

Matematická analýza 1

2024/2025

11. Určitý integrál

Pre správne zobrazenie, fungovanie tooltipov, 2D a 3D animácií je nevyhnutné súbor otvoriť pomocou programu Adobe Reader (zásuvný modul Adobe PDF Plug-In webového prehliadača nestačí).

Kliknutím na text pred ikonou  získate nápomoc.

Kliknutím na skratku v modrej lište vpravo hore sa dostanete na príslušný slajd, druhým kliknutím sa dostanete na koniec tohto slajdu.

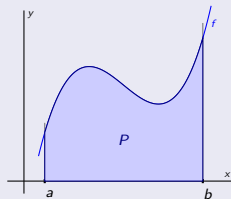
Obsah

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- 2 Riemannov určitý integrál
- 3 Príklady
- 4 Vlastnosti Riemannovho integrálu
- 5 Aditívnosť Riemannovho integrálu
- 6 Výpočet Riemannovho integrálu
- 7 Metóda per partes
- 8 Metóda substitúcie
- 9 Integrovanie párných a nepárnych funkcií
- 10 Integrovanie periodických funkcií

Integrálne súčty – Motivačný príklad

$$y = f(x) > 0, x \in \langle a; b \rangle, a, b \in \mathbb{R}, a < b.$$

Určte plošný obsah množiny $P = \{[x; y] \in \mathbb{R}^2; x \in \langle a; b \rangle, 0 \leq y \leq f(x)\}$.

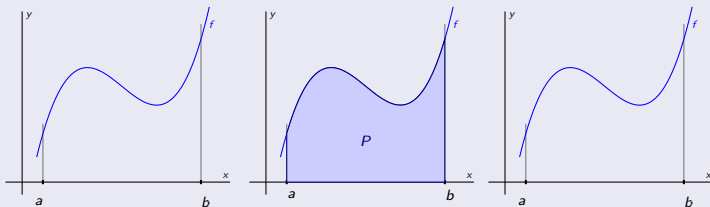


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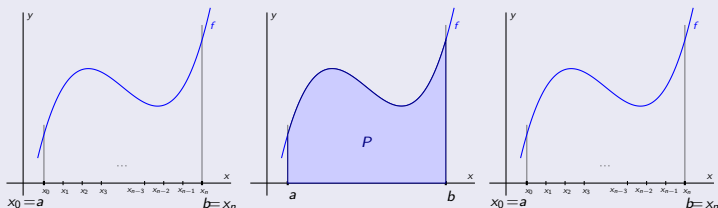


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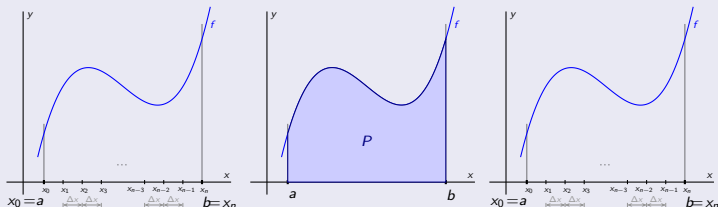


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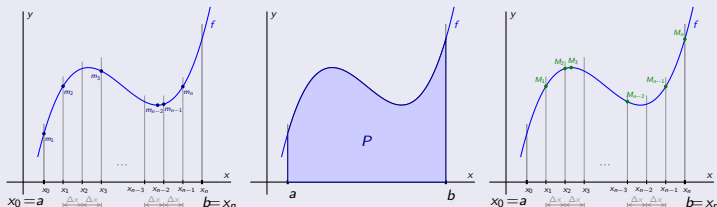
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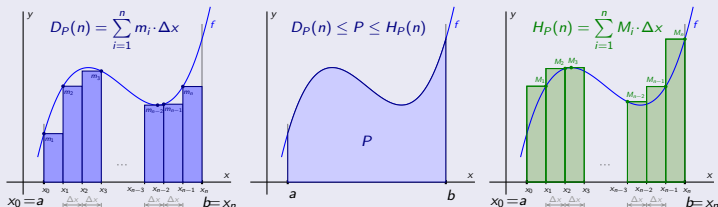
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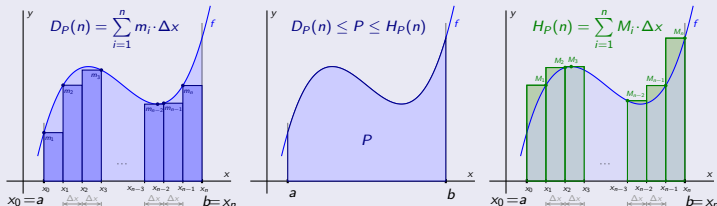
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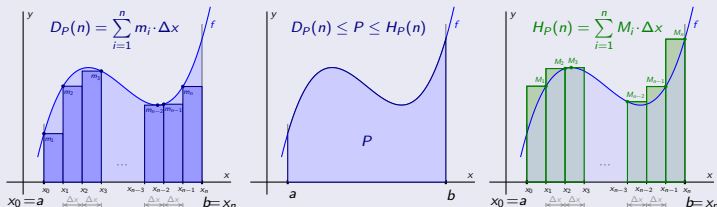
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Ak $n \rightarrow \infty$, t. j. $\Delta x = \frac{b-a}{n} \rightarrow 0$, potom $D_P(n) \rightarrow P$ (dolný odhad), $H_P(n) \rightarrow P$ (horný odhad).

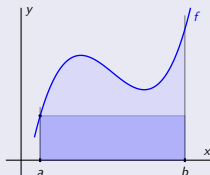
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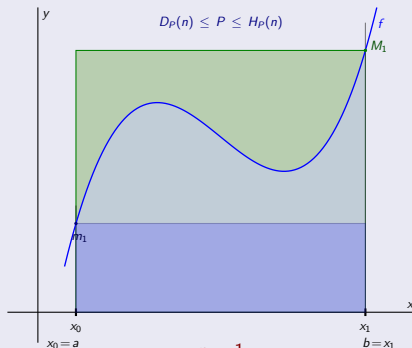
$$D_P(n) = \sum_{i=1}^n m_i \cdot \Delta x_i \rightarrow P$$

pre $n \rightarrow \infty, \epsilon_j, \Delta x = \frac{b-a}{n} \rightarrow 0$



$$D_P(1) = \sum_{i=1}^1 m_i \cdot \Delta x_i$$

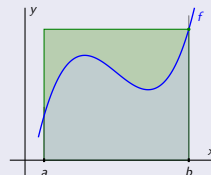
Dolný odhad



$n = 1$

$$P \leftarrow \sum_{i=1}^n M_i \cdot \Delta x_i = H_P(n)$$

pre $n \rightarrow \infty, \epsilon_j, \Delta x = \frac{b-a}{n} \rightarrow 0$



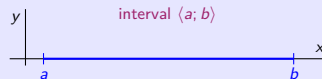
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Horný odhad



Integrálne súčty

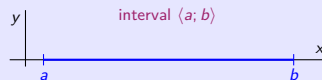
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Delením intervalu $\langle a; b \rangle$



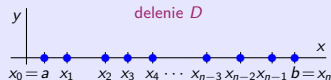
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Delením intervalu $\langle a; b \rangle$ nazývame každú konečnú množinu bodov

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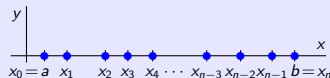
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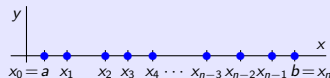
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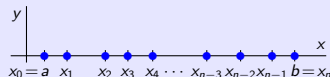
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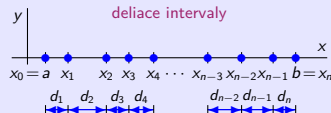
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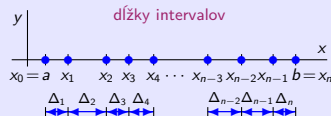
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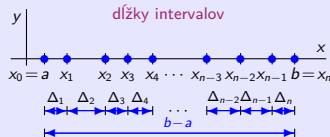
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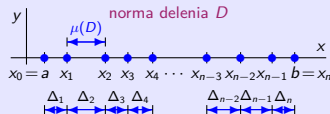
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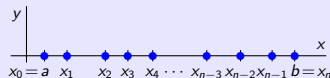
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Ak $D^*, D^{**} \in \mathfrak{D}_{\langle a; b \rangle}$, $D^* \subset D^{**}$, potom delenie D^{**} sa nazýva **zjemnenie delenia D^*** .

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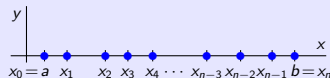
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Každé delenie $D \in \mathfrak{D}_{\langle a; b \rangle}$ má nekonečne veľa zjemnení.

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kde $a = x_0 < x_1 < x_2 < \dots < x_{n-1} < x_n = b$ sú **deliace body**. ↗ jednoznačne určujú delenie

Množinu všetkých delení $\langle a; b \rangle$ označujeme $\mathfrak{D}_{\langle a; b \rangle} = \{D; D \text{ je delenie } \langle a; b \rangle\}$.

Deliace intervaly: $d_i = \langle x_{i-1}; x_i \rangle$, $i = 1, 2, \dots, n$,

dĺžky intervalov: $\Delta x_i = x_i - x_{i-1}$, $i = 1, 2, \dots, n$,

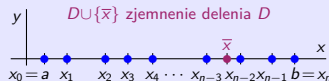
pričom
$$\sum_{i=1}^n \Delta x_i = \sum_{i=1}^n (x_i - x_{i-1}) = x_n - x_0 = b - a.$$

Norma delenia D : $\mu(D) = \max \{\Delta x_i, i = 1, 2, \dots, n\}$ (dĺžka najdlhšieho z intervalov d_i).

Ak $D^*, D^{**} \in \mathfrak{D}_{\langle a; b \rangle}$, $D^* \subset D^{**}$, potom delenie D^{**} sa nazýva **zjemnenie delenia D^*** .

Každé delenie $D \in \mathfrak{D}_{\langle a; b \rangle}$ má nekonečne veľa zjemnení.

Stačí zvoliť ľubovoľný bod $\bar{x} \in \langle a; b \rangle$, $\bar{x} \notin D$ a delenie $D \cup \{\bar{x}\}$ je zjemnením delenia D .



Integrálne súčty

$\langle a; b \rangle$ je nedegenerovaný interval, t. j. $a < b$.

Delením intervalu $\langle a; b \rangle$ nazývame každú konečnú množinu bodov

$$D = \{x_0, x_1, x_2, \dots, x_{n-1}, x_n\} = \{x_i\}_{i=0}^n, \quad n \in \mathbb{N},$$

↖ jednoznačne určujú delenie

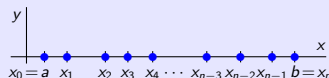
kde $a = x_0 < x_1 < x_2 < \dots < x_{n-1} < x_n = b$ sú **deliace body**.

Množinu všetkých delení $\langle a; b \rangle$ označujeme $\mathfrak{D}_{\langle a; b \rangle} = \{D; D \text{ je delenie } \langle a; b \rangle\}$.

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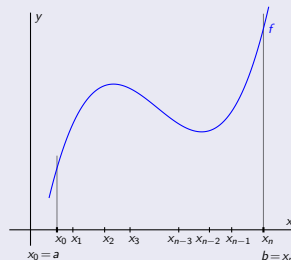
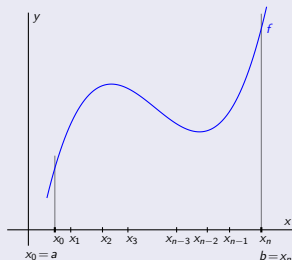
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Ak $D^*, D^{**} \in \mathfrak{D}_{\langle a; b \rangle}$, potom $D = D^* \cup D^{**} \in \mathfrak{D}_{\langle a; b \rangle}$ sa nazýva **spoločné zjemnenie D^*, D^{**}** .

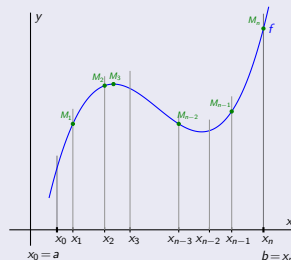
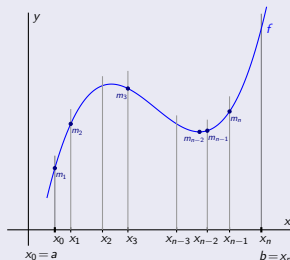
Integrálne súčty

$y = f(x)$, $x \in \langle a; b \rangle$ je ohraničená funkcia, $D \in \mathfrak{D}_{\langle a; b \rangle}$, $D = \{x_i\}_{i=0}^n$, $n \in \mathbb{N}$,



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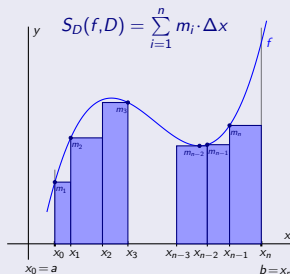
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Dolným Riemannovým (integrálnym) súčtom $S_D(f, D)$ funkcie f pri delení D

nazývame

$$S_D(f, D) = \sum_{i=1}^n m_i \cdot \Delta x_i$$



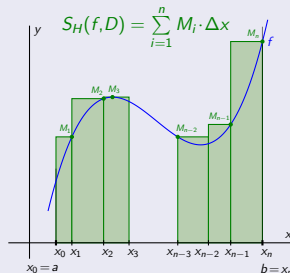
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horným Riemannovým (integrálnym) súčtom $S_H(f, D)$ funkcie f pri delení D

nazývame

$$S_H(f, D) = \sum_{i=1}^n M_i \cdot \Delta x_i$$

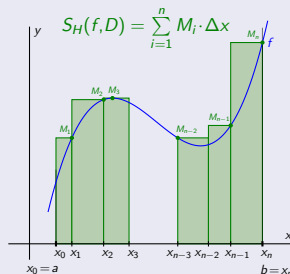
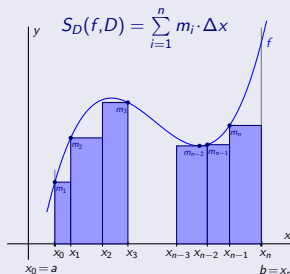


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nazývame čísla: $S_D(f, D) = \sum_{i=1}^n m_i \cdot \Delta x_i$, $S_H(f, D) = \sum_{i=1}^n M_i \cdot \Delta x_i$.



Integrálne súčty

$y = f(x)$, $x \in \langle a; b \rangle$ je ohraničená

Integrálne súčty

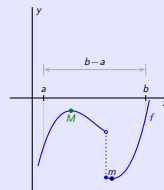
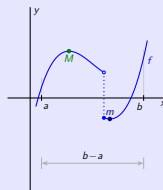
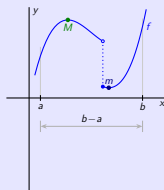
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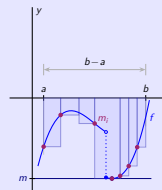
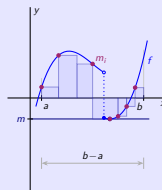
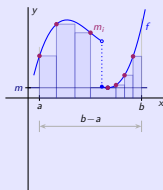


Označme $m = \inf \{f(x), x \in \langle a; b \rangle\}$, $M = \sup \{f(x), x \in \langle a; b \rangle\}$.

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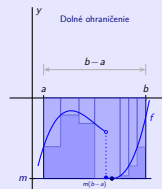
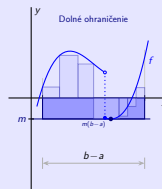
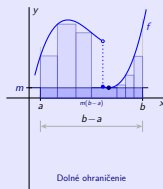
Pre každé $D = \{x_i\}_{i=0}^n \in \mathfrak{D}_{\langle a; b \rangle}$, $n \in \mathbb{N}$ platí $m \leq m_i$

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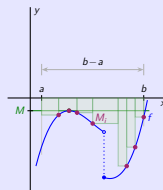
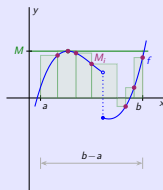
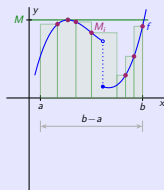
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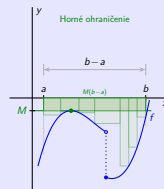
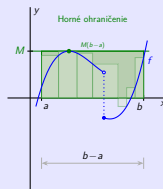
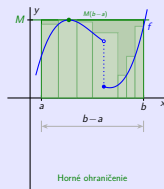
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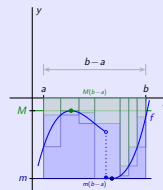
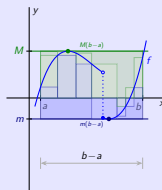
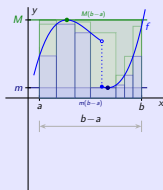
$M_i \leq M$ pre všetky $i = 1, 2, \dots, n$.

$$S_H(f,D) = \sum_{i=1}^n M_i \Delta x_i \leq \sum_{i=1}^n M \Delta x_i = M \sum_{i=1}^n \Delta x_i = M(b-a).$$

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Pre každé $D = \{x_i\}_{i=0}^n \in \mathfrak{D}_{\langle a;b \rangle}$, $n \in \mathbb{N}$ platí $m \leq m_i \leq M_i \leq M$ pre všetky $i = 1, 2, \dots, n$.

$$\begin{aligned} m(b-a) &= m \sum_{i=1}^n \Delta x_i = \sum_{i=1}^n m \Delta x_i \leq \sum_{i=1}^n m_i \Delta x_i = S_D(f,D) \leq \\ &\leq S_H(f,D) = \sum_{i=1}^n M_i \Delta x_i \leq \sum_{i=1}^n M \Delta x_i = M \sum_{i=1}^n \Delta x_i = M(b-a). \end{aligned}$$

Integrálne súčty

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a matematickou indukciou rozšíriť na počet pridaných bodov.

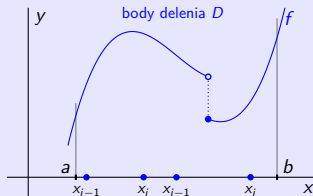
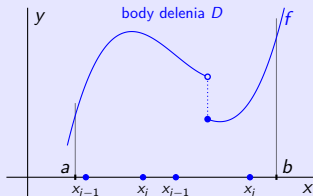
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Integrálne súčty

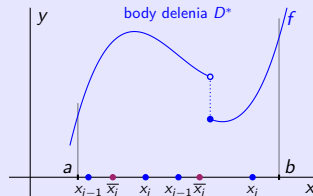
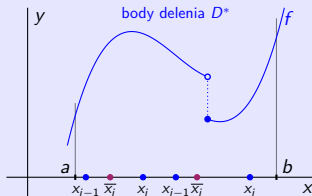
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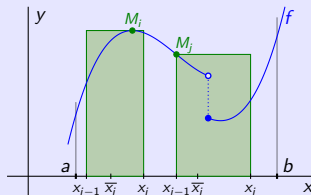
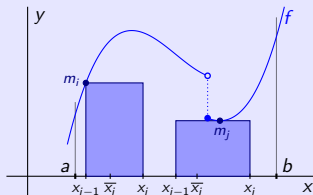


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 a matematickou indukciou rozšíriť na počet pridaných bodov.

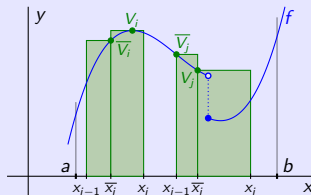
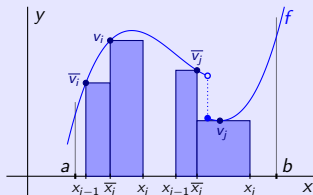


Integrálne súčty

$y = f(x)$, $x \in \langle a; b \rangle$ je ohraničená, $D, D^* \in \mathfrak{D}_{\langle a; b \rangle}$, $D \subset D^*$ (D^* je zjemnením delenia D)

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D^* vznikne z $D = \{x_i\}_{i=0}^n$, $n \in \mathbb{N}$ pridaním najviac spočítateľného počtu bodov,
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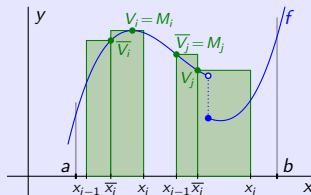
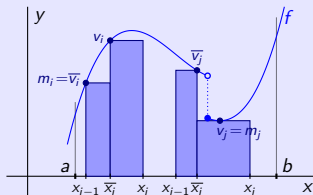


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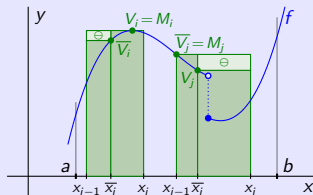
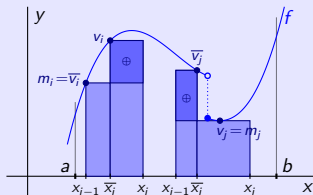


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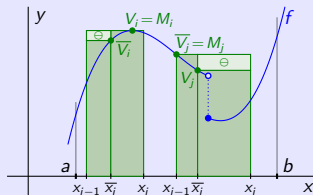
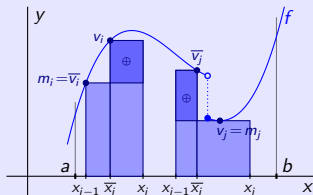
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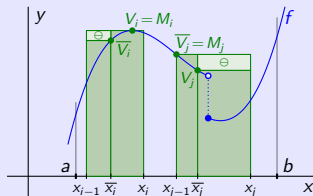
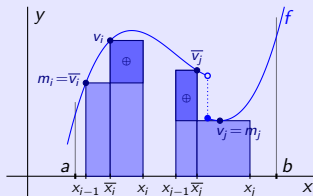


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Dirichletova funkcia $\chi(x) = 1$ pre $x \in \langle 0; 1 \rangle$, $x \in \mathbb{Q}$, $\chi(x) = 0$ pre $x \in \langle 0; 1 \rangle$, $x \notin \mathbb{Q}$.

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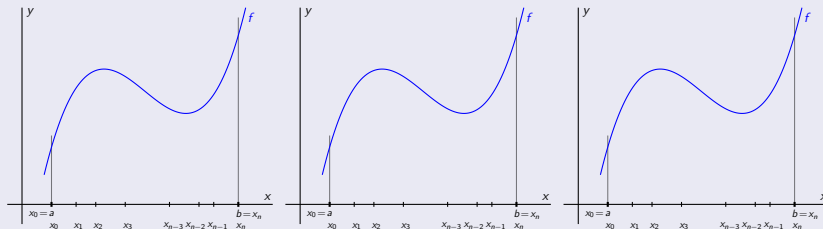
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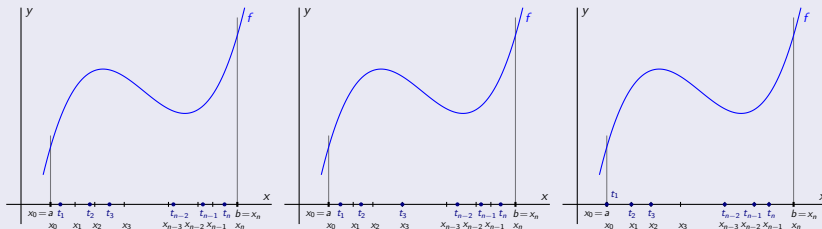
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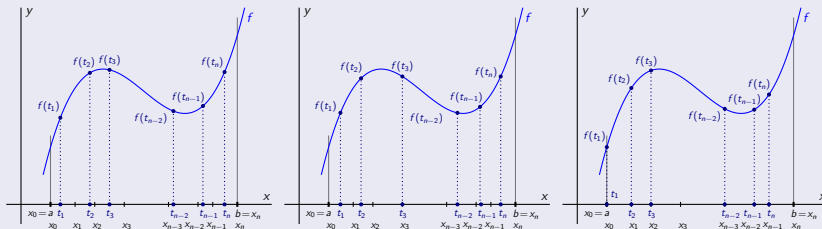
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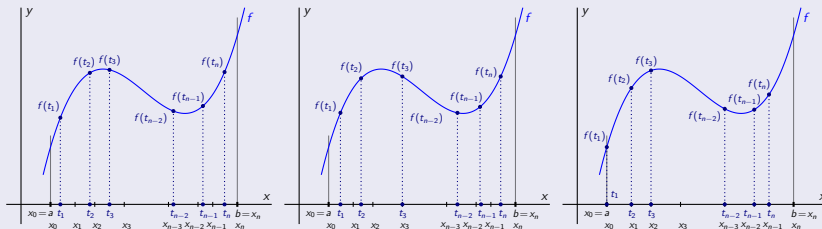
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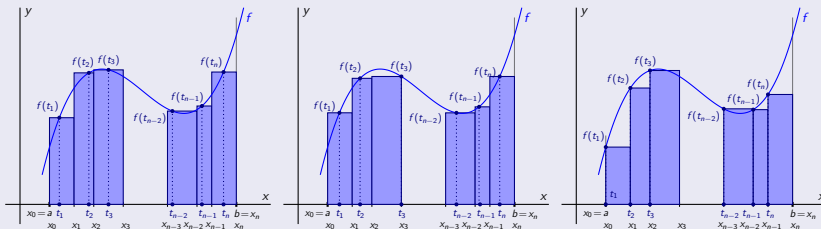
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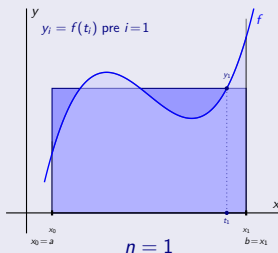
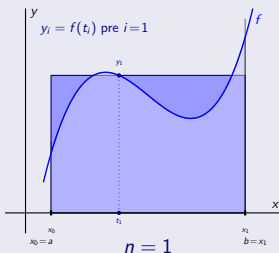
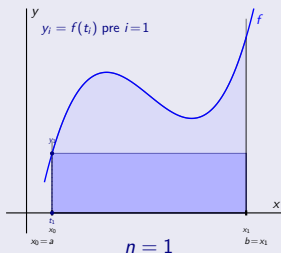
nazývame číslo $S_T(f, D) = \sum_{i=1}^n f(t_i) \cdot \Delta x_i$.



Riemannov určitý integrál

$y = f(x)$, $x \in \langle a; b \rangle$ je ohraničená, $D \in \mathfrak{D}_{\langle a; b \rangle}$, $D = \{x_i\}_{i=0}^n$, $n \in \mathbb{N}$,

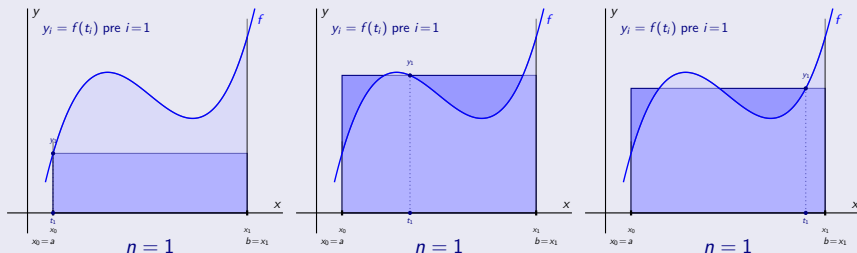
$T = \{t_1, t_2, \dots, t_n\}$ pričom $t_i \in \langle x_{i-1}; x_i \rangle$ a existuje $f(t_i)$ pre $i = 1, 2, \dots, n$.



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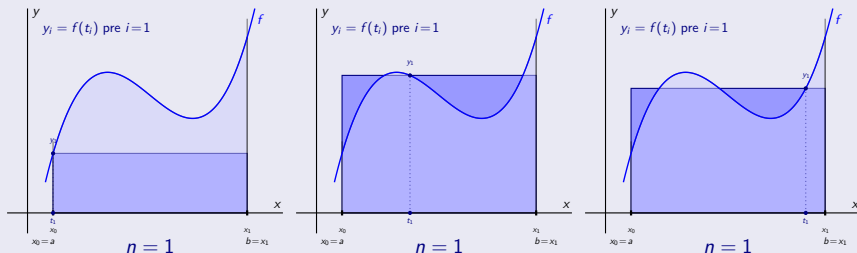


Pre integrálny súčet $S_T(f, D)$ pri ľubovoľnej voľbe bodov $T = \{t_1, t_2, \dots, t_n\}$

Riemannov určitý integrál

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Pre integrálny súčet $S_T(f, D)$ pri ľubovoľnej voľbe bodov $T = \{t_1, t_2, \dots, t_n\}$

platí $m(b-a) \leq S_D(f, D) \leq S_T(f, D) \leq S_H(f, D) \leq M(b-a)$,

pričom $m = \inf \{f(x); x \in \langle a; b \rangle\}$, $M = \sup \{f(x); x \in \langle a; b \rangle\}$.

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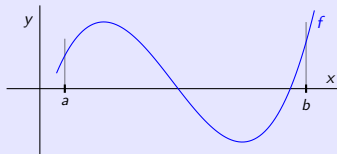
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Geometricky predstavuje $\int_a^b f(x) dx$

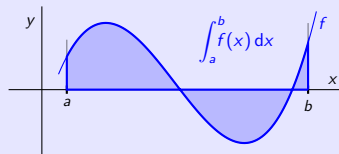


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plochu krivočiareho lichobežníka
určeného funkciou f a intervalom $\langle a; b \rangle$.



Riemannov určitý integrál

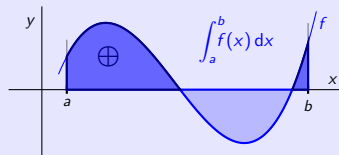
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Pre $f(x) \geq 0$ je táto plocha kladná.



Riemannov určitý integrál

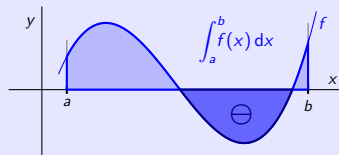
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Pre $f(x) \leq 0$ je táto plocha záporná.

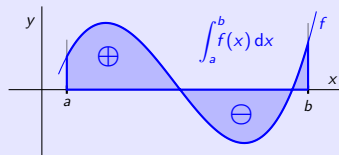


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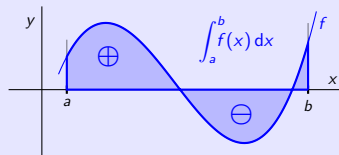
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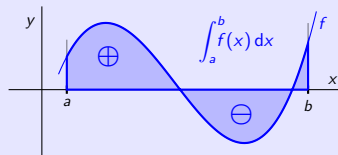
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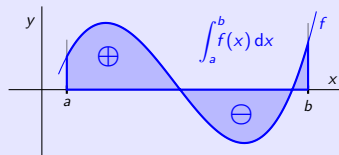
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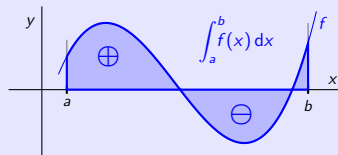
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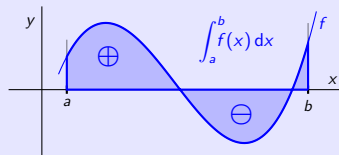
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$\Rightarrow S_D(f, D)$, $S_H(f, D)$ sú Riemannovými integrálnymi súčtami
pre konkrétne voľby bodov $T_D = \{u_1, u_2, \dots, u_n\}$, $T_H = \{v_1, v_2, \dots, v_n\}$.

Riemannov určitý integrál

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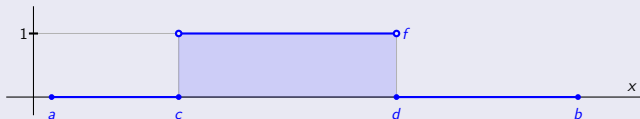
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Príklady

$$\int_a^b f(x) dx$$

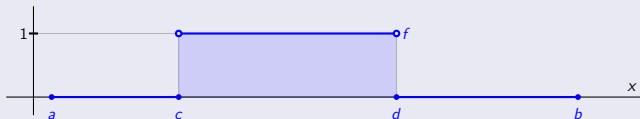
$$, \text{ pričom } f(x) = \begin{cases} 1 & \text{pre } x \in (c; d), \\ 0 & \text{pre } x \in \langle a; c \rangle \cup \langle d; b \rangle, \end{cases} \quad \begin{matrix} a < b < c < d, \\ a, b, c, d \in \mathbb{R}. \end{matrix}$$



Príklady

$$\int_a^b f(x) dx, \text{ pričom } f(x) = \begin{cases} 1 & \text{pre } x \in (c; d), \\ 0 & \text{pre } x \in \langle a; c \rangle \cup \langle d; b \rangle, \end{cases} \quad \begin{matrix} a < b < c < d, \\ a, b, c, d \in \mathbb{R}. \end{matrix}$$

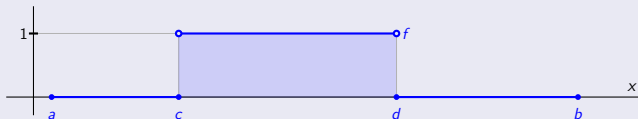
f je po častiach spojitá na $\langle a; b \rangle$



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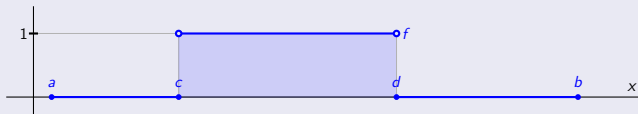
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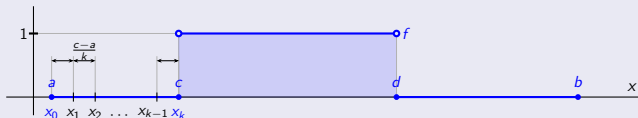


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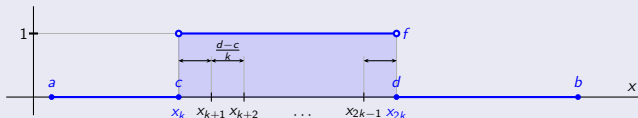


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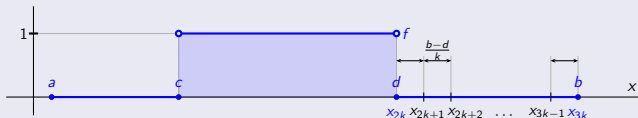


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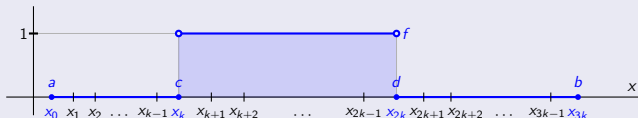
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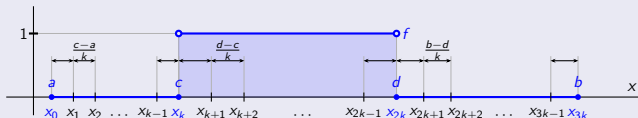
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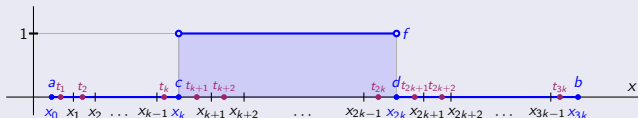
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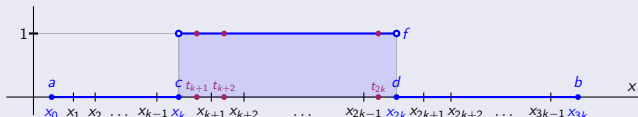
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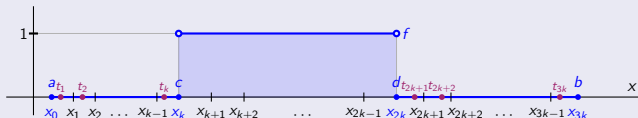
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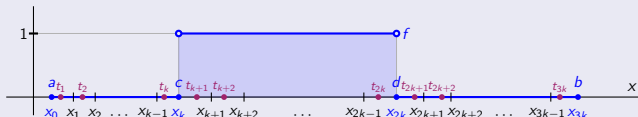
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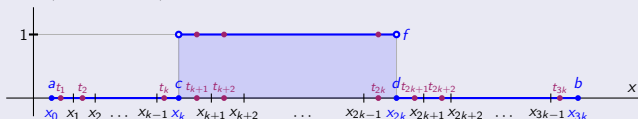
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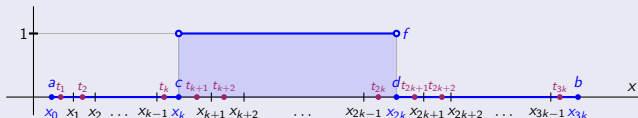
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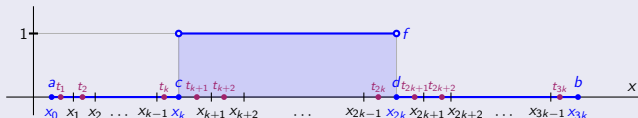
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$$\Rightarrow S_T(f, D_k) = \sum_{i=1}^k 0 \cdot \frac{c-a}{k} + \sum_{i=k+1}^{2k} 1 \cdot \frac{d-c}{k} + \sum_{i=2k+1}^{3k} 0 \cdot \frac{b-d}{k} = \sum_{i=k+1}^{2k} \frac{d-c}{k} = k \cdot \frac{d-c}{k}$$



Príklady

$$\int_a^b f(x) dx, \text{ pričom } f(x) = \begin{cases} 1 & \text{pre } x \in (c; d), \\ 0 & \text{pre } x \in \langle a; c \rangle \cup \langle d; b \rangle, \end{cases} \quad a < b < c < d, \quad a, b, c, d \in \mathbb{R}.$$

f je po častiach spojitá na $\langle a; b \rangle \Rightarrow f \in R_{\langle a; b \rangle}$. Pre $k \in \mathbb{N}$ zvolíme $D_k = \{x_i\}_{i=0}^{3k} \in \mathcal{D}_{\langle a; b \rangle}$ nasledovne:

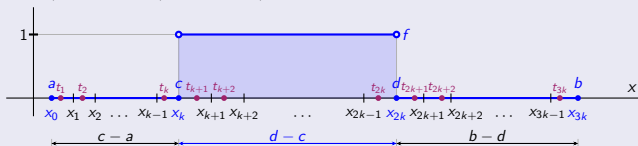
- Rozdelíme $\langle a; c \rangle$ bodmi $a = x_0, x_1, \dots, x_k = c$ na k intervalov rovnakej dĺžky $\Delta x_1 = \frac{c-a}{k}$.
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- Rozdelíme $\langle d; b \rangle$ bodmi $d = x_{2k}, x_{2k+1}, \dots, x_{3k} = b$ na k intervalov rovnakej dĺžky $\Delta x_3 = \frac{b-d}{k}$.

$\{D_k\}_{k=1}^{\infty}$ je normálna, pretože $\mu(D_k) = \max \{\Delta x_1, \Delta x_2, \Delta x_3\} = \max \left\{ \frac{c-a}{k}, \frac{d-c}{k}, \frac{b-d}{k} \right\} \rightarrow 0$ pre $k \rightarrow \infty$.

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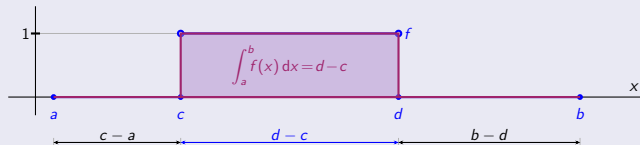
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$$S_D(f, D_k) = \sum_{i=1}^k m_i \cdot \Delta x_i = \sum_{i=1}^k \frac{i-1}{2k \cdot k} = \frac{0+1+\dots+(k-1)}{2k^2} = \frac{(k-1)k}{2 \cdot 2k^2} = \frac{k-1}{4k} \rightarrow \frac{1}{4},$$

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Príklady

$$\int_0^1 \frac{x dx}{2} = \frac{1}{4}.$$

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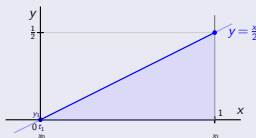
$$\int_0^1 \frac{x \, dx}{2} = \lim_{k \rightarrow \infty} S_D(f, D) = \lim_{k \rightarrow \infty} \frac{k-1}{4k} = \frac{1}{4}$$

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$$S_D(f, D) = \sum_{i=1}^n m_i \Delta x_i = \sum_{i=1}^1 \frac{i-1}{2 \cdot 1^2} = 0$$

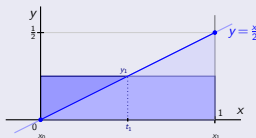
$$\int_0^1 \frac{x \, dx}{2} = \lim_{k \rightarrow \infty} S_T(f, D) = \lim_{k \rightarrow \infty} \frac{1}{4} = \frac{1}{4}$$

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$$S_T(f, D) = \sum_{i=1}^n f(t_i) \Delta x_i = \sum_{i=1}^1 \frac{2i-1}{4 \cdot 1^2} = 0.25$$

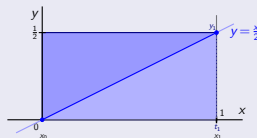
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Príklady

$$\int_0^1 x^2 dx$$

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$$\int_0^1 x^2 dx = \begin{cases} \lim_{k \rightarrow \infty} S_H(f, D_k) \\ \lim_{k \rightarrow \infty} S_D(f, D_k) \end{cases}$$

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$$S_H(f, D_k) = S_{T_2}(f, D_k) = \sum_{i=1}^k M_i \cdot \Delta x_i = \sum_{i=1}^k \frac{i^2}{k^2 \cdot k} = \frac{1^2 + 2^2 + \dots + k^2}{k^3} = \frac{k(k+1)(2k+1)}{6 \cdot k^3} = \frac{(k+1)(2k+1)}{6k^2},$$

$$\begin{aligned} S_D(f, D_k) = S_{T_1}(f, D_k) &= \sum_{i=1}^k m_i \cdot \Delta x_i = \sum_{i=1}^k \frac{(i-1)^2}{k^2 \cdot k} = \frac{0 + 1^2 + \dots + (k-1)^2}{k^3} = \frac{1^2 + \dots + (k-1)^2}{k^3} \\ &= \frac{(k-1)k[2(k-1)+1]}{6 \cdot k^3} = \frac{(k-1)k(2k-1)}{6k^3} = \frac{(k-1)(2k-1)}{6k^2} \text{ pre } k \in \mathbb{N}. \end{aligned}$$

$$\int_0^1 x^2 dx = \begin{cases} \lim_{k \rightarrow \infty} S_H(f, D_k) = \lim_{k \rightarrow \infty} \frac{(k+1)(2k+1)}{6k^2} \\ \lim_{k \rightarrow \infty} S_D(f, D_k) = \lim_{k \rightarrow \infty} \frac{(k-1)(2k-1)}{6k^2} \end{cases}$$

Príklady

$$\int_0^1 x^2 dx$$

$f(x) = x^2$, $x \in \langle 0; 1 \rangle$ je rastúca, spojitá $\Rightarrow f \in R_{\langle 0; 1 \rangle}$.

$D_k = \left\{ \frac{i}{k} \right\}_{i=0}^k \subset \mathfrak{D}_{\langle 0; 1 \rangle}$, $\Delta x_i = \frac{1}{k} \rightarrow 0$ pre $k \rightarrow \infty$, t. j. $\{D_k\}_{k=1}^\infty$ je normálna.

$m_i = f(x_{i-1}) = \frac{(i-1)^2}{k^2}$ pre $i = 1, 2, \dots, k$, t. j. voľba $T_1 = \{t_i = x_{i-1}\}_{i=1}^k$ [ľavé hranice intervalov].

$M_i = f(x_i) = \frac{i^2}{k^2}$ pre $i = 1, 2, \dots, k$, t. j. voľba $T_2 = \{t_i = x_i\}_{i=1}^k$ [pravé hranice intervalov].

$$S_H(f, D_k) = S_{T_2}(f, D_k) = \sum_{i=1}^k M_i \cdot \Delta x_i = \sum_{i=1}^k \frac{i^2}{k^2 \cdot k} = \frac{1^2 + 2^2 + \dots + k^2}{k^3} = \frac{k(k+1)(2k+1)}{6 \cdot k^3} = \frac{(k+1)(2k+1)}{6k^2},$$

$$\begin{aligned} S_D(f, D_k) = S_{T_1}(f, D_k) &= \sum_{i=1}^k m_i \cdot \Delta x_i = \sum_{i=1}^k \frac{(i-1)^2}{k^2 \cdot k} = \frac{0 + 1^2 + \dots + (k-1)^2}{k^3} = \frac{1^2 + \dots + (k-1)^2}{k^3} \\ &= \frac{(k-1)k[2(k-1)+1]}{6 \cdot k^3} = \frac{(k-1)k(2k-1)}{6k^3} = \frac{(k-1)(2k-1)}{6k^2} \text{ pre } k \in \mathbb{N}. \end{aligned}$$

$$\int_0^1 x^2 dx = \begin{cases} \lim_{k \rightarrow \infty} S_H(f, D_k) = \lim_{k \rightarrow \infty} \frac{(k+1)(2k+1)}{6k^2} = \lim_{k \rightarrow \infty} \frac{(1+\frac{1}{k})(2+\frac{1}{k})}{6} \\ \lim_{k \rightarrow \infty} S_D(f, D_k) = \lim_{k \rightarrow \infty} \frac{(k-1)(2k-1)}{6k^2} = \lim_{k \rightarrow \infty} \frac{(1-\frac{1}{k})(2-\frac{1}{k})}{6} \end{cases}$$

Príklady

$$\int_0^1 x^2 dx$$

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$$\int_0^1 x^2 dx = \begin{cases} \lim_{k \rightarrow \infty} S_H(f, D_k) = \lim_{k \rightarrow \infty} \frac{(k+1)(2k+1)}{6k^2} = \lim_{k \rightarrow \infty} \frac{(1+\frac{1}{k})(2+\frac{1}{k})}{6} = \frac{(1+0)(2+0)}{6} \\ \lim_{k \rightarrow \infty} S_D(f, D_k) = \lim_{k \rightarrow \infty} \frac{(k-1)(2k-1)}{6k^2} = \lim_{k \rightarrow \infty} \frac{(1-\frac{1}{k})(2-\frac{1}{k})}{6} = \frac{(1-0)(2-0)}{6} \end{cases}$$

Príklady

$$\int_0^1 x^2 dx = \frac{1}{3}.$$

$f(x) = x^2$, $x \in \langle 0; 1 \rangle$ je rastúca, spojitá $\Rightarrow f \in R_{\langle 0; 1 \rangle}$.

$D_k = \left\{ \frac{i}{k} \right\}_{i=0}^k \subset \mathfrak{D}_{\langle 0; 1 \rangle}$, $\Delta x_i = \frac{1}{k} \rightarrow 0$ pre $k \rightarrow \infty$, t. j. $\{D_k\}_{k=1}^\infty$ je normálna.

$m_i = f(x_{i-1}) = \frac{(i-1)^2}{k^2}$ pre $i = 1, 2, \dots, k$, t. j. voľba $T_1 = \{t_i = x_{i-1}\}_{i=1}^k$ [ľavé hranice intervalov].

$M_i = f(x_i) = \frac{i^2}{k^2}$ pre $i = 1, 2, \dots, k$, t. j. voľba $T_2 = \{t_i = x_i\}_{i=1}^k$ [pravé hranice intervalov].

$$S_H(f, D_k) = S_{T_2}(f, D_k) = \sum_{i=1}^k M_i \cdot \Delta x_i = \sum_{i=1}^k \frac{i^2}{k^2 \cdot k} = \frac{1^2 + 2^2 + \dots + k^2}{k^3} = \frac{k(k+1)(2k+1)}{6 \cdot k^3} = \frac{(k+1)(2k+1)}{6k^2},$$

$$\begin{aligned} S_D(f, D_k) = S_{T_1}(f, D_k) &= \sum_{i=1}^k m_i \cdot \Delta x_i = \sum_{i=1}^k \frac{(i-1)^2}{k^2 \cdot k} = \frac{0 + 1^2 + \dots + (k-1)^2}{k^3} = \frac{1^2 + \dots + (k-1)^2}{k^3} \\ &= \frac{(k-1)k[2(k-1)+1]}{6 \cdot k^3} = \frac{(k-1)k(2k-1)}{6k^3} = \frac{(k-1)(2k-1)}{6k^2} \quad \text{pre } k \in \mathbb{N}. \end{aligned}$$

$$\int_0^1 x^2 dx = \begin{cases} \lim_{k \rightarrow \infty} S_H(f, D_k) = \lim_{k \rightarrow \infty} \frac{(k+1)(2k+1)}{6k^2} = \lim_{k \rightarrow \infty} \frac{(1+\frac{1}{k})(2+\frac{1}{k})}{6} = \frac{(1+0)(2+0)}{6} = \frac{1}{3}, \\ \lim_{k \rightarrow \infty} S_D(f, D_k) = \lim_{k \rightarrow \infty} \frac{(k-1)(2k-1)}{6k^2} = \lim_{k \rightarrow \infty} \frac{(1-\frac{1}{k})(2-\frac{1}{k})}{6} = \frac{(1-0)(2-0)}{6} = \frac{1}{3}. \end{cases}$$

Príklady

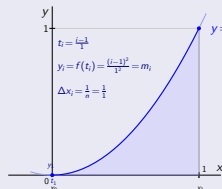
$$\int_0^1 x^2 dx = \frac{1}{3}.$$

$f(x) = x^2$, $x \in \langle 0; 1 \rangle$ je rastúca, spojitá $\Rightarrow f \in R_{\langle 0; 1 \rangle}$.

$D_k = \left\{ \frac{i}{k} \right\}_{i=0}^k \subset \mathfrak{D}_{\langle 0; 1 \rangle}$, $\Delta x_i = \frac{1}{k} \rightarrow 0$ pre $k \rightarrow \infty$, t. j. $\{D_k\}_{k=1}^\infty$ je normálna.

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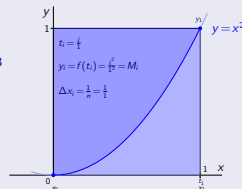


$$\int_0^1 x^2 dx = \frac{1}{3} \approx 0.333333$$

$$n = 1$$

$$i = 1$$

$$S_{T_1}(f, D) = \sum_{i=1}^1 f(t_i) \Delta x_i = S_D(f, D) = \sum_{i=1}^1 m_i \Delta x_i = \sum_{i=1}^1 \frac{(i-1)^2}{1^2} = 0$$



$$S_{T_2}(f, D) = \sum_{i=1}^1 f(t_i) \Delta x_i = S_H(f, D) = \sum_{i=1}^1 M_i \Delta x_i = \sum_{i=1}^1 \frac{1^2}{1^2} = 1$$



Vlastnosti Riemannovho integrálu

$$f, g \in R_{\langle a; b \rangle}, c \in R.$$

Vlastnosti Riemannovho integrálu

$$f, g \in R_{\langle a; b \rangle}, c \in \mathbb{R}.$$

$$\Rightarrow cf, f+g \in R_{\langle a; b \rangle}$$

Vlastnosti Riemannovho integrálu

$f, g \in R_{\langle a; b \rangle}, c \in \mathbb{R}.$

$\Rightarrow cf, f+g \in R_{\langle a; b \rangle}$ a platí

$$\int_a^b cf(x) dx = c \int_a^b f(x) dx,$$

Vlastnosti Riemannovho integrálu

$f, g \in R_{\langle a; b \rangle}, c \in \mathbb{R}.$

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$$\int_a^b cf(x) dx = c \int_a^b f(x) dx, \quad \int_a^b [f(x) + g(x)] dx = \int_a^b f(x) dx + \int_a^b g(x) dx.$$

Vlastnosti Riemannovho integrálu

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$f \in R_{\langle a; b \rangle}, \varphi: \langle m; M \rangle \rightarrow R$ je spojitá,

pričom $m = \inf \{f(x); x \in \langle a; b \rangle\}, M = \sup \{f(x); x \in \langle a; b \rangle\}.$

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$\Rightarrow$ (zložená funkcia) $\varphi(f) \in R_{\langle a; b \rangle}.$

Vlastnosti Riemannovho integrálu

$$f, g \in R_{\langle a; b \rangle}, c \in R.$$

$$\Rightarrow cf, f+g \in R_{\langle a; b \rangle} \text{ a platí}$$

$$\int_a^b cf(x) dx = c \int_a^b f(x) dx, \int_a^b [f(x) + g(x)] dx = \int_a^b f(x) dx + \int_a^b g(x) dx.$$

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$$\Rightarrow \text{(zložená funkcia)} \varphi(f) \in R_{\langle a; b \rangle}.$$

$$f, g \in R_{\langle a; b \rangle}.$$

Vlastnosti Riemannovho integrálu

$$f, g \in R_{\langle a; b \rangle}, c \in R.$$

$$\Rightarrow cf, f+g \in R_{\langle a; b \rangle} \text{ a platí}$$

$$\int_a^b cf(x) dx = c \int_a^b f(x) dx, \int_a^b [f(x) + g(x)] dx = \int_a^b f(x) dx + \int_a^b g(x) dx.$$

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$$f, g \in R_{\langle a; b \rangle}.$$

$$\Rightarrow |f|, f^2, fg \in R_{\langle a; b \rangle}.$$

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$$\int_a^b cf(x) dx = c \int_a^b f(x) dx, \int_a^b [f(x) + g(x)] dx = \int_a^b f(x) dx + \int_a^b g(x) dx.$$

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$$f, g \in R_{\langle a; b \rangle}, \text{ pričom } \inf \{g(x); x \in \langle a; b \rangle\} > 0, \text{ resp. } \sup \{g(x); x \in \langle a; b \rangle\} < 0.$$

Vlastnosti Riemannovho integrálu

$$f, g \in R_{\langle a; b \rangle}, c \in R.$$

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$$\int_a^b cf(x) dx = c \int_a^b f(x) dx, \int_a^b [f(x) + g(x)] dx = \int_a^b f(x) dx + \int_a^b g(x) dx.$$

$$f \in R_{\langle a; b \rangle}, \varphi: \langle m; M \rangle \rightarrow R \text{ je spojitá,}$$

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$$\Rightarrow \frac{1}{g}, \frac{f}{g} \in R_{\langle a; b \rangle}.$$

Vlastnosti Riemannovho integrálu

$$f \in R_{\langle a; b \rangle}, f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle.$$

Vlastnosti Riemannovho integrálu

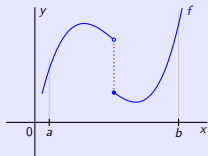
$f \in R_{\langle a; b \rangle}, f(x) \geq 0$ pre všetky $x \in \langle a; b \rangle$.

$$\Rightarrow \int_a^b f(x) dx \geq 0.$$

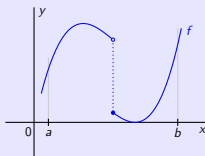
Vlastnosti Riemannovho integrálu

$$f \in R_{\langle a; b \rangle}, f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle.$$

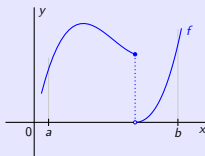
$$\Rightarrow \int_a^b f(x) dx \geq 0.$$



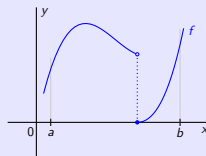
$$f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



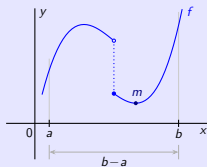
$$f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle$$

$$f(x) \geq 0, x \in \langle a; b \rangle$$

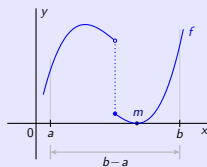
Vlastnosti Riemannovho integrálu

$$f \in R_{\langle a; b \rangle}, f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle.$$

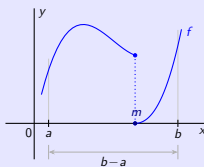
$$\Rightarrow \int_a^b f(x) dx \geq 0.$$



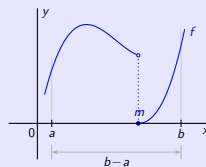
$$f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle$$



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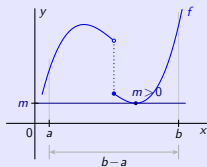
$$f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle$$

$$f(x) \geq 0, x \in \langle a; b \rangle \Rightarrow m = \inf \{f(x); x \in \langle a; b \rangle\} \geq 0$$

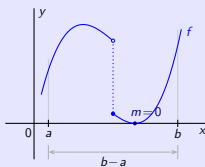
Vlastnosti Riemannovho integrálu

$$f \in R_{\langle a; b \rangle}, f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle.$$

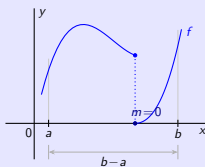
$$\Rightarrow \int_a^b f(x) dx \geq 0.$$



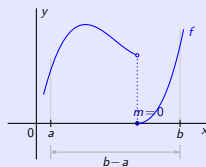
$$f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



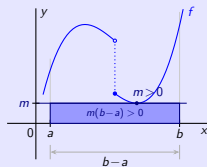
$$f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle$$

$$f(x) \geq 0, x \in \langle a; b \rangle \Rightarrow m = \inf \{f(x); x \in \langle a; b \rangle\} \geq 0$$

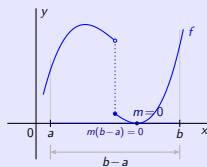
Vlastnosti Riemannovho integrálu

$$f \in R_{\langle a; b \rangle}, f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle.$$

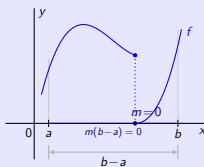
$$\Rightarrow \int_a^b f(x) dx \geq 0.$$



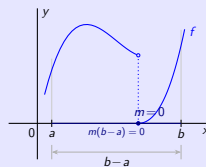
$$f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle$$

$$f(x) \geq 0, x \in \langle a; b \rangle \Rightarrow m = \inf \{f(x); x \in \langle a; b \rangle\} \geq 0$$

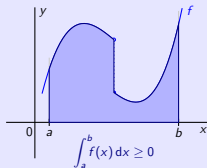


$$m(b-a) \geq 0.$$

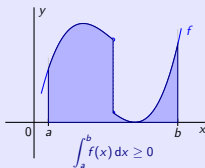
Vlastnosti Riemannovho integrálu

$$f \in R_{\langle a; b \rangle}, f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle.$$

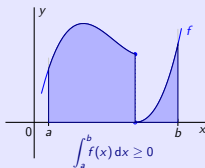
$$\Rightarrow \int_a^b f(x) dx \geq 0.$$



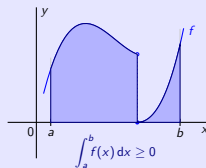
$$f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$f(x) \geq 0 \text{ pre všetky } x \in \langle a; b \rangle$$

$$f(x) \geq 0, x \in \langle a; b \rangle \Rightarrow m = \inf \{f(x); x \in \langle a; b \rangle\} \geq 0$$

$$\Rightarrow \int_a^b f(x) dx \geq m(b-a) \geq 0.$$

Vlastnosti Riemannovho integrálu

$f, g \in R_{\langle a; b \rangle}, g(x) \geq f(x)$ pre všetky $x \in \langle a; b \rangle$.



Vlastnosti Riemannovho integrálu

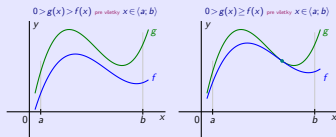
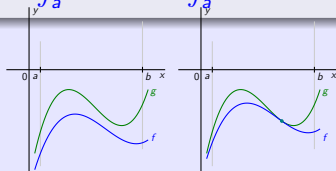
$f, g \in R_{\langle a; b \rangle}, g(x) \geq f(x)$ pre všetky $x \in \langle a; b \rangle$.

$$\Rightarrow \int_a^b g(x) dx \geq \int_a^b f(x) dx.$$

Vlastnosti Riemannovho integrálu

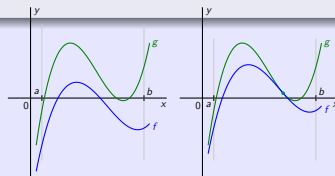
$$f, g \in R_{\langle a; b \rangle}, g(x) \geq f(x) \text{ pre všetky } x \in \langle a; b \rangle.$$

$$\Rightarrow \int_a^b g(x) dx \geq \int_a^b f(x) dx.$$



$$g(x) > f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$

$$g(x) \geq f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



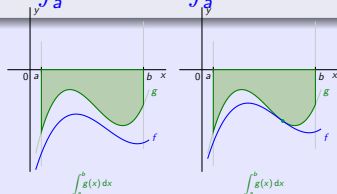
$$g(x) > f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

$$g(x) \geq f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

Vlastnosti Riemannovho integrálu

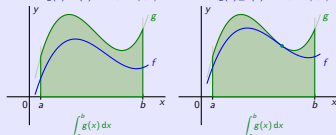
$f, g \in R_{\langle a; b \rangle}$, $g(x) \geq f(x)$ pre všetky $x \in \langle a; b \rangle$.

$$\Rightarrow \int_a^b g(x) dx \geq \int_a^b f(x) dx.$$



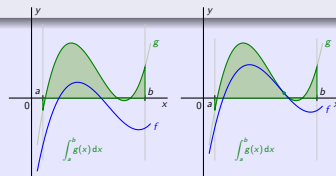
$0 > g(x) > f(x)$ pre všetky $x \in (a; b)$

$0 > g(x) \geq f(x)$ pre všetky $x \in (a; b)$



$g(x) > f(x) > 0$ pre všetky $x \in (a; b)$

$g(x) \geq f(x) > 0$ pre všetky $x \in (a; b)$



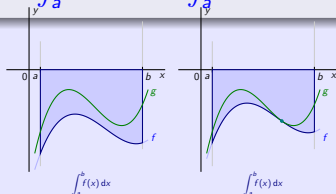
$g(x) > f(x)$ pre všetky $x \in (a; b)$

$g(x) \geq f(x)$ pre všetky $x \in (a; b)$

Vlastnosti Riemannovho integrálu

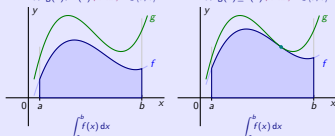
$$f, g \in R_{\langle a; b \rangle}, g(x) \geq f(x) \text{ pre všetky } x \in \langle a; b \rangle.$$

$$\Rightarrow \int_a^b g(x) dx \geq \int_a^b f(x) dx.$$



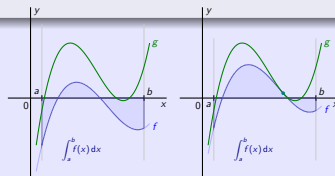
$$0 < g(x) > f(x) \text{ pre všetky } x \in (a; b)$$

$$0 < g(x) \geq f(x) \text{ pre všetky } x \in (a; b)$$



$$g(x) > f(x) > 0 \text{ pre všetky } x \in (a; b)$$

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$$g(x) > f(x) \text{ pre všetky } x \in (a; b)$$

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Vlastnosti Riemannovho integrálu

$f, g \in R_{\langle a; b \rangle}$, $g(x) \geq f(x)$ pre všetky $x \in \langle a; b \rangle$.

$$\Rightarrow \int_a^b g(x) dx \geq \int_a^b f(x) dx.$$



$$0 > \int_a^b g(x) dx > \int_a^b f(x) dx$$

$$0 > g(x) > f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

$$0 > \int_a^b g(x) dx \geq \int_a^b f(x) dx$$

$$0 > g(x) \geq f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

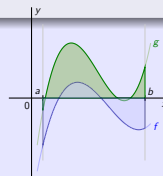


$$\int_a^b g(x) dx > \int_a^b f(x) dx > 0$$

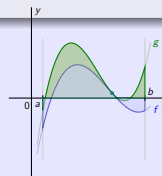
$$g(x) > f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$

$$\int_a^b g(x) dx \geq \int_a^b f(x) dx > 0$$

$$g(x) \geq f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$g(x) > f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

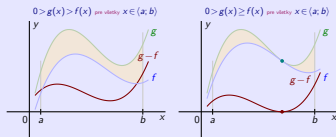
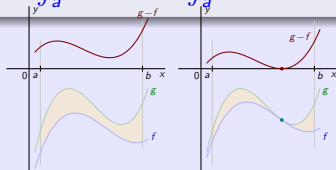


$$g(x) \geq f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

Vlastnosti Riemannovho integrálu

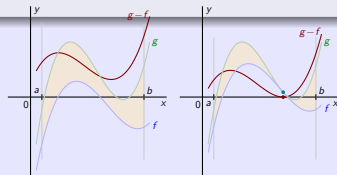
$f, g \in R_{\langle a; b \rangle}$, $g(x) \geq f(x)$ pre všetky $x \in \langle a; b \rangle$.

$$\Rightarrow \int_a^b g(x) dx \geq \int_a^b f(x) dx.$$



$g(x) > f(x) > 0$ pre všetky $x \in \langle a; b \rangle$

$g(x) \geq f(x) > 0$ pre všetky $x \in \langle a; b \rangle$



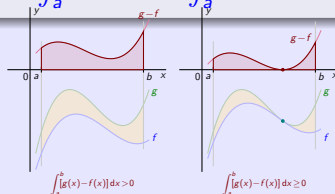
$g(x) > f(x)$ pre všetky $x \in \langle a; b \rangle$

$g(x) \geq f(x)$ pre všetky $x \in \langle a; b \rangle$

Vlastnosti Riemannovho integrálu

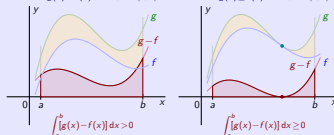
$$f, g \in R_{\langle a, b \rangle}, g(x) \geq f(x) \text{ pre všetky } x \in \langle a, b \rangle.$$

$$\Rightarrow \int_a^b g(x) dx \geq \int_a^b f(x) dx.$$



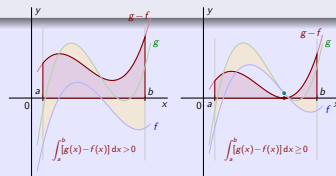
$$0 < g(x) > f(x) \text{ pre všetky } x \in \langle a, b \rangle$$

$$0 < g(x) \geq f(x) \text{ pre všetky } x \in \langle a, b \rangle$$



$$g(x) > f(x) > 0 \text{ pre všetky } x \in \langle a, b \rangle$$

$$g(x) \geq f(x) > 0 \text{ pre všetky } x \in \langle a, b \rangle$$



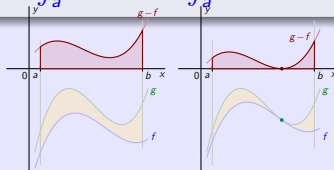
$$g(x) > f(x) \text{ pre všetky } x \in \langle a, b \rangle$$

$$g(x) \geq f(x) \text{ pre všetky } x \in \langle a, b \rangle$$

Vlastnosti Riemannovho integrálu

$f, g \in R_{[a,b]}$, $g(x) \geq f(x)$ pre všetky $x \in [a; b]$.

$$\Rightarrow \int_a^b g(x) dx \geq \int_a^b f(x) dx.$$

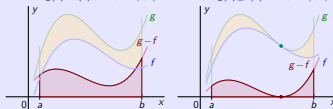


$$0 < \int_a^b g(x) dx - \int_a^b f(x) dx$$

$$0 < g(x) > f(x) \text{ pre všetky } x \in (a; b)$$

$$0 < \int_a^b g(x) dx - \int_a^b f(x) dx$$

$$0 < g(x) \geq f(x) \text{ pre všetky } x \in (a; b)$$

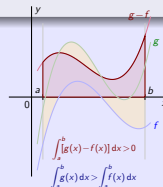


$$\int_a^b g(x) dx > \int_a^b f(x) dx > 0$$

$$g(x) > f(x) > 0 \text{ pre všetky } x \in (a; b)$$

$$\int_a^b g(x) dx \geq \int_a^b f(x) dx > 0$$

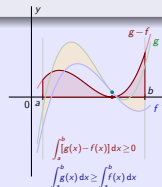
$$g(x) \geq f(x) > 0 \text{ pre všetky } x \in (a; b)$$



$$\int_a^b [g(x) - f(x)] dx > 0$$

$$\int_a^b g(x) dx > \int_a^b f(x) dx$$

$$g(x) > f(x) \text{ pre všetky } x \in (a; b)$$



$$\int_a^b [g(x) - f(x)] dx \geq 0$$

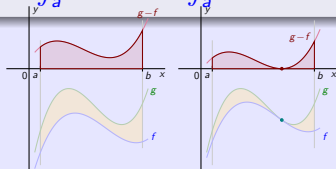
$$\int_a^b g(x) dx \geq \int_a^b f(x) dx$$

$$g(x) \geq f(x) \text{ pre všetky } x \in (a; b)$$

Vlastnosti Riemannovho integrálu

$$f, g \in R_{\langle a; b \rangle}, g(x) \geq f(x) \text{ pre všetky } x \in \langle a; b \rangle.$$

$$\Rightarrow \int_a^b g(x) dx \geq \int_a^b f(x) dx.$$

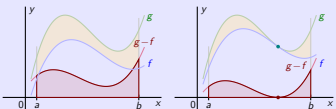


$$0 < \int_a^b g(x) dx > \int_a^b f(x) dx$$

$$0 < g(x) > f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

$$0 < \int_a^b g(x) dx \geq \int_a^b f(x) dx$$

$$0 < g(x) \geq f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

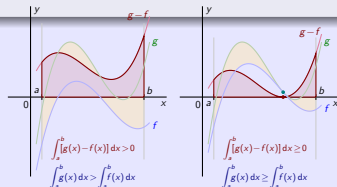


$$\int_a^b g(x) dx > \int_a^b f(x) dx > 0$$

$$g(x) > f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$

$$\int_a^b g(x) dx \geq \int_a^b f(x) dx > 0$$

$$g(x) \geq f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$\int_a^b [g(x) - f(x)] dx > 0$$

$$g(x) > f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

$$\int_a^b [g(x) - f(x)] dx \geq 0$$

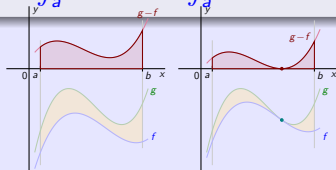
$$g(x) \geq f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

$$f \in R_{\langle a; b \rangle}.$$

Vlastnosti Riemannovho integrálu

$$f, g \in R_{\langle a; b \rangle}, g(x) \geq f(x) \text{ pre všetky } x \in \langle a; b \rangle.$$

$$\Rightarrow \int_a^b g(x) dx \geq \int_a^b f(x) dx.$$

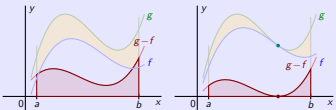


$$0 < \int_a^b g(x) dx > \int_a^b f(x) dx$$

$$0 < g(x) > f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

$$0 < \int_a^b g(x) dx \geq \int_a^b f(x) dx$$

$$0 < g(x) \geq f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

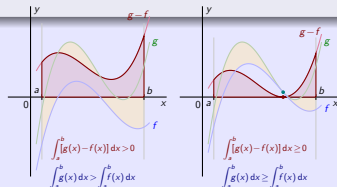


$$\int_a^b g(x) dx > \int_a^b f(x) dx > 0$$

$$g(x) > f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$

$$\int_a^b g(x) dx \geq \int_a^b f(x) dx > 0$$

$$g(x) \geq f(x) > 0 \text{ pre všetky } x \in \langle a; b \rangle$$



$$\int_a^b [g(x) - f(x)] dx > 0$$

$$g(x) > f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

$$\int_a^b [g(x) - f(x)] dx \geq 0$$

$$g(x) \geq f(x) \text{ pre všetky } x \in \langle a; b \rangle$$

$$f \in R_{\langle a; b \rangle}.$$

$$\Rightarrow \int_a^b |f(x)| dx \geq \left| \int_a^b f(x) dx \right|.$$

Vlastnosti Riemannovho integrálu

$y = f(x)$, $y = g(x)$, $x \in \langle a; b \rangle$ sú ohraničené,

$f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

Vlastnosti Riemannovho integrálu

$y = f(x)$, $y = g(x)$, $x \in \langle a; b \rangle$ sú ohraničené,

$f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

$$\Rightarrow \int_a^b f(x) dx = \int_a^b g(x) dx, \quad \overline{\int_a^b f(x) dx} = \overline{\int_a^b g(x) dx}.$$

Vlastnosti Riemannovho integrálu

$y = f(x)$, $y = g(x)$, $x \in \langle a; b \rangle$ sú ohraničené,
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$f \in R_{\langle a; b \rangle}$, $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

Vlastnosti Riemannovho integrálu

$y = f(x)$, $y = g(x)$, $x \in \langle a; b \rangle$ sú ohraničené,
 $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

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$f \in R_{\langle a; b \rangle}$, $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

$$\Rightarrow g \in R_{\langle a; b \rangle}$$

Vlastnosti Riemannovho integrálu

$y = f(x)$, $y = g(x)$, $x \in \langle a; b \rangle$ sú ohraničené,
 $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

$$\Rightarrow \int_a^b f(x) dx = \int_a^b g(x) dx, \quad \overline{\int_a^b f(x) dx} = \overline{\int_a^b g(x) dx}.$$

$f \in R_{\langle a; b \rangle}$, $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

$$\Rightarrow g \in R_{\langle a; b \rangle} \text{ a platí } \int_a^b f(x) dx = \int_a^b g(x) dx.$$

Vlastnosti Riemannovho integrálu

$y = f(x)$, $y = g(x)$, $x \in \langle a; b \rangle$ sú ohraničené,
 $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

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$f \in R_{\langle a; b \rangle}$, $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

$$\Rightarrow g \in R_{\langle a; b \rangle} \text{ a platí } \int_a^b f(x) dx = \int_a^b g(x) dx.$$

- Konečný počet bodov nemá vplyv na vlastnosti Riemannovho integrálu.

Vlastnosti Riemannovho integrálu

$y = f(x)$, $y = g(x)$, $x \in \langle a; b \rangle$ sú ohraničené,
 $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

$$\Rightarrow \int_a^b f(x) dx = \int_a^b g(x) dx, \quad \overline{\int_a^b f(x) dx} = \overline{\int_a^b g(x) dx}.$$

$f \in R_{\langle a; b \rangle}$, $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

$$\Rightarrow g \in R_{\langle a; b \rangle} \text{ a platí } \int_a^b f(x) dx = \int_a^b g(x) dx.$$

- Konečný počet bodov nemá vplyv na vlastnosti Riemannovho integrálu.
- Riemannov integrál môžeme definovať pre ohraničenú funkciu, nedefinovanú v konečnom počte bodov

Vlastnosti Riemannovho integrálu

$y = f(x)$, $y = g(x)$, $x \in \langle a; b \rangle$ sú ohraničené,
 $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

$$\Rightarrow \int_a^b f(x) dx = \int_a^b g(x) dx, \quad \overline{\int_a^b f(x) dx} = \overline{\int_a^b g(x) dx}.$$

$f \in R_{\langle a; b \rangle}$, $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

$$\Rightarrow g \in R_{\langle a; b \rangle} \text{ a platí } \int_a^b f(x) dx = \int_a^b g(x) dx.$$

- Konečný počet bodov nemá vplyv na vlastnosti Riemannovho integrálu.
- Riemannov integrál môžeme definovať pre ohraničenú funkciu, nedefinovanú v konečnom počte bodov – vrátane krajných bodov a , b .

Vlastnosti Riemannovho integrálu

$y = f(x)$, $y = g(x)$, $x \in \langle a; b \rangle$ sú ohraničené,
 $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

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$f \in R_{\langle a; b \rangle}$, $f(x) = g(x)$ pre všetky $x \in \langle a; b \rangle$ okrem konečného počtu bodov.

$$\Rightarrow g \in R_{\langle a; b \rangle} \text{ a platí } \int_a^b f(x) dx = \int_a^b g(x) dx.$$

- Konečný počet bodov nemá vplyv na vlastnosti Riemannovho integrálu.
- Riemannov integrál môžeme definovať pre ohraničenú funkciu,
 nedefinovanú v konečnom počte bodov – vrátane krajných bodov a , b .
- Riemannov integrál na intervale I s krajnými bodmi a , b môžeme značiť

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$$\int_a^b f(x) dx, \text{ resp. } \int_I f(x) dx, \int_{\langle a; b \rangle} f(x) dx, \int_{[a; b]} f(x) dx, \int_{(a; b)} f(x) dx, \int_{[a; b)} f(x) dx.$$

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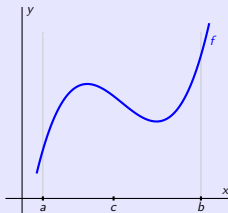
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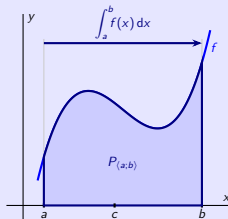
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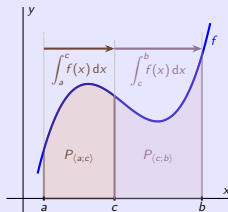
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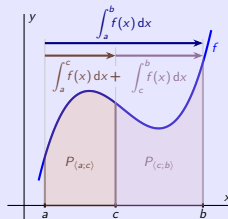
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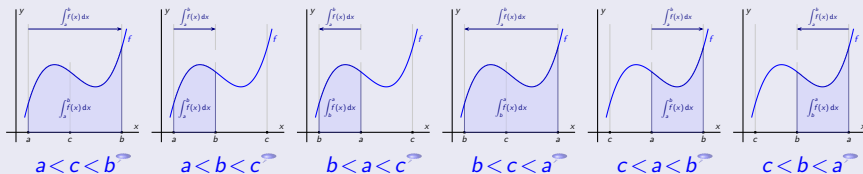
resp. $\overrightarrow{ab} = \overrightarrow{ac} + \overrightarrow{cb} = \overrightarrow{ac} - \overrightarrow{bc}.$

Aditivnosť Riemannovho integrálu

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 Riemannov (určitý) integrál funkcie f na množine A definujeme vzťahom

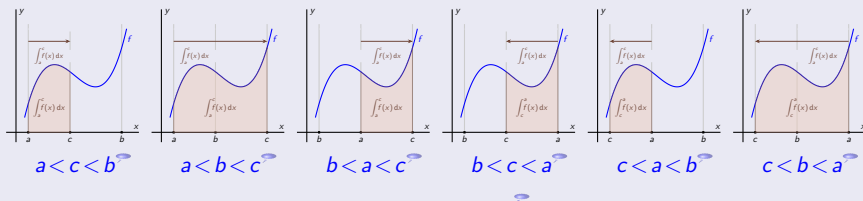
$$\int_A f(x) dx = \int_{I_1} f(x) dx + \int_{I_2} f(x) dx + \dots + \int_{I_k} f(x) dx.$$

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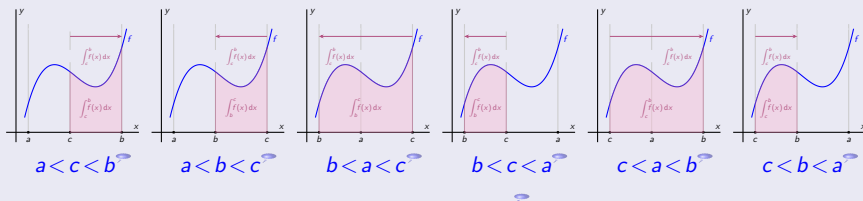
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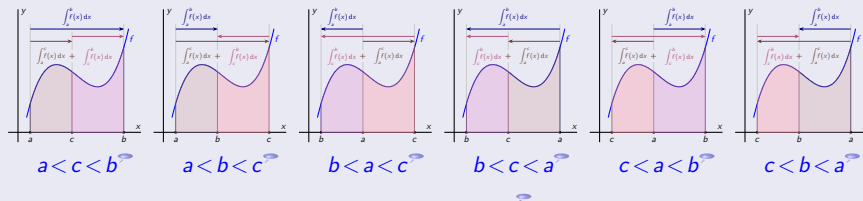
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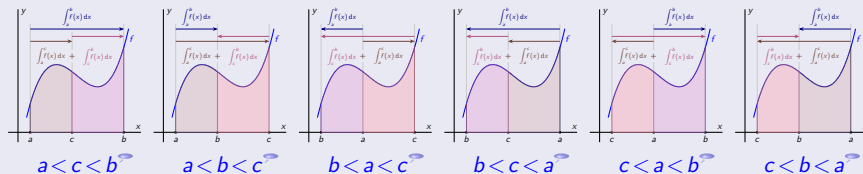
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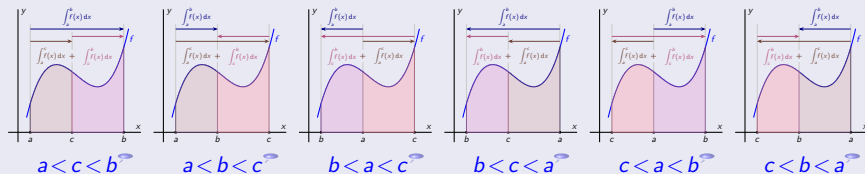
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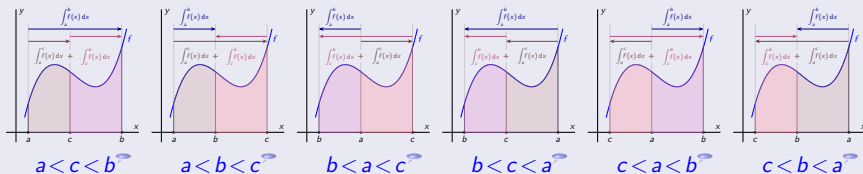
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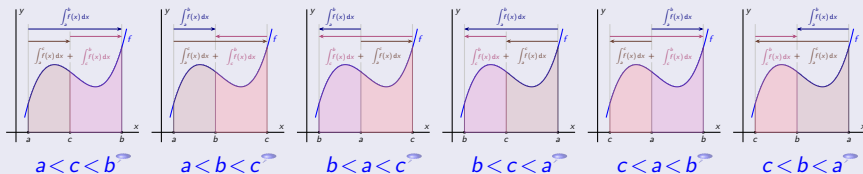
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$A = I_1 \cup I_2 \cup \dots \cup I_k$, kde $I_1, I_2, \dots, I_k$, $k \in \mathbb{N}$ sú nedegenerované ohraničené po dvoch disjunktné (t. j. $I_i \cap I_j = \emptyset$ pre $i \neq j$) intervaly, $f \in R_{I_1}, f \in R_{I_2}, \dots, f \in R_{I_k}$. Riemannov (určitý) integrál funkcie f na množine A definujeme vzťahom

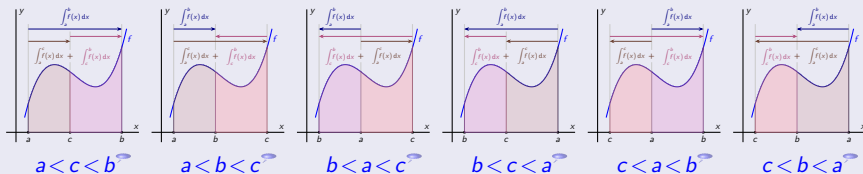
$$\int_A f(x) dx = \int_{I_1} f(x) dx + \int_{I_2} f(x) dx + \dots + \int_{I_k} f(x) dx$$

Aditivnosť Riemannovho integrálu

Aditivnosť

$f \in R_I$, $I \subset \mathbb{R}$ je ohraničený interval, ľubovoľné body $a, b, c \in I$.

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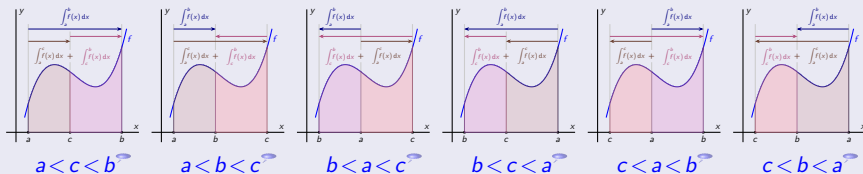
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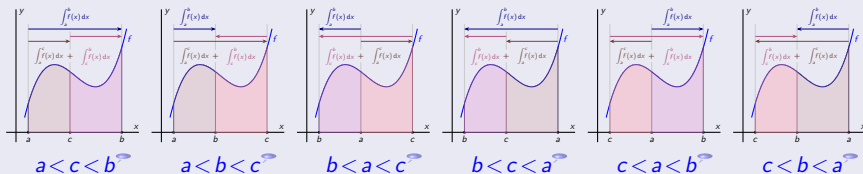
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Výpočet Riemannovho integrálu

Neurčitý Riemannov integrál funkcie f na intervale $\langle a; b \rangle$

$$f \in R_{\langle a; b \rangle}$$

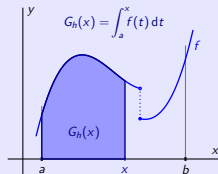


Výpočet Riemannovho integrálu

Neurčitý Riemannov integrál funkcie f na intervale $\langle a; b \rangle$

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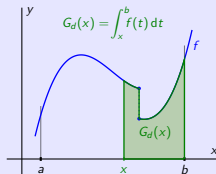


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Funkcia $G_d(x) = \int_x^b f(t) dt$, $x \in \langle a; b \rangle$ sa nazýva
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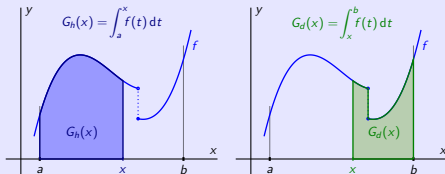
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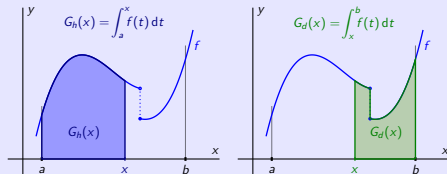
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Geometricky predstavujú funkčné hodnoty $G_h(x)$, resp. $G_d(x)$
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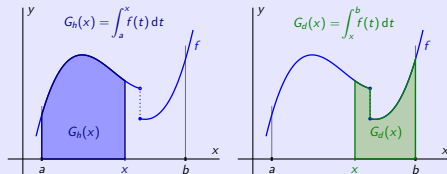
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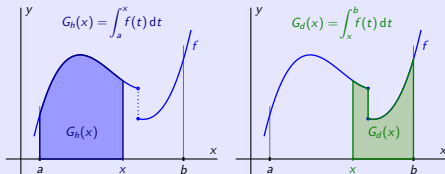
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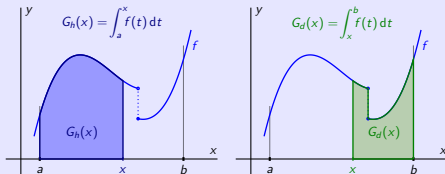
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Výpočet Riemannovho integrálu

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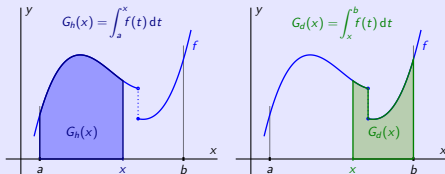
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Funkciu G_h , resp. G_d nazývame tiež neurčitý Riemannov integrál funkcie f na $\langle a; b \rangle$.

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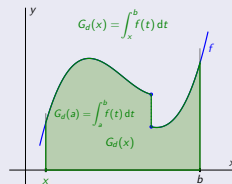
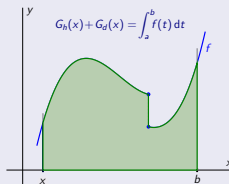
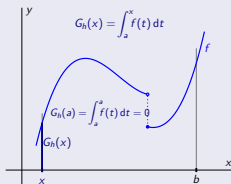
$$G_h(b) = \int_a^b f(t) dt = G_d(a).$$

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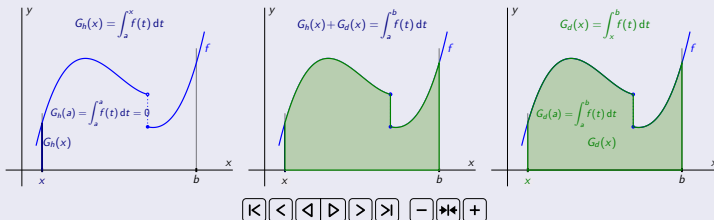
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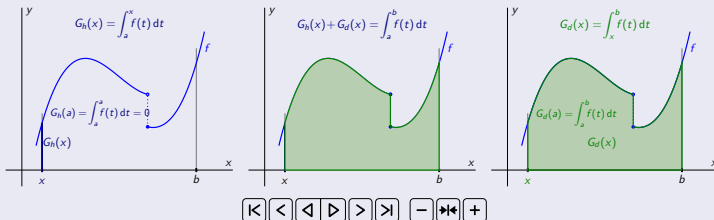


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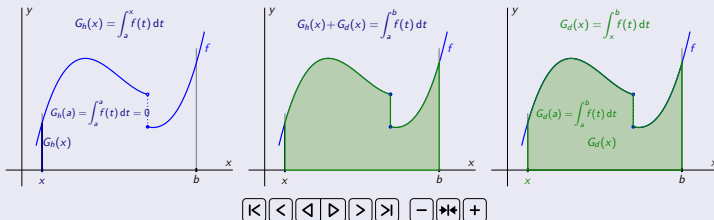
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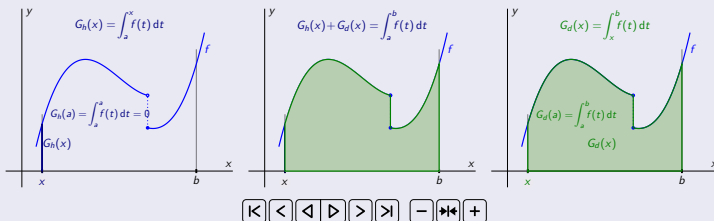
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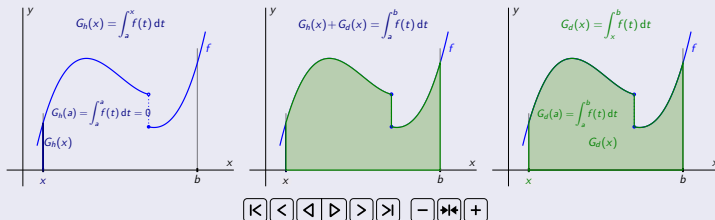
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- Funkcie $G_h, -G_d$ sú **primitívne** k f na $\langle a; b \rangle$.

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$$f(x) = \frac{x}{2}, x \in \langle 0; 1 \rangle, F(x) = \frac{x^2}{2 \cdot 2} + c, x \in \langle 0; 1 \rangle \text{ je primitívna k } f,$$

Výpočet Riemannovho integrálu

Nasledujúci vzorec je základom pre výpočet určitého integrálu,
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Výpočet Riemannovho integrálu

$$\int_0^1 x^2 dx$$

$$\int_{-1}^0 x^2 dx$$

Výpočet Riemannovho integrálu

$$\int_0^1 x^2 dx$$

$$= \left[\frac{x^3}{3} \right]_0^1$$

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Výpočet Riemannovho integrálu

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$$\int_0^{2\pi} \cos x dx$$

Výpočet Riemannovho integrálu

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Výpočet Riemannovho integrálu

$$\int_{-1}^1 \frac{dx}{x}$$



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Výpočet Riemannovho integrálu

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$$\int_0^{\frac{\pi}{4}} \operatorname{tg}^2 x \, dx$$

Výpočet Riemannovho integrálu

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$$= \int_0^{\frac{\pi}{4}} \operatorname{tg}^2 x \, dx = \int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^2 x} \, dx = \int_0^{\frac{\pi}{4}} \frac{1 - \cos^2 x}{\cos^2 x} \, dx$$

Výpočet Riemannovho integrálu

$$\int_{-1}^1 \frac{dx}{x} = ???$$

~~$$= \left[\ln |x| \right]_{-1}^1 = \ln |1| - \ln |-1| = \ln 1 - \ln 1 = 0.$$~~

Newton-Leibnizov vzorec nemôžeme použiť.

Neplatí $\frac{1}{x} \in R_{(-1;1)}$, pretože $y = \frac{1}{x}$ nie je spojitá v bode 0,

v bode 0 je neohraničená, $\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$, $\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$.

$$\int_0^{\frac{\pi}{4}} \operatorname{tg}^2 x \, dx$$

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$$\int_0^{\frac{\pi}{4}} \operatorname{tg}^2 x \, dx = 1 - \frac{\pi}{4}$$

$$= \int_0^{\frac{\pi}{4}} \operatorname{tg}^2 x \, dx = \int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^2 x} \, dx = \int_0^{\frac{\pi}{4}} \frac{1 - \cos^2 x}{\cos^2 x} \, dx = \int_0^{\frac{\pi}{4}} \left(\frac{1}{\cos^2 x} - 1 \right) dx$$

$$= \left[\operatorname{tg} x - x \right]_0^{\frac{\pi}{4}} = \operatorname{tg} \frac{\pi}{4} - \frac{\pi}{4} - (\operatorname{tg} 0 - 0) = 1 - \frac{\pi}{4} - (0 - 0) = 1 - \frac{\pi}{4}.$$

Metóda per partes

Metóda per partes

$$u, v \in R_{\langle a; b \rangle}, u', v' \in R_{\langle a; b \rangle}.$$



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$$\begin{aligned} u(x)v(x) &= \int [u(x)v(x)]' dx \\ &= \int [u'(x)v(x) + u(x)v'(x)] dx \end{aligned}$$

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$$\int_0^{2\pi} x \sin x dx$$

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$$\int_0^{2\pi} x \sin x dx$$

$$= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin x & v = -\cos x \end{array} \right]$$



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$$= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin x & v = -\cos x \end{array} \right] = \left[-x \cos x \right]_0^{2\pi} + \int_0^{2\pi} \cos x dx$$



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$$\begin{aligned} &= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin x & v = -\cos x \end{array} \right] = \left[-x \cos x \right]_0^{2\pi} + \int_0^{2\pi} \cos x dx \\ &= \left[-2\pi \cos 2\pi + 0 \cdot \cos 0 \right] + \left[\sin x \right]_0^{2\pi} \end{aligned}$$

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$$\int_0^{2\pi} x \sin x dx = -2\pi$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x \\ v' = \sin x \end{array} \left| \begin{array}{l} u' = 1 \\ v = -\cos x \end{array} \right. \right] = \left[-x \cos x \right]_0^{2\pi} + \int_0^{2\pi} \cos x dx \\ &= \left[-2\pi \cos 2\pi + 0 \cdot \cos 0 \right] + \left[\sin x \right]_0^{2\pi} = -2\pi + \left[\sin 2\pi - \sin 0 \right] = -2\pi. \end{aligned}$$

Metóda per partes

$$\int_0^{2\pi} x^2 \sin x \, dx$$

Metóda per partes

$$\int_0^{2\pi} x^2 \sin x \, dx$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sin x & v = -\cos x \end{array} \right]$$

Metóda per partes

$$\int_0^{2\pi} x^2 \sin x \, dx$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sin x & v = -\cos x \end{array} \right] = \left[-x^2 \cos x \right]_0^{2\pi} - \int_0^{2\pi} (-2x \cos x) \, dx$$

Metóda per partes

$$\int_0^{2\pi} x^2 \sin x \, dx$$

$$\begin{aligned} &= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sin x & v = -\cos x \end{array} \right] = \left[-x^2 \cos x \right]_0^{2\pi} - \int_0^{2\pi} (-2x \cos x) \, dx \\ &= \left[-4\pi^2 \cos 2\pi - (-0^2 \cdot \cos 0) \right] + 2 \int_0^{2\pi} x \cos x \, dx \end{aligned}$$

Metóda per partes

$$\int_0^{2\pi} x^2 \sin x \, dx$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \\ v' = \sin x \end{array} \middle| \begin{array}{l} u' = 2x \\ v = -\cos x \end{array} \right] = \left[-x^2 \cos x \right]_0^{2\pi} - \int_0^{2\pi} (-2x \cos x) \, dx \\ &= \left[-4\pi^2 \cos 2\pi - (-0^2 \cdot \cos 0) \right] + 2 \int_0^{2\pi} x \cos x \, dx = \left[\begin{array}{l} u = x \\ v' = \cos x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \sin x \end{array} \right] \end{aligned}$$

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 &= \left[-4\pi^2 \cdot 1 + 0 \cdot 1 \right] + 2 \left[\left[x \sin x \right]_0^{2\pi} - \int_0^{2\pi} \sin x \, dx \right] \\
 &= -4\pi^2 + 2 \left[2\pi \sin 2\pi - 0 \cdot \sin 0 \right] - 2 \left[-\cos x \right]_0^{2\pi}
 \end{aligned}$$

Metóda per partes

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 &= \left[-4\pi^2 \cos 2\pi - (-0^2 \cdot \cos 0) \right] + 2 \int_0^{2\pi} x \cos x \, dx = \left[\begin{array}{l} u = x \\ v' = \cos x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \sin x \end{array} \right] \\
 &= \left[-4\pi^2 \cdot 1 + 0 \cdot 1 \right] + 2 \left[\left[x \sin x \right]_0^{2\pi} - \int_0^{2\pi} \sin x \, dx \right] \\
 &= -4\pi^2 + 2 \left[2\pi \sin 2\pi - 0 \cdot \sin 0 \right] - 2 \left[-\cos x \right]_0^{2\pi} \\
 &= -4\pi^2 + 2 \left[2\pi \cdot 0 - 0 \right] - 2 \left[-\cos 2\pi + \cos 0 \right] = -4\pi^2 + 0 - 2 \left[-1 + 1 \right]
 \end{aligned}$$



Metóda per partes

$$\int_0^{2\pi} x^2 \sin x \, dx = -4\pi^2$$

$$\begin{aligned}
 &= \left[\begin{array}{l} u = x^2 \\ v' = \sin x \end{array} \middle| \begin{array}{l} u' = 2x \\ v = -\cos x \end{array} \right] = \left[-x^2 \cos x \right]_0^{2\pi} - \int_0^{2\pi} (-2x \cos x) \, dx \\
 &= \left[-4\pi^2 \cos 2\pi - (-0^2 \cdot \cos 0) \right] + 2 \int_0^{2\pi} x \cos x \, dx = \left[\begin{array}{l} u = x \\ v' = \cos x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \sin x \end{array} \right] \\
 &= \left[-4\pi^2 \cdot 1 + 0 \cdot 1 \right] + 2 \left[\left[x \sin x \right]_0^{2\pi} - \int_0^{2\pi} \sin x \, dx \right] \\
 &= -4\pi^2 + 2 \left[2\pi \sin 2\pi - 0 \cdot \sin 0 \right] - 2 \left[-\cos x \right]_0^{2\pi} \\
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 \end{aligned}$$



Metóda per partes

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 &= \left[-4\pi^2 \cos 2\pi - (-0^2 \cdot \cos 0) \right] + 2 \int_0^{2\pi} x \cos x \, dx = \left[\begin{array}{l} u = x \\ v' = \cos x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \sin x \end{array} \right] \\
 &= \left[-4\pi^2 \cdot 1 + 0 \cdot 1 \right] + 2 \left[\left[x \sin x \right]_0^{2\pi} - \int_0^{2\pi} \sin x \, dx \right] \\
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 \end{aligned}$$

Najprv vypočítame primitívnu funkciu a potom dosadíme do Newton-Leibnizovho vzorca.



Metóda per partes

$$\int_0^{2\pi} x^2 \sin x \, dx = -4\pi^2$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \\ v' = \sin x \end{array} \middle| \begin{array}{l} u' = 2x \\ v = -\cos x \end{array} \right] = \left[-x^2 \cos x \right]_0^{2\pi} - \int_0^{2\pi} (-2x \cos x) \, dx \\ &= \left[-4\pi^2 \cos 2\pi - (-0^2 \cdot \cos 0) \right] + 2 \int_0^{2\pi} x \cos x \, dx = \left[\begin{array}{l} u = x \\ v' = \cos x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \sin x \end{array} \right] \\ &= \left[-4\pi^2 \cdot 1 + 0 \cdot 1 \right] + 2 \left[\left[x \sin x \right]_0^{2\pi} - \int_0^{2\pi} \sin x \, dx \right] \\ &= -4\pi^2 + 2 \left[2\pi \sin 2\pi - 0 \cdot \sin 0 \right] - 2 \left[-\cos x \right]_0^{2\pi} \\ &= -4\pi^2 + 2 \left[2\pi \cdot 0 - 0 \right] - 2 \left[-\cos 2\pi + \cos 0 \right] = -4\pi^2 + 0 - 2 \left[-1 + 1 \right] = -4\pi^2. \end{aligned}$$

Najprv vypočítame primitívnu funkciu a potom dosadíme do Newton-Leibnizovho vzorca.

$$= \left[-x^2 \cos x + 2x \sin x + 2 \cos x \right]_0^{2\pi}$$

Metóda per partes

$$\int_0^{2\pi} x^2 \sin x \, dx = -4\pi^2$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \\ v' = \sin x \end{array} \middle| \begin{array}{l} u' = 2x \\ v = -\cos x \end{array} \right] = \left[-x^2 \cos x \right]_0^{2\pi} - \int_0^{2\pi} (-2x \cos x) \, dx \\ &= \left[-4\pi^2 \cos 2\pi - (-0^2 \cdot \cos 0) \right] + 2 \int_0^{2\pi} x \cos x \, dx = \left[\begin{array}{l} u = x \\ v' = \cos x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \sin x \end{array} \right] \\ &= \left[-4\pi^2 \cdot 1 + 0 \cdot 1 \right] + 2 \left[\left[x \sin x \right]_0^{2\pi} - \int_0^{2\pi} \sin x \, dx \right] \\ &= -4\pi^2 + 2 \left[2\pi \sin 2\pi - 0 \cdot \sin 0 \right] - 2 \left[-\cos x \right]_0^{2\pi} \\ &= -4\pi^2 + 2 \left[2\pi \cdot 0 - 0 \right] - 2 \left[-\cos 2\pi + \cos 0 \right] = -4\pi^2 + 0 - 2 \left[-1 + 1 \right] = -4\pi^2. \end{aligned}$$

Najprv vypočítame primitívnu funkciu a potom dosadíme do Newton-Leibnizovho vzorca.

$$\begin{aligned} &= \left[-x^2 \cos x + 2x \sin x + 2 \cos x \right]_0^{2\pi} \\ &= -4\pi^2 \cos 2\pi + 2 \cdot 2\pi \sin 2\pi + 2 \cos 2\pi - (-4 \cdot 0^2 \cos 0 + 2 \cdot 0 \sin 0 + 2 \cos 0) \end{aligned}$$

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$y = f(x)$ je spojitá na intervale I s hranicami a, b ,
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Metóda substitúcie

$$\int_{-1}^1 \sqrt{1-x^2} dx$$



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$$= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in \langle -1; 1 \rangle \mid x = 1 \mapsto t = \frac{\pi}{2} \mid 1 = \sin \frac{\pi}{2} \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \\ dx = \cos t dt \mid t \in \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle \mid x = -1 \mapsto t = -\frac{\pi}{2} \mid -1 = \sin(-\frac{\pi}{2}) \mid \cos t \geq 0 \text{ pre všetky } t \in \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle \end{array} \right]$$

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$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt$$

Predpoklady metódy sú splnené.

$f(x) = \sqrt{1-x^2}$ je spojitá na $I = \langle -1; 1 \rangle$, $x = \varphi(t) = \sin t$ má na $J = \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle$ spojitú deriváciu $\varphi'(t) = \cos t$, $\varphi(J) = I$, $\varphi(\pm \frac{\pi}{2}) = \pm 1$.

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 &= \frac{1}{2} \left[\frac{\pi}{2} + \frac{\sin \pi}{2} - \left(-\frac{\pi}{2} + \frac{\sin(-\pi)}{2} \right) \right] = \frac{1}{2} \left[\frac{\pi}{2} + 0 - \left(-\frac{\pi}{2} + 0 \right) \right]
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$$\int_{-1}^1 \sqrt{1-x^2} dx$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in \langle -1; 1 \rangle \mid x=1 \mapsto t = \frac{\pi}{2} \mid 1 = \sin \frac{\pi}{2} \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \\ dx = \cos t dt \mid t \in \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle \mid x=-1 \mapsto t = -\frac{\pi}{2} \mid -1 = \sin(-\frac{\pi}{2}) \mid \cos t \geq 0 \text{ pre všetky } t \in \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle \end{array} \right] \\
 &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2} dt = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [1 + \cos 2t] dt = \frac{1}{2} \left[t + \frac{\sin 2t}{2} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \frac{1}{2} \left[\frac{\pi}{2} + \frac{\sin \pi}{2} - \left(-\frac{\pi}{2} + \frac{\sin(-\pi)}{2} \right) \right] = \frac{1}{2} \left[\frac{\pi}{2} + 0 - \left(-\frac{\pi}{2} + 0 \right) \right] = \frac{1}{2} \cdot \pi
 \end{aligned}$$

Predpoklady metódy sú splnené.

$f(x) = \sqrt{1-x^2}$ je spojitá na $I = \langle -1; 1 \rangle$, $x = \varphi(t) = \sin t$ má na $J = \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle$ spojitú deriváciu $\varphi'(t) = \cos t$, $\varphi(J) = I$, $\varphi(\pm \frac{\pi}{2}) = \pm 1$.

Metóda substitúcie

$$\int_{-1}^1 \sqrt{1-x^2} dx = \frac{\pi}{2}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in \langle -1; 1 \rangle \mid x=1 \mapsto t = \frac{\pi}{2} \mid 1 = \sin \frac{\pi}{2} \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \\ dx = \cos t dt \mid t \in \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle \mid x=-1 \mapsto t = -\frac{\pi}{2} \mid -1 = \sin(-\frac{\pi}{2}) \mid \cos t \geq 0 \text{ pre všetky } t \in \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle \end{array} \right] \\
 &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2} dt = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [1 + \cos 2t] dt = \frac{1}{2} \left[t + \frac{\sin 2t}{2} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \frac{1}{2} \left[\frac{\pi}{2} + \frac{\sin \pi}{2} - \left(-\frac{\pi}{2} + \frac{\sin(-\pi)}{2} \right) \right] = \frac{1}{2} \left[\frac{\pi}{2} + 0 - \left(-\frac{\pi}{2} + 0 \right) \right] = \frac{1}{2} \cdot \pi = \frac{\pi}{2}.
 \end{aligned}$$

Predpoklady metódy sú splnené.

$f(x) = \sqrt{1-x^2}$ je spojitá na $I = \langle -1; 1 \rangle$, $x = \varphi(t) = \sin t$ má na $J = \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle$ spojitú deriváciu $\varphi'(t) = \cos t$, $\varphi(J) = I$, $\varphi(\pm \frac{\pi}{2}) = \pm 1$.



Metóda substitúcie

$$\int_{-1}^1 \sqrt{1-x^2} dx = \frac{\pi}{2}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in \langle -1; 1 \rangle \mid x=1 \mapsto t = \frac{\pi}{2} \mid 1 = \sin \frac{\pi}{2} \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \\ dx = \cos t dt \mid t \in \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle \mid x=-1 \mapsto t = -\frac{\pi}{2} \mid -1 = \sin(-\frac{\pi}{2}) \mid \cos t \geq 0 \text{ pre všetky } t \in \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle \end{array} \right] \\
 &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2} dt = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [1 + \cos 2t] dt = \frac{1}{2} \left[t + \frac{\sin 2t}{2} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \frac{1}{2} \left[\frac{\pi}{2} + \frac{\sin \pi}{2} - \left(-\frac{\pi}{2} + \frac{\sin(-\pi)}{2} \right) \right] = \frac{1}{2} \left[\frac{\pi}{2} + 0 - \left(-\frac{\pi}{2} + 0 \right) \right] = \frac{1}{2} \cdot \pi = \frac{\pi}{2}.
 \end{aligned}$$

Predpoklady metódy sú splnené.

$f(x) = \sqrt{1-x^2}$ je spojitá na $I = \langle -1; 1 \rangle$, $x = \varphi(t) = \sin t$ má na $J = \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle$ spojitú deriváciu $\varphi'(t) = \cos t$, $\varphi(J) = I$, $\varphi(\pm \frac{\pi}{2}) = \pm 1$.

Najprv vypočítame primitívnu funkciu a potom dosadíme do Newton-Leibnizovho vzorca.



Metóda substitúcie

$$\int_{-1}^1 \sqrt{1-x^2} dx = \frac{\pi}{2}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in (-1; 1) \mid x=1 \mapsto t = \frac{\pi}{2} \mid 1 = \sin \frac{\pi}{2} \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \\ dx = \cos t dt \mid t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \mid x=-1 \mapsto t = -\frac{\pi}{2} \mid -1 = \sin\left(-\frac{\pi}{2}\right) \mid \cos t \geq 0 \text{ pre všetky } t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \end{array} \right] \\
 &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2} dt = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [1 + \cos 2t] dt = \frac{1}{2} \left[t + \frac{\sin 2t}{2} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \frac{1}{2} \left[\frac{\pi}{2} + \frac{\sin \pi}{2} - \left(-\frac{\pi}{2} + \frac{\sin(-\pi)}{2} \right) \right] = \frac{1}{2} \left[\frac{\pi}{2} + 0 - \left(-\frac{\pi}{2} + 0 \right) \right] = \frac{1}{2} \cdot \pi = \frac{\pi}{2}.
 \end{aligned}$$

Predpoklady metódy sú splnené.

$f(x) = \sqrt{1-x^2}$ je spojitá na $I = (-1; 1)$, $x = \varphi(t) = \sin t$ má na $J = \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ spojitú deriváciu $\varphi'(t) = \cos t$, $\varphi(J) = I$, $\varphi(\pm \frac{\pi}{2}) = \pm 1$.

Najprv vypočítame primitívnu funkciu a potom dosadíme do Newton-Leibnizovho vzorca.

$$= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{\arcsin x}{2} \right]_{-1}^1$$

Metóda substitúcie

$$\int_{-1}^1 \sqrt{1-x^2} dx = \frac{\pi}{2}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in (-1; 1) \mid x=1 \mapsto t = \frac{\pi}{2} \mid 1 = \sin \frac{\pi}{2} \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \\ dx = \cos t dt \mid t \in \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle \mid x=-1 \mapsto t = -\frac{\pi}{2} \mid -1 = \sin(-\frac{\pi}{2}) \mid \cos t \geq 0 \text{ pre všetky } t \in \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle \end{array} \right] \\
 &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2} dt = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [1 + \cos 2t] dt = \frac{1}{2} \left[t + \frac{\sin 2t}{2} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \frac{1}{2} \left[\frac{\pi}{2} + \frac{\sin \pi}{2} - \left(-\frac{\pi}{2} + \frac{\sin(-\pi)}{2} \right) \right] = \frac{1}{2} \left[\frac{\pi}{2} + 0 - \left(-\frac{\pi}{2} + 0 \right) \right] = \frac{1}{2} \cdot \pi = \frac{\pi}{2}.
 \end{aligned}$$

Predpoklady metódy sú splnené.

$f(x) = \sqrt{1-x^2}$ je spojitá na $I = (-1; 1)$, $x = \varphi(t) = \sin t$ má na $J = \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle$ spojitú deriváciu $\varphi'(t) = \cos t$, $\varphi(J) = I$, $\varphi(\pm \frac{\pi}{2}) = \pm 1$.

Najprv vypočítame primitívnu funkciu a potom dosadíme do Newton-Leibnizovho vzorca.

$$\begin{aligned}
 &= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{\arcsin x}{2} \right]_{-1}^1 = \frac{1 \cdot \sqrt{1-1^2}}{2} + \frac{\arcsin 1}{2} - \left(\frac{-1 \cdot \sqrt{1-(-1)^2}}{2} + \frac{\arcsin(-1)}{2} \right)
 \end{aligned}$$

Metóda substitúcie

$$\int_{-1}^1 \sqrt{1-x^2} dx = \frac{\pi}{2}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in (-1; 1) \mid x=1 \mapsto t = \frac{\pi}{2} \mid 1 = \sin \frac{\pi}{2} \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \\ dx = \cos t dt \mid t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \mid x=-1 \mapsto t = -\frac{\pi}{2} \mid -1 = \sin\left(-\frac{\pi}{2}\right) \mid \cos t \geq 0 \text{ pre všetky } t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \end{array} \right] \\
 &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2} dt = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [1 + \cos 2t] dt = \frac{1}{2} \left[t + \frac{\sin 2t}{2} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \frac{1}{2} \left[\frac{\pi}{2} + \frac{\sin \pi}{2} - \left(-\frac{\pi}{2} + \frac{\sin(-\pi)}{2} \right) \right] = \frac{1}{2} \left[\frac{\pi}{2} + 0 - \left(-\frac{\pi}{2} + 0 \right) \right] = \frac{1}{2} \cdot \pi = \frac{\pi}{2}.
 \end{aligned}$$

Predpoklady metódy sú splnené.

$f(x) = \sqrt{1-x^2}$ je spojitá na $I = (-1; 1)$, $x = \varphi(t) = \sin t$ má na $J = \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ spojitú deriváciu $\varphi'(t) = \cos t$, $\varphi(J) = I$, $\varphi(\pm \frac{\pi}{2}) = \pm 1$.

Najprv vypočítame primitívnu funkciu a potom dosadíme do Newton-Leibnizovho vzorca.

$$\begin{aligned}
 &= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{\arcsin x}{2} \right]_{-1}^1 = \frac{1 \cdot \sqrt{1-1^2}}{2} + \frac{\arcsin 1}{2} - \left(\frac{-1 \cdot \sqrt{1-(-1)^2}}{2} + \frac{\arcsin(-1)}{2} \right) \\
 &= \frac{1 \cdot 0}{2} + \frac{\frac{\pi}{2}}{2} - \left(\frac{-1 \cdot 0}{2} + \frac{-\frac{\pi}{2}}{2} \right)
 \end{aligned}$$

Metóda substitúcie

$$\int_{-1}^1 \sqrt{1-x^2} dx = \frac{\pi}{2}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in (-1; 1) \mid x=1 \mapsto t = \frac{\pi}{2} \mid 1 = \sin \frac{\pi}{2} \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \\ dx = \cos t dt \mid t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \mid x=-1 \mapsto t = -\frac{\pi}{2} \mid -1 = \sin\left(-\frac{\pi}{2}\right) \mid \cos t \geq 0 \text{ pre všetky } t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \end{array} \right] \\
 &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2} dt = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [1 + \cos 2t] dt = \frac{1}{2} \left[t + \frac{\sin 2t}{2} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \frac{1}{2} \left[\frac{\pi}{2} + \frac{\sin \pi}{2} - \left(-\frac{\pi}{2} + \frac{\sin(-\pi)}{2} \right) \right] = \frac{1}{2} \left[\frac{\pi}{2} + 0 - \left(-\frac{\pi}{2} + 0 \right) \right] = \frac{1}{2} \cdot \pi = \frac{\pi}{2}.
 \end{aligned}$$

Predpoklady metódy sú splnené.

$f(x) = \sqrt{1-x^2}$ je spojitá na $I = (-1; 1)$, $x = \varphi(t) = \sin t$ má na $J = \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ spojitú deriváciu $\varphi'(t) = \cos t$, $\varphi(J) = I$, $\varphi(\pm \frac{\pi}{2}) = \pm 1$.

Najprv vypočítame primitívnu funkciu a potom dosadíme do Newton-Leibnizovho vzorca.

$$\begin{aligned}
 &= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{\arcsin x}{2} \right]_{-1}^1 = \frac{1 \cdot \sqrt{1-1^2}}{2} + \frac{\arcsin 1}{2} - \left(\frac{-1 \cdot \sqrt{1-(-1)^2}}{2} + \frac{\arcsin(-1)}{2} \right) \\
 &= \frac{1 \cdot 0}{2} + \frac{\frac{\pi}{2}}{2} - \left(\frac{-1 \cdot 0}{2} + \frac{-\frac{\pi}{2}}{2} \right) = 0 + \frac{\pi}{4} - 0 + \frac{\pi}{4}
 \end{aligned}$$

Metóda substitúcie

$$\int_{-1}^1 \sqrt{1-x^2} dx = \frac{\pi}{2}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid x \in (-1; 1) \mid x=1 \mapsto t = \frac{\pi}{2} \mid 1 = \sin \frac{\pi}{2} \mid \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \\ dx = \cos t dt \mid t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \mid x=-1 \mapsto t = -\frac{\pi}{2} \mid -1 = \sin\left(-\frac{\pi}{2}\right) \mid \cos t \geq 0 \text{ pre všetky } t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \end{array} \right] \\
 &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2} dt = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [1 + \cos 2t] dt = \frac{1}{2} \left[t + \frac{\sin 2t}{2} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \frac{1}{2} \left[\frac{\pi}{2} + \frac{\sin \pi}{2} - \left(-\frac{\pi}{2} + \frac{\sin(-\pi)}{2} \right) \right] = \frac{1}{2} \left[\frac{\pi}{2} + 0 - \left(-\frac{\pi}{2} + 0 \right) \right] = \frac{1}{2} \cdot \pi = \frac{\pi}{2}.
 \end{aligned}$$

Predpoklady metódy sú splnené.

$f(x) = \sqrt{1-x^2}$ je spojitá na $I = (-1; 1)$, $x = \varphi(t) = \sin t$ má na $J = \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ spojitú deriváciu $\varphi'(t) = \cos t$, $\varphi(J) = I$, $\varphi(\pm \frac{\pi}{2}) = \pm 1$.

Najprv vypočítame primitívnu funkciu a potom dosadíme do Newton-Leibnizovho vzorca.

$$\begin{aligned}
 &= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{\arcsin x}{2} \right]_{-1}^1 = \frac{1 \cdot \sqrt{1-1^2}}{2} + \frac{\arcsin 1}{2} - \left(\frac{-1 \cdot \sqrt{1-(-1)^2}}{2} + \frac{\arcsin(-1)}{2} \right) \\
 &= \frac{1 \cdot 0}{2} + \frac{\frac{\pi}{2}}{2} - \left(\frac{-1 \cdot 0}{2} + \frac{-\frac{\pi}{2}}{2} \right) = 0 + \frac{\pi}{4} - 0 + \frac{\pi}{4} = \frac{\pi}{2}.
 \end{aligned}$$

Metóda substitúcie

$$\int_{-1}^2 t \sin(t^2 + 1) dt$$

Metóda substitúcie

$$\int_{-1}^2 t \sin(t^2 + 1) dt$$

$$= \left[\begin{array}{l} \text{Subst. } x = t^2 + 1 \mid t \in \langle -1; 2 \rangle \mid t = 2 \mapsto x = 2^2 + 1 = 5 \\ \quad \quad \quad dx = 2 dt \mid x \in \langle 1; 5 \rangle \mid t = -1 \mapsto x = (-1)^2 + 1 = 2 \end{array} \right]$$

Metóda substitúcie

$$\int_{-1}^2 t \sin(t^2+1) dt$$

$$= \left[\begin{array}{l} \text{Subst. } x=t^2+1 \mid t \in \langle -1; 2 \rangle \mid t=2 \mapsto x=2^2+1=5 \\ \quad \quad \quad dx=2dt \mid x \in \langle 1; 5 \rangle \mid t=-1 \mapsto x=(-1)^2+1=2 \end{array} \right] = \frac{1}{2} \int_2^5 \sin x dx = \frac{1}{2} \left[-\cos x \right]_2^5$$

Predpoklady metódy sú splnené.

$f(x)=\sin x$ je spojitá na $I=\langle 2; 5 \rangle$, $x=\varphi(t)=t^2+1$ má na $J=\langle -1; 2 \rangle$ spojité deriváciu $\varphi'(t)=2t$, $\varphi(J)=I$, $\varphi(-1)=2$, $\varphi(2)=5$.

Metóda substitúcie

$$\int_{-1}^2 t \sin(t^2+1) dt$$

$$= \left[\begin{array}{l} \text{Subst. } x=t^2+1 \mid t \in \langle -1; 2 \rangle \mid t=2 \mapsto x=2^2+1=5 \\ \quad \quad \quad dx=2dt \mid x \in \langle 1; 5 \rangle \mid t=-1 \mapsto x=(-1)^2+1=2 \end{array} \right] = \frac{1}{2} \int_2^5 \sin x dx = \frac{1}{2} \left[-\cos x \right]_2^5$$

$$= \frac{1}{2} \left[-\cos 5 + \cos 2 \right]$$

Predpoklady metódy sú splnené.

$f(x)=\sin x$ je spojitá na $I=\langle 2; 5 \rangle$, $x=\varphi(t)=t^2+1$ má na $J=\langle -1; 2 \rangle$ spojité deriváciu $\varphi'(t)=2t$, $\varphi(J)=I$, $\varphi(-1)=2$, $\varphi(2)=5$.

Metóda substitúcie

$$\int_{-1}^2 t \sin(t^2+1) dt = \frac{\cos 2 - \cos 5}{2}$$

$$= \left[\begin{array}{l} \text{Subst. } x=t^2+1 \mid t \in \langle -1; 2 \rangle \mid t=2 \mapsto x=2^2+1=5 \\ \quad \quad \quad dx=2dt \mid x \in \langle 1; 5 \rangle \mid t=-1 \mapsto x=(-1)^2+1=2 \end{array} \right] = \frac{1}{2} \int_2^5 \sin x dx = \frac{1}{2} \left[-\cos x \right]_2^5$$

$$= \frac{1}{2} \left[-\cos 5 + \cos 2 \right] = \frac{\cos 2 - \cos 5}{2}.$$

Predpoklady metódy sú splnené.

$f(x) = \sin x$ je spojitá na $I = \langle 2; 5 \rangle$, $x = \varphi(t) = t^2 + 1$ má na $J = \langle -1; 2 \rangle$ spojitú deriváciu $\varphi'(t) = 2t$, $\varphi(J) = I$, $\varphi(-1) = 2$, $\varphi(2) = 5$.

$$\int_0^{\frac{\pi}{2}} \sin^3 t \cdot \cos t dt$$

$$\left[\begin{array}{l} \text{Subst. } x = \sin t \mid t \in \langle 0; \frac{\pi}{2} \rangle \mid t = \frac{\pi}{2} \mapsto x = \sin \frac{\pi}{2} = 1 \\ \quad \quad \quad dx = \cos t dt \mid x \in \langle 0; 1 \rangle \mid t = 0 \mapsto x = \sin 0 = 0 \end{array} \right]$$

Metóda substitúcie

$$\int_{-1}^2 t \sin(t^2+1) dt = \frac{\cos 2 - \cos 5}{2}$$

$$= \left[\begin{array}{l} \text{Subst. } x=t^2+1 \mid t \in \langle -1; 2 \rangle \mid t=2 \mapsto x=2^2+1=5 \\ \quad \quad \quad dx=2dt \mid x \in \langle 1; 5 \rangle \mid t=-1 \mapsto x=(-1)^2+1=2 \end{array} \right] = \frac{1}{2} \int_2^5 \sin x dx = \frac{1}{2} \left[-\cos x \right]_2^5$$

$$= \frac{1}{2} \left[-\cos 5 + \cos 2 \right] = \frac{\cos 2 - \cos 5}{2}.$$

Predpoklady metódy sú splnené.

$f(x) = \sin x$ je spojitá na $I = \langle 2; 5 \rangle$, $x = \varphi(t) = t^2 + 1$ má na $J = \langle -1; 2 \rangle$ spojitú deriváciu $\varphi'(t) = 2t$, $\varphi(J) = I$, $\varphi(-1) = 2$, $\varphi(2) = 5$.

$$\int_0^{\frac{\pi}{2}} \sin^3 t \cdot \cos t dt$$

$$= \left[\begin{array}{l} \text{Subst. } x = \sin t \mid t \in \langle 0; \frac{\pi}{2} \rangle \mid t = \frac{\pi}{2} \mapsto x = \sin \frac{\pi}{2} = 1 \\ \quad \quad \quad dx = \cos t dt \mid x \in \langle 0; 1 \rangle \mid t = 0 \mapsto x = \sin 0 = 0 \end{array} \right]$$

Metóda substitúcie

$$\int_{-1}^2 t \sin(t^2+1) dt = \frac{\cos 2 - \cos 5}{2}$$

$$= \left[\text{Subst. } x=t^2+1 \mid t \in \langle -1; 2 \rangle \mid t=2 \mapsto x=2^2+1=5 \right. \\ \left. dx=2dt \mid x \in \langle 1; 5 \rangle \mid t=-1 \mapsto x=(-1)^2+1=2 \right] = \frac{1}{2} \int_2^5 \sin x dx = \frac{1}{2} \left[-\cos x \right]_2^5 \\ = \frac{1}{2} \left[-\cos 5 + \cos 2 \right] = \frac{\cos 2 - \cos 5}{2}.$$

Predpoklady metódy sú splnené.

$f(x) = \sin x$ je spojitá na $I = \langle 2; 5 \rangle$, $x = \varphi(t) = t^2 + 1$ má na $J = \langle -1; 2 \rangle$ spojitú deriváciu $\varphi'(t) = 2t$, $\varphi(J) = I$, $\varphi(-1) = 2$, $\varphi(2) = 5$.

$$\int_0^{\frac{\pi}{2}} \sin^3 t \cdot \cos t dt$$

$$= \left[\text{Subst. } x = \sin t \mid t \in \langle 0; \frac{\pi}{2} \rangle \mid t = \frac{\pi}{2} \mapsto x = \sin \frac{\pi}{2} = 1 \right. \\ \left. dx = \cos t dt \mid x \in \langle 0; 1 \rangle \mid t = 0 \mapsto x = \sin 0 = 0 \right] = \int_0^1 x^3 dx$$

Predpoklady metódy sú splnené.

$f(x) = x^3$ je spojitá na $I = \langle 0; 1 \rangle$, $x = \varphi(t) = \sin t$ má na $J = \langle 0; \frac{\pi}{2} \rangle$ spojitú deriváciu $\varphi'(t) = \cos t$, $\varphi(J) = I$, $\varphi(0) = 0$, $\varphi(\frac{\pi}{2}) = 1$.

Metóda substitúcie

$$\int_{-1}^2 t \sin(t^2+1) dt = \frac{\cos 2 - \cos 5}{2}$$

$$= \left[\text{Subst. } x=t^2+1 \mid t \in \langle -1; 2 \rangle \mid t=2 \mapsto x=2^2+1=5 \right. \\ \left. dx=2dt \mid x \in \langle 1; 5 \rangle \mid t=-1 \mapsto x=(-1)^2+1=2 \right] = \frac{1}{2} \int_2^5 \sin x dx = \frac{1}{2} \left[-\cos x \right]_2^5 \\ = \frac{1}{2} \left[-\cos 5 + \cos 2 \right] = \frac{\cos 2 - \cos 5}{2}.$$

Predpoklady metódy sú splnené.

$f(x) = \sin x$ je spojitá na $I = \langle 2; 5 \rangle$, $x = \varphi(t) = t^2 + 1$ má na $J = \langle -1; 2 \rangle$ spojitú deriváciu $\varphi'(t) = 2t$, $\varphi(J) = I$, $\varphi(-1) = 2$, $\varphi(2) = 5$.

$$\int_0^{\frac{\pi}{2}} \sin^3 t \cdot \cos t dt$$

$$= \left[\text{Subst. } x = \sin t \mid t \in \langle 0; \frac{\pi}{2} \rangle \mid t = \frac{\pi}{2} \mapsto x = \sin \frac{\pi}{2} = 1 \right. \\ \left. dx = \cos t dt \mid x \in \langle 0; 1 \rangle \mid t = 0 \mapsto x = \sin 0 = 0 \right] = \int_0^1 x^3 dx = \left[\frac{x^4}{4} \right]_0^1$$

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$$\int_0^{\frac{\pi}{2}} \sin^3 t \cdot \cos t dt = \frac{1}{4}$$

$$= \left[\text{Subst. } x = \sin t \mid t \in \langle 0; \frac{\pi}{2} \rangle \mid t = \frac{\pi}{2} \mapsto x = \sin \frac{\pi}{2} = 1 \right. \\ \left. dx = \cos t dt \mid x \in \langle 0; 1 \rangle \mid t = 0 \mapsto x = \sin 0 = 0 \right] = \int_0^1 x^3 dx = \left[\frac{x^4}{4} \right]_0^1 = \frac{1}{4} - \frac{0}{4} = \frac{1}{4}.$$

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$$\int_1^e \frac{\ln x \, dx}{x}$$

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$$\int_{\pi}^0 \sin^3 x \, dx = \int_{\pi}^0 \sin^2 x \cdot \sin x \, dx$$

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$f(t)=t$ je spojitá na $I=\langle 0; 1 \rangle$, $t=\varphi(x)=\ln x$ má na $J=\langle 1; e \rangle$ spojitú deriváciu $\varphi'(x)=\frac{1}{x}$, $\varphi(J)=I$, $\varphi(1)=0$, $\varphi(e)=1$.

$$\int_{\pi}^0 \sin^3 x \, dx = \int_{\pi}^0 \sin^2 x \cdot \sin x \, dx$$

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Metóda substitúcie

$$\int_1^e \frac{\ln x \, dx}{x} = \frac{1}{2}$$

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$f(t) = 1 - t^2$ je spojitá na $I = \langle -1; 1 \rangle$, $t = \varphi(x) = \cos x$ má na $J = \langle 0; \pi \rangle$ spojitú deriváciu $\varphi'(x) = -\sin x$, $\varphi(J) = I$, $\varphi(0) = 1$, $\varphi(\pi) = -1$.

Metóda substitúcie

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Metóda substitúcie

$$\int_1^e \frac{\ln x \, dx}{x} = \frac{1}{2}$$

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$f(t) = 1 - t^2$ je spojitá na $I = \langle -1; 1 \rangle$, $t = \varphi(x) = \cos x$ má na $J = \langle 0; \pi \rangle$ spojitú deriváciu $\varphi'(x) = -\sin x$, $\varphi(J) = I$, $\varphi(0) = 1$, $\varphi(\pi) = -1$.

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Metóda substitúcie

$$\int_1^e \frac{\ln x \, dx}{x} = \frac{1}{2}$$

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$$\int_{\pi}^0 \sin^3 x \, dx = \int_{\pi}^0 \sin^2 x \cdot \sin x \, dx = -\frac{4}{3}$$

$$= \int_{\pi}^0 (1 - \cos^2 x) \sin x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in \langle 0; \pi \rangle \mid x=0 \mapsto t=\cos 0=1 \\ \quad \quad \quad dt = -\sin x \, dx \mid t \in \langle -1; 1 \rangle \mid x=\pi \mapsto t=\cos \pi=-1 \end{array} \right] = - \int_{-1}^1 (1 - t^2) \, dt$$

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Predpoklady metódy sú splnené.

$f(t) = 1 - t^2$ je spojitá na $I = \langle -1; 1 \rangle$, $t = \varphi(x) = \cos x$ má na $J = \langle 0; \pi \rangle$ spojitú deriváciu $\varphi'(x) = -\sin x$, $\varphi(J) = I$, $\varphi(0) = 1$, $\varphi(\pi) = -1$.

Metóda substitúcie

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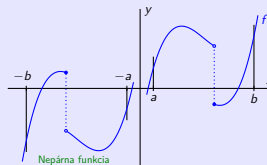
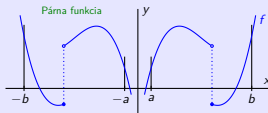
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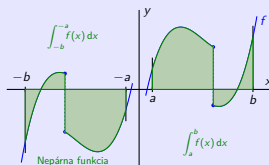
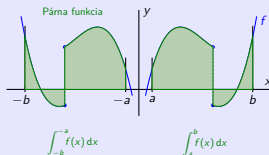
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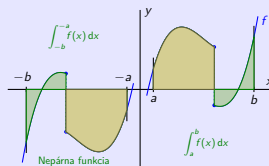
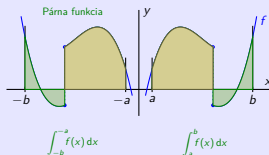
Integrovanie párných a nepárných funkcií

$$f(x) \in R_{\langle a; b \rangle}.$$

$$\Rightarrow f(-x) \in R_{\langle -b; -a \rangle} \text{ a platí } \int_a^b f(x) dx = \int_{-b}^{-a} f(-x) dx.$$

$$\int_a^b f(x) dx = \left[\begin{array}{l} \text{Subst. } x = -t \\ x \in \langle a; b \rangle \\ x = a \mapsto t = -a \\ x = b \mapsto t = -b \end{array} \quad \begin{array}{l} dx = -dt \\ t \in \langle -b; -a \rangle \\ x = b \mapsto t = -a \\ x = a \mapsto t = -b \end{array} \right] = - \int_{-a}^{-b} f(-t) dt = \int_{-b}^{-a} f(-t) dt = \int_{-b}^{-a} f(-x) dx.$$

- $f \in R_{\langle a; b \rangle}$ je párna. $\int_a^b f(x) dx = \int_{-b}^{-a} f(x) dx$
- $f \in R_{\langle a; b \rangle}$ je nepárna. $\int_a^b f(x) dx = - \int_{-b}^{-a} f(x) dx$



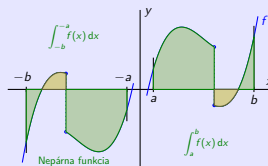
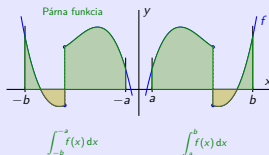
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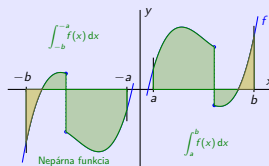
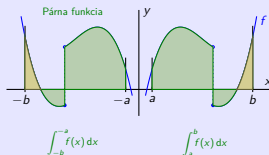
Integrovanie párných a nepárnych funkcií

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Integrovanie párných a nepárných funkcií

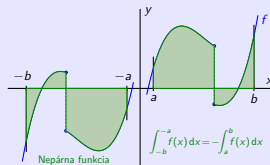
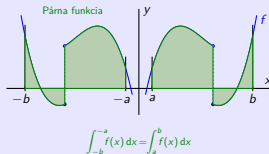
$$f(x) \in R_{\langle a; b \rangle}.$$

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$$\bullet f \in R_{\langle a; b \rangle} \text{ je párna.} \Rightarrow \int_a^b f(x) dx = \int_{-b}^{-a} f(-x) dx = \int_{-b}^{-a} f(x) dx.$$

$$\bullet f \in R_{\langle a; b \rangle} \text{ je nepárna.} \Rightarrow \int_a^b f(x) dx = \int_{-b}^{-a} f(-x) dx = - \int_{-b}^{-a} f(x) dx.$$



Integrovanie párných a nepárnych funkcií

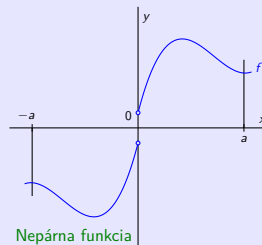
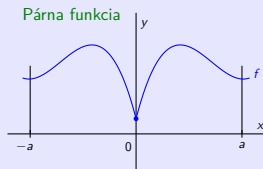
$$f \in R_{\langle -a; a \rangle}, a > 0.$$

Integrovanie párných a nepárnych funkcií

$$f \in R_{\langle -a; a \rangle}, a > 0.$$

f je párna

f je nepárna

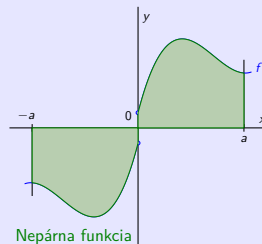
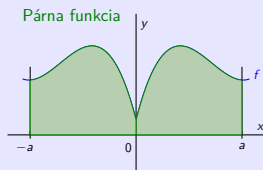


Integrovanie párných a nepárnych funkcií

$$f \in R_{\langle -a; a \rangle}, a > 0.$$

$$f \text{ je párna} \Rightarrow \int_{-a}^a f(x) dx$$

$$f \text{ je nepárna} \Rightarrow \int_{-a}^a f(x) dx$$

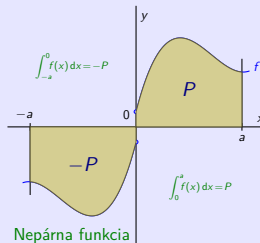
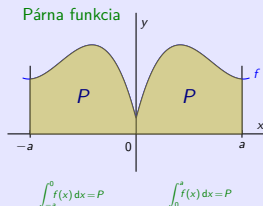


Integrovanie párných a nepárných funkcií

$$f \in R_{\langle -a; a \rangle}, a > 0.$$

$$f \text{ je párna} \Rightarrow \int_{-a}^a f(x) dx = \int_{-a}^0 f(x) dx + \int_0^a f(x) dx$$

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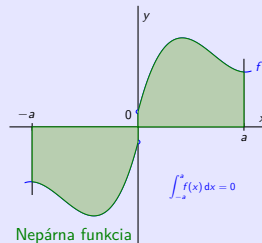
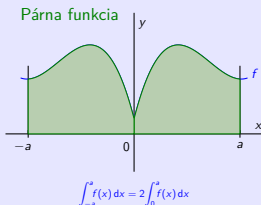


Integrovanie párných a nepárných funkcií

$$f \in R_{\langle -a; a \rangle}, a > 0.$$

$$f \text{ je párna} \Rightarrow \int_{-a}^a f(x) dx = \int_{-a}^0 f(x) dx + \int_0^a f(x) dx = 2 \int_0^a f(x) dx.$$

$$f \text{ je nepárna} \Rightarrow \int_{-a}^a f(x) dx = \int_{-a}^0 f(x) dx + \int_0^a f(x) dx = 0.$$



Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 \frac{x^3 \sqrt[3]{x^2 + \sin^2 x}}{x^4 + 1} dx$$

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Kedže je funkcia $f \in R_{\langle -1; 1 \rangle}$ nepárna, primitívnu funkciu nemusíme počítať.

$$\int_{-\pi}^{\pi} \sin |x| dx$$

$$= \left[\begin{array}{l} f(x) = \sin |x| \text{ je spojitá na } \langle -\pi; \pi \rangle \Rightarrow f \in R_{\langle -\pi; \pi \rangle} \\ f(-x) = \sin |-x| = \sin |x| = f(x) \Rightarrow f \text{ je párna} \end{array} \right] = 2 \int_0^{\pi} \sin |x| dx = \left[\begin{array}{l} x \in \langle 0; \pi \rangle, |x| = x \\ \sin |x| = \sin x \end{array} \right]$$

$$= 2 \int_0^{\pi} \sin x dx = 2 \left[-\cos x \right]_0^{\pi} = -2 \left[\cos x \right]_0^{\pi} = -2 \left[\cos \pi - \cos 0 \right]$$

$$= -2 \left[-1 - 1 \right]$$

Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 \frac{x^3 \sqrt[3]{x^2 + \sin^2 x}}{x^4 + 1} dx = 0$$

$$= \left[\begin{array}{l} f(x) = \frac{x^3 \sqrt[3]{x^2 + \sin^2 x}}{x^4 + 1} \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} \\ f(-x) = \frac{(-x)^3 \sqrt[3]{(-x)^2 + \sin^2(-x)}}{(-x)^4 + 1} = \frac{-x^3 \sqrt[3]{x^2 + \sin^2 x}}{x^4 + 1} = -f(x) \Rightarrow f \text{ je nepárna} \end{array} \right] = 0.$$

Kedže je funkcia $f \in R_{\langle -1; 1 \rangle}$ nepárna, primitívnu funkciu nemusíme počítať.

$$\int_{-\pi}^{\pi} \sin |x| dx$$

$$= \left[\begin{array}{l} f(x) = \sin |x| \text{ je spojitá na } \langle -\pi; \pi \rangle \Rightarrow f \in R_{\langle -\pi; \pi \rangle} \\ f(-x) = \sin |-x| = \sin |x| = f(x) \Rightarrow f \text{ je párna} \end{array} \right] = 2 \int_0^{\pi} \sin |x| dx = \left[\begin{array}{l} x \in \langle 0; \pi \rangle, |x| = x \\ \sin |x| = \sin x \end{array} \right]$$

$$= 2 \int_0^{\pi} \sin x dx = 2 \left[-\cos x \right]_0^{\pi} = -2 \left[\cos x \right]_0^{\pi} = -2 \left[\cos \pi - \cos 0 \right]$$

$$= -2 \left[-1 - 1 \right] = (-2) \cdot (-2)$$

Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 \frac{x^3 \sqrt[3]{x^2 + \sin^2 x}}{x^4 + 1} dx = 0$$

$$= \left[\begin{array}{l} f(x) = \frac{x^3 \sqrt[3]{x^2 + \sin^2 x}}{x^4 + 1} \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} \\ f(-x) = \frac{(-x)^3 \sqrt[3]{(-x)^2 + \sin^2(-x)}}{(-x)^4 + 1} = \frac{-x^3 \sqrt[3]{x^2 + \sin^2 x}}{x^4 + 1} = -f(x) \Rightarrow f \text{ je nepárna} \end{array} \right] = 0.$$

Kedže je funkcia $f \in R_{\langle -1; 1 \rangle}$ nepárna, primitívnu funkciu nemusíme počítať.

$$\int_{-\pi}^{\pi} \sin |x| dx = 4$$

$$= \left[\begin{array}{l} f(x) = \sin |x| \text{ je spojitá na } \langle -\pi; \pi \rangle \Rightarrow f \in R_{\langle -\pi; \pi \rangle} \\ f(-x) = \sin |-x| = \sin |x| = f(x) \Rightarrow f \text{ je párna} \end{array} \right] = 2 \int_0^{\pi} \sin |x| dx = \left[\begin{array}{l} x \in \langle 0; \pi \rangle, |x| = x \\ \sin |x| = \sin x \end{array} \right]$$

$$= 2 \int_0^{\pi} \sin x dx = 2 \left[-\cos x \right]_0^{\pi} = -2 \left[\cos x \right]_0^{\pi} = -2 \left[\cos \pi - \cos 0 \right]$$

$$= -2 \left[-1 - 1 \right] = (-2) \cdot (-2) = 4.$$

Integrovanie párných a nepárnych funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx$$

Integrovanie párných a nepárnych funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx$$

$$= \left[f(x) = x^4 \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} \mid g(x) = \sin^3 4x \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow g \in R_{\langle -1; 1 \rangle} \right]$$

Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx$$

$$= \left[\begin{array}{l|l} f(x) = x^4 \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} & g(x) = \sin^3 4x \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow g \in R_{\langle -1; 1 \rangle} \\ f(-x) = (-x)^4 = x^4 = f(x) \Rightarrow f \text{ je párna} & g(-x) = \sin^3(-4x) = (-\sin 4x)^3 = -\sin^3 4x = -g(x) \Rightarrow g \text{ je nepárna} \end{array} \right]$$

Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx$$

$$= \left[\begin{array}{l|l} f(x) = x^4 \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} & g(x) = \sin^3 4x \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow g \in R_{\langle -1; 1 \rangle} \\ f(-x) = (-x)^4 = x^4 = f(x) \Rightarrow f \text{ je párna} & g(-x) = \sin^3(-4x) = (-\sin 4x)^3 = -\sin^3 4x = -g(x) \Rightarrow g \text{ je nepárna} \end{array} \right]$$

$$= \int_{-1}^1 x^4 dx - \int_{-1}^1 \sin^3 4x dx$$

Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx$$

$$= \left[\begin{array}{l|l} f(x)=x^4 \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} & g(x)=\sin^3 4x \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow g \in R_{\langle -1; 1 \rangle} \\ f(-x)=(-x)^4=x^4=f(x) \Rightarrow f \text{ je párna} & g(-x)=\sin^3(-4x)=(-\sin 4x)^3=-\sin^3 4x=-g(x) \Rightarrow g \text{ je nepárna} \end{array} \right]$$

$$= \int_{-1}^1 x^4 dx - \int_{-1}^1 \sin^3 4x dx = 2 \int_0^1 x^4 dx - 0$$

Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx$$

$$= \left[\begin{array}{l|l} f(x)=x^4 \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} & g(x)=\sin^3 4x \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow g \in R_{\langle -1; 1 \rangle} \\ f(-x)=(-x)^4=x^4=f(x) \Rightarrow f \text{ je párna} & g(-x)=\sin^3(-4x)=(-\sin 4x)^3=-\sin^3 4x=-g(x) \Rightarrow g \text{ je nepárna} \end{array} \right]$$

$$= \int_{-1}^1 x^4 dx - \int_{-1}^1 \sin^3 4x dx = 2 \int_0^1 x^4 dx - 0 = 2 \left[\frac{x^5}{5} \right]_0^1$$

Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx$$

$$= \left[\begin{array}{l} f(x) = x^4 \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} \quad \left| \quad g(x) = \sin^3 4x \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow g \in R_{\langle -1; 1 \rangle} \right. \\ f(-x) = (-x)^4 = x^4 = f(x) \Rightarrow f \text{ je párna} \quad \left| \quad g(-x) = \sin^3(-4x) = (-\sin 4x)^3 = -\sin^3 4x = -g(x) \Rightarrow g \text{ je nepárna} \right. \end{array} \right]$$

$$= \int_{-1}^1 x^4 dx - \int_{-1}^1 \sin^3 4x dx = 2 \int_0^1 x^4 dx - 0 = 2 \left[\frac{x^5}{5} \right]_0^1 = 2 \left[\frac{1^5}{5} - \frac{0^5}{5} \right]$$

Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx = \frac{2}{5}$$

$$= \left[\begin{array}{l} f(x) = x^4 \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} \quad \left| \quad g(x) = \sin^3 4x \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow g \in R_{\langle -1; 1 \rangle} \right. \\ f(-x) = (-x)^4 = x^4 = f(x) \Rightarrow f \text{ je párna} \quad \left| \quad g(-x) = \sin^3(-4x) = (-\sin 4x)^3 = -\sin^3 4x = -g(x) \Rightarrow g \text{ je nepárna} \right. \end{array} \right]$$

$$= \int_{-1}^1 x^4 dx - \int_{-1}^1 \sin^3 4x dx = 2 \int_0^1 x^4 dx - 0 = 2 \left[\frac{x^5}{5} \right]_0^1 = 2 \left[\frac{1^5}{5} - \frac{0^5}{5} \right] = \frac{2}{5}.$$

Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx = \frac{2}{5}$$

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$$\int_{-\pi}^{\pi} x \sin x^2 dx$$

Integrovanie párných a nepárnych funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx = \frac{2}{5}$$

$$= \left[\begin{array}{l} f(x) = x^4 \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} \quad \left| \quad g(x) = \sin^3 4x \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow g \in R_{\langle -1; 1 \rangle} \right. \\ f(-x) = (-x)^4 = x^4 = f(x) \Rightarrow f \text{ je párna} \quad \left| \quad g(-x) = \sin^3(-4x) = (-\sin 4x)^3 = -\sin^3 4x = -g(x) \Rightarrow g \text{ je nepárna} \right. \end{array} \right]$$

$$= \int_{-1}^1 x^4 dx - \int_{-1}^1 \sin^3 4x dx = 2 \int_0^1 x^4 dx - 0 = 2 \left[\frac{x^5}{5} \right]_0^1 = 2 \left[\frac{1^5}{5} - \frac{0^5}{5} \right] = \frac{2}{5}.$$

$$\int_{-\pi}^{\pi} x \sin x^2 dx$$

$$= \left[f(x) = x \sin x^2 \text{ je spojitá na } \langle -\pi; \pi \rangle \Rightarrow f \in R_{\langle -\pi; \pi \rangle} \right]$$

Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx = \frac{2}{5}$$

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$$\int_{-\pi}^{\pi} x \sin x^2 dx$$

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Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx = \frac{2}{5}$$

$$= \left[\begin{array}{l|l} f(x) = x^4 \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} & g(x) = \sin^3 4x \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow g \in R_{\langle -1; 1 \rangle} \\ f(-x) = (-x)^4 = x^4 = f(x) \Rightarrow f \text{ je párna} & g(-x) = \sin^3(-4x) = (-\sin 4x)^3 = -\sin^3 4x = -g(x) \Rightarrow g \text{ je nepárna} \end{array} \right]$$

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Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx = \frac{2}{5}$$

$$= \left[\begin{array}{l|l} f(x) = x^4 \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} & g(x) = \sin^3 4x \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow g \in R_{\langle -1; 1 \rangle} \\ f(-x) = (-x)^4 = x^4 = f(x) \Rightarrow f \text{ je párna} & g(-x) = \sin^3(-4x) = (-\sin 4x)^3 = -\sin^3 4x = -g(x) \Rightarrow g \text{ je nepárna} \end{array} \right]$$

$$= \int_{-1}^1 x^4 dx - \int_{-1}^1 \sin^3 4x dx = 2 \int_0^1 x^4 dx - 0 = 2 \left[\frac{x^5}{5} \right]_0^1 = 2 \left[\frac{1^5}{5} - \frac{0^5}{5} \right] = \frac{2}{5}.$$

$$\int_{-\pi}^{\pi} x \sin x^2 dx = 0$$

$$= \left[\begin{array}{l|l} f(x) = x \sin x^2 \text{ je spojitá na } \langle -\pi; \pi \rangle \Rightarrow f \in R_{\langle -\pi; \pi \rangle} & \\ f(-x) = -x \sin(-x)^2 = -x \sin x^2 = -f(x) \Rightarrow f \text{ je nepárna} & \end{array} \right] = 0.$$

$$\int_{-\pi}^{\pi} x \cos x^2 dx$$

Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx = \frac{2}{5}$$

$$= \left[\begin{array}{l} f(x) = x^4 \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow f \in R_{\langle -1; 1 \rangle} \quad \left| \quad g(x) = \sin^3 4x \text{ je spojitá na } \langle -1; 1 \rangle \Rightarrow g \in R_{\langle -1; 1 \rangle} \right. \\ f(-x) = (-x)^4 = x^4 = f(x) \Rightarrow f \text{ je párna} \quad \left| \quad g(-x) = \sin^3(-4x) = (-\sin 4x)^3 = -\sin^3 4x = -g(x) \Rightarrow g \text{ je nepárna} \right. \end{array} \right]$$

$$= \int_{-1}^1 x^4 dx - \int_{-1}^1 \sin^3 4x dx = 2 \int_0^1 x^4 dx - 0 = 2 \left[\frac{x^5}{5} \right]_0^1 = 2 \left[\frac{1^5}{5} - \frac{0^5}{5} \right] = \frac{2}{5}.$$

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$$\int_{-\pi}^{\pi} x \cos x^2 dx$$

$$= \left[\begin{array}{l} f(x) = x \cos x^2 \text{ je spojitá na } \langle -\pi; \pi \rangle \Rightarrow f \in R_{\langle -\pi; \pi \rangle} \end{array} \right]$$

Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx = \frac{2}{5}$$

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$$= \int_{-1}^1 x^4 dx - \int_{-1}^1 \sin^3 4x dx = 2 \int_0^1 x^4 dx - 0 = 2 \left[\frac{x^5}{5} \right]_0^1 = 2 \left[\frac{1^5}{5} - \frac{0^5}{5} \right] = \frac{2}{5}.$$

$$\int_{-\pi}^{\pi} x \sin x^2 dx = 0$$

$$= \left[\begin{array}{l} f(x) = x \sin x^2 \text{ je spojitá na } \langle -\pi; \pi \rangle \Rightarrow f \in R_{\langle -\pi; \pi \rangle} \\ f(-x) = -x \sin (-x)^2 = -x \sin x^2 = -f(x) \Rightarrow f \text{ je nepárna} \end{array} \right] = 0.$$

$$\int_{-\pi}^{\pi} x \cos x^2 dx$$

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Integrovanie párných a nepárných funkcií

$$\int_{-1}^1 (x^4 - \sin^3 4x) dx = \frac{2}{5}$$

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$$= \int_{-1}^1 x^4 dx - \int_{-1}^1 \sin^3 4x dx = 2 \int_0^1 x^4 dx - 0 = 2 \left[\frac{x^5}{5} \right]_0^1 = 2 \left[\frac{1^5}{5} - \frac{0^5}{5} \right] = \frac{2}{5}.$$

$$\int_{-\pi}^{\pi} x \sin x^2 dx = 0$$

$$= \left[\begin{array}{l} f(x) = x \sin x^2 \text{ je spojitá na } \langle -\pi; \pi \rangle \Rightarrow f \in R_{\langle -\pi; \pi \rangle} \\ f(-x) = -x \sin (-x)^2 = -x \sin x^2 = -f(x) \Rightarrow f \text{ je nepárna} \end{array} \right] = 0.$$

$$\int_{-\pi}^{\pi} x \cos x^2 dx = 0$$

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Integrovanie párných a nepárných funkcií

$$\int_{-2\pi}^{2\pi} x \sin x \, dx$$

Integrovanie párných a nepárnych funkcií

$$\int_{-2\pi}^{2\pi} x \sin x \, dx$$

$$= \left[f(x) = x \sin x \text{ je spojitá na } \langle -2\pi; 2\pi \rangle \Rightarrow f \in R_{\langle -2\pi; 2\pi \rangle} \right]$$

Integrovanie párných a nepárnych funkcií

$$\int_{-2\pi}^{2\pi} x \sin x \, dx$$

$$= \left[\begin{array}{l} f(x) = x \sin x \text{ je spojitá na } \langle -2\pi; 2\pi \rangle \Rightarrow f \in R_{\langle -2\pi; 2\pi \rangle} \\ f(-x) = -x \sin(-x) \\ \quad = -x(-\sin x) = x \sin x = f(x) \Rightarrow f \text{ je párna} \end{array} \right]$$

Integrovanie párných a nepárnych funkcií

$$\int_{-2\pi}^{2\pi} x \sin x \, dx$$

$$= \left[\begin{array}{l} f(x) = x \sin x \text{ je spojitá na } \langle -2\pi; 2\pi \rangle \Rightarrow f \in R_{\langle -2\pi; 2\pi \rangle} \\ f(-x) = -x \sin(-x) \\ \quad = -x(-\sin x) = x \sin x = f(x) \Rightarrow f \text{ je párna} \end{array} \right] = 2 \int_0^{2\pi} x \sin x \, dx$$

Integrovanie párných a nepárných funkcií

$$\int_{-2\pi}^{2\pi} x \sin x \, dx$$

$$= \left[\begin{array}{l} f(x) = x \sin x \text{ je spojitá na } \langle -2\pi; 2\pi \rangle \Rightarrow f \in R_{\langle -2\pi; 2\pi \rangle} \\ f(-x) = -x \sin(-x) \\ \quad = -x(-\sin x) = x \sin x = f(x) \Rightarrow f \text{ je párna} \end{array} \right] = 2 \int_0^{2\pi} x \sin x \, dx = \left[\begin{array}{l} u = x \quad u' = 1 \\ v' = \sin x \quad v = -\cos x \end{array} \right]$$

$$= \left[\begin{array}{l} u = x \quad u' = 1 \\ v' = \sin x \quad v = -\cos x \end{array} \right]$$

Integrovanie párných a nepárných funkcií

$$\int_{-2\pi}^{2\pi} x \sin x \, dx$$

$$= \left[\begin{array}{l} f(x) = x \sin x \text{ je spojitá na } \langle -2\pi; 2\pi \rangle \Rightarrow f \in R_{\langle -2\pi; 2\pi \rangle} \\ f(-x) = -x \sin(-x) \\ \quad = -x(-\sin x) = x \sin x = f(x) \Rightarrow f \text{ je párna} \end{array} \right] = 2 \int_0^{2\pi} x \sin x \, dx = \left[\begin{array}{l} u = x \quad u' = 1 \\ v' = \sin x \quad v = -\cos x \end{array} \right]$$

$$= 2 \left[-x \cos x \right]_0^{2\pi} + 2 \int_0^{2\pi} \cos x \, dx$$

$$= \left[\begin{array}{l} u = x \quad u' = 1 \\ v' = \sin x \quad v = -\cos x \end{array} \right] = \left[-x \cos x \right]_{-2\pi}^{2\pi} + \int_{-2\pi}^{2\pi} \cos x \, dx$$

Integrovanie párných a nepárných funkcií

$$\int_{-2\pi}^{2\pi} x \sin x \, dx$$

$$= \left[\begin{array}{l} f(x) = x \sin x \text{ je spojitá na } \langle -2\pi; 2\pi \rangle \Rightarrow f \in R_{\langle -2\pi; 2\pi \rangle} \\ f(-x) = -x \sin(-x) \\ \quad = -x(-\sin x) = x \sin x = f(x) \Rightarrow f \text{ je párna} \end{array} \right] = 2 \int_0^{2\pi} x \sin x \, dx = \left[\begin{array}{l} u = x \quad u' = 1 \\ v' = \sin x \quad v = -\cos x \end{array} \right]$$

$$= 2 \left[-x \cos x \right]_0^{2\pi} + 2 \int_0^{2\pi} \cos x \, dx = 2 \left[-2\pi \cos 2\pi + 0 \cdot \cos 0 \right] + 2 \left[\sin x \right]_0^{2\pi}$$

$$= \left[\begin{array}{l} u = x \quad u' = 1 \\ v' = \sin x \quad v = -\cos x \end{array} \right] = \left[-x \cos x \right]_{-2\pi}^{2\pi} + \int_{-2\pi}^{2\pi} \cos x \, dx$$

$$= \left[-2\pi \cos 2\pi + (-2\pi) \cos(-2\pi) \right] + \left[\sin x \right]_{-2\pi}^{2\pi}$$

Integrovanie párných a nepárných funkcií

$$\int_{-2\pi}^{2\pi} x \sin x \, dx$$

$$\begin{aligned}
 &= \left[\begin{array}{l} f(x) = x \sin x \text{ je spojitá na } \langle -2\pi; 2\pi \rangle \Rightarrow f \in R_{\langle -2\pi; 2\pi \rangle} \\ f(-x) = -x \sin(-x) \\ \quad = -x(-\sin x) = x \sin x = f(x) \Rightarrow f \text{ je párna} \end{array} \right] = 2 \int_0^{2\pi} x \sin x \, dx = \left[\begin{array}{l} u = x \quad u' = 1 \\ v' = \sin x \quad v = -\cos x \end{array} \right] \\
 &= 2 \left[-x \cos x \right]_0^{2\pi} + 2 \int_0^{2\pi} \cos x \, dx = 2 \left[-2\pi \cos 2\pi + 0 \cdot \cos 0 \right] + 2 \left[\sin x \right]_0^{2\pi} \\
 &= 2 \left[-2\pi \cdot 1 + 0 \right] + 2 \left[\sin 2\pi - \sin 0 \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} u = x \quad u' = 1 \\ v' = \sin x \quad v = -\cos x \end{array} \right] = \left[-x \cos x \right]_{-2\pi}^{2\pi} + \int_{-2\pi}^{2\pi} \cos x \, dx \\
 &= \left[-2\pi \cos 2\pi + (-2\pi) \cos(-2\pi) \right] + \left[\sin x \right]_{-2\pi}^{2\pi} \\
 &= \left[-2\pi \cdot 1 - 2\pi \cdot 1 \right] + \left[\sin 2\pi - \sin(-2\pi) \right]
 \end{aligned}$$

Integrovanie párných a nepárných funkcií

$$\int_{-2\pi}^{2\pi} x \sin x \, dx$$

$$\begin{aligned}
 &= \left[\begin{array}{l} f(x) = x \sin x \text{ je spojitá na } \langle -2\pi; 2\pi \rangle \Rightarrow f \in R_{\langle -2\pi; 2\pi \rangle} \\ f(-x) = -x \sin(-x) \\ \quad = -x(-\sin x) = x \sin x = f(x) \Rightarrow f \text{ je párna} \end{array} \right] = 2 \int_0^{2\pi} x \sin x \, dx = \left[\begin{array}{l} u = x \\ v' = \sin x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\cos x \end{array} \right] \\
 &= 2 \left[-x \cos x \right]_0^{2\pi} + 2 \int_0^{2\pi} \cos x \, dx = 2 \left[-2\pi \cos 2\pi + 0 \cdot \cos 0 \right] + 2 \left[\sin x \right]_0^{2\pi} \\
 &= 2 \left[-2\pi \cdot 1 + 0 \right] + 2 \left[\sin 2\pi - \sin 0 \right] = -4\pi + 2 \left[0 - 0 \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} u = x \\ v' = \sin x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\cos x \end{array} \right] = \left[-x \cos x \right]_{-2\pi}^{2\pi} + \int_{-2\pi}^{2\pi} \cos x \, dx \\
 &= \left[-2\pi \cos 2\pi + (-2\pi) \cos(-2\pi) \right] + \left[\sin x \right]_{-2\pi}^{2\pi} \\
 &= \left[-2\pi \cdot 1 - 2\pi \cdot 1 \right] + \left[\sin 2\pi - \sin(-2\pi) \right] = -4\pi + \left[0 - 0 \right]
 \end{aligned}$$

Integrovanie párných a nepárných funkcií

$$\int_{-2\pi}^{2\pi} x \sin x \, dx = -4\pi$$

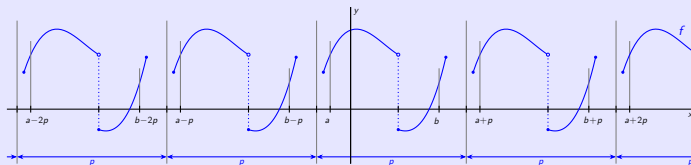
$$\begin{aligned}
 &= \left[\begin{array}{l} f(x) = x \sin x \text{ je spojitá na } \langle -2\pi; 2\pi \rangle \Rightarrow f \in R_{\langle -2\pi; 2\pi \rangle} \\ f(-x) = -x \sin(-x) \\ \quad = -x(-\sin x) = x \sin x = f(x) \Rightarrow f \text{ je párna} \end{array} \right] = 2 \int_0^{2\pi} x \sin x \, dx = \left[\begin{array}{l} u = x \\ v' = \sin x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\cos x \end{array} \right] \\
 &= 2 \left[-x \cos x \right]_0^{2\pi} + 2 \int_0^{2\pi} \cos x \, dx = 2 \left[-2\pi \cos 2\pi + 0 \cdot \cos 0 \right] + 2 \left[\sin x \right]_0^{2\pi} \\
 &= 2 \left[-2\pi \cdot 1 + 0 \right] + 2 \left[\sin 2\pi - \sin 0 \right] = -4\pi + 2 \left[0 - 0 \right] = -4\pi.
 \end{aligned}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} u = x \\ v' = \sin x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\cos x \end{array} \right] = \left[-x \cos x \right]_{-2\pi}^{2\pi} + \int_{-2\pi}^{2\pi} \cos x \, dx \\
 &= \left[-2\pi \cos 2\pi + (-2\pi) \cos(-2\pi) \right] + \left[\sin x \right]_{-2\pi}^{2\pi} \\
 &= \left[-2\pi \cdot 1 - 2\pi \cdot 1 \right] + \left[\sin 2\pi - \sin(-2\pi) \right] = -4\pi + \left[0 - 0 \right] = -4\pi.
 \end{aligned}$$

Integrovanie periodických funkcií

$f(x) \in R_{\langle a; b \rangle}$, f je periodická s periódou $p > 0$,

t. j. $f(x + kp) = f(x)$ pre všetky $k \in \mathbb{Z}$, $x \in \langle a; b \rangle$.

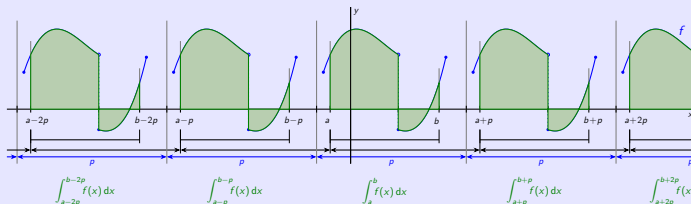


Integrovanie periodických funkcií

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t. j. $f(x + kp) = f(x)$ pre všetky $k \in \mathbb{Z}$, $x \in \langle a; b \rangle$.

$\Rightarrow f(x + kp) \in R_{\langle a+kp; b+kp \rangle}$ pre všetky $k \in \mathbb{Z}$

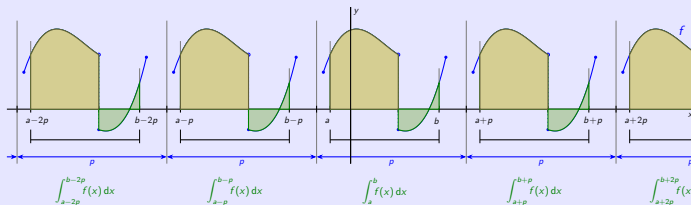


Integrovanie periodických funkcií

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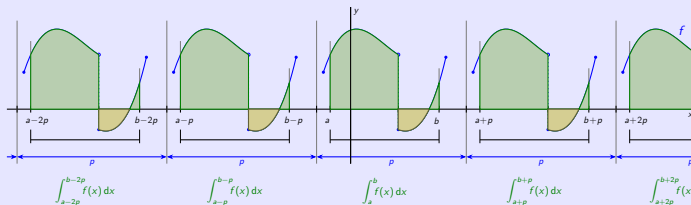


Integrovanie periodických funkcií

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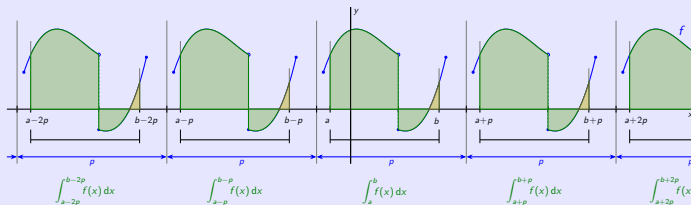


Integrovanie periodických funkcií

$f(x) \in R_{\langle a; b \rangle}$, f je periodická s periódou $p > 0$,

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$\Rightarrow f(x + kp) \in R_{\langle a+kp; b+kp \rangle}$ pre všetky $k \in \mathbb{Z}$

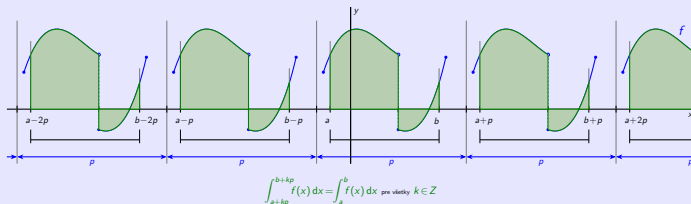


Integrovanie periodických funkcií

$f(x) \in R_{\langle a; b \rangle}$, f je periodická s periódou $p > 0$,

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$$\Rightarrow f(x + kp) \in R_{\langle a+kp; b+kp \rangle} \text{ pre všetky } k \in \mathbb{Z} \text{ a platí } \int_a^b f(x) dx = \int_{a+kp}^{b+kp} f(x) dx.$$



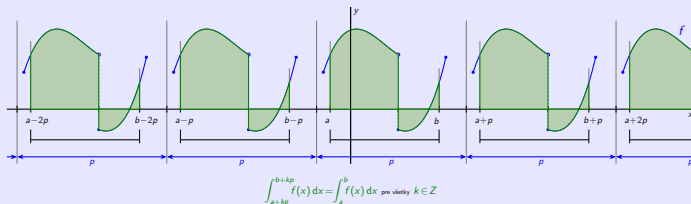
Integrovanie periodických funkcií

$f(x) \in R_{\langle a; b \rangle}$, f je periodická s periódou $p > 0$,

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$$\int_{a+kp}^{b+kp} f(x) dx$$



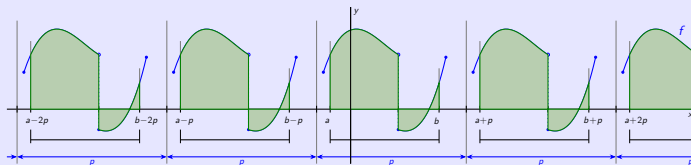
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$f(x) \in R_{\langle a; b \rangle}$, f je periodická s periódou $p > 0$,

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$$\Rightarrow f(x + kp) \in R_{\langle a+kp; b+kp \rangle} \text{ pre všetky } k \in \mathbb{Z} \text{ a platí } \int_a^b f(x) dx = \int_{a+kp}^{b+kp} f(x) dx.$$

$$\int_{a+kp}^{b+kp} f(x) dx = \left[\begin{array}{l} \text{Subst. } x = t + kp \mid x \in \langle a+kp; b+kp \rangle \mid x = b+kp \mapsto t = b \\ t = x - kp, dx = dt \mid t \in \langle a; b \rangle \mid x = a+kp \mapsto t = a \end{array} \right]$$



$$\int_{a+kp}^{b+kp} f(x) dx = \int_a^b f(x) dx \text{ pre všetky } k \in \mathbb{Z}$$

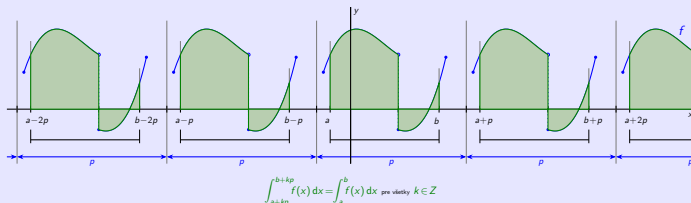
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$$\begin{aligned} \int_{a+kp}^{b+kp} f(x) dx &= \left[\begin{array}{l} \text{Subst. } x = t + kp \mid x \in \langle a + kp; b + kp \rangle \\ t = x - kp, dx = dt \mid t \in \langle a; b \rangle \end{array} \right] \begin{array}{l} x = b + kp \mapsto t = b \\ x = a + kp \mapsto t = a \end{array} \\ &= \int_a^b f(t + kp) dt \end{aligned}$$



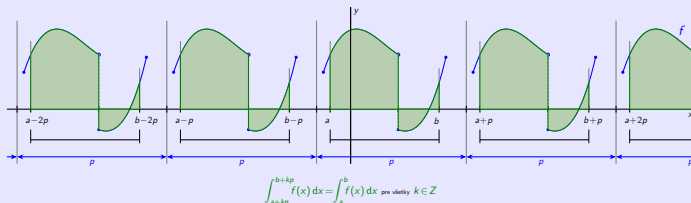
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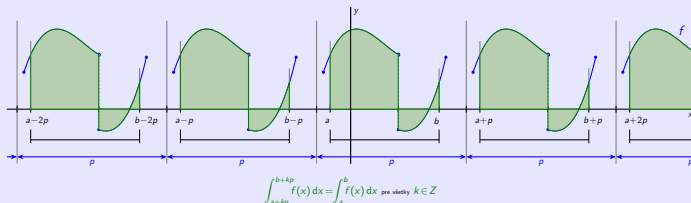
Integrovanie periodických funkcií

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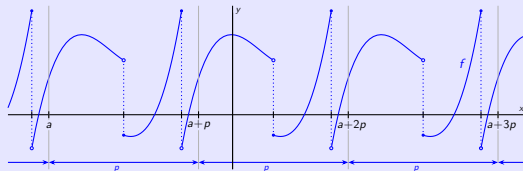
$$\Rightarrow f(x + kp) \in R_{\langle a+kp; b+kp \rangle} \text{ pre všetky } k \in \mathbb{Z} \text{ a platí } \int_a^b f(x) dx = \int_{a+kp}^{b+kp} f(x) dx.$$

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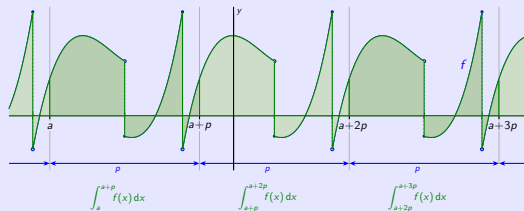
Integrovanie periodických funkcií

f je periodická s periódou $p > 0$, $a \in \mathbb{R}$.



Integrovanie periodických funkcií

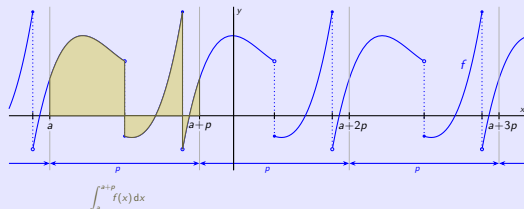
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Integrovanie periodických funkcií

f je periodická s periódou $p > 0$, $a \in \mathbb{R}$.

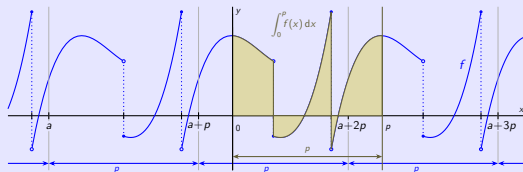
$$f \in R_{\langle a; a+p \rangle}$$



Integrovanie periodických funkcií

f je periodická s periódou $p > 0$, $a \in \mathbb{R}$.

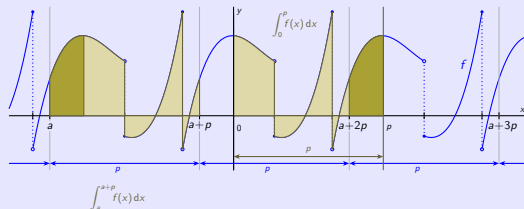
$$f \in R_{\langle a; a+p \rangle} \Leftrightarrow f \in R_{\langle 0; p \rangle}$$



Integrovanie periodických funkcií

f je periodická s periódou $p > 0$, $a \in \mathbb{R}$.

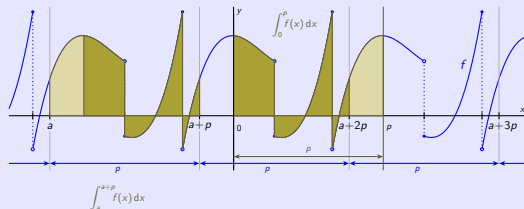
$$f \in R_{\langle a; a+p \rangle} \Leftrightarrow f \in R_{\langle 0; p \rangle}$$



Integrovanie periodických funkcií

f je periodická s periódou $p > 0$, $a \in \mathbb{R}$.

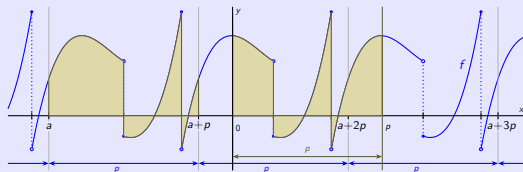
$$f \in R_{\langle a; a+p \rangle} \Leftrightarrow f \in R_{\langle 0; p \rangle}$$



Integrovanie periodických funkcií

f je periodická s periódou $p > 0$, $a \in \mathbb{R}$.

$$f \in R_{\langle a; a+p \rangle} \Leftrightarrow f \in R_{\langle 0; p \rangle} \text{ a pokiaľ existujú, platí } \int_0^p f(x) dx = \int_a^{a+p} f(x) dx.$$



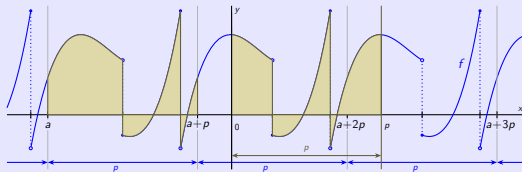
$$\int_a^{a+p} f(x) dx = \int_0^p f(x) dx \text{ pre všetky } a \in \mathbb{R}$$

Integrovanie periodických funkcií

f je periodická s periódou $p > 0$, $a \in \mathbb{R}$.

$$f \in R_{\langle a; a+p \rangle} \Leftrightarrow f \in R_{\langle 0; p \rangle} \text{ a pokiaľ existujú, platí } \int_0^p f(x) dx = \int_a^{a+p} f(x) dx.$$

$k \in \mathbb{Z}$ je také, že $a \in \langle (k-1)p; kp \rangle$.



$$\int_a^{a+p} f(x) dx = \int_0^p f(x) dx \text{ pre všetky } a \in \mathbb{R}$$

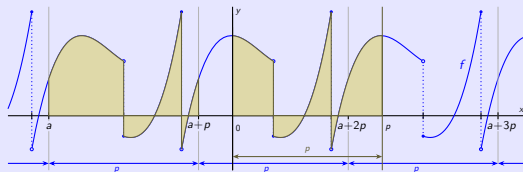
Integrovanie periodických funkcií

f je periodická s periódou $p > 0$, $a \in \mathbb{R}$.

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$$\Rightarrow \int_a^{a+p} f(x) dx$$



$$\int_a^{a+p} f(x) dx = \int_0^p f(x) dx \text{ pre všetky } a \in \mathbb{R}$$

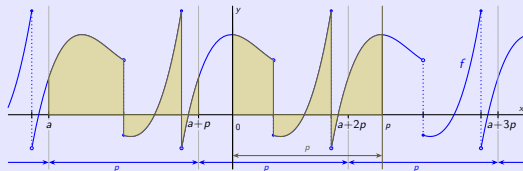
Integrovanie periodických funkcií

f je periodická s periódou $p > 0$, $a \in \mathbb{R}$.

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$k \in \mathbb{Z}$ je také, že $a \in \langle (k-1)p; kp \rangle$.

$$\Rightarrow \int_a^{a+p} f(x) dx = \int_a^{kp} f(x) dx + \int_{kp}^{a+p} f(x) dx$$



$$\int_a^{a+p} f(x) dx = \int_0^p f(x) dx \text{ pre všetky } a \in \mathbb{R}$$

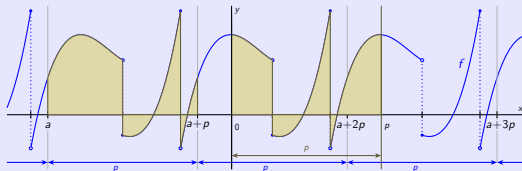
Integrovanie periodických funkcií

f je periodická s periódou $p > 0$, $a \in \mathbb{R}$.

$$f \in R_{\langle a; a+p \rangle} \Leftrightarrow f \in R_{\langle 0; p \rangle} \text{ a pokiaľ existujú, platí } \int_0^p f(x) dx = \int_a^{a+p} f(x) dx.$$

$k \in \mathbb{Z}$ je také, že $a \in \langle (k-1)p; kp \rangle$.

$$\begin{aligned} \Rightarrow \int_a^{a+p} f(x) dx &= \int_a^{kp} f(x) dx + \int_{kp}^{a+p} f(x) dx \\ &= \int_a^{kp} f(x) dx + \int_{kp-p}^a f(x) dx \end{aligned}$$



$$\int_a^{a+p} f(x) dx = \int_0^p f(x) dx \text{ pre všetky } a \in \mathbb{R}$$

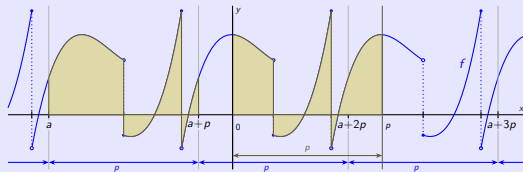
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$$f \in R_{\langle a; a+p \rangle} \Leftrightarrow f \in R_{\langle 0; p \rangle} \text{ a pokiaľ existujú, platí } \int_0^p f(x) dx = \int_a^{a+p} f(x) dx.$$

$k \in \mathbb{Z}$ je také, že $a \in \langle (k-1)p; kp \rangle$.

$$\begin{aligned} \Rightarrow \int_a^{a+p} f(x) dx &= \int_a^{kp} f(x) dx + \int_{kp}^{a+p} f(x) dx \\ &= \int_a^{kp} f(x) dx + \int_{kp-p}^a f(x) dx = \int_{kp-p}^{kp} f(x) dx \end{aligned}$$



$$\int_a^{a+p} f(x) dx = \int_0^p f(x) dx \text{ pre všetky } a \in \mathbb{R}$$

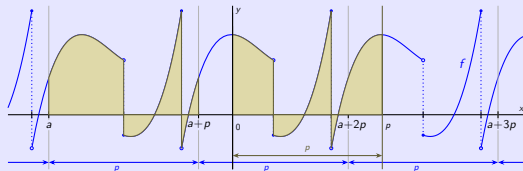
Integrovanie periodických funkcií

f je periodická s periódou $p > 0$, $a \in \mathbb{R}$.

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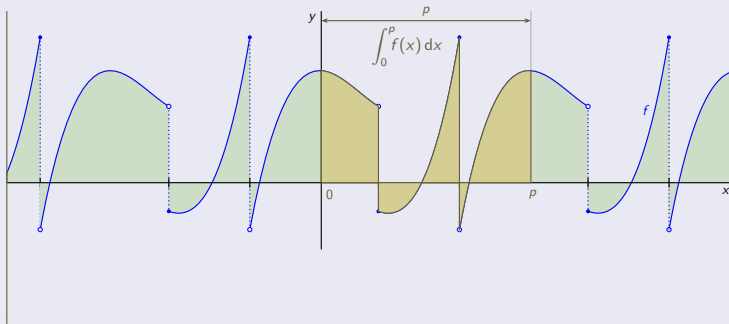


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Integrovanie periodických funkcií

$$\int_a^{a+2\pi} \sin^2(nx) \, dx$$

$$a \in \mathbb{R}, n \in \mathbb{N}$$

$$\int_a^{a+2\pi} \cos^2(nx) \, dx$$

$$a \in \mathbb{R}, n \in \mathbb{N}$$

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$$a \in \mathbb{R}, n \in \mathbb{N}$$

$$= \left[f(x) = \sin^2(nx) \text{ je spojitá na } \mathbb{R} \Rightarrow f \in R_{(a; a+2\pi)} \right]$$

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Integrovanie periodických funkcií

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 &= \int_0^{2\pi} \left[\frac{1}{2} - \frac{\cos(2nx)}{2} \right] \, dx
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 &= \pi - 0 - 0 + 0
 \end{aligned}$$

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 &= \pi + 0 - 0 - 0
 \end{aligned}$$

Integrovanie periodických funkcií

$$\int_a^{a+2\pi} \sin^2(nx) dx = \pi$$

$$a \in \mathbb{R}, n \in \mathbb{N}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} f(x) = \sin^2(nx) \text{ je spojitá na } \mathbb{R} \Rightarrow f \in R_{(a; a+2\pi)} \\ f(x) \text{ je periodická s periódou } 2\pi \Rightarrow f \in R_{(0; 2\pi)} \end{array} \right] = \int_0^{2\pi} \sin^2(nx) dx = \int_0^{2\pi} \frac{1 - \cos(2nx)}{2} dx \\
 &= \int_0^{2\pi} \left[\frac{x}{2} - \frac{\cos(2nx)}{2} \right] dx = \left[\frac{x}{2} - \frac{\sin(2nx)}{2 \cdot 2n} \right]_0^{2\pi} = \frac{2\pi}{2} - \frac{\sin(2n \cdot 2\pi)}{4n} - \frac{0}{2} + \frac{\sin 0}{4n} \\
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 \end{aligned}$$

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Integrovanie periodických funkcií

$$\int_a^{a+2\pi} \sin(mx) \cdot \sin(nx) dx$$

$$a \in \mathbb{R}, m, n \in \mathbb{N}, m \neq n$$

Integrovanie periodických funkcií

$$\int_a^{a+2\pi} \sin(mx) \cdot \sin(nx) dx$$

$$a \in \mathbb{R}, m, n \in \mathbb{N}, m \neq n$$

$$= \left[f(x) = \sin(mx) \cdot \sin(nx) \text{ je spojitá na } \mathbb{R} \Rightarrow f \in R_{(a; a+2\pi)} \right]$$



Integrovanie periodických funkcií

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$$a \in \mathbb{R}, m, n \in \mathbb{N}, m \neq n$$

$$= \left[\begin{array}{ll} f(x) = \sin(mx) \cdot \sin(nx) \text{ je spojitá na } \mathbb{R} \Rightarrow f \in R_{(a; a+2\pi)} & \text{?} \\ f(x) \text{ je periodická s periódou } 2\pi & \Rightarrow f \in R_{(0; 2\pi)} \end{array} \right]$$

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$$= \int_0^{2\pi} \frac{\cos(mx - nx) - \cos(mx + nx)}{2} dx$$

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$$= \int_0^{2\pi} \frac{\cos(mx - nx) - \cos(mx + nx)}{2} dx = \frac{1}{2} \int_0^{2\pi} [\cos(m-n)x - \cos(m+n)x] dx$$

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$$= \frac{1}{2} \left[\frac{\sin(m-n)x}{m-n} - \frac{\sin(m+n)x}{m+n} \right]_0^{2\pi}$$

Integrovanie periodických funkcií

$$\int_a^{a+2\pi} \sin(mx) \cdot \sin(nx) dx$$

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$$= \frac{1}{2} \left[\frac{\sin(m-n)x}{m-n} - \frac{\sin(m+n)x}{m+n} \right]_0^{2\pi} = \frac{1}{2} \left[\frac{\sin(m-n) \cdot 2\pi}{m-n} - \frac{\sin(m+n) \cdot 2\pi}{m+n} - \frac{\sin 0}{m-n} + \frac{\sin 0}{m+n} \right]$$

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$$= \frac{1}{2} [0 - 0 - 0 + 0]$$

Integrovanie periodických funkcií

$$\int_a^{a+2\pi} \sin(mx) \cdot \sin(nx) dx = 0$$

$$a \in \mathbb{R}, m, n \in \mathbb{N}, m \neq n$$

$$\begin{aligned}
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 &= \frac{1}{2} \left[\frac{\sin(m-n)x}{m-n} - \frac{\sin(m+n)x}{m+n} \right]_0^{2\pi} = \frac{1}{2} \left[\frac{\sin(m-n) \cdot 2\pi}{m-n} - \frac{\sin(m+n) \cdot 2\pi}{m+n} - \frac{\sin 0}{m-n} + \frac{\sin 0}{m+n} \right] \\
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$$a \in \mathbb{R}, m, n \in \mathbb{N}, m \neq n$$



Integrovanie periodických funkcií

$$\int_a^{a+2\pi} \sin(mx) \cdot \sin(nx) dx = 0$$

$$a \in R, m, n \in N, m \neq n$$

$$= \left[\begin{array}{l} f(x) = \sin(mx) \cdot \sin(nx) \text{ je spojitá na } R \Rightarrow f \in R_{(a; a+2\pi)} \\ f(x) \text{ je periodická s periódou } 2\pi \Rightarrow f \in R_{(0; 2\pi)} \end{array} \right] = \int_0^{2\pi} \sin(mx) \cdot \sin(nx) dx$$

$$= \int_0^{2\pi} \frac{\cos(mx-nx) - \cos(mx+nx)}{2} dx = \frac{1}{2} \int_0^{2\pi} [\cos(m-n)x - \cos(m+n)x] dx$$

$$= \frac{1}{2} \left[\frac{\sin(m-n)x}{m-n} - \frac{\sin(m+n)x}{m+n} \right]_0^{2\pi} = \frac{1}{2} \left[\frac{\sin(m-n) \cdot 2\pi}{m-n} - \frac{\sin(m+n) \cdot 2\pi}{m+n} - \frac{\sin 0}{m-n} + \frac{\sin 0}{m+n} \right]$$

$$= \frac{1}{2} [0 - 0 - 0 + 0] = 0.$$

$$\int_a^{a+2\pi} \sin(mx) \cdot \cos(nx) dx$$

$$a \in R, m, n \in N, m \neq n$$

$$= \left[f(x) = \sin(mx) \cdot \cos(nx) \text{ je spojitá na } R \Rightarrow f \in R_{(a; a+2\pi)} \right]$$

Integrovanie periodických funkcií

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 &= \left[\begin{array}{l} f(x) = \sin(mx) \cdot \sin(nx) \text{ je spojitá na } R \Rightarrow f \in R_{(a; a+2\pi)} \\ f(x) \text{ je periodická s periódou } 2\pi \Rightarrow f \in R_{(0; 2\pi)} \end{array} \right] \stackrel{!}{=} \int_0^{2\pi} \sin(mx) \cdot \sin(nx) dx \\
 &= \int_0^{2\pi} \frac{\cos(mx-nx) - \cos(mx+nx)}{2} dx = \frac{1}{2} \int_0^{2\pi} [\cos(m-n)x - \cos(m+n)x] dx \\
 &= \frac{1}{2} \left[\frac{\sin(m-n)x}{m-n} - \frac{\sin(m+n)x}{m+n} \right]_0^{2\pi} = \frac{1}{2} \left[\frac{\sin(m-n) \cdot 2\pi}{m-n} - \frac{\sin(m+n) \cdot 2\pi}{m+n} - \frac{\sin 0}{m-n} + \frac{\sin 0}{m+n} \right] \\
 &= \frac{1}{2} [0 - 0 - 0 + 0] = 0.
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Integrovanie periodických funkcií

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Integrovanie periodických funkcií

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 &= \frac{1}{2} \left[\frac{\sin(m-n)x}{m-n} + \frac{\sin(m+n)x}{m+n} \right]_0^{2\pi}
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$$a \in \mathbb{R}, n \in \mathbb{N}$$



Integrovanie periodických funkcií

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$$\int_a^{a+2\pi} \sin(nx) \cdot \cos(nx) dx$$

$$a \in \mathbb{R}, n \in \mathbb{N}$$

$$= \left[\begin{array}{l} f(x) = \sin(nx) \cdot \cos(nx) \text{ je spojitá na } \mathbb{R} \Rightarrow f \in R_{[a; a+2\pi]} \\ f(x) \text{ je periodická s periódou } 2\pi \Rightarrow f \in R_{(-\pi; \pi)} \end{array} \right] \stackrel{b}{=} \int_{-\pi}^{\pi} \sin(nx) \cdot \cos(nx) dx = \left[\begin{array}{l} f \text{ je} \\ \text{nepárna} \end{array} \right]$$

Integrovanie periodických funkcií

$$\int_a^{a+2\pi} \cos(mx) \cdot \cos(nx) dx = 0$$

$$a \in \mathbb{R}, m, n \in \mathbb{N}, m \neq n$$

$$= \left[\begin{array}{l} f(x) = \cos(mx) \cdot \cos(nx) \text{ je spojitá na } \mathbb{R} \Rightarrow f \in R_{[a; a+2\pi]} \\ f(x) \text{ je periodická s periódou } 2\pi \Rightarrow f \in R_{(0; 2\pi)} \end{array} \right] \stackrel{b}{=} \int_0^{2\pi} \cos(mx) \cdot \cos(nx) dx$$

$$= \int_0^{2\pi} \frac{\cos(mx-nx) + \cos(mx+nx)}{2} dx = \frac{1}{2} \int_0^{2\pi} [\cos(m-n)x + \cos(m+n)x] dx$$

$$= \frac{1}{2} \left[\frac{\sin(m-n)x}{m-n} + \frac{\sin(m+n)x}{m+n} \right]_0^{2\pi} = \frac{1}{2} \left[\frac{\sin(m-n) \cdot 2\pi}{m-n} + \frac{\sin(m+n) \cdot 2\pi}{m+n} - \frac{\sin 0}{m-n} - \frac{\sin 0}{m+n} \right]$$

$$= \frac{1}{2} [0 + 0 - 0 - 0] = 0.$$

$$\int_a^{a+2\pi} \sin(nx) \cdot \cos(nx) dx = 0$$

$$a \in \mathbb{R}, n \in \mathbb{N}$$

$$= \left[\begin{array}{l} f(x) = \sin(nx) \cdot \cos(nx) \text{ je spojitá na } \mathbb{R} \Rightarrow f \in R_{[a; a+2\pi]} \\ f(x) \text{ je periodická s periódou } 2\pi \Rightarrow f \in R_{(-\pi; \pi)} \end{array} \right] \stackrel{b}{=} \int_{-\pi}^{\pi} \sin(nx) \cdot \cos(nx) dx = \left[\begin{array}{l} f \text{ je} \\ \text{nepárna} \end{array} \right] = 0.$$

Koniec 11. časti

Ďakujem za pozornosť.