časti príklady b – 192, 193

$$\int \frac{x^5 dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2+1}}$$

$$\int \frac{x^5 dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3+1}}$$

Koniec 10. časti príklady b – 192, 193

$$\int \frac{x^5 dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2+1 \mid x \in (0; \infty), t \in (1; \infty) \\ dt=2x dx \mid x \in (-\infty; 0), t \in (1; \infty) \end{array} \right]$$

$$\int \frac{x^5 dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3+1 \mid x \in (-1; \infty) \\ dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 192, 193

$$\int \frac{x^5 dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2+1 \mid x \in (0; \infty), t \in (1; \infty) \\ dt=2x dx \mid x \in (-\infty; 0), t \in (1; \infty) \end{array} \right] = \frac{1}{2} \int \frac{(t-1)^2 dt}{\sqrt{t}}$$

$$\int \frac{x^5 dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3+1 \mid x \in (-1; \infty) \\ dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right] = \frac{1}{3} \int \frac{(t-1) dt}{\sqrt{t}}$$

Koniec 10. časti príklady b – 192, 193

$$\int \frac{x^5 dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2+1 \mid x \in (0; \infty), t \in (1; \infty) \\ dt=2x dx \mid x \in (-\infty; 0), t \in (1; \infty) \end{array} \right] = \frac{1}{2} \int \frac{(t-1)^2 dt}{\sqrt{t}} = \frac{1}{2} \int \frac{t^2-2t+1}{t^{\frac{1}{2}}} dt$$

$$\int \frac{x^5 dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3+1 \mid x \in (-1; \infty) \\ dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right] = \frac{1}{3} \int \frac{(t-1) dt}{\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}} dt$$

Koniec 10. časti príklady b – 192, 193

$$\int \frac{x^5 dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2+1 \mid x \in (0; \infty), t \in (1; \infty) \\ dt=2x dx \mid x \in (-\infty; 0), t \in (1; \infty) \end{array} \right] = \frac{1}{2} \int \frac{(t-1)^2 dt}{\sqrt{t}} = \frac{1}{2} \int \frac{t^2-2t+1}{t^{\frac{1}{2}}} dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{\frac{2}{2}} - t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{\frac{2}{2}} \right] dt$$

$$\int \frac{x^5 dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3+1 \mid x \in (-1; \infty) \\ dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right] = \frac{1}{3} \int \frac{(t-1) dt}{\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}} dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3} \right] dt$$

Koniec 10. časti príklady b – 192, 193

$$\int \frac{x^5 dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2+1 \mid x \in (0; \infty), t \in (1; \infty) \\ \quad \quad \quad dt=2x dx \mid x \in (-\infty; 0), t \in (1; \infty) \end{array} \right] = \frac{1}{2} \int \frac{(t-1)^2 dt}{\sqrt{t}} = \frac{1}{2} \int \frac{t^2 - 2t + 1}{t^{\frac{1}{2}}} dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{\frac{2}{2}} - t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{\frac{2}{2}} \right] dt = \frac{t^{\frac{5}{2}}}{2 \cdot \frac{5}{2}} - \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{2 \cdot \frac{1}{2}} + c$$

$$\int \frac{x^5 dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3+1 \mid x \in (-1; \infty) \\ \quad \quad \quad dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right] = \frac{1}{3} \int \frac{(t-1) dt}{\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}} dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3} \right] dt$$

$$= \frac{t^{\frac{3}{2}}}{3 \cdot \frac{3}{2}} - \frac{t^{\frac{1}{2}}}{3 \cdot \frac{1}{2}} + c$$

Koniec 10. časti príklady b – 192, 193

$$\int \frac{x^5 dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2+1 \mid x \in (0; \infty), t \in (1; \infty) \\ dt=2x dx \mid x \in (-\infty; 0), t \in (1; \infty) \end{array} \right] = \frac{1}{2} \int \frac{(t-1)^2 dt}{\sqrt{t}} = \frac{1}{2} \int \frac{t^2-2t+1}{t^{\frac{1}{2}}} dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{\frac{1}{2}} \right] dt = \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{\sqrt{t^5}}{5} - \frac{2\sqrt{t^3}}{3} + \sqrt{t} + c$$

$$\int \frac{x^5 dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3+1 \mid x \in (-1; \infty) \\ dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right] = \frac{1}{3} \int \frac{(t-1) dt}{\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}} dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3} \right] dt$$

$$= \frac{t^{\frac{3}{2}}}{3 \cdot \frac{3}{2}} - \frac{t^{\frac{1}{2}}}{3 \cdot \frac{1}{2}} + c = \frac{2\sqrt{t^3}}{9} - \frac{2\sqrt{t}}{3} + c$$

Koniec 10. časti príklady b – 192, 193

$$\int \frac{x^5 dx}{\sqrt{x^2+1}} = \frac{\sqrt{(x^2+1)^5}}{5} - \frac{2\sqrt{(x^2+1)^3}}{3} + \sqrt{x^2+1} + c$$

$$\begin{aligned} &= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2+1 \mid x \in (0; \infty), t \in (1; \infty) \\ dt=2x dx \mid x \in (-\infty; 0), t \in (1; \infty) \end{array} \right] = \frac{1}{2} \int \frac{(t-1)^2 dt}{\sqrt{t}} = \frac{1}{2} \int \frac{t^2-2t+1}{t^{\frac{1}{2}}} dt \\ &= \int \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{\frac{1}{2}} \right] dt = \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{\sqrt{t^5}}{5} - \frac{2\sqrt{t^3}}{3} + \sqrt{t} + c \\ &= \frac{\sqrt{(x^2+1)^5}}{5} - \frac{2\sqrt{(x^2+1)^3}}{3} + \sqrt{x^2+1} + c, x \in R, c \in R. \end{aligned}$$

$$\int \frac{x^5 dx}{\sqrt{x^3+1}} = \frac{2\sqrt{(x^3+1)^3}}{9} - \frac{2\sqrt{x^3+1}}{3} + c$$

$$\begin{aligned} &= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3+1 \mid x \in (-1; \infty) \\ dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right] = \frac{1}{3} \int \frac{(t-1) dt}{\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}} dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3} \right] dt \\ &= \frac{t^{\frac{3}{2}}}{3 \cdot \frac{3}{2}} - \frac{t^{\frac{1}{2}}}{3 \cdot \frac{1}{2}} + c = \frac{2\sqrt{t^3}}{9} - \frac{2\sqrt{t}}{3} + c = \frac{2\sqrt{(x^3+1)^3}}{9} - \frac{2\sqrt{x^3+1}}{3} + c, \\ &\hspace{15em} x \in (-1; \infty), c \in R. \end{aligned}$$

Koniec 10. časti príklady b – 194, 195

$$\int \frac{x^5 dx}{\sqrt{x^2-1}}$$

$$\int \frac{x^5 dx}{\sqrt{x^3-1}}$$

Koniec 10. časti príklady b – 194, 195

$$\int \frac{x^5 dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2-1}}$$

$$\int \frac{x^5 dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3-1}}$$

Koniec 10. časti príklady b – 194, 195

$$\int \frac{x^5 dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2-1}} = \left[\begin{array}{l} \text{Subst. } t=x^2-1 \mid x \in (1; \infty), t \in (0; \infty) \\ dt=2x dx \mid x \in (-\infty; -1), t \in (0; \infty) \end{array} \right]$$

$$\int \frac{x^5 dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3-1}} = \left[\begin{array}{l} \text{Subst. } t=x^3-1 \mid x \in (1; \infty) \\ dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 194, 195

$$\int \frac{x^5 dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2-1}} = \left[\begin{array}{l} \text{Subst. } t=x^2-1 \mid x \in (1; \infty), t \in (0; \infty) \\ dt=2x dx \mid x \in (-\infty; -1), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{(t+1)^2 dt}{\sqrt{t}}$$

$$\int \frac{x^5 dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3-1}} = \left[\begin{array}{l} \text{Subst. } t=x^3-1 \mid x \in (1; \infty) \\ dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right] = \frac{1}{3} \int \frac{(t+1) dt}{\sqrt{t}}$$

Koniec 10. časti príklady b – 194, 195

$$\int \frac{x^5 dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2-1}} = \left[\begin{array}{l} \text{Subst. } t=x^2-1 \mid x \in (1; \infty), t \in (0; \infty) \\ dt=2x dx \mid x \in (-\infty; -1), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{(t+1)^2 dt}{\sqrt{t}} = \frac{1}{2} \int \frac{t^2+2t+1}{t^{\frac{1}{2}}} dt$$

$$\int \frac{x^5 dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3-1}} = \left[\begin{array}{l} \text{Subst. } t=x^3-1 \mid x \in (1; \infty) \\ dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right] = \frac{1}{3} \int \frac{(t+1) dt}{\sqrt{t}} = \int \frac{t+1}{3t^{\frac{1}{2}}} dt$$

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$$\int \frac{x^5 dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2-1}} = \left[\begin{array}{l} \text{Subst. } t=x^2-1 \mid x \in (1; \infty), t \in (0; \infty) \\ dt=2x dx \mid x \in (-\infty; -1), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{(t+1)^2 dt}{\sqrt{t}} = \frac{1}{2} \int \frac{t^2+2t+1}{t^{\frac{1}{2}}} dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{\frac{2}{2}} + t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{\frac{2}{2}} \right] dt$$

$$\int \frac{x^5 dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3-1}} = \left[\begin{array}{l} \text{Subst. } t=x^3-1 \mid x \in (1; \infty) \\ dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right] = \frac{1}{3} \int \frac{(t+1) dt}{\sqrt{t}} = \int \frac{t+1}{3t^{\frac{1}{2}}} dt = \int \left[\frac{t^{\frac{1}{2}}}{3} + \frac{t^{-\frac{1}{2}}}{3} \right] dt$$

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$$\int \frac{x^5 dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2-1}} = \left[\begin{array}{l} \text{Subst. } t=x^2-1 \mid x \in (1; \infty), t \in (0; \infty) \\ dt=2x dx \mid x \in (-\infty; -1), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{(t+1)^2 dt}{\sqrt{t}} = \frac{1}{2} \int \frac{t^2+2t+1}{t^{\frac{1}{2}}} dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{\frac{2}{2}} + t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{\frac{2}{2}} \right] dt = \frac{t^{\frac{5}{2}}}{2 \cdot \frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{2 \cdot \frac{1}{2}} + c$$

$$\int \frac{x^5 dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3-1}} = \left[\begin{array}{l} \text{Subst. } t=x^3-1 \mid x \in (1; \infty) \\ dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right] = \frac{1}{3} \int \frac{(t+1) dt}{\sqrt{t}} = \int \frac{t+1}{3t^{\frac{1}{2}}} dt = \int \left[\frac{t^{\frac{1}{2}}}{3} + \frac{t^{-\frac{1}{2}}}{3} \right] dt$$

$$= \frac{t^{\frac{3}{2}}}{3 \cdot \frac{3}{2}} + \frac{t^{\frac{1}{2}}}{3 \cdot \frac{1}{2}} + c$$

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$$\int \frac{x^5 dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2-1}} = \left[\begin{array}{l} \text{Subst. } t=x^2-1 \mid x \in (1; \infty), t \in (0; \infty) \\ \quad \quad \quad dt=2x dx \mid x \in (-\infty; -1), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{(t+1)^2 dt}{\sqrt{t}} = \frac{1}{2} \int \frac{t^2+2t+1}{t^{\frac{1}{2}}} dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{\frac{2}{2}} + t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{\frac{2}{2}} \right] dt = \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{\frac{2}{2}} + c = \frac{\sqrt{t^5}}{5} + \frac{2\sqrt{t^3}}{3} + \sqrt{t} + c$$

$$\int \frac{x^5 dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3-1}} = \left[\begin{array}{l} \text{Subst. } t=x^3-1 \mid x \in (1; \infty) \\ \quad \quad \quad dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right] = \frac{1}{3} \int \frac{(t+1) dt}{\sqrt{t}} = \int \frac{t+1}{3t^{\frac{1}{2}}} dt = \int \left[\frac{t^{\frac{1}{2}}}{3} + \frac{t^{-\frac{1}{2}}}{3} \right] dt$$

$$= \frac{t^{\frac{3}{2}}}{3 \cdot \frac{3}{2}} + \frac{t^{\frac{1}{2}}}{3 \cdot \frac{1}{2}} + c = \frac{2\sqrt{t^3}}{9} + \frac{2\sqrt{t}}{3} + c$$

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$$\int \frac{x^5 dx}{\sqrt{x^2-1}} = \frac{\sqrt{(x^2-1)^5}}{5} + \frac{2\sqrt{(x^2-1)^3}}{3} + \sqrt{x^2-1} + c$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{x^2-1}} = \left[\begin{array}{l} \text{Subst. } t=x^2-1 \mid x \in (1; \infty), t \in (0; \infty) \\ dt=2x dx \mid x \in (-\infty; -1), t \in (0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{(t+1)^2 dt}{\sqrt{t}} = \frac{1}{2} \int \frac{t^2+2t+1}{t^{\frac{1}{2}}} dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{\frac{2}{2}} + t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{\frac{2}{2}} \right] dt = \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{\sqrt{t^5}}{5} + \frac{2\sqrt{t^3}}{3} + \sqrt{t} + c$$

$$= \frac{\sqrt{(x^2-1)^5}}{5} + \frac{2\sqrt{(x^2-1)^3}}{3} + \sqrt{x^2-1} + c, x \in (-\infty; -1) \cup (1; \infty), c \in \mathbb{R}.$$

$$\int \frac{x^5 dx}{\sqrt{x^3-1}} = \frac{2\sqrt{(x^3-1)^3}}{9} + \frac{2\sqrt{x^3-1}}{3} + c$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{x^3-1}} = \left[\begin{array}{l} \text{Subst. } t=x^3-1 \mid x \in (1; \infty) \\ dt=3x^2 dx \mid x \in (0; \infty) \end{array} \right] = \frac{1}{3} \int \frac{(t+1) dt}{\sqrt{t}} = \int \frac{t+1}{3t^{\frac{1}{2}}} dt = \int \left[\frac{t^{\frac{1}{2}}}{3} + \frac{t^{-\frac{1}{2}}}{3} \right] dt$$

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2\sqrt{t^3}}{9} + \frac{2\sqrt{t}}{3} + c = \frac{2\sqrt{(x^3-1)^3}}{9} + \frac{2\sqrt{x^3-1}}{3} + c,$$

$$x \in (1; \infty), c \in \mathbb{R}.$$

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$$\int \frac{x^5 dx}{\sqrt{1-x^2}}$$

$$\int \frac{x^5 dx}{\sqrt{1-x^3}}$$

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$$\int \frac{x^5 dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{1-x^2}}$$

$$\int \frac{x^5 dx}{\sqrt{1-x^3}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{1-x^3}}$$

Koniec 10. časti príklady b – 196, 197

$$\int \frac{x^5 dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{1-x^2}} = \left[\begin{array}{l} \text{Subst. } t=1-x^2 \mid x \in (0; 1), t \in (0; 1) \\ dt = -2x dx \mid x \in (-1; 0), t \in (0; 1) \end{array} \right]$$

$$\int \frac{x^5 dx}{\sqrt{1-x^3}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{1-x^3}} = \left[\begin{array}{l} \text{Subst. } t=1-x^3 \mid x \in (-\infty; 1) \\ dt = -3x^2 dx \mid x \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 196, 197

$$\int \frac{x^5 dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{1-x^2}} = \left[\begin{array}{l} \text{Subst. } t=1-x^2 \mid x \in (0;1), t \in (0;1) \\ dt = -2x dx \mid x \in (-1;0), t \in (0;1) \end{array} \right] = -\frac{1}{2} \int \frac{(1-t)^2 dt}{\sqrt{t}}$$

$$\int \frac{x^5 dx}{\sqrt{1-x^3}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{1-x^3}} = \left[\begin{array}{l} \text{Subst. } t=1-x^3 \mid x \in (-\infty;1) \\ dt = -3x^2 dx \mid x \in (0;\infty) \end{array} \right] = -\int \frac{(1-t) dt}{3\sqrt{t}}$$

Koniec 10. časti príklady b – 196, 197

$$\int \frac{x^5 dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{1-x^2}} = \left[\begin{array}{l} \text{Subst. } t=1-x^2 \mid x \in (0;1), t \in (0;1) \\ dt = -2x dx \mid x \in (-1;0), t \in (0;1) \end{array} \right] = -\frac{1}{2} \int \frac{(1-t)^2 dt}{\sqrt{t}} = -\frac{1}{2} \int \frac{1-2t+t^2}{t^{\frac{1}{2}}} dt$$

$$\int \frac{x^5 dx}{\sqrt{1-x^3}}$$

$$= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{1-x^3}} = \left[\begin{array}{l} \text{Subst. } t=1-x^3 \mid x \in (-\infty;1) \\ dt = -3x^2 dx \mid x \in (0;\infty) \end{array} \right] = -\int \frac{(1-t) dt}{3\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}} dt$$

Koniec 10. časti príklady b – 196, 197

$$\int \frac{x^5 dx}{\sqrt{1-x^2}}$$

$$\begin{aligned} &= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{1-x^2}} = \left[\begin{array}{l} \text{Subst. } t=1-x^2 \mid x \in (0;1), t \in (0;1) \\ dt = -2x dx \mid x \in (-1;0), t \in (0;1) \end{array} \right] = -\frac{1}{2} \int \frac{(1-t)^2 dt}{\sqrt{t}} = -\frac{1}{2} \int \frac{1-2t+t^2}{t^{\frac{1}{2}}} dt \\ &= -\int \left[\frac{t^{-\frac{1}{2}}}{2} - t^{\frac{1}{2}} + \frac{t^{\frac{3}{2}}}{2} \right] dt \end{aligned}$$

$$\int \frac{x^5 dx}{\sqrt{1-x^3}}$$

$$\begin{aligned} &= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{1-x^3}} = \left[\begin{array}{l} \text{Subst. } t=1-x^3 \mid x \in (-\infty;1) \\ dt = -3x^2 dx \mid x \in (0;\infty) \end{array} \right] = -\int \frac{(1-t) dt}{3\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}} dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3} \right] dt \end{aligned}$$

Koniec 10. časti príklady b – 196, 197

$$\int \frac{x^5 dx}{\sqrt{1-x^2}}$$

$$\begin{aligned} &= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{1-x^2}} = \left[\begin{array}{l} \text{Subst. } t=1-x^2 \mid x \in (0;1), t \in (0;1) \\ dt = -2x dx \mid x \in (-1;0), t \in (0;1) \end{array} \right] = -\frac{1}{2} \int \frac{(1-t)^2 dt}{\sqrt{t}} = -\frac{1}{2} \int \frac{1-2t+t^2}{t^{\frac{1}{2}}} dt \\ &= -\int \left[\frac{t^{-\frac{1}{2}}}{2} - t^{\frac{1}{2}} + \frac{t^{\frac{3}{2}}}{2} \right] dt = -\frac{t^{\frac{1}{2}}}{2 \cdot \frac{1}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{2 \cdot \frac{5}{2}} + c \end{aligned}$$

$$\int \frac{x^5 dx}{\sqrt{1-x^3}}$$

$$\begin{aligned} &= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{1-x^3}} = \left[\begin{array}{l} \text{Subst. } t=1-x^3 \mid x \in (-\infty;1) \\ dt = -3x^2 dx \mid x \in (0;\infty) \end{array} \right] = -\int \frac{(1-t) dt}{3\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}} dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3} \right] dt \\ &= \frac{t^{\frac{3}{2}}}{3 \cdot \frac{3}{2}} - \frac{t^{\frac{1}{2}}}{3 \cdot \frac{1}{2}} + c \end{aligned}$$

Koniec 10. časti príklady b – 196, 197

$$\int \frac{x^5 dx}{\sqrt{1-x^2}}$$

$$\begin{aligned} &= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{1-x^2}} = \left[\begin{array}{l} \text{Subst. } t=1-x^2 \mid x \in (0;1), t \in (0;1) \\ dt = -2x dx \mid x \in (-1;0), t \in (0;1) \end{array} \right] = -\frac{1}{2} \int \frac{(1-t)^2 dt}{\sqrt{t}} = -\frac{1}{2} \int \frac{1-2t+t^2}{t^{\frac{1}{2}}} dt \\ &= -\int \left[\frac{t^{-\frac{1}{2}}}{2} - t^{\frac{1}{2}} + \frac{t^{\frac{3}{2}}}{2} \right] dt = -\frac{t^{\frac{1}{2}}}{2 \cdot \frac{1}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{2 \cdot \frac{5}{2}} + C = -\sqrt{t} + \frac{2\sqrt{t^3}}{3} - \frac{\sqrt{t^5}}{5} + C \end{aligned}$$

$$\int \frac{x^5 dx}{\sqrt{1-x^3}}$$

$$\begin{aligned} &= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{1-x^3}} = \left[\begin{array}{l} \text{Subst. } t=1-x^3 \mid x \in (-\infty;1) \\ dt = -3x^2 dx \mid x \in (0;\infty) \end{array} \right] = -\int \frac{(1-t) dt}{3\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}} dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3} \right] dt \\ &= \frac{t^{\frac{3}{2}}}{3 \cdot \frac{3}{2}} - \frac{t^{\frac{1}{2}}}{3 \cdot \frac{1}{2}} + C = \frac{2\sqrt{t^3}}{9} - \frac{2\sqrt{t}}{3} + C \end{aligned}$$

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$$\int \frac{x^5 dx}{\sqrt{1-x^2}} = -\frac{\sqrt{(1-x^2)^5}}{5} + \frac{2\sqrt{(1-x^2)^3}}{3} - \sqrt{1-x^2} + c$$

$$\begin{aligned} &= \frac{1}{2} \int \frac{x^4 \cdot 2x dx}{\sqrt{1-x^2}} = \left[\text{Subst. } t=1-x^2 \mid x \in (0;1), t \in (0;1) \right. \\ &\quad \left. dt = -2x dx \mid x \in (-1;0), t \in (0;1) \right] = -\frac{1}{2} \int \frac{(1-t)^2 dt}{\sqrt{t}} = -\frac{1}{2} \int \frac{1-2t+t^2}{t^{\frac{1}{2}}} dt \\ &= -\int \left[\frac{t^{-\frac{1}{2}}}{2} - t^{\frac{1}{2}} + \frac{t^{\frac{3}{2}}}{2} \right] dt = -\frac{t^{\frac{1}{2}}}{2 \cdot \frac{1}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{2 \cdot \frac{5}{2}} + c = -\sqrt{t} + \frac{2\sqrt{t^3}}{3} - \frac{\sqrt{t^5}}{5} + c \\ &= -\frac{\sqrt{(1-x^2)^5}}{5} + \frac{2\sqrt{(1-x^2)^3}}{3} - \sqrt{1-x^2} + c, x \in (-1;1), c \in \mathbb{R}. \end{aligned}$$

$$\int \frac{x^5 dx}{\sqrt{1-x^3}} = \frac{2\sqrt{(1-x^3)^3}}{9} - \frac{2\sqrt{1-x^3}}{3} + c$$

$$\begin{aligned} &= \frac{1}{3} \int \frac{x^3 \cdot 3x^2 dx}{\sqrt{1-x^3}} = \left[\text{Subst. } t=1-x^3 \mid x \in (-\infty;1) \right. \\ &\quad \left. dt = -3x^2 dx \mid x \in (0;\infty) \right] = -\int \frac{(1-t) dt}{3\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}} dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3} \right] dt \\ &= \frac{t^{\frac{3}{2}}}{3 \cdot \frac{3}{2}} - \frac{t^{\frac{1}{2}}}{3 \cdot \frac{1}{2}} + c = \frac{2\sqrt{t^3}}{9} - \frac{2\sqrt{t}}{3} + c = \frac{2\sqrt{(1-x^3)^3}}{9} - \frac{2\sqrt{1-x^3}}{3} + c, \\ &\quad x \in (-\infty;1), c \in \mathbb{R}. \end{aligned}$$

Koniec 10. časti príklady b – 198

$$\int \frac{\sqrt{1-x^2}}{x^2} dx$$

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$$\int \frac{\sqrt{1-x^2}}{x^2} dx$$

$$= \left[\begin{array}{l} u = \sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}} \quad u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}} \\ v' = \frac{1}{x^2} = x^{-2} \quad v = \frac{x^{-1}}{-1} = -\frac{1}{x} \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in (-1; 0), t \in (-\frac{\pi}{2}; 0) \mid t = \arcsin x, \cos t \geq 0 \\ dx = \cos t dt \mid x \in (0; 1), t \in (0; \frac{\pi}{2}) \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } x = \cos t \mid x \in (-1; 0), t \in (\frac{\pi}{2}; \pi) \mid t = \arccos x, \sin t \geq 0 \\ dx = -\sin t dt \mid x \in (0; 1), t \in (0; \frac{\pi}{2}) \mid \sqrt{1-x^2} = \sqrt{1-\cos^2 t} = \sqrt{\sin^2 t} = |\sin t| = \sin t \end{array} \right]$$

Koniec 10. časti príklady b – 198

$$\int \frac{\sqrt{1-x^2}}{x^2} dx$$

$$= \left[\begin{array}{l} u = \sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}} \\ v' = \frac{1}{x^2} = x^{-2} \end{array} \middle| \begin{array}{l} u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}} \\ v = \frac{x^{-1}}{-1} = -\frac{1}{x} \end{array} \right] = -\frac{\sqrt{1-x^2}}{x} - \int \frac{dx}{\sqrt{1-x^2}}$$

$$= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in \langle -1; 0 \rangle, t \in \langle -\frac{\pi}{2}; 0 \rangle \\ dx = \cos t dt \mid x \in (0; 1), t \in (0; \frac{\pi}{2}) \end{array} \middle| \begin{array}{l} t = \arcsin x, \cos t \geq 0 \\ \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \end{array} \right] = \int \frac{\cos t \cdot \cos t dt}{\sin^2 t}$$

$$= \left[\begin{array}{l} \text{Subst. } x = \cos t \mid x \in \langle -1; 0 \rangle, t \in (\frac{\pi}{2}; \pi) \\ dx = -\sin t dt \mid x \in (0; 1), t \in (0; \frac{\pi}{2}) \end{array} \middle| \begin{array}{l} t = \arccos x, \sin t \geq 0 \\ \sqrt{1-x^2} = \sqrt{1-\cos^2 t} = \sqrt{\sin^2 t} = |\sin t| = \sin t \end{array} \right] = -\int \frac{\sin t \cdot \sin t dt}{\cos^2 t}$$

Koniec 10. časti príklady b – 198

$$\int \frac{\sqrt{1-x^2}}{x^2} dx$$

$$= \left[\begin{array}{l} u = \sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}} \quad u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}} \\ v' = \frac{1}{x^2} = x^{-2} \quad v = \frac{x^{-1}}{-1} = -\frac{1}{x} \end{array} \right] = -\frac{\sqrt{1-x^2}}{x} - \int \frac{dx}{\sqrt{1-x^2}}$$

$$= -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = -\frac{\sqrt{1-x^2}}{x} + \arccos x + c_2, \quad x \in (-1; 1) - \{0\}, \quad c_1, c_2 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in (-1; 0), t \in (-\frac{\pi}{2}; 0) \mid t = \arcsin x, \cos t \geq 0 \\ dx = \cos t dt \mid x \in (0; 1), t \in (0; \frac{\pi}{2}) \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \end{array} \right] = \int \frac{\cos t \cdot \cos t dt}{\sin^2 t}$$

$$= \int \frac{(1-\sin^2 t) dt}{\sin^2 t}$$

$$= \left[\begin{array}{l} \text{Subst. } x = \cos t \mid x \in (-1; 0), t \in (\frac{\pi}{2}; \pi) \mid t = \arccos x, \sin t \geq 0 \\ dx = -\sin t dt \mid x \in (0; 1), t \in (0; \frac{\pi}{2}) \mid \sqrt{1-x^2} = \sqrt{1-\cos^2 t} = \sqrt{\sin^2 t} = |\sin t| = \sin t \end{array} \right] = -\int \frac{\sin t \cdot \sin t dt}{\cos^2 t}$$

$$= -\int \frac{(1-\cos^2 t) dt}{\sin^2 t}$$

Koniec 10. časti príklady b – 198

$$\int \frac{\sqrt{1-x^2}}{x^2} dx = -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = \frac{\sqrt{1-x^2}}{x} + \arccos x + c_2$$

$$= \left[\begin{array}{l} u = \sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}} \quad u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}} \\ v' = \frac{1}{x^2} = x^{-2} \quad v = \frac{x^{-1}}{-1} = -\frac{1}{x} \end{array} \right] = -\frac{\sqrt{1-x^2}}{x} - \int \frac{dx}{\sqrt{1-x^2}}$$

$$= -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = -\frac{\sqrt{1-x^2}}{x} + \arccos x + c_2,$$

$$x \in (-1; 1) - \{0\}, c_1, c_2 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in (-1; 0), t \in (-\frac{\pi}{2}; 0) \mid t = \arcsin x, \cos t \geq 0 \\ dx = \cos t dt \mid x \in (0; 1), t \in (0; \frac{\pi}{2}) \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \end{array} \right] = \int \frac{\cos t \cdot \cos t dt}{\sin^2 t}$$

$$= \int \frac{(1-\sin^2 t) dt}{\sin^2 t} = \int \left[\frac{1}{\sin^2 t} - 1 \right] dt$$

$$= \left[\begin{array}{l} \text{Subst. } x = \cos t \mid x \in (-1; 0), t \in (\frac{\pi}{2}; \pi) \mid t = \arccos x, \sin t \geq 0 \\ dx = -\sin t dt \mid x \in (0; 1), t \in (0; \frac{\pi}{2}) \mid \sqrt{1-x^2} = \sqrt{1-\cos^2 t} = \sqrt{\sin^2 t} = |\sin t| = \sin t \end{array} \right] = -\int \frac{\sin t \cdot \sin t dt}{\cos^2 t}$$

$$= -\int \frac{(1-\cos^2 t) dt}{\sin^2 t} = \int \left[1 - \frac{1}{\cos^2 t} \right] dt$$

Koniec 10. časti príklady b – 198

$$\int \frac{\sqrt{1-x^2}}{x^2} dx = -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = \frac{\sqrt{1-x^2}}{x} + \arccos x + c_2$$

$$= \left[\begin{array}{l} u = \sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}} \quad u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}} \\ v' = \frac{1}{x^2} = x^{-2} \quad v = \frac{x^{-1}}{-1} = -\frac{1}{x} \end{array} \right] = -\frac{\sqrt{1-x^2}}{x} - \int \frac{dx}{\sqrt{1-x^2}}$$

$$= -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = -\frac{\sqrt{1-x^2}}{x} + \arccos x + c_2,$$

$$x \in (-1; 1) - \{0\}, c_1, c_2 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in (-1; 0), t \in (-\frac{\pi}{2}; 0) \mid t = \arcsin x, \cos t \geq 0, \cotg t = \frac{\cos t}{\sin t} = \frac{\sqrt{1-x^2}}{x} \\ dx = \cos t dt \mid x \in (0; 1), t \in (0; \frac{\pi}{2}) \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \end{array} \right] = \int \frac{\cos t \cdot \cos t dt}{\sin^2 t}$$

$$= \int \frac{(1-\sin^2 t) dt}{\sin^2 t} = \int \left[\frac{1}{\sin^2 t} - 1 \right] dt = -\cotg t - t + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } x = \cos t \mid x \in (-1; 0), t \in (\frac{\pi}{2}; \pi) \mid t = \arccos x, \sin t \geq 0, \tg t = \frac{\sin t}{\cos t} = \frac{\sqrt{1-x^2}}{x} \\ dx = -\sin t dt \mid x \in (0; 1), t \in (0; \frac{\pi}{2}) \mid \sqrt{1-x^2} = \sqrt{1-\cos^2 t} = \sqrt{\sin^2 t} = |\sin t| = \sin t \end{array} \right] = -\int \frac{\sin t \cdot \sin t dt}{\cos^2 t}$$

$$= -\int \frac{(1-\cos^2 t) dt}{\sin^2 t} = \int \left[1 - \frac{1}{\cos^2 t} \right] dt = t - \tg t + c_2$$

Koniec 10. časti príklady b – 198

$$\int \frac{\sqrt{1-x^2}}{x^2} dx = -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = \frac{\sqrt{1-x^2}}{x} + \arccos x + c_2$$

$$= \left[\begin{array}{l} u = \sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}} \quad u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}} \\ v' = \frac{1}{x^2} = x^{-2} \quad v = \frac{x^{-1}}{-1} = -\frac{1}{x} \end{array} \right] = -\frac{\sqrt{1-x^2}}{x} - \int \frac{dx}{\sqrt{1-x^2}}$$

$$= -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = -\frac{\sqrt{1-x^2}}{x} + \arccos x + c_2,$$

$$x \in (-1; 1) - \{0\}, c_1, c_2 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in (-1; 0), t \in (-\frac{\pi}{2}; 0) \mid t = \arcsin x, \cos t \geq 0, \cotg t = \frac{\cos t}{\sin t} = \frac{\sqrt{1-x^2}}{x} \\ dx = \cos t dt \mid x \in (0; 1), t \in (0; \frac{\pi}{2}) \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \end{array} \right] = \int \frac{\cos t \cdot \cos t dt}{\sin^2 t}$$

$$= \int \frac{(1-\sin^2 t) dt}{\sin^2 t} = \int \left[\frac{1}{\sin^2 t} - 1 \right] dt = -\cotg t - t + c_1 = -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1,$$

$$x \in (-1; 0) \cup (0; 1), c_1 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } x = \cos t \mid x \in (-1; 0), t \in (\frac{\pi}{2}; \pi) \mid t = \arccos x, \sin t \geq 0, \tg t = \frac{\sin t}{\cos t} = \frac{\sqrt{1-x^2}}{x} \\ dx = -\sin t dt \mid x \in (0; 1), t \in (0; \frac{\pi}{2}) \mid \sqrt{1-x^2} = \sqrt{1-\cos^2 t} = \sqrt{\sin^2 t} = |\sin t| = \sin t \end{array} \right] = -\int \frac{\sin t \cdot \sin t dt}{\cos^2 t}$$

$$= -\int \frac{(1-\cos^2 t) dt}{\sin^2 t} = \int \left[1 - \frac{1}{\cos^2 t} \right] dt = t - \tg t + c_2 = \arccos x - \frac{\sqrt{1-x^2}}{x} + c_2,$$

$$x \in (-1; 0) \cup (0; 1), c_2 \in \mathbb{R}.$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x \, dx}{x^2 \sqrt{x^4+x^2+1}}$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x \, dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1 = (x^2+\frac{1}{2})^2+\frac{3}{4} > 0 \\ dt=2x \, dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1 = (t+\frac{1}{2})^2+\frac{3}{4} > 0 \end{array} \right]$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x \, dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1 = (x^2+\frac{1}{2})^2+\frac{3}{4} > 0 \\ dt=2x \, dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1 = (t+\frac{1}{2})^2+\frac{3}{4} > 0 \end{array} \right] = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x \, dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1 = (x^2+\frac{1}{2})^2+\frac{3}{4} > 0 \\ dt=2x \, dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1 = (t+\frac{1}{2})^2+\frac{3}{4} > 0 \end{array} \right] = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in (0; \infty) \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u} = \frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in (0; \infty) \end{array} \right]$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x \, dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1 = (x^2+\frac{1}{2})^2+\frac{3}{4} > 0 \\ dt=2x \, dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1 = (t+\frac{1}{2})^2+\frac{3}{4} > 0 \end{array} \right] = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in (0; \infty) \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u} = \frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in (0; \infty) \end{array} \right]$$

$$= \frac{1}{2} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u}}$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x \, dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1 = (x^2+\frac{1}{2})^2 + \frac{3}{4} > 0 \\ dt=2x \, dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1 = (t+\frac{1}{2})^2 + \frac{3}{4} > 0 \end{array} \right] = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in (0; \infty) \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u} = \frac{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}}{u} \\ dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in (0; \infty) \end{array} \right]$$

$$= \frac{1}{2} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2} \int \frac{du}{\sqrt{u^2+u+1}}$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x \, dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1 = (x^2+\frac{1}{2})^2+\frac{3}{4} > 0 \\ dt=2x \, dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1 = (t+\frac{1}{2})^2+\frac{3}{4} > 0 \end{array} \right] = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in (0; \infty) \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u} = \frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in (0; \infty) \end{array} \right]$$

$$= \frac{1}{2} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2} \int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2} \int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1 = (x^2+\frac{1}{2})^2+\frac{3}{4} > 0 \\ dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1 = (t+\frac{1}{2})^2+\frac{3}{4} > 0 \end{array} \right] = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=\frac{1}{u}=u^{-1} \mid t \in (0; \infty) \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2}+\frac{1}{u}+1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u} = \frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in (0; \infty) \end{array} \right]$$

$$= \frac{1}{2} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2} \int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2} \int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= -\frac{1}{2} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + c_1$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=\frac{1}{u}=u^{-1} \mid t \in (0; \infty) \mid \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2}du=-\frac{du}{u^2} \mid u \in (0; \infty) \mid \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t}=\frac{\sqrt{x^4+x^2+1}}{x^2} \end{array} \right]$$

$$= \frac{1}{2} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2} \int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2} \int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= -\frac{1}{2} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + C_1 = \left[u = \frac{1}{t} = \frac{1}{x^2} \right]$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x \, dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x \, dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=\frac{1}{u}=u^{-1} \mid t \in (0; \infty) \mid \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2}du=-\frac{du}{u^2} \mid u \in (0; \infty) \mid \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t}=\frac{\sqrt{x^4+x^2+1}}{x^2} \end{array} \right]$$

$$= \frac{1}{2} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2} \int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2} \int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= -\frac{1}{2} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + C_1 = \left[u = \frac{1}{t} = \frac{1}{x^2} \right]$$

$$= -\frac{1}{2} \ln \left| \frac{1}{x^2} + \frac{1}{2} + \frac{\sqrt{x^4+x^2+1}}{x^2} \right| + C_1$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x \, dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x \, dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=\frac{1}{u}=u^{-1} \mid t \in (0; \infty) \mid \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2} du=-\frac{du}{u^2} \mid u \in (0; \infty) \mid \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t}=\frac{\sqrt{x^4+x^2+1}}{x^2} \end{array} \right]$$

$$= \frac{1}{2} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2} \int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2} \int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= -\frac{1}{2} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + C_1 = \left[u = \frac{1}{t} = \frac{1}{x^2} \right]$$

$$= -\frac{1}{2} \ln \left| \frac{1}{x^2} + \frac{1}{2} + \frac{\sqrt{x^4+x^2+1}}{x^2} \right| + C_1 = C_1 - \frac{1}{2} \ln \left| \frac{2+x^2+2\sqrt{x^4+x^2+1}}{2x^2} \right|$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=\frac{1}{u}=u^{-1} \mid t \in (0; \infty) \mid \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2} du=-\frac{du}{u^2} \mid u \in (0; \infty) \mid \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t}=\frac{\sqrt{x^4+x^2+1}}{x^2} \end{array} \right]$$

$$= \frac{1}{2} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2} \int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2} \int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= -\frac{1}{2} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + C_1 = \left[u = \frac{1}{t} = \frac{1}{x^2} \right]$$

$$= -\frac{1}{2} \ln \left| \frac{1}{x^2} + \frac{1}{2} + \frac{\sqrt{x^4+x^2+1}}{x^2} \right| + C_1 = C_1 - \frac{1}{2} \ln \left| \frac{2+x^2+2\sqrt{x^4+x^2+1}}{2x^2} \right|$$

$$= C_1 + \frac{\ln 2}{3} - \frac{1}{2} \ln \left| \frac{2+x^2+2\sqrt{x^4+x^2+1}}{x^2} \right|$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=\frac{1}{u}=u^{-1} \mid t \in (0; \infty) \mid \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2} du=-\frac{du}{u^2} \mid u \in (0; \infty) \mid \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t}=\frac{\sqrt{x^4+x^2+1}}{x^2} \end{array} \right]$$

$$= \frac{1}{2} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2} \int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2} \int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= -\frac{1}{2} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + c_1 = \left[u = \frac{1}{t} = \frac{1}{x^2} \right]$$

$$= -\frac{1}{2} \ln \left| \frac{1}{x^2} + \frac{1}{2} + \frac{\sqrt{x^4+x^2+1}}{x^2} \right| + c_1 = c_1 - \frac{1}{2} \ln \left| \frac{2+x^2+2\sqrt{x^4+x^2+1}}{2x^2} \right|$$

$$= c_1 + \frac{\ln 2}{3} - \frac{1}{2} \ln \left| \frac{2+x^2+2\sqrt{x^4+x^2+1}}{x^2} \right| = \left[\begin{array}{l} c = c_1 + \frac{\ln 2}{3} \\ c_1 \in R, c \in R \end{array} \right]$$

Koniec 10. časti príklady b – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}} = c - \frac{1}{2} \ln \left| \frac{2+x^2+2\sqrt{x^4+x^2+1}}{x^2} \right|$$

$$= \int \frac{x dx}{x^2 \sqrt{x^4+x^2+1}} = \left[\begin{array}{l} \text{Subst. } t=x^2 \mid x \in (-\infty; 0), t \in (0; \infty) \mid x^4+x^2+1 = (x^2+\frac{1}{2})^2 + \frac{3}{4} > 0 \\ dt = 2x dx \mid x \in (0; \infty), t \in (0; \infty) \mid t^2+t+1 = (t+\frac{1}{2})^2 + \frac{3}{4} > 0 \end{array} \right] = \frac{1}{2} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in (0; \infty) \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u} = \frac{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}}{u} \\ dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in (0; \infty) \mid \sqrt{u^2+u+1} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t} = \frac{\sqrt{x^4+x^2+1}}{x^2} \end{array} \right]$$

$$= \frac{1}{2} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2} \int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2} \int \frac{du}{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}}$$

$$= -\frac{1}{2} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + c_1 = \left[u = \frac{1}{t} = \frac{1}{x^2} \right]$$

$$= -\frac{1}{2} \ln \left| \frac{1}{x^2} + \frac{1}{2} + \frac{\sqrt{x^4+x^2+1}}{x^2} \right| + c_1 = c_1 - \frac{1}{2} \ln \left| \frac{2+x^2+2\sqrt{x^4+x^2+1}}{2x^2} \right|$$

$$= c_1 + \frac{\ln 2}{3} - \frac{1}{2} \ln \left| \frac{2+x^2+2\sqrt{x^4+x^2+1}}{x^2} \right| = \left[\begin{array}{l} c = c_1 + \frac{\ln 2}{3} \\ c_1 \in R, c \in R \end{array} \right]$$

$$= c - \frac{1}{2} \ln \left| \frac{2+x^2+2\sqrt{x^4+x^2+1}}{x^2} \right|, x \in R - \{0\}, c \in R.$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3\sqrt{x^6+x^3+1}}$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \\ dt=3x^2 dx \mid t \in \mathbb{R} - \{0\} \mid t^2+t+1 = (t+\frac{1}{2})^2 + \frac{3}{4} > 0 \end{array} \right]$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \\ dt=3x^2 dx \mid t \in \mathbb{R} - \{0\} \mid t^2+t+1 = (t+\frac{1}{2})^2 + \frac{3}{4} > 0 \end{array} \right] = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2 dx \mid t \in \mathbb{R} - \{0\} \mid t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=\frac{1}{u}=u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn } u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u \cdot \text{sgn } u} \\ dt=-u^{-2} du=-\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \end{array} \right]$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \\ dt=3x^2 dx \mid t \in \mathbb{R} - \{0\} \mid t^2+t+1 = (t+\frac{1}{2})^2 + \frac{3}{4} > 0 \end{array} \right] = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn } u} = \frac{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}}{u \cdot \text{sgn } u} \\ dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \end{array} \right]$$

$$= \frac{1}{3} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn } u}}$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \\ dt=3x^2 dx \mid t \in \mathbb{R} - \{0\} \mid t^2+t+1 = (t+\frac{1}{2})^2 + \frac{3}{4} > 0 \end{array} \right] = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn } u} = \frac{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}}{u \cdot \text{sgn } u} \\ dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \end{array} \right]$$

$$= \frac{1}{3} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn } u}} = \frac{1}{3} \int \frac{-\text{sgn } u du}{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}}$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2 dx \mid t \in \mathbb{R} - \{0\} \mid t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=\frac{1}{u}=u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn } u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u \cdot \text{sgn } u} \\ dt=-u^{-2} du=-\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \end{array} \right]$$

$$= \frac{1}{3} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn } u}} = \frac{1}{3} \int \frac{-\text{sgn } u du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\text{sgn } u}{3} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + c_1$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \\ dt=3x^2 dx \mid t \in \mathbb{R} - \{0\} \mid t^2+t+1 = (t+\frac{1}{2})^2 + \frac{3}{4} > 0 \end{array} \right] = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn } u} = \frac{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}}{u \cdot \text{sgn } u} \\ dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \end{array} \right]$$

$$= \frac{1}{3} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn } u}} = \frac{1}{3} \int \frac{-\text{sgn } u du}{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\text{sgn } u}{3} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + c_1 = \left[\begin{array}{l} \text{sgn } u \\ = \text{sgn } x \end{array} \right]$$



Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1 = (x^3 + \frac{1}{2})^2 + \frac{3}{4} > 0 \right] = \frac{1}{3} \int \frac{dt}{t \sqrt{t^2+t+1}}$$

$$= \left[\text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn} u} = \frac{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}}{u \cdot \text{sgn} u}, \quad u = \frac{1}{t} = \frac{1}{x^3} \right]$$

$$dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \mid \sqrt{u^2+u+1} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgn} t} = \frac{\sqrt{x^6+x^3+1}}{x^3 \cdot \text{sgn} x} = \frac{\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3}$$

$$= \frac{1}{3} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn} u}} = \frac{1}{3} \int \frac{-\text{sgn} u du}{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\text{sgn} u}{3} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + c_1 = \left[\text{sgn} u \right]$$

$$= -\frac{\text{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$\begin{aligned}
 &= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right] = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2+t+1}} \\
 &= \left[\text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn} u} = \frac{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}}{u \cdot \text{sgn} u}, \quad u = \frac{1}{t} = \frac{1}{x^3} \right] \\
 &\quad \left[dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \mid \sqrt{u^2+u+1} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgn} t} = \frac{\sqrt{x^6+x^3+1}}{x^3 \cdot \text{sgn} x} = \frac{\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right] \\
 &= \frac{1}{3} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn} u}} = \frac{1}{3} \int \frac{-\text{sgn} u du}{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\text{sgn} u}{3} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + c_1 = \left[\text{sgn} u \right] \\
 &= -\frac{\text{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\text{sgn} x}{3} \ln \left| \frac{2+x^3+2\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right|
 \end{aligned}$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$\begin{aligned}
 &= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right] = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2+t+1}} \\
 &= \left[\text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn} u} = \frac{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}}{u \cdot \text{sgn} u}, \quad u = \frac{1}{t} = \frac{1}{x^3} \right] \\
 &\quad \left[dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \mid \sqrt{u^2+u+1} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgn} t} = \frac{\sqrt{x^6+x^3+1}}{x^3 \cdot \text{sgn} x} = \frac{\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right] \\
 &= \frac{1}{3} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn} u}} = \frac{1}{3} \int \frac{-\text{sgn} u du}{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\text{sgn} u}{3} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + c_1 = \left[\text{sgn} u \right] \\
 &= -\frac{\text{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\text{sgn} x}{3} \ln \left| \frac{2+x^3+2\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right| \\
 &= c_1 - \frac{\text{sgn} x}{3} \ln \left| \frac{2+x^3+2\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + \frac{\text{sgn} x \cdot \ln 2}{3}
 \end{aligned}$$



Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$\begin{aligned}
 &= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right] = \frac{1}{3} \int \frac{dt}{t \sqrt{t^2+t+1}} \\
 &= \left[\text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{u^2+u+1}}{|u|} = \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn} u} = \frac{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}}{u \cdot \text{sgn} u}, \quad u = \frac{1}{t} = \frac{1}{x^3} \right] \\
 &\quad \left[dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \mid \sqrt{u^2+u+1} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgn} t} = \frac{\sqrt{x^6+x^3+1}}{x^3 \cdot \text{sgn} x^3} = \frac{\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right] \\
 &= \frac{1}{3} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u \cdot \text{sgn} u}} = \frac{1}{3} \int \frac{-\text{sgn} u du}{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\text{sgn} u}{3} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + c_1 = \left[\text{sgn} u \right] \\
 &\quad = \text{sgn} x \\
 &= -\frac{\text{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\text{sgn} x}{3} \ln \left| \frac{2+x^3+2\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right| \\
 &= c_1 - \frac{\text{sgn} x}{3} \ln \left| \frac{2+x^3+2\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + \frac{\text{sgn} x \cdot \ln 2}{3} = \left[c_2 = c_1 + \frac{\text{sgn} x \cdot \ln 2}{3} \right] \\
 &\quad \left[c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \right]
 \end{aligned}$$

Koniec 10. časti príklady b – 200

$$\begin{aligned}
 \int \frac{dx}{x\sqrt{x^6+x^3+1}} &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| \\
 &= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right. \\
 &\quad \left. dt=3x^2 dx \mid t \in \mathbb{R} - \{0\} \mid t^2+t+1 = (t+\frac{1}{2})^2 + \frac{3}{4} > 0 \right] = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2+t+1}} \\
 &= \left[\text{Subst. } t = \frac{1}{u} = u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1} = \sqrt{\frac{1}{u^2} + \frac{1}{u} + 1} = \sqrt{\frac{1+u+u^2}{u^2}} = \frac{\sqrt{1+u+u^2}}{|u|} = \frac{\sqrt{u^2+u+1}}{u \cdot \operatorname{sgn} u} = \frac{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}}{u \cdot \operatorname{sgn} u}, \right. \\
 &\quad \left. dt = -u^{-2} du = -\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \mid \sqrt{u^2+u+1} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{1+t+t^2}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \operatorname{sgn} t} = \frac{\sqrt{x^6+x^3+1}}{x^3 \cdot \operatorname{sgn} x^3} = \frac{\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right] \\
 &= \frac{1}{3} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u \cdot \operatorname{sgn} u}} = \frac{1}{3} \int \frac{-\operatorname{sgn} u du}{\sqrt{(u+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\operatorname{sgn} u}{3} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + c_1 = \left[\begin{matrix} \operatorname{sgn} u \\ = \operatorname{sgn} x \end{matrix} \right] \\
 &= -\frac{\operatorname{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right| \\
 &= c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + \frac{\operatorname{sgn} x \cdot \ln 2}{3} = \left[\begin{matrix} c_2 = c_1 + \frac{\operatorname{sgn} x \cdot \ln 2}{3} \\ c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \end{matrix} \right] \\
 &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right|, x \in \mathbb{R} - \{0\}, c_2 \in \mathbb{R}.
 \end{aligned}$$

Koniec 10. časti príklady b – 200

$$\begin{aligned}
 \int \frac{dx}{x\sqrt{x^6+x^3+1}} &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| \\
 &= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \right] = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2+t+1}} \\
 &= \left[\text{Subst. } t=\frac{1}{u}=u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u \cdot \operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u \cdot \operatorname{sgn} u}, u=\frac{1}{t}=\frac{1}{x^3} \right] \\
 &\quad \left[dt=-u^{-2} du=-\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \mid \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t \cdot \operatorname{sgn} t}=\frac{\sqrt{x^6+x^3+1}}{x^3 \cdot \operatorname{sgn} x^3}=\frac{\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right] \\
 &= \frac{1}{3} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u \cdot \operatorname{sgn} u}} = \frac{1}{3} \int \frac{-\operatorname{sgn} u du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} u}{3} \ln \left| u+\frac{1}{2}+\sqrt{u^2+u+1} \right| + c_1 = \left[\begin{matrix} \operatorname{sgn} u \\ = \operatorname{sgn} x \end{matrix} \right] \\
 &= -\frac{\operatorname{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right| \\
 &= c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + \frac{\operatorname{sgn} x \cdot \ln 2}{3} = \left[\begin{matrix} c_2 = c_1 + \frac{\operatorname{sgn} x \cdot \ln 2}{3} \\ c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \end{matrix} \right] \\
 &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right|, x \in \mathbb{R} - \{0\}, c_2 \in \mathbb{R}.
 \end{aligned}$$

$$x \in (0; \infty) \Rightarrow c_2 - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right|$$

$$x \in (-\infty; 0) \Rightarrow c_2 + \frac{1}{3} \ln \left| \frac{2+x^3-2\sqrt{x^6+x^3+1}}{x^3} \right|$$

Koniec 10. časti príklady b – 200

$$\begin{aligned}
 \int \frac{dx}{x\sqrt{x^6+x^3+1}} &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| \\
 &= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2 dx \mid t \in \mathbb{R} - \{0\} \mid t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2+t+1}} \\
 &= \left[\begin{array}{l} \text{Subst. } t=\frac{1}{u}=u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u \cdot \operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u \cdot \operatorname{sgn} u}, \quad u=\frac{1}{t}=\frac{1}{x^3} \\ dt=-u^{-2} du=-\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \mid \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t \cdot \operatorname{sgn} t}=\frac{\sqrt{x^6+x^3+1}}{x^3 \cdot \operatorname{sgn} x^3}=\frac{\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \end{array} \right] \\
 &= \frac{1}{3} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u \cdot \operatorname{sgn} u}} = \frac{1}{3} \int \frac{-\operatorname{sgn} u du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} u}{3} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + c_1 = \left[\begin{array}{l} \operatorname{sgn} u \\ = \operatorname{sgn} x \end{array} \right] \\
 &= -\frac{\operatorname{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right| \\
 &= c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + \frac{\operatorname{sgn} x \cdot \ln 2}{3} = \left[\begin{array}{l} c_2 = c_1 + \frac{\operatorname{sgn} x \cdot \ln 2}{3} \\ c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right] \\
 &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right|, \quad x \in \mathbb{R} - \{0\}, \quad c_2 \in \mathbb{R}.
 \end{aligned}$$

$$\boxed{x \in (0; \infty)} = c_2 - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right| = \left[\begin{array}{l} c = c_2 \\ c \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right]$$

$$\boxed{x \in (-\infty; 0)} = c_2 + \frac{1}{3} \ln \left| \frac{2+x^3-2\sqrt{x^6+x^3+1}}{x^3} \right| = c + \frac{\ln 3}{3} - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right| = \left[\begin{array}{l} c = c_2 + \frac{\ln 3}{3} \\ c \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right]$$

Koniec 10. časti príklady b – 200

$$\begin{aligned}
 \int \frac{dx}{x\sqrt{x^6+x^3+1}} &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| = c - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right| \\
 &= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \in \mathbb{R} - \{0\} \mid x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2 dx \mid t \in \mathbb{R} - \{0\} \mid t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2+t+1}} \\
 &= \left[\begin{array}{l} \text{Subst. } t=\frac{1}{u}=u^{-1} \mid t \in \mathbb{R} - \{0\} \mid \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u \cdot \operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u \cdot \operatorname{sgn} u}, \quad u=\frac{1}{t}=\frac{1}{x^3} \\ dt=-u^{-2} du=-\frac{du}{u^2} \mid u \in \mathbb{R} - \{0\} \mid \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t \cdot \operatorname{sgn} t}=\frac{\sqrt{x^6+x^3+1}}{x^3 \cdot \operatorname{sgn} x}=\frac{\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \end{array} \right] \\
 &= \frac{1}{3} \int \frac{-\frac{du}{u^2}}{\frac{1}{u} \cdot \frac{\sqrt{u^2+u+1}}{u \cdot \operatorname{sgn} u}} = \frac{1}{3} \int \frac{-\operatorname{sgn} u du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} u}{3} \ln \left| u + \frac{1}{2} + \sqrt{u^2+u+1} \right| + c_1 = \left[\begin{array}{l} \operatorname{sgn} u \\ = \operatorname{sgn} x \end{array} \right] \\
 &= -\frac{\operatorname{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right| \\
 &= c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + \frac{\operatorname{sgn} x \cdot \ln 2}{3} = \left[\begin{array}{l} c_2 = c_1 + \frac{\operatorname{sgn} x \cdot \ln 2}{3} \\ c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right] \\
 &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right|, \quad x \in \mathbb{R} - \{0\}, \quad c_2 \in \mathbb{R}.
 \end{aligned}$$

$$\boxed{x \in (0; \infty)} = c_2 - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right| = \left[\begin{array}{l} c = c_2 \\ c \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right] = c - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right|.$$

$$\boxed{x \in (-\infty; 0)} = c_2 + \frac{1}{3} \ln \left| \frac{2+x^3-2\sqrt{x^6+x^3+1}}{x^3} \right| = \left[\begin{array}{l} c = c_2 + \frac{\ln 3}{3} \\ c \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right] = c - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right|.$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3\sqrt{x^6+x^3+1}}$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\begin{array}{l} \text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \\ 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3 + \frac{1}{2})^2 + \frac{3}{4} > 0 \end{array} \right]$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\begin{array}{l} \text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \\ 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \end{array} \right]$$

$$= \left[\sqrt{x^6+x^3+1} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgnt}} = \frac{1}{t \cdot \text{sgnt}} \sqrt{\left(t+\frac{1}{2}\right)^2 + \frac{3}{4}} \right]$$



Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \right. \\ \left. 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right]$$

$$= \left[\sqrt{x^6+x^3+1} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgn} t} = \frac{1}{t \cdot \text{sgn} t} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t} \cdot \frac{1}{t \cdot \text{sgn} t} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}}$$



Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \right. \\ \left. 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3 + \frac{1}{2})^2 + \frac{3}{4} > 0 \right]$$

$$= \left[\sqrt{x^6+x^3+1} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgnt}} = \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t} \cdot \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}}$$

$$= \frac{1}{3} \int \frac{-\text{sgnt} dt}{\sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}}$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \right. \\ \left. 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right]$$

$$= \left[\sqrt{x^6+x^3+1} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgnt}} = \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t} \cdot \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}}$$

$$= \frac{1}{3} \int \frac{-\text{sgnt} dt}{\sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\text{sgnt}}{3} \ln \left| t + \frac{1}{2} + \sqrt{t^2+t+1} \right| + c_1$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \right. \\ \left. 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right]$$

$$= \left[\sqrt{x^6+x^3+1} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgnt}} = \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t} \cdot \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}}$$

$$= \frac{1}{3} \int \frac{-\text{sgnt } dt}{\sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\text{sgnt}}{3} \ln \left| t + \frac{1}{2} + \sqrt{t^2+t+1} \right| + c_1 = \left[\text{sgn } u = \text{sgn } x \right]$$



Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \right. \\ \left. 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right]$$

$$= \left[\frac{\sqrt{x^6+x^3+1}}{\sqrt{t^2+t+1}} = \frac{\sqrt{\frac{1}{t^2} + \frac{1}{t} + 1}}{\sqrt{t^2+t+1}} = \frac{\sqrt{\frac{1+t+t^2}{t^2}}}{\sqrt{t^2+t+1}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgnt}} = \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t} \cdot \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}}$$

$$= \frac{1}{3} \int \frac{-\text{sgnt} dt}{\sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\text{sgnt}}{3} \ln \left| t + \frac{1}{2} + \sqrt{t^2+t+1} \right| + c_1 = \left[\text{sgn} u = \text{sgn} x \right]$$

$$= -\frac{\text{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1$$



Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \right. \\ \left. 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right]$$

$$= \left[\frac{\sqrt{x^6+x^3+1}}{\sqrt{t^2+t+1}} = \sqrt{\frac{\frac{1}{t^2} + \frac{1}{t} + 1}{t^2}} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgnt}} = \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t} \cdot \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}}$$

$$= \frac{1}{3} \int \frac{-\text{sgnt} dt}{\sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\text{sgnt}}{3} \ln \left| t + \frac{1}{2} + \sqrt{t^2+t+1} \right| + c_1 = \left[\text{sgn} u = \text{sgn} x \right]$$

$$= -\frac{\text{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\text{sgn} x}{3} \ln \left| \frac{2+x^3+2\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right|$$



Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \right. \\ \left. 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right]$$

$$= \left[\frac{\sqrt{x^6+x^3+1}}{\sqrt{t^2+t+1}} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgnt}} = \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t} \cdot \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}}$$

$$= \frac{1}{3} \int \frac{-\text{sgnt} dt}{\sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\text{sgnt}}{3} \ln \left| t + \frac{1}{2} + \sqrt{t^2+t+1} \right| + C_1 = \left[\text{sgn} u = \text{sgn} x \right]$$

$$= -\frac{\text{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + C_1 = C_1 - \frac{\text{sgn} x}{3} \ln \left| \frac{2+x^3+2\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right|$$

$$= C_1 - \frac{\text{sgn} x}{3} \ln \left| \frac{2+x^3+2\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + \frac{\text{sgn} x \cdot \ln 2}{3}$$



Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \right. \\ \left. 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right]$$

$$= \left[\frac{\sqrt{x^6+x^3+1}}{\sqrt{t^2+t+1}} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \text{sgnt}} = \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t} \cdot \frac{1}{t \cdot \text{sgnt}} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}}$$

$$= \frac{1}{3} \int \frac{-\text{sgnt} dt}{\sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\text{sgnt}}{3} \ln \left| t + \frac{1}{2} + \sqrt{t^2+t+1} \right| + c_1 = \left[\text{sgn} u = \text{sgn} x \right]$$

$$= -\frac{\text{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\text{sgn} x}{3} \ln \left| \frac{2+x^3+2\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right|$$

$$= c_1 - \frac{\text{sgn} x}{3} \ln \left| \frac{2+x^3+2\text{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + \frac{\text{sgn} x \cdot \ln 2}{3} = \left[c_2 = c_1 + \frac{\text{sgn} x \cdot \ln 2}{3} \right. \\ \left. c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \right]$$



Koniec 10. časti príklady b – 200

$$\begin{aligned}
 \int \frac{dx}{x\sqrt{x^6+x^3+1}} &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| \\
 &= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \right. \\
 &\quad \left. 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3 + \frac{1}{2})^2 + \frac{3}{4} > 0 \right] \\
 &= \left[\frac{\sqrt{x^6+x^3+1}}{\sqrt{t^2+t+1}} = \sqrt{\frac{1}{t^2} + \frac{1}{t} + 1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \operatorname{sgn} t} = \frac{1}{t \cdot \operatorname{sgn} t} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t} \cdot \frac{1}{t \cdot \operatorname{sgn} t} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} \\
 &= \frac{1}{3} \int \frac{-\operatorname{sgn} t dt}{\sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3} \ln \left| t + \frac{1}{2} + \sqrt{t^2+t+1} \right| + c_1 = \left[\operatorname{sgn} u = \operatorname{sgn} x \right] \\
 &= -\frac{\operatorname{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right| \\
 &= c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + \frac{\operatorname{sgn} x \cdot \ln 2}{3} = \left[c_2 = c_1 + \frac{\operatorname{sgn} x \cdot \ln 2}{3} \right. \\
 &\quad \left. c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \right] \\
 &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right|, x \in \mathbb{R} - \{0\}, c_2 \in \mathbb{R}.
 \end{aligned}$$



Koniec 10. časti príklady b – 200

$$\begin{aligned}
 \int \frac{dx}{x\sqrt{x^6+x^3+1}} &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| \\
 &= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \right. \\
 &\quad \left. 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3 + \frac{1}{2})^2 + \frac{3}{4} > 0 \right] \\
 &= \left[\frac{\sqrt{x^6+x^3+1}}{\sqrt{t^2+t+1}} = \sqrt{\frac{1+t+t^2}{t^2+1}} = \sqrt{\frac{t^2+t+1}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \operatorname{sgn} t} = \frac{1}{t \cdot \operatorname{sgn} t} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t} \cdot \frac{1}{t \cdot \operatorname{sgn} t} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} \\
 &= \frac{1}{3} \int \frac{-\operatorname{sgn} t dt}{\sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3} \ln \left| t + \frac{1}{2} + \sqrt{t^2+t+1} \right| + c_1 = \left[\operatorname{sgn} u = \operatorname{sgn} x \right] \\
 &= -\frac{\operatorname{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right| \\
 &= c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + \frac{\operatorname{sgn} x \cdot \ln 2}{3} = \left[c_2 = c_1 + \frac{\operatorname{sgn} x \cdot \ln 2}{3} \right. \\
 &\quad \left. c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \right] \\
 &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right|, x \in \mathbb{R} - \{0\}, c_2 \in \mathbb{R}.
 \end{aligned}$$

$$x \in (0; \infty) \Rightarrow c_2 - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right|$$

$$x \in (-\infty; 0) \Rightarrow c_2 + \frac{1}{3} \ln \left| \frac{2+x^3-2\sqrt{x^6+x^3+1}}{x^3} \right| \quad \text{☞}$$

Koniec 10. časti príklady b – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}} = c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right|$$

$$= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \right. \\ \left. 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right]$$

$$= \left[\frac{\sqrt{x^6+x^3+1}}{\sqrt{t^2+t+1}} = \frac{\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}}{\sqrt{t^2+t+1}} = \frac{\sqrt{\frac{1+t+t^2}{t^2}}}{\sqrt{t^2+t+1}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \operatorname{sgn} t} = \frac{1}{t \cdot \operatorname{sgn} t} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t} \cdot \frac{1}{t \cdot \operatorname{sgn} t} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}}$$

$$= \frac{1}{3} \int \frac{-\operatorname{sgn} t dt}{\sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3} \ln \left| t + \frac{1}{2} + \sqrt{t^2+t+1} \right| + c_1 = \left[\operatorname{sgn} u = \operatorname{sgn} x \right]$$

$$= -\frac{\operatorname{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right|$$

$$= c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + \frac{\operatorname{sgn} x \cdot \ln 2}{3} = \left[c_2 = c_1 + \frac{\operatorname{sgn} x \cdot \ln 2}{3} \right. \\ \left. c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \right]$$

$$= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right|, x \in \mathbb{R} - \{0\}, c_2 \in \mathbb{R}.$$

$$x \in (0; \infty) \Rightarrow c_2 - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right| = \left[c = c_2 \right. \\ \left. c \in \mathbb{R}, c_2 \in \mathbb{R} \right]$$

$$x \in (-\infty; 0) \Rightarrow c_2 + \frac{1}{3} \ln \left| \frac{2+x^3-2\sqrt{x^6+x^3+1}}{x^3} \right| = c + \frac{\ln 3}{3} - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right| = \left[c = c_2 + \frac{\ln 3}{3} \right. \\ \left. c \in \mathbb{R}, c_2 \in \mathbb{R} \right]$$

Koniec 10. časti príklady b – 200

$$\begin{aligned}
 \int \frac{dx}{x\sqrt{x^6+x^3+1}} &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| = c - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right| \\
 &= \int \frac{x^2 dx}{x^3 \sqrt{x^6+x^3+1}} = \left[\text{Subst. } t = \frac{1}{x^3}, x^3 = \frac{1}{t} = t^{-1} \mid x \in \mathbb{R} - \{0\}, t \in \mathbb{R} - \{0\} \right. \\
 &\quad \left. 3x^2 dx = -t^{-2} dt = -\frac{dt}{t^2} \mid x^6+x^3+1 = (x^3+\frac{1}{2})^2 + \frac{3}{4} > 0 \right] \\
 &= \left[\frac{\sqrt{x^6+x^3+1}}{\sqrt{t^2+t+1}} = \frac{\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}}{\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}} = \frac{\sqrt{\frac{1+t+t^2}{t^2}}}{\sqrt{\frac{1+t+t^2}{t^2}}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t \cdot \operatorname{sgn} t} = \frac{1}{t \cdot \operatorname{sgn} t} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t} \cdot \frac{1}{t \cdot \operatorname{sgn} t} \sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} \\
 &= \frac{1}{3} \int \frac{-\operatorname{sgn} t dt}{\sqrt{(t+\frac{1}{2})^2 + \frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3} \ln \left| t + \frac{1}{2} + \sqrt{t^2+t+1} \right| + c_1 = \left[\operatorname{sgn} u = \operatorname{sgn} x \right] \\
 &= -\frac{\operatorname{sgn} x}{3} \ln \left| \frac{1}{x^3} + \frac{1}{2} + \frac{\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + c_1 = c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{2x^3} \right| \\
 &= c_1 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right| + \frac{\operatorname{sgn} x \cdot \ln 2}{3} = \left[\begin{array}{l} c_2 = c_1 + \frac{\operatorname{sgn} x \cdot \ln 2}{3} \\ c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right] \\
 &= c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x \cdot \sqrt{x^6+x^3+1}}{x^3} \right|, x \in \mathbb{R} - \{0\}, c_2 \in \mathbb{R}.
 \end{aligned}$$

$$x \in (0; \infty) \Rightarrow c_2 - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right| = \left[\begin{array}{l} c = c_2 \\ c \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right] = c - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right|.$$

$$x \in (-\infty; 0) \Rightarrow c_2 + \frac{1}{3} \ln \left| \frac{2+x^3-2\sqrt{x^6+x^3+1}}{x^3} \right| = \left[\begin{array}{l} c = c_2 + \frac{\ln 3}{3} \\ c \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right] = c - \frac{1}{3} \ln \left| \frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3} \right|.$$