Matematická analýza 1

2024/2025

10. Neurčitý integrál
Riešené príklady

Pre správne zobrazenie, fungovanie tooltipov, 2D a 3D animácií je nevyhnutné súbor otvoriť pomocou programu Adobe Reader (zásuvný modul Adobe PDF Plug-In webového prehliadača nestačí).

Kliknutím na text pred ikonou získate nápomoc.
Kliknutím na skratku v modrej lište vpravo hore sa dostanete na príslušný slajd, druhým kliknutím sa dostanete na koniec tohto slajdu.

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Obsah – Riešené príklady

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Zoznam integrálov I

001. $\int \operatorname{cotg} x\,dx$ 002. $\int \operatorname{cotg} 3x\,dx$ 003. $\int \operatorname{cotg}(-x)\,dx$ 004. $\int \operatorname{tg} x\,dx$ 005. $\int \operatorname{tg} 3x\,dx$ 006. $\int \operatorname{tg}(-x)\,dx$ 007. $\int \sqrt[5]{x^3}\,dx$ 008. $\int \left[2\cos x + x^3 + \frac{3}{x^2+1}\right]dx$ 009. $\int |x|\,dx$ 010. $\int \min\limits_{x\in(0;\infty)}\{1,\frac{1}{x}\}\,dx$

011. $\int \operatorname{arctg} x\,dx$ 012. $\int \frac{dx}{\sin x\cos x}$ 013. $\int \frac{dx}{\sqrt[3]{x}+1}$ 014. $\int \frac{dx}{\sin^2 x\cos^2 x}$ 015. $\int \frac{(x-1)^2}{x}\,dx$ 016. $\int \frac{x^2}{x-1}\,dx$ 017. $\int \operatorname{tg}^2 x\,dx$ 018. $\int \operatorname{tgh}^2 x\,dx$ 019. $\int \operatorname{cotg}^2 x\,dx$ 020. $\int \operatorname{cotgh}^2 x\,dx$ 021. $\int \sin^3 x\cdot\cos x\,dx$ 022. $\int \frac{x^3\,dx}{x^8+1}$

023. $\int \frac{f'(x)}{f(x)}\,dx$ 024. $\int \frac{f'(t)}{f(t)}\,dt$ 025. $\int f(ax+b)\,dx$ 026. $\int \frac{dx}{x+1}$ 027. $\int \sin(-x)\,dx$ 028. $\int \frac{(x+1)^3}{(1-x)^2}\,dx$ 029. $\int \frac{(x-2)^4}{(x-1)^2}\,dx$ 030. $\int \frac{(x-1)^2}{(x-2)^4}\,dx$ 031. $\int \frac{(x-1)^4}{(x-2)^2}\,dx$ 032. $\int \frac{(x-2)^2}{(x-1)^4}\,dx$ 033. $\int \frac{x\,dx}{x-1}$ 034. $\int \frac{x^2\,dx}{x-1}$ 035. $\int \frac{x^6\,dx}{x-1}$

036. $\int \frac{x^9\,dx}{x-1}$ 037. $\int \frac{x^n\,dx}{x-1}$ 038. $\int \frac{dx}{x-t}$ 039. $\int \frac{dx}{(x-t)^n}$ 040. $\int \frac{dt}{(x-t)^n}$ 041. $\int \frac{x^2\,dx}{x^3+1}$ 042. $\int \frac{x^2\,dx}{x^6+1}$ 043. $\int \frac{x^2\,dx}{x^6-1}$ 044. $\int |x-a|^{98}\,dx$ 045. $\int |x-a|^{99}\,dx$ 046. $\int |x-a|\cdot(x-a)^{98}\,dx$ 047. $\int |x-a|\cdot(x-a)^{99}\,dx$

048. $\int (x-1)(x-2)(x-3)\,dx$ 049. $\int (x-1)(x+2)(x-3)\,dx$ 050. $\int x(x-a)(x-b)\,dx$ 051. $\int \frac{dx}{x^2+ax+b}$ 052. $\int \frac{dx}{x^2+4x+2}$ 053. $\int \frac{dx}{x^2+4x+3}$ 054. $\int \frac{dx}{x^2+4x+4}$ 055. $\int \frac{dx}{x^2+4x+5}$ 056. $\int \frac{dx}{x^2+4x+6}$ 057. $\int \frac{dx}{x^2+11x+4}$

058. $\int \frac{dx}{x^2-11x+4}$ 059. $\int \frac{dx}{x^2+9x+25}$ 060. $\int \frac{dx}{x^2-9x+25}$ 061. $\int \frac{2x+1}{x^2-9x+25}\,dx$ 062. $\int \frac{2x+3}{(x^2+2x+3)^2}\,dx$ 063. $\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx$ 064. $\int \frac{-2x^2+1}{x^3+2x^2+x}\,dx$ 065. $\int \frac{x}{x^2+a^2}\,dx$ 066. $\int \frac{x\,dx}{(x^2+a^2)^n}$ 067. $\int \frac{x}{x^2+3}\,dx$ 068. $\int \frac{x\,dx}{(x^2+3)^6}$

069. $\int \frac{2x\,dx}{(x^2+33)^{2024}}$ 070. $\int \frac{x}{x^2-a^2}\,dx$ 071. $\int \frac{x\,dx}{(x^2-a^2)^n}$ 072. $\int \frac{x}{x^2-3}\,dx$ 073. $\int \frac{x\,dx}{(x^2-3)^6}$ 074. $\int \frac{2x\,dx}{(x^2-33)^{2024}}$ 075. $\int \frac{x^2}{x^2+a^2}\,dx$ 076. $\int \frac{x^2\,dx}{(x^2+a^2)^2}$ 077. $\int \frac{x^2\,dx}{(x^2+3)^2}$ 078. $\int \frac{dx}{x^2+a^2}$ 079. $\int \frac{dx}{(x^2+a^2)^n}$ 080. $\int \frac{dx}{(x^2+a^2)^2}$ 081. $\int \frac{dx}{(x^2+1)^2}$

082. $\int \frac{dx}{(x^2\pm 4x+5)^2}$ 083. $\int \frac{dx}{(x^2+a^2)^3}$ 084. $\int \frac{dx}{(x^2+a^2)^4}$ 085. $\int \frac{dx}{(x^2+3)^6}$ 086. $\int \frac{x^2}{x^2-a^2}\,dx$ 087. $\int \frac{x^2\,dx}{(x^2-a^2)^2}$ 088. $\int \frac{x^2\,dx}{(x^2-3)^2}$ 089. $\int \frac{dx}{x^2-a^2}$ 090. $\int \frac{dx}{(x^2-a^2)^n}$ 091. $\int \frac{dx}{(x^2-a^2)^2}$ 092. $\int \frac{dx}{(x^2-1)^2}$ 093. $\int \frac{dx}{(x^2+ax+b)^n}$

094. $\int \frac{dx}{(x^2\pm 4x+3)^2}$ 095. $\int \frac{dx}{(x^2-a^2)^3}$ 096. $\int \frac{dx}{(x^2-a^2)^4}$ 097. $\int \frac{dx}{(x^2-3)^6}$ 098. $\int \frac{dx}{(x^2-1)^6}$ 099. $\int \frac{dx}{(x^2+4x+3)^2}$ 100. $\int \frac{dx}{(x^2+4x+2)^2}$ 101. $\int \frac{dx}{(x^2+4x+5)^2}$ 102. $\int \frac{dx}{(x^2+4x+6)^2}$ 103. $\int \frac{dx}{(x^2+4x+3)^3}$ 104. $\int \frac{dx}{(x^2+4x+3)^4}$ 105. $\int \frac{dx}{(1-x)x^2}$

106. $\int \frac{dx}{(x+1)x^2}$ 107. $\int \frac{dx}{(x+2)x^2}$ 108. $\int \frac{dx}{x^3-7x+6}$ 109. $\int \frac{dx}{x^3-3x-2}$ 110. $\int \frac{dx}{x^3-3x+2}$ 111. $\int \frac{dx}{x^3+x-2}$ 112. $\int \frac{dx}{x^3-x^2+2}$ 113. $\int \frac{dx}{x^3-x^2-4}$ 114. $\int \frac{dx}{x^3-2x-4}$ 115. $\int \frac{dx}{x^3+6x^2+11x+6}$ 116. $\int \frac{dx}{x^3-6x^2+11x-6}$ 117. $\int \frac{dx}{x^3-2x^2-x+2}$

118. $\int \frac{dx}{x^3-x^2-4x+4}$ 119. $\int \frac{dx}{x^3+x^2-4x-4}$ 120. $\int \frac{dx}{x^3-3x^2-x+3}$ 121. $\int \frac{dx}{x^3-4x^2+x+6}$ 122. $\int \frac{dx}{x^3+4x^2+5x+2}$ 123. $\int \frac{dx}{x^3-4x^2+5x-2}$ 124. $\int \frac{dx}{x^3-5x^2+8x-4}$ 125. $\int \frac{dx}{x^3+5x^2+8x+4}$ 126. $\int \frac{dx}{x^3-7x^2+15x-9}$ 127. $\int \frac{dx}{x^3-x^2-x+1}$

128. $\int \frac{dx}{x^3+x^2-x-1}$ 129. $\int \frac{dx}{x^3-5x^2+7x-3}$ 130. $\int \frac{dx}{x^3-3x^2+4x-4}$ 131. $\int \frac{dx}{x^3-3x^2+4x-2}$ 132. $\int \frac{dx}{x^3+2x^2+3x+2}$ 133. $\int \frac{dx}{x^3+3x^2+4x+2}$ 134. $\int \frac{dx}{x^3-x^2+x+3}$ 135. $\int \frac{dx}{x^3-3x^2+5x-3}$ 136. $\int \frac{dx}{x^3-x+6}$ 137. $\int \frac{dx}{x^3+3x^2+5x+3}$

138. $\int \frac{dx}{x^3-4x^2+7x-6}$ 139. $\int \frac{dx}{x^3+3x^2+6x+4}$ 140. $\int \frac{x-1}{(x^2-2x+2)^3}\,dx$ 141. $\int \frac{x+1}{(x^2+2x+2)^3}\,dx$ 142. $\int \frac{x+1}{(x^2+2x+3)^3}\,dx$ 143. $\int \frac{x^6-x^5+x^4-x^3+x+1}{x^4-2x^3+2x^2-2x+1}\,dx$ 144. $\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx$ 145. $\int \frac{dx}{x^6+1}$ 146. $\int \frac{x\,dx}{x^6+1}$ 147. $\int \frac{x^2\,dx}{x^6+1}$

148. $\int \frac{x^3\,dx}{x^6+1}$ 149. $\int \frac{x^4\,dx}{x^6+1}$ 150. $\int \frac{x^5\,dx}{x^6+1}$ 151. $\int \frac{x^6\,dx}{x^6+1}$ 152. $\int \frac{dx}{x^6(x^2+1)}$ 153. $\int \frac{dx}{\sin x}$ 154. $\int \frac{dx}{1+\sin x}$ 155. $\int \frac{dx}{1-\sin x}$ 156. $\int \frac{dx}{5+4\sin x}$ 157. $\int \frac{dx}{5-4\sin x}$ 158. $\int \frac{dx}{4+5\sin x}$ 159. $\int \frac{dx}{4-5\sin x}$ 160. $\int \frac{dx}{\cos x}$ 161. $\int \frac{dx}{1+\cos x}$ 162. $\int \frac{dx}{1-\cos x}$

163. 164. 165. 166. 167. 168. 169. 170.

Riešené príklady – 001, 002, 003

$\int \operatorname{cotg} x \, dx$ [001]

$\int \operatorname{cotg} 3x \, dx$ [002]

$\int \operatorname{cotg} (-x) \, dx$ [003]

Riešené príklady – 001, 002, 003

$$\int \operatorname{cotg} x \,dx = \int \frac{\cos x}{\sin x}\,dx$$ [001]

$$\int \operatorname{cotg} 3x \,dx = \int \frac{\cos 3x}{\sin 3x}\,dx$$ [002]

$$\int \operatorname{cotg}(-x)\,dx$$ [003]

- $= \displaystyle\int \frac{\cos(-x)}{\sin(-x)}\,dx = -\int \frac{-\cos(-x)}{\sin(-x)}\,dx$

- $= \left[\text{Nepárna funkcia}\right] = -\displaystyle\int \operatorname{cotg} x\,dx = -\int \frac{\cos x}{\sin x}\,dx$

Riešené príklady – 001, 002, 003

$$\int \operatorname{cotg} x \, dx = \int \frac{\cos x}{\sin x} \, dx$$ [001]

- $= \int \frac{[\sin x]'}{\sin x} \, dx$

$$\int \operatorname{cotg} 3x \, dx = \int \frac{\cos 3x}{\sin 3x} \, dx$$ [002]

- $= \frac{1}{3} \int \frac{3 \cos 3x}{\sin 3x} \, dx = \frac{1}{3} \int \frac{[\sin 3x]'}{\sin 3x} \, dx$

$$\int \operatorname{cotg} (-x) \, dx$$ [003]

- $= \int \frac{\cos (-x)}{\sin (-x)} \, dx = -\int \frac{-\cos (-x)}{\sin (-x)} \, dx = -\int \frac{[\sin (-x)]'}{\sin (-x)} \, dx$

- $= \left[\text{Nepárna funkcia} \right] = -\int \operatorname{cotg} x \, dx = -\int \frac{\cos x}{\sin x} \, dx = -\int \frac{[\sin x]'}{\sin x} \, dx$

Riešené príklady – 001, 002, 003

$$\int \operatorname{cotg} x \, dx = \int \frac{\cos x}{\sin x} \, dx = \ln |\sin x| + c$$ [001]

- $= \int \frac{[\sin x]'}{\sin x} \, dx = \ln |\sin x| + c, \ x \in R, \ x \neq k\pi, \ k \in Z, \ c \in R.$

$$\int \operatorname{cotg} 3x \, dx = \int \frac{\cos 3x}{\sin 3x} \, dx = \frac{1}{3} \ln |\sin 3x| + c$$ [002]

- $= \frac{1}{3} \int \frac{3 \cos 3x}{\sin 3x} \, dx = \frac{1}{3} \int \frac{[\sin 3x]'}{\sin 3x} \, dx = \frac{1}{3} \ln |\sin 3x| + c, \ x \in R, \ x \neq \frac{k\pi}{3}, \ k \in Z, \ c \in R.$

$$\int \operatorname{cotg} (-x) \, dx = -\ln |\sin (-x)| + c = -\ln |\sin x| + c$$ [003]

- $= \int \frac{\cos (-x)}{\sin (-x)} \, dx = -\int \frac{-\cos (-x)}{\sin (-x)} \, dx = -\int \frac{[\sin (-x)]'}{\sin (-x)} \, dx = -\ln |\sin (-x)| + c$

- $= \left[\text{Nepárna funkcia} \right] = -\int \operatorname{cotg} x \, dx = -\int \frac{\cos x}{\sin x} \, dx = -\int \frac{[\sin x]'}{\sin x} \, dx = -\ln |\sin x| + c,$ $x \in R, \ x \neq k\pi, \ k \in Z, \ c \in R.$

Riešené príklady – 001, 002, 003

$$\int \operatorname{cotg} x \, dx = \int \frac{\cos x}{\sin x}\, dx = \ln|\sin x| + c$$ [001]

- $= \int \frac{[\sin x]'}{\sin x}\, dx = \ln|\sin x| + c,\ x \in R,\ x \neq k\pi,\ k \in Z,\ c \in R.$

$$\int \operatorname{cotg} 3x \, dx = \int \frac{\cos 3x}{\sin 3x}\, dx = \frac{1}{3}\ln|\sin 3x| + c$$ [002]

- $= \frac{1}{3}\int \frac{3\cos 3x}{\sin 3x}\, dx = \frac{1}{3}\int \frac{[\sin 3x]'}{\sin 3x}\, dx = \frac{1}{3}\ln|\sin 3x| + c,\ x \in R,\ x \neq \frac{k\pi}{3},\ k \in Z,\ c \in R.$

$$\int \operatorname{cotg}(-x)\, dx = -\ln|\sin(-x)| + c = -\ln|\sin x| + c$$ [003]

- $= \int \frac{\cos(-x)}{\sin(-x)}\, dx = -\int \frac{-\cos(-x)}{\sin(-x)}\, dx = -\int \frac{[\sin(-x)]'}{\sin(-x)}\, dx = -\ln|\sin(-x)| + c$
  $= \left[\text{Nepárna funkcia}\right] = -\ln|-\sin x| + c$

- $= \left[\begin{array}{c}\text{Nepárna}\\ \text{funkcia}\end{array}\right] = -\int \operatorname{cotg} x\, dx = -\int \frac{\cos x}{\sin x}\, dx = -\int \frac{[\sin x]'}{\sin x}\, dx = -\ln|\sin x| + c,$
  $x \in R,\ x \neq k\pi,\ k \in Z,\ c \in R.$

Riešené príklady – 001, 002, 003

$$\int \operatorname{cotg} x\,dx = \int \frac{\cos x}{\sin x}\,dx = \ln|\sin x| + c$$ [001]

- $= \int \frac{[\sin x]'}{\sin x}\,dx = \ln|\sin x| + c,\ x\in R,\ x\neq k\pi,\ k\in Z,\ c\in R.$

$$\int \operatorname{cotg} 3x\,dx = \int \frac{\cos 3x}{\sin 3x}\,dx = \frac{1}{3}\ln|\sin 3x| + c$$ [002]

- $= \frac{1}{3}\int \frac{3\cos 3x}{\sin 3x}\,dx = \frac{1}{3}\int \frac{[\sin 3x]'}{\sin 3x}\,dx = \frac{1}{3}\ln|\sin 3x| + c,\ x\in R,\ x\neq \frac{k\pi}{3},\ k\in Z,\ c\in R.$

$$\int \operatorname{cotg}(-x)\,dx = -\ln|\sin(-x)| + c = -\ln|\sin x| + c$$ [003]

- $= \int \frac{\cos(-x)}{\sin(-x)}\,dx = -\int \frac{-\cos(-x)}{\sin(-x)}\,dx = -\int \frac{[\sin(-x)]'}{\sin(-x)}\,dx = -\ln|\sin(-x)| + c$
  $= \left[\text{Nepárna funkcia}\right] = -\ln|-\sin x| + c = -\ln|\sin x| + c,\ x\in R,\ x\neq k\pi,\ k\in Z,\ c\in R.$

- $= \left[\text{Nepárna funkcia}\right] = -\int \operatorname{cotg} x\,dx = -\int \frac{\cos x}{\sin x}\,dx = -\int \frac{[\sin x]'}{\sin x}\,dx = -\ln|\sin x| + c,$
  $x\in R,\ x\neq k\pi,\ k\in Z,\ c\in R.$

Riešené príklady – 004, 005, 006

$\int \operatorname{tg} x \, dx$ [004]

$\int \operatorname{tg} 3x \, dx$ [005]

$\int \operatorname{tg} (-x) \, dx$ [006]

Riešené príklady – 004, 005, 006

$$\int \operatorname{tg} x \, \mathrm{d}x = \int \frac{\sin x}{\cos x} \, \mathrm{d}x$$ [004]

$$\int \operatorname{tg} 3x \, \mathrm{d}x = \int \frac{\sin 3x}{\cos 3x} \, \mathrm{d}x$$ [005]

$$\int \operatorname{tg} (-x) \, \mathrm{d}x$$ [006]

- $= \displaystyle\int \frac{\sin(-x)}{\cos(-x)} \, \mathrm{d}x$

- $= \left[\begin{matrix} \text{Nepárna} \\ \text{funkcia} \end{matrix} \right] = -\displaystyle\int \operatorname{tg} x \, \mathrm{d}x$

Riešené príklady – 004, 005, 006

$$\int \operatorname{tg} x \,\mathrm{d}x = \int \frac{\sin x}{\cos x}\,\mathrm{d}x$$ [004]

- $= -\int \frac{-\sin x}{\cos x}\,\mathrm{d}x = -\int \frac{[\cos x]'}{\cos x}\,\mathrm{d}x$

$$\int \operatorname{tg} 3x \,\mathrm{d}x = \int \frac{\sin 3x}{\cos 3x}\,\mathrm{d}x$$ [005]

- $= -\frac{1}{3}\int \frac{-3\sin 3x}{\cos 3x}\,\mathrm{d}x = -\frac{1}{3}\int \frac{[\cos 3x]'}{\cos 3x}\,\mathrm{d}x$

$$\int \operatorname{tg}(-x)\,\mathrm{d}x$$ [006]

- $= \int \frac{\sin(-x)}{\cos(-x)}\,\mathrm{d}x = \int \frac{[\cos(-x)]'}{\cos(-x)}\,\mathrm{d}x =$

- $= \left[\begin{array}{c}\text{Nepárna}\\ \text{funkcia}\end{array}\right] = -\int \operatorname{tg} x\,\mathrm{d}x = \int \frac{-\sin x}{\cos x}\,\mathrm{d}x = \int \frac{[\cos x]'}{\cos x}\,\mathrm{d}x$

Riešené príklady – 004, 005, 006

$$\int \operatorname{tg} x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\ln|\cos x| + c$$ [004]

- $= -\int \frac{-\sin x}{\cos x} \, dx = -\int \frac{[\cos x]'}{\cos x} \, dx = -\ln|\cos x| + c, \; x \in R, \; x \neq \frac{\pi}{2} + k\pi, \; k \in Z, \; c \in R.$

$$\int \operatorname{tg} 3x \, dx = \int \frac{\sin 3x}{\cos 3x} \, dx = -\frac{1}{3} \ln|\cos 3x| + c$$ [005]

- $= -\frac{1}{3} \int \frac{-3 \sin 3x}{\cos 3x} \, dx = -\frac{1}{3} \int \frac{[\cos 3x]'}{\cos 3x} \, dx = -\frac{1}{3} \ln|\cos 3x| + c,$
  $x \in R, \; x \neq \frac{\pi}{6} + \frac{k\pi}{3}, \; k \in Z, \; c \in R.$

$$\int \operatorname{tg}(-x) \, dx = \ln|\cos(-x)| + c = \ln|\cos x| + c$$ [006]

- $= \int \frac{\sin(-x)}{\cos(-x)} \, dx = \int \frac{[\cos(-x)]'}{\cos(-x)} \, dx = \ln|\cos(-x)| + c$

- $= \left[\text{Nepárna funkcia} \right] = -\int \operatorname{tg} x \, dx = \int \frac{-\sin x}{\cos x} \, dx = \int \frac{[\cos x]'}{\cos x} \, dx = \ln|\cos x| + c,$
  $x \in R, \; x \neq \frac{\pi}{2} + k\pi, \; k \in Z, \; c \in R.$

Riešené príklady – 004, 005, 006

$$\int \operatorname{tg} x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\ln|\cos x| + c$$ [004]

- $= -\int \frac{-\sin x}{\cos x} \, dx = -\int \frac{[\cos x]'}{\cos x} \, dx = -\ln|\cos x| + c, \; x \in R, \; x \neq \frac{\pi}{2} + k\pi, \; k \in Z, \; c \in R.$

$$\int \operatorname{tg} 3x \, dx = \int \frac{\sin 3x}{\cos 3x} \, dx = -\tfrac{1}{3} \ln|\cos 3x| + c$$ [005]

- $= -\frac{1}{3} \int \frac{-3 \sin 3x}{\cos 3x} \, dx = -\frac{1}{3} \int \frac{[\cos 3x]'}{\cos 3x} \, dx = -\tfrac{1}{3} \ln|\cos 3x| + c,$
  $x \in R, \; x \neq \frac{\pi}{6} + \frac{k\pi}{3}, \; k \in Z, \; c \in R.$

$$\int \operatorname{tg}(-x) \, dx = \ln|\cos(-x)| + c = \ln|\cos x| + c$$ [006]

- $= \int \frac{\sin(-x)}{\cos(-x)} \, dx = \int \frac{[\cos(-x)]'}{\cos(-x)} \, dx = \ln|\cos(-x)| + c = \left[\text{Párna funkcia} \right] = \ln|\cos x| + c,$
  $x \in R, \; x \neq \frac{\pi}{2} + k\pi, \; k \in Z, \; c \in R.$

- $= \left[\begin{matrix} \text{Nepárna} \\ \text{funkcia} \end{matrix} \right] = -\int \operatorname{tg} x \, dx = \int \frac{-\sin x}{\cos x} \, dx = \int \frac{[\cos x]'}{\cos x} \, dx = \ln|\cos x| + c,$
  $x \in R, \; x \neq \frac{\pi}{2} + k\pi, \; k \in Z, \; c \in R.$

Riešené príklady – 007, 008, 009

$$\int \sqrt[5]{x^3}\, dx$$ [007]

$$\int \left[2\cos x + x^3 + \frac{3}{x^2+1}\right] dx$$ [008]

$$\int |x|\, dx$$ [009]

Riešené príklady – 007, 008, 009

$\int \sqrt[5]{x^3}\, dx$ [007]

- $= \int x^{\frac{3}{5}}\, dx$

$\int \left[2\cos x + x^3 + \frac{3}{x^2+1}\right] dx$ [008]

- $= 2\int \cos x\, dx + \int x^3\, dx + \int \frac{3\, dx}{x^2+1}$

$\int |x|\, dx$ [009]

Pre $x \in \langle 0; \infty)$ platí $|x| = x$:

- $= \int x\, dx$

Pre $x \in (-\infty; 0\rangle$ platí $|x| = -x$:

- $= \int (-x)\, dx = -\int x\, dx$

Riešené príklady – 007, 008, 009

$\int \sqrt[5]{x^3}\,dx$ [007]

- $= \int x^{\frac{3}{5}}\,dx = \frac{x^{\frac{3}{5}+1}}{\frac{3}{5}+1} + c = \frac{x^{\frac{8}{5}}}{\frac{8}{5}} + c$

$\int \left[2\cos x + x^3 + \frac{3}{x^2+1}\right] dx = 2\sin x + \frac{x^4}{4} + 3\,\mathrm{arctg}\, x + c$ [008]

- $= 2\int \cos x\,dx + \int x^3\,dx + \int \frac{3\,dx}{x^2+1} = 2\sin x + \frac{x^4}{4} + 3\,\mathrm{arctg}\, x + c,\ x \in R,\ c \in R.$

$\int |x|\,dx$ [009]

Pre $x \in \langle 0; \infty)$ platí $|x| = x$:

- $= \int x\,dx = \frac{x^2}{2} + c$

Pre $x \in (-\infty; 0\rangle$ platí $|x| = -x$:

- $= \int (-x)\,dx = -\int x\,dx = -\frac{x^2}{2} + c$

Riešené príklady – 007, 008, 009

$\int \sqrt[5]{x^3}\,dx = \frac{5}{8}x^{\frac{8}{5}} + c = \frac{5}{8}\sqrt[5]{x^8} + c$ [007]

- $= \int x^{\frac{3}{5}}\,dx = \frac{x^{\frac{3}{5}+1}}{\frac{3}{5}+1} + c = \frac{x^{\frac{8}{5}}}{\frac{8}{5}} + c = \frac{5}{8}x^{\frac{8}{5}} + c = \frac{5}{8}\sqrt[5]{x^8} + c,\ x \geq 0,\ c \in R.$

$\int \left[2\cos x + x^3 + \frac{3}{x^2+1}\right] dx = 2\sin x + \frac{x^4}{4} + 3\,\mathrm{arctg}\,x + c$ [008]

- $= 2\int \cos x\,dx + \int x^3\,dx + \int \frac{3\,dx}{x^2+1} = 2\sin x + \frac{x^4}{4} + 3\,\mathrm{arctg}\,x + c,\ x \in R,\ c \in R.$

$\int |x|\,dx$ [009]

Pre $x \in \langle 0; \infty)$ platí $|x| = x$:

- $= \int x\,dx = \frac{x^2}{2} + c = \frac{x \cdot x}{2} + c$

Pre $x \in (-\infty; 0\rangle$ platí $|x| = -x$:

- $= \int (-x)\,dx = -\int x\,dx = -\frac{x^2}{2} + c = \frac{x \cdot (-x)}{2} + c$

Riešené príklady – 007, 008, 009

$$\int \sqrt[5]{x^3}\,dx = \frac{5}{8}x^{\frac{8}{5}} + c = \frac{5}{8}\sqrt[5]{x^8} + c$$ [007]

- $= \int x^{\frac{3}{5}}\,dx = \frac{x^{\frac{3}{5}+1}}{\frac{3}{5}+1} + c = \frac{x^{\frac{8}{5}}}{\frac{8}{5}} + c = \frac{5}{8}x^{\frac{8}{5}} + c = \frac{5}{8}\sqrt[5]{x^8} + c,\ x \geq 0,\ c\in R.$

$$\int \left[2\cos x + x^3 + \frac{3}{x^2+1}\right] dx = 2\sin x + \frac{x^4}{4} + 3\operatorname{arctg} x + c$$ [008]

- $= 2\int \cos x\,dx + \int x^3\,dx + \int \frac{3\,dx}{x^2+1} = 2\sin x + \frac{x^4}{4} + 3\operatorname{arctg} x + c,\ x\in R,\ c\in R.$

$$\int |x|\,dx = \frac{x\,|x|}{2} + c,\ x\in R,\ c\in R.$$ [009]

Pre $x\in\langle 0;\infty)$ platí $|x| = x$:

- $= \int x\,dx = \frac{x^2}{2} + c = \frac{x\cdot x}{2} + c = \frac{x\,|x|}{2} + c,\ c\in R.$

Pre $x\in(-\infty;0\rangle$ platí $|x| = -x$:

- $= \int (-x)\,dx = -\int x\,dx = -\frac{x^2}{2} + c = \frac{x\cdot(-x)}{2} + c = \frac{x\,|x|}{2} + c,\ c\in R.$

Riešené príklady – 010, 011

$$\int \min_{x\in(0;\infty)} \left\{1, \frac{1}{x}\right\} \mathrm{d}x$$ [010]

$$\int \operatorname{arctg} x \, \mathrm{d}x$$ [011]

Riešené príklady – 010, 011

$$\int \min_{x\in(0;\infty)} \left\{1,\frac{1}{x}\right\} dx$$ [010]

Pre $x \in (0; 1\rangle$ platí $\min\left\{1,\frac{1}{x}\right\} = 1$:

- $= \int dx$

Pre $x \in \langle 1; \infty)$ platí $\min\left\{1,\frac{1}{x}\right\} = \frac{1}{x}$,

- $= \int \frac{dx}{x}$

$$\int \operatorname{arctg} x \, dx$$ [011]

- $= \int 1 \cdot \operatorname{arctg} x \, dx = \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \frac{1}{x^2+1} \end{array} \right]$

Riešené príklady – 010, 011

$\displaystyle\int \min_{x\in(0;\infty)} \left\{1, \frac{1}{x}\right\} dx$ [010]

Pre $x \in (0; 1\rangle$ platí $\min\left\{1, \frac{1}{x}\right\} = 1$:

- $\displaystyle = \int dx = x + c,\ c \in R.$

Pre $x \in \langle 1; \infty)$ platí $\min\left\{1, \frac{1}{x}\right\} = \frac{1}{x}$, $|x| = x$:

- $\displaystyle = \int \frac{dx}{x} = \ln|x| + c_1 = \ln x + c_1,\ c_1 \in R.$

$\displaystyle\int \operatorname{arctg} x\, dx$ [011]

- $\displaystyle = \int 1\cdot \operatorname{arctg} x\, dx = \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \frac{1}{x^2+1} \end{array}\right] = x \operatorname{arctg} x - \int \frac{x\, dx}{x^2+1} = x \operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{x^2+1}\, dx$

Riešené príklady – 010, 011

$$\int \min_{x\in(0;\infty)} \left\{1,\tfrac{1}{x}\right\} dx$$

[010]

Pre $x\in(0;1\rangle$ platí $\min\left\{1,\frac{1}{x}\right\}=1$:

- $= \int dx = x + c,\ c\in R.$

Pre $x\in\langle 1;\infty)$ platí $\min\left\{1,\frac{1}{x}\right\}=\frac{1}{x}$, $|x|=x$:

- $= \int \frac{dx}{x} = \ln|x| + c_1 = \ln x + c_1,\ c_1\in R.$

- Vypočítali sme primitívne funkcie, ale iba na intervale $(0;1\rangle$ a na intervale $\langle 1;\infty)$.

$$\int \operatorname{arctg} x\, dx$$

[011]

- $= \int 1\cdot \operatorname{arctg} x\, dx = \left[\begin{array}{l|l} u'=1 & u=x \\ v=\operatorname{arctg} x & v'=\frac{1}{x^2+1} \end{array}\right] = x \operatorname{arctg} x - \int \frac{x\, dx}{x^2+1} = x\operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{x^2+1}\, dx$
  $= x\operatorname{arctg} x - \frac{1}{2}\ln\left|x^2+1\right| + c$

Riešené príklady – 010, 011

$$\int \min_{x\in(0;\infty)} \left\{1,\tfrac{1}{x}\right\} dx$$ [010]

Pre $x\in(0;1\rangle$ platí $\min\left\{1,\frac{1}{x}\right\}=1$:

- $= \int dx = x + c,\ c\in R.$

Pre $x\in\langle 1;\infty)$ platí $\min\left\{1,\frac{1}{x}\right\}=\frac{1}{x}$, $|x|=x$:

- $= \int \frac{dx}{x} = \ln|x| + c_1 = \ln x + c_1,\ c_1\in R.$

- Vypočítali sme primitívne funkcie, ale iba na intervale $(0;1\rangle$ a na intervale $\langle 1;\infty)$.
- Ak chceme určiť primitívnu funkciu na intervale $(0;\infty)$, musíme zabezpečiť spojitosť primitívnej funkcie na celom intervale $(0;\infty)$, t. j. aj v bode $x=1$.

$$\int \operatorname{arctg} x\, dx = x\operatorname{arctg} x - \tfrac{1}{2}\ln\left(x^2+1\right) + c = x\operatorname{arctg} x - \ln\sqrt{x^2+1} + c$$ [011]

- $= \int 1\cdot\operatorname{arctg} x\, dx = \left[\begin{array}{l|l} u'=1 & u=x \\ v=\operatorname{arctg} x & v'=\frac{1}{x^2+1}\end{array}\right] = x\operatorname{arctg} x - \int\frac{x\,dx}{x^2+1} = x\operatorname{arctg} x - \frac{1}{2}\int\frac{2x}{x^2+1}\,dx$
  $= x\operatorname{arctg} x - \frac{1}{2}\ln\left|x^2+1\right| + c = x\operatorname{arctg} x - \frac{1}{2}\ln\left(x^2+1\right) + c$
  $= x\operatorname{arctg} x - \ln\sqrt{x^2+1} + c,\ x\in R,\ c\in R.$

Riešené príklady – 010, 011

$$\int \min_{x\in(0;\infty)} \left\{1, \tfrac{1}{x}\right\} dx$$ [010]

Pre $x\in(0;1\rangle$ platí $\min\{1,\frac{1}{x}\} = 1$:

- $= \int dx = x + c,\ c\in R.$

Pre $x\in\langle 1;\infty)$ platí $\min\{1,\frac{1}{x}\} = \frac{1}{x}$, $|x| = x$:

- $= \int \frac{dx}{x} = \ln|x| + c_1 = \ln x + c_1,\ c_1\in R.$

- Vypočítali sme primitívne funkcie, ale iba na intervale $(0;1\rangle$ a na intervale $\langle 1;\infty)$.
- Ak chceme určiť primitívnu funkciu na intervale $(0;\infty)$, musíme zabezpečiť spojitosť primitívnej funkcie na celom intervale $(0;\infty)$, t. j. aj v bode $x = 1$.
- To znamená, že pre $x = 1$ musí platiť $x + c = \ln x + c_1$, t. j. $1 + c = \ln 1 + c_1 = 0 + c_1 = c_1$,

$$\int \operatorname{arctg} x\, dx = x \operatorname{arctg} x - \tfrac{1}{2}\ln\left(x^2+1\right) + c = x\operatorname{arctg} x - \ln\sqrt{x^2+1} + c$$ [011]

- $= \int 1\cdot \operatorname{arctg} x\, dx = \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \frac{1}{x^2+1}\end{array}\right] = x\operatorname{arctg} x - \int \frac{x\,dx}{x^2+1} = x\operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{x^2+1}\,dx$
  $= x\operatorname{arctg} x - \frac{1}{2}\ln\left|x^2+1\right| + c = x\operatorname{arctg} x - \frac{1}{2}\ln\left(x^2+1\right) + c$
  $= x\operatorname{arctg} x - \ln\sqrt{x^2+1} + c,\ x\in R,\ c\in R.$

Riešené príklady – 010, 011

$\int \min_{x\in(0;\infty)} \{1, \frac{1}{x}\}\, dx$ sa rovná $x + c$ pre $x \in (0; 1\rangle$ a $\ln x + c + 1$ pre $x \in \langle 1; \infty)$, $c \in R$. [010]

Pre $x \in (0; 1\rangle$ platí $\min\{1, \frac{1}{x}\} = 1$:

- $= \int dx = x + c,\ x \in (0; 1\rangle,\ c \in R.$

Pre $x \in \langle 1; \infty)$ platí $\min\{1, \frac{1}{x}\} = \frac{1}{x}$, $|x| = x$:

- $= \int \frac{dx}{x} = \ln|x| + c_1 = \ln x + c_1 = \ln x + c + 1,\ x \in \langle 1; \infty),\ c_1 \in R,\ c \in R.$

- Vypočítali sme primitívne funkcie, ale iba na intervale $(0; 1\rangle$ a na intervale $\langle 1; \infty)$.
- Ak chceme určiť primitívnu funkciu na intervale $(0; \infty)$, musíme zabezpečiť spojitosť primitívnej funkcie na celom intervale $(0; \infty)$, t. j. aj v bode $x = 1$.
- To znamená, že pre $x = 1$ musí platiť $x + c = \ln x + c_1$, t. j. $1 + c = \ln 1 + c_1 = 0 + c_1 = c_1$, t. j. $c_1 = c + 1$.

$\int \operatorname{arctg} x\, dx = x \operatorname{arctg} x - \frac{1}{2} \ln(x^2+1) + c = x \operatorname{arctg} x - \ln\sqrt{x^2+1} + c$ [011]

- $= \int 1 \cdot \operatorname{arctg} x\, dx = \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \frac{1}{x^2+1} \end{array}\right] = x \operatorname{arctg} x - \int \frac{x\, dx}{x^2+1} = x \operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{x^2+1}\, dx$
  $= x \operatorname{arctg} x - \frac{1}{2}\ln|x^2+1| + c = x \operatorname{arctg} x - \frac{1}{2}\ln(x^2+1) + c$
  $= x \operatorname{arctg} x - \ln\sqrt{x^2+1} + c,\ x \in R,\ c \in R.$

Riešené príklady – 012, 013

$$\int \frac{\mathrm{d}x}{\sin x \cos x}$$ [012]

$$\int \frac{\mathrm{d}x}{\sqrt[3]{x}+1}$$ [013]

Riešené príklady – 012, 013

$$\int \frac{dx}{\sin x \cos x}$$ [012]

- $= \int \frac{\frac{1}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}}\, dx$

- $= \int \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}\, dx = \int \left[\frac{\cos^2 x}{\sin x \cos x} + \frac{\sin^2 x}{\sin x \cos x} \right] dx$

$$\int \frac{dx}{\sqrt[3]{x}+1}$$ [013]

- $= \left[\begin{array}{r|l|l} \text{Subst. } x = t^3 & t = \sqrt[3]{t} & x \in \langle 0; \infty) \\ dx = 3t^2\, dt & t^2 = \sqrt[3]{t^2} & t \in \langle 0; \infty) \end{array} \right]$

Riešené príklady – 012, 013

$\int \frac{dx}{\sin x \cos x}$ [012]

- $= \int \frac{\frac{1}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}}\, dx = \int \frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}}\, dx = \int \frac{(\operatorname{tg} x)'}{\operatorname{tg} x}\, dx$

- $= \int \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}\, dx = \int \left[\frac{\cos^2 x}{\sin x \cos x} + \frac{\sin^2 x}{\sin x \cos x}\right] dx = \int \left[\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right] dx$
  $= \int \left[\frac{\cos x}{\sin x} - \frac{-\sin x}{\cos x}\right] dx = \int \left[\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x}\right] dx$

$\int \frac{dx}{\sqrt[3]{x}+1}$ [013]

- $= \left[\begin{array}{c|c|c} \text{Subst. } x = t^3 & t = \sqrt[3]{t} & x \in \langle 0; \infty) \\ dx = 3t^2\, dt & t^2 = \sqrt[3]{t^2} & t \in \langle 0; \infty) \end{array}\right] = \int \frac{3t^2\, dt}{t+1} = 3 \int \frac{t^2}{t+1}\, dt$

Riešené príklady – 012, 013

$$\int \frac{dx}{\sin x \cos x} = \ln|\operatorname{tg} x| + c$$ [012]

- $= \int \frac{\frac{1}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}}\, dx = \int \frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}}\, dx = \int \frac{(\operatorname{tg} x)'}{\operatorname{tg} x}\, dx = \ln|\operatorname{tg} x| + c,\ x \in R,\ x \neq \frac{k\pi}{2},\ k \in Z,\ c \in R.$

- $= \int \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}\, dx = \int \left[\frac{\cos^2 x}{\sin x \cos x} + \frac{\sin^2 x}{\sin x \cos x}\right] dx = \int \left[\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right] dx$

  $= \int \left[\frac{\cos x}{\sin x} - \frac{-\sin x}{\cos x}\right] dx = \int \left[\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x}\right] dx = \ln|\sin x| - \ln|\cos x| + c$

  $= \ln\left|\frac{\sin x}{\cos x}\right| + c = \ln|\operatorname{tg} x| + c,\ x \in R,\ x \neq \frac{k\pi}{2},\ k \in Z,\ c \in R.$

$$\int \frac{dx}{\sqrt[3]{x}+1}$$ [013]

- $= \left[\begin{array}{c|c|c} \text{Subst. } x = t^3 & t = \sqrt[3]{t} & x \in \langle 0;\infty) \\ dx = 3t^2\, dt & t^2 = \sqrt[3]{t^2} & t \in \langle 0;\infty) \end{array}\right] = \int \frac{3t^2\, dt}{t+1} = 3\int \frac{t^2}{t+1}\, dt = 3\int \frac{(t^2+t)-(t+1)+1}{t+1}\, dt$

  $= 3\int \left[t - 1 + \frac{1}{t+1}\right] dt$

Riešené príklady – 012, 013

$$\int \frac{dx}{\sin x \cos x} = \ln|\operatorname{tg} x| + c$$ [012]

- $= \int \frac{\frac{1}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}}\, dx = \int \frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}}\, dx = \int \frac{(\operatorname{tg} x)'}{\operatorname{tg} x}\, dx = \ln|\operatorname{tg} x| + c,\ x \in R,\ x \neq \frac{k\pi}{2},\ k \in Z,\ c \in R.$

- $= \int \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}\, dx = \int \left[\frac{\cos^2 x}{\sin x \cos x} + \frac{\sin^2 x}{\sin x \cos x}\right] dx = \int \left[\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right] dx$

  $= \int \left[\frac{\cos x}{\sin x} - \frac{-\sin x}{\cos x}\right] dx = \int \left[\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x}\right] dx = \ln|\sin x| - \ln|\cos x| + c$

  $= \ln\left|\frac{\sin x}{\cos x}\right| + c = \ln|\operatorname{tg} x| + c,\ x \in R,\ x \neq \frac{k\pi}{2},\ k \in Z,\ c \in R.$

$$\int \frac{dx}{\sqrt[3]{x}+1}$$ [013]

- $= \left[\begin{array}{c|c|c} \text{Subst. } x = t^3 & t = \sqrt[3]{t} & x \in (0;\infty) \\ dx = 3t^2\, dt & t^2 = \sqrt[3]{t^2} & t \in (0;\infty) \end{array}\right] = \int \frac{3t^2\, dt}{t+1} = 3\int \frac{t^2}{t+1}\, dt = 3\int \frac{(t^2+t)-(t+1)+1}{t+1}\, dt$

  $= 3\int \left[t - 1 + \frac{1}{t+1}\right] dt = 3\left[\frac{t^2}{2} - t + \ln|t+1|\right] + c = \frac{3t^2}{2} - 3t + 3\ln(t+1) + c$

Riešené príklady – 012, 013

$$\int \frac{dx}{\sin x \cos x} = \ln|\operatorname{tg} x| + c$$ [012]

- $= \int \frac{\frac{1}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}}\, dx = \int \frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}}\, dx = \int \frac{(\operatorname{tg} x)'}{\operatorname{tg} x}\, dx = \ln|\operatorname{tg} x| + c,\ x\in R,\ x \neq \frac{k\pi}{2},\ k\in Z,\ c\in R.$

- $= \int \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}\, dx = \int \left[\frac{\cos^2 x}{\sin x \cos x} + \frac{\sin^2 x}{\sin x \cos x}\right] dx = \int \left[\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right] dx$
  $= \int \left[\frac{\cos x}{\sin x} - \frac{-\sin x}{\cos x}\right] dx = \int \left[\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x}\right] dx = \ln|\sin x| - \ln|\cos x| + c$
  $= \ln\left|\frac{\sin x}{\cos x}\right| + c = \ln|\operatorname{tg} x| + c,\ x\in R,\ x \neq \frac{k\pi}{2},\ k\in Z,\ c\in R.$

$$\int \frac{dx}{\sqrt[3]{x}+1} = \frac{3\sqrt[3]{x^2}}{2} - 3\sqrt[3]{x} + 3\ln\left(\sqrt[3]{x}+1\right) + c$$ [013]

- $= \left[\begin{array}{c|c|c} \text{Subst. } x = t^3 & t = \sqrt[3]{t} & x\in\langle 0;\infty) \\ dx = 3t^2\, dt & t^2 = \sqrt[3]{t^2} & t\in\langle 0;\infty) \end{array}\right] = \int \frac{3t^2\, dt}{t+1} = 3\int \frac{t^2}{t+1}\, dt = 3\int \frac{(t^2+t)-(t+1)+1}{t+1}\, dt$
  $= 3\int \left[t - 1 + \frac{1}{t+1}\right] dt = 3\left[\frac{t^2}{2} - t + \ln|t+1|\right] + c = \frac{3t^2}{2} - 3t + 3\ln(t+1) + c$
  $= \frac{3\sqrt[3]{x^2}}{2} - 3\sqrt[3]{x} + 3\ln\left(\sqrt[3]{x}+1\right) + c,\ x\in\langle 0;\infty),\ c\in R.$

Riešené príklady – 014, 015, 016

$$\int \frac{dx}{\sin^2 x \cos^2 x}$$ [014]

$$\int \frac{(x-1)^2}{x}\, dx$$ [015]

$$\int \frac{x^2}{x-1}\, dx$$ [016]

Riešené príklady – 014, 015, 016

$$\int \frac{dx}{\sin^2 x \cos^2 x}$$ [014]

- $= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx = \int \left[\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right] dx$

- $= 4 \int \frac{dx}{(2 \sin x \cos x)^2} = 4 \int \frac{dx}{\sin^2 2x}$

$$\int \frac{(x-1)^2}{x}\, dx$$ [015]

- $= \int \frac{x^2 - 2x + 1}{x}\, dx = \int \left[x - 2 + \frac{1}{x} \right] dx$

$$\int \frac{x^2}{x-1}\, dx$$ [016]

- $= \int \frac{(x^2 - x) + (x-1) + 1}{x-1}\, dx = \int \left[x + 1 + \frac{1}{x-1} \right] dx$

Riešené príklady – 014, 015, 016

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c_1 = -2 \operatorname{cotg} 2x + c_2$$ [014]

- $= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx = \int \left[\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right] dx = \operatorname{tg} x - \operatorname{cotg} x + c_1,$
  $x \in R,\ x \neq \frac{k\pi}{2},\ k \in Z,\ c_1 \in R.$

- $= 4 \int \frac{dx}{(2 \sin x \cos x)^2} = 4 \int \frac{dx}{\sin^2 2x} = 4 \cdot \left[-\frac{\operatorname{cotg} 2x}{2}\right] + c = -2 \operatorname{cotg} 2x + c_2,$
  $x \in R,\ x \neq \frac{k\pi}{2},\ k \in Z,\ c_2 \in R.$

$$\int \frac{(x-1)^2}{x}\, dx = \frac{x^2}{2} - 2x + \ln|x| + c$$ [015]

- $= \int \frac{x^2 - 2x + 1}{x}\, dx = \int \left[x - 2 + \frac{1}{x}\right] dx = \frac{x^2}{2} - 2x + \ln|x| + c,\ x \in R,\ x \neq 0,\ c \in R.$

$$\int \frac{x^2}{x-1}\, dx = \frac{x^2}{2} + x + \ln|x-1| + c$$ [016]

- $= \int \frac{(x^2 - x) + (x-1) + 1}{x-1}\, dx = \int \left[x + 1 + \frac{1}{x-1}\right] dx = \frac{x^2}{2} + x + \ln|x-1| + c,$
  $x \in R,\ x \neq 1,\ c \in R.$

Riešené príklady – 017, 018

$$\int \operatorname{tg}^2 x \, \mathrm{d}x$$ [017]

$$\int \operatorname{tgh}^2 x \, \mathrm{d}x$$ [018]

pr10a-01

Riešené príklady – 017, 018

$\int \operatorname{tg}^2 x \, dx$ [017]

- $= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1-\cos^2 x}{\cos^2 x} \, dx$

$\int \operatorname{tgh}^2 x \, dx$ [018]

- $= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx$

- $= \int \left[\frac{e^{2x} - 1}{e^{2x} + 1}\right]^2 dx$ pr10a-01

Riešené príklady – 017, 018

$\int \mathrm{tg}^2 x \, \mathrm{d}x$ [017]

- $= \int \frac{\sin^2 x}{\cos^2 x} \, \mathrm{d}x = \int \frac{1-\cos^2 x}{\cos^2 x} \, \mathrm{d}x = \int \left[\frac{1}{\cos^2 x} - 1 \right] \mathrm{d}x$

$\int \mathrm{tgh}^2 x \, \mathrm{d}x$ [018]

- $= \int \frac{\sinh^2 x}{\cosh^2 x} \, \mathrm{d}x = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, \mathrm{d}x = \int \left[1 - \frac{1}{\cosh^2 x} \right] \mathrm{d}x$

- $= \int \left[\frac{e^{2x} - 1}{e^{2x} + 1} \right]^2 \mathrm{d}x = \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & x = \frac{\ln t}{2} & x \in (-\infty; \infty) \\ 2x = \ln t & \mathrm{d}x = \frac{\mathrm{d}t}{2t} & t \in (0; \infty) \end{array} \right] = \int \left[\frac{t-1}{t+1} \right]^2 \frac{\mathrm{d}t}{2t} = \int \frac{t^2 - 2t + 1}{2t(t+1)^2} \, \mathrm{d}t$

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Riešené príklady – 017, 018

$$\int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + c$$ [017]

- $= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1-\cos^2 x}{\cos^2 x} \, dx = \int \left[\frac{1}{\cos^2 x} - 1\right] dx = \operatorname{tg} x - x + c,$
$x \in R,\ x \neq \frac{\pi}{2} + k\pi,\ k \in Z,\ c \in R.$

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$ [018]

- $= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \left[1 - \frac{1}{\cosh^2 x}\right] dx = x - \operatorname{tgh} x + c_1,\ x \in R,\ c_1 \in R.$

- $= \int \left[\frac{e^{2x}-1}{e^{2x}+1}\right]^2 dx = \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & x = \frac{\ln t}{2} & x \in (-\infty; \infty) \\ 2x = \ln t & dx = \frac{dt}{2t} & t \in (0; \infty) \end{array}\right] = \int \left[\frac{t-1}{t+1}\right]^2 \frac{dt}{2t} = \int \frac{t^2-2t+1}{2t(t+1)^2} \, dt$

pr10a-01

$= \int \frac{(t^2+2t+1)-4t}{2t(t^2+2t+1)} \, dt = \int \left[\frac{1}{2t} - \frac{2}{t^2+2t+1}\right] dt = \int \left[\frac{1}{2t} - 2(t+1)^{-2}\right] dt$

Riešené príklady – 017, 018

$$\int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + c$$ [017]

- $= \int \frac{\sin^2 x}{\cos^2 x}\, dx = \int \frac{1-\cos^2 x}{\cos^2 x}\, dx = \int \left[\frac{1}{\cos^2 x} - 1\right] dx = \operatorname{tg} x - x + c,$
  $x \in R,\ x \neq \frac{\pi}{2} + k\pi,\ k \in Z,\ c \in R.$

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$ [018]

- $= \int \frac{\sinh^2 x}{\cosh^2 x}\, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x}\, dx = \int \left[1 - \frac{1}{\cosh^2 x}\right] dx = x - \operatorname{tgh} x + c_1,\ x \in R,\ c_1 \in R.$

- $= \int \left[\frac{e^{2x}-1}{e^{2x}+1}\right]^2 dx = \left[\begin{array}{c|c|c} \text{Subst. } t = e^{2x} & x = \frac{\ln t}{2} & x \in (-\infty; \infty) \\ 2x = \ln t & dx = \frac{dt}{2t} & t \in (0; \infty) \end{array}\right] = \int \left[\frac{t-1}{t+1}\right]^2 \frac{dt}{2t} = \int \frac{t^2-2t+1}{2t(t+1)^2}\, dt$

  pr10a-01

$$= \int \frac{(t^2+2t+1)-4t}{2t(t^2+2t+1)}\, dt = \int \left[\frac{1}{2t} - \frac{2}{t^2+2t+1}\right] dt = \int \left[\frac{1}{2t} - 2(t+1)^{-2}\right] dt$$

$$= \frac{1}{2} \ln |t| - 2\frac{(t+1)^{-1}}{-1} + c_2 = \frac{1}{2} \ln t + \frac{2}{t+1} + c_2$$

Riešené príklady – 017, 018

$$\int \mathrm{tg}^2 x\,\mathrm{d}x = \mathrm{tg}\,x - x + c$$ [017]

- $= \int \frac{\sin^2 x}{\cos^2 x}\,\mathrm{d}x = \int \frac{1-\cos^2 x}{\cos^2 x}\,\mathrm{d}x = \int \Big[\frac{1}{\cos^2 x} - 1\Big]\,\mathrm{d}x = \mathrm{tg}\,x - x + c,$
$x \in R,\ x \neq \frac{\pi}{2} + k\pi,\ k \in Z,\ c \in R.$

$$\int \mathrm{tgh}^2 x\,\mathrm{d}x = x - \mathrm{tgh}\,x + c_1 = x + \frac{2}{e^{2x}+1} + c_2$$ [018]

- $= \int \frac{\sinh^2 x}{\cosh^2 x}\,\mathrm{d}x = \int \frac{\cosh^2 x - 1}{\cosh^2 x}\,\mathrm{d}x = \int \Big[1 - \frac{1}{\cosh^2 x}\Big]\,\mathrm{d}x = x - \mathrm{tgh}\,x + c_1,\ x \in R,\ c_1 \in R.$

- $= \int \Big[\frac{e^{2x}-1}{e^{2x}+1}\Big]^2 \mathrm{d}x = \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & x = \frac{\ln t}{2} & x \in (-\infty;\infty) \\ 2x = \ln t & \mathrm{d}x = \frac{\mathrm{d}t}{2t} & t \in (0;\infty) \end{array}\right] = \int \Big[\frac{t-1}{t+1}\Big]^2 \frac{\mathrm{d}t}{2t} = \int \frac{t^2-2t+1}{2t(t+1)^2}\,\mathrm{d}t$

pr10a-01

$= \int \frac{(t^2+2t+1)-4t}{2t(t^2+2t+1)}\,\mathrm{d}t = \int \Big[\frac{1}{2t} - \frac{2}{t^2+2t+1}\Big]\,\mathrm{d}t = \int \Big[\frac{1}{2t} - 2(t+1)^{-2}\Big]\,\mathrm{d}t$

$= \frac{1}{2}\ln|t| - 2\frac{(t+1)^{-1}}{-1} + c_2 = \frac{1}{2}\ln t + \frac{2}{t+1} + c_2 = \frac{1}{2}\cdot 2x + \frac{2}{e^{2x}+1} + c_2$

$= x + \frac{2}{e^{2x}+1} + c_2,\ x \in R,\ c_2 \in R.$

Riešené príklady – 019, 020

$\int \operatorname{cotg}^2 x \, dx$ [019]

$\int \operatorname{cotgh}^2 x \, dx$ [020]

pr10a-01

Riešené príklady – 019, 020

$\int \operatorname{cotg}^2 x \, dx$ [019]

- $= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1-\sin^2 x}{\sin^2 x} \, dx$

$\int \operatorname{cotgh}^2 x \, dx$ [020]

- $= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x+1}{\sinh^2 x} \, dx$

- $= \int \left[\frac{e^{2x}+1}{e^{2x}-1}\right]^2 dx$ pr10a-01

Riešené príklady – 019, 020

$\int \operatorname{cotg}^2 x \, dx$ [019]

- $= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1-\sin^2 x}{\sin^2 x} \, dx = \int \Big[\frac{1}{\sin^2 x} - 1 \Big] \, dx$

$\int \operatorname{cotgh}^2 x \, dx$ [020]

- $= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x+1}{\sinh^2 x} \, dx = \int \Big[1 + \frac{1}{\sinh^2 x} \Big] \, dx$

- $= \int \Big[\frac{e^{2x}+1}{e^{2x}-1} \Big]^2 dx = \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & x = \frac{\ln t}{2} & x \in (-\infty; \infty) \\ 2x = \ln t & dx = \frac{dt}{2t} & t \in (0; \infty) \end{array} \right] = \int \Big[\frac{t+1}{t-1} \Big]^2 \frac{dt}{2t} = \int \frac{t^2+2t+1}{2t(t-1)^2} \, dt$

pr10a-01

Riešené príklady – 019, 020

$$\int \operatorname{cotg}^2 x \, dx = -\operatorname{cotg} x - x + c$$ [019]

- $= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1-\sin^2 x}{\sin^2 x} \, dx = \int \Big[\frac{1}{\sin^2 x} - 1 \Big] \, dx = -\operatorname{cotg} x - x + c,$
  $x \in R,\ x \neq k\pi,\ k \in Z,\ c \in R.$

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1$$ [020]

- $= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \Big[1 + \frac{1}{\sinh^2 x} \Big] \, dx = x - \operatorname{cotgh} x + c_1,$
  $x \in R,\ x \neq 0,\ c_1 \in R.$

- $= \int \Big[\frac{e^{2x}+1}{e^{2x}-1} \Big]^2 dx$ pr10a-01 $= \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & x = \frac{\ln t}{2} & x \in (-\infty; \infty) \\ 2x = \ln t & dx = \frac{dt}{2t} & t \in (0; \infty) \end{array} \right] = \int \Big[\frac{t+1}{t-1} \Big]^2 \frac{dt}{2t} = \int \frac{t^2+2t+1}{2t(t-1)^2} \, dt$

  $= \int \frac{(t^2-2t+1)+4t}{2t(t^2-2t+1)} \, dt = \int \Big[\frac{1}{2t} + \frac{2}{t^2-2t+1} \Big] \, dt = \int \Big[\frac{1}{2t} + 2(t-1)^{-2} \Big] \, dt$

Riešené príklady – 019, 020

$$\int \operatorname{cotg}^2 x \, dx = -\operatorname{cotg} x - x + c$$ [019]

- $= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1-\sin^2 x}{\sin^2 x} \, dx = \int \left[\frac{1}{\sin^2 x} - 1\right] dx = -\operatorname{cotg} x - x + c,$
  $x \in R,\ x \neq k\pi,\ k \in Z,\ c \in R.$

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1$$ [020]

- $= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x}\right] dx = x - \operatorname{cotgh} x + c_1,$
  $x \in R,\ x \neq 0,\ c_1 \in R.$

- $= \int \left[\frac{e^{2x}+1}{e^{2x}-1}\right]^2 dx \overset{\text{pr10a-01}}{=} \left[\begin{array}{c|c|c} \text{Subst. } t = e^{2x} & x = \frac{\ln t}{2} & x \in (-\infty; \infty) \\ 2x = \ln t & dx = \frac{dt}{2t} & t \in (0; \infty) \end{array}\right] = \int \left[\frac{t+1}{t-1}\right]^2 \frac{dt}{2t} = \int \frac{t^2+2t+1}{2t(t-1)^2} \, dt$

  $= \int \frac{(t^2-2t+1)+4t}{2t(t^2-2t+1)} \, dt = \int \left[\frac{1}{2t} + \frac{2}{t^2-2t+1}\right] dt = \int \left[\frac{1}{2t} + 2(t-1)^{-2}\right] dt$

  $= \frac{1}{2} \ln |t| + 2\frac{(t-1)^{-1}}{-1} + c_2 = \frac{1}{2} \ln t - \frac{2}{t-1} + c_2$

Riešené príklady – 019, 020

$$\int \operatorname{cotg}^2 x \, dx = -\operatorname{cotg} x - x + c$$ [019]

- $= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1-\sin^2 x}{\sin^2 x} \, dx = \int \left[\frac{1}{\sin^2 x} - 1\right] dx = -\operatorname{cotg} x - x + c,$

  $x \in R,\ x \neq k\pi,\ k \in Z,\ c \in R.$

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$ [020]

- $= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x}\right] dx = x - \operatorname{cotgh} x + c_1,$

  $x \in R,\ x \neq 0,\ c_1 \in R.$

- $= \int \left[\frac{e^{2x}+1}{e^{2x}-1}\right]^2 dx = \left[\begin{array}{c|c|c} \text{Subst. } t = e^{2x} & x = \frac{\ln t}{2} & x \in (-\infty; \infty) \\ 2x = \ln t & dx = \frac{dt}{2t} & t \in (0; \infty) \end{array}\right] = \int \left[\frac{t+1}{t-1}\right]^2 \frac{dt}{2t} = \int \frac{t^2+2t+1}{2t(t-1)^2} \, dt$

  pr10a-01

  $= \int \frac{(t^2-2t+1)+4t}{2t(t^2-2t+1)} \, dt = \int \left[\frac{1}{2t} + \frac{2}{t^2-2t+1}\right] dt = \int \left[\frac{1}{2t} + 2(t-1)^{-2}\right] dt$

  $= \frac{1}{2} \ln |t| + 2\frac{(t-1)^{-1}}{-1} + c_2 = \frac{1}{2} \ln t - \frac{2}{t-1} + c_2 = \frac{1}{2} \cdot 2x - \frac{2}{e^{2x}-1} + c_2$

  $= x - \frac{2}{e^{2x}-1} + c_2,\ x \in R,\ x \neq 0,\ c_2 \in R.$

Riešené príklady – 021, 022, 023, 024

$\int \sin^3 x \cdot \cos x \, dx$ [021]

$\int \frac{x^3 \, dx}{x^8+1}$ [022]

$\int \frac{f'(x)}{f(x)} \, dx$ [023]

$\int \frac{f'(t)}{f(t)} \, dt$ [024]

Riešené príklady – 021, 022, 023, 024

$\int \sin^3 x \cdot \cos x \, dx$ [021]

- $= \left[\begin{array}{r|l} \text{Subst. } t = \sin x & x \in R \\ dt = \cos x \, dx & t \in \langle -1; 1 \rangle \end{array} \right] = \int t^3 \, dt$

$\int \frac{x^3 \, dx}{x^8+1}$ [022]

- $= \frac{1}{4} \int \frac{4x^3 \, dx}{x^8+1}$

$\int \frac{f'(x)}{f(x)} \, dx$ [023]

- $= \left[\begin{array}{r|l} \text{Subst. } t = f(x) & f(x) \neq 0 \\ dt = f'(x) \, dx & t \neq 0 \end{array} \right] = \int \frac{dt}{t}$

$\int \frac{f'(t)}{f(t)} \, dt$ [024]

- $= \left[\begin{array}{r|l} \text{Subst. } x = f(t) & f(t) \neq 0 \\ dx = f'(t) \, dt & x \neq 0 \end{array} \right] = \int \frac{dx}{x}$

Riešené príklady – 021, 022, 023, 024

$$\int \sin^3 x \cdot \cos x \, \mathrm{d}x$$ [021]

- $= \left[\begin{array}{r|l} \text{Subst. } t = \sin x & x \in R \\ \mathrm{d}t = \cos x \, \mathrm{d}x & t \in \langle -1; 1 \rangle \end{array} \right] = \int t^3 \, \mathrm{d}t = \frac{t^4}{4} + c$

$$\int \frac{x^3 \, \mathrm{d}x}{x^8 + 1}$$ [022]

- $= \frac{1}{4} \int \frac{4x^3 \, \mathrm{d}x}{x^8 + 1} = \left[\begin{array}{r|l} \text{Subst. } t = x^4 & x \in R \\ \mathrm{d}t = 4x^3 \, \mathrm{d}x & t \in R \end{array} \right] = \frac{1}{4} \int \frac{\mathrm{d}t}{t^2 + 1}$

$$\int \frac{f'(x)}{f(x)} \, \mathrm{d}x$$ [023]

- $= \left[\begin{array}{r|l} \text{Subst. } t = f(x) & f(x) \neq 0 \\ \mathrm{d}t = f'(x) \, \mathrm{d}x & t \neq 0 \end{array} \right] = \int \frac{\mathrm{d}t}{t}$
  $= \ln |t| + c$

$$\int \frac{f'(t)}{f(t)} \, \mathrm{d}t$$ [024]

- $= \left[\begin{array}{r|l} \text{Subst. } x = f(t) & f(t) \neq 0 \\ \mathrm{d}x = f'(t) \, \mathrm{d}t & x \neq 0 \end{array} \right] = \int \frac{\mathrm{d}x}{x}$
  $= \ln |x| + c$

Riešené príklady – 021, 022, 023, 024

$$\int \sin^3 x \cdot \cos x \, dx = \frac{\sin^4 x}{4} + c$$ [021]

- $= \left[\begin{array}{c|c} \text{Subst. } t = \sin x & x \in R \\ dt = \cos x \, dx & t \in \langle -1; 1 \rangle \end{array} \right] = \int t^3 \, dt = \frac{t^4}{4} + c = \frac{\sin^4 x}{4} + c, \; x \in R, \; c \in R.$

$$\int \frac{x^3 \, dx}{x^8 + 1}$$ [022]

- $= \frac{1}{4} \int \frac{4x^3 \, dx}{x^8 + 1} = \left[\begin{array}{c|c} \text{Subst. } t = x^4 & x \in R \\ dt = 4x^3 \, dx & t \in R \end{array} \right] = \frac{1}{4} \int \frac{dt}{t^2 + 1} = \frac{1}{4} \operatorname{arctg} t + c$

$$\int \frac{f'(x)}{f(x)} \, dx = \ln |f(x)| + c$$ [023]

- $= \left[\begin{array}{c|c} \text{Subst. } t = f(x) & f(x) \neq 0 \\ dt = f'(x) \, dx & t \neq 0 \end{array} \right] = \int \frac{dt}{t}$
  $= \ln |t| + c = \ln |f(x)| + c,$
  $x \in D(f), \; f(x) \neq 0, \; c \in R.$

$$\int \frac{f'(t)}{f(t)} \, dt = \ln |f(t)| + c$$ [024]

- $= \left[\begin{array}{c|c} \text{Subst. } x = f(t) & f(t) \neq 0 \\ dx = f'(t) \, dt & x \neq 0 \end{array} \right] = \int \frac{dx}{x}$
  $= \ln |x| + c = \ln |f(t)| + c,$
  $t \in D(f), \; f(t) \neq 0, \; c \in R.$

Riešené príklady – 021, 022, 023, 024

$$\int \sin^3 x \cdot \cos x \, dx = \frac{\sin^4 x}{4} + c$$ [021]

- $= \left[\begin{array}{c|c} \text{Subst. } t = \sin x & x \in R \\ dt = \cos x \, dx & t \in \langle -1; 1 \rangle \end{array} \right] = \int t^3 \, dt = \frac{t^4}{4} + c = \frac{\sin^4 x}{4} + c, \; x \in R, \; c \in R.$

$$\int \frac{x^3 \, dx}{x^8+1} = \frac{1}{4} \operatorname{arctg} x^4 + c$$ [022]

- $= \frac{1}{4} \int \frac{4x^3 \, dx}{x^8+1} = \left[\begin{array}{c|c} \text{Subst. } t = x^4 & x \in R \\ dt = 4x^3 \, dx & t \in R \end{array} \right] = \frac{1}{4} \int \frac{dt}{t^2+1} = \frac{1}{4} \operatorname{arctg} t + c = \frac{1}{4} \operatorname{arctg} x^4 + c,$
  $x \in R, \; c \in R.$

$$\int \frac{f'(x)}{f(x)} \, dx = \ln |f(x)| + c$$ [023]

- $= \left[\begin{array}{c|c} \text{Subst. } t = f(x) & f(x) \neq 0 \\ dt = f'(x) \, dx & t \neq 0 \end{array} \right] = \int \frac{dt}{t}$
  $= \ln |t| + c = \ln |f(x)| + c,$
  $x \in D(f), \; f(x) \neq 0, \; c \in R.$

$$\int \frac{f'(t)}{f(t)} \, dt = \ln |f(t)| + c$$ [024]

- $= \left[\begin{array}{c|c} \text{Subst. } x = f(t) & f(t) \neq 0 \\ dx = f'(t) \, dt & x \neq 0 \end{array} \right] = \int \frac{dx}{x}$
  $= \ln |x| + c = \ln |f(t)| + c,$
  $t \in D(f), \; f(t) \neq 0, \; c \in R.$

Riešené príklady – 025, 026, 027

$\int f(ax+b)\,\mathrm{d}x$ pre $x\in(\alpha;\beta)$ a $a,b\in R,\ a\neq 0.$ 1 [025]

$\int \frac{\mathrm{d}x}{x+1}$ [026]

$\int \sin(-x)\,\mathrm{d}x$ [027]

Riešené príklady – 025, 026, 027

$\int f(ax+b)\,dx$ pre $x\in(\alpha;\beta)$ a $a,b\in R,\ a\neq 0.$ 2 [025]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t=ax+b & x=\alpha & t=a\alpha+b \\ dt=a\,dx & x=\beta & t=a\beta+b \end{array} \right] = \int \frac{f(t)\,dt}{a}$

Pre $a=1,\ b\in R$ platí: • $\int f(x+b)\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x+b & x\in(\alpha;\beta) \\ dt=dx & t\in(\alpha+b;\beta+b) \end{array} \right] = \int f(t)\,dt$

Pre $a=-1,\ b=0$ platí: • $\int f(-x)\,dx = \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in(\alpha;\beta) \\ dt=-dx & t\in(-\beta;-\alpha) \end{array} \right] = -\int f(t)\,dt$

$\int \frac{dx}{x+1}$ [026]

- $= \left[\begin{array}{r|l} \text{Subst.} t=x+1 & x\in(-1;\infty) \Rightarrow t\in(0;\infty) \\ dt=dx & x\in(-\infty;-1) \Rightarrow t\in(-\infty;0) \end{array} \right] = \int \frac{dt}{t}$

$\int \sin(-x)\,dx$ [027]

- $= \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in R \\ dt=-dx & t\in R \end{array} \right] = -\int \sin t\,dt$

Riešené príklady – 025, 026, 027

$\int f(ax+b)\,dx$ pre $x\in(\alpha;\beta)$ a $a,b\in R,\ a\neq 0.$ 3 [025]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t=ax+b & x=\alpha & t=a\alpha+b \\ dt=a\,dx & x=\beta & t=a\beta+b \end{array}\right] = \int \frac{f(t)\,dt}{a} = \frac{F(t)}{a}+c$

[$F(t),\ t\in J$ je primitívna k $f(t),\ t\in J$ na intervale J s hranicami $a\alpha+b$ a $a\beta+b$.]

Pre $a=1,\ b\in R$ platí: $\int f(x+b)\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x+b & x\in(\alpha;\beta) \\ dt=dx & t\in(\alpha+b;\beta+b) \end{array}\right] = \int f(t)\,dt = F(t)+c$

Pre $a=-1,\ b=0$ platí: $\int f(-x)\,dx = \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in(\alpha;\beta) \\ dt=-dx & t\in(-\beta;-\alpha) \end{array}\right] = -\int f(t)\,dt = -F(t)+c$

$\int \frac{dx}{x+1}$ [026]

- $= \left[\begin{array}{r|l} \text{Subst.} t=x+1 & x\in(-1;\infty)\Rightarrow t\in(0;\infty) \\ dt=dx & x\in(-\infty;-1)\Rightarrow t\in(-\infty;0) \end{array}\right] = \int \frac{dt}{t} = \ln|t|+c$

$\int \sin(-x)\,dx$ [027]

- $= \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in R \\ dt=-dx & t\in R \end{array}\right] = -\int \sin t\,dt = -(-\cos t)+c$

Riešené príklady – 025, 026, 027

$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + c$ pre $x \in (\alpha;\beta)$ a $a,b \in R$, $a \neq 0$. 4 [025]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t = ax+b & x=\alpha & t = a\alpha+b \\ dt = a\,dx & x = \beta & t = a\beta + b \end{array} \right] = \int \frac{f(t)\,dt}{a} = \frac{F(t)}{a} + c = \frac{F(ax+b)}{a} + c$, $x \in (\alpha;\beta)$, $c \in R$.

[$F(t)$, $t \in J$ je primitívna k $f(t)$, $t \in J$ na intervale J s hranicami $a\alpha+b$ a $a\beta+b$.]

Pre $a=1$, $b \in R$ platí: • $\int f(x+b)\,dx = \left[\begin{array}{r|l} \text{Subst. } t = x+b & x \in (\alpha;\beta) \\ dt = dx & t \in (\alpha+b;\beta+b) \end{array} \right] = \int f(t)\,dt = F(t) + c = F(x+b) + c$, $x \in (\alpha;\beta)$, $c \in R$.

Pre $a=-1$, $b=0$ platí: • $\int f(-x)\,dx = \left[\begin{array}{r|l} \text{Subst. } t = -x & x \in (\alpha;\beta) \\ dt = -dx & t \in (-\beta;-\alpha) \end{array} \right] = -\int f(t)\,dt = -F(t) + c = F(-x) + c$, $x \in (\alpha;\beta)$, $c \in R$.

$\int \frac{dx}{x+1} = \ln|x+1| + c$ [026]

- $= \left[\begin{array}{r|l} \text{Subst.} t = x+1 & x \in (-1;\infty) \Rightarrow t \in (0;\infty) \\ dt = dx & x \in (-\infty;-1) \Rightarrow t \in (-\infty;0) \end{array} \right] = \int \frac{dt}{t} = \ln|t| + c = \ln|x+1| + c$, $x \in R$, $x \neq 1$, $c \in R$.

$\int \sin(-x)\,dx = \cos x + c$ [027]

- $= \left[\begin{array}{r|l} \text{Subst. } t = -x & x \in R \\ dt = -dx & t \in R \end{array} \right] = -\int \sin t\,dt = -(-\cos t) + c = \cos(-x) + c = \cos x + c$, $x \in R$, $c \in R$.

Riešené príklady – 028

$$\int \frac{(x+1)^3}{(1-x)^2}\,dx$$ [028]

Riešené príklady – 028

$$\int \frac{(x+1)^3}{(1-x)^2}\,dx$$ [028]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t = 1-x & x = 1-t & x\in(-\infty;1) \Rightarrow t\in(0;\infty) \\ x+1 = 2-t & dx = -dt & x\in(1;\infty) \Rightarrow t\in(-\infty;0) \end{array} \right] = -\int \frac{(2-t)^3}{t^2}\,dt$

- $= \int \frac{(x+1)^3}{(x-1)^2}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t = x-1 & x = t+1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ x+1 = t+2 & dx = dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t+2)^3}{t^2}\,dt$

- $= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[x + 5 + \frac{12}{x-1} + \frac{8}{(x-1)^2} \right] dx$

Riešené príklady – 028

$\int \frac{(x+1)^3}{(1-x)^2}\,dx$ [028]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t=1-x & x=1-t & x\in(-\infty;1) \Rightarrow t\in(0;\infty) \\ x+1=2-t & dx=-dt & x\in(1;\infty) \Rightarrow t\in(-\infty;0) \end{array} \right] = -\int \frac{(2-t)^3}{t^2}\,dt = -\int \frac{2^3-3\cdot 2^2 t+3\cdot 2t^2-t^3}{t^2}\,dt$

  $= \int \left[-8t^{-2} + \frac{12}{t} - 6 + t \right] dt$

- $= \int \frac{(x+1)^3}{(x-1)^2}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ x+1=t+2 & dx=dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t+2)^3}{t^2}\,dt$

  $= \int \frac{t^3+3\cdot 2t^2+3\cdot 2^2 t+2^3}{t^2}\,dt = \int \left[t + 6 + \frac{12}{t} + 8t^{-2} \right] dt$

- $= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[x + 5 + \frac{12}{x-1} + \frac{8}{(x-1)^2} \right] dx = \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ dx=dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right]$

  $= \frac{x^2}{2} + 5x + \int \left[\frac{12}{t} + 8t^{-2} \right] dt$

Riešené príklady – 028

$\int \frac{(x+1)^3}{(1-x)^2}\,dx$ [028]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t=1-x & x=1-t & x\in(-\infty;1) \Rightarrow t\in(0;\infty) \\ x+1=2-t & dx=-dt & x\in(1;\infty) \Rightarrow t\in(-\infty;0) \end{array} \right] = -\int \frac{(2-t)^3}{t^2}\,dt = -\int \frac{2^3-3\cdot 2^2 t+3\cdot 2t^2-t^3}{t^2}\,dt$
  $= \int \left[-8t^{-2} + \frac{12}{t} - 6 + t\right] dt = -\frac{8t^{-1}}{-1} + 12\ln|t| - 6t + \frac{t^2}{2} + c_1$

- $= \int \frac{(x+1)^3}{(x-1)^2}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ x+1=t+2 & dx=dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t+2)^3}{t^2}\,dt$
  $= \int \frac{t^3+3\cdot 2t^2+3\cdot 2^2 t+2^3}{t^2}\,dt = \int \left[t + 6 + \frac{12}{t} + 8t^{-2}\right] dt = \frac{t^2}{2} + 6t + 12\ln|t| + \frac{8t^{-1}}{-1} + c_1$

- $= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[x + 5 + \frac{12}{x-1} + \frac{8}{(x-1)^2}\right] dx = \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ dx=dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right]$
  $= \frac{x^2}{2} + 5x + \int \left[\frac{12}{t} + 8t^{-2}\right] dt = \frac{x^2}{2} + 5x + 12\ln|t| + \frac{8t^{-1}}{-1} + c_2$

Riešené príklady – 028

$$\int \frac{(x+1)^3}{(1-x)^2}\,dx = \frac{(x-1)^2}{2} + 6(x-1) + 12\ln|x-1| + \frac{8}{1-x} + c_1$$ [028]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t=1-x & x=1-t & x\in(-\infty;1)\Rightarrow t\in(0;\infty) \\ x+1=2-t & dx=-dt & x\in(1;\infty)\Rightarrow t\in(-\infty;0) \end{array}\right] = -\int \frac{(2-t)^3}{t^2}\,dt = -\int \frac{2^3-3\cdot 2^2 t+3\cdot 2t^2-t^3}{t^2}\,dt$

  $= \int \left[-8t^{-2} + \frac{12}{t} - 6 + t\right] dt = -\frac{8t^{-1}}{-1} + 12\ln|t| - 6t + \frac{t^2}{2} + c_1$

  $= \frac{8}{1-x} + 12\ln|1-x| - 6(1-x) + \frac{(1-x)^2}{2} + c_1,\ x\in R,\ x\neq 1,\ c_1\in R.$

- $= \int \frac{(x+1)^3}{(x-1)^2}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(-\infty;1)\Rightarrow t\in(-\infty;0) \\ x+1=t+2 & dx=dt & x\in(1;\infty)\Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t+2)^3}{t^2}\,dt$

  $= \int \frac{t^3+3\cdot 2t^2+3\cdot 2^2 t+2^3}{t^2}\,dt = \int \left[t + 6 + \frac{12}{t} + 8t^{-2}\right] dt = \frac{t^2}{2} + 6t + 12\ln|t| + \frac{8t^{-1}}{-1} + c_1$

  $= \frac{(x-1)^2}{2} + 6(x-1) + 12\ln|x-1| - \frac{8}{x-1} + c_1,\ x\in R,\ x\neq 1,\ c_1\in R.$

- $= \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[x + 5 + \frac{12}{x-1} + \frac{8}{(x-1)^2}\right] dx = \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in(-\infty;1)\Rightarrow t\in(-\infty;0) \\ dx=dt & x\in(1;\infty)\Rightarrow t\in(0;\infty) \end{array}\right]$

  $= \frac{x^2}{2} + 5x + \int \left[\frac{12}{t} + 8t^{-2}\right] dt = \frac{x^2}{2} + 5x + 12\ln|t| + \frac{8t^{-1}}{-1} + c_2$

  $= \frac{x^2}{2} + 5x + 12\ln|x-1| - \frac{8}{x-1} + c_2,\ x\in R,\ x\neq 1,\ c_2\in R.$

Riešené príklady – 029, 030

$\int \frac{(x-2)^4}{(x-1)^2}\,\mathrm{d}x$ [029]

$\int \frac{(x-1)^2}{(x-2)^4}\,\mathrm{d}x$ [030]

Riešené príklady – 029, 030

$$\int \frac{(x-2)^4}{(x-1)^2}\,dx$$ [029]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t = x-1 & x = t+1 & x \in (-\infty; 1) \Rightarrow t \in (-\infty; 0) \\ x-2 = t-1 & dx = dt & x \in (1; \infty) \Rightarrow t \in (0; \infty) \end{array}\right] = \int \frac{(t-1)^4}{t^2}\,dt$

$$\int \frac{(x-1)^2}{(x-2)^4}\,dx$$ [030]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t = x-2 & x = t+2 & x \in (-\infty; 2) \Rightarrow t \in (-\infty; 0) \\ x-1 = t+1 & dx = dt & x \in (2; \infty) \Rightarrow t \in (0; \infty) \end{array}\right] = \int \frac{(t+1)^2}{t^4}\,dt$

Riešené príklady – 029, 030

$$\int \frac{(x-2)^4}{(x-1)^2}\,\mathrm{d}x$$ [029]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(-\infty;1)\Rightarrow t\in(-\infty;0) \\ x-2=t-1 & \mathrm{d}x=\mathrm{d}t & x\in(1;\infty)\Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t-1)^4}{t^2}\,\mathrm{d}t = \int \frac{t^4-4t^3+6t^2-4t+1}{t^2}\,\mathrm{d}t$

  $= \int \left[t^2 - 4t + 6 - \frac{4}{t} + t^{-2}\right]\mathrm{d}t$

$$\int \frac{(x-1)^2}{(x-2)^4}\,\mathrm{d}x$$ [030]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t=x-2 & x=t+2 & x\in(-\infty;2)\Rightarrow t\in(-\infty;0) \\ x-1=t+1 & \mathrm{d}x=\mathrm{d}t & x\in(2;\infty)\Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t+1)^2}{t^4}\,\mathrm{d}t = \int \frac{t^2+2t+1}{t^4}\,\mathrm{d}t$

  $= \int \left[t^{-2}+2t^{-3}+t^{-4}\right]\mathrm{d}t$

Riešené príklady – 029, 030

$\int \frac{(x-2)^4}{(x-1)^2}\,dx$ [029]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t=x-1 & x=t+1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ x-2=t-1 & dx=dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t-1)^4}{t^2}\,dt = \int \frac{t^4-4t^3+6t^2-4t+1}{t^2}\,dt$
  $= \int \Big[t^2 - 4t + 6 - \frac{4}{t} + t^{-2} \Big] dt = \frac{t^3}{3} - \frac{4t^2}{2} + 6t - 4\ln|t| + \frac{t^{-1}}{-1} + c$

$\int \frac{(x-1)^2}{(x-2)^4}\,dx$ [030]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t=x-2 & x=t+2 & x\in(-\infty;2) \Rightarrow t\in(-\infty;0) \\ x-1=t+1 & dx=dt & x\in(2;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t+1)^2}{t^4}\,dt = \int \frac{t^2+2t+1}{t^4}\,dt$
  $= \int \Big[t^{-2}+2t^{-3}+t^{-4} \Big] dt = \frac{t^{-1}}{-1} + \frac{2t^{-2}}{-2} + \frac{t^{-3}}{-3} + c$

Riešené príklady – 029, 030

$$\int \frac{(x-2)^4}{(x-1)^2}\,dx = \frac{(x-1)^3}{3} - 2(x-1)^2 + 6(x-1) - 4\ln|x-1| - \frac{1}{x-1} + c$$ [029]

- $= \left[\begin{array}{l|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(-\infty;1)\Rightarrow t\in(-\infty;0) \\ x-2=t-1 & dx=dt & x\in(1;\infty)\Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t-1)^4}{t^2}\,dt = \int \frac{t^4-4t^3+6t^2-4t+1}{t^2}\,dt$

$= \int \left[t^2 - 4t + 6 - \frac{4}{t} + t^{-2}\right] dt = \frac{t^3}{3} - \frac{4t^2}{2} + 6t - 4\ln|t| + \frac{t^{-1}}{-1} + c$

$= \frac{(x-1)^3}{3} - 2(x-1)^2 + 6(x-1) - 4\ln|x-1| - \frac{1}{x-1} + c$

$x\in R,\ x\neq 1,\ c\in R,$

$$\int \frac{(x-1)^2}{(x-2)^4}\,dx = -\frac{1}{x-2} - \frac{1}{(x-2)^2} - \frac{1}{3(x-2)^3} + c$$ [030]

- $= \left[\begin{array}{l|l|l} \text{Subst. } t=x-2 & x=t+2 & x\in(-\infty;2)\Rightarrow t\in(-\infty;0) \\ x-1=t+1 & dx=dt & x\in(2;\infty)\Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t+1)^2}{t^4}\,dt = \int \frac{t^2+2t+1}{t^4}\,dt$

$= \int \left[t^{-2}+2t^{-3}+t^{-4}\right] dt = \frac{t^{-1}}{-1} + \frac{2t^{-2}}{-2} + \frac{t^{-3}}{-3} + c = -\frac{1}{x-2} - \frac{1}{(x-2)^2} - \frac{1}{3(x-2)^3} + c,$

$x\in R,\ x\neq 2,\ c\in R.$

Riešené príklady – 029, 030

$$\int \frac{(x-2)^4}{(x-1)^2}\,dx = \frac{(x-1)^3}{3} - 2(x-1)^2 + 6(x-1) - 4\ln|x-1| - \frac{1}{x-1} + c$$ [029]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t = x-1 & x = t+1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ x-2 = t-1 & dx = dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t-1)^4}{t^2}\,dt = \int \frac{t^4-4t^3+6t^2-4t+1}{t^2}\,dt$

$= \int \left[t^2 - 4t + 6 - \frac{4}{t} + t^{-2}\right] dt = \frac{t^3}{3} - \frac{4t^2}{2} + 6t - 4\ln|t| + \frac{t^{-1}}{-1} + c$

$= \frac{(x-1)^3}{3} - 2(x-1)^2 + 6(x-1) - 4\ln|x-1| - \frac{1}{x-1} + c$

$= \frac{x^3-3x^2+3x-1}{3} - 2(x^2-2x+1) + 6(x-1) - 4\ln|x-1| - \frac{1}{x-1} + c$

$x\in R,\ x\neq 1,\ c\in R,$

$$\int \frac{(x-1)^2}{(x-2)^4}\,dx = -\frac{1}{x-2} - \frac{1}{(x-2)^2} - \frac{1}{3(x-2)^3} + c$$ [030]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t = x-2 & x = t+2 & x\in(-\infty;2) \Rightarrow t\in(-\infty;0) \\ x-1 = t+1 & dx = dt & x\in(2;\infty) \Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t+1)^2}{t^4}\,dt = \int \frac{t^2+2t+1}{t^4}\,dt$

$= \int \left[t^{-2}+2t^{-3}+t^{-4}\right] dt = \frac{t^{-1}}{-1} + \frac{2t^{-2}}{-2} + \frac{t^{-3}}{-3} + c = -\frac{1}{x-2} - \frac{1}{(x-2)^2} - \frac{1}{3(x-2)^3} + c,$

$x\in R,\ x\neq 2,\ c\in R.$

Riešené príklady – 029, 030

$$\int \frac{(x-2)^4}{(x-1)^2}\,dx = \frac{(x-1)^3}{3} - 2(x-1)^2 + 6(x-1) - 4\ln|x-1| - \frac{1}{x-1} + c$$ [029]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t=x-1 & x=t+1 & x\in(-\infty;1)\Rightarrow t\in(-\infty;0) \\ x-2=t-1 & dx=dt & x\in(1;\infty)\Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t-1)^4}{t^2}\,dt = \int \frac{t^4-4t^3+6t^2-4t+1}{t^2}\,dt$

$= \int \left[t^2 - 4t + 6 - \frac{4}{t} + t^{-2}\right]dt = \frac{t^3}{3} - \frac{4t^2}{2} + 6t - 4\ln|t| + \frac{t^{-1}}{-1} + c$

$= \frac{(x-1)^3}{3} - 2(x-1)^2 + 6(x-1) - 4\ln|x-1| - \frac{1}{x-1} + c$

$= \frac{x^3-3x^2+3x-1}{3} - 2(x^2-2x+1) + 6(x-1) - 4\ln|x-1| - \frac{1}{x-1} + c$

$= \frac{x^3}{3} - 3x^2 + 11x - 4\ln|x-1| - \frac{1}{x-1} + c_1$, $x\in R$, $x\neq 1$, $c\in R$, pričom $c_1 = c - \frac{25}{3}$.

$$\int \frac{(x-1)^2}{(x-2)^4}\,dx = -\frac{1}{x-2} - \frac{1}{(x-2)^2} - \frac{1}{3(x-2)^3} + c$$ [030]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t=x-2 & x=t+2 & x\in(-\infty;2)\Rightarrow t\in(-\infty;0) \\ x-1=t+1 & dx=dt & x\in(2;\infty)\Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t+1)^2}{t^4}\,dt = \int \frac{t^2+2t+1}{t^4}\,dt$

$= \int \left[t^{-2}+2t^{-3}+t^{-4}\right]dt = \frac{t^{-1}}{-1} + \frac{2t^{-2}}{-2} + \frac{t^{-3}}{-3} + c = -\frac{1}{x-2} - \frac{1}{(x-2)^2} - \frac{1}{3(x-2)^3} + c$,

$x\in R$, $x\neq 2$, $c\in R$.

Riešené príklady – 031, 032

$$\int \frac{(x-1)^4}{(x-2)^2}\, dx$$ [032]

$$\int \frac{(x-2)^2}{(x-1)^4}\, dx$$ [032]

Riešené príklady – 031, 032

$\int \frac{(x-1)^4}{(x-2)^2}\,dx$ [032]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t = x-2 & x = t+2 & x\in(-\infty;2) \Rightarrow t\in(-\infty;0) \\ x-1 = t+1 & dx = dt & x\in(2;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t+1)^4}{t^2}\,dt$

$\int \frac{(x-2)^2}{(x-1)^4}\,dx$ [032]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t = x-1 & x = t+1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ x-2 = t-1 & dx = dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t-1)^2}{t^4}\,dt$

Riešené príklady – 031, 032

$\int \frac{(x-1)^4}{(x-2)^2}\,dx$ [032]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t=x-2 & x=t+2 & x\in(-\infty;2) \Rightarrow t\in(-\infty;0) \\ x-1=t+1 & dx=dt & x\in(2;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t+1)^4}{t^2}\,dt = \int \frac{t^4+4t^3+6t^2+4t+1}{t^2}\,dt$
  $= \int \left[t^2+4t+6+\frac{4}{t}+t^{-2} \right] dt$

$\int \frac{(x-2)^2}{(x-1)^4}\,dx$ [032]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t=x-1 & x=t+1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ x-2=t-1 & dx=dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t-1)^2}{t^4}\,dt = \int \frac{t^2-2t+1}{t^4}\,dt$
  $= \int \left[t^{-2}-2t^{-3}+t^{-4} \right] dt$

Riešené príklady – 031, 032

$\int \frac{(x-1)^4}{(x-2)^2}\,dx$ [032]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t=x-2 & x=t+2 & x\in(-\infty;2)\Rightarrow t\in(-\infty;0) \\ x-1=t+1 & dx=dt & x\in(2;\infty)\Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t+1)^4}{t^2}\,dt = \int \frac{t^4+4t^3+6t^2+4t+1}{t^2}\,dt$
  $= \int \left[t^2+4t+6+\frac{4}{t}+t^{-2}\right] dt = \frac{t^3}{3}+\frac{4t^2}{2}+6t+4\ln|t|+\frac{t^{-1}}{-1}+c$

$\int \frac{(x-2)^2}{(x-1)^4}\,dx$ [032]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(-\infty;1)\Rightarrow t\in(-\infty;0) \\ x-2=t-1 & dx=dt & x\in(1;\infty)\Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t-1)^2}{t^4}\,dt = \int \frac{t^2-2t+1}{t^4}\,dt$
  $= \int \left[t^{-2}-2t^{-3}+t^{-4}\right] dt = \frac{t^{-1}}{-1}-\frac{2t^{-2}}{-2}+\frac{t^{-3}}{-3}+c$

Riešené príklady – 031, 032

$$\int \frac{(x-1)^4}{(x-2)^2}\,dx = \frac{(x-2)^3}{3} + 2(x-2)^2 + 6(x-2) + 4\ln|x-2| - \frac{1}{x-2} + c$$ [032]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t=x-2 & x=t+2 & x\in(-\infty;2) \Rightarrow t\in(-\infty;0) \\ x-1=t+1 & dx=dt & x\in(2;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t+1)^4}{t^2}\,dt = \int \frac{t^4+4t^3+6t^2+4t+1}{t^2}\,dt$

$$= \int \left[t^2 + 4t + 6 + \frac{4}{t} + t^{-2} \right] dt = \frac{t^3}{3} + \frac{4t^2}{2} + 6t + 4\ln|t| + \frac{t^{-1}}{-1} + c$$

$$= \frac{(x-2)^3}{3} + 2(x-2)^2 + 6(x-2) + 4\ln|x-2| - \frac{1}{x-2} + c$$

$x\in R,\ x\neq 2,\ c\in R,$

$$\int \frac{(x-2)^2}{(x-1)^4}\,dx = -\frac{1}{x-1} + \frac{1}{(x-1)^2} - \frac{1}{3(x-1)^3} + c$$ [032]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t=x-1 & x=t+1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ x-2=t-1 & dx=dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t-1)^2}{t^4}\,dt = \int \frac{t^2-2t+1}{t^4}\,dt$

$$= \int \left[t^{-2} - 2t^{-3} + t^{-4} \right] dt = \frac{t^{-1}}{-1} - \frac{2t^{-2}}{-2} + \frac{t^{-3}}{-3} + c = -\frac{1}{x-1} + \frac{1}{(x-1)^2} - \frac{1}{3(x-1)^3} + c,$$

$x\in R,\ x\neq 1,\ c\in R.$

Riešené príklady – 031, 032

$$\int \frac{(x-1)^4}{(x-2)^2}\,dx = \frac{(x-2)^3}{3} + 2(x-2)^2 + 6(x-2) + 4\ln|x-2| - \frac{1}{x-2} + c$$ [032]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t = x-2 & x = t+2 & x\in(-\infty;2) \Rightarrow t\in(-\infty;0) \\ x-1 = t+1 & dx = dt & x\in(2;\infty) \Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t+1)^4}{t^2}\,dt = \int \frac{t^4+4t^3+6t^2+4t+1}{t^2}\,dt$

$= \int \left[t^2 + 4t + 6 + \frac{4}{t} + t^{-2}\right] dt = \frac{t^3}{3} + \frac{4t^2}{2} + 6t + 4\ln|t| + \frac{t^{-1}}{-1} + c$

$= \frac{(x-2)^3}{3} + 2(x-2)^2 + 6(x-2) + 4\ln|x-2| - \frac{1}{x-2} + c$

$= \frac{x^3-3\cdot 2x^2+3\cdot 2^2x-2^3}{3} + 2(x^2-4x+4) + 6(x-2) + 4\ln|x-2| - \frac{1}{x-2} + c$

$x\in R,\ x\neq 2,\ c\in R,$

$$\int \frac{(x-2)^2}{(x-1)^4}\,dx = -\frac{1}{x-1} + \frac{1}{(x-1)^2} - \frac{1}{3(x-1)^3} + c$$ [032]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t = x-1 & x = t+1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ x-2 = t-1 & dx = dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t-1)^2}{t^4}\,dt = \int \frac{t^2-2t+1}{t^4}\,dt$

$= \int \left[t^{-2}-2t^{-3}+t^{-4}\right] dt = \frac{t^{-1}}{-1} - \frac{2t^{-2}}{-2} + \frac{t^{-3}}{-3} + c = -\frac{1}{x-1} + \frac{1}{(x-1)^2} - \frac{1}{3(x-1)^3} + c,$

$x\in R,\ x\neq 1,\ c\in R.$

Riešené príklady – 031, 032

$$\int \frac{(x-1)^4}{(x-2)^2}\,dx = \frac{(x-2)^3}{3} + 2(x-2)^2 + 6(x-2) + 4\ln|x-2| - \frac{1}{x-2} + c$$ [032]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t=x-2 & x=t+2 & x\in(-\infty;2)\Rightarrow t\in(-\infty;0) \\ x-1=t+1 & dx=dt & x\in(2;\infty)\Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t+1)^4}{t^2}\,dt = \int \frac{t^4+4t^3+6t^2+4t+1}{t^2}\,dt$

$= \int \left[t^2 + 4t + 6 + \frac{4}{t} + t^{-2}\right] dt = \frac{t^3}{3} + \frac{4t^2}{2} + 6t + 4\ln|t| + \frac{t^{-1}}{-1} + c$

$= \frac{(x-2)^3}{3} + 2(x-2)^2 + 6(x-2) + 4\ln|x-2| - \frac{1}{x-2} + c$

$= \frac{x^3-3\cdot 2x^2+3\cdot 2^2x-2^3}{3} + 2(x^2-4x+4) + 6(x-2) + 4\ln|x-2| - \frac{1}{x-2} + c$

$= \frac{x^3}{3} + 2x + 4\ln|x-1| - \frac{1}{x-1} + c_1,\ x\in R,\ x\neq 2,\ c\in R,$ pričom $c_1 = c - \frac{20}{3}$.

$$\int \frac{(x-2)^2}{(x-1)^4}\,dx = -\frac{1}{x-1} + \frac{1}{(x-1)^2} - \frac{1}{3(x-1)^3} + c$$ [032]

- $= \left[\begin{array}{c|c|c} \text{Subst. } t=x-1 & x=t+1 & x\in(-\infty;1)\Rightarrow t\in(-\infty;0) \\ x-2=t-1 & dx=dt & x\in(1;\infty)\Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t-1)^2}{t^4}\,dt = \int \frac{t^2-2t+1}{t^4}\,dt$

$= \int \left[t^{-2}-2t^{-3}+t^{-4}\right] dt = \frac{t^{-1}}{-1} - \frac{2t^{-2}}{-2} + \frac{t^{-3}}{-3} + c = -\frac{1}{x-1} + \frac{1}{(x-1)^2} - \frac{1}{3(x-1)^3} + c,$

$x\in R,\ x\neq 1,\ c\in R.$

Riešené príklady – 033, 034

$\int \frac{x\,dx}{x-1}$ [033]

$\int \frac{x^2\,dx}{x-1}$ [034]

Riešené príklady – 033, 034

$\int \frac{x\,dx}{x-1}$ [033]

- $= \int \frac{(x-1)+1}{x-1}\,dx = \int \left[1 + \frac{1}{x-1}\right] dx$
- $= \left[\begin{array}{r|l} \text{Subst. } t = x-1 & x \in (-\infty; 1) \Rightarrow t \in (-\infty; 0) \\ dx = dt & x \in (1; \infty) \Rightarrow t \in (0; \infty) \end{array} \right] = \int \frac{t+1}{t}\,dt$

$\int \frac{x^2\,dx}{x-1}$ [034]

- $= \int \frac{(x^2-1)+1}{x-1}\,dx = \int \left[x + 1 + \frac{1}{x-1}\right] dx$
- $= \left[\begin{array}{r|l} \text{Subst. } t = x-1 & x \in (-\infty; 1) \Rightarrow t \in (-\infty; 0) \\ dx = dt & x \in (1; \infty) \Rightarrow t \in (0; \infty) \end{array} \right] = \int \frac{(t+1)^2}{t}\,dt$

Riešené príklady – 033, 034

$\int \frac{x\,dx}{x-1} = x + \ln|x-1| + c$ [033]

- $= \int \frac{(x-1)+1}{x-1}\,dx = \int \left[1 + \frac{1}{x-1}\right] dx = x + \ln|x-1| + c, \; x \in R, \; x \neq 1, \; c \in R.$

- $= \left[\begin{array}{r|l} \text{Subst. } t = x-1 & x \in (-\infty;1) \Rightarrow t \in (-\infty;0) \\ dx = dt & x \in (1;\infty) \Rightarrow t \in (0;\infty) \end{array} \right] = \int \frac{t+1}{t}\,dt = \int \left[1 + \frac{1}{t}\right] dt$

$\int \frac{x^2\,dx}{x-1} = \frac{x^2}{2} + x + \ln|x-1| + c$ [034]

- $= \int \frac{(x^2-1)+1}{x-1}\,dx = \int \left[x + 1 + \frac{1}{x-1}\right] dx = \frac{x^2}{2} + x + \ln|x-1| + c, \; x \in R, \; x \neq 1, \; c \in R.$

- $= \left[\begin{array}{r|l} \text{Subst. } t = x-1 & x \in (-\infty;1) \Rightarrow t \in (-\infty;0) \\ dx = dt & x \in (1;\infty) \Rightarrow t \in (0;\infty) \end{array} \right] = \int \frac{(t+1)^2}{t}\,dt = \int \frac{t^2+2t+1}{t}\,dt = \int \left[t + 2 + \frac{1}{t}\right] dt$

Riešené príklady – 033, 034

$$\int \frac{x\,dx}{x-1} = x + \ln|x-1| + c$$ [033]

- $= \int \frac{(x-1)+1}{x-1}\,dx = \int \Big[1 + \frac{1}{x-1}\Big]dx = x + \ln|x-1| + c,\ x\in R,\ x\neq 1,\ c\in R.$

- $= \left[\begin{array}{r|l} \text{Subst. } t = x-1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ dx = dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{t+1}{t}\,dt = \int \Big[1 + \frac{1}{t}\Big]dt = t + \ln|t| + c_1$

$$\int \frac{x^2\,dx}{x-1} = \frac{x^2}{2} + x + \ln|x-1| + c$$ [034]

- $= \int \frac{(x^2-1)+1}{x-1}\,dx = \int \Big[x + 1 + \frac{1}{x-1}\Big]dx = \frac{x^2}{2} + x + \ln|x-1| + c,\ x\in R,\ x\neq 1,\ c\in R.$

- $= \left[\begin{array}{r|l} \text{Subst. } t = x-1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ dx = dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{(t+1)^2}{t}\,dt = \int \frac{t^2+2t+1}{t}\,dt = \int \Big[t + 2 + \frac{1}{t}\Big]dt$

  $= \frac{t^2}{2} + 2t + \ln|t| + c_1$

Riešené príklady – 033, 034

$$\int \frac{x\,dx}{x-1} = x + \ln|x-1| + c$$ [033]

- $= \int \frac{(x-1)+1}{x-1}\,dx = \int \Big[1 + \frac{1}{x-1}\Big] dx = x + \ln|x-1| + c, \; x\in R, \; x\neq 1, \; c\in R.$

- $= \left[\begin{array}{r|l} \text{Subst. } t = x-1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ dx = dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{t+1}{t}\,dt = \int \Big[1 + \frac{1}{t}\Big] dt = t + \ln|t| + c_1$
  $= x-1 + \ln|x-1| + c_1 = x + \ln|x-1| + c, \; x\in R, \; x\neq 1, \; c, c_1\in R,$ pričom $c = c_1 - 1.$

$$\int \frac{x^2\,dx}{x-1} = \frac{x^2}{2} + x + \ln|x-1| + c$$ [034]

- $= \int \frac{(x^2-1)+1}{x-1}\,dx = \int \Big[x + 1 + \frac{1}{x-1}\Big] dx = \frac{x^2}{2} + x + \ln|x-1| + c, \; x\in R, \; x\neq 1, \; c\in R.$

- $= \left[\begin{array}{r|l} \text{Subst. } t = x-1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ dx = dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t+1)^2}{t}\,dt = \int \frac{t^2+2t+1}{t}\,dt = \int \Big[t + 2 + \frac{1}{t}\Big] dt$
  $= \frac{t^2}{2} + 2t + \ln|t| + c_1 = \frac{(x-1)^2}{2} + 2(x-1) + \ln|x-1| + c_1$
  $= \frac{x^2-2x+1}{2} + 2(x-1) + \ln|x-1| + c_1$
  $x\in R, \; x\neq 1, \quad c_1\in R,$

Riešené príklady – 033, 034

$$\int \frac{x\,dx}{x-1} = x + \ln|x-1| + c$$ [033]

- $= \int \frac{(x-1)+1}{x-1}\,dx = \int \left[1 + \frac{1}{x-1}\right] dx = x + \ln|x-1| + c,\ x \in R,\ x \neq 1,\ c \in R.$

- $= \left[\begin{array}{r|l} \text{Subst. } t = x-1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ dx = dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{t+1}{t}\,dt = \int \left[1 + \frac{1}{t}\right] dt = t + \ln|t| + c_1$
  $= x-1 + \ln|x-1| + c_1 = x + \ln|x-1| + c,\ x\in R,\ x\neq 1,\ c, c_1 \in R,$ pričom $c = c_1 - 1.$

$$\int \frac{x^2\,dx}{x-1} = \frac{x^2}{2} + x + \ln|x-1| + c$$ [034]

- $= \int \frac{(x^2-1)+1}{x-1}\,dx = \int \left[x + 1 + \frac{1}{x-1}\right] dx = \frac{x^2}{2} + x + \ln|x-1| + c,\ x\in R,\ x\neq 1,\ c\in R.$

- $= \left[\begin{array}{r|l} \text{Subst. } t = x-1 & x\in(-\infty;1) \Rightarrow t\in(-\infty;0) \\ dx = dt & x\in(1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{(t+1)^2}{t}\,dt = \int \frac{t^2+2t+1}{t}\,dt = \int \left[t + 2 + \frac{1}{t}\right] dt$
  $= \frac{t^2}{2} + 2t + \ln|t| + c_1 = \frac{(x-1)^2}{2} + 2(x-1) + \ln|x-1| + c_1$
  $= \frac{x^2-2x+1}{2} + 2(x-1) + \ln|x-1| + c_1 = \frac{x^2}{2} + x + \ln|x-1| + c,$
  $x\in R,\ x\neq 1,\ c, c_1\in R,$ pričom $c = c_1 - \frac{3}{2}.$

Riešené príklady – 035, 036, 037

$\int \frac{x^6 \, dx}{x-1}$ [035]

$\int \frac{x^9 \, dx}{x-1}$ [036]

$\int \frac{x^n \, dx}{x-1}$ pre $n \in N$. [037]

Riešené príklady – 035, 036, 037

$\int \frac{x^6\,dx}{x-1}$ [035]

- $= \int \frac{(x^6-1)+1}{x-1}\,dx = \int \left[\frac{x^6-1}{x-1} + \frac{1}{x-1}\right] dx$

$\int \frac{x^9\,dx}{x-1}$ [036]

- $= \int \frac{(x^9-1)+1}{x-1}\,dx = \int \left[\frac{x^9-1}{x-1} + \frac{1}{x-1}\right] dx$

$\int \frac{x^n\,dx}{x-1}$ pre $n \in N$. [037]

- $= \int \frac{(x^n-1)+1}{x-1}\,dx = \int \left[\frac{x^n-1}{x-1} + \frac{1}{x-1}\right] dx$

Riešené príklady – 035, 036, 037

$\int \frac{x^6\,dx}{x-1}$ [035]

- $= \int \frac{(x^6-1)+1}{x-1}\,dx = \int \left[\frac{x^6-1}{x-1} + \frac{1}{x-1}\right] dx = \int \left[x^5+x^4+x^3+x^2+x+1 + \frac{1}{x-1}\right] dx$

$\int \frac{x^9\,dx}{x-1}$ [036]

- $= \int \frac{(x^9-1)+1}{x-1}\,dx = \int \left[\frac{x^9-1}{x-1} + \frac{1}{x-1}\right] dx = \int \left[x^8+x^7+\cdots+x^3+x^2+x+1 + \frac{1}{x-1}\right] dx$

$\int \frac{x^n\,dx}{x-1}$ pre $n \in N$. [037]

- $= \int \frac{(x^n-1)+1}{x-1}\,dx = \int \left[\frac{x^n-1}{x-1} + \frac{1}{x-1}\right] dx = \int \left[x^{n-1}+x^{n-2}+\cdots+x+1 + \frac{1}{x-1}\right] dx$

Riešené príklady – 035, 036, 037

$$\int \frac{x^6\,dx}{x-1} = \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + c$$ [035]

- $= \int \frac{(x^6-1)+1}{x-1}\,dx = \int \left[\frac{x^6-1}{x-1} + \frac{1}{x-1}\right]dx = \int \left[x^5+x^4+x^3+x^2+x+1 + \frac{1}{x-1}\right]dx$
  $= \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + c,\ x\in R,\ x\neq 1,\ c\in R.$

$$\int \frac{x^9\,dx}{x-1} = \frac{x^9}{9} + \frac{x^8}{8} + \frac{x^7}{7} + \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + c$$ [036]

- $= \int \frac{(x^9-1)+1}{x-1}\,dx = \int \left[\frac{x^9-1}{x-1} + \frac{1}{x-1}\right]dx = \int \left[x^8+x^7+\cdots+x^3+x^2+x+1 + \frac{1}{x-1}\right]dx$
  $= \frac{x^9}{9} + \frac{x^8}{8} + \frac{x^7}{7} + \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + c,\ x\in R,\ x\neq 1,\ c\in R.$

$$\int \frac{x^n\,dx}{x-1} = \frac{x^n}{n} + \frac{x^{n-1}}{n-1} + \cdots + \frac{x^2}{2} + x + \ln|x-1| + c = \ln|x-1| + \sum_{k=1}^{n} \frac{x^k}{k} \quad \text{pre } n\in N.$$ [037]

- $= \int \frac{(x^n-1)+1}{x-1}\,dx = \int \left[\frac{x^n-1}{x-1} + \frac{1}{x-1}\right]dx = \int \left[x^{n-1}+x^{n-2}+\cdots+x+1 + \frac{1}{x-1}\right]dx$
  $= \frac{x^n}{n} + \frac{x^{n-1}}{n-1} + \cdots + \frac{x^2}{2} + x + \ln|x-1| + c = \ln|x-1| + \sum_{k=1}^{n} \frac{x^k}{k},\ x\in R,\ x\neq 1,\ c\in R.$

Riešené príklady – 038, 039, 040

$\int \frac{dx}{x-t}$ pre $t \in R$ a $n = 1$. [038]

$\int \frac{dx}{(x-t)^n}$ pre $t \in R$ a $n \in N$, $n \neq 1$. [039]

$\int \frac{dt}{(x-t)^n}$ pre $x \in R$ a $n \in N$, $n \neq 1$. [040]

Riešené príklady – 038, 039, 040

$\displaystyle\int \frac{dx}{x-t}$ pre $t \in R$ a $n = 1$. [038]

- $= \left[\begin{array}{r|l} \text{Subst. } u = x-t & x \in (-\infty; t) \Rightarrow u \in (-\infty; 0) \\ du = dx & x \in (t; \infty) \Rightarrow u \in (0; \infty) \end{array} \right] = \displaystyle\int \frac{du}{u}$

$\displaystyle\int \frac{dx}{(x-t)^n}$ pre $t \in R$ a $n \in N$, $n \neq 1$. [039]

- $= \left[\begin{array}{r|l} \text{Subst. } u = x-t & x \in (-\infty; t) \Rightarrow u \in (-\infty; 0) \\ du = dx & x \in (t; \infty) \Rightarrow u \in (0; \infty) \end{array} \right] = \displaystyle\int \frac{du}{u^n} = \int u^{-n}\, du$

$\displaystyle\int \frac{dt}{(x-t)^n}$ pre $x \in R$ a $n \in N$, $n \neq 1$. [040]

- $= \left[\begin{array}{r|l} \text{Subst. } u = x-t & t \in (-\infty; x) \Rightarrow u \in (0; \infty) \\ du = -dt & t \in (x; \infty) \Rightarrow u \in (-\infty; 0) \end{array} \right] = -\displaystyle\int \frac{du}{u^n} = -\int u^{-n}\, du$

Riešené príklady – 038, 039, 040

$\int \frac{dx}{x-t}$ pre $t \in R$ a $n = 1$. [038]

- $= \left[\begin{array}{r|l} \text{Subst. } u = x-t & x \in (-\infty; t) \Rightarrow u \in (-\infty; 0) \\ du = dx & x \in (t; \infty) \Rightarrow u \in (0; \infty) \end{array} \right] = \int \frac{du}{u} = \ln|u| + c$

$\int \frac{dx}{(x-t)^n}$ pre $t \in R$ a $n \in N$, $n \neq 1$. [039]

- $= \left[\begin{array}{r|l} \text{Subst. } u = x-t & x \in (-\infty; t) \Rightarrow u \in (-\infty; 0) \\ du = dx & x \in (t; \infty) \Rightarrow u \in (0; \infty) \end{array} \right] = \int \frac{du}{u^n} = \int u^{-n}\, du = \frac{u^{-n+1}}{-n+1} + c$

$\int \frac{dt}{(x-t)^n}$ pre $x \in R$ a $n \in N$, $n \neq 1$. [040]

- $= \left[\begin{array}{r|l} \text{Subst. } u = x-t & t \in (-\infty; x) \Rightarrow u \in (0; \infty) \\ du = -dt & t \in (x; \infty) \Rightarrow u \in (-\infty; 0) \end{array} \right] = -\int \frac{du}{u^n} = -\int u^{-n}\, du = -\frac{u^{-n+1}}{-n+1} + c$

Riešené príklady – 038, 039, 040

$\displaystyle\int \frac{dx}{x-t} = \ln|x-t| + c$ pre $t \in R$ a $n = 1$. [038]

- $\displaystyle = \left[\begin{array}{r|l} \text{Subst. } u = x-t & x\in(-\infty;t) \Rightarrow u\in(-\infty;0) \\ du = dx & x\in(t;\infty) \Rightarrow u\in(0;\infty) \end{array}\right] = \int \frac{du}{u} = \ln|u| + c = \ln|x-t| + c,\ x\in R,\ x\neq t,\ c\in R.$

$\displaystyle\int \frac{dx}{(x-t)^n} = -\frac{1}{(n-1)\cdot(x-t)^{n-1}} + c$ pre $t \in R$ a $n \in N$, $n \neq 1$. [039]

- $\displaystyle = \left[\begin{array}{r|l} \text{Subst. } u = x-t & x\in(-\infty;t) \Rightarrow u\in(-\infty;0) \\ du = dx & x\in(t;\infty) \Rightarrow u\in(0;\infty) \end{array}\right] = \int \frac{du}{u^n} = \int u^{-n}\,du = \frac{u^{-n+1}}{-n+1} + c$
  $\displaystyle = \frac{(x-t)^{1-n}}{1-n} + c = -\frac{1}{(n-1)\cdot(x-t)^{n-1}} + c,\ x\in R,\ x\neq t,\ c\in R.$

$\displaystyle\int \frac{dt}{(x-t)^n} = \frac{1}{(n-1)\cdot(x-t)^{n-1}} + c$ pre $x \in R$ a $n \in N$, $n \neq 1$. [040]

- $\displaystyle = \left[\begin{array}{r|l} \text{Subst. } u = x-t & t\in(-\infty;x) \Rightarrow u\in(0;\infty) \\ du = -dt & t\in(x;\infty) \Rightarrow u\in(-\infty;0) \end{array}\right] = -\int \frac{du}{u^n} = -\int u^{-n}\,du = -\frac{u^{-n+1}}{-n+1} + c$
  $\displaystyle = \frac{(x-t)^{1-n}}{n-1} + c = \frac{1}{(n-1)\cdot(x-t)^{n-1}} + c,\ t\in R,\ t\neq x,\ c\in R.$

Riešené príklady – 041, 042, 043

$\int \frac{x^2\,dx}{x^3+1}$ [041]

$\int \frac{x^2\,dx}{x^6+1}$ [042]

$\int \frac{x^2\,dx}{x^6-1}$ [043]

Riešené príklady – 041, 042, 043

$\int \frac{x^2\,dx}{x^3+1}$ [041]

- $= \frac{1}{3}\int \frac{3x^2}{x^3+1}\,dx = \frac{1}{3}\int \frac{(x^3+1)'}{x^3+1}\,dx$

- $= \left[\begin{array}{c|c} \text{Subst. } t = x^3+1 & x\in(-\infty;-1) \Rightarrow t\in(-\infty;0) \\ dt = 3x^2\,dx & x\in(-1;\infty) \Rightarrow t\in(0;\infty) \end{array}\right] = \frac{1}{3}\int \frac{dt}{t}$

$\int \frac{x^2\,dx}{x^6+1}$ [042]

- $= \left[\begin{array}{c|c} \text{Subst. } t = x^3 & x\in R \\ dt = 3x^2\,dx & t\in R \end{array}\right] = \frac{1}{3}\int \frac{dt}{t^2+1}$

$\int \frac{x^2\,dx}{x^6-1}$ [043]

- $= \left[\begin{array}{c|c|c|c} \text{Subst. } t = x^3 & x\in(-\infty;-1) & x\in(-1;1) & x\in(1;\infty) \\ dt = 3x^2\,dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array}\right] = \frac{1}{3}\int \frac{dt}{t^2-1}$

Riešené príklady – 041, 042, 043

$$\int \frac{x^2\,dx}{x^3+1} = \frac{1}{3}\ln\left|x^3+1\right| + c$$ [041]

- $= \frac{1}{3}\int \frac{3x^2}{x^3+1}\,dx = \frac{1}{3}\int \frac{(x^3+1)'}{x^3+1}\,dx = \frac{1}{3}\ln\left|x^3+1\right| + c,\ x\in R,\ x\neq -1,\ c\in R.$

- $= \left[\begin{array}{c|c} \text{Subst. } t = x^3+1 & x\in(-\infty;-1) \Rightarrow t\in(-\infty;0) \\ dt = 3x^2\,dx & x\in(-1;\infty) \Rightarrow t\in(0;\infty) \end{array}\right] = \frac{1}{3}\int \frac{dt}{t} = \frac{1}{3}\ln|t| + c$

$$\int \frac{x^2\,dx}{x^6+1}$$ [042]

- $= \left[\begin{array}{c|c} \text{Subst. } t = x^3 & x\in R \\ dt = 3x^2\,dx & t\in R \end{array}\right] = \frac{1}{3}\int \frac{dt}{t^2+1} = \frac{1}{3}\operatorname{arctg} t + c$

$$\int \frac{x^2\,dx}{x^6-1}$$ [043]

- $= \left[\begin{array}{c|c|c|c} \text{Subst. } t = x^3 & x\in(-\infty;-1) & x\in(-1;1) & x\in(1;\infty) \\ dt = 3x^2\,dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array}\right] = \frac{1}{3}\int \frac{dt}{t^2-1} = \frac{1}{3}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c$

Riešené príklady – 041, 042, 043

$\int \frac{x^2\,dx}{x^3+1} = \frac{1}{3} \ln |x^3+1| + c$ [041]

- $= \frac{1}{3} \int \frac{3x^2}{x^3+1}\,dx = \frac{1}{3} \int \frac{(x^3+1)'}{x^3+1}\,dx = \frac{1}{3} \ln |x^3+1| + c,\ x \in R,\ x \neq -1,\ c \in R.$

- $= \left[\begin{array}{c|c} \text{Subst. } t = x^3+1 & x\in(-\infty;-1) \Rightarrow t\in(-\infty;0) \\ dt = 3x^2\,dx & x\in(-1;\infty) \Rightarrow t\in(0;\infty) \end{array} \right] = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln |t| + c = \frac{1}{3} \ln |x^3+1| + c,$
  $x \in R,\ x \neq -1,\ c \in R.$

$\int \frac{x^2\,dx}{x^6+1} = \frac{1}{3} \operatorname{arctg} x^3 + c$ [042]

- $= \left[\begin{array}{c|c} \text{Subst. } t = x^3 & x\in R \\ dt = 3x^2\,dx & t\in R \end{array} \right] = \frac{1}{3} \int \frac{dt}{t^2+1} = \frac{1}{3} \operatorname{arctg} t + c = \frac{1}{3} \operatorname{arctg} x^3 + c,\ x \in R,\ c \in R.$

$\int \frac{x^2\,dx}{x^6-1} = \frac{1}{6} \ln \left| \frac{x^3-1}{x^3+1} \right| + c = \frac{1}{6} \ln |x^3-1| - \frac{1}{6} \ln |x^3+1| + c$ [043]

- $= \left[\begin{array}{c|c|c|c} \text{Subst. } t = x^3 & x\in(-\infty;-1) & x\in(-1;1) & x\in(1;\infty) \\ dt = 3x^2\,dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array} \right] = \frac{1}{3} \int \frac{dt}{t^2-1} = \frac{1}{3} \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c = \frac{1}{6} \ln \left| \frac{x^3-1}{x^3+1} \right| + c$
  $= \frac{1}{6} \ln |x^3-1| - \frac{1}{6} \ln |x^3+1| + c,\ x \in R,\ x \neq \pm 1,\ c \in R.$

Riešené príklady – 044, 045

$\int |x-a|^{98}\,dx$ pre $a \in R$. [044]

$\int |x-a|^{99}\,dx$ pre $a \in R$. [045]

Riešené príklady – 044, 045

$\int |x-a|^{98}\,dx$ pre $a \in R$. [044]

- $= \int (x-a)^{98}\,dx$

$\int |x-a|^{99}\,dx$ pre $a \in R$. [045]

Pre $x \in \langle a; \infty)$ platí $|x-a| = x-a$:

- $= \int (x-a)^{99}\,dx$

Pre $x \in (-\infty; a\rangle$ platí $|x-a| = -(x-a) = a-x$:

- $= \int (a-x)^{99}\,dx$

Riešené príklady – 044, 045

$\int |x-a|^{98}\,dx$ pre $a\in R$. [044]

- $= \int (x-a)^{98}\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x-a & x\in R \\ dt=dx & t\in R \end{array}\right] = \int t^{98}\,dt$

$\int |x-a|^{99}\,dx$ pre $a\in R$. [045]

Pre $x\in\langle a;\infty)$ platí $|x-a|=x-a$:

- $= \int (x-a)^{99}\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x-a & x\in\langle a;\infty) \\ dt=dx & t\in\langle 0;\infty) \end{array}\right] = \int t^{99}\,dt$

Pre $x\in(-\infty;a\rangle$ platí $|x-a|=-(x-a)=a-x$:

- $= \int (a-x)^{99}\,dx = \left[\begin{array}{r|l} \text{Subst. } t=a-x & x\in(-\infty;a\rangle \\ dt=-dx & t\in\langle 0;\infty) \end{array}\right] = -\int t^{99}\,dt$

Riešené príklady – 044, 045

$\int |x-a|^{98}\,dx$ pre $a \in R$. [044]

- $= \int (x-a)^{98}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = x-a & x \in R \\ dt = dx & t \in R \end{array} \right] = \int t^{98}\,dt = \frac{t^{99}}{99} + c$

$\int |x-a|^{99}\,dx$ pre $a \in R$. [045]

Pre $x \in \langle a; \infty)$ platí $|x-a| = x-a$:

- $= \int (x-a)^{99}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = x-a & x \in \langle a; \infty) \\ dt = dx & t \in \langle 0; \infty) \end{array} \right] = \int t^{99}\,dt = \frac{t^{100}}{100} + c$

Pre $x \in (-\infty; a\rangle$ platí $|x-a| = -(x-a) = a-x$:

- $= \int (a-x)^{99}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = a-x & x \in (-\infty; a\rangle \\ dt = -dx & t \in \langle 0; \infty) \end{array} \right] = -\int t^{99}\,dt = -\frac{t^{100}}{100} + c$

Riešené príklady – 044, 045

$$\int |x-a|^{98}\,dx = \frac{(x-a)^{99}}{99} + c \qquad \text{pre } a \in R. \qquad [044]$$

- $= \int (x-a)^{98}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = x-a & x \in R \\ dt = dx & t \in R \end{array} \right] = \int t^{98}\,dt = \frac{t^{99}}{99} + c = \frac{(x-a)^{99}}{99} + c$

$$\int |x-a|^{99}\,dx \qquad \text{pre } a \in R. \qquad [045]$$

Pre $x \in \langle a; \infty)$ platí $|x-a| = x-a$:

- $= \int (x-a)^{99}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = x-a & x \in \langle a; \infty) \\ dt = dx & t \in \langle 0; \infty) \end{array} \right] = \int t^{99}\,dt = \frac{t^{100}}{100} + c = \frac{(x-a)^{100}}{100} + c$

Pre $x \in (-\infty; a\rangle$ platí $|x-a| = -(x-a) = a-x$:

- $= \int (a-x)^{99}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = a-x & x \in (-\infty; a\rangle \\ dt = -dx & t \in \langle 0; \infty) \end{array} \right] = -\int t^{99}\,dt = -\frac{t^{100}}{100} + c = -\frac{(a-x)^{100}}{100} + c$

Riešené príklady – 044, 045

$$\int |x-a|^{98}\,dx = \frac{(x-a)^{99}}{99} + c = \frac{|x-a|^{98}\cdot(x-a)}{99} + c \text{ pre } a\in R.$$ [044]

- $$= \int (x-a)^{98}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = x-a & x\in R \\ dt = dx & t\in R \end{array}\right] = \int t^{98}\,dt = \frac{t^{99}}{99} + c = \frac{(x-a)^{99}}{99} + c$$
$$= \frac{(x-a)^{98}\cdot(x-a)}{99} + c = \frac{|x-a|^{98}\cdot(x-a)}{99} + c,\ x\in R,\ c\in R.$$

$$\int |x-a|^{99}\,dx = \frac{|x-a|^{99}\cdot(x-a)}{100} + c,\ x\in R,\ c\in R \text{ pre } a\in R.$$ [045]

Pre $x\in\langle a;\infty)$ platí $|x-a| = x-a$:

- $$= \int (x-a)^{99}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = x-a & x\in\langle a;\infty) \\ dt = dx & t\in\langle 0;\infty) \end{array}\right] = \int t^{99}\,dt = \frac{t^{100}}{100} + c = \frac{(x-a)^{100}}{100} + c$$
$$= \frac{(x-a)^{99}\cdot(x-a)}{100} + c = \frac{|x-a|^{99}\cdot(x-a)}{100} + c,\ x\in\langle a;\infty),\ c\in R.$$

Pre $x\in(-\infty;a\rangle$ platí $|x-a| = -(x-a) = a-x$:

- $$= \int (a-x)^{99}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = a-x & x\in(-\infty;a\rangle \\ dt = -dx & t\in\langle 0;\infty) \end{array}\right] = -\int t^{99}\,dt = -\frac{t^{100}}{100} + c = -\frac{(a-x)^{100}}{100} + c$$
$$= -\frac{(a-x)^{99}\cdot(a-x)}{100} + c = \frac{(a-x)^{99}\cdot(x-a)}{100} + c = \frac{|x-a|^{99}\cdot(x-a)}{100} + c,\ x\in(-\infty;a\rangle,\ c\in R.$$

Riešené príklady – 046, 047

$$\int |x-a|\cdot(x-a)^{98}\,dx$$ pre $a\in R$. [046]

$$\int |x-a|\cdot(x-a)^{99}\,dx$$ pre $a\in R$. [047]

Riešené príklady – 046, 047

$$\int |x-a|\cdot(x-a)^{98}\,\mathrm{d}x$$ pre $a\in R$. [046]

Pre $x\in\langle a;\infty)$ platí $|x-a|=x-a$:

- $= \int (x-a)^{99}\,\mathrm{d}x$

Pre $x\in(-\infty;a\rangle$ platí $|x-a|=-(x-a)$:

- $= -\int (x-a)^{99}\,\mathrm{d}x$

$$\int |x-a|\cdot(x-a)^{99}\,\mathrm{d}x$$ pre $a\in R$. [047]

Pre $x\in\langle a;\infty)$ platí $|x-a|=x-a$:

- $= \int (x-a)^{100}\,\mathrm{d}x$

Pre $x\in(-\infty;a\rangle$ platí $|x-a|=-(x-a)$:

- $= -\int (x-a)^{100}\,\mathrm{d}x$

Riešené príklady – 046, 047

$$\int |x-a|\cdot(x-a)^{98}\,dx \quad \text{pre } a\in R.$$ [046]

Pre $x\in\langle a;\infty)$ platí $|x-a|=x-a$:

- $= \int (x-a)^{99}\,dx = \frac{(x-a)^{100}}{100} + c$

Pre $x\in(-\infty;a\rangle$ platí $|x-a|=-(x-a)$:

- $= -\int (x-a)^{99}\,dx = -\frac{(x-a)^{100}}{100} + c$

$$\int (x-a)^{99}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x-a & x\in R \\ dt=dx & t\in R \end{array}\right] = \int t^{99}\,dt = \frac{t^{100}}{100}+c = \frac{(x-a)^{100}}{100}+c,\ x\in R,\ c\in R.$$

$$\int |x-a|\cdot(x-a)^{99}\,dx \quad \text{pre } a\in R.$$ [047]

Pre $x\in\langle a;\infty)$ platí $|x-a|=x-a$:

- $= \int (x-a)^{100}\,dx = \frac{(x-a)^{101}}{101} + c$

Pre $x\in(-\infty;a\rangle$ platí $|x-a|=-(x-a)$:

- $= -\int (x-a)^{100}\,dx = -\frac{(x-a)^{101}}{101} + c$

$$\int (x-a)^{100}\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x-a & x\in R \\ dt=dx & t\in R \end{array}\right] = \int t^{100}\,dt = \frac{t^{101}}{101}+c = \frac{(x-a)^{101}}{101}+c,\ x\in R,\ c\in R.$$

Riešené príklady – 046, 047

$$\int |x-a|\cdot(x-a)^{98}\,dx = \frac{|x-a|\cdot(x-a)^{99}}{100} + c \text{ pre } a\in R.$$ [046]

Pre $x\in\langle a;\infty)$ platí $|x-a| = x-a$:

- $= \int (x-a)^{99}\,dx = \frac{(x-a)^{100}}{100} + c = \frac{(x-a)\cdot(x-a)^{99}}{100} + c = \frac{|x-a|\cdot(x-a)^{99}}{100} + c,\ c\in R.$

Pre $x\in(-\infty;a\rangle$ platí $|x-a| = -(x-a)$:

- $= -\int (x-a)^{99}\,dx = -\frac{(x-a)^{100}}{100} + c = \frac{-(x-a)\cdot(x-a)^{99}}{100} + c = \frac{|x-a|\cdot(x-a)^{99}}{100} + c,\ c\in R.$

$$\int (x-a)^{99}\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x-a & x\in R \\ dt=dx & t\in R \end{array}\right] = \int t^{99}\,dt = \frac{t^{100}}{100} + c = \frac{(x-a)^{100}}{100} + c,\ x\in R,\ c\in R.$$

$$\int |x-a|\cdot(x-a)^{99}\,dx = \frac{|x-a|\cdot(x-a)^{100}}{101} + c \text{ pre } a\in R.$$ [047]

Pre $x\in\langle a;\infty)$ platí $|x-a| = x-a$:

- $= \int (x-a)^{100}\,dx = \frac{(x-a)^{101}}{101} + c = \frac{(x-a)\cdot(x-a)^{100}}{101} + c = \frac{|x-a|\cdot(x-a)^{100}}{101} + c,\ c\in R.$

Pre $x\in(-\infty;a\rangle$ platí $|x-a| = -(x-a)$:

- $= -\int (x-a)^{100}\,dx = -\frac{(x-a)^{101}}{101} + c = \frac{-(x-a)\cdot(x-a)^{100}}{101} + c = \frac{|x-a|\cdot(x-a)^{100}}{101} + c,\ c\in R.$

$$\int (x-a)^{100}\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x-a & x\in R \\ dt=dx & t\in R \end{array}\right] = \int t^{100}\,dt = \frac{t^{101}}{101} + c = \frac{(x-a)^{101}}{101} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 048, 049, 050

$$\int (x-1)(x-2)(x-3)\,dx$$

[048]

$$\int (x-1)(x+2)(x-3)\,dx$$

[049]

$$\int x(x-a)(x-b)\,dx$$

pre $a, b \in R$.

[050]

Riešené príklady – 048, 049, 050

$\int (x-1)(x-2)(x-3)\,dx$ [048]

- $= \int (x^2 - 3x + 2)(x-3)\,dx = \int \left[x^3 - 6x^2 + 11x - 6\right] dx$

$\int (x-1)(x+2)(x-3)\,dx$ [049]

- $= \int (x^2 + x - 2)(x-3)\,dx = \int \left[x^3 - 2x^2 - 5x + 6\right] dx$

$\int x(x-a)(x-b)\,dx$ pre $a, b \in R$. [050]

- $= \int x(x^2 - ax - bx + ab)\,dx = \int \left[x^3 - (a+b)x^2 + abx\right] dx$

Riešené príklady – 048, 049, 050

$$\int (x-1)(x-2)(x-3)\,dx = \frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x + c$$ [048]

- $= \int (x^2 - 3x + 2)(x-3)\,dx = \int \Big[x^3 - 6x^2 + 11x - 6\Big]\,dx$
$= \frac{x^4}{4} - \frac{6x^3}{3} + \frac{11x^2}{2} - 6x + c = \frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x + c$, $x \in R$, $c \in R$.

$$\int (x-1)(x+2)(x-3)\,dx = \frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x + c$$ [049]

- $= \int (x^2 + x - 2)(x-3)\,dx = \int \Big[x^3 - 2x^2 - 5x + 6\Big]\,dx$
$= \frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x + c$, $x \in R$, $c \in R$.

$$\int x(x-a)(x-b)\,dx = \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c$$ pre $a, b \in R$. [050]

- $= \int x(x^2 - ax - bx + ab)\,dx = \int \Big[x^3 - (a+b)x^2 + abx\Big]\,dx$
$= \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c$, $x \in R$, $c \in R$.

Riešené príklady – 051

$\int \frac{dx}{x^2+ax+b}$ pre $a, b \in R$. [051]

Riešené príklady – 051

$\int \frac{dx}{x^2+ax+b}$ pre $a, b \in R$. [051]

- $= \int \frac{dx}{\left(x^2+ax+\frac{a^2}{4}\right)+b-\frac{a^2}{4}} = \int \frac{dx}{(x+\frac{a}{2})^2+b-\frac{a^2}{4}} = \int \frac{dx}{\left(\frac{2x+a}{2}\right)^2+\frac{4b-a^2}{4}}$

Riešené príklady – 051

$\int \frac{dx}{x^2+ax+b}$ pre $a,b \in R$. [051]

- $= \int \frac{dx}{\left(x^2+ax+\frac{a^2}{4}\right)+b-\frac{a^2}{4}} = \int \frac{dx}{(x+\frac{a}{2})^2+b-\frac{a^2}{4}} = \int \frac{dx}{\left(\frac{2x+a}{2}\right)^2+\frac{4b-a^2}{4}} = \int \frac{dx}{\frac{(2x+a)^2}{4}+\frac{4b-a^2}{4}}$
  $= 4\int \frac{dx}{(2x+a)^2+(4b-a^2)}$

Pre $4b-a^2>0$ platí $(2x+a)^2+(4b-a^2)$, pričom $x \in R$

- $\int \frac{dx}{x^2+ax+b}$

Pre $4b-a^2=0$ platí $(2x+a)^2+(4b-a^2)$, pričom $x \in R$, $x \neq -\frac{a}{2}$

- $\int \frac{dx}{x^2+ax+b}$

Pre $4b-a^2<0$ platí $(2x+a)^2+(4b-a^2)$, pričom $x \in R$, $x \neq \frac{-a\pm\sqrt{a^2-4b}}{2}$

- $\int \frac{dx}{x^2+ax+b}$

Riešené príklady – 051

$\int \frac{dx}{x^2+ax+b}$ pre $a,b \in R$. [051]

- $= \int \frac{dx}{\left(x^2+ax+\frac{a^2}{4}\right)+b-\frac{a^2}{4}} = \int \frac{dx}{(x+\frac{a}{2})^2+b-\frac{a^2}{4}} = \int \frac{dx}{\left(\frac{2x+a}{2}\right)^2+\frac{4b-a^2}{4}} = \int \frac{dx}{\frac{(2x+a)^2}{4}+\frac{4b-a^2}{4}}$
  $= 4\int \frac{dx}{(2x+a)^2+(4b-a^2)} = \left[\begin{array}{r|l|r} \text{Subst. } t=2x+a & x\in R & x^2+ax+b\neq 0 \Rightarrow (2x+a)^2+(4b-a^2)\neq 0 \\ dt = 2\,dx & t\in R & t^2+(4b-a^2)\neq 0 \end{array} \right] = 2\int \frac{dt}{t^2+(4b-a^2)}$.

Pre $4b-a^2>0$ platí $(2x+a)^2+(4b-a^2)=t^2+(4b-a^2)>t^2$, pričom $x\in R$ a $t\in R$:

- $\int \frac{dx}{x^2+ax+b} = 2\int \frac{dt}{t^2+\left(\sqrt{4b-a^2}\right)^2}$

Pre $4b-a^2=0$ platí $(2x+a)^2+(4b-a^2)=t^2+0=t^2$, pričom $x\in R$, $x\neq -\frac{a}{2}$ a $t\in R$, $t\neq 0$:

- $\int \frac{dx}{x^2+ax+b} = 2\int \frac{dt}{t^2} = 2\int t^{-2}\,dt$

Pre $4b-a^2<0$ platí $(2x+a)^2+(4b-a^2)=t^2+(4b-a^2)<t^2$, pričom $x\in R$, $x\neq \frac{-a\pm\sqrt{a^2-4b}}{2}$ a $t\in R$, $t\neq \pm\sqrt{a^2-4b}$:

- $\int \frac{dx}{x^2+ax+b} = 2\int \frac{dt}{t^2-\left(\sqrt{a^2-4b}\right)^2}$

Riešené príklady – 051

$\int \frac{dx}{x^2+ax+b}$ pre $a,b\in R$. [051]

- $= \int \frac{dx}{\left(x^2+ax+\frac{a^2}{4}\right)+b-\frac{a^2}{4}} = \int \frac{dx}{(x+\frac{a}{2})^2+b-\frac{a^2}{4}} = \int \frac{dx}{\left(\frac{2x+a}{2}\right)^2+\frac{4b-a^2}{4}} = \int \frac{dx}{\frac{(2x+a)^2}{4}+\frac{4b-a^2}{4}}$

$$= 4\int \frac{dx}{(2x+a)^2+(4b-a^2)} = \left[\begin{array}{r|l|r} \text{Subst. } t=2x+a & x\in R & x^2+ax+b\neq 0 \Rightarrow (2x+a)^2+(4b-a^2)\neq 0 \\ dt=2\,dx & t\in R & t^2+(4b-a^2)\neq 0 \end{array}\right] = 2\int \frac{dt}{t^2+(4b-a^2)}.$$

Pre $4b-a^2>0$ platí $(2x+a)^2+(4b-a^2)=t^2+(4b-a^2)>t^2$, pričom $x\in R$ a $t\in R$:

- $\int \frac{dx}{x^2+ax+b} = 2\int \frac{dt}{t^2+\left(\sqrt{4b-a^2}\right)^2} = \frac{2}{\sqrt{4b-a^2}} \operatorname{arctg} \frac{t}{\sqrt{4b-a^2}} + c$

Pre $4b-a^2=0$ platí $(2x+a)^2+(4b-a^2)=t^2+0=t^2$, pričom $x\in R$, $x\neq -\frac{a}{2}$ a $t\in R$, $t\neq 0$:

- $\int \frac{dx}{x^2+ax+b} = 2\int \frac{dt}{t^2} = 2\int t^{-2}\,dt = \frac{2t^{-1}}{-1} + c = -\frac{2}{t} + c$

Pre $4b-a^2<0$ platí $(2x+a)^2+(4b-a^2)=t^2+(4b-a^2)<t^2$, pričom $x\in R$, $x\neq \frac{-a\pm\sqrt{a^2-4b}}{2}$ a $t\in R$, $t\neq \pm\sqrt{a^2-4b}$:

- $\int \frac{dx}{x^2+ax+b} = 2\int \frac{dt}{t^2-\left(\sqrt{a^2-4b}\right)^2} = \frac{1}{\sqrt{a^2-4b}} \ln\left|\frac{t-\sqrt{a^2-4b}}{t+\sqrt{a^2-4b}}\right| + c$

Riešené príklady – 051

$\int \frac{dx}{x^2+ax+b}$ pre $a, b \in R$. [051]

- $= \int \frac{dx}{\left(x^2+ax+\frac{a^2}{4}\right)+b-\frac{a^2}{4}} = \int \frac{dx}{(x+\frac{a}{2})^2+b-\frac{a^2}{4}} = \int \frac{dx}{\left(\frac{2x+a}{2}\right)^2+\frac{4b-a^2}{4}} = \int \frac{dx}{\frac{(2x+a)^2}{4}+\frac{4b-a^2}{4}}$
  $= 4\int \frac{dx}{(2x+a)^2+(4b-a^2)} = \left[\begin{array}{r|l|r} \text{Subst. } t = 2x+a & x\in R & x^2+ax+b \neq 0 \Rightarrow (2x+a)^2+(4b-a^2) \neq 0 \\ dt = 2\,dx & t\in R & t^2+(4b-a^2)\neq 0 \end{array}\right] = 2\int \frac{dt}{t^2+(4b-a^2)}$.

Pre $4b-a^2>0$ platí $(2x+a)^2+(4b-a^2) = t^2+(4b-a^2) > t^2$, pričom $x\in R$ a $t\in R$:

- $\int \frac{dx}{x^2+ax+b} = 2\int \frac{dt}{t^2+\left(\sqrt{4b-a^2}\right)^2} = \frac{2}{\sqrt{4b-a^2}} \operatorname{arctg} \frac{t}{\sqrt{4b-a^2}} + c$
  $= \frac{2}{\sqrt{4b-a^2}} \operatorname{arctg} \frac{2x+a}{\sqrt{4b-a^2}} + c$, $x\in R$, $c\in R$.

Pre $4b-a^2=0$ platí $(2x+a)^2+(4b-a^2) = t^2+0 = t^2$, pričom $x\in R$, $x\neq -\frac{a}{2}$ a $t\in R$, $t\neq 0$:

- $\int \frac{dx}{x^2+ax+b} = 2\int \frac{dt}{t^2} = 2\int t^{-2}\,dt = \frac{2t^{-1}}{-1} + c = -\frac{2}{t} + c = -\frac{2}{2x+a} + c$,
  $x\in R$, $x\neq -\frac{a}{2}$, $c\in R$.

Pre $4b-a^2<0$ platí $(2x+a)^2+(4b-a^2) = t^2+(4b-a^2) < t^2$, pričom $x\in R$, $x\neq \frac{-a\pm\sqrt{a^2-4b}}{2}$ a $t\in R$, $t\neq \pm\sqrt{a^2-4b}$:

- $\int \frac{dx}{x^2+ax+b} = 2\int \frac{dt}{t^2-\left(\sqrt{a^2-4b}\right)^2} = \frac{1}{\sqrt{a^2-4b}} \ln\left|\frac{t-\sqrt{a^2-4b}}{t+\sqrt{a^2-4b}}\right| + c$
  $= \frac{1}{\sqrt{a^2-4b}} \ln\left|\frac{2x+a-\sqrt{a^2-4b}}{2x+a+\sqrt{a^2-4b}}\right| + c$, $x\in R$, $x\neq \frac{-a\pm\sqrt{a^2-4b}}{2}$, $c\in R$.

Riešené príklady – 052, 053

$\int \frac{dx}{x^2+4x+2}$ [052]

$\int \frac{dx}{x^2+4x+3}$ [053]

Riešené príklady – 052, 053

$\int \frac{dx}{x^2+4x+2}$ [052]

- $= \int \frac{dx}{x^2+4x+4-2} = \int \frac{dx}{(x+2)^2-2}$

$\int \frac{dx}{x^2+4x+3}$ [053]

- $= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1}$

- $= \int \frac{dx}{(x+1)\cdot(x+3)}$

Riešené príklady – 052, 053

$\int \frac{dx}{x^2+4x+2}$ [052]

- $= \int \frac{dx}{x^2+4x+4-2} = \int \frac{dx}{(x+2)^2-2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x \in (-\infty; -2-\sqrt{2}) & x \in (-2-\sqrt{2}; -2+\sqrt{2}) & x \in (-2+\sqrt{2}; \infty) \\ dt = dx & t \in (-\infty; -\sqrt{2}) & t \in (-\sqrt{2}; \sqrt{2}) & t \in (\sqrt{2}; \infty) \end{array} \right]$
  $= \int \frac{dt}{t^2-2} = \int \frac{dt}{t^2-(\sqrt{2})^2}$

$\int \frac{dx}{x^2+4x+3}$ [053]

- $= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x \in (-\infty; -3) & x \in (-3; -1) & x \in (-1; \infty) \\ dt = dx & t \in (-\infty; -1) & t \in (-1; 1) & t \in (1; \infty) \end{array} \right] = \int \frac{dt}{t^2-1}$

- $= \int \frac{dx}{(x+1)\cdot(x+3)} = \frac{1}{2} \int \frac{(x+3)-(x+1)}{(x+1)\cdot(x+3)}\, dx$

Riešené príklady – 052, 053

$\int \frac{dx}{x^2+4x+2}$ [052]

- $= \int \frac{dx}{x^2+4x+4-2} = \int \frac{dx}{(x+2)^2-2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x\in(-\infty; -2-\sqrt{2}) & x\in(-2-\sqrt{2}; -2+\sqrt{2}) & x\in(-2+\sqrt{2}; \infty) \\ dt = dx & t\in(-\infty; -\sqrt{2}) & t\in(-\sqrt{2}; \sqrt{2}) & t\in(\sqrt{2}; \infty) \end{array} \right]$
  $= \int \frac{dt}{t^2-2} = \int \frac{dt}{t^2-(\sqrt{2})^2} = \frac{1}{\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c$

$\int \frac{dx}{x^2+4x+3}$ [053]

- $= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x\in(-\infty; -3) & x\in(-3; -1) & x\in(-1; \infty) \\ dt = dx & t\in(-\infty; -1) & t\in(-1; 1) & t\in(1; \infty) \end{array} \right] = \int \frac{dt}{t^2-1}$
  $= \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c$

- $= \int \frac{dx}{(x+1)\cdot(x+3)} = \frac{1}{2} \int \frac{(x+3)-(x+1)}{(x+1)\cdot(x+3)}\, dx = \frac{1}{2} \int \left[\frac{1}{x+1} - \frac{1}{x+3} \right] dx$

Riešené príklady – 052, 053

$$\int \frac{dx}{x^2+4x+2} = \frac{1}{\sqrt{2}} \ln \left| \frac{x+2-\sqrt{2}}{x+2+\sqrt{2}} \right| + c$$ [052]

- $= \int \frac{dx}{x^2+4x+4-2} = \int \frac{dx}{(x+2)^2-2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x \in (-\infty; -2-\sqrt{2}) & x \in (-2-\sqrt{2}; -2+\sqrt{2}) & x \in (-2+\sqrt{2}; \infty) \\ dt = dx & t \in (-\infty; -\sqrt{2}) & t \in (-\sqrt{2}; \sqrt{2}) & t \in (\sqrt{2}; \infty) \end{array} \right]$
  $= \int \frac{dt}{t^2-2} = \int \frac{dt}{t^2-(\sqrt{2})^2} = \frac{1}{\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c = \frac{1}{\sqrt{2}} \ln \left| \frac{x+2-\sqrt{2}}{x+2+\sqrt{2}} \right| + c,$
  $x \in R,\ x \neq -2 \pm \sqrt{2},\ c \in R.$

$$\int \frac{dx}{x^2+4x+3} = \frac{1}{2} \ln \left| \frac{x+1}{x+3} \right| + c$$ [053]

- $= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x \in (-\infty; -3) & x \in (-3; -1) & x \in (-1; \infty) \\ dt = dx & t \in (-\infty; -1) & t \in (-1; 1) & t \in (1; \infty) \end{array} \right] = \int \frac{dt}{t^2-1}$
  $= \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c = \frac{1}{2} \ln \left| \frac{x+2-1}{x+2+1} \right| + c = \frac{1}{2} \ln \left| \frac{x+1}{x+3} \right| + c,\ x \in R,\ x \neq -3,\ x \neq -1,\ c \in R.$

- $= \int \frac{dx}{(x+1)\cdot(x+3)} = \frac{1}{2} \int \frac{(x+3)-(x+1)}{(x+1)\cdot(x+3)}\, dx = \frac{1}{2} \int \left[\frac{1}{x+1} - \frac{1}{x+3} \right] dx$
  $= \frac{1}{2} \Big[\ln|x+1| - \ln|x+3| \Big] + c = \frac{1}{2} \ln \left| \frac{x+1}{x+3} \right| + c,\ x \in R,\ x \neq -3,\ x \neq -1,\ c \in R.$

Riešené príklady – 054, 055, 056

$\int \frac{dx}{x^2+4x+4}$ [054]

$\int \frac{dx}{x^2+4x+5}$ [055]

$\int \frac{dx}{x^2+4x+6}$ [056]

Riešené príklady – 054, 055, 056

$\int \frac{dx}{x^2+4x+4}$ [054]

- $= \int \frac{dx}{(x+2)^2}$

$\int \frac{dx}{x^2+4x+5}$ [055]

- $= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1}$

$\int \frac{dx}{x^2+4x+6}$ [056]

- $= \int \frac{dx}{x^2+4x+4+2} = \int \frac{dx}{(x+2)^2+2}$

Riešené príklady – 054, 055, 056

$\int \frac{dx}{x^2+4x+4}$ [054]

- $= \int \frac{dx}{(x+2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in (-\infty; -2) \Rightarrow t \in (-\infty; 0) \\ dt = dx & x \in (-2; \infty) \Rightarrow t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t^2} = \int t^{-2}\, dt$

$\int \frac{dx}{x^2+4x+5}$ [055]

- $= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{dt}{t^2+1}$

$\int \frac{dx}{x^2+4x+6}$ [056]

- $= \int \frac{dx}{x^2+4x+4+2} = \int \frac{dx}{(x+2)^2+2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{dt}{t^2+(\sqrt{2})^2}$

Riešené príklady – 054, 055, 056

$$\int \frac{dx}{x^2+4x+4}$$ [054]

- $$= \int \frac{dx}{(x+2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in (-\infty; -2) \Rightarrow t \in (-\infty; 0) \\ dt = dx & x \in (-2; \infty) \Rightarrow t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t^2} = \int t^{-2}\, dt = \frac{t^{-2+1}}{-2+1} + c = -\frac{1}{t} + c$$

$$\int \frac{dx}{x^2+4x+5}$$ [055]

- $$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{dt}{t^2+1} = \operatorname{arctg} t + c$$

$$\int \frac{dx}{x^2+4x+6}$$ [056]

- $$= \int \frac{dx}{x^2+4x+4+2} = \int \frac{dx}{(x+2)^2+2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{dt}{t^2+(\sqrt{2})^2} = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + c$$

Riešené príklady – 054, 055, 056

$$\int\frac{dx}{x^2+4x+4} = -\frac{1}{x+2}+c$$ [054]

- $= \int\frac{dx}{(x+2)^2} = \left[\begin{array}{r|l} \text{Subst. } t=x+2 & x\in(-\infty;-2)\Rightarrow t\in(-\infty;0) \\ dt=dx & x\in(-2;\infty)\Rightarrow t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t^2} = \int t^{-2}\,dt = \frac{t^{-2+1}}{-2+1}+c$
  $= -\frac{1}{t}+c = -\frac{1}{x+2}+c,\ x\in R,\ x\neq -2,\ c\in R.$

$$\int\frac{dx}{x^2+4x+5} = \operatorname{arctg}(x+2)+c$$ [055]

- $= \int\frac{dx}{x^2+4x+4+1} = \int\frac{dx}{(x+2)^2+1} = \left[\begin{array}{r|l} \text{Subst. } t=x+2 & x\in R \\ dt=dx & t\in R\end{array}\right] = \int\frac{dt}{t^2+1} = \operatorname{arctg} t+c$
  $= \operatorname{arctg}(x+2)+c,\ x\in R,\ c\in R.$

$$\int\frac{dx}{x^2+4x+6} = \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x+2}{\sqrt{2}}+c$$ [056]

- $= \int\frac{dx}{x^2+4x+4+2} = \int\frac{dx}{(x+2)^2+2} = \left[\begin{array}{r|l} \text{Subst. } t=x+2 & x\in R \\ dt=dx & t\in R\end{array}\right] = \int\frac{dt}{t^2+(\sqrt{2})^2} = \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{t}{\sqrt{2}}+c$
  $= \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x+2}{\sqrt{2}}+c,\ x\in R,\ c\in R.$

Riešené príklady – 057, 058

$\int \frac{dx}{x^2+11x+4}$ [057]

$\int \frac{dx}{x^2-11x+4}$ [058]

Riešené príklady – 057, 058

$$\int \frac{dx}{x^2+11x+4}$$ [057]

- $= \int \frac{dx}{(x^2+11x+\frac{11^2}{4})+4-\frac{11^2}{4}} = \int \frac{dx}{\left(x+\frac{11}{2}\right)^2+\frac{16-121}{4}}$

$$\int \frac{dx}{x^2-11x+4}$$ [058]

- $= \int \frac{dx}{(x-11x+\frac{11^2}{4})+4-\frac{11^2}{4}} = \int \frac{dx}{\left(x-\frac{11}{2}\right)^2+\frac{16-121}{4}}$

Riešené príklady – 057, 058

$\int \frac{dx}{x^2+11x+4}$ [057]

- $= \int \frac{dx}{(x^2+11x+\frac{11^2}{4})+4-\frac{11^2}{4}} = \int \frac{dx}{\left(x+\frac{11}{2}\right)^2+\frac{16-121}{4}} = \int \frac{dx}{\frac{(2x+11)^2}{4}-\frac{105}{4}} = 4\int \frac{dx}{(2x+11)^2-105}$

$\int \frac{dx}{x^2-11x+4}$ [058]

- $= \int \frac{dx}{(x-11x+\frac{11^2}{4})+4-\frac{11^2}{4}} = \int \frac{dx}{\left(x-\frac{11}{2}\right)^2+\frac{16-121}{4}} = \int \frac{dx}{\frac{(2x-11)^2}{4}-\frac{105}{4}} = 4\int \frac{dx}{(2x-11)^2-105}$

Riešené príklady – 057, 058

$\int \frac{dx}{x^2+11x+4}$ [057]

- $= \int \frac{dx}{(x^2+11x+\frac{11^2}{4})+4-\frac{11^2}{4}} = \int \frac{dx}{\left(x+\frac{11}{2}\right)^2+\frac{16-121}{4}} = \int \frac{dx}{\frac{(2x+11)^2}{4}-\frac{105}{4}} = 4\int \frac{dx}{(2x+11)^2-105}$

$= \left[\begin{array}{r|l|l|l} \text{Subst. } t=2x+11 & 2x\in(-\infty;-11-\sqrt{105}) & 2x\in(-11-\sqrt{105};-11+\sqrt{105}) & 2x\in(-11+\sqrt{105};\infty) \\ dt=2\,dx & t\in(-\infty;-\sqrt{105}) & t\in(-\sqrt{105};\sqrt{105}) & t\in(\sqrt{105};\infty) \end{array} \right] = 2\int \frac{dt}{t^2-(\sqrt{105})^2}$

$\int \frac{dx}{x^2-11x+4}$ [058]

- $= \int \frac{dx}{(x-11x+\frac{11^2}{4})+4-\frac{11^2}{4}} = \int \frac{dx}{\left(x-\frac{11}{2}\right)^2+\frac{16-121}{4}} = \int \frac{dx}{\frac{(2x-11)^2}{4}-\frac{105}{4}} = 4\int \frac{dx}{(2x-11)^2-105}$

$= \left[\begin{array}{r|l|l|l} \text{Subst. } t=2x-11 & 2x\in(-\infty;11-\sqrt{105}) & 2x\in(11-\sqrt{105};11+\sqrt{105}) & 2x\in(11+\sqrt{105};\infty) \\ dt=2\,dx & t\in(-\infty;-\sqrt{105}) & t\in(-\sqrt{105};\sqrt{105}) & t\in(\sqrt{105};\infty) \end{array} \right] = 2\int \frac{dt}{t^2-(\sqrt{105})^2}$

Riešené príklady – 057, 058

$$\int \frac{dx}{x^2+11x+4}$$ [057]

- $$= \int \frac{dx}{(x^2+11x+\frac{11^2}{4})+4-\frac{11^2}{4}} = \int \frac{dx}{\left(x+\frac{11}{2}\right)^2+\frac{16-121}{4}} = \int \frac{dx}{\frac{(2x+11)^2}{4}-\frac{105}{4}} = 4\int \frac{dx}{(2x+11)^2-105}$$

$$= \left[\begin{array}{r|l|l|l} \text{Subst. } t=2x+11 & 2x\in(-\infty;-11-\sqrt{105}) & 2x\in(-11-\sqrt{105};-11+\sqrt{105}) & 2x\in(-11+\sqrt{105};\infty) \\ dt=2\,dx & t\in(-\infty;-\sqrt{105}) & t\in(-\sqrt{105};\sqrt{105}) & t\in(\sqrt{105};\infty) \end{array} \right] = 2\int \frac{dt}{t^2-(\sqrt{105})^2}$$

$$= \frac{2}{2\sqrt{105}} \ln\left|\frac{t-\sqrt{105}}{t+\sqrt{105}}\right| + c$$

$$\int \frac{dx}{x^2-11x+4}$$ [058]

- $$= \int \frac{dx}{(x-11x+\frac{11^2}{4})+4-\frac{11^2}{4}} = \int \frac{dx}{\left(x-\frac{11}{2}\right)^2+\frac{16-121}{4}} = \int \frac{dx}{\frac{(2x-11)^2}{4}-\frac{105}{4}} = 4\int \frac{dx}{(2x-11)^2-105}$$

$$= \left[\begin{array}{r|l|l|l} \text{Subst. } t=2x-11 & 2x\in(-\infty;11-\sqrt{105}) & 2x\in(11-\sqrt{105};11+\sqrt{105}) & 2x\in(11+\sqrt{105};\infty) \\ dt=2\,dx & t\in(-\infty;-\sqrt{105}) & t\in(-\sqrt{105};\sqrt{105}) & t\in(\sqrt{105};\infty) \end{array} \right] = 2\int \frac{dt}{t^2-(\sqrt{105})^2}$$

$$= \frac{2}{2\sqrt{105}} \ln\left|\frac{t-\sqrt{105}}{t+\sqrt{105}}\right| + c$$

Riešené príklady – 057, 058

$$\int \frac{dx}{x^2+11x+4} = \frac{1}{\sqrt{105}} \ln \left| \frac{2x+11-\sqrt{105}}{2x+11+\sqrt{105}} \right| + c$$ [057]

- $= \int \frac{dx}{(x^2+11x+\frac{11^2}{4})+4-\frac{11^2}{4}} = \int \frac{dx}{\left(x+\frac{11}{2}\right)^2+\frac{16-121}{4}} = \int \frac{dx}{\frac{(2x+11)^2}{4}-\frac{105}{4}} = 4\int \frac{dx}{(2x+11)^2-105}$

$= \left[\begin{array}{c|c|c|c} \text{Subst. } t=2x+11 & 2x\in(-\infty;-11-\sqrt{105}) & 2x\in(-11-\sqrt{105};-11+\sqrt{105}) & 2x\in(-11+\sqrt{105};\infty) \\ dt=2\,dx & t\in(-\infty;-\sqrt{105}) & t\in(-\sqrt{105};\sqrt{105}) & t\in(\sqrt{105};\infty) \end{array} \right] = 2\int \frac{dt}{t^2-(\sqrt{105})^2}$

$= \frac{2}{2\sqrt{105}} \ln \left| \frac{t-\sqrt{105}}{t+\sqrt{105}} \right| + c = \frac{1}{\sqrt{105}} \ln \left| \frac{2x+11-\sqrt{105}}{2x+11+\sqrt{105}} \right| + c,\ x\in R,\ x\neq \frac{-11\pm\sqrt{105}}{2},\ c\in R.$

$$\int \frac{dx}{x^2-11x+4} = \frac{1}{\sqrt{105}} \ln \left| \frac{2x-11-\sqrt{105}}{2x-11+\sqrt{105}} \right| + c$$ [058]

- $= \int \frac{dx}{(x-11x+\frac{11^2}{4})+4-\frac{11^2}{4}} = \int \frac{dx}{\left(x-\frac{11}{2}\right)^2+\frac{16-121}{4}} = \int \frac{dx}{\frac{(2x-11)^2}{4}-\frac{105}{4}} = 4\int \frac{dx}{(2x-11)^2-105}$

$= \left[\begin{array}{c|c|c|c} \text{Subst. } t=2x-11 & 2x\in(-\infty;11-\sqrt{105}) & 2x\in(11-\sqrt{105};11+\sqrt{105}) & 2x\in(11+\sqrt{105};\infty) \\ dt=2\,dx & t\in(-\infty;-\sqrt{105}) & t\in(-\sqrt{105};\sqrt{105}) & t\in(\sqrt{105};\infty) \end{array} \right] = 2\int \frac{dt}{t^2-(\sqrt{105})^2}$

$= \frac{2}{2\sqrt{105}} \ln \left| \frac{t-\sqrt{105}}{t+\sqrt{105}} \right| + c = \frac{1}{\sqrt{105}} \ln \left| \frac{2x-11-\sqrt{105}}{2x-11+\sqrt{105}} \right| + c,\ x\in R,\ x\neq \frac{11\pm\sqrt{105}}{2},\ c\in R.$

Riešené príklady – 059, 060, 061

$$\int \frac{dx}{x^2+9x+25}$$ [059]

$$\int \frac{dx}{x^2-9x+25}$$ [060]

$$\int \frac{2x+1}{x^2-9x+25}\, dx$$ [061]

Riešené príklady – 059, 060, 061

$\int \frac{dx}{x^2+9x+25}$ [059]

- $= \int \frac{dx}{(x^2+9x+\frac{9^2}{4})+25-\frac{9^2}{4}} = \int \frac{dx}{\left(x+\frac{9}{2}\right)^2+\frac{100-81}{4}}$

$\int \frac{dx}{x^2-9x+25}$ [060]

- $= \int \frac{dx}{(x^2-9x+\frac{9^2}{4})+25-\frac{9^2}{4}} = \int \frac{dx}{\left(x-\frac{9}{2}\right)^2+\frac{100-81}{4}}$

$\int \frac{2x+1}{x^2-9x+25}\, dx = \int \frac{2x-9+10}{x^2-9x+25}\, dx$ [061]

Riešené príklady – 059, 060, 061

$\int \frac{dx}{x^2+9x+25}$ [059]

- $= \int \frac{dx}{(x^2+9x+\frac{9^2}{4})+25-\frac{9^2}{4}} = \int \frac{dx}{\left(x+\frac{9}{2}\right)^2+\frac{100-81}{4}} = 4\int \frac{dx}{(2x+9)^2+19}$

$\int \frac{dx}{x^2-9x+25}$ [060]

- $= \int \frac{dx}{(x^2-9x+\frac{9^2}{4})+25-\frac{9^2}{4}} = \int \frac{dx}{\left(x-\frac{9}{2}\right)^2+\frac{100-81}{4}} = 4\int \frac{dx}{(2x-9)^2+19}$

$\int \frac{2x+1}{x^2-9x+25}\,dx = \int \frac{2x-9+10}{x^2-9x+25}\,dx$ [061]

- $= \int \frac{(x^2-9x+25)'}{x^2-9x+25}\,dx + \int \frac{10\,dx}{x^2-9x+25}$

Riešené príklady – 059, 060, 061

$$\int \frac{dx}{x^2+9x+25}$$ [059]

- $$= \int \frac{dx}{(x^2+9x+\frac{9^2}{4})+25-\frac{9^2}{4}} = \int \frac{dx}{\left(x+\frac{9}{2}\right)^2+\frac{100-81}{4}} = 4\int \frac{dx}{(2x+9)^2+19} = \left[\begin{array}{c|c} \text{Subst. } t=2x+9 & x\in R \\ dt=2\,dx & t\in R \end{array}\right] = 2\int \frac{dt}{t^2+(\sqrt{19})^2}$$

$$\int \frac{dx}{x^2-9x+25}$$ [060]

- $$= \int \frac{dx}{(x^2-9x+\frac{9^2}{4})+25-\frac{9^2}{4}} = \int \frac{dx}{\left(x-\frac{9}{2}\right)^2+\frac{100-81}{4}} = 4\int \frac{dx}{(2x-9)^2+19} = \left[\begin{array}{c|c} \text{Subst. } t=2x-9 & x\in R \\ dt=2\,dx & t\in R \end{array}\right] = 2\int \frac{dt}{t^2+(\sqrt{19})^2}$$

$$\int \frac{2x+1}{x^2-9x+25}\,dx = \int \frac{2x-9+10}{x^2-9x+25}\,dx$$ [061]

- $$= \int \frac{(x^2-9x+25)'}{x^2-9x+25}\,dx + \int \frac{10\,dx}{x^2-9x+25} = \ln\left(x^2-9x+25\right) +$$

Riešené príklady – 059, 060, 061

$\int \frac{dx}{x^2+9x+25}$ [059]

- $= \int \frac{dx}{(x^2+9x+\frac{9^2}{4})+25-\frac{9^2}{4}} = \int \frac{dx}{\left(x+\frac{9}{2}\right)^2+\frac{100-81}{4}} = 4\int \frac{dx}{(2x+9)^2+19} = \left[\begin{array}{r|l} \text{Subst. } t=2x+9 & x\in R \\ dt = 2\,dx & t\in R \end{array}\right]$
  $= 2\int \frac{dt}{t^2+(\sqrt{19})^2} = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{t}{\sqrt{19}} + c$

$\int \frac{dx}{x^2-9x+25}$ [060]

- $= \int \frac{dx}{(x^2-9x+\frac{9^2}{4})+25-\frac{9^2}{4}} = \int \frac{dx}{\left(x-\frac{9}{2}\right)^2+\frac{100-81}{4}} = 4\int \frac{dx}{(2x-9)^2+19} = \left[\begin{array}{r|l} \text{Subst. } t=2x-9 & x\in R \\ dt = 2\,dx & t\in R \end{array}\right]$
  $= 2\int \frac{dt}{t^2+(\sqrt{19})^2} = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{t}{\sqrt{19}} + c$

$\int \frac{2x+1}{x^2-9x+25}\,dx = \int \frac{2x-9+10}{x^2-9x+25}\,dx$ [061]

- $= \int \frac{(x^2-9x+25)'}{x^2-9x+25}\,dx + \int \frac{10\,dx}{x^2-9x+25}$ $= \ln\left(x^2-9x+25\right) +$

Riešené príklady – 059, 060, 061

$$\int \frac{dx}{x^2+9x+25} = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{2x+9}{\sqrt{19}} + c$$ [059]

- $= \int \frac{dx}{(x^2+9x+\frac{9^2}{4})+25-\frac{9^2}{4}} = \int \frac{dx}{\left(x+\frac{9}{2}\right)^2+\frac{100-81}{4}} = 4\int \frac{dx}{(2x+9)^2+19} = \left[\begin{array}{c|c} \text{Subst. } t=2x+9 & x\in R \\ dt = 2\,dx & t\in R \end{array} \right]$
  $= 2\int \frac{dt}{t^2+(\sqrt{19})^2} = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{t}{\sqrt{19}} + c = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{2x+9}{\sqrt{19}} + c,\ x\in R,\ c\in R.$

$$\int \frac{dx}{x^2-9x+25} = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{2x-9}{\sqrt{19}} + c$$ [060]

- $= \int \frac{dx}{(x^2-9x+\frac{9^2}{4})+25-\frac{9^2}{4}} = \int \frac{dx}{\left(x-\frac{9}{2}\right)^2+\frac{100-81}{4}} = 4\int \frac{dx}{(2x-9)^2+19} = \left[\begin{array}{c|c} \text{Subst. } t=2x-9 & x\in R \\ dt = 2\,dx & t\in R \end{array} \right]$
  $= 2\int \frac{dt}{t^2+(\sqrt{19})^2} = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{t}{\sqrt{19}} + c = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{2x-9}{\sqrt{19}} + c,\ x\in R,\ c\in R.$

$$\int \frac{2x+1}{x^2-9x+25}\,dx = \int \frac{2x-9+10}{x^2-9x+25}\,dx$$ [061]

- $= \int \frac{(x^2-9x+25)'}{x^2-9x+25}\,dx + \int \frac{10\,dx}{x^2-9x+25} \qquad = \ln\left(x^2-9x+25\right) +$

Riešené príklady – 059, 060, 061

$$\int \frac{dx}{x^2+9x+25} = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{2x+9}{\sqrt{19}} + c$$ [059]

- $= \int \frac{dx}{(x^2+9x+\frac{9^2}{4})+25-\frac{9^2}{4}} = \int \frac{dx}{\left(x+\frac{9}{2}\right)^2+\frac{100-81}{4}} = 4\int \frac{dx}{(2x+9)^2+19} = \left[\begin{array}{r|l} \text{Subst. } t=2x+9 & x\in R \\ dt = 2\,dx & t\in R \end{array}\right]$
$= 2\int \frac{dt}{t^2+(\sqrt{19})^2} = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{t}{\sqrt{19}} + c = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{2x+9}{\sqrt{19}} + c,\ x\in R,\ c\in R.$

$$\int \frac{dx}{x^2-9x+25} = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{2x-9}{\sqrt{19}} + c$$ [060]

- $= \int \frac{dx}{(x^2-9x+\frac{9^2}{4})+25-\frac{9^2}{4}} = \int \frac{dx}{\left(x-\frac{9}{2}\right)^2+\frac{100-81}{4}} = 4\int \frac{dx}{(2x-9)^2+19} = \left[\begin{array}{r|l} \text{Subst. } t=2x-9 & x\in R \\ dt = 2\,dx & t\in R \end{array}\right]$
$= 2\int \frac{dt}{t^2+(\sqrt{19})^2} = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{t}{\sqrt{19}} + c = \frac{2}{\sqrt{19}} \operatorname{arctg} \frac{2x-9}{\sqrt{19}} + c,\ x\in R,\ c\in R.$

$$\int \frac{2x+1}{x^2-9x+25}\,dx = \int \frac{2x-9+10}{x^2-9x+25}\,dx = \ln\left(x^2-9x+25\right) + \frac{20}{\sqrt{19}} \operatorname{arctg} \frac{2x-9}{\sqrt{19}} + c$$ [061]

- $= \int \frac{(x^2-9x+25)'}{x^2-9x+25}\,dx + \int \frac{10\,dx}{x^2-9x+25} = \left[\text{Pr. 060.}\right] = \ln\left(x^2-9x+25\right) + \frac{20}{\sqrt{19}} \operatorname{arctg} \frac{2x-9}{\sqrt{19}} + c,$
$x\in R,\ c\in R.$

Riešené príklady – 062

$$\int \frac{2x+3}{(x^2+2x+3)^2}\,\mathrm{d}x$$ 1 [062]

Riešené príklady – 062

$$\int \frac{2x+3}{(x^2+2x+3)^2}\,dx$$

2 [062]

- $= \int \frac{(2x+2+1)\,dx}{(x^2+2x+1+2)^2} = \int \frac{(2x+2)\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2}$

Riešené príklady – 062

$\int \frac{2x+3}{(x^2+2x+3)^2}\,dx$ 3 [062]

- $= \int \frac{(2x+2+1)\,dx}{(x^2+2x+1+2)^2} = \int \frac{(2x+2)\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2} = \int \frac{(x^2+2x+3)'\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2}$

Riešené príklady – 062

$$\int \frac{2x+3}{(x^2+2x+3)^2}\,dx$$ 4 [062]

- $$= \int \frac{(2x+2+1)\,dx}{(x^2+2x+1+2)^2} = \int \frac{(2x+2)\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2} = \int \frac{(x^2+2x+3)'\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2}$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = x^2+2x+3 & x\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = (2x+2)\,dx & x\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array} \right] + \left[\begin{array}{r|l|l} \text{Subst. } z = x+1 & x\in R & z^2+2 \\ dz = dx & z\in R & = x^2+2x+3 \end{array} \right] = \int \frac{dt}{t^2} + \int \frac{dz}{(z^2+2)^2}$$

Riešené príklady – 062

$\int \frac{2x+3}{(x^2+2x+3)^2}\,dx$ 5 [062]

- $= \int \frac{(2x+2+1)\,dx}{(x^2+2x+1+2)^2} = \int \frac{(2x+2)\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2} = \int \frac{(x^2+2x+3)'\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2}$
  $= \left[\begin{array}{r|l} \text{Subst. } t = x^2+2x+3 & x \in (-\infty; -1\rangle \Rightarrow t \in \langle 2; \infty) \\ dt = (2x+2)\,dx & x \in \langle -1; \infty) \Rightarrow t \in \langle 2; \infty) \end{array} \right] + \left[\begin{array}{r|l|l} \text{Subst. } z = x+1 & x \in R & z^2+2 \\ dz = dx & z \in R & = x^2+2x+3 \end{array} \right] = \int \frac{dt}{t^2} + \int \frac{dz}{(z^2+2)^2}$

- $\int \frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} + c_1 = -\frac{1}{t} + c_1,\ c_1 \in R.$

Riešené príklady – 062

$\int \frac{2x+3}{(x^2+2x+3)^2}\,dx$ 6 [062]

- $= \int \frac{(2x+2+1)\,dx}{(x^2+2x+1+2)^2} = \int \frac{(2x+2)\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2} = \int \frac{(x^2+2x+3)'\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2}$

  $= \left[\begin{array}{r|l} \text{Subst. } t = x^2+2x+3 & x\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = (2x+2)\,dx & x\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array} \right] + \left[\begin{array}{r|l|l} \text{Subst. } z = x+1 & x\in R & z^2+2 \\ dz = dx & z\in R & = x^2+2x+3 \end{array} \right] = \int \frac{dt}{t^2} + \int \frac{dz}{(z^2+2)^2}$

- $\int \frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} + c_1 = -\frac{1}{t} + c_1,\ c_1\in R.$

- $\int \frac{dz}{(z^2+2)^2} = \frac{1}{2}\int \frac{(z^2+2-z^2)\,dz}{(z^2+2)^2} = \frac{1}{2}\int \frac{dz}{z^2+2} - \frac{1}{4}\int \frac{z\cdot 2z\,dz}{(z^2+2)^2}$

Riešené príklady – 062

$$\int \frac{2x+3}{(x^2+2x+3)^2}\,dx$$ 7 [062]

- $$= \int \frac{(2x+2+1)\,dx}{(x^2+2x+1+2)^2} = \int \frac{(2x+2)\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2} = \int \frac{(x^2+2x+3)'\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2}$$
  $$= \left[\begin{array}{r|l} \text{Subst. } t = x^2+2x+3 & x\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = (2x+2)\,dx & x\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array} \right] + \left[\begin{array}{r|l|l} \text{Subst. } z = x+1 & x\in R & z^2+2 \\ dz = dx & z\in R & \ = x^2+2x+3 \end{array} \right] = \int \frac{dt}{t^2} + \int \frac{dz}{(z^2+2)^2}$$

- $\int \frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} + c_1 = -\frac{1}{t} + c_1,\ c_1\in R.$

- $$\int \frac{dz}{(z^2+2)^2} = \frac{1}{2}\int \frac{(z^2+2-z^2)\,dz}{(z^2+2)^2} = \frac{1}{2}\int \frac{dz}{z^2+2} - \frac{1}{4}\int \frac{z\cdot 2z\,dz}{(z^2+2)^2} = \left[\begin{array}{l|l} v' = \frac{2z}{(z^2+2)^2} & v = \int \frac{2z\,dx}{(z^2+2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = z^2+2 & z\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = 2z\,dz & z\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array} \right] = \int \frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{z^2+2} \\ u = z & u' = 1 \end{array} \right]$$
  $$= \frac{1}{2}\int \frac{dz}{z^2+2} - \frac{1}{4}\left[-\frac{z}{z^2+2} + \int \frac{dz}{z^2+2} \right] = \frac{1}{4}\cdot\frac{z}{z^2+2} + \frac{1}{4}\int \frac{dz}{z^2+(\sqrt{2})^2}$$

Riešené príklady – 062

$$\int \frac{2x+3}{(x^2+2x+3)^2}\,dx$$ 8 [062]

- $$= \int \frac{(2x+2+1)\,dx}{(x^2+2x+1+2)^2} = \int \frac{(2x+2)\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2} = \int \frac{(x^2+2x+3)'\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2}$$
  $$= \left[\begin{array}{r|l} \text{Subst. } t = x^2+2x+3 & x\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = (2x+2)\,dx & x\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array} \right] + \left[\begin{array}{r|l|l} \text{Subst. } z = x+1 & x\in R & z^2+2 \\ dz = dx & z\in R & \; = x^2+2x+3 \end{array} \right] = \int \frac{dt}{t^2} + \int \frac{dz}{(z^2+2)^2}$$

- $$\int \frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} + c_1 = -\frac{1}{t} + c_1,\ c_1\in R.$$

- $$\int \frac{dz}{(z^2+2)^2} = \frac{1}{2}\int \frac{(z^2+2-z^2)\,dz}{(z^2+2)^2} = \frac{1}{2}\int \frac{dz}{z^2+2} - \frac{1}{4}\int \frac{z\cdot 2z\,dz}{(z^2+2)^2} = \left[\begin{array}{l|l} v' = \frac{2z}{(z^2+2)^2} & v = \int \frac{2z\,dx}{(z^2+2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = z^2+2 & z\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = 2z\,dz & z\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array} \right] = \int \frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{z^2+2} \\ u = z & u' = 1 \end{array} \right]$$
  $$= \frac{1}{2}\int \frac{dz}{z^2+2} - \frac{1}{4}\left[-\frac{z}{z^2+2} + \int \frac{dz}{z^2+2} \right] = \frac{1}{4}\cdot\frac{z}{z^2+2} + \frac{1}{4}\int \frac{dz}{z^2+(\sqrt{2})^2} = \frac{z}{4(z^2+2)} + \frac{1}{4\sqrt{2}}\,\mathrm{arctg}\,\frac{z}{\sqrt{2}} + c_2,\ c_2\in R.$$

Riešené príklady – 062

$$\int \frac{2x+3}{(x^2+2x+3)^2}\,dx$$

9 [062]

- $$= \int \frac{(2x+2+1)\,dx}{(x^2+2x+1+2)^2} = \int \frac{(2x+2)\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2} = \int \frac{(x^2+2x+3)'\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2}$$
$$= \left[\begin{array}{r|l} \text{Subst. } t = x^2+2x+3 & x\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = (2x+2)\,dx & x\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array}\right] + \left[\begin{array}{r|l|l} \text{Subst. } z = x+1 & x\in R & z^2+2 \\ dz = dx & z\in R & = x^2+2x+3 \end{array}\right] = \int \frac{dt}{t^2} + \int \frac{dz}{(z^2+2)^2} = \circledast$$

- $$\int \frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} + c_1 = -\frac{1}{t} + c_1,\ c_1\in R.$$

- $$\int \frac{dz}{(z^2+2)^2} = \frac{1}{2}\int \frac{(z^2+2-z^2)\,dz}{(z^2+2)^2} = \frac{1}{2}\int \frac{dz}{z^2+2} - \frac{1}{4}\int \frac{z\cdot 2z\,dz}{(z^2+2)^2} = \left[\begin{array}{l|l} v' = \frac{2z}{(z^2+2)^2} & v = \int \frac{2z\,dx}{(z^2+2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = z^2+2 & z\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = 2z\,dz & z\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array}\right] = \int \frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{z^2+2} \\ u = z & u' = 1 \end{array}\right]$$
$$= \frac{1}{2}\int \frac{dz}{z^2+2} - \frac{1}{4}\left[-\frac{z}{z^2+2} + \int \frac{dz}{z^2+2}\right] = \frac{1}{4}\cdot\frac{z}{z^2+2} + \frac{1}{4}\int \frac{dz}{z^2+(\sqrt{2})^2} = \frac{z}{4(z^2+2)} + \frac{1}{4\sqrt{2}}\operatorname{arctg}\frac{z}{\sqrt{2}} + c_2,\ c_2\in R.$$

$$\circledast = -\frac{1}{t} + \frac{z}{4(z^2+2)} + \frac{1}{4\sqrt{2}}\operatorname{arctg}\frac{z}{\sqrt{2}} + c$$

Riešené príklady – 062

$\displaystyle\int \frac{2x+3}{(x^2+2x+3)^2}\,dx$ 10 [062]

- $\displaystyle = \int \frac{(2x+2+1)\,dx}{(x^2+2x+1+2)^2} = \int \frac{(2x+2)\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2} = \int \frac{(x^2+2x+3)'\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2}$

  $\displaystyle = \left[\begin{array}{r|l} \text{Subst. } t = x^2+2x+3 & x\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = (2x+2)\,dx & x\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array}\right] + \left[\begin{array}{r|l|l} \text{Subst. } z = x+1 & x\in R & z^2+2 \\ dz = dx & z\in R & = x^2+2x+3 \end{array}\right] = \int \frac{dt}{t^2} + \int \frac{dz}{(z^2+2)^2} = \circledast$

- $\displaystyle \int \frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} + c_1 = -\frac{1}{t} + c_1,\ c_1\in R.$

- $\displaystyle \int \frac{dz}{(z^2+2)^2} = \frac{1}{2}\int \frac{(z^2+2-z^2)\,dz}{(z^2+2)^2} = \frac{1}{2}\int \frac{dz}{z^2+2} - \frac{1}{4}\int \frac{z\cdot 2z\,dz}{(z^2+2)^2} = \left[\begin{array}{l|l} v' = \frac{2z}{(z^2+2)^2} & v = \int \frac{2z\,dx}{(z^2+2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = z^2+2 & z\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = 2z\,dz & z\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array}\right] = \int \frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{z^2+2} \\ u = z & u' = 1 \end{array}\right]$

  $\displaystyle = \frac{1}{2}\int \frac{dz}{z^2+2} - \frac{1}{4}\left[-\frac{z}{z^2+2} + \int \frac{dz}{z^2+2}\right] = \frac{1}{4}\cdot\frac{z}{z^2+2} + \frac{1}{4}\int \frac{dz}{z^2+(\sqrt{2})^2} = \frac{z}{4(z^2+2)} + \frac{1}{4\sqrt{2}}\operatorname{arctg}\frac{z}{\sqrt{2}} + c_2,\ c_2\in R.$

$\displaystyle \circledast = -\frac{1}{t} + \frac{z}{4(z^2+2)} + \frac{1}{4\sqrt{2}}\operatorname{arctg}\frac{z}{\sqrt{2}} + c$

$\displaystyle = -\frac{1}{x^2+2x+3} + \frac{x+1}{4(x^2+2x+3)} + \frac{1}{4\sqrt{2}}\operatorname{arctg}\frac{x+1}{\sqrt{2}} + c$

$\displaystyle = \frac{-4}{4(x^2+2x+3)} + \frac{x+1}{4(x^2+2x+3)} + \frac{1}{4\sqrt{2}}\operatorname{arctg}\frac{x+1}{\sqrt{2}} + c$

Riešené príklady – 062

$$\int \frac{2x+3}{(x^2+2x+3)^2}\,dx = \frac{x-3}{4(x^2+2x+3)} + \frac{1}{4\sqrt{2}} \operatorname{arctg} \frac{x+1}{\sqrt{2}} + c$$ 11 [062]

- $$= \int \frac{(2x+2+1)\,dx}{(x^2+2x+1+2)^2} = \int \frac{(2x+2)\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2} = \int \frac{(x^2+2x+3)'\,dx}{(x^2+2x+3)^2} + \int \frac{dx}{[(x+1)^2+2]^2}$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = x^2+2x+3 & x\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = (2x+2)\,dx & x\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array}\right] + \left[\begin{array}{r|l|l} \text{Subst. } z = x+1 & x\in R & z^2+2 \\ dz = dx & z\in R & = x^2+2x+3 \end{array}\right] = \int \frac{dt}{t^2} + \int \frac{dz}{(z^2+2)^2} = \circledast$$

- $$\int \frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} + c_1 = -\frac{1}{t} + c_1,\ c_1\in R.$$

- $$\int \frac{dz}{(z^2+2)^2} = \frac{1}{2}\int \frac{(z^2+2-z^2)\,dz}{(z^2+2)^2} = \frac{1}{2}\int \frac{dz}{z^2+2} - \frac{1}{4}\int \frac{z\cdot 2z\,dz}{(z^2+2)^2} = \left[\begin{array}{l|l} v' = \frac{2z}{(z^2+2)^2} & v = \int \frac{2z\,dx}{(z^2+2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = z^2+2 & z\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = 2z\,dz & z\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array}\right] = \int\frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{z^2+2} \\ u = z & u' = 1 \end{array}\right]$$

$$= \frac{1}{2}\int \frac{dz}{z^2+2} - \frac{1}{4}\left[-\frac{z}{z^2+2} + \int \frac{dz}{z^2+2}\right] = \frac{1}{4}\cdot\frac{z}{z^2+2} + \frac{1}{4}\int \frac{dz}{z^2+(\sqrt{2})^2} = \frac{z}{4(z^2+2)} + \frac{1}{4\sqrt{2}}\operatorname{arctg}\frac{z}{\sqrt{2}} + c_2,\ c_2\in R.$$

$$\circledast = -\frac{1}{t} + \frac{z}{4(z^2+2)} + \frac{1}{4\sqrt{2}}\operatorname{arctg}\frac{z}{\sqrt{2}} + c$$

$$= -\frac{1}{x^2+2x+3} + \frac{x+1}{4(x^2+2x+3)} + \frac{1}{4\sqrt{2}}\operatorname{arctg}\frac{x+1}{\sqrt{2}} + c$$

$$= \frac{-4}{4(x^2+2x+3)} + \frac{x+1}{4(x^2+2x+3)} + \frac{1}{4\sqrt{2}}\operatorname{arctg}\frac{x+1}{\sqrt{2}} + c$$

$$= \frac{x-3}{4(x^2+2x+3)} + \frac{1}{4\sqrt{2}}\operatorname{arctg}\frac{x+1}{\sqrt{2}} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 063, 064

$$\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx$$ [063]

$$\int \frac{-2x^2+1}{x^3+2x^2+x}\,dx$$ [064]

Riešené príklady – 063, 064

$\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx$ [063]

- $= \int \frac{-2x^3+1}{x^2(x^2+2x+1)}\,dx = \int \frac{-2x^3+1}{x^2(x+1)^2}\,dx$

$\int \frac{-2x^2+1}{x^3+2x^2+x}\,dx$ [064]

- $= \int \frac{-2x^2+1}{x(x^2+2x+1)}\,dx = \int \frac{-2x^2+1}{x(x+1)^2}\,dx$

Riešené príklady – 063, 064

$$\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx$$ [063]

- $= \int \frac{-2x^3+1}{x^2(x^2+2x+1)}\,dx = \int \frac{-2x^3+1}{x^2(x+1)^2}\,dx =$ [Rozklad na parciálne zlomky] $= \int \left[-\frac{2}{x} + \frac{1}{x^2} + \frac{3}{(x+1)^2} \right] dx$

$$\int \frac{-2x^2+1}{x^3+2x^2+x}\,dx$$ [064]

- $= \int \frac{-2x^2+1}{x(x^2+2x+1)}\,dx = \int \frac{-2x^2+1}{x(x+1)^2}\,dx =$ [Rozklad na parciálne zlomky] $= \int \left[\frac{1}{x} + \frac{1}{(x+1)^2} - \frac{3}{x+1} \right] dx$

Riešené príklady – 063, 064

$\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx$ [063]

- $= \int \frac{-2x^3+1}{x^2(x^2+2x+1)}\,dx = \int \frac{-2x^3+1}{x^2(x+1)^2}\,dx = \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[-\frac{2}{x} + \frac{1}{x^2} + \frac{3}{(x+1)^2}\Big]dx$

  $= \left[\begin{array}{r|l} \text{Subst. } t = x+1 & x \in \langle -1;\infty) \Rightarrow t \in \langle 0;\infty) \\ dt = dx & x \in (-\infty;-1\rangle \Rightarrow t \in (-\infty;0\rangle \end{array}\right] = -2\ln|x| + \frac{x^{-1}}{-1} + \int \frac{3\,dt}{t^2} = -2\ln|x| - \frac{1}{x} + 3\int t^{-2}\,dt$

$\int \frac{-2x^2+1}{x^3+2x^2+x}\,dx$ [064]

- $= \int \frac{-2x^2+1}{x(x^2+2x+1)}\,dx = \int \frac{-2x^2+1}{x(x+1)^2}\,dx = \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[\frac{1}{x} + \frac{1}{(x+1)^2} - \frac{3}{x+1}\Big]dx$

  $= \left[\begin{array}{r|l} \text{Subst. } t = x+1 & x \in \langle -1;\infty) \Rightarrow t \in \langle 0;\infty) \\ dt = dx & x \in (-\infty;-1\rangle \Rightarrow t \in (-\infty;0\rangle \end{array}\right] = \ln|x| + \int \Big[\frac{1}{t^2} - \frac{3}{t}\Big]dt = \ln|x| + \int \Big[t^{-2} - \frac{3}{t}\Big]dt$

Riešené príklady – 063, 064

$$\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx$$ [063]

- $= \int \frac{-2x^3+1}{x^2(x^2+2x+1)}\,dx = \int \frac{-2x^3+1}{x^2(x+1)^2}\,dx = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[-\frac{2}{x} + \frac{1}{x^2} + \frac{3}{(x+1)^2}\right] dx$
  $= \left[\begin{array}{r|l} \text{Subst. } t = x+1 & x \in (-1;\infty) \Rightarrow t \in (0;\infty) \\ dt = dx & x \in (-\infty;-1) \Rightarrow t \in (-\infty;0) \end{array}\right] = -2\ln|x| + \frac{x^{-1}}{-1} + \int \frac{3\,dt}{t^2} = -2\ln|x| - \frac{1}{x} + 3\int t^{-2}\,dt$
  $= -2\ln|x| - \frac{1}{x} + \frac{3t^{-1}}{-1} + c = -2\ln|x| - \frac{1}{x} - \frac{3}{t} + c$

$$\int \frac{-2x^2+1}{x^3+2x^2+x}\,dx$$ [064]

- $= \int \frac{-2x^2+1}{x(x^2+2x+1)}\,dx = \int \frac{-2x^2+1}{x(x+1)^2}\,dx = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{x} + \frac{1}{(x+1)^2} - \frac{3}{x+1}\right] dx$
  $= \left[\begin{array}{r|l} \text{Subst. } t = x+1 & x \in (-1;\infty) \Rightarrow t \in (0;\infty) \\ dt = dx & x \in (-\infty;-1) \Rightarrow t \in (-\infty;0) \end{array}\right] = \ln|x| + \int \left[\frac{1}{t^2} - \frac{3}{t}\right] dt = \ln|x| + \int \left[t^{-2} - \frac{3}{t}\right] dt$
  $= \ln|x| + \frac{t^{-1}}{-1} - 3\ln|t| + c = \ln|x| - 3\ln|t| - \frac{1}{t} + c$

Riešené príklady – 063, 064

$$\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx = -2\ln|x| - \frac{1}{x} - \frac{3}{x+1} + c$$ [063]

- $= \int \frac{-2x^3+1}{x^2(x^2+2x+1)}\,dx = \int \frac{-2x^3+1}{x^2(x+1)^2}\,dx = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[-\frac{2}{x} + \frac{1}{x^2} + \frac{3}{(x+1)^2}\right] dx$
  $= \left[\begin{array}{r|l} \text{Subst. } t = x+1 & x \in (-1;\infty) \Rightarrow t \in (0;\infty) \\ dt = dx & x \in (-\infty;-1) \Rightarrow t \in (-\infty;0) \end{array}\right] = -2\ln|x| + \frac{x^{-1}}{-1} + \int \frac{3\,dt}{t^2} = -2\ln|x| - \frac{1}{x} + 3\int t^{-2}\,dt$
  $= -2\ln|x| - \frac{1}{x} + \frac{3t^{-1}}{-1} + c = -2\ln|x| - \frac{1}{x} - \frac{3}{t} + c = -2\ln|x| - \frac{1}{x} - \frac{3}{x+1} + c,$
  $x \in R,\ x \neq 0,\ x \neq -1,\ c \in R.$

$$\int \frac{-2x^2+1}{x^3+2x^2+x}\,dx = \ln|x| - 3\ln|x+1| - \frac{1}{x+1} + c$$ [064]

- $= \int \frac{-2x^2+1}{x(x^2+2x+1)}\,dx = \int \frac{-2x^2+1}{x(x+1)^2}\,dx = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{x} + \frac{1}{(x+1)^2} - \frac{3}{x+1}\right] dx$
  $= \left[\begin{array}{r|l} \text{Subst. } t = x+1 & x \in (-1;\infty) \Rightarrow t \in (0;\infty) \\ dt = dx & x \in (-\infty;-1) \Rightarrow t \in (-\infty;0) \end{array}\right] = \ln|x| + \int \left[\frac{1}{t^2} - \frac{3}{t}\right] dt = \ln|x| + \int \left[t^{-2} - \frac{3}{t}\right] dt$
  $= \ln|x| + \frac{t^{-1}}{-1} - 3\ln|t| + c = \ln|x| - 3\ln|t| - \frac{1}{t} + c$
  $= \ln|x| - 3\ln|x+1| - \frac{1}{x+1} + c,\ x \in R,\ x \neq 0,\ x \neq -1,\ c \in R.$

Riešené príklady – 065, 066

$\int \frac{x}{x^2+a^2}\,dx$ pre $a>0$. [065]

$\int \frac{x\,dx}{(x^2+a^2)^n}$ pre $a>0,\ n\in N,\ n\neq 1$. [066]

Riešené príklady – 065, 066

$\int \frac{x}{x^2+a^2}\,dx$ pre $a > 0$. [065]

- $= \frac{1}{2}\int \frac{2x}{x^2+a^2}\,dx = \frac{1}{2}\int \frac{(x^2+a^2)'}{x^2+a^2}\,dx$

- $= \frac{1}{2}\int \frac{2x}{x^2+a^2}\,dx$

$\int \frac{x\,dx}{(x^2+a^2)^n}$ pre $a > 0,\ n \in N,\ n \neq 1$. [066]

- $= \frac{1}{2}\int \frac{2x}{(x^2+a^2)^n}\,dx$

Riešené príklady – 065, 066

$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln\left(x^2+a^2\right)+c$ pre $a>0$. [065]

- $= \frac{1}{2}\int \frac{2x}{x^2+a^2}\,dx = \frac{1}{2}\int \frac{(x^2+a^2)'}{x^2+a^2}\,dx = \frac{1}{2}\ln\left|x^2+a^2\right|+c = \frac{1}{2}\ln\left(x^2+a^2\right)+c,$ $x\in R,\ c\in R.$

- $= \frac{1}{2}\int \frac{2x}{x^2+a^2}\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x^2+a^2 & x\in(0;\infty)\Rightarrow t\in\langle a^2;\infty) \\ dt=2x\,dx & x\in(-\infty;0\rangle\Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t}$

$\int \frac{x\,dx}{(x^2+a^2)^n}$ pre $a>0,\ n\in N,\ n\neq 1.$ [066]

- $= \frac{1}{2}\int \frac{2x}{(x^2+a^2)^n}\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x^2+a^2 & x\in(0;\infty)\Rightarrow t\in\langle a^2;\infty) \\ dt=2x\,dx & x\in(-\infty;0\rangle\Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^n} = \frac{1}{2}\int t^{-n}\,dt$

Riešené príklady – 065, 066

$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln(x^2+a^2)+c$ pre $a>0$. [065]

- $= \frac{1}{2}\int \frac{2x}{x^2+a^2}\,dx = \frac{1}{2}\int \frac{(x^2+a^2)'}{x^2+a^2}\,dx = \frac{1}{2}\ln|x^2+a^2|+c = \frac{1}{2}\ln(x^2+a^2)+c,$ $x\in R,\ c\in R.$

- $= \frac{1}{2}\int \frac{2x}{x^2+a^2}\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x^2+a^2 & x\in\langle 0;\infty) \Rightarrow t\in\langle a^2;\infty) \\ dt = 2x\,dx & x\in(-\infty;0\rangle \Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t}$

$$= \frac{1}{2}\ln|t|+c = \frac{1}{2}\ln t + c$$

$\int \frac{x\,dx}{(x^2+a^2)^n}$ pre $a>0,\ n\in N,\ n\neq 1$. [066]

- $= \frac{1}{2}\int \frac{2x}{(x^2+a^2)^n}\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x^2+a^2 & x\in\langle 0;\infty) \Rightarrow t\in\langle a^2;\infty) \\ dt = 2x\,dx & x\in(-\infty;0\rangle \Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^n} = \frac{1}{2}\int t^{-n}\,dt$

$$= \frac{t^{1-n}}{2(1-n)} + c = -\frac{1}{2(n-1)t^{n-1}} + c$$

Riešené príklady – 065, 066

$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln\left(x^2+a^2\right)+c$ pre $a>0$. [065]

- $= \frac{1}{2}\int \frac{2x}{x^2+a^2}\,dx = \frac{1}{2}\int \frac{(x^2+a^2)'}{x^2+a^2}\,dx = \frac{1}{2}\ln\left|x^2+a^2\right|+c = \frac{1}{2}\ln\left(x^2+a^2\right)+c,$ $x\in R,\ c\in R.$

- $= \frac{1}{2}\int \frac{2x}{x^2+a^2}\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x^2+a^2 & x\in\langle 0;\infty) \Rightarrow t\in\langle a^2;\infty) \\ dt=2x\,dx & x\in(-\infty;0\rangle \Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t}$ $= \frac{1}{2}\ln|t|+c = \frac{1}{2}\ln t+c = \frac{1}{2}\ln\left(x^2+a^2\right)+c,\ x\in R,\ c\in R.$

$\int \frac{x\,dx}{(x^2+a^2)^n} = -\frac{1}{2(n-1)(x^2+a^2)^{n-1}}+c$ pre $a>0,\ n\in N,\ n\neq 1$. [066]

- $= \frac{1}{2}\int \frac{2x}{(x^2+a^2)^n}\,dx = \left[\begin{array}{r|l} \text{Subst. } t=x^2+a^2 & x\in\langle 0;\infty) \Rightarrow t\in\langle a^2;\infty) \\ dt=2x\,dx & x\in(-\infty;0\rangle \Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^n} = \frac{1}{2}\int t^{-n}\,dt$ $= \frac{t^{1-n}}{2(1-n)}+c = -\frac{1}{2(n-1)t^{n-1}}+c = -\frac{1}{2(n-1)(x^2+a^2)^{n-1}}+c,\ x\in R,\ c\in R.$

Riešené príklady – 067, 068, 069

$$\int \frac{x}{x^2+3}\,\mathrm{d}x$$ [067]

$$\int \frac{x\,\mathrm{d}x}{(x^2+3)^6}$$ [068]

$$\int \frac{2x\,\mathrm{d}x}{(x^2+33)^{2024}}$$ [069]

Riešené príklady – 067, 068, 069

$\int \frac{x}{x^2+3}\,dx$ [067]

- $= \frac{1}{2}\int \frac{2x}{x^2+3}\,dx = \frac{1}{2}\int \frac{(x^2+3)'}{x^2+3}\,dx$

$\int \frac{x\,dx}{(x^2+3)^6}$ [068]

- $= \frac{1}{2}\int \frac{2x}{(x^2+3)^6}\,dx$

$\int \frac{2x\,dx}{(x^2+33)^{2024}}$ [069]

- $= \left[\begin{array}{r|l} \text{Subst. } t = x^2+33 & x\in\langle 0;\infty) \Rightarrow t\in\langle 33;\infty) \\ dt = 2x\,dx & x\in(-\infty;0\rangle \Rightarrow t\in\langle 33;\infty) \end{array} \right] = \int \frac{dt}{t^{2024}} = \int t^{-2024}\,dt$

Riešené príklady – 067, 068, 069

$\int \frac{x}{x^2+3}\,\mathrm{d}x$ [067]

- $= \frac{1}{2}\int \frac{2x}{x^2+3}\,\mathrm{d}x = \frac{1}{2}\int \frac{(x^2+3)'}{x^2+3}\,\mathrm{d}x = \frac{1}{2}\ln\left|x^2+3\right| + c$

$\int \frac{x\,\mathrm{d}x}{(x^2+3)^6}$ [068]

- $= \frac{1}{2}\int \frac{2x}{(x^2+3)^6}\,\mathrm{d}x = \left[\begin{array}{r|l} \text{Subst. } t = x^2+3 & x\in\langle 0;\infty) \Rightarrow t\in\langle 3;\infty) \\ \mathrm{d}t = 2x\,\mathrm{d}x & x\in(-\infty;0\rangle \Rightarrow t\in\langle 3;\infty) \end{array}\right] = \frac{1}{2}\int \frac{\mathrm{d}t}{t^6} = \frac{1}{2}\int t^{-6}\,\mathrm{d}t$

$\int \frac{2x\,\mathrm{d}x}{(x^2+33)^{2024}}$ [069]

- $= \left[\begin{array}{r|l} \text{Subst. } t = x^2+33 & x\in\langle 0;\infty) \Rightarrow t\in\langle 33;\infty) \\ \mathrm{d}t = 2x\,\mathrm{d}x & x\in(-\infty;0\rangle \Rightarrow t\in\langle 33;\infty) \end{array}\right] = \int \frac{\mathrm{d}t}{t^{2024}} = \int t^{-2024}\,\mathrm{d}t$

$$= \frac{t^{-2023}}{-2023} + c = -\frac{1}{2023t^{2023}} + c$$

Riešené príklady – 067, 068, 069

$\int \frac{x}{x^2+3}\,dx = \frac{1}{2}\ln(x^2+3) + c$ [067]

- $= \frac{1}{2}\int \frac{2x}{x^2+3}\,dx = \frac{1}{2}\int \frac{(x^2+3)'}{x^2+3}\,dx = \frac{1}{2}\ln|x^2+3| + c = \frac{1}{2}\ln(x^2+3) + c,\ x\in R,\ c\in R.$

$\int \frac{x\,dx}{(x^2+3)^6}$ [068]

- $= \frac{1}{2}\int \frac{2x}{(x^2+3)^6}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = x^2+3 & x\in\langle 0;\infty) \Rightarrow t\in\langle 3;\infty) \\ dt = 2x\,dx & x\in(-\infty;0\rangle \Rightarrow t\in\langle 3;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^6} = \frac{1}{2}\int t^{-6}\,dt$

$$= \frac{t^{-5}}{2\cdot(-5)} + c = -\frac{1}{10t^5} + c$$

$\int \frac{2x\,dx}{(x^2+33)^{2024}} = -\frac{1}{2023(x^2+3)^{2023}} + c$ [069]

- $= \left[\begin{array}{r|l} \text{Subst. } t = x^2+33 & x\in\langle 0;\infty) \Rightarrow t\in\langle 33;\infty) \\ dt = 2x\,dx & x\in(-\infty;0\rangle \Rightarrow t\in\langle 33;\infty) \end{array}\right] = \int \frac{dt}{t^{2024}} = \int t^{-2024}\,dt$

$$= \frac{t^{-2023}}{-2023} + c = -\frac{1}{2023t^{2023}} + c = -\frac{1}{2023(x^2+3)^{2023}} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 067, 068, 069

$$\int \frac{x}{x^2+3}\,dx = \frac{1}{2}\ln\left(x^2+3\right) + c$$ [067]

- $= \frac{1}{2}\int \frac{2x}{x^2+3}\,dx = \frac{1}{2}\int \frac{(x^2+3)'}{x^2+3}\,dx = \frac{1}{2}\ln\left|x^2+3\right| + c = \frac{1}{2}\ln\left(x^2+3\right) + c,\ x\in R,\ c\in R.$

$$\int \frac{x\,dx}{(x^2+3)^6} = -\frac{1}{10(x^2+3)^5} + c$$ [068]

- $= \frac{1}{2}\int \frac{2x}{(x^2+3)^6}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = x^2+3 & x\in\langle 0;\infty) \Rightarrow t\in\langle 3;\infty) \\ dt = 2x\,dx & x\in(-\infty;0\rangle \Rightarrow t\in\langle 3;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^6} = \frac{1}{2}\int t^{-6}\,dt$

  $= \frac{t^{-5}}{2\cdot(-5)} + c = -\frac{1}{10t^5} + c = -\frac{1}{10(x^2+3)^5} + c,\ x\in R,\ c\in R.$

$$\int \frac{2x\,dx}{(x^2+33)^{2024}} = -\frac{1}{2023(x^2+3)^{2023}} + c$$ [069]

- $= \left[\begin{array}{r|l} \text{Subst. } t = x^2+33 & x\in\langle 0;\infty) \Rightarrow t\in\langle 33;\infty) \\ dt = 2x\,dx & x\in(-\infty;0\rangle \Rightarrow t\in\langle 33;\infty) \end{array}\right] = \int \frac{dt}{t^{2024}} = \int t^{-2024}\,dt$

  $= \frac{t^{-2023}}{-2023} + c = -\frac{1}{2023t^{2023}} + c = -\frac{1}{2023(x^2+3)^{2023}} + c,\ x\in R,\ c\in R.$

Riešené príklady – 070, 071

$\int \frac{x}{x^2-a^2}\,dx$ pre $a>0.$ [070]

$\int \frac{x\,dx}{(x^2-a^2)^n}$ pre $a>0,\ n\in N,\ n\neq 1.$ [071]

Riešené príklady – 070, 071

$\int \frac{x}{x^2-a^2}\,dx$ pre $a > 0.$ [070]

- $= \frac{1}{2}\int \frac{2x}{x^2-a^2}\,dx = \frac{1}{2}\int \frac{(x^2-a^2)'}{x^2-a^2}\,dx$

- $= \frac{1}{2}\int \frac{2x}{x^2-a^2}\,dx$

$\int \frac{x\,dx}{(x^2-a^2)^n}$ pre $a > 0,\ n \in N,\ n \neq 1.$ [071]

- $= \frac{1}{2}\int \frac{2x}{(x^2-a^2)^n}\,dx$

Riešené príklady – 070, 071

$\int \frac{x}{x^2-a^2}\,dx = \frac{1}{2}\ln\left|x^2-a^2\right| + c$ pre $a>0$. [070]

- $= \frac{1}{2}\int \frac{2x}{x^2-a^2}\,dx = \frac{1}{2}\int \frac{(x^2-a^2)'}{x^2-a^2}\,dx = \frac{1}{2}\ln\left|x^2-a^2\right| + c,\ x\in R,\ x\neq \pm a,\ c\in R.$

- $= \frac{1}{2}\int \frac{2x}{x^2-a^2}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;a)\Rightarrow t\in\langle -a^2;0) & x\in(a;\infty)\Rightarrow t\in(0;\infty) \\ dt=2x\,dx & x\in(-a;0\rangle\Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a)\Rightarrow t\in(0;\infty)\end{array}\right] = \frac{1}{2}\int \frac{dt}{t}$

$\int \frac{x\,dx}{(x^2-a^2)^n}$ pre $a>0,\ n\in N,\ n\neq 1$. [071]

- $= \frac{1}{2}\int \frac{2x}{(x^2-a^2)^n}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;a)\Rightarrow t\in\langle -a^2;0) & x\in(a;\infty)\Rightarrow t\in(0;\infty) \\ dt=2x\,dx & x\in(-a;0\rangle\Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a)\Rightarrow t\in(0;\infty)\end{array}\right]$

  $= \frac{1}{2}\int \frac{dt}{t^n} = \frac{1}{2}\int t^{-n}\,dt$

Riešené príklady – 070, 071

$$\int \frac{x}{x^2-a^2}\,dx = \frac{1}{2}\ln\left|x^2-a^2\right| + c \text{ pre } a>0.$$ [070]

- $= \frac{1}{2}\int \frac{2x}{x^2-a^2}\,dx = \frac{1}{2}\int \frac{(x^2-a^2)'}{x^2-a^2}\,dx = \frac{1}{2}\ln\left|x^2-a^2\right| + c,\ x\in R,\ x\neq \pm a,\ c\in R.$

- $= \frac{1}{2}\int \frac{2x}{x^2-a^2}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;a) \Rightarrow t\in\langle -a^2;0) & x\in(a;\infty) \Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-a;0\rangle \Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a) \Rightarrow t\in(0;\infty) \end{array}\right] = \frac{1}{2}\int\frac{dt}{t}$

  $= \frac{1}{2}\ln|t| + c$

$$\int \frac{x\,dx}{(x^2-a^2)^n} \text{ pre } a>0,\ n\in N,\ n\neq 1.$$ [071]

- $= \frac{1}{2}\int \frac{2x}{(x^2-a^2)^n}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;a) \Rightarrow t\in\langle -a^2;0) & x\in(a;\infty) \Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-a;0\rangle \Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a) \Rightarrow t\in(0;\infty) \end{array}\right]$

  $= \frac{1}{2}\int\frac{dt}{t^n} = \frac{1}{2}\int t^{-n}\,dt = \frac{t^{1-n}}{2(1-n)} + c = -\frac{1}{2(n-1)t^{n-1}} + c$

Riešené príklady – 070, 071

$$\int \frac{x}{x^2-a^2}\,dx = \frac{1}{2}\ln\left|x^2-a^2\right| + c \text{ pre } a>0.$$ [070]

- $= \frac{1}{2}\int \frac{2x}{x^2-a^2}\,dx = \frac{1}{2}\int \frac{(x^2-a^2)'}{x^2-a^2}\,dx = \frac{1}{2}\ln\left|x^2-a^2\right| + c,\ x\in R,\ x\neq \pm a,\ c\in R.$

- $= \frac{1}{2}\int \frac{2x}{x^2-a^2}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;a) \Rightarrow t\in\langle -a^2;0) & x\in(a;\infty) \Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-a;0\rangle \Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a) \Rightarrow t\in(0;\infty) \end{array}\right] = \frac{1}{2}\int\frac{dt}{t}$

  $= \frac{1}{2}\ln|t| + c = \frac{1}{2}\ln\left|x^2-a^2\right| + c,\ x\in R,\ x\neq \pm a,\ c\in R.$

$$\int \frac{x\,dx}{(x^2-a^2)^n} = -\frac{1}{2(n-1)(x^2-a^2)^{n-1}} + c \text{ pre } a>0,\ n\in N,\ n\neq 1.$$ [071]

- $= \frac{1}{2}\int \frac{2x}{(x^2-a^2)^n}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;a) \Rightarrow t\in\langle -a^2;0) & x\in(a;\infty) \Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-a;0\rangle \Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a) \Rightarrow t\in(0;\infty) \end{array}\right]$

  $= \frac{1}{2}\int\frac{dt}{t^n} = \frac{1}{2}\int t^{-n}\,dt = \frac{t^{1-n}}{2(1-n)} + c = -\frac{1}{2(n-1)t^{n-1}} + c$

  $= -\frac{1}{2(n-1)(x^2-a^2)^{n-1}} + c,\ x\in R,\ x\neq \pm a,\ c\in R.$

Riešené príklady – 072, 073, 074

$$\int \frac{x}{x^2-3}\,dx$$

1

[072]

$$\int \frac{x\,dx}{(x^2-3)^6}$$

[073]

$$\int \frac{2x\,dx}{(x^2-33)^{2024}}$$

[074]

Riešené príklady – 072, 073, 074

$\int \frac{x}{x^2-3}\,dx$ 2 [072]

- $= \frac{1}{2}\int \frac{2x}{x^2-3}\,dx = \frac{1}{2}\int \frac{(x^2-3)'}{x^2-3}\,dx$

$\int \frac{x\,dx}{(x^2-3)^6}$ [073]

- $= \frac{1}{2}\int \frac{2x}{(x^2-3)^6}\,dx$

$\int \frac{2x\,dx}{(x^2-33)^{2024}}$ [074]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t = x^2 - a^2 & x\in\langle 0;\sqrt{33}) \Rightarrow t\in\langle -3;0) & x\in(\sqrt{33};\infty) \Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-\sqrt{33};0\rangle \Rightarrow t\in\langle -3;0) & x\in(-\infty;-\sqrt{33}) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{dt}{t^{2024}} = \int t^{-2024}\,dt$

Riešené príklady – 072, 073, 074

$\displaystyle\int \frac{x}{x^2-3}\,dx = \frac{1}{2}\ln\left|x^2-3\right| + c$ 3 [072]

- $= \displaystyle\frac{1}{2}\int \frac{2x}{x^2-3}\,dx = \frac{1}{2}\int \frac{(x^2-3)'}{x^2-3}\,dx = \frac{1}{2}\ln\left|x^2-3\right| + c,\ x\in R,\ x\neq \pm\sqrt{3},\ c\in R.$

$\displaystyle\int \frac{x\,dx}{(x^2-3)^6}$ [073]

- $= \displaystyle\frac{1}{2}\int \frac{2x}{(x^2-3)^6}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;\sqrt{3}) \Rightarrow t\in\langle -3;0) & x\in(\sqrt{3};\infty) \Rightarrow t\in(0;\infty) \\ dt=2x\,dx & x\in(-\sqrt{3};0\rangle \Rightarrow t\in\langle -3;0) & x\in(-\infty;-\sqrt{3}) \Rightarrow t\in(0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^6}$
  $= \displaystyle\frac{1}{2}\int t^{-6}\,dt$

$\displaystyle\int \frac{2x\,dx}{(x^2-33)^{2024}}$ [074]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;\sqrt{33}) \Rightarrow t\in\langle -3;0) & x\in(\sqrt{33};\infty) \Rightarrow t\in(0;\infty) \\ dt=2x\,dx & x\in(-\sqrt{33};0\rangle \Rightarrow t\in\langle -3;0) & x\in(-\infty;-\sqrt{33}) \Rightarrow t\in(0;\infty) \end{array}\right] = \displaystyle\int \frac{dt}{t^{2024}} = \int t^{-2024}\,dt$
  $= \displaystyle\frac{t^{-2023}}{-2023} + c = -\frac{1}{2023t^{2023}} + c$

Riešené príklady – 072, 073, 074

$$\int \frac{x}{x^2-3}\,dx = \frac{1}{2}\ln\left|x^2-3\right| + c$$ 4 [072]

- $= \frac{1}{2}\int \frac{2x}{x^2-3}\,dx = \frac{1}{2}\int \frac{(x^2-3)'}{x^2-3}\,dx = \frac{1}{2}\ln\left|x^2-3\right| + c,\ x\in R,\ x\neq\pm\sqrt{3},\ c\in R.$

$$\int \frac{x\,dx}{(x^2-3)^6}$$ [073]

- $= \frac{1}{2}\int \frac{2x}{(x^2-3)^6}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;\sqrt{3}) \Rightarrow t\in\langle -3;0) & x\in(\sqrt{3};\infty) \Rightarrow t\in(0;\infty) \\ dt=2x\,dx & x\in(-\sqrt{3};0\rangle \Rightarrow t\in\langle -3;0) & x\in(-\infty;-\sqrt{3}) \Rightarrow t\in(0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^6}$

  $= \frac{1}{2}\int t^{-6}\,dt = \frac{t^{-5}}{2\cdot(-5)} + c = -\frac{1}{10t^5} + c$

$$\int \frac{2x\,dx}{(x^2-33)^{2024}} = -\frac{1}{2023(x^2-3)^{2023}} + c$$ [074]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;\sqrt{33}) \Rightarrow t\in\langle -3;0) & x\in(\sqrt{33};\infty) \Rightarrow t\in(0;\infty) \\ dt=2x\,dx & x\in(-\sqrt{33};0\rangle \Rightarrow t\in\langle -3;0) & x\in(-\infty;-\sqrt{33}) \Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t^{2024}} = \int t^{-2024}\,dt$

  $= \frac{t^{-2023}}{-2023} + c = -\frac{1}{2023t^{2023}} + c = -\frac{1}{2023(x^2-3)^{2023}} + c,\ x\in R,\ x\neq\pm\sqrt{33},\ c\in R.$

Riešené príklady – 072, 073, 074

$$\int \frac{x}{x^2-3}\,dx = \frac{1}{2}\ln|x^2-3| + c$$ 5 [072]

- $= \frac{1}{2}\int \frac{2x}{x^2-3}\,dx = \frac{1}{2}\int \frac{(x^2-3)'}{x^2-3}\,dx = \frac{1}{2}\ln|x^2-3| + c,\ x\in R,\ x\neq \pm\sqrt{3},\ c\in R.$

$$\int \frac{x\,dx}{(x^2-3)^6} = -\frac{1}{10(x^2-3)^5} + c$$ [073]

- $= \frac{1}{2}\int \frac{2x}{(x^2-3)^6}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;\sqrt{3}) \Rightarrow t\in\langle -3;0) & x\in(\sqrt{3};\infty) \Rightarrow t\in(0;\infty) \\ dt=2x\,dx & x\in(-\sqrt{3};0\rangle \Rightarrow t\in\langle -3;0) & x\in(-\infty;-\sqrt{3}) \Rightarrow t\in(0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^6}$

  $= \frac{1}{2}\int t^{-6}\,dt = \frac{t^{-5}}{2\cdot(-5)} + c = -\frac{1}{10t^5} + c = -\frac{1}{10(x^2-3)^5} + c,\ x\in R,\ x\neq \pm\sqrt{3},\ c\in R.$

$$\int \frac{2x\,dx}{(x^2-33)^{2024}} = -\frac{1}{2023(x^2-3)^{2023}} + c$$ [074]

- $= \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;\sqrt{33}) \Rightarrow t\in\langle -3;0) & x\in(\sqrt{33};\infty) \Rightarrow t\in(0;\infty) \\ dt=2x\,dx & x\in(-\sqrt{33};0\rangle \Rightarrow t\in\langle -3;0) & x\in(-\infty;-\sqrt{33}) \Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t^{2024}} = \int t^{-2024}\,dt$

  $= \frac{t^{-2023}}{-2023} + c = -\frac{1}{2023t^{2023}} + c = -\frac{1}{2023(x^2-3)^{2023}} + c,\ x\in R,\ x\neq \pm\sqrt{33},\ c\in R.$

Riešené príklady – 075, 076, 077

$\int \frac{x^2}{x^2+a^2}\,\mathrm{d}x$ pre $a > 0$. [075]

$\int \frac{x^2\,\mathrm{d}x}{(x^2+a^2)^2}$ pre $a > 0$. [076]

$\int \frac{x^2\,\mathrm{d}x}{(x^2+3)^2}$ [077]

Riešené príklady – 075, 076, 077

$$\int \frac{x^2}{x^2+a^2}\,dx = \int \frac{x^2+a^2-a^2}{x^2+a^2}\,dx$$ pre $a>0$. [075]

$$\int \frac{x^2\,dx}{(x^2+a^2)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\,dx$$ pre $a>0$. [076]

$$\int \frac{x^2\,dx}{(x^2+3)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+3)^2}\,dx$$ [077]

Riešené príklady – 075, 076, 077

$$\int \frac{x^2}{x^2+a^2}\,dx = \int \frac{x^2+a^2-a^2}{x^2+a^2}\,dx$$ pre $a>0$. [075]

- $= \int \left[1 - \frac{a^2}{x^2+a^2}\right] dx$

$$\int \frac{x^2\,dx}{(x^2+a^2)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\,dx$$ pre $a>0$. [076]

- $= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+a^2)^2} & u = \int \frac{2x\,dx}{(x^2+a^2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x^2+a^2 & x\in(-\infty;0\rangle \Rightarrow t\in\langle a^2;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ & = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+a^2} \\ v = x & v' = 1 \end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2}\right]$

$$\int \frac{x^2\,dx}{(x^2+3)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+3)^2}\,dx$$ [077]

- $= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+3)^2} & u = \int \frac{2x\,dx}{(x^2+3)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x^2+3 & x\in(-\infty;0\rangle \Rightarrow t\in\langle 3;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 3;\infty) \end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ & = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+3} \\ v = x & v' = 1 \end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2+3} + \int \frac{dx}{x^2+3}\right]$

Riešené príklady – 075, 076, 077

$$\int \frac{x^2}{x^2+a^2}\,dx = \int \frac{x^2+a^2-a^2}{x^2+a^2}\,dx$$ pre $a > 0$. [075]

- $= \int \left[1 - \frac{a^2}{x^2+a^2}\right] dx = x - \frac{a^2}{a}\,\text{arctg}\,\frac{x}{a} + c$

$$\int \frac{x^2\,dx}{(x^2+a^2)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\,dx$$ pre $a > 0$. [076]

- $= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+a^2)^2} & u = \int \frac{2x\,dx}{(x^2+a^2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x^2+a^2 & x\in(-\infty;0) \Rightarrow t\in\langle a^2;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ & = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+a^2} \\ v = x & v' = 1 \end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2}\right]$

  $= -\frac{x}{2(x^2+a^2)} + \frac{1}{2}\cdot\frac{1}{a}\,\text{arctg}\,\frac{x}{a} + c$

$$\int \frac{x^2\,dx}{(x^2+3)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+3)^2}\,dx$$ [077]

- $= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+3)^2} & u = \int \frac{2x\,dx}{(x^2+3)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x^2+3 & x\in(-\infty;0) \Rightarrow t\in\langle 3;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 3;\infty) \end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ & = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+3} \\ v = x & v' = 1 \end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2+3} + \int \frac{dx}{x^2+3}\right]$

  $= -\frac{x}{2(x^2+3)} + \frac{1}{2}\cdot\frac{1}{\sqrt{3}}\,\text{arctg}\,\frac{x}{\sqrt{3}} + c$

Riešené príklady – 075, 076, 077

$$\int \frac{x^2}{x^2+a^2}\,dx = \int \frac{x^2+a^2-a^2}{x^2+a^2}\,dx = x - a\cdot\operatorname{arctg}\frac{x}{a} + c \text{ pre } a>0.$$ [075]

- $= \int \left[1 - \frac{a^2}{x^2+a^2}\right] dx = x - \frac{a^2}{a}\operatorname{arctg}\frac{x}{a} + c = x - a\cdot\operatorname{arctg}\frac{x}{a} + c,\ x\in R,\ c\in R.$

$$\int \frac{x^2\,dx}{(x^2+a^2)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\,dx = \frac{1}{2a}\operatorname{arctg}\frac{x}{a} - \frac{x}{2(x^2+a^2)} + c \text{ pre } a>0.$$ [076]

- $= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+a^2)^2} & u = \int \frac{2x\,dx}{(x^2+a^2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x^2+a^2 & x\in(-\infty;0\rangle \Rightarrow t\in\langle a^2;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ v = x & v' = 1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+a^2} \end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2}\right]$

  $= -\frac{x}{2(x^2+a^2)} + \frac{1}{2}\cdot\frac{1}{a}\operatorname{arctg}\frac{x}{a} + c = \frac{1}{2a}\operatorname{arctg}\frac{x}{a} - \frac{x}{2(x^2+a^2)} + c,\ x\in R,\ c\in R.$

$$\int \frac{x^2\,dx}{(x^2+3)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+3)^2}\,dx = \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}} - \frac{x}{2(x^2+3)} + c$$ [077]

- $= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+3)^2} & u = \int \frac{2x\,dx}{(x^2+3)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x^2+3 & x\in(-\infty;0\rangle \Rightarrow t\in\langle 3;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 3;\infty) \end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ v = x & v' = 1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+3} \end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2+3} + \int \frac{dx}{x^2+3}\right]$

  $= -\frac{x}{2(x^2+3)} + \frac{1}{2}\cdot\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}} + c = \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}} - \frac{x}{2(x^2+3)} + c,\ x\in R,\ c\in R.$

Riešené príklady – 078, 079

$I_1 = \int \frac{dx}{x^2+a^2}$ pre $a > 0$ a $n = 1$. [078]

$I_n = \int \frac{dx}{(x^2+a^2)^n}$ pre $a > 0$ a $n \in N$, $n \neq 1$. [079]

Riešené príklady – 078, 079

$I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c,\ x \in R,\ c \in R$ pre $a > 0$ a $n = 1.$ [078]

$I_n = \int \frac{dx}{(x^2+a^2)^n}$ pre $a > 0$ a $n \in N,\ n \neq 1.$ [079]

- $= \frac{1}{a^2} \int \frac{a^2}{(x^2+a^2)^n}\, dx = \frac{1}{a^2} \int \frac{(x^2+a^2)-x^2}{(x^2+a^2)^n}\, dx$

Riešené príklady – 078, 079

$I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c,\ x \in R,\ c \in R$ pre $a > 0$ a $n = 1.$ [078]

$I_n = \int \frac{dx}{(x^2+a^2)^n}$ pre $a > 0$ a $n \in N,\ n \neq 1.$ [079]

- $= \frac{1}{a^2} \int \frac{a^2}{(x^2+a^2)^n}\, dx = \frac{1}{a^2} \int \frac{(x^2+a^2)-x^2}{(x^2+a^2)^n}\, dx = \frac{1}{a^2} \int \frac{x^2+a^2}{(x^2+a^2)^n}\, dx - \frac{1}{a^2} \int \frac{x^2}{(x^2+a^2)^n}\, dx$

Riešené príklady – 078, 079

$I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c,\ x \in R,\ c \in R$ pre $a > 0$ a $n = 1.$ [078]

$I_n = \int \frac{dx}{(x^2+a^2)^n}$ pre $a > 0$ a $n \in N,\ n \neq 1.$ [079]

- $= \frac{1}{a^2} \int \frac{a^2}{(x^2+a^2)^n}\, dx = \frac{1}{a^2} \int \frac{(x^2+a^2)-x^2}{(x^2+a^2)^n}\, dx = \frac{1}{a^2} \int \frac{x^2+a^2}{(x^2+a^2)^n}\, dx - \frac{1}{a^2} \int \frac{x^2}{(x^2+a^2)^n}\, dx$
  $= \frac{1}{a^2} \int \frac{dx}{(x^2+a^2)^{n-1}} - \frac{1}{2a^2} \int \frac{x \cdot 2x}{(x^2+a^2)^n}\, dx = \frac{1}{a^2} \cdot I_{n-1} - \frac{1}{2a^2} \int \frac{x \cdot 2x}{(x^2+a^2)^n}\, dx$

Riešené príklady – 078, 079

$I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c,\ x \in R,\ c \in R$ pre $a > 0$ a $n = 1.$ [078]

$I_n = \int \frac{dx}{(x^2+a^2)^n}$ pre $a > 0$ a $n \in N,\ n \neq 1.$ [079]

- $= \frac{1}{a^2} \int \frac{a^2}{(x^2+a^2)^n}\,dx = \frac{1}{a^2} \int \frac{(x^2+a^2)-x^2}{(x^2+a^2)^n}\,dx = \frac{1}{a^2} \int \frac{x^2+a^2}{(x^2+a^2)^n}\,dx - \frac{1}{a^2} \int \frac{x^2}{(x^2+a^2)^n}\,dx$

$= \frac{1}{a^2} \int \frac{dx}{(x^2+a^2)^{n-1}} - \frac{1}{2a^2} \int \frac{x \cdot 2x}{(x^2+a^2)^n}\,dx = \frac{1}{a^2} \cdot I_{n-1} - \frac{1}{2a^2} \int \frac{x \cdot 2x}{(x^2+a^2)^n}\,dx$

$= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+a^2)^n} & u = \int \frac{2x}{(x^2+a^2)^n}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = x^2+a^2 & x \in (-\infty; 0) \Rightarrow t \in \langle a^2; \infty) \\ dt = 2x\,dx & x \in \langle 0; \infty) \Rightarrow t \in \langle a^2; \infty) \end{array} \right] = \int \frac{dt}{t^n} = \int t^{-n}\,dt = \frac{t^{-n+1}}{-n+1} = \frac{1}{(1-n)t^{n-1}} = \frac{1}{(1-n)(x^2+a^2)^{n-1}} \\ v = x & v' = 1 \end{array} \right]$

$= \frac{1}{a^2} \cdot I_{n-1} - \frac{1}{2a^2} \left[\frac{x}{(1-n)(x^2+a^2)^{n-1}} - \frac{1}{1-n} \int \frac{dx}{(x^2+a^2)^{n-1}} \right]$

Riešené príklady – 078, 079

$I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c,\ x \in R,\ c \in R$ pre $a > 0$ a $n = 1.$ [078]

$I_n = \int \frac{dx}{(x^2+a^2)^n}$ pre $a > 0$ a $n \in N,\ n \neq 1.$ [079]

- $$= \frac{1}{a^2}\int \frac{a^2}{(x^2+a^2)^n}\,dx = \frac{1}{a^2}\int \frac{(x^2+a^2)-x^2}{(x^2+a^2)^n}\,dx = \frac{1}{a^2}\int \frac{x^2+a^2}{(x^2+a^2)^n}\,dx - \frac{1}{a^2}\int \frac{x^2}{(x^2+a^2)^n}\,dx$$
$$= \frac{1}{a^2}\int \frac{dx}{(x^2+a^2)^{n-1}} - \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2+a^2)^n}\,dx = \frac{1}{a^2}\cdot I_{n-1} - \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2+a^2)^n}\,dx$$
$$= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+a^2)^n} & u = \int \frac{2x}{(x^2+a^2)^n}\,dx = \left[\begin{array}{r|l} \text{Subst. } t = x^2+a^2 & x\in(-\infty;0) \Rightarrow t\in\langle a^2;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \int \frac{dt}{t^n} = \int t^{-n}\,dt = \frac{t^{-n+1}}{-n+1} = \frac{1}{(1-n)t^{n-1}} = \frac{1}{(1-n)(x^2+a^2)^{n-1}} \\ v = x & v' = 1 \end{array}\right]$$
$$= \frac{1}{a^2}\cdot I_{n-1} - \frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2+a^2)^{n-1}} - \frac{1}{1-n}\int \frac{dx}{(x^2+a^2)^{n-1}}\right]$$
$$= \frac{1}{a^2}\cdot I_{n-1} - \frac{x}{2a^2(1-n)(x^2+a^2)^{n-1}} + \frac{1}{2a^2(1-n)}\cdot I_{n-1}$$
$$= \frac{2(1-n)+1}{2a^2(1-n)}\cdot I_{n-1} - \frac{x}{2a^2(1-n)(x^2+a^2)^{n-1}}$$

Riešené príklady – 078, 079

$$I_1=\int\frac{dx}{x^2+a^2}=\frac{1}{a}\operatorname{arctg}\frac{x}{a}+c,\ x\in R,\ c\in R \text{ pre } a>0 \text{ a } n=1.$$ [078]

$$I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}\int\frac{dx}{(x^2+a^2)^{n-1}}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \text{ pre } a>0 \text{ a } n\in N,\ n\neq 1.$$ [079]

- $= \frac{1}{a^2}\int\frac{a^2}{(x^2+a^2)^n}\,dx=\frac{1}{a^2}\int\frac{(x^2+a^2)-x^2}{(x^2+a^2)^n}\,dx=\frac{1}{a^2}\int\frac{x^2+a^2}{(x^2+a^2)^n}\,dx-\frac{1}{a^2}\int\frac{x^2}{(x^2+a^2)^n}\,dx$

$= \frac{1}{a^2}\int\frac{dx}{(x^2+a^2)^{n-1}}-\frac{1}{2a^2}\int\frac{x\cdot 2x}{(x^2+a^2)^n}\,dx=\frac{1}{a^2}\cdot I_{n-1}-\frac{1}{2a^2}\int\frac{x\cdot 2x}{(x^2+a^2)^n}\,dx$

$= \left[\begin{array}{l|l} u'=\frac{2x}{(x^2+a^2)^n} & u=\int\frac{2x}{(x^2+a^2)^n}\,dx=\left[\begin{array}{r|l}\text{Subst. } t=x^2+a^2 & x\in(-\infty;0)\Rightarrow t\in\langle a^2;\infty)\\ dt=2x\,dx & x\in\langle 0;\infty)\Rightarrow t\in\langle a^2;\infty)\end{array}\right]=\int\frac{dt}{t^n}=\int t^{-n}\,dt=\frac{t^{-n+1}}{-n+1}=\frac{1}{(1-n)t^{n-1}}=\frac{1}{(1-n)(x^2+a^2)^{n-1}} \\ v=x & v'=1\end{array}\right]$

$= \frac{1}{a^2}\cdot I_{n-1}-\frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2+a^2)^{n-1}}-\frac{1}{1-n}\int\frac{dx}{(x^2+a^2)^{n-1}}\right]$

$= \frac{1}{a^2}\cdot I_{n-1}-\frac{x}{2a^2(1-n)(x^2+a^2)^{n-1}}+\frac{1}{2a^2(1-n)}\cdot I_{n-1}$

$= \frac{2(1-n)+1}{2a^2(1-n)}\cdot I_{n-1}-\frac{x}{2a^2(1-n)(x^2+a^2)^{n-1}}=\frac{3-2n}{2a^2(1-n)}\cdot I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$

$= \frac{2n-3}{2a^2(n-1)}\int\frac{dx}{(x^2+a^2)^{n-1}}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}},\ x\in R.$

Riešené príklady – 080, 081

$I_2 = \int \frac{dx}{(x^2+a^2)^2}$ pre $a > 0$. [080]

$\int \frac{dx}{(x^2+1)^2}$ [081]

Riešené príklady – 080, 081

$$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{a^2} \int \frac{a^2}{(x^2+a^2)^2}\, dx$$ pre $a > 0$. [080]

$$\int \frac{dx}{(x^2+1)^2} = \int \frac{1}{(x^2+1)^2}\, dx$$ [081]

Riešené príklady – 080, 081

$$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{a^2}\int \frac{a^2}{(x^2+a^2)^2}\,dx$$ pre $a > 0$. [080]

- $= \frac{1}{a^2}\int \frac{x^2+a^2-x^2}{(x^2+a^2)^2}\,dx = \frac{1}{a^2}\int \frac{x^2+a^2}{(x^2+a^2)^2}\,dx - \frac{1}{a^2}\int \frac{x^2}{(x^2+a^2)^2}\,dx$

$$\int \frac{dx}{(x^2+1)^2} = \int \frac{1}{(x^2+1)^2}\,dx = \int \frac{x^2+1-x^2}{(x^2+1)^2}\,dx$$ [081]

- $= \int \frac{dx}{x^2+1} - \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+1)^2}\,dx$

Riešené príklady – 080, 081

$$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{a^2}\int \frac{a^2}{(x^2+a^2)^2}\,dx$$ pre $a > 0$. [080]

- $= \frac{1}{a^2}\int \frac{x^2+a^2-x^2}{(x^2+a^2)^2}\,dx = \frac{1}{a^2}\int \frac{x^2+a^2}{(x^2+a^2)^2}\,dx - \frac{1}{a^2}\int \frac{x^2}{(x^2+a^2)^2}\,dx = \frac{1}{a^2}\int \frac{dx}{x^2+a^2} - \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\,dx$
  $= \frac{1}{a^2}\cdot I_1 - \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\,dx$

$$\int \frac{dx}{(x^2+1)^2} = \int \frac{1}{(x^2+1)^2}\,dx = \int \frac{x^2+1-x^2}{(x^2+1)^2}\,dx$$ [081]

- $= \int \frac{dx}{x^2+1} - \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+1)^2}\,dx = \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+1)^2} & u = \int \frac{2x\,dx}{(x^2+1)^2} = \left[\begin{array}{l|l} \text{Subst. } t = x^2+1 & x\in(-\infty;0) \Rightarrow t\in\langle 1;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 1;\infty) \end{array} \right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ v = x & v' = 1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+1} \end{array} \right]$
  $= \int \frac{dx}{x^2+1} - \frac{1}{2}\left[-\frac{x}{x^2+1} + \int \frac{dx}{x^2+1} \right]$

Riešené príklady – 080, 081

$$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{a^2}\int \frac{a^2}{(x^2+a^2)^2}\, dx$$ pre $a > 0$. [080]

- $= \frac{1}{a^2}\int \frac{x^2+a^2-x^2}{(x^2+a^2)^2}\, dx = \frac{1}{a^2}\int \frac{x^2+a^2}{(x^2+a^2)^2}\, dx - \frac{1}{a^2}\int \frac{x^2}{(x^2+a^2)^2}\, dx = \frac{1}{a^2}\int \frac{dx}{x^2+a^2} - \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\, dx$

  $= \frac{1}{a^2}\cdot I_1 - \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\, dx = \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+a^2)^2} & u = \int \frac{2x\, dx}{(x^2+a^2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x^2+a^2 & x\in(-\infty;0\rangle \Rightarrow t\in\langle a^2;\infty) \\ dt = 2x\, dx & x\in\langle 0;\infty) \Rightarrow t\in\langle a^2;\infty) \end{array} \right] = \int \frac{dt}{t^2} = \int t^{-2}\, dt \\ v = x & v' = 1 \qquad\qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+a^2} \end{array} \right]$

  $= \frac{1}{a^2}\cdot I_1 - \frac{1}{2a^2}\left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2} \right]$

$$\int \frac{dx}{(x^2+1)^2} = \int \frac{1}{(x^2+1)^2}\, dx = \int \frac{x^2+1-x^2}{(x^2+1)^2}\, dx$$ [081]

- $= \int \frac{dx}{x^2+1} - \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+1)^2}\, dx = \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+1)^2} & u = \int \frac{2x\, dx}{(x^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x^2+1 & x\in(-\infty;0\rangle \Rightarrow t\in\langle 1;\infty) \\ dt = 2x\, dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 1;\infty) \end{array} \right] = \int \frac{dt}{t^2} = \int t^{-2}\, dt \\ v = x & v' = 1 \qquad\qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+1} \end{array} \right]$

  $= \int \frac{dx}{x^2+1} - \frac{1}{2}\left[-\frac{x}{x^2+1} + \int \frac{dx}{x^2+1} \right] = \frac{1}{2}\int \frac{dx}{x^2+1} + \frac{x}{2(x^2+1)}$

Riešené príklady – 080, 081

$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{a^2}\int \frac{a^2}{(x^2+a^2)^2}\,dx$ pre $a > 0$. [080]

- $= \frac{1}{a^2}\int \frac{x^2+a^2-x^2}{(x^2+a^2)^2}\,dx = \frac{1}{a^2}\int \frac{x^2+a^2}{(x^2+a^2)^2}\,dx - \frac{1}{a^2}\int \frac{x^2}{(x^2+a^2)^2}\,dx = \frac{1}{a^2}\int \frac{dx}{x^2+a^2} - \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\,dx$

  $= \frac{1}{a^2}\cdot I_1 - \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\,dx = \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+a^2)^2} & u = \int \frac{2x\,dx}{(x^2+a^2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x^2+a^2 & x\in(-\infty;0) \Rightarrow t\in\langle a^2;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ v = x & v' = 1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+a^2} \end{array}\right]$

  $= \frac{1}{a^2}\cdot I_1 - \frac{1}{2a^2}\left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2}\right] = \frac{1}{a^2}\cdot I_1 + \frac{x}{2a^2(x^2+a^2)} - \frac{1}{2a^2}\cdot I_1 = \frac{1}{2a^2}\cdot I_1 + \frac{x}{2a^2(x^2+a^2)}$

$\int \frac{dx}{(x^2+1)^2} = \int \frac{1}{(x^2+1)^2}\,dx = \int \frac{x^2+1-x^2}{(x^2+1)^2}\,dx = \frac{1}{2}\operatorname{arctg} x + \frac{x}{2(x^2+1)} + c$ [081]

- $= \int \frac{dx}{x^2+1} - \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+1)^2}\,dx = \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+1)^2} & u = \int \frac{2x\,dx}{(x^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x^2+1 & x\in(-\infty;0) \Rightarrow t\in\langle 1;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 1;\infty) \end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ v = x & v' = 1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+1} \end{array}\right]$

  $= \int \frac{dx}{x^2+1} - \frac{1}{2}\left[-\frac{x}{x^2+1} + \int \frac{dx}{x^2+1}\right] = \frac{1}{2}\int \frac{dx}{x^2+1} + \frac{x}{2(x^2+1)} = \frac{1}{2}\operatorname{arctg} x + \frac{x}{2(x^2+1)} + c,$

  $x\in R,\ c\in R.$

Riešené príklady – 080, 081

$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{a^2}\int \frac{a^2}{(x^2+a^2)^2}\,dx$ pre $a > 0$. [080]

- $= \frac{1}{a^2}\int \frac{x^2+a^2-x^2}{(x^2+a^2)^2}\,dx = \frac{1}{a^2}\int \frac{x^2+a^2}{(x^2+a^2)^2}\,dx - \frac{1}{a^2}\int \frac{x^2}{(x^2+a^2)^2}\,dx = \frac{1}{a^2}\int \frac{dx}{x^2+a^2} - \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\,dx$
  $= \frac{1}{a^2}\cdot I_1 - \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\,dx = \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+a^2)^2} & u = \int \frac{2x\,dx}{(x^2+a^2)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x^2+a^2 & x\in(-\infty;0) \Rightarrow t\in\langle a^2;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ v = x & v' = 1 \quad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+a^2} \end{array}\right]$
  $= \frac{1}{a^2}\cdot I_1 - \frac{1}{2a^2}\left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2}\right] = \frac{1}{a^2}\cdot I_1 + \frac{x}{2a^2(x^2+a^2)} - \frac{1}{2a^2}\cdot I_1 = \frac{1}{2a^2}\cdot I_1 + \frac{x}{2a^2(x^2+a^2)}$
  $= \frac{1}{2a^2}\int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)}$

$\int \frac{dx}{(x^2+1)^2} = \int \frac{1}{(x^2+1)^2}\,dx = \int \frac{x^2+1-x^2}{(x^2+1)^2}\,dx = \frac{1}{2}\operatorname{arctg} x + \frac{x}{2(x^2+1)} + c$ [081]

- $= \int \frac{dx}{x^2+1} - \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+1)^2}\,dx = \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+1)^2} & u = \int \frac{2x\,dx}{(x^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x^2+1 & x\in(-\infty;0) \Rightarrow t\in\langle 1;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 1;\infty) \end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ v = x & v' = 1 \quad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+1} \end{array}\right]$
  $= \int \frac{dx}{x^2+1} - \frac{1}{2}\left[-\frac{x}{x^2+1} + \int \frac{dx}{x^2+1}\right] = \frac{1}{2}\int \frac{dx}{x^2+1} + \frac{x}{2(x^2+1)} = \frac{1}{2}\operatorname{arctg} x + \frac{x}{2(x^2+1)} + c,$
  $x\in R,\ c\in R.$

Riešené príklady – 080, 081

$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{a^2}\int \frac{a^2}{(x^2+a^2)^2}\,dx = \frac{1}{2a^3}\operatorname{arctg}\frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c$ pre $a > 0$. [080]

- $$= \frac{1}{a^2}\int \frac{x^2+a^2-x^2}{(x^2+a^2)^2}\,dx = \frac{1}{a^2}\int \frac{x^2+a^2}{(x^2+a^2)^2}\,dx - \frac{1}{a^2}\int \frac{x^2}{(x^2+a^2)^2}\,dx = \frac{1}{a^2}\int \frac{dx}{x^2+a^2} - \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\,dx$$
  $$= \frac{1}{a^2}\cdot I_1 - \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2+a^2)^2}\,dx = \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+a^2)^2} & u = \int \frac{2x\,dx}{(x^2+a^2)^2} = \left[\begin{array}{l|l} \text{Subst. } t = x^2+a^2 & x\in(-\infty;0) \Rightarrow t\in\langle a^2;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle a^2;\infty) \end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ v = x & v' = 1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+a^2} \end{array}\right]$$
  $$= \frac{1}{a^2}\cdot I_1 - \frac{1}{2a^2}\left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2}\right] = \frac{1}{a^2}\cdot I_1 + \frac{x}{2a^2(x^2+a^2)} - \frac{1}{2a^2}\cdot I_1 = \frac{1}{2a^2}\cdot I_1 + \frac{x}{2a^2(x^2+a^2)}$$
  $$= \frac{1}{2a^2}\int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} = \frac{1}{2a^2}\cdot\frac{1}{a}\operatorname{arctg}\frac{x}{a} + \frac{x}{2a^2(x^2+a^2)}$$
  $$= \frac{1}{2a^3}\operatorname{arctg}\frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c,\ x\in R,\ c\in R.$$

$\int \frac{dx}{(x^2+1)^2} = \int \frac{1}{(x^2+1)^2}\,dx = \int \frac{x^2+1-x^2}{(x^2+1)^2}\,dx = \frac{1}{2}\operatorname{arctg} x + \frac{x}{2(x^2+1)} + c$ [081]

- $$= \int \frac{dx}{x^2+1} - \frac{1}{2}\int \frac{x\cdot 2x}{(x^2+1)^2}\,dx = \left[\begin{array}{l|l} u' = \frac{2x}{(x^2+1)^2} & u = \int \frac{2x\,dx}{(x^2+1)^2} = \left[\begin{array}{l|l} \text{Subst. } t = x^2+1 & x\in(-\infty;0) \Rightarrow t\in\langle 1;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 1;\infty) \end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt \\ v = x & v' = 1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2+1} \end{array}\right]$$
  $$= \int \frac{dx}{x^2+1} - \frac{1}{2}\left[-\frac{x}{x^2+1} + \int \frac{dx}{x^2+1}\right] = \frac{1}{2}\int \frac{dx}{x^2+1} + \frac{x}{2(x^2+1)} = \frac{1}{2}\operatorname{arctg} x + \frac{x}{2(x^2+1)} + c,$$
  $$x\in R,\ c\in R.$$

Riešené príklady – 082

$$\int \frac{dx}{(x^2 \pm 4x + 5)^2}$$ [082]

Riešené príklady – 082

$\int \frac{dx}{(x^2 \pm 4x+5)^2}$ [082]

- $= \int \frac{dx}{(x^2 \pm 4x+4+1)^2} = \int \frac{dx}{((x \pm 2)^2+1)^2}$

- $= \int \frac{dx}{(x^2 \pm 4x+4+1)^2} = \int \frac{dx}{((x \pm 2)^2+1)^2}$

Riešené príklady – 082

$\int \frac{dx}{(x^2 \pm 4x+5)^2}$ [082]

- $= \int \frac{dx}{(x^2 \pm 4x+4+1)^2} = \int \frac{dx}{((x \pm 2)^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x \pm 2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{1}{(t^2+1)^2}\, dt$

- $= \int \frac{dx}{(x^2 \pm 4x+4+1)^2} = \int \frac{dx}{((x \pm 2)^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x \pm 2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{1}{(t^2+1)^2}\, dt$

Riešené príklady – 082

$\int \frac{dx}{(x^2 \pm 4x+5)^2}$ [082]

- $= \int \frac{dx}{(x^2 \pm 4x+4+1)^2} = \int \frac{dx}{((x \pm 2)^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x \pm 2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{1}{(t^2+1)^2}\, dt = \int \frac{t^2+1-t^2}{(t^2+1)^2}\, dt$
  $= \int \frac{dt}{t^2+1} - \frac{1}{2} \int \frac{2t^2\, dt}{(t^2+1)^2}$

- $= \int \frac{dx}{(x^2 \pm 4x+4+1)^2} = \int \frac{dx}{((x \pm 2)^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x \pm 2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{1}{(t^2+1)^2}\, dt$
  $= \left[\begin{array}{r} \text{Pr. 079: } \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} \cdot \int \frac{dt}{(t^2+a^2)^{n-1}} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \\ \text{pre } a > 0 \text{ a } n \in N,\ n \neq 1. \end{array} \right] = \frac{2 \cdot 2-3}{2(2-1)} \int \frac{dt}{(t^2+1)^{2-1}} + \frac{t}{2(2-1)(t^2+1)}$

Riešené príklady – 082

$$\int \frac{dx}{(x^2 \pm 4x+5)^2}$$ [082]

- $= \int \frac{dx}{(x^2\pm 4x+4+1)^2} = \int \frac{dx}{((x\pm 2)^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x\pm 2 & x\in R \\ dt = dx & t\in R \end{array} \right] = \int \frac{1}{(t^2+1)^2}\,dt = \int \frac{t^2+1-t^2}{(t^2+1)^2}\,dt$

$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\int \frac{2t^2\,dt}{(t^2+1)^2} = \left[\begin{array}{l|l} u' = \frac{2t}{(t^2+1)^2} & u = \int \frac{2t\,dt}{(t^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } z = t^2+1 & t\in(-\infty;0\rangle \Rightarrow z\in\langle 1;\infty) \\ dz = 2t\,dt & t\in\langle 0;\infty) \Rightarrow z\in\langle 1;\infty) \end{array} \right] = \int \frac{dz}{z^2} = \int z^{-2}\,dz \\ v = t & v' = 1 \qquad = -z^{-1} = -\frac{1}{z} = -\frac{1}{t^2+1} \end{array} \right]$$

$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right]$$

- $= \int \frac{dx}{(x^2\pm 4x+4+1)^2} = \int \frac{dx}{((x\pm 2)^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x\pm 2 & x\in R \\ dt = dx & t\in R \end{array} \right] = \int \frac{1}{(t^2+1)^2}\,dt$

$$= \left[\begin{array}{r} \text{Pr. 079: } \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} \cdot \int \frac{dt}{(t^2+a^2)^{n-1}} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \\ \text{pre } a>0 \text{ a } n\in N,\ n\neq 1. \end{array} \right] = \frac{2\cdot 2-3}{2(2-1)} \int \frac{dt}{(t^2+1)^{2-1}} + \frac{t}{2(2-1)(t^2+1)}$$

$$= \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)}$$

Riešené príklady – 082

$$\int \frac{dx}{(x^2\pm 4x+5)^2}$$ [082]

- $$= \int \frac{dx}{(x^2\pm 4x+4+1)^2} = \int \frac{dx}{((x\pm 2)^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x\pm 2 & x\in R \\ dt = dx & t\in R \end{array} \right] = \int \frac{1}{(t^2+1)^2}\, dt = \int \frac{t^2+1-t^2}{(t^2+1)^2}\, dt$$
$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\int \frac{2t^2\, dt}{(t^2+1)^2} = \left[\begin{array}{l|l} u' = \frac{2t}{(t^2+1)^2} & u = \int \frac{2t\, dt}{(t^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } z = t^2+1 & t\in(-\infty;0\rangle \Rightarrow z\in\langle 1;\infty) \\ dz = 2t\, dt & t\in\langle 0;\infty) \Rightarrow z\in\langle 1;\infty) \end{array} \right] = \int \frac{dz}{z^2} = \int z^{-2}\, dz \\ v = t & v' = 1 \qquad = -z^{-1} = -\frac{1}{z} = -\frac{1}{t^2+1} \end{array} \right]$$
$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right] = \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\,\mathrm{arctg}\, t + \frac{t}{2(t^2+1)} + c$$

- $$= \int \frac{dx}{(x^2\pm 4x+4+1)^2} = \int \frac{dx}{((x\pm 2)^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x\pm 2 & x\in R \\ dt = dx & t\in R \end{array} \right] = \int \frac{1}{(t^2+1)^2}\, dt$$
$$= \left[\begin{array}{l} \text{Pr. 079: } \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} \cdot \int \frac{dt}{(t^2+a^2)^{n-1}} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \\ \text{pre } a>0 \text{ a } n\in N,\ n\neq 1. \end{array} \right] = \frac{2\cdot 2-3}{2(2-1)}\int \frac{dt}{(t^2+1)^{2-1}} + \frac{t}{2(2-1)(t^2+1)}$$
$$= \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\,\mathrm{arctg}\, t + \frac{t}{2(t^2+1)} + c$$

Riešené príklady – 082

$$\int\frac{dx}{(x^2\pm4x+5)^2}=\frac{1}{2}\operatorname{arctg}(x\pm2)+\frac{x\pm2}{2(x^2\pm4x+5)}+c$$ [082]

- $$=\int\frac{dx}{(x^2\pm4x+4+1)^2}=\int\frac{dx}{((x\pm2)^2+1)^2}=\left[\begin{array}{r|l}\text{Subst. } t=x\pm2 & x\in R\\ dt=dx & t\in R\end{array}\right]=\int\frac{1}{(t^2+1)^2}\,dt=\int\frac{t^2+1-t^2}{(t^2+1)^2}\,dt$$
$$=\int\frac{dt}{t^2+1}-\frac{1}{2}\int\frac{2t^2\,dt}{(t^2+1)^2}=\left[\begin{array}{l|l}u'=\frac{2t}{(t^2+1)^2} & u=\int\frac{2t\,dt}{(t^2+1)^2}=\left[\begin{array}{r|l}\text{Subst. } z=t^2+1 & t\in(-\infty;0\rangle\Rightarrow z\in\langle1;\infty)\\ dz=2t\,dt & t\in\langle0;\infty)\Rightarrow z\in\langle1;\infty)\end{array}\right]=\int\frac{dz}{z^2}=\int z^{-2}\,dz\\ & =-z^{-1}=-\frac{1}{z}=-\frac{1}{t^2+1}\\ v=t & v'=1\end{array}\right]$$
$$=\int\frac{dt}{t^2+1}-\frac{1}{2}\left[-\frac{t}{t^2+1}+\int\frac{dt}{t^2+1}\right]=\frac{1}{2}\int\frac{dt}{t^2+1}+\frac{t}{2(t^2+1)}=\frac{1}{2}\operatorname{arctg}t+\frac{t}{2(t^2+1)}+c$$
$$=\left[\begin{array}{r}t^2+1=(x\pm2)^2+1\\=x^2\pm4x+5\end{array}\right]=\frac{1}{2}\operatorname{arctg}(x\pm2)+\frac{x\pm2}{2(x^2\pm4x+5)}+c,\ x\in R,\ c\in R.$$

- $$=\int\frac{dx}{(x^2\pm4x+4+1)^2}=\int\frac{dx}{((x\pm2)^2+1)^2}=\left[\begin{array}{r|l}\text{Subst. } t=x\pm2 & x\in R\\ dt=dx & t\in R\end{array}\right]=\int\frac{1}{(t^2+1)^2}\,dt$$
$$=\left[\begin{array}{r}\text{Pr. 079: }\int\frac{dt}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}\cdot\int\frac{dt}{(t^2+a^2)^{n-1}}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\\ \text{pre } a>0 \text{ a } n\in N,\ n\neq1.\end{array}\right]=\frac{2\cdot2-3}{2(2-1)}\int\frac{dt}{(t^2+1)^{2-1}}+\frac{t}{2(2-1)(t^2+1)}$$
$$=\frac{1}{2}\int\frac{dt}{t^2+1}+\frac{t}{2(t^2+1)}=\frac{1}{2}\operatorname{arctg}t+\frac{t}{2(t^2+1)}+c$$
$$=\left[t^2+1=(x\pm2)^2+1=x^2\pm4x+5\right]=\frac{1}{2}\operatorname{arctg}(x\pm2)+\frac{x\pm2}{2(x^2\pm4x+5)}+c,\ x\in R,\ c\in R.$$

Riešené príklady – 082

$$\int \frac{dx}{(x^2+4x+5)^2} = \frac{1}{2}\operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c$$ [082]

- $$= \int \frac{dx}{(x^2+4x+4+1)^2} = \int \frac{dx}{((x+2)^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x\in R \\ dt = dx & t\in R \end{array}\right] = \int \frac{1}{(t^2+1)^2}\,dt = \int \frac{t^2+1-t^2}{(t^2+1)^2}\,dt$$
$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\int \frac{2t^2\,dt}{(t^2+1)^2} = \left[\begin{array}{l|l} u' = \frac{2t}{(t^2+1)^2} & u = \int \frac{2t\,dt}{(t^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } z=t^2+1 & t\in(-\infty;0) \Rightarrow z\in\langle 1;\infty) \\ dz = 2t\,dt & t\in\langle 0;\infty) \Rightarrow z\in\langle 1;\infty) \end{array}\right] = \int\frac{dz}{z^2} = \int z^{-2}\,dz \\ v = t & v' = 1 \qquad = -z^{-1} = -\frac{1}{z} = -\frac{1}{t^2+1} \end{array}\right]$$
$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1}\right] = \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\operatorname{arctg} t + \frac{t}{2(t^2+1)} + c$$
$$= \left[\begin{array}{r} t^2+1 = (x+2)^2+1 \\ = x^2+4x+5 \end{array}\right] = \frac{1}{2}\operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c,\ x\in R,\ c\in R.$$

- $$= \int \frac{dx}{(x^2+4x+4+1)^2} = \int \frac{dx}{((x+2)^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x\in R \\ dt = dx & t\in R \end{array}\right] = \int \frac{1}{(t^2+1)^2}\,dt$$
$$= \left[\begin{array}{r} \text{Pr. 079: } \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)}\cdot\int\frac{dt}{(t^2+a^2)^{n-1}} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \\ \text{pre } a>0 \text{ a } n\in N,\ n\neq 1. \end{array}\right] = \frac{2\cdot 2-3}{2(2-1)}\int \frac{dt}{(t^2+1)^{2-1}} + \frac{t}{2(2-1)(t^2+1)}$$
$$= \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\operatorname{arctg} t + \frac{t}{2(t^2+1)} + c$$
$$= \left[\, t^2+1 = (x+2)^2+1 = x^2+4x+5 \,\right] = \frac{1}{2}\operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 082

$$\int \frac{dx}{(x^2-4x+5)^2} = \frac{1}{2}\operatorname{arctg}(x-2) + \frac{x-2}{2(x^2-4x+5)} + c$$ [082]

- $$= \int \frac{dx}{(x^2-4x+4+1)^2} = \int \frac{dx}{((x-2)^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t=x-2 & x\in R \\ dt=dx & t\in R \end{array}\right] = \int \frac{1}{(t^2+1)^2}\,dt = \int \frac{t^2+1-t^2}{(t^2+1)^2}\,dt$$
$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\int \frac{2t^2\,dt}{(t^2+1)^2} = \left[\begin{array}{l|l} u' = \frac{2t}{(t^2+1)^2} & u = \int \frac{2t\,dt}{(t^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } z=t^2+1 & t\in(-\infty;0\rangle \Rightarrow z\in\langle 1;\infty) \\ dz=2t\,dt & t\in\langle 0;\infty) \Rightarrow z\in\langle 1;\infty) \end{array}\right] = \int \frac{dz}{z^2} = \int z^{-2}\,dz \\ v = t & v'=1 \qquad = -z^{-1} = -\frac{1}{z} = -\frac{1}{t^2+1} \end{array}\right]$$
$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1}\right] = \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\operatorname{arctg} t + \frac{t}{2(t^2+1)} + c$$
$$= \left[\begin{array}{r} t^2+1=(x-2)^2+1 \\ =x^2-4x+5 \end{array}\right] = \frac{1}{2}\operatorname{arctg}(x-2) + \frac{x-2}{2(x^2-4x+5)} + c,\ x\in R,\ c\in R.$$

- $$= \int \frac{dx}{(x^2-4x+4+1)^2} = \int \frac{dx}{((x-2)^2+1)^2} = \left[\begin{array}{r|l} \text{Subst. } t=x-2 & x\in R \\ dt=dx & t\in R \end{array}\right] = \int \frac{1}{(t^2+1)^2}\,dt$$
$$= \left[\begin{array}{r} \text{Pr. 079: } \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)}\cdot\int \frac{dt}{(t^2+a^2)^{n-1}} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \\ \text{pre } a>0 \text{ a } n\in N,\ n\neq 1. \end{array}\right] = \frac{2\cdot 2-3}{2(2-1)}\int \frac{dt}{(t^2+1)^{2-1}} + \frac{t}{2(2-1)(t^2+1)}$$
$$= \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\operatorname{arctg} t + \frac{t}{2(t^2+1)} + c$$
$$= \left[t^2+1=(x-2)^2+1=x^2-4x+5\right] = \frac{1}{2}\operatorname{arctg}(x-2) + \frac{x-2}{2(x^2-4x+5)} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 082

$$\int\frac{dx}{(x^2\pm4x+5)^2}=\frac{1}{2}\operatorname{arctg}(x\pm2)+\frac{x\pm2}{2(x^2\pm4x+5)}+c$$ [082]

- $$=\int\frac{dx}{(x^2\pm4x+4+1)^2}=\int\frac{dx}{((x\pm2)^2+1)^2}=\left[\begin{array}{r|l}\text{Subst. } t=x\pm2 & x\in R\\ dt=dx & t\in R\end{array}\right]=\int\frac{1}{(t^2+1)^2}\,dt=\int\frac{t^2+1-t^2}{(t^2+1)^2}\,dt$$
$$=\int\frac{dt}{t^2+1}-\frac{1}{2}\int\frac{2t^2\,dt}{(t^2+1)^2}=\left[\begin{array}{l|l}u'=\frac{2t}{(t^2+1)^2} & u=\int\frac{2t\,dt}{(t^2+1)^2}=\left[\begin{array}{r|l}\text{Subst. } z=t^2+1 & t\in(-\infty;0)\Rightarrow z\in\langle1;\infty)\\ dz=2t\,dt & t\in\langle0;\infty)\Rightarrow z\in\langle1;\infty)\end{array}\right]=\int\frac{dz}{z^2}=\int z^{-2}\,dz\\ v=t & v'=1 \qquad =-z^{-1}=-\frac{1}{z}=-\frac{1}{t^2+1}\end{array}\right]$$
$$=\int\frac{dt}{t^2+1}-\frac{1}{2}\left[-\frac{t}{t^2+1}+\int\frac{dt}{t^2+1}\right]=\frac{1}{2}\int\frac{dt}{t^2+1}+\frac{t}{2(t^2+1)}=\frac{1}{2}\operatorname{arctg}t+\frac{t}{2(t^2+1)}+c$$
$$=\left[\begin{array}{r}t^2+1=(x\pm2)^2+1\\ =x^2\pm4x+5\end{array}\right]=\frac{1}{2}\operatorname{arctg}(x\pm2)+\frac{x\pm2}{2(x^2\pm4x+5)}+c,\ x\in R,\ c\in R.$$

- $$=\int\frac{dx}{(x^2\pm4x+4+1)^2}=\int\frac{dx}{((x\pm2)^2+1)^2}=\left[\begin{array}{r|l}\text{Subst. } t=x\pm2 & x\in R\\ dt=dx & t\in R\end{array}\right]=\int\frac{1}{(t^2+1)^2}\,dt$$
$$=\left[\begin{array}{r}\text{Pr. 079: }\int\frac{dt}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}\cdot\int\frac{dt}{(t^2+a^2)^{n-1}}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\\ \text{pre } a>0 \text{ a } n\in N,\ n\neq1.\end{array}\right]=\frac{2\cdot2-3}{2(2-1)}\int\frac{dt}{(t^2+1)^{2-1}}+\frac{t}{2(2-1)(t^2+1)}$$
$$=\frac{1}{2}\int\frac{dt}{t^2+1}+\frac{t}{2(t^2+1)}=\frac{1}{2}\operatorname{arctg}t+\frac{t}{2(t^2+1)}+c$$
$$=\left[t^2+1=(x\pm2)^2+1=x^2\pm4x+5\right]=\frac{1}{2}\operatorname{arctg}(x\pm2)+\frac{x\pm2}{2(x^2\pm4x+5)}+c,\ x\in R,\ c\in R.$$

Riešené príklady – 083, 084

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$, • $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} \cdot I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 078, Pr 079.]

$I_3 = \int \frac{dx}{(x^2+a^2)^3}$ pre $a>0$ a $n=3$. [083]

$I_4 = \int \frac{dx}{(x^2+a^2)^4}$ pre $a>0$ a $n=4$. [084]

Riešené príklady – 083, 084

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} \cdot I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 078, Pr 079.]

$I_3 = \int \frac{dx}{(x^2+a^2)^3}$ pre $a>0$ a $n=3$. [083]

- $= \frac{2\cdot 3-3}{2a^2(3-1)} \cdot I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^{3-1}}$

$I_4 = \int \frac{dx}{(x^2+a^2)^4}$ pre $a>0$ a $n=4$. [084]

- $= \frac{2\cdot 4-3}{2a^2(4-1)} \cdot I_3 + \frac{x}{2a^2(4-1)(x^2+a^2)^{4-1}}$

Riešené príklady – 083, 084

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a}\,\text{arctg}\,\frac{x}{a} + c$, • $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} \cdot I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 078, Pr 079.]

$I_3 = \int \frac{dx}{(x^2+a^2)^3}$ pre $a>0$ a $n=3$. [083]

- $= \frac{2\cdot3-3}{2a^2(3-1)} \cdot I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^{3-1}} = \frac{3}{4a^2} \cdot \left[\frac{2\cdot2-3}{2a^2(2-1)} \cdot I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)^{2-1}}\right] + \frac{x}{4a^2(x^2+a^2)^2}$

$I_4 = \int \frac{dx}{(x^2+a^2)^4}$ pre $a>0$ a $n=4$. [084]

- $= \frac{2\cdot4-3}{2a^2(4-1)} \cdot I_3 + \frac{x}{2a^2(4-1)(x^2+a^2)^{4-1}} = \frac{5}{6a^2} \cdot \left[\frac{2\cdot3-3}{2a^2(3-1)} \cdot I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^{3-1}}\right] + \frac{x}{6a^2(x^2+a^2)^3}$

Riešené príklady – 083, 084

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} \cdot I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 078, Pr 079.]

$I_3 = \int \frac{dx}{(x^2+a^2)^3}$ pre $a>0$ a $n=3$. [083]

- $$= \frac{2\cdot 3-3}{2a^2(3-1)} \cdot I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^{3-1}} = \frac{3}{4a^2} \cdot \left[\frac{2\cdot 2-3}{2a^2(2-1)} \cdot I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)^{2-1}} \right] + \frac{x}{4a^2(x^2+a^2)^2}$$
  $$= \frac{3}{4a^2} \cdot \left[\frac{1}{2a^2} \cdot I_1 + \frac{x}{2a^2(x^2+a^2)} \right] + \frac{x}{4a^2(x^2+a^2)^2} = \frac{3}{8a^4} \cdot I_1 + \frac{3x}{8a^4(x^2+a^2)} + \frac{x}{4a^2(x^2+a^2)^2}$$

$I_4 = \int \frac{dx}{(x^2+a^2)^4}$ pre $a>0$ a $n=4$. [084]

- $$= \frac{2\cdot 4-3}{2a^2(4-1)} \cdot I_3 + \frac{x}{2a^2(4-1)(x^2+a^2)^{4-1}} = \frac{5}{6a^2} \cdot \left[\frac{2\cdot 3-3}{2a^2(3-1)} \cdot I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^{3-1}} \right] + \frac{x}{6a^2(x^2+a^2)^3}$$
  $$= \frac{5}{6a^2} \cdot \left[\frac{3}{4a^2} \cdot I_2 + \frac{x}{4a^2(x^2+a^2)^2} \right] + \frac{x}{6a^2(x^2+a^2)^3} = \frac{5}{8a^4} \cdot I_2 + \frac{5x}{24a^4(x^2+a^2)^4} + \frac{x}{6a^2(x^2+a^2)^3}$$

Riešené príklady – 083, 084

- $I_1=\int\frac{dx}{x^2+a^2}=\frac{1}{a}\operatorname{arctg}\frac{x}{a}+c$, • $I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}\cdot I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a>0$ a $n\in N$, $n\neq 1$.

[Viď Pr 078, Pr 079.]

$$I_3=\int\frac{dx}{(x^2+a^2)^3}=\frac{3\operatorname{arctg}\frac{x}{a}}{8a^5}+\frac{3x}{8a^4(x^2+a^2)}+\frac{x}{4a^2(x^2+a^2)^2}+c \text{ pre } a>0 \text{ a } n=3.$$ [083]

- $$=\frac{2\cdot 3-3}{2a^2(3-1)}\cdot I_2+\frac{x}{2a^2(3-1)(x^2+a^2)^{3-1}}=\frac{3}{4a^2}\cdot\left[\frac{2\cdot 2-3}{2a^2(2-1)}\cdot I_1+\frac{x}{2a^2(2-1)(x^2+a^2)^{2-1}}\right]+\frac{x}{4a^2(x^2+a^2)^2}$$
$$=\frac{3}{4a^2}\cdot\left[\frac{1}{2a^2}\cdot I_1+\frac{x}{2a^2(x^2+a^2)}\right]+\frac{x}{4a^2(x^2+a^2)^2}=\frac{3}{8a^4}\cdot I_1+\frac{3x}{8a^4(x^2+a^2)}+\frac{x}{4a^2(x^2+a^2)^2}$$
$$=\frac{3}{8a^5}\operatorname{arctg}\frac{x}{a}+\frac{3x}{8a^4(x^2+a^2)}+\frac{x}{4a^2(x^2+a^2)^2}+c,\ x\in R,\ c\in R.$$

$$I_4=\int\frac{dx}{(x^2+a^2)^4} \text{ pre } a>0 \text{ a } n=4.$$ [084]

- $$=\frac{2\cdot 4-3}{2a^2(4-1)}\cdot I_3+\frac{x}{2a^2(4-1)(x^2+a^2)^{4-1}}=\frac{5}{6a^2}\cdot\left[\frac{2\cdot 3-3}{2a^2(3-1)}\cdot I_2+\frac{x}{2a^2(3-1)(x^2+a^2)^{3-1}}\right]+\frac{x}{6a^2(x^2+a^2)^3}$$
$$=\frac{5}{6a^2}\cdot\left[\frac{3}{4a^2}\cdot I_2+\frac{x}{4a^2(x^2+a^2)^2}\right]+\frac{x}{6a^2(x^2+a^2)^3}=\frac{5}{8a^4}\cdot I_2+\frac{5x}{24a^4(x^2+a^2)^4}+\frac{x}{6a^2(x^2+a^2)^3}$$
$$=\frac{5}{8a^4}\cdot\left[\frac{2\cdot 2-3}{2a^2(2-1)}\cdot I_1+\frac{x}{2a^2(2-1)(x^2+a^2)^{2-1}}\right]+\frac{5x}{24a^4(x^2+a^2)^4}+\frac{x}{6a^2(x^2+a^2)^3}$$

Riešené príklady – 083, 084

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + c$, • $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} \cdot I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a>0$ a $n\in N$, $n\neq 1$.

[Viď Pr 078, Pr 079.]

$$I_3 = \int \frac{dx}{(x^2+a^2)^3} = \frac{3\operatorname{arctg}\frac{x}{a}}{8a^5} + \frac{3x}{8a^4(x^2+a^2)} + \frac{x}{4a^2(x^2+a^2)^2} + c \text{ pre } a>0 \text{ a } n=3.$$ [083]

- $$= \frac{2\cdot3-3}{2a^2(3-1)}\cdot I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^{3-1}} = \frac{3}{4a^2}\cdot\left[\frac{2\cdot2-3}{2a^2(2-1)}\cdot I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)^{2-1}}\right] + \frac{x}{4a^2(x^2+a^2)^2}$$
$$= \frac{3}{4a^2}\cdot\left[\frac{1}{2a^2}\cdot I_1 + \frac{x}{2a^2(x^2+a^2)}\right] + \frac{x}{4a^2(x^2+a^2)^2} = \frac{3}{8a^4}\cdot I_1 + \frac{3x}{8a^4(x^2+a^2)} + \frac{x}{4a^2(x^2+a^2)^2}$$
$$= \frac{3}{8a^5}\operatorname{arctg}\frac{x}{a} + \frac{3x}{8a^4(x^2+a^2)} + \frac{x}{4a^2(x^2+a^2)^2} + c,\ x\in R,\ c\in R.$$

$$I_4 = \int \frac{dx}{(x^2+a^2)^4} = \frac{5\operatorname{arctg}\frac{x}{a}}{16a^7} + \frac{5x}{16a^6(x^2+a^2)} + \frac{5x}{24a^4(x^2+a^2)^2} + \frac{x}{6a^2(x^2+a^2)^3} + c \text{ pre } a>0 \text{ a } n=4.$$ [084]

- $$= \frac{2\cdot4-3}{2a^2(4-1)}\cdot I_3 + \frac{x}{2a^2(4-1)(x^2+a^2)^{4-1}} = \frac{5}{6a^2}\cdot\left[\frac{2\cdot3-3}{2a^2(3-1)}\cdot I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^{3-1}}\right] + \frac{x}{6a^2(x^2+a^2)^3}$$
$$= \frac{5}{6a^2}\cdot\left[\frac{3}{4a^2}\cdot I_2 + \frac{x}{4a^2(x^2+a^2)^2}\right] + \frac{x}{6a^2(x^2+a^2)^3} = \frac{5}{8a^4}\cdot I_2 + \frac{5x}{24a^4(x^2+a^2)^4} + \frac{x}{6a^2(x^2+a^2)^3}$$
$$= \frac{5}{8a^4}\cdot\left[\frac{2\cdot2-3}{2a^2(2-1)}\cdot I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)^{2-1}}\right] + \frac{5x}{24a^4(x^2+a^2)^4} + \frac{x}{6a^2(x^2+a^2)^3}$$
$$= \frac{5}{16a^7}\operatorname{arctg}\frac{x}{a} + \frac{5x}{16a^6(x^2+a^2)} + \frac{5x}{24a^4(x^2+a^2)^2} + \frac{x}{6a^2(x^2+a^2)^3} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 085

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2+3} = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + c$,
- $I_n = \int \frac{dx}{(x^2+3)^n} = \frac{2n-3}{6(n-1)} I_{n-1} + \frac{x}{6(n-1)(x^2+3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 078, Pr 079.]

$I_6 = \int \frac{dx}{(x^2+3)^6}$ pre $n=6$ [085]

Riešené príklady – 085

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2+3} = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + c$,
- $I_n = \int \frac{dx}{(x^2+3)^n} = \frac{2n-3}{6(n-1)} I_{n-1} + \frac{x}{6(n-1)(x^2+3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 078, Pr 079.]

$I_6 = \int \frac{dx}{(x^2+3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2+3$. [085]

Riešené príklady – 085

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2+3} = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + c$,
- $I_n = \int \frac{dx}{(x^2+3)^n} = \frac{2n-3}{6(n-1)} I_{n-1} + \frac{x}{6(n-1)(x^2+3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 078, Pr 079.]

$I_6 = \int \frac{dx}{(x^2+3)^6} = \int \frac{dx}{\alpha}^{6}$ pre $n=6$ a označenie $\alpha = x^2+3$. [085]

- $= \frac{9}{6\cdot 5} I_5 + \frac{x}{6\cdot 5\alpha^5}$

$= \frac{9}{6\cdot 5} \Big[I_5 \qquad \Big] + \frac{x}{6\cdot 5\alpha^5}$

Riešené príklady – 085

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c,$
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N,\ n \neq 1.$
- $I_1 = \int \frac{dx}{x^2+3} = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + c,$
- $I_n = \int \frac{dx}{(x^2+3)^n} = \frac{2n-3}{6(n-1)} I_{n-1} + \frac{x}{6(n-1)(x^2+3)^{n-1}}$ pre $n \in N,\ n \neq 1.$ [Viď Pr 078, Pr 079.]

$I_6 = \int \frac{dx}{(x^2+3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2+3.$ [085]

- $= \frac{9}{6{\cdot}5} I_5 + \frac{x}{6{\cdot}5\alpha^5} = \frac{9}{6{\cdot}5} \left[\frac{7}{6{\cdot}4} I_4 + \frac{x}{6{\cdot}4\alpha^4} \right] + \frac{x}{6{\cdot}5\alpha^5}$

$= \frac{9}{6{\cdot}5} \left[\frac{7}{6{\cdot}4} \left[I_4 \quad \right] + \frac{x}{6{\cdot}4\alpha^4} \right] + \frac{x}{6{\cdot}5\alpha^5}$

Riešené príklady – 085

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2+3} = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + c$,
- $I_n = \int \frac{dx}{(x^2+3)^n} = \frac{2n-3}{6(n-1)} I_{n-1} + \frac{x}{6(n-1)(x^2+3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 078, Pr 079.]

$I_6 = \int \frac{dx}{(x^2+3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2+3$. [085]

- $= \frac{9}{6{\cdot}5} I_5 + \frac{x}{6{\cdot}5\alpha^5} = \frac{9}{6{\cdot}5}\left[\frac{7}{6{\cdot}4} I_4 + \frac{x}{6{\cdot}4\alpha^4}\right] + \frac{x}{6{\cdot}5\alpha^5}$

$= \frac{9}{6{\cdot}5}\left[\frac{7}{6{\cdot}4}\left[\frac{5}{6{\cdot}3} I_3 + \frac{x}{6{\cdot}3\alpha^3}\right] + \frac{x}{6{\cdot}4\alpha^4}\right] + \frac{x}{6{\cdot}5\alpha^5}$

$= \frac{9}{6{\cdot}5}\left[\frac{7}{6{\cdot}4}\left[\frac{5}{6{\cdot}3}\left[I_3 \qquad\qquad\qquad \right] + \frac{x}{6{\cdot}3\alpha^3}\right] + \frac{x}{6{\cdot}4\alpha^4}\right] + \frac{x}{6{\cdot}5\alpha^5}$

Riešené príklady – 085

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2+3} = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + c$,
- $I_n = \int \frac{dx}{(x^2+3)^n} = \frac{2n-3}{6(n-1)} I_{n-1} + \frac{x}{6(n-1)(x^2+3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 078, Pr 079.]

$I_6 = \int \frac{dx}{(x^2+3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2+3$. [085]

- $= \frac{9}{6\cdot5} I_5 + \frac{x}{6\cdot5\alpha^5} = \frac{9}{6\cdot5}\left[\frac{7}{6\cdot4} I_4 + \frac{x}{6\cdot4\alpha^4}\right] + \frac{x}{6\cdot5\alpha^5}$

$= \frac{9}{6\cdot5}\left[\frac{7}{6\cdot4}\left[\frac{5}{6\cdot3} I_3 + \frac{x}{6\cdot3\alpha^3}\right] + \frac{x}{6\cdot4\alpha^4}\right] + \frac{x}{6\cdot5\alpha^5}$

$= \frac{9}{6\cdot5}\left[\frac{7}{6\cdot4}\left[\frac{5}{6\cdot3}\left[\frac{3}{6\cdot2} I_2 + \frac{x}{6\cdot2\alpha^2}\right] + \frac{x}{6\cdot3\alpha^3}\right] + \frac{x}{6\cdot4\alpha^4}\right] + \frac{x}{6\cdot5\alpha^5}$

$= \frac{9}{6\cdot5}\left[\frac{7}{6\cdot4}\left[\frac{5}{6\cdot3}\left[\frac{3}{6\cdot2}\left[I_2 \qquad \right] + \frac{x}{6\cdot2\alpha^2}\right] + \frac{x}{6\cdot3\alpha^3}\right] + \frac{x}{6\cdot4\alpha^4}\right] + \frac{x}{6\cdot5\alpha^5}$

Riešené príklady – 085

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2+3} = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + c$,
- $I_n = \int \frac{dx}{(x^2+3)^n} = \frac{2n-3}{6(n-1)} I_{n-1} + \frac{x}{6(n-1)(x^2+3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 078, Pr 079.]

$I_6 = \int \frac{dx}{(x^2+3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2+3$. [085]

- $$= \frac{9}{6\cdot 5} I_5 + \frac{x}{6\cdot 5\alpha^5} = \frac{9}{6\cdot 5}\left[\frac{7}{6\cdot 4} I_4 + \frac{x}{6\cdot 4\alpha^4}\right] + \frac{x}{6\cdot 5\alpha^5}$$
$$= \frac{9}{6\cdot 5}\left[\frac{7}{6\cdot 4}\left[\frac{5}{6\cdot 3} I_3 + \frac{x}{6\cdot 3\alpha^3}\right] + \frac{x}{6\cdot 4\alpha^4}\right] + \frac{x}{6\cdot 5\alpha^5}$$
$$= \frac{9}{6\cdot 5}\left[\frac{7}{6\cdot 4}\left[\frac{5}{6\cdot 3}\left[\frac{3}{6\cdot 2} I_2 + \frac{x}{6\cdot 2\alpha^2}\right] + \frac{x}{6\cdot 3\alpha^3}\right] + \frac{x}{6\cdot 4\alpha^4}\right] + \frac{x}{6\cdot 5\alpha^5}$$
$$= \frac{9}{6\cdot 5}\left[\frac{7}{6\cdot 4}\left[\frac{5}{6\cdot 3}\left[\frac{3}{6\cdot 2}\left[\frac{1}{6\cdot 1} I_1 + \frac{x}{6\cdot 1\alpha}\right] + \frac{x}{6\cdot 2\alpha^2}\right] + \frac{x}{6\cdot 3\alpha^3}\right] + \frac{x}{6\cdot 4\alpha^4}\right] + \frac{x}{6\cdot 5\alpha^5}$$

$$= \frac{9}{6\cdot 5}\left[\frac{7}{6\cdot 4}\left[\frac{5}{6\cdot 3}\left[\frac{3}{6\cdot 2}\left[\frac{1}{6\cdot 1} I_1 + \frac{x}{6\cdot 1\alpha}\right] + \frac{x}{6\cdot 2\alpha^2}\right] + \frac{x}{6\cdot 3\alpha^3}\right] + \frac{x}{6\cdot 4\alpha^4}\right] + \frac{x}{6\cdot 5\alpha^5}$$

Riešené príklady – 085

- $I_1=\int\frac{dx}{x^2+a^2}=\frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a}+c,$
- $I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a>0$ a $n\in N,\ n\neq 1.$
- $I_1=\int\frac{dx}{x^2+3}=\frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{x}{\sqrt{3}}+c,$
- $I_n=\int\frac{dx}{(x^2+3)^n}=\frac{2n-3}{6(n-1)}I_{n-1}+\frac{x}{6(n-1)(x^2+3)^{n-1}}$ pre $n\in N,\ n\neq 1.$ [Viď Pr 078, Pr 079.]

$I_6=\int\frac{dx}{(x^2+3)^6}=\int\frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha=x^2+3.$ [085]

- $=\frac{9}{6\cdot5}I_5+\frac{x}{6\cdot5\alpha^5}=\frac{9}{6\cdot5}\left[\frac{7}{6\cdot4}I_4+\frac{x}{6\cdot4\alpha^4}\right]+\frac{x}{6\cdot5\alpha^5}$

$$=\frac{9}{6\cdot5}\left[\frac{7}{6\cdot4}\left[\frac{5}{6\cdot3}I_3+\frac{x}{6\cdot3\alpha^3}\right]+\frac{x}{6\cdot4\alpha^4}\right]+\frac{x}{6\cdot5\alpha^5}$$

$$=\frac{9}{6\cdot5}\left[\frac{7}{6\cdot4}\left[\frac{5}{6\cdot3}\left[\frac{3}{6\cdot2}I_2+\frac{x}{6\cdot2\alpha^2}\right]+\frac{x}{6\cdot3\alpha^3}\right]+\frac{x}{6\cdot4\alpha^4}\right]+\frac{x}{6\cdot5\alpha^5}$$

$$=\frac{9}{6\cdot5}\left[\frac{7}{6\cdot4}\left[\frac{5}{6\cdot3}\left[\frac{3}{6\cdot2}\left[\frac{1}{6\cdot1}I_1+\frac{x}{6\cdot1\alpha}\right]+\frac{x}{6\cdot2\alpha^2}\right]+\frac{x}{6\cdot3\alpha^3}\right]+\frac{x}{6\cdot4\alpha^4}\right]+\frac{x}{6\cdot5\alpha^5}$$

$$=\frac{9}{6\cdot5}\left[\frac{7}{6\cdot4}\left[\frac{5}{6\cdot3}\left[\frac{3}{6\cdot2}\left[\frac{1}{6\cdot1}I_1+\frac{x}{6\cdot1\alpha}\right]+\frac{x}{6\cdot2\alpha^2}\right]+\frac{x}{6\cdot3\alpha^3}\right]+\frac{x}{6\cdot4\alpha^4}\right]+\frac{x}{6\cdot5\alpha^5}$$

$$=\frac{9\cdot7\cdot5\cdot3\cdot1}{6^5\cdot5\cdot4\cdot3\cdot2\cdot1\cdot\sqrt{3}}\,\mathrm{arctg}\,\frac{x}{\sqrt{3}}+\frac{9\cdot7\cdot5\cdot3x}{6^5\cdot5\cdot4\cdot3\cdot2\cdot1\alpha}+\frac{9\cdot7\cdot5x}{6^4\cdot5\cdot4\cdot3\cdot2\alpha^2}+\frac{9\cdot7x}{6^3\cdot5\cdot4\cdot3\alpha^3}+\frac{9x}{6^2\cdot5\cdot4\alpha^4}+\frac{x}{6\cdot5\alpha^5}+c$$

Riešené príklady – 085

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2+3} = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + c$,
- $I_n = \int \frac{dx}{(x^2+3)^n} = \frac{2n-3}{6(n-1)} I_{n-1} + \frac{x}{6(n-1)(x^2+3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 078, Pr 079.]

$I_6 = \int \frac{dx}{(x^2+3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2+3$. [085]

- $$= \frac{9}{6{\cdot}5} I_5 + \frac{x}{6{\cdot}5\alpha^5} = \frac{9}{6{\cdot}5}\left[\frac{7}{6{\cdot}4} I_4 + \frac{x}{6{\cdot}4\alpha^4}\right] + \frac{x}{6{\cdot}5\alpha^5}$$

$$= \frac{9}{6{\cdot}5}\left[\frac{7}{6{\cdot}4}\left[\frac{5}{6{\cdot}3} I_3 + \frac{x}{6{\cdot}3\alpha^3}\right] + \frac{x}{6{\cdot}4\alpha^4}\right] + \frac{x}{6{\cdot}5\alpha^5}$$

$$= \frac{9}{6{\cdot}5}\left[\frac{7}{6{\cdot}4}\left[\frac{5}{6{\cdot}3}\left[\frac{3}{6{\cdot}2} I_2 + \frac{x}{6{\cdot}2\alpha^2}\right] + \frac{x}{6{\cdot}3\alpha^3}\right] + \frac{x}{6{\cdot}4\alpha^4}\right] + \frac{x}{6{\cdot}5\alpha^5}$$

$$= \frac{9}{6{\cdot}5}\left[\frac{7}{6{\cdot}4}\left[\frac{5}{6{\cdot}3}\left[\frac{3}{6{\cdot}2}\left[\frac{1}{6{\cdot}1} I_1 + \frac{x}{6{\cdot}1\alpha}\right] + \frac{x}{6{\cdot}2\alpha^2}\right] + \frac{x}{6{\cdot}3\alpha^3}\right] + \frac{x}{6{\cdot}4\alpha^4}\right] + \frac{x}{6{\cdot}5\alpha^5}$$

$$= \frac{9}{6{\cdot}5}\left[\frac{7}{6{\cdot}4}\left[\frac{5}{6{\cdot}3}\left[\frac{3}{6{\cdot}2}\left[\frac{1}{6{\cdot}1} I_1 + \frac{x}{6{\cdot}1\alpha}\right] + \frac{x}{6{\cdot}2\alpha^2}\right] + \frac{x}{6{\cdot}3\alpha^3}\right] + \frac{x}{6{\cdot}4\alpha^4}\right] + \frac{x}{6{\cdot}5\alpha^5}$$

$$= \frac{9{\cdot}7{\cdot}5{\cdot}3{\cdot}1}{6^5{\cdot}5{\cdot}4{\cdot}3{\cdot}2{\cdot}1{\cdot}\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + \frac{9{\cdot}7{\cdot}5{\cdot}3x}{6^5{\cdot}5{\cdot}4{\cdot}3{\cdot}2{\cdot}1\alpha} + \frac{9{\cdot}7{\cdot}5x}{6^4{\cdot}5{\cdot}4{\cdot}3{\cdot}2\alpha^2} + \frac{9{\cdot}7x}{6^3{\cdot}5{\cdot}4{\cdot}3\alpha^3} + \frac{9x}{6^2{\cdot}5{\cdot}4\alpha^4} + \frac{x}{6{\cdot}5\alpha^5} + c$$

$$= \frac{9!!}{6^5{\cdot}5!{\cdot}\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + \frac{9!!}{6^5{\cdot}5!}\left[\frac{x}{x^2+3} + \frac{6{\cdot}1!x}{3!!(x^2+3)^2} + \frac{6^2{\cdot}2!x}{5!!(x^2+3)^3} + \frac{6^3{\cdot}3!x}{7!!(x^2+3)^4} + \frac{6^4{\cdot}4!x}{9!!(x^2+3)^5}\right] + c,$$

[$n!! = n{\cdot}(n-2){\cdot}(n-4)\cdots$, $n \in N$, napr. $9!! = 9{\cdot}7{\cdot}5{\cdot}3{\cdot}1$, $8!! = 8{\cdot}6{\cdot}4{\cdot}2$, $7!! = 7{\cdot}5{\cdot}3{\cdot}1$, $6!! = 6{\cdot}4{\cdot}2$, $\ldots$] $x \in R$, $c \in R$.

Riešené príklady – 086, 087, 088

$$\int \frac{x^2}{x^2-a^2}\,dx$$ pre $a > 0$. [086]

$$\int \frac{x^2\,dx}{(x^2-a^2)^2}$$ pre $a > 0$. [087]

$$\int \frac{x^2\,dx}{(x^2-3)^2}$$ [088]

Riešené príklady – 086, 087, 088

$$\int \frac{x^2}{x^2-a^2}\,dx = \int \frac{x^2-a^2+a^2}{x^2-a^2}\,dx$$ pre $a > 0.$ [086]

$$\int \frac{x^2\,dx}{(x^2-a^2)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2-a^2)^2}\,dx$$ pre $a > 0.$ [087]

$$\int \frac{x^2\,dx}{(x^2-3)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2-3)^2}\,dx$$ [088]

Riešené príklady – 086, 087, 088

$$\int \frac{x^2}{x^2-a^2}\,dx = \int \frac{x^2-a^2+a^2}{x^2-a^2}\,dx \quad \text{pre } a>0. \qquad [086]$$

- $= \int \left[1 + \frac{a^2}{x^2-a^2}\right] dx$

$$\int \frac{x^2\,dx}{(x^2-a^2)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2-a^2)^2}\,dx \quad \text{pre } a>0. \qquad [087]$$

- $= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2-a^2)^2} & u = \int \frac{2x\,dx}{(x^2-a^2)^2} = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2-a^2 & x\in\langle 0;a) \Rightarrow t\in\langle -a^2;0) & x\in(a;\infty) \Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-a;0\rangle \Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a) \Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t^2} \\ v = x & v' = 1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2-a^2} \end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2-a^2} + \int \frac{dx}{x^2-a^2}\right]$

$$\int \frac{x^2\,dx}{(x^2-3)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2-3)^2}\,dx \qquad [088]$$

- $= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2-3)^2} & u = \int \frac{2x\,dx}{(x^2-3)^2} = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2-3 & x\in\langle 0;\sqrt{3}) \Rightarrow t\in\langle -3;0) & x\in(\sqrt{3};\infty) \Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-\sqrt{3};0\rangle \Rightarrow t\in\langle -3;0) & x\in(-\infty;-\sqrt{3}) \Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t^2} \\ v = x & v' = 1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2-3} \end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2-3} + \int \frac{dx}{x^2-3}\right]$

Riešené príklady – 086, 087, 088

$$\int \frac{x^2}{x^2-a^2}\,dx = \int \frac{x^2-a^2+a^2}{x^2-a^2}\,dx \quad \text{pre } a>0. \qquad [086]$$

- $= \int \left[1+\frac{a^2}{x^2-a^2}\right]dx = x+\frac{a^2}{2a}\ln\left|\frac{x-a}{x+a}\right|+c$

$$\int \frac{x^2\,dx}{(x^2-a^2)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2-a^2)^2}\,dx \quad \text{pre } a>0. \qquad [087]$$

- $= \left[\begin{array}{l|l} u'=\frac{2x}{(x^2-a^2)^2} & u=\int\frac{2x\,dx}{(x^2-a^2)^2} = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;a)\Rightarrow t\in\langle -a^2;0) & x\in(a;\infty)\Rightarrow t\in(0;\infty) \\ dt=2x\,dx & x\in(-a;0\rangle\Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a)\Rightarrow t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t^2} \\ v=x & v'=1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2-a^2}\end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2-a^2}+\int\frac{dx}{x^2-a^2}\right]$

  $= -\frac{x}{2(x^2-a^2)}+\frac{1}{2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c$

$$\int \frac{x^2\,dx}{(x^2-3)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2-3)^2}\,dx \qquad [088]$$

- $= \left[\begin{array}{l|l} u'=\frac{2x}{(x^2-3)^2} & u=\int\frac{2x\,dx}{(x^2-3)^2} = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-3 & x\in\langle 0;\sqrt{3})\Rightarrow t\in\langle -3;0) & x\in(\sqrt{3};\infty)\Rightarrow t\in(0;\infty) \\ dt=2x\,dx & x\in(-\sqrt{3};0\rangle\Rightarrow t\in\langle -3;0) & x\in(-\infty;-\sqrt{3})\Rightarrow t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t^2} \\ v=x & v'=1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2-3}\end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2-3}+\int\frac{dx}{x^2-3}\right]$

  $= -\frac{x}{2(x^2-3)}+\frac{1}{2}\cdot\frac{1}{2\sqrt{3}}\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right|+c$

Riešené príklady – 086, 087, 088

$$\int \frac{x^2}{x^2-a^2}\,dx = \int \frac{x^2-a^2+a^2}{x^2-a^2}\,dx = x + \frac{a}{2}\ln\left|\frac{x-a}{x+a}\right| + c \text{ pre } a>0.$$ [086]

- $= \int\left[1+\frac{a^2}{x^2-a^2}\right]dx = x + \frac{a^2}{2a}\ln\left|\frac{x-a}{x+a}\right| + c = x + \frac{a}{2}\ln\left|\frac{x-a}{x+a}\right| + c,\ x\in R,\ x\neq\pm a,\ c\in R.$

$$\int \frac{x^2\,dx}{(x^2-a^2)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2-a^2)^2}\,dx = \frac{1}{4a}\ln\left|\frac{x-a}{x+a}\right| - \frac{x}{2(x^2-a^2)} + c \text{ pre } a>0.$$ [087]

- $= \left[\begin{array}{l|l} u'=\frac{2x}{(x^2-a^2)^2} & u=\int\frac{2x\,dx}{(x^2-a^2)^2} = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-a^2 & x\in\langle 0;a)\Rightarrow t\in\langle -a^2;0) & x\in(a;\infty)\Rightarrow t\in(0;\infty) \\ dt=2x\,dx & x\in(-a;0\rangle\Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a)\Rightarrow t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t^2} \\ v=x & v'=1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2-a^2}\end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2-a^2} + \int\frac{dx}{x^2-a^2}\right]$

  $= -\frac{x}{2(x^2-a^2)} + \frac{1}{2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c = \frac{1}{4a}\ln\left|\frac{x-a}{x+a}\right| - \frac{x}{2(x^2-a^2)} + c,\ x\in R,\ x\neq\pm a,\ c\in R.$

$$\int \frac{x^2\,dx}{(x^2-3)^2} = \frac{1}{2}\int \frac{x\cdot 2x}{(x^2-3)^2}\,dx = \frac{1}{4\sqrt{3}}\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| - \frac{x}{2(x^2-3)} + c$$ [088]

- $= \left[\begin{array}{l|l} u'=\frac{2x}{(x^2-3)^2} & u=\int\frac{2x\,dx}{(x^2-3)^2} = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2-3 & x\in\langle 0;\sqrt{3})\Rightarrow t\in\langle -3;0) & x\in(\sqrt{3};\infty)\Rightarrow t\in(0;\infty) \\ dt=2x\,dx & x\in(-\sqrt{3};0\rangle\Rightarrow t\in\langle -3;0) & x\in(-\infty;-\sqrt{3})\Rightarrow t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t^2} \\ v=x & v'=1 \qquad = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2-3}\end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2-3} + \int\frac{dx}{x^2-3}\right]$

  $= -\frac{x}{2(x^2-3)} + \frac{1}{2}\cdot\frac{1}{2\sqrt{3}}\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + c = \frac{1}{4\sqrt{3}}\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| - \frac{x}{2(x^2-3)} + c,$

  $x\in R,\ x\neq\pm\sqrt{3},\ c\in R.$

Riešené príklady – 089, 090

$I_1 = \int \frac{dx}{x^2 - a^2}$ pre $a > 0$ a $n = 1$. [089]

$I_n = \int \frac{dx}{(x^2 - a^2)^n}$ pre $a > 0$ a $n \in N$, $n \neq 1$. [090]

Riešené príklady – 089, 090

$I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c,\ x\in R,\ x\neq \pm a,\ c\in R$ pre $a>0$ a $n=1$. [089]

$I_n = \int \frac{dx}{(x^2-a^2)^n}$ pre $a>0$ a $n\in N,\ n\neq 1$. [090]

- $= -\frac{1}{a^2} \int \frac{-a^2}{(x^2-a^2)^n}\, dx$

Riešené príklady – 089, 090

$I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c,\ x \in R,\ x \neq \pm a,\ c \in R$ pre $a > 0$ a $n = 1.$ [089]

$I_n = \int \frac{dx}{(x^2-a^2)^n}$ pre $a > 0$ a $n \in N,\ n \neq 1.$ [090]

- $= -\frac{1}{a^2} \int \frac{-a^2}{(x^2-a^2)^n}\, dx = -\frac{1}{a^2} \int \frac{(x^2-a^2)-x^2}{(x^2-a^2)^n}\, dx = -\frac{1}{a^2} \int \frac{x^2-a^2}{(x^2-a^2)^n}\, dx + \frac{1}{a^2} \int \frac{x^2}{(x^2-a^2)^n}\, dx$

Riešené príklady – 089, 090

$I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c,\ x \in R,\ x \neq \pm a,\ c \in R$ pre $a > 0$ a $n = 1.$ [089]

$I_n = \int \frac{dx}{(x^2-a^2)^n}$ pre $a > 0$ a $n \in N,\ n \neq 1.$ [090]

- $= -\frac{1}{a^2} \int \frac{-a^2}{(x^2-a^2)^n}\, dx = -\frac{1}{a^2} \int \frac{(x^2-a^2)-x^2}{(x^2-a^2)^n}\, dx = -\frac{1}{a^2} \int \frac{x^2-a^2}{(x^2-a^2)^n}\, dx + \frac{1}{a^2} \int \frac{x^2}{(x^2-a^2)^n}\, dx$
  $= -\frac{1}{a^2} \int \frac{dx}{(x^2-a^2)^{n-1}} + \frac{1}{2a^2} \int \frac{x \cdot 2x}{(x^2-a^2)^n}\, dx = -\frac{1}{a^2} \cdot I_{n-1} + \frac{1}{2a^2} \int \frac{x \cdot 2x}{(x^2-a^2)^n}\, dx$

Riešené príklady – 089, 090

$$I_1=\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c,\ x\in R,\ x\neq\pm a,\ c\in R \text{ pre } a>0 \text{ a } n=1.$$ [089]

$$I_n=\int\frac{dx}{(x^2-a^2)^n} \quad \text{pre } a>0 \text{ a } n\in N,\ n\neq 1.$$ [090]

- $$=-\frac{1}{a^2}\int\frac{-a^2}{(x^2-a^2)^n}\,dx=-\frac{1}{a^2}\int\frac{(x^2-a^2)-x^2}{(x^2-a^2)^n}\,dx=-\frac{1}{a^2}\int\frac{x^2-a^2}{(x^2-a^2)^n}\,dx+\frac{1}{a^2}\int\frac{x^2}{(x^2-a^2)^n}\,dx$$

$$=-\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{n-1}}+\frac{1}{2a^2}\int\frac{x\cdot 2x}{(x^2-a^2)^n}\,dx=-\frac{1}{a^2}\cdot I_{n-1}+\frac{1}{2a^2}\int\frac{x\cdot 2x}{(x^2-a^2)^n}\,dx$$

$$=\left[\begin{array}{l|l} u'=\frac{2x}{(x^2-a^2)^n} & u=\int\frac{2x}{(x^2-a^2)^n}\,dx=\left[\begin{array}{r|l|l}\text{Subst. } t=x^2-a^2 & x\in\langle 0;a)\Rightarrow t\in\langle -a^2;0) & x\in(a;\infty)\Rightarrow t\in(0;\infty)\\ dt=2x\,dx & x\in(-a;0\rangle\Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a)\Rightarrow t\in(0;\infty)\end{array}\right]=\int\frac{dt}{t^n}=\int t^{-n}\,dt \\ & \qquad =\frac{t^{-n+1}}{-n+1}=\frac{1}{(1-n)t^{n-1}}=\frac{1}{(1-n)(x^2-a^2)^{n-1}} \\ v=x & v'=1\end{array}\right]$$

$$=-\frac{1}{a^2}\cdot I_{n-1}+\frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2-a^2)^{n-1}}-\frac{1}{1-n}\int\frac{dx}{(x^2-a^2)^{n-1}}\right]$$

Riešené príklady – 089, 090

$I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c,\ x\in R,\ x \neq \pm a,\ c\in R$ pre $a > 0$ a $n = 1.$ [089]

$I_n = \int \frac{dx}{(x^2-a^2)^n}$ pre $a > 0$ a $n\in N,\ n \neq 1.$ [090]

- $= -\frac{1}{a^2}\int \frac{-a^2}{(x^2-a^2)^n}\,dx = -\frac{1}{a^2}\int \frac{(x^2-a^2)-x^2}{(x^2-a^2)^n}\,dx = -\frac{1}{a^2}\int \frac{x^2-a^2}{(x^2-a^2)^n}\,dx + \frac{1}{a^2}\int \frac{x^2}{(x^2-a^2)^n}\,dx$

$= -\frac{1}{a^2}\int \frac{dx}{(x^2-a^2)^{n-1}} + \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2-a^2)^n}\,dx = -\frac{1}{a^2}\cdot I_{n-1} + \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2-a^2)^n}\,dx$

$= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2-a^2)^n} & u = \int \frac{2x}{(x^2-a^2)^n}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2-a^2 & x\in\langle 0;a)\Rightarrow t\in\langle -a^2;0) & x\in(a;\infty)\Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-a;0\rangle\Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a)\Rightarrow t\in(0;\infty) \end{array} \right] = \int\frac{dt}{t^n} = \int t^{-n}\,dt \\ & \qquad = \frac{t^{-n+1}}{-n+1} = \frac{1}{(1-n)t^{n-1}} = \frac{1}{(1-n)(x^2-a^2)^{n-1}} \\ v = x & v' = 1 \end{array} \right]$

$= -\frac{1}{a^2}\cdot I_{n-1} + \frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2-a^2)^{n-1}} - \frac{1}{1-n}\int \frac{dx}{(x^2-a^2)^{n-1}} \right]$

$= -\frac{1}{a^2}\cdot I_{n-1} + \frac{x}{2a^2(1-n)(x^2-a^2)^{n-1}} - \frac{1}{2a^2(1-n)}\cdot I_{n-1}$

$= -\frac{2(1-n)+1}{2a^2(1-n)}\cdot I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$

Riešené príklady – 089, 090

$I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$, $x\in R$, $x \neq \pm a$, $c \in R$ pre $a>0$ a $n=1$. [089]

$I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} \int \frac{dx}{(x^2-a^2)^{n-1}} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a>0$ a $n \in N$, $n \neq 1$. [090]

- $$= -\frac{1}{a^2}\int \frac{-a^2}{(x^2-a^2)^n}\,dx = -\frac{1}{a^2}\int \frac{(x^2-a^2)-x^2}{(x^2-a^2)^n}\,dx = -\frac{1}{a^2}\int \frac{x^2-a^2}{(x^2-a^2)^n}\,dx + \frac{1}{a^2}\int \frac{x^2}{(x^2-a^2)^n}\,dx$$

$$= -\frac{1}{a^2}\int \frac{dx}{(x^2-a^2)^{n-1}} + \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2-a^2)^n}\,dx = -\frac{1}{a^2}\cdot I_{n-1} + \frac{1}{2a^2}\int \frac{x\cdot 2x}{(x^2-a^2)^n}\,dx$$

$$= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2-a^2)^n} & u = \int \frac{2x}{(x^2-a^2)^n}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2-a^2 & x\in(0;a) \Rightarrow t\in\langle -a^2;0) & x\in(a;\infty) \Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-a;0\rangle \Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a) \Rightarrow t\in(0;\infty)\end{array}\right] = \int \frac{dt}{t^n} = \int t^{-n}\,dt \\ & \qquad = \frac{t^{-n+1}}{-n+1} = \frac{1}{(1-n)t^{n-1}} = \frac{1}{(1-n)(x^2-a^2)^{n-1}} \\ v = x & v' = 1 \end{array}\right]$$

$$= -\frac{1}{a^2}\cdot I_{n-1} + \frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2-a^2)^{n-1}} - \frac{1}{1-n}\int \frac{dx}{(x^2-a^2)^{n-1}}\right]$$

$$= -\frac{1}{a^2}\cdot I_{n-1} + \frac{x}{2a^2(1-n)(x^2-a^2)^{n-1}} - \frac{1}{2a^2(1-n)}\cdot I_{n-1}$$

$$= -\frac{2(1-n)+1}{2a^2(1-n)}\cdot I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}} = -\frac{3-2n}{2a^2(1-n)}\cdot I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$$

$$= \frac{3-2n}{2a^2(n-1)}\int \frac{dx}{(x^2-a^2)^{n-1}} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}},\ x\in R,\ x\neq \pm a.$$

Riešené príklady – 091, 092

$I_2 = \int \frac{dx}{(x^2-a^2)^2}$ pre $a > 0$. [091]

$\int \frac{dx}{(x^2-1)^2}$ [092]

Riešené príklady – 091, 092

$I_2 = \int \frac{dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{-a^2\,dx}{(x^2-a^2)^2}$ pre $a > 0$. [091]

- $= -\frac{1}{a^2}\int \frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2}$

$\int \frac{dx}{(x^2-1)^2}$ [092]

- $= -\int \frac{-1\,dx}{(x^2-1)^2} = -\int \frac{(x^2-1-x^2)\,dx}{(x^2-1)^2}$

Riešené príklady – 091, 092

$I_2 = \int \frac{dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{-a^2\,dx}{(x^2-a^2)^2}$ pre $a > 0$. [091]

- $= -\frac{1}{a^2}\int \frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{(x^2-a^2)\,dx}{(x^2-a^2)^2} + \frac{1}{a^2}\int \frac{x^2\,dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{dx}{x^2-a^2} + \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2-a^2)^2}$

$\int \frac{dx}{(x^2-1)^2} =$ [092]

- $= -\int \frac{-1\,dx}{(x^2-1)^2} = -\int \frac{(x^2-1-x^2)\,dx}{(x^2-1)^2} = -\int \frac{dx}{x^2-1} + \frac{1}{2}\int \frac{x\cdot 2x\,dx}{(x^2-1)^2}$

Riešené príklady – 091, 092

$$I_2 = \int \frac{dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{-a^2\,dx}{(x^2-a^2)^2} \quad \text{pre } a>0.$$ [091]

- $$= -\frac{1}{a^2}\int \frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{(x^2-a^2)\,dx}{(x^2-a^2)^2} + \frac{1}{a^2}\int \frac{x^2\,dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{dx}{x^2-a^2} + \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2-a^2)^2}$$

$$= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2-a^2)^2} & u = \int \frac{2x\,dx}{(x^2-a^2)^2} = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2-a^2 & x\in\langle 0;a) \Rightarrow t\in\langle -a^2;0) & x\in(a;\infty) \Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-a;0\rangle \Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2-a^2} \\ v = x & v' = 1 \end{array} \right]$$

$$\int \frac{dx}{(x^2-1)^2} =$$ [092]

- $$= -\int \frac{-1\,dx}{(x^2-1)^2} = -\int \frac{(x^2-1-x^2)\,dx}{(x^2-1)^2} = -\int \frac{dx}{x^2-1} + \frac{1}{2}\int \frac{x\cdot 2x\,dx}{(x^2-1)^2}$$

$$= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2-1)^2} & u = \int \frac{2x\,dx}{(x^2-1)^2} = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2-1 & x\in\langle 0;1) \Rightarrow t\in\langle -1;0) & x\in(1;\infty) \Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-1;0\rangle \Rightarrow t\in\langle -1;0) & x\in(-\infty;-1) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2-1} \\ v = x & v' = 1 \end{array} \right]$$

Riešené príklady – 091, 092

$$I_2 = \int \frac{dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{-a^2\,dx}{(x^2-a^2)^2}$$ pre $a > 0$. [091]

- $$= -\frac{1}{a^2}\int \frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{(x^2-a^2)\,dx}{(x^2-a^2)^2} + \frac{1}{a^2}\int \frac{x^2\,dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{dx}{x^2-a^2} + \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2-a^2)^2}$$

$$= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2-a^2)^2} & u = \int \frac{2x\,dx}{(x^2-a^2)^2} = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2-a^2 & x\in\langle 0;a) \Rightarrow t\in\langle -a^2;0) & x\in(a;\infty) \Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-a;0\rangle \Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a) \Rightarrow t\in(0;\infty)\end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2-a^2} \\ v = x & v' = 1 \end{array}\right]$$

$$= -\frac{1}{a^2}\int \frac{dx}{x^2-a^2} + \frac{1}{2a^2}\left[-\frac{x}{x^2-a^2} + \int \frac{dx}{x^2-a^2}\right]$$

$$\int \frac{dx}{(x^2-1)^2} =$$ [092]

- $$= -\int \frac{-1\,dx}{(x^2-1)^2} = -\int \frac{(x^2-1-x^2)\,dx}{(x^2-1)^2} = -\int \frac{dx}{x^2-1} + \frac{1}{2}\int \frac{x\cdot 2x\,dx}{(x^2-1)^2}$$

$$= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2-1)^2} & u = \int \frac{2x\,dx}{(x^2-1)^2} = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2-1 & x\in\langle 0;1) \Rightarrow t\in\langle -1;0) & x\in(1;\infty) \Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-1;0\rangle \Rightarrow t\in\langle -1;0) & x\in(-\infty;-1) \Rightarrow t\in(0;\infty)\end{array}\right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2-1} \\ v = x & v' = 1 \end{array}\right]$$

$$= -\int \frac{dx}{x^2-1} + \frac{1}{2}\left[-\frac{x}{x^2-1} + \int \frac{dx}{x^2-1}\right]$$

Riešené príklady – 091, 092

$$I_2=\int\frac{dx}{(x^2-a^2)^2}=-\frac{1}{a^2}\int\frac{-a^2\,dx}{(x^2-a^2)^2}$$ pre $a>0$. [091]

- $$=-\frac{1}{a^2}\int\frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2}=-\frac{1}{a^2}\int\frac{(x^2-a^2)\,dx}{(x^2-a^2)^2}+\frac{1}{a^2}\int\frac{x^2\,dx}{(x^2-a^2)^2}=-\frac{1}{a^2}\int\frac{dx}{x^2-a^2}+\frac{1}{2a^2}\int\frac{x\cdot 2x\,dx}{(x^2-a^2)^2}$$

$$=\left[\begin{array}{l|l} u'=\frac{2x}{(x^2-a^2)^2} & u=\int\frac{2x\,dx}{(x^2-a^2)^2}=\left[\begin{array}{r|l|l}\text{Subst. } t=x^2-a^2 & x\in\langle 0;a)\Rightarrow t\in\langle -a^2;0) & x\in(a;\infty)\Rightarrow t\in(0;\infty)\\ dt=2x\,dx & x\in(-a;0\rangle\Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a)\Rightarrow t\in(0;\infty)\end{array}\right]=\int\frac{dt}{t^2}=\int t^{-2}\,dt=-t^{-1}=-\frac{1}{t}=-\frac{1}{x^2-a^2}\\ v=x & v'=1\end{array}\right]$$

$$=-\frac{1}{a^2}\int\frac{dx}{x^2-a^2}+\frac{1}{2a^2}\left[-\frac{x}{x^2-a^2}+\int\frac{dx}{x^2-a^2}\right]=-\frac{1}{a^2}I_1-\frac{x}{2a^2(x^2-a^2)}+\frac{1}{2a^2}I_1$$

$$=-\frac{1}{2a^2}I_1-\frac{x}{2a^2(x^2-a^2)}$$

$$\int\frac{dx}{(x^2-1)^2}=$$ [092]

- $$=-\int\frac{-1\,dx}{(x^2-1)^2}=-\int\frac{(x^2-1-x^2)\,dx}{(x^2-1)^2}=-\int\frac{dx}{x^2-1}+\frac{1}{2}\int\frac{x\cdot 2x\,dx}{(x^2-1)^2}$$

$$=\left[\begin{array}{l|l} u'=\frac{2x}{(x^2-1)^2} & u=\int\frac{2x\,dx}{(x^2-1)^2}=\left[\begin{array}{r|l|l}\text{Subst. } t=x^2-1 & x\in\langle 0;1)\Rightarrow t\in\langle -1;0) & x\in(1;\infty)\Rightarrow t\in(0;\infty)\\ dt=2x\,dx & x\in(-1;0\rangle\Rightarrow t\in\langle -1;0) & x\in(-\infty;-1)\Rightarrow t\in(0;\infty)\end{array}\right]=\int\frac{dt}{t^2}=\int t^{-2}\,dt=-t^{-1}=-\frac{1}{t}=-\frac{1}{x^2-1}\\ v=x & v'=1\end{array}\right]$$

$$=-\int\frac{dx}{x^2-1}+\frac{1}{2}\left[-\frac{x}{x^2-1}+\int\frac{dx}{x^2-1}\right]=-\frac{1}{2}\int\frac{dx}{x^2-1}-\frac{x}{2(x^2-1)}$$

Riešené príklady – 091, 092

$I_2 = \int \frac{dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{-a^2\,dx}{(x^2-a^2)^2} = \frac{1}{4a^3}\ln\left|\frac{x+a}{x-a}\right| - \frac{x}{2a^2(x^2-a^2)} + c$ pre $a > 0$. [091]

- $= -\frac{1}{a^2}\int \frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{(x^2-a^2)\,dx}{(x^2-a^2)^2} + \frac{1}{a^2}\int \frac{x^2\,dx}{(x^2-a^2)^2} = -\frac{1}{a^2}\int \frac{dx}{x^2-a^2} + \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2-a^2)^2}$

$$= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2-a^2)^2} & u = \int \frac{2x\,dx}{(x^2-a^2)^2} = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2-a^2 & x\in\langle 0;a)\Rightarrow t\in\langle -a^2;0) & x\in(a;\infty)\Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-a;0\rangle\Rightarrow t\in\langle -a^2;0) & x\in(-\infty;-a)\Rightarrow t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2-a^2} \\ v = x & v' = 1 \end{array}\right]$$

$$= -\frac{1}{a^2}\int \frac{dx}{x^2-a^2} + \frac{1}{2a^2}\left[-\frac{x}{x^2-a^2} + \int \frac{dx}{x^2-a^2}\right] = -\frac{1}{a^2}I_1 - \frac{x}{2a^2(x^2-a^2)} + \frac{1}{2a^2}I_1$$

$$= -\frac{1}{2a^2}I_1 - \frac{x}{2a^2(x^2-a^2)} = -\frac{1}{2a^2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| - \frac{x}{2a^2(x^2-a^2)} + c = \frac{1}{4a^3}\ln\left|\frac{x+a}{x-a}\right| - \frac{x}{2a^2(x^2-a^2)} + c,$$

$x\in R,\ x\neq \pm a,\ c\in R.$

$\int \frac{dx}{(x^2-1)^2} = \frac{1}{4}\ln\left|\frac{x+1}{x-1}\right| - \frac{x}{2(x^2-1)} + c$ [092]

- $= -\int \frac{-1\,dx}{(x^2-1)^2} = -\int \frac{(x^2-1-x^2)\,dx}{(x^2-1)^2} = -\int \frac{dx}{x^2-1} + \frac{1}{2}\int \frac{x\cdot 2x\,dx}{(x^2-1)^2}$

$$= \left[\begin{array}{l|l} u' = \frac{2x}{(x^2-1)^2} & u = \int \frac{2x\,dx}{(x^2-1)^2} = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2-1 & x\in\langle 0;1)\Rightarrow t\in\langle -1;0) & x\in(1;\infty)\Rightarrow t\in(0;\infty) \\ dt = 2x\,dx & x\in(-1;0\rangle\Rightarrow t\in\langle -1;0) & x\in(-\infty;-1)\Rightarrow t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t^2} = \int t^{-2}\,dt = -t^{-1} = -\frac{1}{t} = -\frac{1}{x^2-1} \\ v = x & v' = 1 \end{array}\right]$$

$$= -\int \frac{dx}{x^2-1} + \frac{1}{2}\left[-\frac{x}{x^2-1} + \int \frac{dx}{x^2-1}\right] = -\frac{1}{2}\int \frac{dx}{x^2-1} - \frac{x}{2(x^2-1)} = -\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| - \frac{x}{2(x^2-1)} + c,$$

$x\in R,\ x\neq \pm 1,\ c\in R.$

Riešené príklady – 093

$I_n = \int \frac{dx}{(x^2+ax+b)^n}$ pre $a, b \in R$ a $n \in N$, $n \neq 1$. [093]

Riešené príklady – 093

$I_n = \int \frac{dx}{(x^2+ax+b)^n}$ pre $a,b \in R$ a $n \in N$, $n \neq 1$. [093]

- $= \int \frac{dx}{\left[\left(x^2+ax+\frac{a^2}{4}\right)+b-\frac{a^2}{4}\right]^n} = \int \frac{dx}{\left[(x+\frac{a}{2})^2+b-\frac{a^2}{4}\right]^n}$

Riešené príklady – 093

$I_n = \int \frac{dx}{(x^2+ax+b)^n}$ pre $a,b \in R$ a $n \in N$, $n \neq 1$. [093]

- $= \int \frac{dx}{\left[\left(x^2+ax+\frac{a^2}{4}\right)+b-\frac{a^2}{4}\right]^n} = \int \frac{dx}{\left[(x+\frac{a}{2})^2+b-\frac{a^2}{4}\right]^n} = \left[\text{Subst. } t = x+\frac{a}{2} \;\middle|\; x \in R \atop dt = dx \;\middle|\; t \in R \right] = \int \frac{dt}{\left[t^2+\left(b-\frac{a^2}{4}\right)\right]^n}.$

Riešené príklady – 093

$I_n = \int \frac{dx}{(x^2+ax+b)^n}$ pre $a,b\in R$ a $n\in N$, $n\neq 1$. [093]

- $= \int \frac{dx}{\left[\left(x^2+ax+\frac{a^2}{4}\right)+b-\frac{a^2}{4}\right]^n} = \int \frac{dx}{\left[(x+\frac{a}{2})^2+b-\frac{a^2}{4}\right]^n} = \left[\begin{array}{r|l} \text{Subst. } t=x+\frac{a}{2} & x\in R \\ dt=dx & t\in R \end{array}\right] = \int \frac{dt}{\left[t^2+\left(b-\frac{a^2}{4}\right)\right]^n}.$

Pre $b-\frac{a^2}{4}>0$ označme $b-\frac{a^2}{4}=\alpha^2$, $\alpha>0$, t. j. $4b-a^2=4\alpha^2$. Potom $x^2+ax+b=(x+\frac{a}{2})^2+(b-\frac{a^2}{4})=t^2+\alpha^2>t^2$, pričom $x\in R$, $t\in R$:

- $I_n = \int \frac{dt}{[t^2+\alpha^2]^n}$

Pre $b-\frac{a^2}{4}=0$ platí $x^2+ax+b=(x+\frac{a}{2})^2+(b-\frac{a^2}{4})=t^2$, pričom $x\in R$, $x\neq -\frac{a}{2}$, $t\in R$, $t\neq 0$:

- $I_n = \int \frac{dt}{[t^2]^n} = \int t^{-2n}\,dt$

Pre $b-\frac{a^2}{4}<0$ označme $b-\frac{a^2}{4}=-\alpha^2$, $\alpha>0$, t. j. $a^2-4b=4\alpha^2$. Potom $x^2+ax+b=(x+\frac{a}{2})^2+(b-\frac{a^2}{4})=t^2-\alpha^2<t^2$, pričom $x\in R$, $t\in R$, $t\neq\pm\alpha$:

- $I_n = \int \frac{dt}{[t^2-\alpha^2]^n}$

Riešené príklady – 093

$I_n = \int \frac{dx}{(x^2+ax+b)^n}$ pre $a, b \in R$ a $n \in N$, $n \neq 1$. [093]

- $= \int \frac{dx}{\left[\left(x^2+ax+\frac{a^2}{4}\right)+b-\frac{a^2}{4}\right]^n} = \int \frac{dx}{\left[(x+\frac{a}{2})^2+b-\frac{a^2}{4}\right]^n} = \left[\begin{array}{r|l} \text{Subst. } t = x+\frac{a}{2} & x \in R \\ dt = dx & t \in R \end{array}\right] = \int \frac{dt}{\left[t^2+\left(b-\frac{a^2}{4}\right)\right]^n}.$

Pre $b-\frac{a^2}{4} > 0$ označme $b-\frac{a^2}{4} = \alpha^2$, $\alpha > 0$, t. j. $4b-a^2 = 4\alpha^2$. Potom $x^2+ax+b = (x+\frac{a}{2})^2+(b-\frac{a^2}{4}) = t^2+\alpha^2 > t^2$, pričom $x \in R$, $t \in R$:

- $I_n = \int \frac{dt}{[t^2+\alpha^2]^n} = \left[\text{Pr. 079.}\right] = \frac{2n-3}{2\alpha^2(n-1)} \int \frac{dt}{[t^2+\alpha^2]^{n-1}} + \frac{t}{2\alpha^2(n-1)(t^2+\alpha^2)^{n-1}}$

Pre $b-\frac{a^2}{4} = 0$ platí $x^2+ax+b = (x+\frac{a}{2})^2+(b-\frac{a^2}{4}) = t^2$, pričom $x \in R$, $x \neq -\frac{a}{2}$, $t \in R$, $t \neq 0$:

- $I_n = \int \frac{dt}{[t^2]^n} = \int t^{-2n}\, dt = \frac{t^{-2n+1}}{-2n+1} + c$

Pre $b-\frac{a^2}{4} < 0$ označme $b-\frac{a^2}{4} = -\alpha^2$, $\alpha > 0$, t. j. $a^2-4b = 4\alpha^2$. Potom $x^2+ax+b = (x+\frac{a}{2})^2+(b-\frac{a^2}{4}) = t^2-\alpha^2 < t^2$, pričom $x \in R$, $t \in R$, $t \neq \pm\alpha$:

- $I_n = \int \frac{dt}{[t^2-\alpha^2]^n} = \left[\text{Pr. 090.}\right] = \frac{3-2n}{2\alpha^2(n-1)} \int \frac{dt}{[t^2-\alpha^2]^{n-1}} - \frac{t}{2\alpha^2(n-1)(t^2-\alpha^2)^{n-1}}$

Riešené príklady – 093

$I_n = \int \frac{dx}{(x^2+ax+b)^n}$ pre $a,b \in R$ a $n \in N$, $n \neq 1$. [093]

- $= \int \frac{dx}{\left[\left(x^2+ax+\frac{a^2}{4}\right)+b-\frac{a^2}{4}\right]^n} = \int \frac{dx}{\left[(x+\frac{a}{2})^2+b-\frac{a^2}{4}\right]^n} = \left[\begin{array}{r|l} \text{Subst. } t = x+\frac{a}{2} & x \in R \\ dt = dx & t \in R \end{array}\right] = \int \frac{dt}{\left[t^2+\left(b-\frac{a^2}{4}\right)\right]^n}.$

Pre $b-\frac{a^2}{4} > 0$ označme $b-\frac{a^2}{4} = \alpha^2$, $\alpha > 0$, t. j. $4b-a^2 = 4\alpha^2$. Potom $x^2+ax+b = (x+\frac{a}{2})^2+(b-\frac{a^2}{4}) = t^2+\alpha^2 > t^2$, pričom $x \in R$, $t \in R$:

- $$I_n = \int \frac{dt}{[t^2+\alpha^2]^n} = \left[\text{Pr. 079.}\right] = \frac{2n-3}{2\alpha^2(n-1)} \int \frac{dt}{[t^2+\alpha^2]^{n-1}} + \frac{t}{2\alpha^2(n-1)(t^2+\alpha^2)^{n-1}} = \left[\begin{array}{l} \frac{1}{2\alpha^2} = \frac{2}{4\alpha^2} = \frac{2}{4b-a^2} \\ \frac{t}{2\alpha^2} = \frac{2t}{4\alpha^2} = \frac{2x+a}{4b-a^2} \end{array}\right]$$
  $$= \left[\int \frac{dt}{[t^2+\alpha^2]^{n-1}} = \int \frac{dx}{(x^2+ax+b)^{n-1}} = I_{n-1}\right] = \frac{2(2n-3)}{(4b-a^2)(n-1)} I_{n-1} + \frac{2x+a}{(4b-a^2)(n-1)(x^2+ax+b)^{n-1}},$$
  $x \in R$,

Pre $b-\frac{a^2}{4} = 0$ platí $x^2+ax+b = (x+\frac{a}{2})^2+(b-\frac{a^2}{4}) = t^2$, pričom $x \in R$, $x \neq -\frac{a}{2}$, $t \in R$, $t \neq 0$:

- $$I_n = \int \frac{dt}{[t^2]^n} = \int t^{-2n}\, dt = \frac{t^{-2n+1}}{-2n+1} + c = \left[\begin{array}{l} t^{-2n+1} = \frac{1}{t^{2n-1}} = \frac{2^{2n-1}}{2^{2n-1}(x+\frac{a}{2})^{2n-1}} = \frac{2^{2n-1}}{(2x+a)^{2n-1}} = \frac{4^n}{2(2x+a)^{2n-1}} \\ 2^{2n-1} = \frac{2^{2n}}{2} = \frac{(2^2)^n}{2} = \frac{4^n}{2} \end{array}\right]$$
  $$= -\frac{4^n}{2(2n-1)(2x+a)^{2n-1}} + c,\ x \in R,\ x \neq -\frac{a}{2},\ c \in R.$$

Pre $b-\frac{a^2}{4} < 0$ označme $b-\frac{a^2}{4} = -\alpha^2$, $\alpha > 0$, t. j. $a^2-4b = 4\alpha^2$. Potom $x^2+ax+b = (x+\frac{a}{2})^2+(b-\frac{a^2}{4}) = t^2-\alpha^2 < t^2$, pričom $x \in R$, $t \in R$, $t \neq \pm\alpha$:

- $$I_n = \int \frac{dt}{[t^2-\alpha^2]^n} = \left[\text{Pr. 090.}\right] = \frac{3-2n}{2\alpha^2(n-1)} \int \frac{dt}{[t^2-\alpha^2]^{n-1}} - \frac{t}{2\alpha^2(n-1)(t^2-\alpha^2)^{n-1}} = \left[\begin{array}{l} \frac{1}{2\alpha^2} = \frac{2}{4\alpha^2} = \frac{2}{a^2-4b} \\ \frac{t}{2\alpha^2} = \frac{2t}{4\alpha^2} = \frac{2x+a}{a^2-4b} \end{array}\right]$$
  $$= \left[\int \frac{dt}{[t^2-\alpha^2]^{n-1}} = \int \frac{dx}{(x^2+ax+b)^{n-1}} = I_{n-1}\right] = \frac{2(3-2n)}{(a^2-4b)(n-1)} I_{n-1} - \frac{2x+a}{(a^2-4b)(n-1)(x^2+ax+b)^{n-1}},$$
  $x \in R$, $x \neq \frac{-a\pm\sqrt{a^2-4b}}{2}$,

Riešené príklady – 093

$$I_n=\int\frac{dx}{(x^2+ax+b)^n}$$ pre $a,b\in R$ a $n\in N$, $n\neq 1$. [093]

- $$=\int\frac{dx}{\Big[\Big(x^2+ax+\frac{a^2}{4}\Big)+b-\frac{a^2}{4}\Big]^n}=\int\frac{dx}{\Big[(x+\frac{a}{2})^2+b-\frac{a^2}{4}\Big]^n}=\left[\begin{array}{c|c}\text{Subst. } t=x+\frac{a}{2} & x\in R\\ dt=dx & t\in R\end{array}\right]=\int\frac{dt}{\Big[t^2+\Big(b-\frac{a^2}{4}\Big)\Big]^n}.$$

Pre $b-\frac{a^2}{4}>0$ označme $b-\frac{a^2}{4}=\alpha^2$, $\alpha>0$, t. j. $4b-a^2=4\alpha^2$. Potom $x^2+ax+b=(x+\frac{a}{2})^2+(b-\frac{a^2}{4})=t^2+\alpha^2>t^2$, pričom $x\in R$, $t\in R$:

- $$I_n=\int\frac{dt}{[t^2+\alpha^2]^n}=[\text{Pr. 079.}]=\frac{2n-3}{2\alpha^2(n-1)}\int\frac{dt}{[t^2+\alpha^2]^{n-1}}+\frac{t}{2\alpha^2(n-1)(t^2+\alpha^2)^{n-1}}=\left[\begin{array}{c}\frac{1}{2\alpha^2}=\frac{2}{4\alpha^2}=\frac{2}{4b-a^2}\\ \frac{t}{2\alpha^2}=\frac{2t}{4\alpha^2}=\frac{2x+a}{4b-a^2}\end{array}\right]$$
$$=\left[\int\frac{dt}{[t^2+\alpha^2]^{n-1}}=\int\frac{dx}{(x^2+ax+b)^{n-1}}=I_{n-1}\right]=\frac{2(2n-3)}{(4b-a^2)(n-1)}I_{n-1}+\frac{2x+a}{(4b-a^2)(n-1)(x^2+ax+b)^{n-1}},$$
pričom $$I_1=[\text{Pr. 051.}]=\frac{2}{\sqrt{4b-a^2}}\operatorname{arctg}\frac{2x+a}{\sqrt{4b-a^2}}+c,\ x\in R,\ c\in R.$$

Pre $b-\frac{a^2}{4}=0$ platí $x^2+ax+b=(x+\frac{a}{2})^2+(b-\frac{a^2}{4})=t^2$, pričom $x\in R$, $x\neq-\frac{a}{2}$, $t\in R$, $t\neq 0$:

- $$I_n=\int\frac{dt}{[t^2]^n}=\int t^{-2n}\,dt=\frac{t^{-2n+1}}{-2n+1}+c=\left[\begin{array}{l}t^{-2n+1}=\frac{1}{t^{2n-1}}=\frac{2^{2n-1}}{2^{2n-1}(x+\frac{a}{2})^{2n-1}}=\frac{2^{2n-1}}{(2x+a)^{2n-1}}=\frac{4^n}{2(2x+a)^{2n-1}}\\ 2^{2n-1}=\frac{2^{2n}}{2}=\frac{(2^2)^n}{2}=\frac{4^n}{2}\end{array}\right]$$
$$=-\frac{4^n}{2(2n-1)(2x+a)^{2n-1}}+c,\ x\in R,\ x\neq-\frac{a}{2},\ c\in R.$$

Pre $b-\frac{a^2}{4}<0$ označme $b-\frac{a^2}{4}=-\alpha^2$, $\alpha>0$, t. j. $a^2-4b=4\alpha^2$. Potom $x^2+ax+b=(x+\frac{a}{2})^2+(b-\frac{a^2}{4})=t^2-\alpha^2<t^2$, pričom $x\in R$, $t\in R$, $t\neq\pm\alpha$:

- $$I_n=\int\frac{dt}{[t^2-\alpha^2]^n}=[\text{Pr. 090.}]=\frac{3-2n}{2\alpha^2(n-1)}\int\frac{dt}{[t^2-\alpha^2]^{n-1}}-\frac{t}{2\alpha^2(n-1)(t^2-\alpha^2)^{n-1}}=\left[\begin{array}{c}\frac{1}{2\alpha^2}=\frac{2}{4\alpha^2}=\frac{2}{a^2-4b}\\ \frac{t}{2\alpha^2}=\frac{2t}{4\alpha^2}=\frac{2x+a}{a^2-4b}\end{array}\right]$$
$$=\left[\int\frac{dt}{[t^2-\alpha^2]^{n-1}}=\int\frac{dx}{(x^2+ax+b)^{n-1}}=I_{n-1}\right]=\frac{2(3-2n)}{(a^2-4b)(n-1)}I_{n-1}-\frac{2x+a}{(a^2-4b)(n-1)(x^2+ax+b)^{n-1}},$$
pričom $$I_1=[\text{Pr. 051.}]=\frac{1}{\sqrt{a^2-4b}}\ln\left|\frac{2x+a-\sqrt{a^2-4b}}{2x+a+\sqrt{a^2-4b}}\right|+c,\ x\in R,\ x\neq\frac{-a\pm\sqrt{a^2-4b}}{2},\ c\in R.$$

Riešené príklady – 094

$$\int \frac{dx}{(x^2 \pm 4x + 3)^2}$$

[094]

Riešené príklady – 094

$\int \frac{dx}{(x^2 \pm 4x + 3)^2}$ [094]

- $= \int \frac{dx}{[x^2 \pm 4x + 4 - 1]^2} = \int \frac{dx}{[(x \pm 2)^2 - 1]^2}$

- $= \int \frac{dx}{[x^2 \pm 4x + 4 - 1]^2} = \int \frac{dx}{[(x \pm 2)^2 - 1]^2}$

Riešené príklady – 094

$\int \frac{dx}{(x^2 \pm 4x+3)^2}$ [094]

- $= \int \frac{dx}{[x^2 \pm 4x+4-1]^2} = \int \frac{dx}{[(x\pm 2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x\pm 2 & x\in(-\infty; \mp 2-1) & x\in(\mp 2-1; \mp 2+1) & x\in(\mp 2+1; \infty) \\ dt = dx & t\in(-\infty; -1) & t\in(-1; 1) & t\in(1; \infty) \end{array} \right]$
  $= \int \frac{1}{(t^2-1)^2}\, dt$

- $= \int \frac{dx}{[x^2 \pm 4x+4-1]^2} = \int \frac{dx}{[(x\pm 2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x\pm 2 & x\in(-\infty; \mp 2-1) & x\in(\mp 2-1; \mp 2+1) & x\in(\mp 2+1; \infty) \\ dt = dx & t\in(-\infty; -1) & t\in(-1; 1) & t\in(1; \infty) \end{array} \right]$
  $= \int \frac{dt}{(t^2-1)^2}$

Riešené príklady – 094

$$\int \frac{dx}{(x^2 \pm 4x + 3)^2}$$ [094]

- $$= \int \frac{dx}{[x^2 \pm 4x + 4 - 1]^2} = \int \frac{dx}{[(x \pm 2)^2 - 1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x \pm 2 & x \in (-\infty; \mp 2 - 1) & x \in (\mp 2 - 1; \mp 2 + 1) & x \in (\mp 2 + 1; \infty) \\ dt = dx & t \in (-\infty; -1) & t \in (-1; 1) & t \in (1; \infty) \end{array} \right]$$
$$= \int \frac{1}{(t^2-1)^2}\, dt = -\int \frac{-1}{(t^2-1)^2}\, dt = -\int \frac{(t^2-1)-t^2}{(t^2-1)^2}\, dt = -\int \frac{dt}{t^2-1} + \frac{1}{2} \int \frac{2t^2\, dt}{(t^2-1)^2}$$

- $$= \int \frac{dx}{[x^2 \pm 4x + 4 - 1]^2} = \int \frac{dx}{[(x \pm 2)^2 - 1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x \pm 2 & x \in (-\infty; \mp 2 - 1) & x \in (\mp 2 - 1; \mp 2 + 1) & x \in (\mp 2 + 1; \infty) \\ dt = dx & t \in (-\infty; -1) & t \in (-1; 1) & t \in (1; \infty) \end{array} \right]$$
$$= \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr 090: } \int \frac{dt}{(t^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} \cdot \int \frac{dt}{(t^2-a^2)^{n-1}} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \text{ pre } a > 0 \text{ a } n \in N,\ n \neq 1. \right]$$

Riešené príklady – 094

$$\int \frac{dx}{(x^2 \pm 4x + 3)^2}$$ [094]

- $$= \int \frac{dx}{[x^2 \pm 4x + 4 - 1]^2} = \int \frac{dx}{[(x \pm 2)^2 - 1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x \pm 2 & x \in (-\infty; \mp 2 - 1) & x \in (\mp 2 - 1; \mp 2 + 1) & x \in (\mp 2 + 1; \infty) \\ dt = dx & t \in (-\infty; -1) & t \in (-1; 1) & t \in (1; \infty) \end{array} \right]$$
$$= \int \frac{1}{(t^2-1)^2}\, dt = -\int \frac{-1}{(t^2-1)^2}\, dt = -\int \frac{(t^2-1) - t^2}{(t^2-1)^2}\, dt = -\int \frac{dt}{t^2-1} + \frac{1}{2} \int \frac{2t^2\, dt}{(t^2-1)^2}$$
$$= \left[\begin{array}{l|l} u' = \frac{2t}{(t^2-1)^2} & u = \int \frac{2t\, dt}{(t^2-1)^2} = \left[\begin{array}{r|l|l} \text{Subst. } z = t^2 - 1 & t \in \langle 0; 1) \Rightarrow z \in \langle -1; 0) & t \in (1; \infty) \Rightarrow z \in (0; \infty) \\ dt = 2x\, dx & t \in (-1; 0\rangle \Rightarrow z \in \langle -1; 0) & t \in (-\infty; -1) \Rightarrow z \in (0; \infty) \end{array} \right] = \int \frac{dz}{z^2} = \int z^{-2}\, dz = -z^{-1} = -\frac{1}{z} = -\frac{1}{t^2-1} \\ v = t & v' = 1 \end{array} \right]$$
$$= -\int \frac{dt}{t^2-1} + \frac{1}{2} \left[-\frac{t}{t^2-1} + \int \frac{dt}{t^2-1} \right]$$

- $$= \int \frac{dx}{[x^2 \pm 4x + 4 - 1]^2} = \int \frac{dx}{[(x \pm 2)^2 - 1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x \pm 2 & x \in (-\infty; \mp 2 - 1) & x \in (\mp 2 - 1; \mp 2 + 1) & x \in (\mp 2 + 1; \infty) \\ dt = dx & t \in (-\infty; -1) & t \in (-1; 1) & t \in (1; \infty) \end{array} \right]$$
$$= \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr 090: } \int \frac{dt}{(t^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} \cdot \int \frac{dt}{(t^2-a^2)^{n-1}} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \text{ pre } a > 0 \text{ a } n \in N,\ n \neq 1. \right] = \frac{3 - 2\cdot 2}{2(2-1)} \int \frac{dt}{(t^2-1)^{2-1}} - \frac{t}{2(2-1)(t^2-1)}$$

Riešené príklady – 094

$\int \frac{dx}{(x^2 \pm 4x+3)^2}$ [094]

- $= \int \frac{dx}{[x^2 \pm 4x+4-1]^2} = \int \frac{dx}{[(x\pm 2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x\pm 2 & x\in(-\infty;\mp 2-1) & x\in(\mp 2-1;\mp 2+1) & x\in(\mp 2+1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array} \right]$
$= \int \frac{1}{(t^2-1)^2}\, dt = -\int \frac{-1}{(t^2-1)^2}\, dt = -\int \frac{(t^2-1)-t^2}{(t^2-1)^2}\, dt = -\int \frac{dt}{t^2-1} + \frac{1}{2}\int \frac{2t^2\, dt}{(t^2-1)^2}$
$= \left[\begin{array}{l|l} u' = \frac{2t}{(t^2-1)^2} & u = \int \frac{2t\, dt}{(t^2-1)^2} = \left[\begin{array}{r|l|l} \text{Subst. } z = t^2-1 & t\in\langle 0;1) \Rightarrow z\in\langle -1;0) & t\in(1;\infty) \Rightarrow z\in(0;\infty) \\ dt = 2x\, dx & t\in(-1;0\rangle \Rightarrow z\in\langle -1;0) & t\in(-\infty;-1) \Rightarrow z\in(0;\infty) \end{array} \right] = \int \frac{dz}{z^2} = \int z^{-2}\, dz = -z^{-1} = -\frac{1}{z} = -\frac{1}{t^2-1} \\ v = t & v' = 1 \end{array} \right]$
$= -\int \frac{dt}{t^2-1} + \frac{1}{2}\left[-\frac{t}{t^2-1} + \int \frac{dt}{t^2-1} \right] = -\frac{1}{2}\int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)}$

- $= \int \frac{dx}{[x^2 \pm 4x+4-1]^2} = \int \frac{dx}{[(x\pm 2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x\pm 2 & x\in(-\infty;\mp 2-1) & x\in(\mp 2-1;\mp 2+1) & x\in(\mp 2+1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array} \right]$
$= \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr 090: } \int \frac{dt}{(t^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} \cdot \int \frac{dt}{(t^2-a^2)^{n-1}} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \text{ pre } a>0 \text{ a } n\in N,\ n\neq 1. \right] = \frac{3-2\cdot 2}{2(2-1)} \int \frac{dt}{(t^2-1)^{2-1}} - \frac{t}{2(2-1)(t^2-1)}$
$= -\frac{1}{2}\int \frac{dt}{t^2-1} + \frac{t}{2(t^2-1)}$

Riešené príklady – 094

$$\int \frac{dx}{(x^2\pm 4x+3)^2}$$ [094]

- $= \int \frac{dx}{[x^2\pm 4x+4-1]^2} = \int \frac{dx}{[(x\pm 2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t=x\pm 2 & x\in(-\infty;\mp 2-1) & x\in(\mp 2-1;\mp 2+1) & x\in(\mp 2+1;\infty) \\ dt=dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array}\right]$
$= \int \frac{1}{(t^2-1)^2}\,dt = -\int \frac{-1}{(t^2-1)^2}\,dt = -\int \frac{(t^2-1)-t^2}{(t^2-1)^2}\,dt = -\int \frac{dt}{t^2-1} + \frac{1}{2}\int \frac{2t^2\,dt}{(t^2-1)^2}$
$= \left[\begin{array}{l|l} u'=\frac{2t}{(t^2-1)^2} & u=\int\frac{2t\,dt}{(t^2-1)^2} = \left[\begin{array}{r|l|l} \text{Subst. } z=t^2-1 & t\in\langle 0;1)\Rightarrow z\in\langle -1;0) & t\in(1;\infty)\Rightarrow z\in(0;\infty) \\ dt=2x\,dx & t\in(-1;0\rangle\Rightarrow z\in\langle -1;0) & t\in(-\infty;-1)\Rightarrow z\in(0;\infty) \end{array}\right] = \int\frac{dz}{z^2} = \int z^{-2}\,dz = -z^{-1} = -\frac{1}{z} = -\frac{1}{t^2-1} \\ v=t & v'=1 \end{array}\right]$
$= -\int \frac{dt}{t^2-1} + \frac{1}{2}\left[-\frac{t}{t^2-1} + \int\frac{dt}{t^2-1}\right] = -\frac{1}{2}\int\frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} = -\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - \frac{t}{2(t^2-1)} + c$

- $= \int \frac{dx}{[x^2\pm 4x+4-1]^2} = \int \frac{dx}{[(x\pm 2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t=x\pm 2 & x\in(-\infty;\mp 2-1) & x\in(\mp 2-1;\mp 2+1) & x\in(\mp 2+1;\infty) \\ dt=dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array}\right]$
$= \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr 090: } \int\frac{dt}{(t^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)}\cdot\int\frac{dt}{(t^2-a^2)^{n-1}} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \text{ pre } a>0 \text{ a } n\in N,\ n\neq 1.\right] = \frac{3-2\cdot 2}{2(2-1)}\int\frac{dt}{(t^2-1)^{2-1}} - \frac{t}{2(2-1)(t^2-1)}$
$= -\frac{1}{2}\int\frac{dt}{t^2-1} + \frac{t}{2(t^2-1)} = -\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - \frac{t}{2(t^2-1)} + c$

Riešené príklady – 094

$$\int\frac{dx}{(x^2\pm4x+3)^2}=-\frac{1}{4}\ln\left|\frac{x\pm2-1}{x\pm2+1}\right|-\frac{x\pm2}{2(x^2\pm4x+3)}+c$$ [094]

- $$=\int\frac{dx}{[x^2\pm4x+4-1]^2}=\int\frac{dx}{[(x\pm2)^2-1]^2}=\left[\begin{array}{r|l|l|l}\text{Subst. } t=x\pm2 & x\in(-\infty;\mp2-1) & x\in(\mp2-1;\mp2+1) & x\in(\mp2+1;\infty)\\ dt=dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty)\end{array}\right]$$
$$=\int\frac{1}{(t^2-1)^2}\,dt=-\int\frac{-1}{(t^2-1)^2}\,dt=-\int\frac{(t^2-1)-t^2}{(t^2-1)^2}\,dt=-\int\frac{dt}{t^2-1}+\frac{1}{2}\int\frac{2t^2\,dt}{(t^2-1)^2}$$
$$=\left[\begin{array}{l|l}u'=\frac{2t}{(t^2-1)^2} & u=\int\frac{2t\,dt}{(t^2-1)^2}=\left[\begin{array}{r|l|l}\text{Subst. } z=t^2-1 & t\in\langle0;1)\Rightarrow z\in\langle-1;0) & t\in(1;\infty)\Rightarrow z\in(0;\infty)\\ dt=2x\,dx & t\in(-1;0\rangle\Rightarrow z\in\langle-1;0) & t\in(-\infty;-1)\Rightarrow z\in(0;\infty)\end{array}\right]=\int\frac{dz}{z^2}=\int z^{-2}\,dz=-z^{-1}=-\frac{1}{z}=-\frac{1}{t^2-1}\\ v=t & v'=1\end{array}\right]$$
$$=-\int\frac{dt}{t^2-1}+\frac{1}{2}\left[-\frac{t}{t^2-1}+\int\frac{dt}{t^2-1}\right]=-\frac{1}{2}\int\frac{dt}{t^2-1}-\frac{t}{2(t^2-1)}=-\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|-\frac{t}{2(t^2-1)}+c$$
$$=\left[\begin{array}{l}t^2-1=(x\pm2)^2-1\\ \quad=x^2\pm4x+3\end{array}\right]=-\frac{1}{4}\ln\left|\frac{x\pm2-1}{x\pm2+1}\right|-\frac{x\pm2}{2(x^2\pm4x+3)}+c,\ x\in R,\ x\neq\mp2\pm1,\ c\in R.$$

- $$=\int\frac{dx}{[x^2\pm4x+4-1]^2}=\int\frac{dx}{[(x\pm2)^2-1]^2}=\left[\begin{array}{r|l|l|l}\text{Subst. } t=x\pm2 & x\in(-\infty;\mp2-1) & x\in(\mp2-1;\mp2+1) & x\in(\mp2+1;\infty)\\ dt=dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty)\end{array}\right]$$
$$=\int\frac{dt}{(t^2-1)^2}=\left[\text{Pr 090: }\int\frac{dt}{(t^2-a^2)^n}=\frac{3-2n}{2a^2(n-1)}\cdot\int\frac{dt}{(t^2-a^2)^{n-1}}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\text{ pre } a>0 \text{ a } n\in N,\ n\neq1.\right]=\frac{3-2\cdot2}{2(2-1)}\int\frac{dt}{(t^2-1)^{2-1}}-\frac{t}{2(2-1)(t^2-1)}$$
$$=-\frac{1}{2}\int\frac{dt}{t^2-1}+\frac{t}{2(t^2-1)}=-\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|-\frac{t}{2(t^2-1)}+c$$
$$=\left[\begin{array}{l}t^2-1=(x\pm2)^2-1\\ \quad=x^2\pm4x+3\end{array}\right]=-\frac{1}{4}\ln\left|\frac{x\pm2-1}{x\pm2+1}\right|-\frac{x\pm2}{2(x^2\pm4x+3)}+c,\ x\in R,\ x\neq\mp2\pm1,\ c\in R.$$

Riešené príklady – 094

$$\int \frac{dx}{(x^2+4x+3)^2} = -\frac{1}{4}\ln\left|\frac{x+1}{x+3}\right| - \frac{x+2}{2(x^2+4x+3)} + c$$ [094]

- $$= \int \frac{dx}{[x^2+4x+4-1]^2} = \int \frac{dx}{[(x+2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t=x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt=dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array}\right]$$
$$= \int \frac{1}{(t^2-1)^2}\,dt = -\int \frac{-1}{(t^2-1)^2}\,dt = -\int \frac{(t^2-1)-t^2}{(t^2-1)^2}\,dt = -\int \frac{dt}{t^2-1} + \frac{1}{2}\int \frac{2t^2\,dt}{(t^2-1)^2}$$
$$= \left[\begin{array}{l|l} u'=\frac{2t}{(t^2-1)^2} & u=\int\frac{2t\,dt}{(t^2-1)^2} = \left[\begin{array}{r|l|l} \text{Subst. } z=t^2-1 & t\in\langle 0;1) \Rightarrow z\in\langle -1;0) & t\in(1;\infty)\Rightarrow z\in(0;\infty) \\ dt=2x\,dx & t\in(-1;0\rangle \Rightarrow z\in\langle -1;0) & t\in(-\infty;-1)\Rightarrow z\in(0;\infty) \end{array}\right] = \int\frac{dz}{z^2} = \int z^{-2}\,dz = -z^{-1} = -\frac{1}{z} = -\frac{1}{t^2-1} \\ v=t & v'=1 \end{array}\right]$$
$$= -\int \frac{dt}{t^2-1} + \frac{1}{2}\left[-\frac{t}{t^2-1} + \int\frac{dt}{t^2-1}\right] = -\frac{1}{2}\int\frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} = -\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - \frac{t}{2(t^2-1)} + c$$
$$= \left[\begin{array}{l} t^2-1=(x+2)^2-1 \\ \quad = x^2+4x+3 \end{array}\right] = -\frac{1}{4}\ln\left|\frac{x+1}{x+3}\right| - \frac{x+2}{2(x^2+4x+3)} + c,\ x\in R,\ x\neq -2\pm 1,\ c\in R.$$

- $$= \int \frac{dx}{[x^2+4x+4-1]^2} = \int \frac{dx}{[(x+2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t=x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt=dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array}\right]$$
$$= \int \frac{dt}{(t^2-1)^2} = \left[\text{Pr 090: } \int\frac{dt}{(t^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)}\cdot\int\frac{dt}{(t^2-a^2)^{n-1}} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \text{ pre } a>0 \text{ a } n\in N,\ n\neq 1.\right] = \frac{3-2\cdot 2}{2(2-1)}\int\frac{dt}{(t^2-1)^{2-1}} - \frac{t}{2(2-1)(t^2-1)}$$
$$= -\frac{1}{2}\int\frac{dt}{t^2-1} + \frac{t}{2(t^2-1)} = -\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - \frac{t}{2(t^2-1)} + c$$
$$= \left[\begin{array}{l} t^2-1=(x+2)^2-1 \\ \quad = x^2+4x+3 \end{array}\right] = -\frac{1}{4}\ln\left|\frac{x+1}{x+3}\right| - \frac{x+2}{2(x^2+4x+3)} + c,\ x\in R,\ x\neq -2\pm 1,\ c\in R.$$

Riešené príklady – 094

$$\int \frac{dx}{(x^2-4x+3)^2} = -\frac{1}{4}\ln\left|\frac{x-3}{x-1}\right| - \frac{x-2}{2(x^2-4x+3)} + c$$ [094]

- $= \int \frac{dx}{[x^2-4x+4-1]^2} = \int \frac{dx}{[(x-2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t=x-2 & x\in(-\infty;1) & x\in(1;3) & x\in(3;\infty) \\ dt=dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty)\end{array}\right]$
$= \int \frac{1}{(t^2-1)^2}\,dt = -\int \frac{-1}{(t^2-1)^2}\,dt = -\int \frac{(t^2-1)-t^2}{(t^2-1)^2}\,dt = -\int \frac{dt}{t^2-1} + \frac{1}{2}\int \frac{2t^2\,dt}{(t^2-1)^2}$
$= \left[\begin{array}{l|l} u'=\frac{2t}{(t^2-1)^2} & u=\int\frac{2t\,dt}{(t^2-1)^2} = \left[\begin{array}{r|l|l} \text{Subst. } z=t^2-1 & t\in\langle 0;1)\Rightarrow z\in\langle -1;0) & t\in(1;\infty)\Rightarrow z\in(0;\infty) \\ dt=2x\,dx & t\in(-1;0\rangle\Rightarrow z\in\langle -1;0) & t\in(-\infty;-1)\Rightarrow z\in(0;\infty)\end{array}\right] = \int\frac{dz}{z^2} = \int z^{-2}\,dz = -z^{-1} = -\frac{1}{z} = -\frac{1}{t^2-1} \\ v=t & v'=1\end{array}\right]$
$= -\int \frac{dt}{t^2-1} + \frac{1}{2}\left[-\frac{t}{t^2-1} + \int\frac{dt}{t^2-1}\right] = -\frac{1}{2}\int\frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} = -\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - \frac{t}{2(t^2-1)} + c$
$= \left[\begin{array}{l} t^2-1=(x-2)^2-1 \\ \quad =x^2-4x+3\end{array}\right] = -\frac{1}{4}\ln\left|\frac{x-3}{x-1}\right| - \frac{x-2}{2(x^2-4x+3)} + c,\ x\in R,\ x\neq +2\pm 1,\ c\in R.$

- $= \int \frac{dx}{[x^2-4x+4-1]^2} = \int \frac{dx}{[(x-2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t=x-2 & x\in(-\infty;1) & x\in(1;3) & x\in(3;\infty) \\ dt=dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty)\end{array}\right]$
$= \int \frac{dt}{(t^2-1)^2} = \left[\begin{array}{l} \text{Pr 090: } \int\frac{dt}{(t^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)}\cdot\int\frac{dt}{(t^2-a^2)^{n-1}} \\ \quad -\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \text{ pre } a>0 \text{ a } n\in N,\ n\neq 1.\end{array}\right] = \frac{3-2\cdot 2}{2(2-1)}\int\frac{dt}{(t^2-1)^{2-1}} - \frac{t}{2(2-1)(t^2-1)}$
$= -\frac{1}{2}\int\frac{dt}{t^2-1} + \frac{t}{2(t^2-1)} = -\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - \frac{t}{2(t^2-1)} + c$
$= \left[\begin{array}{l} t^2-1=(x-2)^2-1 \\ \quad =x^2-4x+3\end{array}\right] = -\frac{1}{4}\ln\left|\frac{x-3}{x-1}\right| - \frac{x-2}{2(x^2-4x+3)} + c,\ x\in R,\ x\neq +2\pm 1,\ c\in R.$

Riešené príklady – 094

$$\int \frac{dx}{(x^2\pm 4x+3)^2} = -\frac{1}{4}\ln\left|\frac{x\pm 2-1}{x\pm 2+1}\right| - \frac{x\pm 2}{2(x^2\pm 4x+3)} + c$$ [094]

- $= \int \frac{dx}{[x^2\pm 4x+4-1]^2} = \int \frac{dx}{[(x\pm 2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x\pm 2 & x\in(-\infty;\mp 2-1) & x\in(\mp 2-1;\mp 2+1) & x\in(\mp 2+1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty)\end{array}\right]$

  $= \int \frac{1}{(t^2-1)^2}\,dt = -\int \frac{-1}{(t^2-1)^2}\,dt = -\int \frac{(t^2-1)-t^2}{(t^2-1)^2}\,dt = -\int \frac{dt}{t^2-1} + \frac{1}{2}\int \frac{2t^2\,dt}{(t^2-1)^2}$

  $= \left[\begin{array}{l|l} u' = \frac{2t}{(t^2-1)^2} & u = \int \frac{2t\,dt}{(t^2-1)^2} = \left[\begin{array}{r|l|l} \text{Subst. } z = t^2-1 & t\in\langle 0;1) \Rightarrow z\in\langle -1;0) & t\in(1;\infty) \Rightarrow z\in(0;\infty) \\ dt = 2x\,dx & t\in(-1;0\rangle \Rightarrow z\in\langle -1;0) & t\in(-\infty;-1) \Rightarrow z\in(0;\infty)\end{array}\right] = \int \frac{dz}{z^2} = \int z^{-2}\,dz = -z^{-1} = -\frac{1}{z} = -\frac{1}{t^2-1} \\ v = t & v' = 1 \end{array}\right]$

  $= -\int \frac{dt}{t^2-1} + \frac{1}{2}\left[-\frac{t}{t^2-1} + \int \frac{dt}{t^2-1}\right] = -\frac{1}{2}\int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} = -\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - \frac{t}{2(t^2-1)} + c$

  $= \left[\begin{array}{r} t^2-1 = (x\pm 2)^2-1 \\ = x^2\pm 4x+3 \end{array}\right] = -\frac{1}{4}\ln\left|\frac{x\pm 2-1}{x\pm 2+1}\right| - \frac{x\pm 2}{2(x^2\pm 4x+3)} + c,\ x\in R,\ x\neq \mp 2\pm 1,\ c\in R.$

- $= \int \frac{dx}{[x^2\pm 4x+4-1]^2} = \int \frac{dx}{[(x\pm 2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x\pm 2 & x\in(-\infty;\mp 2-1) & x\in(\mp 2-1;\mp 2+1) & x\in(\mp 2+1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty)\end{array}\right]$

  $= \int \frac{dt}{(t^2-1)^2} = \left[\begin{array}{l} \text{Pr 090: } \int \frac{dt}{(t^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)}\cdot\int \frac{dt}{(t^2-a^2)^{n-1}} \\ \quad -\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \text{ pre } a>0 \text{ a } n\in N,\ n\neq 1. \end{array}\right] = \frac{3-2\cdot 2}{2(2-1)}\int \frac{dt}{(t^2-1)^{2-1}} - \frac{t}{2(2-1)(t^2-1)}$

  $= -\frac{1}{2}\int \frac{dt}{t^2-1} + \frac{t}{2(t^2-1)} = -\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - \frac{t}{2(t^2-1)} + c$

  $= \left[\begin{array}{r} t^2-1 = (x\pm 2)^2-1 \\ = x^2\pm 4x+3 \end{array}\right] = -\frac{1}{4}\ln\left|\frac{x\pm 2-1}{x\pm 2+1}\right| - \frac{x\pm 2}{2(x^2\pm 4x+3)} + c,\ x\in R,\ x\neq \mp 2\pm 1,\ c\in R.$

Riešené príklady – 095, 096

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left|\frac{x-a}{x+a}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} \cdot I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$I_3 = \int \frac{dx}{(x^2-a^2)^3}$ pre $a>0$ a $n=3$. [095]

$I_4 = \int \frac{dx}{(x^2-a^2)^4}$ pre $a>0$ a $n=4$. [096]

Riešené príklady – 095, 096

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} \cdot I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$I_3 = \int \frac{dx}{(x^2-a^2)^3}$ pre $a>0$ a $n=3$. [095]

- $= \frac{3-2\cdot 3}{2a^2(3-1)} \cdot I_2 - \frac{x}{2a^2(3-1)(x^2-a^2)^{3-1}}$

$I_4 = \int \frac{dx}{(x^2-a^2)^4}$ pre $a>0$ a $n=4$. [096]

- $= \frac{3-2\cdot 4}{2a^2(4-1)} \cdot I_3 - \frac{x}{2a^2(4-1)(x^2-a^2)^{4-1}}$

Riešené príklady – 095, 096

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c,$
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} \cdot I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N,\ n \neq 1.$

[Viď Pr 089, Pr 090.]

$I_3 = \int \frac{dx}{(x^2-a^2)^3}$ pre $a > 0$ a $n = 3.$ [095]

- $= \frac{3-2\cdot 3}{2a^2(3-1)} \cdot I_2 - \frac{x}{2a^2(3-1)(x^2-a^2)^{3-1}} = \frac{-3}{4a^2} \cdot \left[\frac{3-2\cdot 2}{2a^2(2-1)} \cdot I_1 - \frac{x}{2a^2(2-1)(x^2-a^2)^{2-1}} \right] - \frac{x}{4a^2(x^2-a^2)^2}$

$I_4 = \int \frac{dx}{(x^2-a^2)^4}$ pre $a > 0$ a $n = 4.$ [096]

- $= \frac{3-2\cdot 4}{2a^2(4-1)} \cdot I_3 - \frac{x}{2a^2(4-1)(x^2-a^2)^{4-1}} = \frac{-5}{6a^2} \cdot \left[\frac{3-2\cdot 3}{2a^2(3-1)} \cdot I_2 - \frac{x}{2a^2(3-1)(x^2-a^2)^{3-1}} \right] - \frac{x}{6a^2(x^2-a^2)^3}$

  $= -\frac{5}{6a^2} \cdot \left[\frac{-3}{4a^2} \cdot I_2 - \frac{x}{4a^2(x^2-a^2)^2} \right] - \frac{x}{6a^2(x^2-a^2)^3}$

Riešené príklady – 095, 096

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} \cdot I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$I_3 = \int \frac{dx}{(x^2-a^2)^3}$ pre $a>0$ a $n=3$. [095]

- $= \frac{3-2\cdot3}{2a^2(3-1)} \cdot I_2 - \frac{x}{2a^2(3-1)(x^2-a^2)^{3-1}} = \frac{-3}{4a^2} \cdot \left[\frac{3-2\cdot2}{2a^2(2-1)} \cdot I_1 - \frac{x}{2a^2(2-1)(x^2-a^2)^{2-1}}\right] - \frac{x}{4a^2(x^2-a^2)^2}$
$= -\frac{3}{4a^2} \cdot \left[\frac{-1}{2a^2} \cdot I_1 - \frac{x}{2a^2(x^2-a^2)}\right] - \frac{x}{4a^2(x^2-a^2)^2} = \frac{3}{8a^4} \cdot I_1 + \frac{3x}{8a^4(x^2-a^2)} - \frac{x}{4a^2(x^2-a^2)^2}$

$I_4 = \int \frac{dx}{(x^2-a^2)^4}$ pre $a>0$ a $n=4$. [096]

- $= \frac{3-2\cdot4}{2a^2(4-1)} \cdot I_3 - \frac{x}{2a^2(4-1)(x^2-a^2)^{4-1}} = \frac{-5}{6a^2} \cdot \left[\frac{3-2\cdot3}{2a^2(3-1)} \cdot I_2 - \frac{x}{2a^2(3-1)(x^2-a^2)^{3-1}}\right] - \frac{x}{6a^2(x^2-a^2)^3}$
$= -\frac{5}{6a^2} \cdot \left[\frac{-3}{4a^2} \cdot I_2 - \frac{x}{4a^2(x^2-a^2)^2}\right] - \frac{x}{6a^2(x^2-a^2)^3} = \frac{5}{8a^4} \cdot I_2 + \frac{5x}{24a^4(x^2-a^2)^4} - \frac{x}{6a^2(x^2-a^2)^3}$

Riešené príklady – 095, 096

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$, • $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} \cdot I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$$I_3 = \int \frac{dx}{(x^2-a^2)^3} = \frac{3\ln\left|\frac{x-a}{x+a}\right|}{16a^5} + \frac{3x}{8a^4(x^2-a^2)} - \frac{x}{4a^2(x^2-a^2)^2} + c$$ pre $a>0$ a $n=3$. [095]

- $$= \frac{3-2\cdot 3}{2a^2(3-1)} \cdot I_2 - \frac{x}{2a^2(3-1)(x^2-a^2)^{3-1}} = \frac{-3}{4a^2} \cdot \left[\frac{3-2\cdot 2}{2a^2(2-1)} \cdot I_1 - \frac{x}{2a^2(2-1)(x^2-a^2)^{2-1}}\right] - \frac{x}{4a^2(x^2-a^2)^2}$$
$$= -\frac{3}{4a^2} \cdot \left[\frac{-1}{2a^2} \cdot I_1 - \frac{x}{2a^2(x^2-a^2)}\right] - \frac{x}{4a^2(x^2-a^2)^2} = \frac{3}{8a^4} \cdot I_1 + \frac{3x}{8a^4(x^2-a^2)} - \frac{x}{4a^2(x^2-a^2)^2}$$
$$= \frac{3}{16a^5} \ln\left|\frac{x-a}{x+a}\right| + \frac{3x}{8a^4(x^2-a^2)} - \frac{x}{4a^2(x^2-a^2)^2} + c, \; x \in R, \; x \neq \pm a, \; c \in R.$$

$$I_4 = \int \frac{dx}{(x^2-a^2)^4}$$ pre $a>0$ a $n=4$. [096]

- $$= \frac{3-2\cdot 4}{2a^2(4-1)} \cdot I_3 - \frac{x}{2a^2(4-1)(x^2-a^2)^{4-1}} = \frac{-5}{6a^2} \cdot \left[\frac{3-2\cdot 3}{2a^2(3-1)} \cdot I_2 - \frac{x}{2a^2(3-1)(x^2-a^2)^{3-1}}\right] - \frac{x}{6a^2(x^2-a^2)^3}$$
$$= -\frac{5}{6a^2} \cdot \left[\frac{-3}{4a^2} \cdot I_2 - \frac{x}{4a^2(x^2-a^2)^2}\right] - \frac{x}{6a^2(x^2-a^2)^3} = \frac{5}{8a^4} \cdot I_2 + \frac{5x}{24a^4(x^2-a^2)^4} - \frac{x}{6a^2(x^2-a^2)^3}$$
$$= \frac{5}{8a^4} \cdot \left[\frac{3-2\cdot 2}{2a^2(2-1)} \cdot I_1 - \frac{x}{2a^2(2-1)(x^2-a^2)^{2-1}}\right] + \frac{5x}{24a^4(x^2-a^2)^4} - \frac{x}{6a^2(x^2-a^2)^3}$$

Riešené príklady – 095, 096

• $I_1=\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c$, • $I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{3-2n}{2a^2(n-1)}\cdot I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a>0$ a $n\in N,\ n\neq 1$.

[Viď Pr 089, Pr 090.]

$I_3=\int\frac{dx}{(x^2-a^2)^3}=\frac{3\ln\left|\frac{x-a}{x+a}\right|}{16a^5}+\frac{3x}{8a^4(x^2-a^2)}-\frac{x}{4a^2(x^2-a^2)^2}+c$ pre $a>0$ a $n=3$. [095]

• $$=\frac{3-2\cdot3}{2a^2(3-1)}\cdot I_2-\frac{x}{2a^2(3-1)(x^2-a^2)^{3-1}}=\frac{-3}{4a^2}\cdot\left[\frac{3-2\cdot2}{2a^2(2-1)}\cdot I_1-\frac{x}{2a^2(2-1)(x^2-a^2)^{2-1}}\right]-\frac{x}{4a^2(x^2-a^2)^2}$$

$$=-\frac{3}{4a^2}\cdot\left[\frac{-1}{2a^2}\cdot I_1-\frac{x}{2a^2(x^2-a^2)}\right]-\frac{x}{4a^2(x^2-a^2)^2}=\frac{3}{8a^4}\cdot I_1+\frac{3x}{8a^4(x^2-a^2)}-\frac{x}{4a^2(x^2-a^2)^2}$$

$$=\frac{3}{16a^5}\ln\left|\frac{x-a}{x+a}\right|+\frac{3x}{8a^4(x^2-a^2)}-\frac{x}{4a^2(x^2-a^2)^2}+c,\ x\in R,\ x\neq\pm a,\ c\in R.$$

$I_4=\int\frac{dx}{(x^2-a^2)^4}=-\frac{5\ln\left|\frac{x-a}{x+a}\right|}{32a^7}-\frac{5x}{16a^6(x^2-a^2)}+\frac{5x}{24a^4(x^2-a^2)^2}-\frac{x}{6a^2(x^2-a^2)^3}+c$ pre $a>0$ a $n=4$. [096]

• $$=\frac{3-2\cdot4}{2a^2(4-1)}\cdot I_3-\frac{x}{2a^2(4-1)(x^2-a^2)^{4-1}}=\frac{-5}{6a^2}\cdot\left[\frac{3-2\cdot3}{2a^2(3-1)}\cdot I_2-\frac{x}{2a^2(3-1)(x^2-a^2)^{3-1}}\right]-\frac{x}{6a^2(x^2-a^2)^3}$$

$$=-\frac{5}{6a^2}\cdot\left[\frac{-3}{4a^2}\cdot I_2-\frac{x}{4a^2(x^2-a^2)^2}\right]-\frac{x}{6a^2(x^2-a^2)^3}=\frac{5}{8a^4}\cdot I_2+\frac{5x}{24a^4(x^2-a^2)^4}-\frac{x}{6a^2(x^2-a^2)^3}$$

$$=\frac{5}{8a^4}\cdot\left[\frac{3-2\cdot2}{2a^2(2-1)}\cdot I_1-\frac{x}{2a^2(2-1)(x^2-a^2)^{2-1}}\right]+\frac{5x}{24a^4(x^2-a^2)^4}-\frac{x}{6a^2(x^2-a^2)^3}$$

$$=-\frac{5}{32a^7}\ln\left|\frac{x-a}{x+a}\right|-\frac{5x}{16a^6(x^2-a^2)}+\frac{5x}{24a^4(x^2-a^2)^2}-\frac{x}{6a^2(x^2-a^2)^3}+c,\ x\in R,\ x\neq\pm a,\ c\in R.$$

Riešené príklady – 097

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$, • $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2-3} = \frac{1}{2\sqrt{3}} \ln \left| \frac{x-\sqrt{3}}{x+\sqrt{3}} \right| + c$, • $I_n = \int \frac{dx}{(x^2-3)^n} = \frac{3-2n}{6(n-1)} I_{n-1} - \frac{x}{6(n-1)(x^2-3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-3)^6}$ pre $n=6$ [097]

Riešené príklady – 097

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2-3} = \frac{1}{2\sqrt{3}} \ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-3)^n} = \frac{3-2n}{6(n-1)} I_{n-1} - \frac{x}{6(n-1)(x^2-3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-3$. [097]

Riešené príklady – 097

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2-3} = \frac{1}{2\sqrt{3}} \ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-3)^n} = \frac{3-2n}{6(n-1)} I_{n-1} - \frac{x}{6(n-1)(x^2-3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-3$. [097]

- $= \frac{-9}{6\cdot 5} I_5 - \frac{x}{6\cdot 5\alpha^5}$

$$= \frac{-9}{6\cdot 5}\Big[I_5 \qquad \Big] - \frac{x}{6\cdot 5\alpha^5}$$

Riešené príklady – 097

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c,$
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N,\ n \neq 1.$
- $I_1 = \int \frac{dx}{x^2-3} = \frac{1}{2\sqrt{3}} \ln \left| \frac{x-\sqrt{3}}{x+\sqrt{3}} \right| + c,$
- $I_n = \int \frac{dx}{(x^2-3)^n} = \frac{3-2n}{6(n-1)} I_{n-1} - \frac{x}{6(n-1)(x^2-3)^{n-1}}$ pre $n \in N,\ n \neq 1.$ [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-3.$ [097]

- $= \frac{-9}{6{\cdot}5} I_5 - \frac{x}{6{\cdot}5\alpha^5} = \frac{-9}{6{\cdot}5} \left[\frac{-7}{6{\cdot}4} I_4 - \frac{x}{6{\cdot}4\alpha^4} \right] - \frac{x}{6{\cdot}5\alpha^5}$

$$= \frac{-9}{6{\cdot}5} \left[\frac{-7}{6{\cdot}4} \left[I_4 \qquad \right] - \frac{x}{6{\cdot}4\alpha^4} \right] - \frac{x}{6{\cdot}5\alpha^5}$$

Riešené príklady – 097

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left|\frac{x-a}{x+a}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2-3} = \frac{1}{2\sqrt{3}} \ln \left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-3)^n} = \frac{3-2n}{6(n-1)} I_{n-1} - \frac{x}{6(n-1)(x^2-3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-3$. [097]

- $= \frac{-9}{6\cdot5} I_5 - \frac{x}{6\cdot5\alpha^5} = \frac{-9}{6\cdot5}\left[\frac{-7}{6\cdot4} I_4 - \frac{x}{6\cdot4\alpha^4}\right] - \frac{x}{6\cdot5\alpha^5}$

  $= \frac{-9}{6\cdot5}\left[\frac{-7}{6\cdot4}\left[\frac{-5}{6\cdot3} I_3 - \frac{x}{6\cdot3\alpha^3}\right] - \frac{x}{6\cdot4\alpha^4}\right] - \frac{x}{6\cdot5\alpha^5}$

  $= \frac{-9}{6\cdot5}\left[\frac{-7}{6\cdot4}\left[\frac{-5}{6\cdot3}\left[I_3 \qquad\qquad \right] - \frac{x}{6\cdot3\alpha^3}\right] - \frac{x}{6\cdot4\alpha^4}\right] - \frac{x}{6\cdot5\alpha^5}$

Riešené príklady – 097

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$, • $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2-3} = \frac{1}{2\sqrt{3}} \ln \left| \frac{x-\sqrt{3}}{x+\sqrt{3}} \right| + c$, • $I_n = \int \frac{dx}{(x^2-3)^n} = \frac{3-2n}{6(n-1)} I_{n-1} - \frac{x}{6(n-1)(x^2-3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-3$. [097]

- $= \frac{-9}{6{\cdot}5} I_5 - \frac{x}{6{\cdot}5\alpha^5} = \frac{-9}{6{\cdot}5}\left[\frac{-7}{6{\cdot}4} I_4 - \frac{x}{6{\cdot}4\alpha^4}\right] - \frac{x}{6{\cdot}5\alpha^5}$

$= \frac{-9}{6{\cdot}5}\left[\frac{-7}{6{\cdot}4}\left[\frac{-5}{6{\cdot}3} I_3 - \frac{x}{6{\cdot}3\alpha^3}\right] - \frac{x}{6{\cdot}4\alpha^4}\right] - \frac{x}{6{\cdot}5\alpha^5}$

$= \frac{-9}{6{\cdot}5}\left[\frac{-7}{6{\cdot}4}\left[\frac{-5}{6{\cdot}3}\left[\frac{-3}{6{\cdot}2} I_2 - \frac{x}{6{\cdot}2\alpha^2}\right] - \frac{x}{6{\cdot}3\alpha^3}\right] - \frac{x}{6{\cdot}4\alpha^4}\right] - \frac{x}{6{\cdot}5\alpha^5}$

$= \frac{-9}{6{\cdot}5}\left[\frac{-7}{6{\cdot}4}\left[\frac{-5}{6{\cdot}3}\left[\frac{-3}{6{\cdot}2}\left[I_2 \qquad\qquad \right] - \frac{x}{6{\cdot}2\alpha^2}\right] - \frac{x}{6{\cdot}3\alpha^3}\right] - \frac{x}{6{\cdot}4\alpha^4}\right] - \frac{x}{6{\cdot}5\alpha^5}$

Riešené príklady – 097

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2-3} = \frac{1}{2\sqrt{3}} \ln \left| \frac{x-\sqrt{3}}{x+\sqrt{3}} \right| + c$,
- $I_n = \int \frac{dx}{(x^2-3)^n} = \frac{3-2n}{6(n-1)} I_{n-1} - \frac{x}{6(n-1)(x^2-3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-3$. [097]

- $= \frac{-9}{6\cdot 5} I_5 - \frac{x}{6\cdot 5\alpha^5} = \frac{-9}{6\cdot 5}\left[\frac{-7}{6\cdot 4} I_4 - \frac{x}{6\cdot 4\alpha^4}\right] - \frac{x}{6\cdot 5\alpha^5}$

$= \frac{-9}{6\cdot 5}\left[\frac{-7}{6\cdot 4}\left[\frac{-5}{6\cdot 3} I_3 - \frac{x}{6\cdot 3\alpha^3}\right] - \frac{x}{6\cdot 4\alpha^4}\right] - \frac{x}{6\cdot 5\alpha^5}$

$= \frac{-9}{6\cdot 5}\left[\frac{-7}{6\cdot 4}\left[\frac{-5}{6\cdot 3}\left[\frac{-3}{6\cdot 2} I_2 - \frac{x}{6\cdot 2\alpha^2}\right] - \frac{x}{6\cdot 3\alpha^3}\right] - \frac{x}{6\cdot 4\alpha^4}\right] - \frac{x}{6\cdot 5\alpha^5}$

$= \frac{-9}{6\cdot 5}\left[\frac{-7}{6\cdot 4}\left[\frac{-5}{6\cdot 3}\left[\frac{-3}{6\cdot 2}\left[\frac{-1}{6\cdot 1} I_1 - \frac{x}{6\cdot 1\alpha}\right] - \frac{x}{6\cdot 2\alpha^2}\right] - \frac{x}{6\cdot 3\alpha^3}\right] - \frac{x}{6\cdot 4\alpha^4}\right] - \frac{x}{6\cdot 5\alpha^5}$

$= \frac{-9}{6\cdot 5}\left[\frac{-7}{6\cdot 4}\left[\frac{-5}{6\cdot 3}\left[\frac{-3}{6\cdot 2}\left[\frac{-1}{6\cdot 1} I_1 - \frac{x}{6\cdot 1\alpha}\right] - \frac{x}{6\cdot 2\alpha^2}\right] - \frac{x}{6\cdot 3\alpha^3}\right] - \frac{x}{6\cdot 4\alpha^4}\right] - \frac{x}{6\cdot 5\alpha^5}$

Riešené príklady – 097

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$, • $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2-3} = \frac{1}{2\sqrt{3}} \ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + c$, • $I_n = \int \frac{dx}{(x^2-3)^n} = \frac{3-2n}{6(n-1)} I_{n-1} - \frac{x}{6(n-1)(x^2-3)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-3)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-3$. [097]

- $$= \frac{-9}{6\cdot5} I_5 - \frac{x}{6\cdot5\alpha^5} = \frac{-9}{6\cdot5}\left[\frac{-7}{6\cdot4} I_4 - \frac{x}{6\cdot4\alpha^4}\right] - \frac{x}{6\cdot5\alpha^5}$$
$$= \frac{-9}{6\cdot5}\left[\frac{-7}{6\cdot4}\left[\frac{-5}{6\cdot3} I_3 - \frac{x}{6\cdot3\alpha^3}\right] - \frac{x}{6\cdot4\alpha^4}\right] - \frac{x}{6\cdot5\alpha^5}$$
$$= \frac{-9}{6\cdot5}\left[\frac{-7}{6\cdot4}\left[\frac{-5}{6\cdot3}\left[\frac{-3}{6\cdot2} I_2 - \frac{x}{6\cdot2\alpha^2}\right] - \frac{x}{6\cdot3\alpha^3}\right] - \frac{x}{6\cdot4\alpha^4}\right] - \frac{x}{6\cdot5\alpha^5}$$
$$= \frac{-9}{6\cdot5}\left[\frac{-7}{6\cdot4}\left[\frac{-5}{6\cdot3}\left[\frac{-3}{6\cdot2}\left[\frac{-1}{6\cdot1} I_1 - \frac{x}{6\cdot1\alpha}\right] - \frac{x}{6\cdot2\alpha^2}\right] - \frac{x}{6\cdot3\alpha^3}\right] - \frac{x}{6\cdot4\alpha^4}\right] - \frac{x}{6\cdot5\alpha^5}$$

$$= \frac{-9}{6\cdot5}\left[\frac{-7}{6\cdot4}\left[\frac{-5}{6\cdot3}\left[\frac{-3}{6\cdot2}\left[\frac{-1}{6\cdot1} I_1 - \frac{x}{6\cdot1\alpha}\right] - \frac{x}{6\cdot2\alpha^2}\right] - \frac{x}{6\cdot3\alpha^3}\right] - \frac{x}{6\cdot4\alpha^4}\right] - \frac{x}{6\cdot5\alpha^5}$$

$$= -\frac{9\cdot7\cdot5\cdot3\cdot1}{6^5\cdot5\cdot4\cdot3\cdot2\cdot1\cdot2\sqrt{3}} \ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| - \frac{9\cdot7\cdot5\cdot3x}{6^5\cdot5\cdot4\cdot3\cdot2\cdot1\alpha} + \frac{9\cdot7\cdot5x}{6^4\cdot5\cdot4\cdot3\cdot2\alpha^2} - \frac{9\cdot7x}{6^3\cdot5\cdot4\cdot3\alpha^3} + \frac{9x}{6^2\cdot5\cdot4\alpha^4} - \frac{x}{6\cdot5\alpha^5} + c$$

Riešené príklady – 097

- $I_1=\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c$, • $I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{3-2n}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a>0$ a $n\in N$, $n\neq 1$.
- $I_1=\int\frac{dx}{x^2-3}=\frac{1}{2\sqrt{3}}\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right|+c$, • $I_n=\int\frac{dx}{(x^2-3)^n}=\frac{3-2n}{6(n-1)}I_{n-1}-\frac{x}{6(n-1)(x^2-3)^{n-1}}$ pre $n\in N$, $n\neq 1$. [Viď Pr 089, Pr 090.]

$I_6=\int\frac{dx}{(x^2-3)^6}=\int\frac{dx}{\alpha}{}^6$ pre $n=6$ a označenie $\alpha=x^2-3$. [097]

- $$=\frac{-9}{6{\cdot}5}I_5-\frac{x}{6{\cdot}5\alpha^5}=\frac{-9}{6{\cdot}5}\left[\frac{-7}{6{\cdot}4}I_4-\frac{x}{6{\cdot}4\alpha^4}\right]-\frac{x}{6{\cdot}5\alpha^5}$$

$$=\frac{-9}{6{\cdot}5}\left[\frac{-7}{6{\cdot}4}\left[\frac{-5}{6{\cdot}3}I_3-\frac{x}{6{\cdot}3\alpha^3}\right]-\frac{x}{6{\cdot}4\alpha^4}\right]-\frac{x}{6{\cdot}5\alpha^5}$$

$$=\frac{-9}{6{\cdot}5}\left[\frac{-7}{6{\cdot}4}\left[\frac{-5}{6{\cdot}3}\left[\frac{-3}{6{\cdot}2}I_2-\frac{x}{6{\cdot}2\alpha^2}\right]-\frac{x}{6{\cdot}3\alpha^3}\right]-\frac{x}{6{\cdot}4\alpha^4}\right]-\frac{x}{6{\cdot}5\alpha^5}$$

$$=\frac{-9}{6{\cdot}5}\left[\frac{-7}{6{\cdot}4}\left[\frac{-5}{6{\cdot}3}\left[\frac{-3}{6{\cdot}2}\left[\frac{-1}{6{\cdot}1}I_1-\frac{x}{6{\cdot}1\alpha}\right]-\frac{x}{6{\cdot}2\alpha^2}\right]-\frac{x}{6{\cdot}3\alpha^3}\right]-\frac{x}{6{\cdot}4\alpha^4}\right]-\frac{x}{6{\cdot}5\alpha^5}$$

$$=\frac{-9}{6{\cdot}5}\left[\frac{-7}{6{\cdot}4}\left[\frac{-5}{6{\cdot}3}\left[\frac{-3}{6{\cdot}2}\left[\frac{-1}{6{\cdot}1}I_1-\frac{x}{6{\cdot}1\alpha}\right]-\frac{x}{6{\cdot}2\alpha^2}\right]-\frac{x}{6{\cdot}3\alpha^3}\right]-\frac{x}{6{\cdot}4\alpha^4}\right]-\frac{x}{6{\cdot}5\alpha^5}$$

$$=-\frac{9{\cdot}7{\cdot}5{\cdot}3{\cdot}1}{6^5{\cdot}5{\cdot}4{\cdot}3{\cdot}2{\cdot}1{\cdot}2\sqrt{3}}\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right|-\frac{9{\cdot}7{\cdot}5{\cdot}3x}{6^5{\cdot}5{\cdot}4{\cdot}3{\cdot}2{\cdot}1\alpha}+\frac{9{\cdot}7{\cdot}5x}{6^4{\cdot}5{\cdot}4{\cdot}3{\cdot}2\alpha^2}-\frac{9{\cdot}7x}{6^3{\cdot}5{\cdot}4{\cdot}3\alpha^3}+\frac{9x}{6^2{\cdot}5{\cdot}4\alpha^4}-\frac{x}{6{\cdot}5\alpha^5}+c$$

$$=\frac{9!!}{6^5{\cdot}5!{\cdot}2\sqrt{3}}\ln\left|\frac{x+\sqrt{3}}{x-\sqrt{3}}\right|-\frac{9!!}{6^5{\cdot}5!}\left[\frac{x}{x^2-3}-\frac{6{\cdot}1!x}{3!!(x^2-3)^2}+\frac{6^2{\cdot}2!x}{5!!(x^2-3)^3}-\frac{6^3{\cdot}3!x}{7!!(x^2-3)^4}+\frac{6^4{\cdot}4!x}{9!!(x^2-3)^5}\right]+c,$$

[$n!!=n{\cdot}(n-2){\cdot}(n-4)\cdots$, $n\in N$, napr. $9!!=9{\cdot}7{\cdot}5{\cdot}3{\cdot}1$, $8!!=8{\cdot}6{\cdot}4{\cdot}2$, $7!!=7{\cdot}5{\cdot}3{\cdot}1$, $6!!=6{\cdot}4{\cdot}2$, …] $x\in R$, $x\neq-\pm\sqrt{3}$, $c\in R$.

Riešené príklady – 098

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left|\frac{x-a}{x+a}\right| + c,$
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N,\ n \neq 1.$
- $I_1 = \int \frac{dx}{x^2-1} = \frac{1}{2} \ln \left|\frac{x-1}{x+1}\right| + c,$
- $I_n = \int \frac{dx}{(x^2-1)^n} = \frac{3-2n}{2(n-1)} I_{n-1} - \frac{x}{2(n-1)(x^2-1)^{n-1}}$ pre $n \in N,\ n \neq 1.$ [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-1)^6}$ pre $n=6$ [098]

Riešené príklady – 098

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$, • $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2-1} = \frac{1}{2} \ln\left|\frac{x-1}{x+1}\right| + c$, • $I_n = \int \frac{dx}{(x^2-1)^n} = \frac{3-2n}{2(n-1)} I_{n-1} - \frac{x}{2(n-1)(x^2-1)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-1)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-1$. [098]

Riešené príklady – 098

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c,$
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a>0$ a $n \in N,\ n \neq 1.$
- $I_1 = \int \frac{dx}{x^2-1} = \frac{1}{2} \ln\left|\frac{x-1}{x+1}\right| + c,$
- $I_n = \int \frac{dx}{(x^2-1)^n} = \frac{3-2n}{2(n-1)} I_{n-1} - \frac{x}{2(n-1)(x^2-1)^{n-1}}$ pre $n \in N,\ n \neq 1.$ [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-1)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-1.$ [098]

- $= \frac{-9}{2\cdot 5} I_5 - \frac{x}{2\cdot 5\alpha^5}$

$$= \frac{-9}{2\cdot 5}\Big[I_5 \qquad \Big] - \frac{x}{2\cdot 5\alpha^5}$$

Riešené príklady – 098

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c,$
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N,\ n \neq 1.$
- $I_1 = \int \frac{dx}{x^2-1} = \frac{1}{2} \ln\left|\frac{x-1}{x+1}\right| + c,$
- $I_n = \int \frac{dx}{(x^2-1)^n} = \frac{3-2n}{2(n-1)} I_{n-1} - \frac{x}{2(n-1)(x^2-1)^{n-1}}$ pre $n \in N,\ n \neq 1.$ [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-1)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-1.$ [098]

- $= \frac{-9}{2\cdot 5} I_5 - \frac{x}{2\cdot 5\alpha^5} = \frac{-9}{2\cdot 5}\left[\frac{-7}{2\cdot 4} I_4 - \frac{x}{2\cdot 4\alpha^4}\right] - \frac{x}{2\cdot 5\alpha^5}$

$$= \frac{-9}{2\cdot 5}\left[\frac{-7}{2\cdot 4}\left[I_4 \qquad \right] - \frac{x}{2\cdot 4\alpha^4}\right] - \frac{x}{2\cdot 5\alpha^5}$$

Riešené príklady – 098

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a>0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2-1} = \frac{1}{2} \ln\left|\frac{x-1}{x+1}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-1)^n} = \frac{3-2n}{2(n-1)} I_{n-1} - \frac{x}{2(n-1)(x^2-1)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-1)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-1$. [098]

- $= \frac{-9}{2\cdot 5} I_5 - \frac{x}{2\cdot 5\alpha^5} = \frac{-9}{2\cdot 5}\left[\frac{-7}{2\cdot 4} I_4 - \frac{x}{2\cdot 4\alpha^4}\right] - \frac{x}{2\cdot 5\alpha^5}$

$= \frac{-9}{2\cdot 5}\left[\frac{-7}{2\cdot 4}\left[\frac{-5}{2\cdot 3} I_3 - \frac{x}{2\cdot 3\alpha^3}\right] - \frac{x}{2\cdot 4\alpha^4}\right] - \frac{x}{2\cdot 5\alpha^5}$

$= \frac{-9}{2\cdot 5}\left[\frac{-7}{2\cdot 4}\left[\frac{-5}{2\cdot 3}\left[I_3 \qquad \right] - \frac{x}{2\cdot 3\alpha^3}\right] - \frac{x}{2\cdot 4\alpha^4}\right] - \frac{x}{2\cdot 5\alpha^5}$

Riešené príklady – 098

- $I_1=\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c,$
- $I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{3-2n}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a>0$ a $n\in N,\ n\neq 1.$
- $I_1=\int\frac{dx}{x^2-1}=\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+c,$
- $I_n=\int\frac{dx}{(x^2-1)^n}=\frac{3-2n}{2(n-1)}I_{n-1}-\frac{x}{2(n-1)(x^2-1)^{n-1}}$ pre $n\in N,\ n\neq 1.$ [Viď Pr 089, Pr 090.]

$I_6=\int\frac{dx}{(x^2-1)^6}=\int\frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha=x^2-1.$ [098]

- $$=\frac{-9}{2\cdot5}I_5-\frac{x}{2\cdot5\alpha^5}=\frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4}I_4-\frac{x}{2\cdot4\alpha^4}\right]-\frac{x}{2\cdot5\alpha^5}$$
$$=\frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4}\left[\frac{-5}{2\cdot3}I_3-\frac{x}{2\cdot3\alpha^3}\right]-\frac{x}{2\cdot4\alpha^4}\right]-\frac{x}{2\cdot5\alpha^5}$$
$$=\frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4}\left[\frac{-5}{2\cdot3}\left[\frac{-3}{2\cdot2}I_2-\frac{x}{2\cdot2\alpha^2}\right]-\frac{x}{2\cdot3\alpha^3}\right]-\frac{x}{2\cdot4\alpha^4}\right]-\frac{x}{2\cdot5\alpha^5}$$

$$=\frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4}\left[\frac{-5}{2\cdot3}\left[\frac{-3}{2\cdot2}\left[I_2\qquad\right]-\frac{x}{2\cdot2\alpha^2}\right]-\frac{x}{2\cdot3\alpha^3}\right]-\frac{x}{2\cdot4\alpha^4}\right]-\frac{x}{2\cdot5\alpha^5}$$

Riešené príklady – 098

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c,$
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N,\ n \neq 1.$
- $I_1 = \int \frac{dx}{x^2-1} = \frac{1}{2} \ln\left|\frac{x-1}{x+1}\right| + c,$
- $I_n = \int \frac{dx}{(x^2-1)^n} = \frac{3-2n}{2(n-1)} I_{n-1} - \frac{x}{2(n-1)(x^2-1)^{n-1}}$ pre $n \in N,\ n \neq 1.$ [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-1)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-1.$ [098]

- $$= \frac{-9}{2\cdot5} I_5 - \frac{x}{2\cdot5\alpha^5} = \frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4} I_4 - \frac{x}{2\cdot4\alpha^4}\right] - \frac{x}{2\cdot5\alpha^5}$$
$$= \frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4}\left[\frac{-5}{2\cdot3} I_3 - \frac{x}{2\cdot3\alpha^3}\right] - \frac{x}{2\cdot4\alpha^4}\right] - \frac{x}{2\cdot5\alpha^5}$$
$$= \frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4}\left[\frac{-5}{2\cdot3}\left[\frac{-3}{2\cdot2} I_2 - \frac{x}{2\cdot2\alpha^2}\right] - \frac{x}{2\cdot3\alpha^3}\right] - \frac{x}{2\cdot4\alpha^4}\right] - \frac{x}{2\cdot5\alpha^5}$$
$$= \frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4}\left[\frac{-5}{2\cdot3}\left[\frac{-3}{2\cdot2}\left[\frac{-1}{2\cdot1} I_1 - \frac{x}{2\cdot1\alpha}\right] - \frac{x}{2\cdot2\alpha^2}\right] - \frac{x}{2\cdot3\alpha^3}\right] - \frac{x}{2\cdot4\alpha^4}\right] - \frac{x}{2\cdot5\alpha^5}$$

$$= \frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4}\left[\frac{-5}{2\cdot3}\left[\frac{-3}{2\cdot2}\left[\frac{-1}{2\cdot1} I_1 - \frac{x}{2\cdot1\alpha}\right] - \frac{x}{2\cdot2\alpha^2}\right] - \frac{x}{2\cdot3\alpha^3}\right] - \frac{x}{2\cdot4\alpha^4}\right] - \frac{x}{2\cdot5\alpha^5}$$

Riešené príklady – 098

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2-1} = \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + c$,
- $I_n = \int \frac{dx}{(x^2-1)^n} = \frac{3-2n}{2(n-1)} I_{n-1} - \frac{x}{2(n-1)(x^2-1)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-1)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-1$. [098]

- $= \frac{-9}{2\cdot 5} I_5 - \frac{x}{2\cdot 5\alpha^5} = \frac{-9}{2\cdot 5} \left[\frac{-7}{2\cdot 4} I_4 - \frac{x}{2\cdot 4\alpha^4} \right] - \frac{x}{2\cdot 5\alpha^5}$

$= \frac{-9}{2\cdot 5} \left[\frac{-7}{2\cdot 4} \left[\frac{-5}{2\cdot 3} I_3 - \frac{x}{2\cdot 3\alpha^3} \right] - \frac{x}{2\cdot 4\alpha^4} \right] - \frac{x}{2\cdot 5\alpha^5}$

$= \frac{-9}{2\cdot 5} \left[\frac{-7}{2\cdot 4} \left[\frac{-5}{2\cdot 3} \left[\frac{-3}{2\cdot 2} I_2 - \frac{x}{2\cdot 2\alpha^2} \right] - \frac{x}{2\cdot 3\alpha^3} \right] - \frac{x}{2\cdot 4\alpha^4} \right] - \frac{x}{2\cdot 5\alpha^5}$

$= \frac{-9}{2\cdot 5} \left[\frac{-7}{2\cdot 4} \left[\frac{-5}{2\cdot 3} \left[\frac{-3}{2\cdot 2} \left[\frac{-1}{2\cdot 1} I_1 - \frac{x}{2\cdot 1\alpha} \right] - \frac{x}{2\cdot 2\alpha^2} \right] - \frac{x}{2\cdot 3\alpha^3} \right] - \frac{x}{2\cdot 4\alpha^4} \right] - \frac{x}{2\cdot 5\alpha^5}$

$= \frac{-9}{2\cdot 5} \left[\frac{-7}{2\cdot 4} \left[\frac{-5}{2\cdot 3} \left[\frac{-3}{2\cdot 2} \left[\frac{-1}{2\cdot 1} I_1 - \frac{x}{2\cdot 1\alpha} \right] - \frac{x}{2\cdot 2\alpha^2} \right] - \frac{x}{2\cdot 3\alpha^3} \right] - \frac{x}{2\cdot 4\alpha^4} \right] - \frac{x}{2\cdot 5\alpha^5}$

$= -\frac{9\cdot 7\cdot 5\cdot 3\cdot 1}{2^5\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 2} \ln \left| \frac{x-1}{x+1} \right| - \frac{9\cdot 7\cdot 5\cdot 3x}{2^5\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\alpha} + \frac{9\cdot 7\cdot 5x}{2^4\cdot 5\cdot 4\cdot 3\cdot 2\alpha^2} - \frac{9\cdot 7x}{2^3\cdot 5\cdot 4\cdot 3\alpha^3} + \frac{9x}{2^2\cdot 5\cdot 4\alpha^4} - \frac{x}{2\cdot 5\alpha^5} + c$

Riešené príklady – 098

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.
- $I_1 = \int \frac{dx}{x^2-1} = \frac{1}{2} \ln\left|\frac{x-1}{x+1}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-1)^n} = \frac{3-2n}{2(n-1)} I_{n-1} - \frac{x}{2(n-1)(x^2-1)^{n-1}}$ pre $n \in N$, $n \neq 1$. [Viď Pr 089, Pr 090.]

$I_6 = \int \frac{dx}{(x^2-1)^6} = \int \frac{dx}{\alpha}^6$ pre $n=6$ a označenie $\alpha = x^2-1$. [098]

$$= \frac{-9}{2\cdot5} I_5 - \frac{x}{2\cdot5\alpha^5} = \frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4} I_4 - \frac{x}{2\cdot4\alpha^4}\right] - \frac{x}{2\cdot5\alpha^5}$$

$$= \frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4}\left[\frac{-5}{2\cdot3} I_3 - \frac{x}{2\cdot3\alpha^3}\right] - \frac{x}{2\cdot4\alpha^4}\right] - \frac{x}{2\cdot5\alpha^5}$$

$$= \frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4}\left[\frac{-5}{2\cdot3}\left[\frac{-3}{2\cdot2} I_2 - \frac{x}{2\cdot2\alpha^2}\right] - \frac{x}{2\cdot3\alpha^3}\right] - \frac{x}{2\cdot4\alpha^4}\right] - \frac{x}{2\cdot5\alpha^5}$$

$$= \frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4}\left[\frac{-5}{2\cdot3}\left[\frac{-3}{2\cdot2}\left[\frac{-1}{2\cdot1} I_1 - \frac{x}{2\cdot1\alpha}\right] - \frac{x}{2\cdot2\alpha^2}\right] - \frac{x}{2\cdot3\alpha^3}\right] - \frac{x}{2\cdot4\alpha^4}\right] - \frac{x}{2\cdot5\alpha^5}$$

$$= \frac{-9}{2\cdot5}\left[\frac{-7}{2\cdot4}\left[\frac{-5}{2\cdot3}\left[\frac{-3}{2\cdot2}\left[\frac{-1}{2\cdot1} I_1 - \frac{x}{2\cdot1\alpha}\right] - \frac{x}{2\cdot2\alpha^2}\right] - \frac{x}{2\cdot3\alpha^3}\right] - \frac{x}{2\cdot4\alpha^4}\right] - \frac{x}{2\cdot5\alpha^5}$$

$$= -\frac{9\cdot7\cdot5\cdot3\cdot1}{2^5\cdot5\cdot4\cdot3\cdot2\cdot1\cdot2} \ln\left|\frac{x-1}{x+1}\right| - \frac{9\cdot7\cdot5\cdot3x}{2^5\cdot5\cdot4\cdot3\cdot2\cdot1\alpha} + \frac{9\cdot7\cdot5x}{2^4\cdot5\cdot4\cdot3\cdot2\alpha^2} - \frac{9\cdot7x}{2^3\cdot5\cdot4\cdot3\alpha^3} + \frac{9x}{2^2\cdot5\cdot4\alpha^4} - \frac{x}{2\cdot5\alpha^5} + c$$

$$= \frac{9!!}{2^6\cdot5!} \ln\left|\frac{x+1}{x-1}\right| - \frac{9!!}{2^5\cdot5!}\left[\frac{x}{x^2-1} - \frac{2\cdot1!x}{3!!(x^2-1)^2} + \frac{2^2\cdot2!x}{5!!(x^2-1)^3} - \frac{2^3\cdot3!x}{7!!(x^2-1)^4} + \frac{2^4\cdot4!x}{9!!(x^2-1)^5}\right] + c,$$

[$n!! = n\cdot(n-2)\cdot(n-4)\cdots$, $n \in N$, napr. $9!! = 9\cdot7\cdot5\cdot3\cdot1$, $8!! = 8\cdot6\cdot4\cdot2$, $7!! = 7\cdot5\cdot3\cdot1$, $6!! = 6\cdot4\cdot2$, $\ldots$] $x \in R$, $x \neq -\pm\sqrt{3}$, $c \in R$.

Riešené príklady – 099, 100

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^2}$$ [099]

$$\int \frac{dx}{(x^2+4x+2)^2}$$ [100]

Riešené príklady – 099, 100

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left|\frac{x-a}{x+a}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^2}$$ [099]

- $= \int \frac{dx}{[(x^2+4x+4)-1]^2} = \int \frac{dx}{[(x+2)^2-1]^2}$

$$\int \frac{dx}{(x^2+4x+2)^2}$$ [100]

- $= \int \frac{dx}{[(x^2+4x+4)-2]^2} = \int \frac{dx}{[(x+2)^2-2]^2}$

Riešené príklady – 099, 100

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$, • $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^2}$$ [099]

- $= \int \frac{dx}{[(x^2+4x+4)-1]^2} = \int \frac{dx}{[(x+2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x \in (-\infty; -3) & x \in (-3; -1) & x \in (-1; \infty) \\ dt = dx & t \in (-\infty; -1) & t \in (-1; 1) & t \in (1; \infty) \end{array} \right]$
$= \int \frac{dt}{(t^2-1)^2}$

$$\int \frac{dx}{(x^2+4x+2)^2}$$ [100]

- $= \int \frac{dx}{[(x^2+4x+4)-2]^2} = \int \frac{dx}{[(x+2)^2-2]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x \in (-\infty; -2-\sqrt{2}) & x \in (-2-\sqrt{2}; -2+\sqrt{2}) & x \in (-2+\sqrt{2}; \infty) \\ dt = dx & t \in (-\infty; -1) & t \in (-1; 1) & t \in (1; \infty) \end{array} \right]$
$= \int \frac{dt}{(t^2-2)^2}$

Riešené príklady – 099, 100

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^2}$$ [099]

- $= \int \frac{dx}{[(x^2+4x+4)-1]^2} = \int \frac{dx}{[(x+2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty)\end{array}\right]$
  $= \int \frac{dt}{(t^2-1)^2} = \left[\begin{array}{r} \text{Pr 090: } n=2, \\ a=1. \end{array}\right] = -\frac{1}{2}\int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)}$

$$\int \frac{dx}{(x^2+4x+2)^2}$$ [100]

- $= \int \frac{dx}{[(x^2+4x+4)-2]^2} = \int \frac{dx}{[(x+2)^2-2]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x\in(-\infty;-2-\sqrt{2}) & x\in(-2-\sqrt{2};-2+\sqrt{2}) & x\in(-2+\sqrt{2};\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty)\end{array}\right]$
  $= \int \frac{dt}{(t^2-2)^2} = \left[\begin{array}{r} \text{Pr 090: } n=2, \\ a=\sqrt{2}. \end{array}\right] = -\frac{1}{4}\int \frac{dt}{t^2-2} - \frac{t}{4(t^2-2)}$

Riešené príklady – 099, 100

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^2}$$ [099]

- $= \int \frac{dx}{[(x^2+4x+4)-1]^2} = \int \frac{dx}{[(x+2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x \in (-\infty; -3) & x \in (-3; -1) & x \in (-1; \infty) \\ dt = dx & t \in (-\infty; -1) & t \in (-1; 1) & t \in (1; \infty) \end{array} \right]$
  $= \int \frac{dt}{(t^2-1)^2} = \left[\begin{array}{r} \text{Pr 090: } n = 2, \\ a = 1. \end{array} \right] = -\frac{1}{2} \int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} = -\frac{1}{2\cdot 2} \ln\left|\frac{t-1}{t+1}\right| - \frac{t}{2(t^2-1)} + c$

$$\int \frac{dx}{(x^2+4x+2)^2}$$ [100]

- $= \int \frac{dx}{[(x^2+4x+4)-2]^2} = \int \frac{dx}{[(x+2)^2-2]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x \in (-\infty; -2-\sqrt{2}) & x \in (-2-\sqrt{2}; -2+\sqrt{2}) & x \in (-2+\sqrt{2}; \infty) \\ dt = dx & t \in (-\infty; -1) & t \in (-1; 1) & t \in (1; \infty) \end{array} \right]$
  $= \int \frac{dt}{(t^2-2)^2} = \left[\begin{array}{r} \text{Pr 090: } n = 2, \\ a = \sqrt{2}. \end{array} \right] = -\frac{1}{4} \int \frac{dt}{t^2-2} - \frac{t}{4(t^2-2)} = -\frac{1}{4\cdot 2} \ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| - \frac{t}{4(t^2-4)} + c$

Riešené príklady – 099, 100

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left|\frac{x-a}{x+a}\right| + c$, • $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^2} = \frac{1}{4} \ln \left|\frac{x+3}{x+1}\right| - \frac{x+2}{2(x^2+4x+3)} + c$$ [099]

- $= \int \frac{dx}{[(x^2+4x+4)-1]^2} = \int \frac{dx}{[(x+2)^2-1]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x \in (-\infty; -3) & x \in (-3; -1) & x \in (-1; \infty) \\ dt = dx & t \in (-\infty; -1) & t \in (-1; 1) & t \in (1; \infty) \end{array} \right]$

$= \int \frac{dt}{(t^2-1)^2} = \left[\begin{array}{r} \text{Pr 090: } n = 2, \\ a = 1. \end{array} \right] = -\frac{1}{2} \int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} = -\frac{1}{2 \cdot 2} \ln \left|\frac{t-1}{t+1}\right| - \frac{t}{2(t^2-1)} + c$

$= \left[\begin{array}{l} t-1 = x+2-1 = x+1 \\ t+1 = x+2+1 = x+3 \end{array} \right] = -\frac{1}{4} \ln \left|\frac{x+1}{x+3}\right| - \frac{x+2}{2(x^2+4x+3)} + c$, $x \in R$, $x \neq -1$, $x \neq -3$, $c \in R$.

$$\int \frac{dx}{(x^2+4x+2)^2} = \frac{1}{8\sqrt{2}} \ln \left|\frac{x+2+\sqrt{2}}{x+2-\sqrt{2}}\right| - \frac{x+2}{4(x^2+4x+2)} + c$$ [100]

- $= \int \frac{dx}{[(x^2+4x+4)-2]^2} = \int \frac{dx}{[(x+2)^2-2]^2} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x \in (-\infty; -2-\sqrt{2}) & x \in (-2-\sqrt{2}; -2+\sqrt{2}) & x \in (-2+\sqrt{2}; \infty) \\ dt = dx & t \in (-\infty; -1) & t \in (-1; 1) & t \in (1; \infty) \end{array} \right]$

$= \int \frac{dt}{(t^2-2)^2} = \left[\begin{array}{r} \text{Pr 090: } n = 2, \\ a = \sqrt{2}. \end{array} \right] = -\frac{1}{4} \int \frac{dt}{t^2-2} - \frac{t}{4(t^2-2)} = -\frac{1}{4 \cdot 2} \ln \left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| - \frac{t}{4(t^2-4)} + c$

$= \left[\begin{array}{l} t-\sqrt{2} = x+2-\sqrt{2} \\ t+\sqrt{2} = x+2+\sqrt{2} \end{array} \right] = -\frac{1}{8\sqrt{2}} \ln \left|\frac{x+2-\sqrt{2}}{x+2+\sqrt{2}}\right| - \frac{x+2}{4(x^2+4x+3)} + c$, $x \in R$, $x \neq -2 \pm \sqrt{2}$, $c \in R$.

Riešené príklady – 101, 102

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c,$
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N,\ n \neq 1.$

[Viď Pr 078, Pr 079.]

$$\int \frac{dx}{(x^2+4x+5)^2}$$ 1 [101]

$$\int \frac{dx}{(x^2+4x+6)^2}$$ [102]

Riešené príklady – 101, 102

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 078, Pr 079.]

$$\int \frac{dx}{(x^2+4x+5)^2}$$ 2 [101]

- $= \int \frac{dx}{[(x^2+4x+4)+1]^2} = \int \frac{dx}{[(x+2)^2+1]^2}$

$$\int \frac{dx}{(x^2+4x+6)^2}$$ [102]

- $= \int \frac{dx}{[(x^2+4x+4)+2]^2} = \int \frac{dx}{[(x+2)^2+2]^2}$

Riešené príklady – 101, 102

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 078, Pr 079.]

$$\int \frac{dx}{(x^2+4x+5)^2}$$ 3 [101]

- $= \int \frac{dx}{[(x^2+4x+4)+1]^2} = \int \frac{dx}{[(x+2)^2+1]^2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2}$

$$\int \frac{dx}{(x^2+4x+6)^2}$$ [102]

- $= \int \frac{dx}{[(x^2+4x+4)+2]^2} = \int \frac{dx}{[(x+2)^2+2]^2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{dt}{(t^2+2)^2}$

Riešené príklady – 101, 102

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c,$
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 078, Pr 079.]

$\int \frac{dx}{(x^2+4x+5)^2}$ 4 [101]

- $= \int \frac{dx}{[(x^2+4x+4)+1]^2} = \int \frac{dx}{[(x+2)^2+1]^2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2}$

  $= \left[\begin{array}{r} \text{Pr 079: } n = 2, \\ a = 1. \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)}$

$\int \frac{dx}{(x^2+4x+6)^2}$ [102]

- $= \int \frac{dx}{[(x^2+4x+4)+2]^2} = \int \frac{dx}{[(x+2)^2+2]^2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{dt}{(t^2+2)^2}$

  $= \left[\begin{array}{r} \text{Pr 079: } n = 2, \\ a = \sqrt{2}. \end{array} \right] = \frac{1}{4} \int \frac{dt}{t^2+2} + \frac{t}{4(t^2+2)}$

Riešené príklady – 101, 102

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 078, Pr 079.]

$\int \frac{dx}{(x^2+4x+5)^2}$ 5 [101]

- $= \int \frac{dx}{[(x^2+4x+4)+1]^2} = \int \frac{dx}{[(x+2)^2+1]^2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2}$

$= \left[\begin{array}{r} \text{Pr 079: } n = 2, \\ a = 1. \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = -\frac{1}{2} \operatorname{arctg} t + \frac{t}{2(t^2+1)} + c$

$\int \frac{dx}{(x^2+4x+6)^2}$ [102]

- $= \int \frac{dx}{[(x^2+4x+4)+2]^2} = \int \frac{dx}{[(x+2)^2+2]^2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{dt}{(t^2+2)^2}$

$= \left[\begin{array}{r} \text{Pr 079: } n = 2, \\ a = \sqrt{2}. \end{array} \right] = \frac{1}{4} \int \frac{dt}{t^2+2} + \frac{t}{4(t^2+2)} = \frac{1}{4\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + \frac{t}{4(t^2+2)} + c$

Riešené príklady – 101, 102

- $I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$,
- $I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 078, Pr 079.]

$$\int \frac{dx}{(x^2+4x+5)^2} = \frac{1}{2} \operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c$$ 6 [101]

- $= \int \frac{dx}{[(x^2+4x+4)+1]^2} = \int \frac{dx}{[(x+2)^2+1]^2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2}$

$= \left[\begin{array}{r} \text{Pr 079: } n = 2, \\ a = 1. \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = -\frac{1}{2} \operatorname{arctg} t + \frac{t}{2(t^2+1)} + c$

$= \frac{1}{2} \operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c,\ x \in R,\ c \in R.$

$$\int \frac{dx}{(x^2+4x+6)^2} = \frac{1}{4\sqrt{2}} \operatorname{arctg} \frac{x+2}{\sqrt{2}} + \frac{x+2}{4(x^2+4x+6)} + c$$ [102]

- $= \int \frac{dx}{[(x^2+4x+4)+2]^2} = \int \frac{dx}{[(x+2)^2+2]^2} = \left[\begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right] = \int \frac{dt}{(t^2+2)^2}$

$= \left[\begin{array}{r} \text{Pr 079: } n = 2, \\ a = \sqrt{2}. \end{array} \right] = \frac{1}{4} \int \frac{dt}{t^2+2} + \frac{t}{4(t^2+2)} = \frac{1}{4\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + \frac{t}{4(t^2+2)} + c$

$= \frac{1}{4\sqrt{2}} \operatorname{arctg} \frac{x+2}{\sqrt{2}} + \frac{x+2}{4(x^2+4x+5)} + c,\ x \in R,\ c \in R.$

Riešené príklady – 103, 104

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^3}$$ [103]

$$\int \frac{dx}{(x^2+4x+3)^4}$$ [104]

Riešené príklady – 103, 104

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$,
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^3}$$ [103]

- $= \int \frac{dx}{[(x^2+4x+4)-1]^3} = \int \frac{dx}{[(x+2)^2-1]^3}$

$$\int \frac{dx}{(x^2+4x+3)^4}$$ [104]

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Riešené príklady – 103, 104

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c,$
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N,\ n \neq 1.$

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^3}$$ [103]

- $= \int \frac{dx}{[(x^2+4x+4)-1]^3} = \int \frac{dx}{[(x+2)^2-1]^3} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array} \right] = \int \frac{dt}{(t^2-1)^3}$

$$\int \frac{dx}{(x^2+4x+3)^4}$$ [104]

- $= \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array} \right] = \int \frac{dt}{(t^2-1)^4}$

Riešené príklady – 103, 104

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left|\frac{x-a}{x+a}\right| + c,$
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N,\ n \neq 1.$

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^3}$$ [103]

- $= \int \frac{dx}{[(x^2+4x+4)-1]^3} = \int \frac{dx}{[(x+2)^2-1]^3} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array} \right] = \int \frac{dt}{(t^2-1)^3}$

  $= \left[\begin{array}{l} \text{Pr 090: } a = 1, \\ n = 3, \end{array} \right] = -\frac{3}{4} \int \frac{dt}{(t^2-1)^2} - \frac{t}{4(t^2-1)^2}$

$$\int \frac{dx}{(x^2+4x+3)^4}$$ [104]

- $= \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array} \right] = \int \frac{dt}{(t^2-1)^4} = \left[\begin{array}{r} \text{Pr 090: } a = 1, \\ n = 4. \end{array} \right] = -\frac{5}{6} \int \frac{dt}{(t^2-1)^3} - \frac{t}{6(t^2-1)^3}$

Riešené príklady – 103, 104

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c,$
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^3}$$ [103]

- $$= \int \frac{dx}{[(x^2+4x+4)-1]^3} = \int \frac{dx}{[(x+2)^2-1]^3} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array} \right] = \int \frac{dt}{(t^2-1)^3}$$
$$= \left[\begin{array}{c} \text{Pr 090: } a=1, \\ n=3,\ n=2. \end{array} \right] = -\frac{3}{4}\int \frac{dt}{(t^2-1)^2} - \frac{t}{4(t^2-1)^2} = -\frac{3}{4}\left[-\frac{1}{2}\int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)}\right] - \frac{t}{4(t^2-1)^2}$$

$$\int \frac{dx}{(x^2+4x+3)^4}$$ [104]

- $$= \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array} \right] = \int \frac{dt}{(t^2-1)^4} = \left[\begin{array}{r} \text{Pr 090: } a=1, \\ n=4. \end{array} \right] = -\frac{5}{6}\int \frac{dt}{(t^2-1)^3} - \frac{t}{6(t^2-1)^3}$$

Riešené príklady – 103, 104

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c,$
- $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a > 0$ a $n \in N,\ n \neq 1.$

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^3}$$ [103]

- $= \int \frac{dx}{[(x^2+4x+4)-1]^3} = \int \frac{dx}{[(x+2)^2-1]^3} = \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array} \right] = \int \frac{dt}{(t^2-1)^3}$

$= \left[\begin{array}{r} \text{Pr 090: } a = 1, \\ n = 3,\ n = 2. \end{array} \right] = -\frac{3}{4} \int \frac{dt}{(t^2-1)^2} - \frac{t}{4(t^2-1)^2} = -\frac{3}{4} \left[-\frac{1}{2} \int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} \right] - \frac{t}{4(t^2-1)^2}$

$= \frac{3}{8} \int \frac{dt}{t^2-1} + \frac{3t}{8(t^2-1)} - \frac{t}{4(t^2-1)^2}$

$$\int \frac{dx}{(x^2+4x+3)^4}$$ [104]

- $= \left[\begin{array}{r|l|l|l} \text{Subst. } t = x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt = dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array} \right] = \int \frac{dt}{(t^2-1)^4} = \left[\begin{array}{r} \text{Pr 090: } a = 1, \\ n = 4. \end{array} \right] = -\frac{5}{6} \int \frac{dt}{(t^2-1)^3} - \frac{t}{6(t^2-1)^3}$

Riešené príklady – 103, 104

- $I_1 = \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$, • $I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{3-2n}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a>0$ a $n \in N$, $n \neq 1$.

[Viď Pr 089, Pr 090.]

$$\int \frac{dx}{(x^2+4x+3)^3}$$ [103]

- $= \int \frac{dx}{[(x^2+4x+4)-1]^3} = \int \frac{dx}{[(x+2)^2-1]^3} = \left[\begin{array}{r|l|l|l} \text{Subst. } t=x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt=dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array}\right] = \int \frac{dt}{(t^2-1)^3}$

$= \left[\begin{array}{c} \text{Pr 090: } a=1, \\ n=3,\ n=2. \end{array}\right] = -\frac{3}{4}\int \frac{dt}{(t^2-1)^2} - \frac{t}{4(t^2-1)^2} = -\frac{3}{4}\left[-\frac{1}{2}\int\frac{dt}{t^2-1} - \frac{t}{2(t^2-1)}\right] - \frac{t}{4(t^2-1)^2}$

$= \frac{3}{8}\int\frac{dt}{t^2-1} + \frac{3t}{8(t^2-1)} - \frac{t}{4(t^2-1)^2} = \frac{3}{8\cdot 2}\ln\left|\frac{t-1}{t+1}\right| + \frac{3t}{8(t^2-1)} - \frac{t}{4(t^2-1)^2} + c$

$$\int \frac{dx}{(x^2+4x+3)^4}$$ [104]

- $= \left[\begin{array}{r|l|l|l} \text{Subst. } t=x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty) \\ dt=dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty) \end{array}\right] = \int \frac{dt}{(t^2-1)^4} = \left[\begin{array}{c} \text{Pr 090: } a=1, \\ n=4. \end{array}\right] = -\frac{5}{6}\int\frac{dt}{(t^2-1)^3} - \frac{t}{6(t^2-1)^3}$

$= -\frac{5}{6}\left[\frac{3}{8\cdot 2}\ln\left|\frac{t-1}{t+1}\right| + \frac{3t}{8(t^2-1)} - \frac{t}{4(t^2-1)^2}\right] - \frac{t}{6(t^2-1)^3} + c$

Riešené príklady – 103, 104

- $I_1=\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c$, • $I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{3-2n}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$ pre $a>0$ a $n\in N$, $n\neq 1$.

[Viď Pr 089, Pr 090.]

$$\int\frac{dx}{(x^2+4x+3)^3}=\frac{3}{16}\ln\left|\frac{x+1}{x+3}\right|+\frac{3(x+2)}{8(x^2+4x+3)}-\frac{x+2}{4(x^2+4x+3)^2}+c$$ [103]

- $=\int\frac{dx}{[(x^2+4x+4)-1]^3}=\int\frac{dx}{[(x+2)^2-1]^3}=\left[\begin{array}{r|l|l|l}\text{Subst. } t=x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty)\\ dt=dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty)\end{array}\right]=\int\frac{dt}{(t^2-1)^3}$

$=\left[\begin{array}{l}\text{Pr 090: } a=1,\\ n=3,\ n=2.\end{array}\right]=-\frac{3}{4}\int\frac{dt}{(t^2-1)^2}-\frac{t}{4(t^2-1)^2}=-\frac{3}{4}\left[-\frac{1}{2}\int\frac{dt}{t^2-1}-\frac{t}{2(t^2-1)}\right]-\frac{t}{4(t^2-1)^2}$

$=\frac{3}{8}\int\frac{dt}{t^2-1}+\frac{3t}{8(t^2-1)}-\frac{t}{4(t^2-1)^2}=\frac{3}{8\cdot 2}\ln\left|\frac{t-1}{t+1}\right|+\frac{3t}{8(t^2-1)}-\frac{t}{4(t^2-1)^2}+c=\left[\begin{array}{l}t-1=x+2-1=x+1\\ t+1=x+2+1=x+3\end{array}\right]$

$=\frac{3}{16}\ln\left|\frac{x+1}{x+3}\right|+\frac{3(x+2)}{8(x^2+4x+3)}-\frac{x+2}{4(x^2+4x+3)^2}+c$, $x\in R-\{-1,-3\}$, $c\in R$.

$$\int\frac{dx}{(x^2+4x+3)^4}=\frac{5}{32}\ln\left|\frac{x+3}{x+1}\right|-\frac{5(x+2)}{16(x^2+4x+3)}+\frac{5(x+2)}{24(x^2+4x+3)^2}-\frac{x+2}{6(x^2+4x+3)^3}$$ [104]

- $=\left[\begin{array}{r|l|l|l}\text{Subst. } t=x+2 & x\in(-\infty;-3) & x\in(-3;-1) & x\in(-1;\infty)\\ dt=dx & t\in(-\infty;-1) & t\in(-1;1) & t\in(1;\infty)\end{array}\right]=\int\frac{dt}{(t^2-1)^4}=\left[\begin{array}{r}\text{Pr 090: } a=1,\\ n=4.\end{array}\right]=-\frac{5}{6}\int\frac{dt}{(t^2-1)^3}-\frac{t}{6(t^2-1)^3}$

$=-\frac{5}{6}\left[\frac{3}{8\cdot 2}\ln\left|\frac{t-1}{t+1}\right|+\frac{3t}{8(t^2-1)}-\frac{t}{4(t^2-1)^2}\right]-\frac{t}{6(t^2-1)^3}+c=\left[\begin{array}{l}t-1=x+2-1=x+1\\ t+1=x+2+1=x+3\end{array}\right]$

$=-\frac{5}{32}\ln\left|\frac{x+1}{x+3}\right|-\frac{5(x+2)}{16(x^2+4x+3)}+\frac{5(x+2)}{24(x^2+4x+3)^2}-\frac{x+2}{6(x^2+4x+3)^3}+c$, $x\in R-\{-1,-3\}$, $c\in R$.

Riešené príklady – 105, 106, 107

$$\int \frac{dx}{(1-x)x^2}$$ [105]

$$\int \frac{dx}{(x+1)x^2}$$ [106]

$$\int \frac{dx}{(x+2)x^2}$$ [107]

Riešené príklady – 105, 106, 107

$$\int \frac{dx}{(1-x)x^2}$$

[105]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \displaystyle\int \left[\frac{1}{x} + \frac{1}{x^2} - \frac{1}{x-1}\right] dx$

$$\int \frac{dx}{(x+1)x^2}$$

[106]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \displaystyle\int \left[\frac{1}{x+1} - \frac{1}{x} + \frac{1}{x^2}\right] dx$

$$\int \frac{dx}{(x+2)x^2}$$

[107]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \displaystyle\int \left[\frac{1}{4(x+2)} - \frac{1}{4x} + \frac{1}{2x^2}\right] dx$

Riešené príklady – 105, 106, 107

$$\int \frac{dx}{(1-x)x^2}$$ [105]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \displaystyle\int \Big[\frac{1}{x} + \frac{1}{x^2} - \frac{1}{x-1}\Big] dx = \int \Big[\frac{1}{x} - \frac{1}{x-1} + x^{-2}\Big] dx$

$$\int \frac{dx}{(x+1)x^2}$$ [106]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \displaystyle\int \Big[\frac{1}{x+1} - \frac{1}{x} + \frac{1}{x^2}\Big] dx = \int \Big[\frac{1}{x+1} - \frac{1}{x} + x^{-2}\Big] dx$

$$\int \frac{dx}{(x+2)x^2}$$ [107]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \displaystyle\int \Big[\frac{1}{4(x+2)} - \frac{1}{4x} + \frac{1}{2x^2}\Big] dx = \int \Big[\frac{1}{4(x+2)} - \frac{1}{4x} + \frac{x^{-2}}{2}\Big] dx$

Riešené príklady – 105, 106, 107

$$\int \frac{dx}{(1-x)x^2} = \ln\left|\frac{x}{x-1}\right| - \frac{1}{x} + c$$ [105]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \left[\frac{1}{x} + \frac{1}{x^2} - \frac{1}{x-1}\right] dx = \int \left[\frac{1}{x} - \frac{1}{x-1} + x^{-2}\right] dx$
  $= \ln|x| - \ln|x-1| + \frac{x^{-1}}{-1} + c = \ln\left|\frac{x}{x-1}\right| - \frac{1}{x} + c,\ x \in R,\ x \neq 0,\ x \neq 1,\ c \in R.$

$$\int \frac{dx}{(x+1)x^2} = \ln\left|\frac{x+1}{x}\right| - \frac{1}{x} + c$$ [106]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \left[\frac{1}{x+1} - \frac{1}{x} + \frac{1}{x^2}\right] dx = \int \left[\frac{1}{x+1} - \frac{1}{x} + x^{-2}\right] dx$
  $= \ln|x+1| - \ln|x| + \frac{x^{-1}}{-1} + c = \ln\left|\frac{x+1}{x}\right| - \frac{1}{x} + c,\ x \in R,\ x \neq 0,\ x \neq -1,\ c \in R.$

$$\int \frac{dx}{(x+2)x^2} = \frac{1}{4}\ln\left|\frac{x+2}{x}\right| - \frac{1}{2x} + c$$ [107]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \left[\frac{1}{4(x+2)} - \frac{1}{4x} + \frac{1}{2x^2}\right] dx = \int \left[\frac{1}{4(x+2)} - \frac{1}{4x} + \frac{x^{-2}}{2}\right] dx$
  $= \frac{\ln|x+2|}{4} - \frac{\ln|x|}{4} + \frac{x^{-2}}{-1\cdot 2} + c = \frac{1}{4}\ln\left|\frac{x+2}{x}\right| - \frac{1}{2x} + c,\ x \in R,\ x \neq 0,\ x \neq -2,\ c \in R.$

Riešené príklady – 108, 109, 110

$\int \frac{dx}{x^3-7x+6}$ [108]

$\int \frac{dx}{x^3-3x-2}$ [109]

$\int \frac{dx}{x^3-3x+2}$ [110]

Riešené príklady – 108, 109, 110

$$\int \frac{dx}{x^3-7x+6} = \int \frac{dx}{(x-1)(x-2)(x+3)}$$ [108]

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x+1)^2(x-2)}$$ [109]

$$\int \frac{dx}{x^3-3x+2} = \int \frac{dx}{(x-1)^2(x+2)}$$ [110]

Riešené príklady – 108, 109, 110

$$\int \frac{dx}{x^3-7x+6} = \int \frac{dx}{(x-1)(x-2)(x+3)}$$ [108]

- $= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[-\frac{1}{4(x-1)} + \frac{1}{5(x-2)} + \frac{1}{20(x+3)} \right] dx$

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x+1)^2(x-2)}$$ [109]

- $= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[-\frac{1}{9(x+1)} - \frac{1}{3(x+1)^2} + \frac{1}{9(x-2)} \right] dx$

$$\int \frac{dx}{x^3-3x+2} = \int \frac{dx}{(x-1)^2(x+2)}$$ [110]

- $= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[-\frac{1}{9(x-1)} + \frac{1}{3(x-1)^2} + \frac{1}{9(x+2)} \right] dx$

Riešené príklady – 108, 109, 110

$$\int \frac{dx}{x^3-7x+6} = \int \frac{dx}{(x-1)(x-2)(x+3)} = -\frac{1}{4}\ln|x-1| + \frac{1}{5}\ln|x-2| + \frac{1}{20}\ln|x+3| + c$$ [108]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[-\frac{1}{4(x-1)} + \frac{1}{5(x-2)} + \frac{1}{20(x+3)} \Big] dx$

$$= -\frac{1}{4}\ln|x-1| + \frac{1}{5}\ln|x-2| + \frac{1}{20}\ln|x+3| + c,\ x\in R,\ x\neq 1,\ x\neq 2,\ x\neq 3,\ c\in R.$$

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x+1)^2(x-2)} = -\frac{1}{9}\ln|x+1| + \frac{1}{3(x+1)} + \frac{1}{9}\ln|x-2| + c$$ [109]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[-\frac{1}{9(x+1)} - \frac{1}{3(x+1)^2} + \frac{1}{9(x-2)} \Big] dx = \Big[\int \frac{dx}{(x+1)^2} = \int (x+1)^{-2}\, dx = \frac{(x+1)^{-1}}{-1} = -\frac{1}{x+1} \Big]$

$$= -\frac{1}{9}\ln|x+1| + \frac{1}{3(x+1)} + \frac{1}{9}\ln|x-2| + c,\ x\in R,\ x\neq -1,\ x\neq 2,\ c\in R.$$

$$\int \frac{dx}{x^3-3x+2} = \int \frac{dx}{(x-1)^2(x+2)} = -\frac{1}{9}\ln|x-1| - \frac{1}{3(x-1)} + \frac{1}{9}\ln|x+2| + c$$ [110]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[-\frac{1}{9(x-1)} + \frac{1}{3(x-1)^2} + \frac{1}{9(x+2)} \Big] dx = \Big[\int \frac{dx}{(x-1)^2} = \int (x-1)^{-2}\, dx = \frac{(x-1)^{-1}}{-1} = -\frac{1}{x-1} \Big]$

$$= -\frac{1}{9}\ln|x-1| - \frac{1}{3(x-1)} + \frac{1}{9}\ln|x+2| + c,\ x\in R,\ x\neq 1,\ x\neq -2,\ c\in R.$$

Riešené príklady – 111, 112

$$\int \frac{dx}{x^3+x-2}$$ [111]

pr10a-02

$$\int \frac{dx}{x^3-x^2+2}$$ [112]

pr10a-03

Riešené príklady – 111, 112

$$\int \frac{dx}{x^3+x-2}$$ [111]

- $= \int \frac{dx}{(x^2+x+2)(x-1)}$

pr10a-02

$= \left[\begin{aligned} x^2+x+2 &= (x^2+x+\tfrac{1}{4})+2-\tfrac{1}{4} \\ &= (x+\tfrac{1}{2})^2+\tfrac{7}{4} \geq \tfrac{7}{4} > 0 \end{aligned} \right]$

$$\int \frac{dx}{x^3-x^2+2}$$ [112]

- $= \left[\begin{aligned} &x^2-2x+2 \\ &\quad = (x-1)^2+1 > 0 \end{aligned} \right] = \int \frac{dx}{(x^2-2x+2)(x+1)}$

pr10a-03

Riešené príklady – 111, 112

$\int \frac{dx}{x^3+x-2}$ [111]

- $= \int \frac{dx}{(x^2+x+2)(x-1)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{4(x-1)} - \frac{x+2}{4(x^2+x+2)} \right] dx$

pr10a-02

$\int \frac{dx}{x^3-x^2+2}$ [112]

- $= \left[\begin{array}{l} x^2-2x+2 \\ \quad = (x-1)^2+1 > 0 \end{array} \right] = \int \frac{dx}{(x^2-2x+2)(x+1)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{5(x+1)} - \frac{x-3}{5(x^2-2x+2)} \right] dx$

pr10a-03

Riešené príklady – 111, 112

$$\int \frac{dx}{x^3+x-2}$$ [111]

- $$= \int \frac{dx}{(x^2+x+2)(x-1)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{4(x-1)} - \frac{x+2}{4(x^2+x+2)} \right] dx$$
$$= \frac{1}{4}\int \frac{dx}{x-1} - \frac{1}{8}\int \frac{2x+4}{x^2+x+2}\, dx = \frac{1}{4}\int \frac{dx}{x-1} - \frac{1}{8}\int \frac{2x+1}{x^2+x+2}\, dx - \frac{1}{8}\int \frac{3}{(x+\frac{1}{2})^2+\frac{7}{4}}\, dx$$
pr10a-02
$$= \left[\begin{aligned} x^2+x+2 &= (x^2+x+\tfrac{1}{4})+2-\tfrac{1}{4} \\ &= (x+\tfrac{1}{2})^2+\tfrac{7}{4} \geq \tfrac{7}{4} > 0 \end{aligned} \right]$$

$$\int \frac{dx}{x^3-x^2+2}$$ [112]

- $$= \left[\begin{aligned} x^2 &- 2x+2 \\ &= (x-1)^2+1 > 0 \end{aligned} \right] = \int \frac{dx}{(x^2-2x+2)(x+1)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{5(x+1)} - \frac{x-3}{5(x^2-2x+2)} \right] dx$$
$$= \frac{1}{5}\int \frac{dx}{x+1} - \frac{1}{10}\int \frac{2x-6}{x^2-2x+2}\, dx = \frac{1}{5}\int \frac{dx}{x+1} - \frac{1}{10}\int \frac{2x-2}{x^2-2x+2}\, dx - \frac{1}{10}\int \frac{-4}{(x-1)^2+1}\, dx$$
pr10a-03

Riešené príklady – 111, 112

$$\int \frac{dx}{x^3+x-2} = \frac{1}{4}\ln|x-1| - \frac{1}{8}\ln(x^2+x+2) - \frac{3}{4\sqrt{7}}\,\mathrm{arctg}\,\frac{2x+1}{\sqrt{7}} + c$$ [111]

- $= \int \frac{dx}{(x^2+x+2)(x-1)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{4(x-1)} - \frac{x+2}{4(x^2+x+2)}\right] dx$

$$= \frac{1}{4}\int \frac{dx}{x-1} - \frac{1}{8}\int \frac{2x+4}{x^2+x+2}\,dx = \frac{1}{4}\int \frac{dx}{x-1} - \frac{1}{8}\int \frac{2x+1}{x^2+x+2}\,dx - \frac{1}{8}\int \frac{3}{(x+\frac{1}{2})^2+\frac{7}{4}}\,dx$$

pr10a-02

$$= \left[\begin{aligned} x^2+x+2 &= (x^2+x+\tfrac{1}{4})+2-\tfrac{1}{4} \\ &= (x+\tfrac{1}{2})^2+\tfrac{7}{4} \geq \tfrac{7}{4} > 0 \end{aligned}\right] = \frac{1}{4}\ln|x-1| - \frac{1}{8}\ln|x^2+x+2| - \frac{3}{8}\cdot\frac{1}{\frac{\sqrt{7}}{2}}\,\mathrm{arctg}\,\frac{x+\frac{1}{2}}{\frac{\sqrt{7}}{2}} + c$$

$$= \frac{1}{4}\ln|x-1| - \frac{1}{8}\ln(x^2+x+2) - \frac{3}{4\sqrt{7}}\,\mathrm{arctg}\,\frac{2x+1}{\sqrt{7}} + c,\ x\in R,\ x \neq 1,\ c\in R.$$

$$\int \frac{dx}{x^3-x^2+2} = \frac{1}{5}\ln|x+1| - \frac{1}{10}\ln(x^2-2x+2) + \frac{2}{5}\,\mathrm{arctg}\,(x-1) + c$$ [112]

- $= \left[\begin{aligned} x^2-2x+2 \\ = (x-1)^2+1 > 0 \end{aligned}\right] = \int \frac{dx}{(x^2-2x+2)(x+1)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{5(x+1)} - \frac{x-3}{5(x^2-2x+2)}\right] dx$

$$= \frac{1}{5}\int \frac{dx}{x+1} - \frac{1}{10}\int \frac{2x-6}{x^2-2x+2}\,dx = \frac{1}{5}\int \frac{dx}{x+1} - \frac{1}{10}\int \frac{2x-2}{x^2-2x+2}\,dx - \frac{1}{10}\int \frac{-4}{(x-1)^2+1}\,dx$$

pr10a-03

$$= \frac{1}{5}\ln|x+1| - \frac{1}{10}\ln(x^2-2x+2) + \frac{4}{10}\,\mathrm{arctg}\,(x-1) + c,\ x\in R,\ x \neq -1,\ c\in R.$$

Riešené príklady – 113, 114

$$\int \frac{dx}{x^3 - x^2 - 4}$$ [113]

pr10a-04

$$\int \frac{dx}{x^3 - 2x - 4}$$ [114]

pr10a-05

Riešené príklady – 113, 114

$\int \frac{dx}{x^3-x^2-4}$ [113]

- $= \int \frac{dx}{(x^2+x+2)(x-2)}$

pr10a-04

$= \left[\begin{aligned} x^2+x+2 &= (x^2+x+\tfrac{1}{4})+2-\tfrac{1}{4} \\ &= (x+\tfrac{1}{2})^2+\tfrac{7}{4} \geq \tfrac{7}{4} > 0 \end{aligned} \right]$

$\int \frac{dx}{x^3-2x-4}$ [114]

- $= \left[\begin{aligned} &x^2-2x+2 \\ &= (x-1)^2+1 > 0 \end{aligned} \right] = \int \frac{dx}{(x^2+2x+2)(x-2)}$

pr10a-05

Riešené príklady – 113, 114

$$\int \frac{dx}{x^3-x^2-4}$$ [113]

- $= \int \frac{dx}{(x^2+x+2)(x-2)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{8(x-2)} - \frac{x+3}{8(x^2+x+2)} \right] dx$

pr10a-04

$$\int \frac{dx}{x^3-2x-4}$$ [114]

- $= \left[\begin{array}{l} x^2-2x+2 \\ = (x-1)^2+1>0 \end{array} \right] = \int \frac{dx}{(x^2+2x+2)(x-2)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{10(x-2)} - \frac{x+4}{10(x^2+2x+2)} \right] dx$

pr10a-05

Riešené príklady – 113, 114

$\int \frac{dx}{x^3-x^2-4}$ [113]

- $= \int \frac{dx}{(x^2+x+2)(x-2)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{8(x-2)} - \frac{x+3}{8(x^2+x+2)} \right] dx$

  $= \frac{1}{8} \int \frac{dx}{x-2} - \frac{1}{16} \int \frac{2x+6}{x^2+x+2}\, dx = \frac{1}{8} \int \frac{dx}{x-2} - \frac{1}{16} \int \frac{2x+1}{x^2+x+2}\, dx - \frac{1}{16} \int \frac{5}{(x+\frac{1}{2})^2+\frac{7}{4}}\, dx$ pr10a-04

  $= \left[\begin{array}{l} x^2+x+2 = (x^2+x+\frac{1}{4})+2-\frac{1}{4} \\ \qquad = (x+\frac{1}{2})^2+\frac{7}{4} \geq \frac{7}{4} > 0 \end{array} \right]$

$\int \frac{dx}{x^3-2x-4}$ [114]

- $= \left[\begin{array}{l} x^2-2x+2 \\ = (x-1)^2+1 > 0 \end{array} \right] = \int \frac{dx}{(x^2+2x+2)(x-2)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{10(x-2)} - \frac{x+4}{10(x^2+2x+2)} \right] dx$

  $= \frac{1}{10} \int \frac{dx}{x-2} - \frac{1}{20} \int \frac{2x+8}{x^2+2x+2}\, dx = \frac{1}{10} \int \frac{dx}{x-2} - \frac{1}{20} \int \frac{2x+2}{x^2+2x+2}\, dx - \frac{1}{20} \int \frac{6}{(x+1)^2+1}\, dx$ pr10a-05

Riešené príklady – 113, 114

$$\int \frac{dx}{x^3-x^2-4} = \frac{1}{8}\ln|x-2| - \frac{1}{16}\ln\left(x^2+x+2\right) - \frac{5}{8\sqrt{7}}\operatorname{arctg}\frac{2x+1}{\sqrt{7}} + c \qquad [113]$$

- $= \int \frac{dx}{(x^2+x+2)(x-2)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{8(x-2)} - \frac{x+3}{8(x^2+x+2)}\right] dx$

$= \frac{1}{8}\int \frac{dx}{x-2} - \frac{1}{16}\int \frac{2x+6}{x^2+x+2}\,dx = \frac{1}{8}\int \frac{dx}{x-2} - \frac{1}{16}\int \frac{2x+1}{x^2+x+2}\,dx - \frac{1}{16}\int \frac{5}{(x+\frac{1}{2})^2+\frac{7}{4}}\,dx$ pr10a-04

$= \left[\begin{array}{l} x^2+x+2 = (x^2+x+\frac{1}{4})+2-\frac{1}{4} \\ \quad = (x+\frac{1}{2})^2+\frac{7}{4} \geq \frac{7}{4} > 0 \end{array}\right] = \frac{1}{8}\ln|x-2| - \frac{1}{16}\ln\left|x^2+x+2\right| - \frac{5}{16}\cdot\frac{1}{\frac{\sqrt{7}}{2}}\operatorname{arctg}\frac{x+\frac{1}{2}}{\frac{\sqrt{7}}{2}} + c$

$= \frac{1}{8}\ln|x-2| - \frac{1}{16}\ln\left(x^2+x+2\right) - \frac{5}{8\sqrt{7}}\operatorname{arctg}\frac{2x+1}{\sqrt{7}} + c,\ x\in R,\ x\neq 1,\ c\in R.$

$$\int \frac{dx}{x^3-2x-4} = \frac{1}{10}\ln|x-2| - \frac{1}{20}\ln\left(x^2+2x+2\right) - \frac{3}{10}\operatorname{arctg}(x+1) + c \qquad [114]$$

- $= \left[\begin{array}{l} x^2-2x+2 \\ = (x-1)^2+1 > 0 \end{array}\right] = \int \frac{dx}{(x^2+2x+2)(x-2)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{10(x-2)} - \frac{x+4}{10(x^2+2x+2)}\right] dx$

$= \frac{1}{10}\int \frac{dx}{x-2} - \frac{1}{20}\int \frac{2x+8}{x^2+2x+2}\,dx = \frac{1}{10}\int \frac{dx}{x-2} - \frac{1}{20}\int \frac{2x+2}{x^2+2x+2}\,dx - \frac{1}{20}\int \frac{6}{(x+1)^2+1}\,dx$ pr10a-05

$= \frac{1}{10}\ln|x-2| - \frac{1}{20}\ln\left(x^2+2x+2\right) - \frac{6}{20}\operatorname{arctg}(x+1) + c,\ x\in R,\ x\neq 2,\ c\in R.$

Riešené príklady – 115, 116, 117

$$\int \frac{dx}{x^3+6x^2+11x+6}$$ [115]

$$\int \frac{dx}{x^3-6x^2+11x-6}$$ [116]

$$\int \frac{dx}{x^3-2x^2-x+2}$$ [117]

Riešené príklady – 115, 116, 117

$$\int \frac{dx}{x^3+6x^2+11x+6} = \int \frac{dx}{(x+1)(x+2)(x+3)}$$ [115]

$$\int \frac{dx}{x^3-6x^2+11x-6} = \int \frac{dx}{(x-1)(x-2)(x-3)}$$ [116]

$$\int \frac{dx}{x^3-2x^2-x+2} \qquad \int \frac{dx}{(x-1)(x+1)(x-2)}$$ [117]

Riešené príklady – 115, 116, 117

$$\int \frac{dx}{x^3+6x^2+11x+6} = \int \frac{dx}{(x+1)(x+2)(x+3)}$$ [115]

- $=$ [Rozklad na parciálne zlomky] $= \int \left[\frac{1}{2(x+1)} - \frac{1}{x+2} + \frac{1}{2(x+3)} \right] dx$

$$\int \frac{dx}{x^3-6x^2+11x-6} = \int \frac{dx}{(x-1)(x-2)(x-3)}$$ [116]

- $=$ [Rozklad na parciálne zlomky] $= \int \left[\frac{1}{2(x-1)} - \frac{1}{x-2} + \frac{1}{2(x-3)} \right] dx$

$$\int \frac{dx}{x^3-2x^2-x+2} = \int \frac{dx}{(x-1)(x+1)(x-2)}$$ [117]

- $=$ [Rozklad na parciálne zlomky] $= \int \left[-\frac{1}{2(x-1)} + \frac{1}{6(x+1)} + \frac{1}{3(x-2)} \right] dx$

Riešené príklady – 115, 116, 117

$$\int \frac{dx}{x^3+6x^2+11x+6} = \int \frac{dx}{(x+1)(x+2)(x+3)} = \frac{1}{2}\ln|x+1| - \ln|x+2| + \frac{1}{2}\ln|x+3| + c$$ [115]

- $= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{2(x+1)} - \frac{1}{x+2} + \frac{1}{2(x+3)} \right] dx$

$= \frac{1}{2}\ln|x+1| - \ln|x+2| + \frac{1}{2}\ln|x+3| + c,\ x \in R,\ x \neq -1,\ x \neq -2,\ x \neq -3,\ c \in R.$

$$\int \frac{dx}{x^3-6x^2+11x-6} = \int \frac{dx}{(x-1)(x-2)(x-3)} = \frac{1}{2}\ln|x-1| - \ln|x-2| + \frac{1}{2}\ln|x-3| + c$$ [116]

- $= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{2(x-1)} - \frac{1}{x-2} + \frac{1}{2(x-3)} \right] dx$

$= \frac{1}{2}\ln|x-1| - \ln|x-2| + \frac{1}{2}\ln|x-3| + c,\ x \in R,\ x \neq 1,\ x \neq 2,\ x \neq 3,\ c \in R.$

$$\int \frac{dx}{x^3-2x^2-x+2} = \int \frac{dx}{(x-1)(x+1)(x-2)} = -\frac{1}{2}\ln|x-1| + \frac{1}{6}\ln|x+1| + \frac{1}{3}\ln|x-2| + c$$ [117]

- $= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[-\frac{1}{2(x-1)} + \frac{1}{6(x+1)} + \frac{1}{3(x-2)} \right] dx$

$= -\frac{1}{2}\ln|x-1| + \frac{1}{6}\ln|x+1| + \frac{1}{3}\ln|x-2| + c,\ x \in R,\ x \neq \pm 1,\ x \neq 2,\ c \in R.$

Riešené príklady – 118, 119, 120

$$\int \frac{dx}{x^3-x^2-4x+4}$$ [118]

$$\int \frac{dx}{x^3+x^2-4x-4}$$ [119]

$$\int \frac{dx}{x^3-3x^2-x+3}$$ [120]

Riešené príklady – 118, 119, 120

$$\int \frac{dx}{x^3-x^2-4x+4} = \int \frac{dx}{(x-1)(x-2)(x+2)}$$ [118]

$$\int \frac{dx}{x^3+x^2-4x-4} = \int \frac{dx}{(x+1)(x-2)(x+2)}$$ [119]

$$\int \frac{dx}{x^3-3x^2-x+3} = \int \frac{dx}{(x-1)(x+1)(x-3)}$$ [120]

Riešené príklady – 118, 119, 120

$$\int \frac{dx}{x^3-x^2-4x+4} = \int \frac{dx}{(x-1)(x-2)(x+2)}$$ [118]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \left[-\frac{1}{3(x-1)} + \frac{1}{4(x-2)} + \frac{1}{12(x+2)}\right] dx$

$$\int \frac{dx}{x^3+x^2-4x-4} = \int \frac{dx}{(x+1)(x-2)(x+2)}$$ [119]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \left[-\frac{1}{3(x+1)} + \frac{1}{12(x-2)} + \frac{1}{4(x+2)}\right] dx$

$$\int \frac{dx}{x^3-3x^2-x+3} = \int \frac{dx}{(x-1)(x+1)(x-3)}$$ [120]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \left[-\frac{1}{4(x-1)} + \frac{1}{8(x+1)} + \frac{1}{8(x-3)}\right] dx$

Riešené príklady – 118, 119, 120

$$\int \frac{dx}{x^3-x^2-4x+4} = \int \frac{dx}{(x-1)(x-2)(x+2)} = -\frac{1}{3}\ln|x-1| + \frac{1}{4}\ln|x-2| + \frac{1}{12}\ln|x+2| + c \qquad [118]$$

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[-\frac{1}{3(x-1)} + \frac{1}{4(x-2)} + \frac{1}{12(x+2)}\Big]dx$
  $= -\frac{1}{3}\ln|x-1| + \frac{1}{4}\ln|x-2| + \frac{1}{12}\ln|x+2| + c,\ x\in R,\ x\neq 1,\ x\neq \pm 2,\ c\in R.$

$$\int \frac{dx}{x^3+x^2-4x-4} = \int \frac{dx}{(x+1)(x-2)(x+2)} = -\frac{1}{3}\ln|x+1| + \frac{1}{12}\ln|x-2| + \frac{1}{4}\ln|x+2| + c \qquad [119]$$

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[-\frac{1}{3(x+1)} + \frac{1}{12(x-2)} + \frac{1}{4(x+2)}\Big]dx$
  $= -\frac{1}{3}\ln|x+1| + \frac{1}{12}\ln|x-2| + \frac{1}{4}\ln|x+2| + c,\ x\in R,\ x\neq -1,\ x\neq \pm 2,\ c\in R.$

$$\int \frac{dx}{x^3-3x^2-x+3} = \int \frac{dx}{(x-1)(x+1)(x-3)} = -\frac{1}{4}\ln|x-1| + \frac{1}{8}\ln|x+1| + \frac{1}{8}\ln|x-3| + c \qquad [120]$$

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[-\frac{1}{4(x-1)} + \frac{1}{8(x+1)} + \frac{1}{8(x-3)}\Big]dx$
  $= -\frac{1}{4}\ln|x-1| + \frac{1}{8}\ln|x+1| + \frac{1}{8}\ln|x-3| + c,\ x\in R,\ x\neq \pm 1,\ x\neq 3,\ c\in R.$

Riešené príklady – 121, 122, 123

$$\int \frac{\mathrm{d}x}{x^3-4x^2+x+6}$$ [121]

$$\int \frac{\mathrm{d}x}{x^3+4x^2+5x+2}$$ [122]

$$\int \frac{\mathrm{d}x}{x^3-4x^2+5x-2}$$ [123]

Riešené príklady – 121, 122, 123

$$\int \frac{dx}{x^3-4x^2+x+6} = \int \frac{dx}{(x+1)(x-2)(x-3)}$$ [121]

$$\int \frac{dx}{x^3+4x^2+5x+2} = \int \frac{dx}{(x+1)^2(x+2)}$$ [122]

$$\int \frac{dx}{x^3-4x^2+5x-2} = \int \frac{dx}{(x-1)^2(x-2)}$$ [123]

Riešené príklady – 121, 122, 123

$$\int \frac{dx}{x^3-4x^2+x+6} = \int \frac{dx}{(x+1)(x-2)(x-3)}$$ [121]

- $=$ [Rozklad na parciálne zlomky] $= \displaystyle\int \left[\frac{1}{12(x+1)} - \frac{1}{3(x-2)} + \frac{1}{4(x-3)} \right] dx$

$$\int \frac{dx}{x^3+4x^2+5x+2} = \int \frac{dx}{(x+1)^2(x+2)}$$ [122]

- $=$ [Rozklad na parciálne zlomky] $= \displaystyle\int \left[-\frac{1}{x+1} + \frac{1}{(x+1)^2} + \frac{1}{x+2} \right] dx$

$$\int \frac{dx}{x^3-4x^2+5x-2} = \int \frac{dx}{(x-1)^2(x-2)}$$ [123]

- $=$ [Rozklad na parciálne zlomky] $= \displaystyle\int \left[-\frac{1}{x-1} - \frac{1}{(x-1)^2} + \frac{1}{x-2} \right] dx$

Riešené príklady – 121, 122, 123

$$\int \frac{dx}{x^3-4x^2+x+6} = \int \frac{dx}{(x+1)(x-2)(x-3)} = \frac{1}{12}\ln|x+1| - \frac{1}{3}\ln|x-2| + \frac{1}{4}\ln|x-3| + c$$ [121]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \displaystyle\int \Big[\frac{1}{12(x+1)} - \frac{1}{3(x-2)} + \frac{1}{4(x-3)}\Big] dx$

$$= \frac{1}{12}\ln|x+1| - \frac{1}{3}\ln|x-2| + \frac{1}{4}\ln|x-3| + c,\ x\in R,\ x\neq -1,\ x\neq 2,\ x\neq 3,\ c\in R.$$

$$\int \frac{dx}{x^3+4x^2+5x+2} = \int \frac{dx}{(x+1)^2(x+2)} = -\ln|x+1| - \frac{1}{x+1} + \ln|x+2| + c$$ [122]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \displaystyle\int \Big[-\frac{1}{x+1} + \frac{1}{(x+1)^2} + \frac{1}{x+2}\Big] dx = \Big[\int \frac{dx}{(x+1)^2} = \int (x+1)^{-2}\,dx = \frac{(x+1)^{-1}}{-1} = -\frac{1}{x+1}\Big]$

$$= -\ln|x+1| - \frac{1}{x+1} + \ln|x+2| + c,\ x\in R,\ x\neq -1,\ x\neq -2,\ c\in R.$$

$$\int \frac{dx}{x^3-4x^2+5x-2} = \int \frac{dx}{(x-1)^2(x-2)} = -\ln|x-1| + \frac{1}{x-1} + \ln|x-2| + c$$ [123]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \displaystyle\int \Big[-\frac{1}{x-1} - \frac{1}{(x-1)^2} + \frac{1}{x-2}\Big] dx = \Big[\int \frac{dx}{(x-1)^2} = \int (x-1)^{-2}\,dx = \frac{(x-1)^{-1}}{-1} = -\frac{1}{x-1}\Big]$

$$= -\ln|x-1| + \frac{1}{x-1} + \ln|x-2| + c,\ x\in R,\ x\neq 1,\ x\neq 2,\ c\in R.$$

Riešené príklady – 124, 125, 126

$$\int \frac{dx}{x^3-5x^2+8x-4}$$ [124]

$$\int \frac{dx}{x^3+5x^2+8x+4}$$ [125]

$$\int \frac{dx}{x^3-7x^2+15x-9}$$ [126]

Riešené príklady – 124, 125, 126

$$\int \frac{\mathrm{d}x}{x^3-5x^2+8x-4} = \int \frac{\mathrm{d}x}{(x-2)^2(x-1)}$$ [124]

$$\int \frac{\mathrm{d}x}{x^3+5x^2+8x+4} = \int \frac{\mathrm{d}x}{(x+2)^2(x+1)}$$ [125]

$$\int \frac{\mathrm{d}x}{x^3-7x^2+15x-9} = \int \frac{\mathrm{d}x}{(x-3)^2(x-1)}$$ [126]

Riešené príklady – 124, 125, 126

$$\int \frac{dx}{x^3-5x^2+8x-4} = \int \frac{dx}{(x-2)^2(x-1)}$$ [124]

- $=$ [Rozklad na parciálne zlomky] $= \int \left[-\frac{1}{x-2} + \frac{1}{(x-2)^2} + \frac{1}{x-1} \right] dx$

$$\int \frac{dx}{x^3+5x^2+8x+4} = \int \frac{dx}{(x+2)^2(x+1)}$$ [125]

- $=$ [Rozklad na parciálne zlomky] $= \int \left[-\frac{1}{x+2} - \frac{1}{(x+2)^2} + \frac{1}{x+1} \right] dx$

$$\int \frac{dx}{x^3-7x^2+15x-9} = \int \frac{dx}{(x-3)^2(x-1)}$$ [126]

- $=$ [Rozklad na parciálne zlomky] $= \int \left[-\frac{1}{4(x-3)} + \frac{1}{2(x-3)^2} + \frac{1}{4(x-1)} \right] dx$

Riešené príklady – 124, 125, 126

$$\int \frac{dx}{x^3-5x^2+8x-4} = \int \frac{dx}{(x-2)^2(x-1)} = -\ln|x-2| - \frac{1}{x-2} + \ln|x-1| + c$$ [124]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[-\frac{1}{x-2} + \frac{1}{(x-2)^2} + \frac{1}{x-1}\Big] dx = \Big[\int \frac{dx}{(x-2)^2} = \int (x-2)^{-2}\,dx = \frac{(x-2)^{-1}}{-1} = -\frac{1}{x-2}\Big]$
  $= -\ln|x-2| - \frac{1}{x-2} + \ln|x-1| + c,\ x \in R,\ x \neq 1,\ x \neq 2,\ c \in R.$

$$\int \frac{dx}{x^3+5x^2+8x+4} = \int \frac{dx}{(x+2)^2(x+1)} = -\ln|x+2| + \frac{1}{x+2} + \ln|x+1| + c$$ [125]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[-\frac{1}{x+2} - \frac{1}{(x+2)^2} + \frac{1}{x+1}\Big] dx = \Big[\int \frac{dx}{(x+2)^2} = \int (x+2)^{-2}\,dx = \frac{(x+2)^{-1}}{-1} = -\frac{1}{x+2}\Big]$
  $= -\ln|x+2| + \frac{1}{x+2} + \ln|x+1| + c,\ x \in R,\ x \neq -1,\ x \neq -2,\ c \in R.$

$$\int \frac{dx}{x^3-7x^2+15x-9} = \int \frac{dx}{(x-3)^2(x-1)} = -\frac{1}{4}\ln|x-3| - \frac{1}{2(x-3)} + \frac{1}{4}\ln|x-1| + c$$ [126]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[-\frac{1}{4(x-3)} + \frac{1}{2(x-3)^2} + \frac{1}{4(x-1)}\Big] dx = \Big[\int \frac{dx}{(x-3)^2} = \int (x-3)^{-2}\,dx = \frac{(x-3)^{-1}}{-1} = -\frac{1}{x-3}\Big]$
  $= -\frac{1}{4}\ln|x-3| - \frac{1}{2(x-3)} + \frac{1}{4}\ln|x-1| + c,\ x \in R,\ x \neq 1,\ x \neq 3,\ c \in R.$

Riešené príklady – 127, 128, 129

$$\int \frac{dx}{x^3 - x^2 - x + 1}$$ [127]

$$\int \frac{dx}{x^3 + x^2 - x - 1}$$ [128]

$$\int \frac{dx}{x^3 - 5x^2 + 7x - 3}$$ [129]

Riešené príklady – 127, 128, 129

$$\int \frac{dx}{x^3-x^2-x+1} = \int \frac{dx}{(x-1)^2(x+1)}$$ [127]

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x+1)^2(x-1)}$$ [128]

$$\int \frac{dx}{x^3-5x^2+7x-3} = \int \frac{dx}{(x-1)^2(x-3)}$$ [129]

Riešené príklady – 127, 128, 129

$$\int \frac{dx}{x^3-x^2-x+1} = \int \frac{dx}{(x-1)^2(x+1)}$$ [127]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \left[-\frac{1}{4(x-1)} + \frac{1}{2(x-1)^2} + \frac{1}{4(x+1)} \right] dx$

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x+1)^2(x-1)}$$ [128]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \left[-\frac{1}{4(x+1)} - \frac{1}{2(x+1)^2} + \frac{1}{4(x-1)} \right] dx$

$$\int \frac{dx}{x^3-5x^2+7x-3} = \int \frac{dx}{(x-1)^2(x-3)}$$ [129]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int \left[-\frac{1}{4(x-1)} - \frac{1}{2(x-1)^2} + \frac{1}{4(x-3)} \right] dx$

Riešené príklady – 127, 128, 129

$$\int\frac{dx}{x^3-x^2-x+1}=\int\frac{dx}{(x-1)^2(x+1)}=-\frac{1}{4}\ln|x-1|-\frac{1}{2(x-1)}+\frac{1}{4}\ln|x+1|+c$$ [127]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int\Big[-\frac{1}{4(x-1)}+\frac{1}{2(x-1)^2}+\frac{1}{4(x+1)}\Big]dx = \Big[\int\frac{dx}{(x-1)^2}=\int(x-1)^{-2}\,dx=\frac{(x-1)^{-1}}{-1}=-\frac{1}{x-1}\Big]$
$= -\frac{1}{4}\ln|x-1|-\frac{1}{2(x-1)}+\frac{1}{4}\ln|x+1|+c,\ x\in R,\ x\neq\pm1,\ c\in R.$

$$\int\frac{dx}{x^3+x^2-x-1}=\int\frac{dx}{(x+1)^2(x-1)}=-\frac{1}{4}\ln|x+1|+\frac{1}{2(x+1)}+\frac{1}{4}\ln|x-1|+c$$ [128]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int\Big[-\frac{1}{4(x+1)}-\frac{1}{2(x+1)^2}+\frac{1}{4(x-1)}\Big]dx = \Big[\int\frac{dx}{(x+1)^2}=\int(x+1)^{-2}\,dx=\frac{(x+1)^{-1}}{-1}=-\frac{1}{x+1}\Big]$
$= -\frac{1}{4}\ln|x+1|+\frac{1}{2(x+1)}+\frac{1}{4}\ln|x-1|+c,\ x\in R,\ x\neq\pm1,\ c\in R.$

$$\int\frac{dx}{x^3-5x^2+7x-3}=\int\frac{dx}{(x-1)^2(x-3)}=-\frac{1}{4}\ln|x-1|+\frac{1}{2(x-1)}+\frac{1}{4}\ln|x-3|+c$$ [129]

- $= \Big[$ Rozklad na parciálne zlomky $\Big] = \int\Big[-\frac{1}{4(x-1)}-\frac{1}{2(x-1)^2}+\frac{1}{4(x-3)}\Big]dx = \Big[\int\frac{dx}{(x-1)^2}=\int(x-1)^{-2}\,dx=\frac{(x-1)^{-1}}{-1}=-\frac{1}{x-1}\Big]$
$= -\frac{1}{4}\ln|x-1|+\frac{1}{2(x-1)}+\frac{1}{4}\ln|x-3|+c,\ x\in R,\ x\neq1,\ x\neq3,\ c\in R.$

Riešené príklady – 130, 131

$$\int \frac{dx}{x^3-3x^2+4x-4}$$ [130]

pr10a-06

$$\int \frac{dx}{x^3-3x^2+4x-2}$$ [131]

Riešené príklady – 130, 131

$$\int \frac{dx}{x^3-3x^2+4x-4}$$ [130]

- $= \int \frac{dx}{(x^2-x+2)(x-2)}$

pr10a-06

$= \left[\begin{aligned} x^2-x+2 &= (x^2-x+\tfrac{1}{4})+2-\tfrac{1}{4} \\ &= (x-\tfrac{1}{2})^2+\tfrac{7}{4} \geq \tfrac{7}{4} > 0 \end{aligned} \right]$

$$\int \frac{dx}{x^3-3x^2+4x-2}$$ [131]

- $= \int \frac{dx}{(x^2-2x+2)(x-1)}$

$\left[\begin{aligned} x^2-2x+2 &= (x^2-2x+1)+1 \\ &= (x-1)^2+1 \geq 1 > 0 \end{aligned} \right]$

Riešené príklady – 130, 131

$$\int \frac{\mathrm{d}x}{x^3-3x^2+4x-4}$$ [130]

- $= \int \frac{\mathrm{d}x}{(x^2-x+2)(x-2)} = $ [Rozklad na parciálne zlomky] $= \int \left[\frac{1}{4(x-2)} - \frac{x+1}{4(x^2-x+2)} \right] \mathrm{d}x$

pr10a-06

$$\int \frac{\mathrm{d}x}{x^3-3x^2+4x-2}$$ [131]

- $= \int \frac{\mathrm{d}x}{(x^2-2x+2)(x-1)} = $ [Rozklad na parciálne zlomky] $= \int \left[\frac{1}{x-1} - \frac{x-1}{x^2-2x+2} \right] \mathrm{d}x$

Riešené príklady – 130, 131

$$\int \frac{dx}{x^3-3x^2+4x-4}$$ [130]

- $$= \int \frac{dx}{(x^2-x+2)(x-2)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{4(x-2)} - \frac{x+1}{4(x^2-x+2)}\right] dx$$

$$= \frac{1}{4}\int \frac{dx}{x-2} - \frac{1}{8}\int \frac{2x+2}{x^2-x+2}\, dx = \frac{1}{4}\int \frac{dx}{x-2} - \frac{1}{8}\int \frac{2x-1}{x^2-x+2}\, dx - \frac{1}{8}\int \frac{3}{(x-\frac{1}{2})^2+\frac{7}{4}}\, dx$$

pr10a-06

$$= \left[\begin{aligned} x^2-x+2 &= (x^2-x+\tfrac{1}{4})+2-\tfrac{1}{4} \\ &= (x-\tfrac{1}{2})^2+\tfrac{7}{4} \geq \tfrac{7}{4} > 0 \end{aligned}\right]$$

$$\int \frac{dx}{x^3-3x^2+4x-2}$$ [131]

- $$= \int \frac{dx}{(x^2-2x+2)(x-1)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{x-1} - \frac{x-1}{x^2-2x+2}\right] dx$$

$$= \int \frac{dx}{x-1} - \frac{1}{2}\int \frac{2x-2}{x^2-2x+2}\, dx = \int \frac{dx}{x-1} - \frac{1}{2}\int \frac{(x^2-2x+2)'}{x^2-2x+2}\, dx = \left[\begin{aligned} x^2-2x+2 &= (x^2-2x+1)+1 \\ &= (x-1)^2+1 \geq 1 > 0 \end{aligned}\right]$$

Riešené príklady – 130, 131

$$\int \frac{\mathrm{d}x}{x^3-3x^2+4x-4} = \frac{1}{4}\ln|x-2| - \frac{1}{8}\ln\left(x^2-x+2\right) - \frac{3}{4\sqrt{7}}\operatorname{arctg}\frac{2x-1}{\sqrt{7}} + c$$ [130]

- $= \int \frac{\mathrm{d}x}{(x^2-x+2)(x-2)} = \left[\begin{array}{c}\text{Rozklad na}\\ \text{parciálne zlomky}\end{array}\right] = \int \left[\frac{1}{4(x-2)} - \frac{x+1}{4(x^2-x+2)}\right]\mathrm{d}x$

$$= \frac{1}{4}\int\frac{\mathrm{d}x}{x-2} - \frac{1}{8}\int\frac{2x+2}{x^2-x+2}\,\mathrm{d}x = \frac{1}{4}\int\frac{\mathrm{d}x}{x-2} - \frac{1}{8}\int\frac{2x-1}{x^2-x+2}\,\mathrm{d}x - \frac{1}{8}\int\frac{3}{(x-\frac{1}{2})^2+\frac{7}{4}}\,\mathrm{d}x$$

pr10a-06

$$= \left[\begin{array}{l} x^2-x+2 = (x^2-x+\tfrac{1}{4})+2-\tfrac{1}{4} \\ \qquad = (x-\tfrac{1}{2})^2+\tfrac{7}{4} \geq \tfrac{7}{4} > 0\end{array}\right] = \frac{1}{4}\ln|x-2| - \frac{1}{8}\ln\left|x^2-x+2\right| - \frac{3}{8}\cdot\frac{1}{\frac{\sqrt{7}}{2}}\operatorname{arctg}\frac{x-\frac{1}{2}}{\frac{\sqrt{7}}{2}} + c$$

$$= \frac{1}{4}\ln|x-2| - \frac{1}{8}\ln\left(x^2-x+2\right) - \frac{3}{4\sqrt{7}}\operatorname{arctg}\frac{2x-1}{\sqrt{7}} + c,\ x\in R,\ x\neq 2,\ c\in R.$$

$$\int \frac{\mathrm{d}x}{x^3-3x^2+4x-2} = \ln|x-1| - \frac{1}{2}\ln\left(x^2-2x+2\right) + c$$ [131]

- $= \int \frac{\mathrm{d}x}{(x^2-2x+2)(x-1)} = \left[\begin{array}{c}\text{Rozklad na}\\ \text{parciálne zlomky}\end{array}\right] = \int \left[\frac{1}{x-1} - \frac{x-1}{x^2-2x+2}\right]\mathrm{d}x$

$$= \int\frac{\mathrm{d}x}{x-1} - \frac{1}{2}\int\frac{2x-2}{x^2-2x+2}\,\mathrm{d}x = \int\frac{\mathrm{d}x}{x-1} - \frac{1}{2}\int\frac{(x^2-2x+2)'}{x^2-2x+2}\,\mathrm{d}x = \left[\begin{array}{l} x^2-2x+2 = (x^2-2x+1)+1 \\ \qquad = (x-1)^2+1 \geq 1 > 0\end{array}\right]$$

$$= \ln|x-1| - \frac{1}{2}\ln\left(x^2-2x+2\right) + c,\ x\in R,\ x\neq 1,\ c\in R.$$

Riešené príklady – 132, 133

$$\int \frac{dx}{x^3+2x^2+3x+2}$$ [132]

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$$\int \frac{dx}{x^3+3x^2+4x+2}$$ [133]

Riešené príklady – 132, 133

$$\int \frac{dx}{x^3+2x^2+3x+2}$$ [132]

- $= \int \frac{dx}{(x^2+x+2)(x+1)}$

pr10a-07

$$= \left[\begin{aligned} x^2+x+2 &= (x^2+x+\tfrac{1}{4})+2-\tfrac{1}{4} \\ &= (x+\tfrac{1}{2})^2+\tfrac{7}{4} \geq \tfrac{7}{4} > 0 \end{aligned} \right]$$

$$\int \frac{dx}{x^3+3x^2+4x+2}$$ [133]

- $= \int \frac{dx}{(x^2+2x+2)(x+1)}$

$$\left[\begin{aligned} x^2+2x+2 &= (x^2+2x+1)+1 \\ &= (x+1)^2+1 \geq 1 > 0 \end{aligned} \right]$$

Riešené príklady – 132, 133

$\int \frac{dx}{x^3+2x^2+3x+2}$ [132]

- $= \int \frac{dx}{(x^2+x+2)(x+1)} =$ [Rozklad na parciálne zlomky] $= \int \left[\frac{1}{2(x+1)} - \frac{x}{2(x^2+x+2)} \right] dx$

pr10a-07

$\int \frac{dx}{x^3+3x^2+4x+2}$ [133]

- $= \int \frac{dx}{(x^2+2x+2)(x+1)} =$ [Rozklad na parciálne zlomky] $= \int \left[\frac{1}{x+1} - \frac{x+1}{x^2+2x+2} \right] dx$

Riešené príklady – 132, 133

$$\int \frac{dx}{x^3+2x^2+3x+2}$$ [132]

- $= \int \frac{dx}{(x^2+x+2)(x+1)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{2(x+1)} - \frac{x}{2(x^2+x+2)} \right] dx$
  $= \frac{1}{2}\int \frac{dx}{x+1} - \frac{1}{4}\int \frac{2x}{x^2+x+2}\, dx = \frac{1}{2}\int \frac{dx}{x+1} - \frac{1}{4}\int \frac{2x+1}{x^2+x+2}\, dx - \frac{1}{4}\int \frac{-1}{(x+\frac{1}{2})^2+\frac{7}{4}}\, dx$
  $= \left[\begin{aligned} x^2+x+2 &= (x^2+x+\tfrac{1}{4})+2-\tfrac{1}{4} \\ &= (x+\tfrac{1}{2})^2+\tfrac{7}{4} \geq \tfrac{7}{4} > 0 \end{aligned} \right]$

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$$\int \frac{dx}{x^3+3x^2+4x+2}$$ [133]

- $= \int \frac{dx}{(x^2+2x+2)(x+1)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{x+1} - \frac{x+1}{x^2+2x+2} \right] dx$
  $= \int \frac{dx}{x+1} - \frac{1}{2}\int \frac{2x+2}{x^2+2x+2}\, dx = \int \frac{dx}{x+1} - \frac{1}{2}\int \frac{(x^2+2x+2)'}{x^2+2x+2}\, dx = \left[\begin{aligned} x^2+2x+2 &= (x^2+2x+1)+1 \\ &= (x+1)^2+1 \geq 1 > 0 \end{aligned} \right]$

Riešené príklady – 132, 133

$$\int \frac{dx}{x^3+2x^2+3x+2} = \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln\left(x^2+x+2\right) + \frac{1}{2\sqrt{7}}\,\mathrm{arctg}\,\frac{2x+1}{\sqrt{7}} + c$$ [132]

- $= \int \frac{dx}{(x^2+x+2)(x+1)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{2(x+1)} - \frac{x}{2(x^2+x+2)}\right] dx$

$= \frac{1}{2}\int \frac{dx}{x+1} - \frac{1}{4}\int \frac{2x}{x^2+x+2}\,dx = \frac{1}{2}\int \frac{dx}{x+1} - \frac{1}{4}\int \frac{2x+1}{x^2+x+2}\,dx - \frac{1}{4}\int \frac{-1}{(x+\frac{1}{2})^2+\frac{7}{4}}\,dx$ pr10a-07

$= \left[\begin{array}{l} x^2+x+2 = (x^2+x+\frac{1}{4})+2-\frac{1}{4} \\ \quad = (x+\frac{1}{2})^2+\frac{7}{4} \geq \frac{7}{4} > 0 \end{array}\right] = \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln\left|x^2+x+2\right| + \frac{1}{4}\cdot\frac{1}{\frac{\sqrt{7}}{2}}\,\mathrm{arctg}\,\frac{x+\frac{1}{2}}{\frac{\sqrt{7}}{2}} + c$

$= \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln\left(x^2+x+2\right) + \frac{1}{2\sqrt{7}}\,\mathrm{arctg}\,\frac{2x+1}{\sqrt{7}} + c,\ x\in R,\ x\neq -1,\ c\in R.$

$$\int \frac{dx}{x^3+3x^2+4x+2} = \ln|x+1| - \frac{1}{2}\ln\left(x^2+2x+2\right) + c$$ [133]

- $= \int \frac{dx}{(x^2+2x+2)(x+1)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{x+1} - \frac{x+1}{x^2+2x+2}\right] dx$

$= \int \frac{dx}{x+1} - \frac{1}{2}\int \frac{2x+2}{x^2+2x+2}\,dx = \int \frac{dx}{x+1} - \frac{1}{2}\int \frac{(x^2+2x+2)'}{x^2+2x+2}\,dx = \left[\begin{array}{l} x^2+2x+2 = (x^2+2x+1)+1 \\ \quad = (x+1)^2+1 \geq 1 > 0 \end{array}\right]$

$= \ln|x+1| - \frac{1}{2}\ln\left(x^2+2x+2\right) + c,\ x\in R,\ x\neq -1,\ c\in R.$

Riešené príklady – 134, 135

$$\int \frac{dx}{x^3-x^2+x+3}$$ [134]

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$$\int \frac{dx}{x^3-3x^2+5x-3}$$ [135]

Riešené príklady – 134, 135

$$\int \frac{dx}{x^3-x^2+x+3}$$ [134]

- $= \int \frac{dx}{(x^2-2x+3)(x+1)}$

pr10a-08

$$= \left[\begin{aligned} x^2-2x+3 &= (x^2-2x+1)+2 \\ &= (x-1)^2+2 \geq 2 > 0 \end{aligned} \right]$$

$$\int \frac{dx}{x^3-3x^2+5x-3}$$ [135]

- $= \int \frac{dx}{(x^2-2x+3)(x-1)}$

$$\left[\begin{aligned} x^2-2x+3 &= (x^2-2x+1)+2 \\ &= (x-1)^2+2 \geq 2 > 0 \end{aligned} \right]$$

Riešené príklady – 134, 135

$$\int \frac{dx}{x^3-x^2+x+3}$$ [134]

- $= \int \frac{dx}{(x^2-2x+3)(x+1)} =$ [Rozklad na parciálne zlomky] $= \int \left[\frac{1}{6(x+1)} - \frac{x-3}{6(x^2-2x+3)} \right] dx$

pr10a-08

$$\int \frac{dx}{x^3-3x^2+5x-3}$$ [135]

- $= \int \frac{dx}{(x^2-2x+3)(x-1)} =$ [Rozklad na parciálne zlomky] $= \int \left[\frac{1}{2(x-1)} - \frac{x-1}{2(x^2-2x+3)} \right] dx$

Riešené príklady – 134, 135

$$\int \frac{dx}{x^3-x^2+x+3}$$ [134]

- $= \int \frac{dx}{(x^2-2x+3)(x+1)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{6(x+1)} - \frac{x-3}{6(x^2-2x+3)} \right] dx$
  $= \frac{1}{6}\int \frac{dx}{x+1} - \frac{1}{12}\int \frac{2x-6}{x^2-2x+3}\,dx = \frac{1}{6}\int \frac{dx}{x+1} - \frac{1}{12}\int \frac{2x-2}{x^2-2x+3}\,dx - \frac{1}{12}\int \frac{-4}{(x-1)^2+2}\,dx$
  $= \left[\begin{array}{l} x^2-2x+3 = (x^2-2x+1)+2 \\ \qquad = (x-1)^2+2 \geq 2 > 0 \end{array} \right]$

pr10a-08

$$\int \frac{dx}{x^3-3x^2+5x-3}$$ [135]

- $= \int \frac{dx}{(x^2-2x+3)(x-1)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{2(x-1)} - \frac{x-1}{2(x^2-2x+3)} \right] dx$
  $= \frac{1}{2}\int \frac{dx}{x-1} - \frac{1}{4}\int \frac{2x-2}{x^2-2x+3}\,dx = \frac{1}{2}\int \frac{dx}{x-1} - \frac{1}{4}\int \frac{(x^2-2x+3)'}{x^2-2x+3}\,dx = \left[\begin{array}{l} x^2-2x+3 = (x^2-2x+1)+2 \\ \qquad = (x-1)^2+2 \geq 2 > 0 \end{array} \right]$

Riešené príklady – 134, 135

$$\int \frac{dx}{x^3-x^2+x+3} = \frac{1}{6}\ln|x+1| - \frac{1}{12}\ln\left(x^2-2x+3\right) + \frac{1}{3\sqrt{2}}\operatorname{arctg}\frac{x-1}{\sqrt{2}} + c \qquad [134]$$

- $= \int \frac{dx}{(x^2-2x+3)(x+1)} = \left[\begin{array}{l}\text{Rozklad na}\\ \text{parciálne zlomky}\end{array}\right] = \int \left[\frac{1}{6(x+1)} - \frac{x-3}{6(x^2-2x+3)}\right] dx$

$= \frac{1}{6}\int \frac{dx}{x+1} - \frac{1}{12}\int \frac{2x-6}{x^2-2x+3}\,dx = \frac{1}{6}\int \frac{dx}{x+1} - \frac{1}{12}\int \frac{2x-2}{x^2-2x+3}\,dx - \frac{1}{12}\int \frac{-4}{(x-1)^2+2}\,dx$

pr10a-08

$= \left[\begin{array}{l} x^2-2x+3 = (x^2-2x+1)+2 \\ \qquad = (x-1)^2+2 \geq 2 > 0\end{array}\right] = \frac{1}{6}\ln|x+1| - \frac{1}{12}\ln\left|x^2-2x+3\right| + \frac{4}{12}\cdot\frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x-1}{\sqrt{2}} + c$

$= \frac{1}{6}\ln|x+1| - \frac{1}{12}\ln\left(x^2-2x+3\right) + \frac{1}{3\sqrt{2}}\operatorname{arctg}\frac{x-1}{\sqrt{2}} + c,\ x \in R,\ x \neq -1,\ c \in R.$

$$\int \frac{dx}{x^3-3x^2+5x-3} = \frac{1}{2}\ln|x-1| - \frac{1}{4}\ln\left(x^2-2x+3\right) + c \qquad [135]$$

- $= \int \frac{dx}{(x^2-2x+3)(x-1)} = \left[\begin{array}{l}\text{Rozklad na}\\ \text{parciálne zlomky}\end{array}\right] = \int \left[\frac{1}{2(x-1)} - \frac{x-1}{2(x^2-2x+3)}\right] dx$

$= \frac{1}{2}\int \frac{dx}{x-1} - \frac{1}{4}\int \frac{2x-2}{x^2-2x+3}\,dx = \frac{1}{2}\int \frac{dx}{x-1} - \frac{1}{4}\int \frac{(x^2-2x+3)'}{x^2-2x+3}\,dx = \left[\begin{array}{l} x^2-2x+3 = (x^2-2x+1)+2 \\ \qquad = (x-1)^2+2 \geq 2 > 0\end{array}\right]$

$= \frac{1}{2}\ln|x-1| - \frac{1}{4}\ln\left(x^2-2x+3\right) + c,\ x \in R,\ x \neq 1,\ c \in R.$

Riešené príklady – 136, 137

$$\int \frac{\mathrm{d}x}{x^3 - x + 6}$$ [136]

pr10a-09

$$\int \frac{\mathrm{d}x}{x^3 + 3x^2 + 5x + 3}$$ [137]

Riešené príklady – 136, 137

$$\int \frac{dx}{x^3-x+6}$$ [136]

- $= \int \frac{dx}{(x^2-2x+3)(x+2)}$

pr10a-09

$= \left[\begin{aligned} x^2-2x+3 &= (x^2-2x+1)+2 \\ &= (x-1)^2+2 \geq 2 > 0 \end{aligned} \right]$

$$\int \frac{dx}{x^3+3x^2+5x+3}$$ [137]

- $= \int \frac{dx}{(x^2+2x+3)(x+1)}$

$\left[\begin{aligned} x^2+2x+3 &= (x^2+2x+1)+2 \\ &= (x+1)^2+2 \geq 2 > 0 \end{aligned} \right]$

Riešené príklady – 136, 137

$\int \frac{dx}{x^3-x+6}$ [136]

- $= \int \frac{dx}{(x^2-2x+3)(x+2)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{11(x+2)} - \frac{x-4}{11(x^2-2x+3)}\right] dx$

pr10a-09

$\int \frac{dx}{x^3+3x^2+5x+3}$ [137]

- $= \int \frac{dx}{(x^2+2x+3)(x+1)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{2(x+1)} - \frac{x+1}{2(x^2+2x+3)}\right] dx$

Riešené príklady – 136, 137

$$\int \frac{dx}{x^3-x+6}$$ [136]

- $$= \int \frac{dx}{(x^2-2x+3)(x+2)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{11(x+2)} - \frac{x-4}{11(x^2-2x+3)} \right] dx$$
$$= \frac{1}{11}\int \frac{dx}{x+2} - \frac{1}{22}\int \frac{2x-8}{x^2-2x+3}\,dx = \frac{1}{11}\int \frac{dx}{x+2} - \frac{1}{22}\int \frac{2x-2}{x^2-2x+3}\,dx - \frac{1}{22}\int \frac{-6}{(x-1)^2+2}\,dx$$
pr10a-09
$$= \left[\begin{aligned} x^2-2x+3 &= (x^2-2x+1)+2 \\ &= (x-1)^2+2 \geq 2 > 0 \end{aligned} \right]$$

$$\int \frac{dx}{x^3+3x^2+5x+3}$$ [137]

- $$= \int \frac{dx}{(x^2+2x+3)(x+1)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{2(x+1)} - \frac{x+1}{2(x^2+2x+3)} \right] dx$$
$$= \frac{1}{2}\int \frac{dx}{x+1} - \frac{1}{4}\int \frac{2x+2}{x^2+2x+3}\,dx = \frac{1}{2}\int \frac{dx}{x+1} - \frac{1}{4}\int \frac{(x^2+2x+3)'}{x^2+2x+3}\,dx = \left[\begin{aligned} x^2+2x+3 &= (x^2+2x+1)+2 \\ &= (x+1)^2+2 \geq 2 > 0 \end{aligned} \right]$$

Riešené príklady – 136, 137

$$\int \frac{dx}{x^3-x+6} = \frac{1}{11}\ln|x+2| - \frac{1}{22}\ln\left(x^2-2x+3\right) + \frac{3}{11\sqrt{2}}\operatorname{arctg}\frac{x-1}{\sqrt{2}} + c$$ [136]

- $= \int \frac{dx}{(x^2-2x+3)(x+2)} = \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[\frac{1}{11(x+2)} - \frac{x-4}{11(x^2-2x+3)}\Big] dx$
  $= \frac{1}{11}\int \frac{dx}{x+2} - \frac{1}{22}\int \frac{2x-8}{x^2-2x+3}\, dx = \frac{1}{11}\int \frac{dx}{x+2} - \frac{1}{22}\int \frac{2x-2}{x^2-2x+3}\, dx - \frac{1}{22}\int \frac{-6}{(x-1)^2+2}\, dx$ pr10a-09
  $= \Big[x^2-2x+3 = (x^2-2x+1)+2 = (x-1)^2+2 \geq 2 > 0 \Big] = \frac{1}{11}\ln|x+2| - \frac{1}{22}\ln\left|x^2-2x+3\right| + \frac{6}{22}\cdot\frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x-1}{\sqrt{2}} + c$
  $= \frac{1}{11}\ln|x+2| - \frac{1}{22}\ln\left(x^2-2x+3\right) + \frac{3}{11\sqrt{2}}\operatorname{arctg}\frac{x-1}{\sqrt{2}} + c,\ x\in R,\ x\neq -2,\ c\in R.$

$$\int \frac{dx}{x^3+3x^2+5x+3} = \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln\left(x^2+2x+3\right) + c$$ [137]

- $= \int \frac{dx}{(x^2+2x+3)(x+1)} = \Big[$ Rozklad na parciálne zlomky $\Big] = \int \Big[\frac{1}{2(x+1)} - \frac{x+1}{2(x^2+2x+3)}\Big] dx$
  $= \frac{1}{2}\int \frac{dx}{x+1} - \frac{1}{4}\int \frac{2x+2}{x^2+2x+3}\, dx = \frac{1}{2}\int \frac{dx}{x+1} - \frac{1}{4}\int \frac{(x^2+2x+3)'}{x^2+2x+3}\, dx = \Big[x^2+2x+3 = (x^2+2x+1)+2 = (x+1)^2+2 \geq 2 > 0 \Big]$
  $= \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln\left(x^2+2x+3\right) + c,\ x\in R,\ x\neq -1,\ c\in R.$

Riešené príklady – 138, 139

$$\int \frac{dx}{x^3-4x^2+7x-6}$$ [138]

pr10a-10

$$\int \frac{dx}{x^3+3x^2+6x+4}$$ [139]

Riešené príklady – 138, 139

$$\int \frac{dx}{x^3-4x^2+7x-6}$$ [138]

- $= \int \frac{dx}{(x^2-2x+3)(x-2)}$

pr10a-10

$= \left[\begin{array}{l} x^2-2x+3 = (x^2-2x+1)+2 \\ \qquad = (x-1)^2+2 \geq 2 > 0 \end{array} \right]$

$$\int \frac{dx}{x^3+3x^2+6x+4}$$ [139]

- $= \int \frac{dx}{(x^2+2x+4)(x+1)}$

$\left[\begin{array}{l} x^2+2x+4 = (x^2+2x+1)+3 \\ \qquad = (x+1)^2+3 \geq 3 > 0 \end{array} \right]$

Riešené príklady – 138, 139

$$\int \frac{dx}{x^3-4x^2+7x-6}$$ [138]

- $= \displaystyle\int \frac{dx}{(x^2-2x+3)(x-2)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{3(x-2)} - \frac{x}{3(x^2-2x+3)}\right] dx$

pr10a-10

$$\int \frac{dx}{x^3+3x^2+6x+4}$$ [139]

- $= \displaystyle\int \frac{dx}{(x^2+2x+4)(x+1)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[\frac{1}{3(x+1)} - \frac{x+1}{3(x^2+2x+4)}\right] dx$

Riešené príklady – 138, 139

$\int \frac{dx}{x^3-4x^2+7x-6}$ [138]

- $= \int \frac{dx}{(x^2-2x+3)(x-2)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{3(x-2)} - \frac{x}{3(x^2-2x+3)} \right] dx$

$= \frac{1}{3} \int \frac{dx}{x-2} - \frac{1}{6} \int \frac{2x}{x^2-2x+3}\, dx = \frac{1}{3} \int \frac{dx}{x-2} - \frac{1}{6} \int \frac{2x-2}{x^2-2x+3}\, dx - \frac{1}{6} \int \frac{2}{(x-1)^2+2}\, dx$

pr10a-10

$= \left[\begin{aligned} x^2-2x+3 &= (x^2-2x+1)+2 \\ &= (x-1)^2+2 \geq 2 > 0 \end{aligned} \right]$

$\int \frac{dx}{x^3+3x^2+6x+4}$ [139]

- $= \int \frac{dx}{(x^2+2x+4)(x+1)} = \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{3(x+1)} - \frac{x+1}{3(x^2+2x+4)} \right] dx$

$= \frac{1}{3} \int \frac{dx}{x+1} - \frac{1}{6} \int \frac{2x+2}{x^2+2x+4}\, dx = \frac{1}{3} \int \frac{dx}{x+1} - \frac{1}{6} \int \frac{(x^2+2x+4)'}{x^2+2x+4}\, dx = \left[\begin{aligned} x^2+2x+4 &= (x^2+2x+1)+3 \\ &= (x+1)^2+3 \geq 3 > 0 \end{aligned} \right]$

Riešené príklady – 138, 139

$$\int \frac{dx}{x^3-4x^2+7x-6} = \frac{1}{3}\ln|x-2| - \frac{1}{6}\ln\left(x^2-2x+3\right) - \frac{1}{3\sqrt{2}}\operatorname{arctg}\frac{x-1}{\sqrt{2}} + c$$ [138]

- $= \int \frac{dx}{(x^2-2x+3)(x-2)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int\left[\frac{1}{3(x-2)} - \frac{x}{3(x^2-2x+3)}\right]dx$

$= \frac{1}{3}\int\frac{dx}{x-2} - \frac{1}{6}\int\frac{2x}{x^2-2x+3}\,dx = \frac{1}{3}\int\frac{dx}{x-2} - \frac{1}{6}\int\frac{2x-2}{x^2-2x+3}\,dx - \frac{1}{6}\int\frac{2}{(x-1)^2+2}\,dx$ pr10a-10

$= \left[\begin{array}{l} x^2-2x+3=(x^2-2x+1)+2 \\ \qquad =(x-1)^2+2\geq 2>0 \end{array}\right] = \frac{1}{3}\ln|x-2| - \frac{1}{6}\ln\left|x^2-2x+3\right| - \frac{2}{6}\cdot\frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x-1}{\sqrt{2}} + c$

$= \frac{1}{3}\ln|x-2| - \frac{1}{6}\ln\left(x^2-2x+3\right) - \frac{1}{3\sqrt{2}}\operatorname{arctg}\frac{x-1}{\sqrt{2}} + c,\ x\in R,\ x\neq 2,\ c\in R.$

$$\int \frac{dx}{x^3+3x^2+6x+4} = \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln\left(x^2+2x+4\right) + c$$ [139]

- $= \int \frac{dx}{(x^2+2x+4)(x+1)} = \left[\text{Rozklad na parciálne zlomky}\right] = \int\left[\frac{1}{3(x+1)} - \frac{x+1}{3(x^2+2x+4)}\right]dx$

$= \frac{1}{3}\int\frac{dx}{x+1} - \frac{1}{6}\int\frac{2x+2}{x^2+2x+4}\,dx = \frac{1}{3}\int\frac{dx}{x+1} - \frac{1}{6}\int\frac{(x^2+2x+4)'}{x^2+2x+4}\,dx = \left[\begin{array}{l} x^2+2x+4=(x^2+2x+1)+3 \\ \qquad =(x+1)^2+3\geq 3>0 \end{array}\right]$

$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln\left(x^2+2x+4\right) + c,\ x\in R,\ x\neq -1,\ c\in R.$

Riešené príklady – 140, 141, 142

$$\int \frac{x-1}{(x^2-2x+2)^3}\,\mathrm{d}x$$ [140]

$$\int \frac{x+1}{(x^2+2x+2)^3}\,\mathrm{d}x$$ [141]

$$\int \frac{x+1}{(x^2+2x+3)^3}\,\mathrm{d}x$$ [142]

Riešené príklady – 140, 141, 142

$$\int \frac{x-1}{(x^2-2x+2)^3}\,dx$$ [140]

- $= \frac{1}{2}\int \frac{2x-2}{(x^2-2x+2)^3}\,dx$

$$\int \frac{x+1}{(x^2+2x+2)^3}\,dx$$ [141]

- $= \frac{1}{2}\int \frac{2x+2}{(x^2+2x+2)^3}\,dx$

$$\int \frac{x+1}{(x^2+2x+3)^3}\,dx$$ [142]

- $= \frac{1}{2}\int \frac{2x+2}{(x^2+2x+3)^3}\,dx$

Riešené príklady – 140, 141, 142

$$\int \frac{x-1}{(x^2-2x+2)^3}\,dx$$ [140]

- $= \frac{1}{2}\int \frac{2x-2}{(x^2-2x+2)^3}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2-2x+2 & x^2-2x+2 & x\in(-\infty;1\rangle \Rightarrow t\in\langle 1;\infty) \\ dt = (2x-2)\,dx & \quad = (x-1)^2+1 \geq 1 > 0 & x\in\langle 1;\infty) \Rightarrow t\in\langle 1;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^3}$
  $= \frac{1}{2}\int t^{-3}\,dt$

$$\int \frac{x+1}{(x^2+2x+2)^3}\,dx$$ [141]

- $= \frac{1}{2}\int \frac{2x+2}{(x^2+2x+2)^3}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2+2x+2 & x^2+2x+2 & x\in(-\infty;-1\rangle \Rightarrow t\in\langle 1;\infty) \\ dt = (2x+2)\,dx & \quad = (x+1)^2+1 \geq 1 > 0 & x\in\langle -1;\infty) \Rightarrow t\in\langle 1;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^3}$
  $= \frac{1}{2}\int t^{-3}\,dt$

$$\int \frac{x+1}{(x^2+2x+3)^3}\,dx$$ [142]

- $= \frac{1}{2}\int \frac{2x+2}{(x^2+2x+3)^3}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2+2x+3 & x^2+2x+3 & x\in(-\infty;-1\rangle \Rightarrow t\in\langle 2;\infty) \\ dt = (2x+2)\,dx & \quad = (x+1)^2+2 \geq 2 > 0 & x\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^3}$
  $= \frac{1}{2}\int t^{-3}\,dt$

Riešené príklady – 140, 141, 142

$$\int \frac{x-1}{(x^2-2x+2)^3}\,dx = -\frac{1}{4(x^2-2x+2)^2} + c$$ [140]

- $= \frac{1}{2}\int \frac{2x-2}{(x^2-2x+2)^3}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2-2x+2 & x^2-2x+2 & x\in(-\infty;1) \Rightarrow t\in\langle 1;\infty) \\ dt = (2x-2)\,dx & = (x-1)^2+1 \geq 1 > 0 & x\in\langle 1;\infty) \Rightarrow t\in\langle 1;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^3}$
$= \frac{1}{2}\int t^{-3}\,dt = \frac{1}{2}\cdot\frac{t^{-2}}{-2} + c = -\frac{1}{4t^2} + c = -\frac{1}{4(x^2-2x+2)^2} + c,\ x\in R,\ c\in R.$

$$\int \frac{x+1}{(x^2+2x+2)^3}\,dx = -\frac{1}{4(x^2+2x+2)^2} + c$$ [141]

- $= \frac{1}{2}\int \frac{2x+2}{(x^2+2x+2)^3}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2+2x+2 & x^2+2x+2 & x\in(-\infty;-1) \Rightarrow t\in\langle 1;\infty) \\ dt = (2x+2)\,dx & = (x+1)^2+1 \geq 1 > 0 & x\in\langle -1;\infty) \Rightarrow t\in\langle 1;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^3}$
$= \frac{1}{2}\int t^{-3}\,dt = \frac{1}{2}\cdot\frac{t^{-2}}{-2} + c = -\frac{1}{4t^2} + c = -\frac{1}{4(x^2+2x+2)^2} + c,\ x\in R,\ c\in R.$

$$\int \frac{x+1}{(x^2+2x+3)^3}\,dx = -\frac{1}{4(x^2+2x+3)^2} + c$$ [142]

- $= \frac{1}{2}\int \frac{2x+2}{(x^2+2x+3)^3}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t = x^2+2x+3 & x^2+2x+3 & x\in(-\infty;-1) \Rightarrow t\in\langle 2;\infty) \\ dt = (2x+2)\,dx & = (x+1)^2+2 \geq 2 > 0 & x\in\langle -1;\infty) \Rightarrow t\in\langle 2;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^3}$
$= \frac{1}{2}\int t^{-3}\,dt = \frac{1}{2}\cdot\frac{t^{-2}}{-2} + c = -\frac{1}{4t^2} + c = -\frac{1}{4(x^2+2x+3)^2} + c,\ x\in R,\ c\in R.$

Riešené príklady – 143, 144

$$\int \frac{x^6-x^5+x^4-x^3+x+1}{x^4-2x^3+2x^2-2x+1}\,dx$$ [143]

$$\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx$$ [144]

Riešené príklady – 143, 144

$$\int \frac{x^6-x^5+x^4-x^3+x+1}{x^4-2x^3+2x^2-2x+1}\,dx$$ [143]

- $=$ [Rozklad na parciálne zlomky] $= \int \Big[x^2+x+1+\frac{1}{2(x-1)}+\frac{1}{(x-1)^2}+\frac{x-1}{2(x^2+1)}\Big]dx$

$$\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx$$ [144]

- $=$ [Rozklad na parciálne zlomky] $= \int \Big[-\frac{2}{x}+\frac{1}{x^2}+\frac{3}{(x+1)^2}\Big]dx = -2\int\frac{dx}{x}+\int x^{-2}\,dx+3\int(x+1)^{-2}\,dx$

Riešené príklady – 143, 144

$$\int \frac{x^6-x^5+x^4-x^3+x+1}{x^4-2x^3+2x^2-2x+1}\,dx$$ [143]

- $= \left[\text{Rozklad na parciálne zlomky}\right] = \int\left[x^2+x+1+\frac{1}{2(x-1)}+\frac{1}{(x-1)^2}+\frac{x-1}{2(x^2+1)}\right]dx$

  $= \frac{x^3}{3}+\frac{x^2}{2}+x+\frac{1}{2}\int\frac{dx}{x-1}+\int(x-1)^{-2}\,dx+\frac{1}{4}\int\frac{2x}{x^2+1}\,dx-\frac{1}{2}\int\frac{dx}{x^2+1}$

$$\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx$$ [144]

- $= \left[\text{Rozklad na parciálne zlomky}\right] = \int\left[-\frac{2}{x}+\frac{1}{x^2}+\frac{3}{(x+1)^2}\right]dx = -2\int\frac{dx}{x}+\int x^{-2}\,dx+3\int(x+1)^{-2}\,dx$

  $= \left[\begin{array}{r|l}\text{Subst. } t=x+1 & x\in R \\ dt=dx & t\in R\end{array}\right] = -2\int\frac{dx}{x}+\int x^{-2}\,dx+3\int t^{-2}\,dt$

Riešené príklady – 143, 144

$\int \frac{x^6-x^5+x^4-x^3+x+1}{x^4-2x^3+2x^2-2x+1}\,dx$ [143]

- $= \left[\text{Rozklad na parciálne zlomky}\right] = \int \left[x^2+x+1+\frac{1}{2(x-1)}+\frac{1}{(x-1)^2}+\frac{x-1}{2(x^2+1)}\right]dx$

$= \frac{x^3}{3}+\frac{x^2}{2}+x+\frac{1}{2}\int\frac{dx}{x-1}+\int(x-1)^{-2}\,dx+\frac{1}{4}\int\frac{2x}{x^2+1}\,dx-\frac{1}{2}\int\frac{dx}{x^2+1}$

$= \frac{x^3}{3}+\frac{x^2}{2}+x+\frac{1}{2}\ln|x-1|-\frac{1}{x-1}+\frac{1}{4}\ln|x^2+1|-\frac{1}{2}\operatorname{arctg}x+c$

$\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx$ [144]

- $= \left[\text{Rozklad na parciálne zlomky}\right] = \int\left[-\frac{2}{x}+\frac{1}{x^2}+\frac{3}{(x+1)^2}\right]dx = -2\int\frac{dx}{x}+\int x^{-2}\,dx+3\int(x+1)^{-2}\,dx$

$= \left[\begin{array}{c|c}\text{Subst. } t=x+1 & x\in R\\ dt=dx & t\in R\end{array}\right] = -2\int\frac{dx}{x}+\int x^{-2}\,dx+3\int t^{-2}\,dt = -2\ln|x|+\frac{x^{-1}}{-1}+\frac{3t^{-1}}{-1}+c$

Riešené príklady – 143, 144

$$\int \frac{x^6-x^5+x^4-x^3+x+1}{x^4-2x^3+2x^2-2x+1}\,dx = \frac{x^3}{3}+\frac{x^2}{2}+x+\frac{1}{4}\ln\left[(x-1)^2(x^2+1)\right]-\frac{1}{x-1}-\frac{1}{2}\operatorname{arctg} x + c \quad [143]$$

- $= \left[\begin{array}{c}\text{Rozklad na}\\ \text{parciálne zlomky}\end{array}\right] = \int\left[x^2+x+1+\frac{1}{2(x-1)}+\frac{1}{(x-1)^2}+\frac{x-1}{2(x^2+1)}\right]dx$

$$= \frac{x^3}{3}+\frac{x^2}{2}+x+\frac{1}{2}\int\frac{dx}{x-1}+\int(x-1)^{-2}\,dx+\frac{1}{4}\int\frac{2x}{x^2+1}\,dx-\frac{1}{2}\int\frac{dx}{x^2+1}$$

$$= \frac{x^3}{3}+\frac{x^2}{2}+x+\frac{1}{2}\ln|x-1|-\frac{1}{x-1}+\frac{1}{4}\ln|x^2+1|-\frac{1}{2}\operatorname{arctg} x+c$$

$$= \frac{x^3}{3}+\frac{x^2}{2}+x+\frac{1}{4}\ln\left[(x-1)^2(x^2+1)\right]-\frac{1}{x-1}-\frac{1}{2}\operatorname{arctg} x+c,\ x\in R,\ x\neq 1,\ c\in R.$$

$$\int \frac{-2x^3+1}{x^4+2x^3+x^2}\,dx = -2\ln|x|-\frac{1}{x}-\frac{3}{x+1}+c = -\ln x^2-\frac{1}{x}-\frac{3}{x+1}+c \quad [144]$$

- $= \left[\begin{array}{c}\text{Rozklad na}\\ \text{parciálne zlomky}\end{array}\right] = \int\left[-\frac{2}{x}+\frac{1}{x^2}+\frac{3}{(x+1)^2}\right]dx = -2\int\frac{dx}{x}+\int x^{-2}\,dx+3\int(x+1)^{-2}\,dx$

$$= \left[\begin{array}{r|l}\text{Subst. } t=x+1 & x\in R\\ dt=dx & t\in R\end{array}\right] = -2\int\frac{dx}{x}+\int x^{-2}\,dx+3\int t^{-2}\,dt = -2\ln|x|+\frac{x^{-1}}{-1}+\frac{3t^{-1}}{-1}+c$$

$$= -2\ln|x|-\frac{1}{x}-\frac{3}{x+1}+c = -\ln x^2-\frac{1}{x}-\frac{3}{x+1}+c,\ x\in R,\ x\neq 0,\ x\neq -1,\ c\in R.$$

Riešené príklady – 145

$$\int \frac{dx}{x^6+1}$$

[145]

Riešené príklady – 145

$$\int \frac{dx}{x^6+1}$$ [145]

- $=$ [Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\pm i$. $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0$, $(x-i)(x+i)=x^2+1>0$. $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)$.]

Riešené príklady – 145

$\int \frac{dx}{x^6+1}$ [145]

- $=$ [Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\pm i$. | $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0,$ $(x-i)(x+i)=x^2+1>0.$ $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1).$]

  $=$ [Rozklad na parciálne zlomky] $= \int\left[\frac{\frac{1}{3}}{x^2+1}+\frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1}-\frac{\frac{x}{2\sqrt{3}}-\frac{1}{3}}{x^2-\sqrt{3}x+1}\right]dx$

Riešené príklady – 145

$\int \frac{dx}{x^6+1}$ [145]

- $= \Big[$ Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\pm i$. $\Big|$ $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0$, $(x-i)(x+i) = x^2+1>0$. $\Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)$. $\Big]$

$= \Big[$ Rozklad na parciálne zlomky $\Big]$ $= \displaystyle\int\Big[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} - \frac{\frac{x}{2\sqrt{3}}-\frac{1}{3}}{x^2-\sqrt{3}x+1}\Big]dx$

$$= \left[\begin{array}{l|l} (x^2+\sqrt{3}x+1)' = 2x+\sqrt{3} & \frac{x}{2\sqrt{3}}+\frac{1}{3} = \frac{1}{4\sqrt{3}}(2x+\frac{4\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{4\sqrt{3}}{3}-\sqrt{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{4\sqrt{3}}{3}-\frac{3\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}) + \frac{1}{12} \\ (x^2-\sqrt{3}x+1)' = 2x-\sqrt{3} & \frac{x}{2\sqrt{3}}-\frac{1}{3} = \frac{1}{4\sqrt{3}}(2x-\frac{4\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{4\sqrt{3}}{3}+\sqrt{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{4\sqrt{3}}{3}+\frac{3\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}) - \frac{1}{12} \end{array}\right]$$

$$= \frac{1}{3}\int\frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int\frac{dx}{x^2+\sqrt{3}x+1} - \frac{1}{4\sqrt{3}}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1} + \frac{1}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$$

Riešené príklady – 145

$$\int \frac{dx}{x^6+1}$$ [145]

- $= \Big[$ Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i$. $\Big|$ $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0$, $(x-i)(x+i)=x^2+1>0$. $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)$. $\Big]$

$= \Big[$ Rozklad na parciálne zlomky $\Big]$ $$= \int\Bigg[\frac{\frac{1}{3}}{x^2+1}+\frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1}-\frac{\frac{x}{2\sqrt{3}}-\frac{1}{3}}{x^2-\sqrt{3}x+1}\Bigg]dx$$

$$= \left[\begin{array}{l|l} (x^2+\sqrt{3}x+1)'=2x+\sqrt{3} & \frac{x}{2\sqrt{3}}+\frac{1}{3}=\frac{1}{4\sqrt{3}}(2x+\frac{4\sqrt{3}}{3})=\frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{4\sqrt{3}}{3}-\sqrt{3})=\frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{4\sqrt{3}}{3}-\frac{3\sqrt{3}}{3})=\frac{1}{4\sqrt{3}}(2x+\sqrt{3})+\frac{1}{12} \\ (x^2-\sqrt{3}x+1)'=2x-\sqrt{3} & \frac{x}{2\sqrt{3}}-\frac{1}{3}=\frac{1}{4\sqrt{3}}(2x-\frac{4\sqrt{3}}{3})=\frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{4\sqrt{3}}{3}+\sqrt{3})=\frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{4\sqrt{3}}{3}+\frac{3\sqrt{3}}{3})=\frac{1}{4\sqrt{3}}(2x-\sqrt{3})-\frac{1}{12} \end{array}\right]$$

$$= \frac{1}{3}\int\frac{dx}{x^2+1}+\frac{1}{4\sqrt{3}}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1}+\frac{1}{12}\int\frac{dx}{x^2+\sqrt{3}x+1}-\frac{1}{4\sqrt{3}}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1}+\frac{1}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x^2+\sqrt{3}x+1=(x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4}=(x+\frac{\sqrt{3}}{2})^2+\frac{1}{4}=u^2+\frac{1}{4}>0 & x\in R,\ u\in R \\ \phantom{\text{Subst. }} v=x-\frac{\sqrt{3}}{2} & dv=dx & x^2-\sqrt{3}x+1=(x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4}=(x-\frac{\sqrt{3}}{2})^2+\frac{1}{4}=v^2+\frac{1}{4}>0 & x\in R,\ v\in R \end{array}\right]$$

$$= \frac{1}{3}\int\frac{dx}{x^2+1}+\frac{1}{4\sqrt{3}}\int\frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1}+\frac{1}{12}\int\frac{du}{u^2+\frac{1}{4}}-\frac{1}{4\sqrt{3}}\int\frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1}+\frac{1}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

Riešené príklady – 145

$$\int \frac{dx}{x^6+1}$$ [145]

- $= \Big[$ Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i$. $\Big|$ $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0,$ $(x-i)(x+i)=x^2+1>0.$ $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1).\Big]$

$= \Big[$ Rozklad na parciálne zlomky $\Big]$ $= \displaystyle\int\left[\frac{\frac{1}{3}}{x^2+1}+\frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1}-\frac{\frac{x}{2\sqrt{3}}-\frac{1}{3}}{x^2-\sqrt{3}x+1}\right]dx$

$= \Big[$ $(x^2+\sqrt{3}x+1)'=2x+\sqrt{3}$ $\Big|$ $\frac{x}{2\sqrt{3}}+\frac{1}{3}=\frac{1}{4\sqrt{3}}(2x+\frac{4\sqrt{3}}{3})=\frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{4\sqrt{3}}{3}-\sqrt{3})=\frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{4\sqrt{3}}{3}-\frac{3\sqrt{3}}{3})=\frac{1}{4\sqrt{3}}(2x+\sqrt{3})+\frac{1}{12}$
$(x^2-\sqrt{3}x+1)'=2x-\sqrt{3}$ $\Big|$ $\frac{x}{2\sqrt{3}}-\frac{1}{3}=\frac{1}{4\sqrt{3}}(2x-\frac{4\sqrt{3}}{3})=\frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{4\sqrt{3}}{3}+\sqrt{3})=\frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{4\sqrt{3}}{3}+\frac{3\sqrt{3}}{3})=\frac{1}{4\sqrt{3}}(2x-\sqrt{3})-\frac{1}{12}$ $\Big]$

$$= \frac{1}{3}\int\frac{dx}{x^2+1}+\frac{1}{4\sqrt{3}}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1}+\frac{1}{12}\int\frac{dx}{x^2+\sqrt{3}x+1}-\frac{1}{4\sqrt{3}}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1}+\frac{1}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$$

$= \Big[$ Subst. $u=x+\frac{\sqrt{3}}{2}$ $\Big|$ $du=dx$ $\Big|$ $x^2+\sqrt{3}x+1=(x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4}=(x+\frac{\sqrt{3}}{2})^2+\frac{1}{4}=u^2+\frac{1}{4}>0$ $\Big|$ $x\in R,\ u\in R$
$v=x-\frac{\sqrt{3}}{2}$ $\Big|$ $dv=dx$ $\Big|$ $x^2-\sqrt{3}x+1=(x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4}=(x-\frac{\sqrt{3}}{2})^2+\frac{1}{4}=v^2+\frac{1}{4}>0$ $\Big|$ $x\in R,\ v\in R$ $\Big]$

$$= \frac{1}{3}\int\frac{dx}{x^2+1}+\frac{1}{4\sqrt{3}}\int\frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1}+\frac{1}{12}\int\frac{du}{u^2+\frac{1}{4}}-\frac{1}{4\sqrt{3}}\int\frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1}+\frac{1}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{1}{3}\operatorname{arctg} x+\frac{\ln(x^2+\sqrt{3}x+1)}{4\sqrt{3}}+\frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}}-\frac{\ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}}+\frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}}+c$$

$$= \frac{1}{3}\operatorname{arctg} x+\frac{\ln(x^2+\sqrt{3}x+1)-\ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}}+\frac{1}{6}\operatorname{arctg} 2u+\frac{1}{6}\operatorname{arctg} 2v+c$$

Riešené príklady – 145

$$\int \frac{dx}{x^6+1} = \frac{1}{3}\,\mathrm{arctg}\,x + \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c$$ [145]

- $= \Big[$ Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i.$ $\Big|$ $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0,$ $(x-i)(x+i)=x^2+1>0.$ $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1).$ $\Big]$

$= \Big[$ Rozklad na parciálne zlomky $\Big]$ $$= \int\left[\frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} - \frac{\frac{x}{2\sqrt{3}}-\frac{1}{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \left[\begin{array}{l|l} (x^2+\sqrt{3}x+1)'=2x+\sqrt{3} & \frac{x}{2\sqrt{3}}+\frac{1}{3} = \frac{1}{4\sqrt{3}}(2x+\frac{4\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{4\sqrt{3}}{3}-\sqrt{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{4\sqrt{3}}{3}-\frac{3\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3})+\frac{1}{12} \\ (x^2-\sqrt{3}x+1)'=2x-\sqrt{3} & \frac{x}{2\sqrt{3}}-\frac{1}{3} = \frac{1}{4\sqrt{3}}(2x-\frac{4\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{4\sqrt{3}}{3}+\sqrt{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{4\sqrt{3}}{3}+\frac{3\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3})-\frac{1}{12} \end{array}\right]$$

$$= \frac{1}{3}\int\frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int\frac{dx}{x^2+\sqrt{3}x+1} - \frac{1}{4\sqrt{3}}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1} + \frac{1}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x^2+\sqrt{3}x+1=(x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4}=(x+\frac{\sqrt{3}}{2})^2+\frac{1}{4}=u^2+\frac{1}{4}>0 & x\in R,\ u\in R \\ \phantom{\text{Subst. }} v=x-\frac{\sqrt{3}}{2} & dv=dx & x^2-\sqrt{3}x+1=(x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4}=(x-\frac{\sqrt{3}}{2})^2+\frac{1}{4}=v^2+\frac{1}{4}>0 & x\in R,\ v\in R \end{array}\right]$$

$$= \frac{1}{3}\int\frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}}\int\frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int\frac{du}{u^2+\frac{1}{4}} - \frac{1}{4\sqrt{3}}\int\frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1} + \frac{1}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x + \frac{\ln(x^2+\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{u}{\frac{1}{2}} - \frac{\ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{v}{\frac{1}{2}} + c$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x + \frac{\ln(x^2+\sqrt{3}x+1)-\ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{6}\,\mathrm{arctg}\,2u + \frac{1}{6}\,\mathrm{arctg}\,2v + c$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x + \frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 146

$$\int \frac{x\,dx}{x^6+1}$$ [146]

Riešené príklady – 146

$\int \frac{x\,dx}{x^6+1}$ [146]

- $=$ [Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\pm i$. | $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2})=(x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4}=(x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4}=x^2\pm\sqrt{3}x+1>0$, $(x-i)(x+i)=x^2+1>0$. $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)$.]

Riešené príklady – 146

$\int \frac{x\,dx}{x^6+1}$ [146]

- $=$ [Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i$. | $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0,$ $(x-i)(x+i)=x^2+1>0.$ $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1).$]

$=$ [Rozklad na parciálne zlomky] $= \int\left[\frac{\frac{x}{3}}{x^2+1} - \frac{\frac{x}{6}+\frac{1}{2\sqrt{3}}}{x^2+\sqrt{3}x+1} - \frac{\frac{x}{6}-\frac{1}{2\sqrt{3}}}{x^2-\sqrt{3}x+1}\right]dx$

Riešené príklady – 146

$\int \frac{x\,dx}{x^6+1}$ [146]

- $= \left[\begin{array}{l} \text{Korene rovnice } x^6+1=0 \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i\,. \end{array} \middle| \begin{array}{l} (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ (x-i)(x+i) = x^2+1>0. \qquad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

$= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{\frac{x}{3}}{x^2+1} - \frac{\frac{x}{6}+\frac{1}{2\sqrt{3}}}{x^2+\sqrt{3}x+1} - \frac{\frac{x}{6}-\frac{1}{2\sqrt{3}}}{x^2-\sqrt{3}x+1} \right] dx$

$= \left[\begin{array}{l} (x^2+\sqrt{3}x+1)' = 2x+\sqrt{3} \\ (x^2-\sqrt{3}x+1)' = 2x-\sqrt{3} \end{array} \middle| \begin{array}{l} \frac{x}{6}+\frac{1}{2\sqrt{3}} = \frac{1}{12}(2x+\frac{6}{\sqrt{3}}) = \frac{1}{12}(2x+\frac{2\cdot 3}{\sqrt{3}}) = \frac{1}{12}(2x+2\sqrt{3}) = \frac{1}{12}(2x+\sqrt{3}+\sqrt{3}) = \frac{1}{12}(2x+\sqrt{3})+\frac{\sqrt{3}}{12} \\ \frac{x}{6}-\frac{1}{2\sqrt{3}} = \frac{1}{12}(2x-\frac{6}{\sqrt{3}}) = \frac{1}{12}(2x-\frac{2\cdot 3}{\sqrt{3}}) = \frac{1}{12}(2x-2\sqrt{3}) = \frac{1}{12}(2x-\sqrt{3}-\sqrt{3}) = \frac{1}{12}(2x-\sqrt{3})-\frac{\sqrt{3}}{12} \end{array} \right]$

$= \frac{1}{6}\int\frac{2x\,dx}{x^2+1} - \frac{1}{12}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{12}\int\frac{dx}{x^2+\sqrt{3}x+1} - \frac{1}{12}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$

Riešené príklady – 146

$$\int \frac{x\,dx}{x^6+1}$$ [146]

- $= \left[\begin{array}{l|l} \text{Korene rovnice } x^6+1=0 & (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. & (x-i)(x+i) = x^2+1>0. \quad \Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

$$= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{\frac{x}{3}}{x^2+1} - \frac{\frac{x}{6}+\frac{1}{2\sqrt{3}}}{x^2+\sqrt{3}x+1} - \frac{\frac{x}{6}-\frac{1}{2\sqrt{3}}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l|l} (x^2+\sqrt{3}x+1)' = 2x+\sqrt{3} & \frac{x}{6}+\frac{1}{2\sqrt{3}} = \frac{1}{12}(2x+\frac{6}{\sqrt{3}}) = \frac{1}{12}(2x+\frac{2\cdot3}{\sqrt{3}}) = \frac{1}{12}(2x+2\sqrt{3}) = \frac{1}{12}(2x+\sqrt{3}+\sqrt{3}) = \frac{1}{12}(2x+\sqrt{3})+\frac{\sqrt{3}}{12} \\ (x^2-\sqrt{3}x+1)' = 2x-\sqrt{3} & \frac{x}{6}-\frac{1}{2\sqrt{3}} = \frac{1}{12}(2x-\frac{6}{\sqrt{3}}) = \frac{1}{12}(2x-\frac{2\cdot3}{\sqrt{3}}) = \frac{1}{12}(2x-2\sqrt{3}) = \frac{1}{12}(2x-\sqrt{3}-\sqrt{3}) = \frac{1}{12}(2x-\sqrt{3})-\frac{\sqrt{3}}{12} \end{array} \right]$$

$$= \frac{1}{6}\int\frac{2x\,dx}{x^2+1} - \frac{1}{12}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{12}\int\frac{dx}{x^2+\sqrt{3}x+1} - \frac{1}{12}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} & du = dx & x^2+\sqrt{3}x+1 = (x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x+\frac{\sqrt{3}}{2})^2+\frac{1}{4} = u^2+\frac{1}{4}>0 & x\in R,\ u\in R \\ \phantom{\text{Subst. }} v = x-\frac{\sqrt{3}}{2} & dv = dx & x^2-\sqrt{3}x+1 = (x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x-\frac{\sqrt{3}}{2})^2+\frac{1}{4} = v^2+\frac{1}{4}>0 & x\in R,\ v\in R \end{array} \right]$$

$$= \frac{1}{6}\int\frac{(x^2+1)'\,dx}{x^2+1} - \frac{1}{12}\int\frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{12}\int\frac{du}{u^2+\frac{1}{4}} - \frac{1}{12}\int\frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

Riešené príklady – 146

$$\int \frac{x\,dx}{x^6+1}$$ [146]

- $= \Big[$ Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\pm i$. $\Big|$ $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2})=(x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4}=(x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4}=x^2\pm\sqrt{3}x+1>0$, $(x-i)(x+i)=x^2+1>0$. $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)$. $\Big]$

$= \Big[$ Rozklad na parciálne zlomky $\Big]$ $= \displaystyle\int\Big[\frac{\frac{x}{3}}{x^2+1}-\frac{\frac{x}{6}+\frac{1}{2\sqrt{3}}}{x^2+\sqrt{3}x+1}-\frac{\frac{x}{6}-\frac{1}{2\sqrt{3}}}{x^2-\sqrt{3}x+1}\Big]dx$

$= \Big[$ $(x^2+\sqrt{3}x+1)'=2x+\sqrt{3}$ $\Big|$ $\frac{x}{6}+\frac{1}{2\sqrt{3}}=\frac{1}{12}(2x+\frac{6}{\sqrt{3}})=\frac{1}{12}(2x+\frac{2\cdot 3}{\sqrt{3}})=\frac{1}{12}(2x+2\sqrt{3})=\frac{1}{12}(2x+\sqrt{3}+\sqrt{3})=\frac{1}{12}(2x+\sqrt{3})+\frac{\sqrt{3}}{12}$
$(x^2-\sqrt{3}x+1)'=2x-\sqrt{3}$ $\Big|$ $\frac{x}{6}-\frac{1}{2\sqrt{3}}=\frac{1}{12}(2x-\frac{6}{\sqrt{3}})=\frac{1}{12}(2x-\frac{2\cdot 3}{\sqrt{3}})=\frac{1}{12}(2x-2\sqrt{3})=\frac{1}{12}(2x-\sqrt{3}-\sqrt{3})=\frac{1}{12}(2x-\sqrt{3})-\frac{\sqrt{3}}{12}$ $\Big]$

$$= \frac{1}{6}\int\frac{2x\,dx}{x^2+1}-\frac{1}{12}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1}-\frac{\sqrt{3}}{12}\int\frac{dx}{x^2+\sqrt{3}x+1}-\frac{1}{12}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1}+\frac{\sqrt{3}}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$$

$= \Big[$ Subst. $u=x+\frac{\sqrt{3}}{2}$ $\Big|$ $du=dx$ $\Big|$ $x^2+\sqrt{3}x+1=(x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4}=(x+\frac{\sqrt{3}}{2})^2+\frac{1}{4}=u^2+\frac{1}{4}>0$ $\Big|$ $x\in R,\ u\in R$
$v=x-\frac{\sqrt{3}}{2}$ $\Big|$ $dv=dx$ $\Big|$ $x^2-\sqrt{3}x+1=(x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4}=(x-\frac{\sqrt{3}}{2})^2+\frac{1}{4}=v^2+\frac{1}{4}>0$ $\Big|$ $x\in R,\ v\in R$ $\Big]$

$$= \frac{1}{6}\int\frac{(x^2+1)'\,dx}{x^2+1}-\frac{1}{12}\int\frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1}-\frac{\sqrt{3}}{12}\int\frac{du}{u^2+\frac{1}{4}}-\frac{1}{12}\int\frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1}+\frac{\sqrt{3}}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{1}{6}\ln\left(x^2+1\right)-\frac{\ln(x^2+\sqrt{3}x+1)}{12}-\frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}}-\frac{\ln(x^2-\sqrt{3}x+1)}{12}+\frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}}+c$$

$$= \frac{\ln(x^2+1)^2}{12}-\frac{\ln(x^2+\sqrt{3}x+1)+\ln(x^2-\sqrt{3}x+1)}{12}-\frac{\sqrt{3}}{6}\operatorname{arctg}2u+\frac{\sqrt{3}}{6}\operatorname{arctg}2v+c$$

Riešené príklady – 146

$$\int \frac{x\,dx}{x^6+1} = \frac{1}{12}\ln\frac{(x^2+1)^2}{x^4-x^2+1} - \frac{\sqrt{3}\,\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\sqrt{3}\,\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c \qquad [146]$$

- $= \Big[$ Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i$. $\Big|\ (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0,$ $(x-i)(x+i)=x^2+1>0.$ $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1).\Big]$

$$= \Big[\text{Rozklad na parciálne zlomky}\Big] = \int\left[\frac{\frac{x}{3}}{x^2+1} - \frac{\frac{x}{6}+\frac{1}{2\sqrt{3}}}{x^2+\sqrt{3}x+1} - \frac{\frac{x}{6}-\frac{1}{2\sqrt{3}}}{x^2-\sqrt{3}x+1}\right]dx$$

$= \Big[(x^2+\sqrt{3}x+1)' = 2x+\sqrt{3} \ \Big|\ \frac{x}{6}+\frac{1}{2\sqrt{3}} = \frac{1}{12}(2x+\frac{6}{\sqrt{3}}) = \frac{1}{12}(2x+\frac{2\cdot 3}{\sqrt{3}}) = \frac{1}{12}(2x+2\sqrt{3}) = \frac{1}{12}(2x+\sqrt{3}+\sqrt{3}) = \frac{1}{12}(2x+\sqrt{3})+\frac{\sqrt{3}}{12}$

$(x^2-\sqrt{3}x+1)' = 2x-\sqrt{3} \ \Big|\ \frac{x}{6}-\frac{1}{2\sqrt{3}} = \frac{1}{12}(2x-\frac{6}{\sqrt{3}}) = \frac{1}{12}(2x-\frac{2\cdot 3}{\sqrt{3}}) = \frac{1}{12}(2x-2\sqrt{3}) = \frac{1}{12}(2x-\sqrt{3}-\sqrt{3}) = \frac{1}{12}(2x-\sqrt{3})-\frac{\sqrt{3}}{12} \Big]$

$$= \frac{1}{6}\int\frac{2x\,dx}{x^2+1} - \frac{1}{12}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{12}\int\frac{dx}{x^2+\sqrt{3}x+1} - \frac{1}{12}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$$

$= \Big[$ Subst. $u=x+\frac{\sqrt{3}}{2}$ | $du=dx$ | $x^2+\sqrt{3}x+1=(x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4}=(x+\frac{\sqrt{3}}{2})^2+\frac{1}{4}=u^2+\frac{1}{4}>0$ | $x\in R,\ u\in R$

$v=x-\frac{\sqrt{3}}{2}$ | $dv=dx$ | $x^2-\sqrt{3}x+1=(x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4}=(x-\frac{\sqrt{3}}{2})^2+\frac{1}{4}=v^2+\frac{1}{4}>0$ | $x\in R,\ v\in R \Big]$

$$= \frac{1}{6}\int\frac{(x^2+1)'\,dx}{x^2+1} - \frac{1}{12}\int\frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{12}\int\frac{du}{u^2+\frac{1}{4}} - \frac{1}{12}\int\frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{1}{6}\ln(x^2+1) - \frac{\ln(x^2+\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{u}{\frac{1}{2}} - \frac{\ln(x^2-\sqrt{3}x+1)}{12} + \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{v}{\frac{1}{2}} + c$$

$$= \frac{\ln(x^2+1)^2}{12} - \frac{\ln(x^2+\sqrt{3}x+1)+\ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{6}\,\mathrm{arctg}\,2u + \frac{\sqrt{3}}{6}\,\mathrm{arctg}\,2v + c$$

$$= \Big[(x^2+1+\sqrt{3}x)(x^2+1-\sqrt{3}x) = (x^2+1)^2-3x^2 = x^4-x^2+1\Big] = \frac{1}{12}\ln\frac{(x^2+1)^2}{x^4-x^2+1} - \frac{\sqrt{3}\,\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\sqrt{3}\,\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c,$$

$x\in R,\ c\in R.$

Riešené príklady – 146

$$\int \frac{x\,dx}{x^6+1}$$

[Iné riešenie 146]

Riešené príklady – 146

$\int \frac{x\,dx}{x^6+1}$

[Iné riešenie 146]

- $= \left[\begin{array}{r|l} \text{Subst. } t = x^2 & x \in (-\infty; 0\rangle \Rightarrow t \in \langle 0; \infty) \\ dt = 2x\,dx & x \in \langle 0; \infty) \Rightarrow t \in \langle 0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^3+1}$

Riešené príklady – 146

$\int \frac{x\,dx}{x^6+1}$

[Iné riešenie 146]

- $= \left[\begin{array}{c|c} \text{Subst. } t = x^2 & x \in (-\infty; 0\rangle \Rightarrow t \in \langle 0; \infty) \\ dt = 2x\,dx & x \in \langle 0; \infty) \Rightarrow t \in \langle 0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{dt}{t^3+1}$

  $= \left[\begin{array}{c} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array} \right] = \frac{1}{2} \int \left[\frac{\frac{1}{3}}{t+1} - \frac{\frac{t}{3} - \frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6} \int \frac{dt}{t+1} - \frac{1}{6} \int \frac{t-2}{t^2-t+1}\, dt$

Riešené príklady – 146

$\int \frac{x\,dx}{x^6+1}$ [Iné riešenie 146]

- $= \left[\begin{array}{c|c} \text{Subst. } t = x^2 & x\in(-\infty;0\rangle \Rightarrow t\in\langle 0;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{dt}{t^3+1}$

$= \left[\text{Rozklad na parciálne zlomky} \right] = \frac{1}{2}\int \left[\frac{\frac{1}{3}}{t+1} - \frac{\frac{t}{3}-\frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{6}\int \frac{t-2}{t^2-t+1}\,dt$

$= \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-4}{t^2-t+1}\,dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-1-3}{t^2-t+1}\,dt$

$= \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-1}{t^2-t+1}\,dt + \frac{1}{12}\int \frac{3\,dt}{t^2-t+1}$

Riešené príklady – 146

$\displaystyle\int \frac{x\,dx}{x^6+1}$

[Iné riešenie 146]

- $$= \left[\begin{array}{r|l} \text{Subst. } t = x^2 & x\in(-\infty;0\rangle \Rightarrow t\in\langle 0;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^3+1}$$

$$= \left[\begin{array}{c} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array}\right] = \frac{1}{2}\int \left[\frac{\frac{1}{3}}{t+1} - \frac{\frac{t}{3}-\frac{2}{3}}{t^2-t+1}\right] dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{6}\int \frac{t-2}{t^2-t+1}\,dt$$

$$= \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-4}{t^2-t+1}\,dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-1-3}{t^2-t+1}\,dt$$

$$= \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-1}{t^2-t+1}\,dt + \frac{1}{12}\int \frac{3\,dt}{t^2-t+1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u = t - \frac{1}{2} & t\in\langle 0;\infty) \Rightarrow u\in\langle -\frac{1}{2};\infty) & (t^2-t+1)' = 2t-1 \\ du = dt & t^2-t+1 = (t-\frac{1}{2})^2 + 1 - \frac{1}{4} = (t-\frac{1}{2})^2 + \frac{3}{4} = u^2 + \frac{3}{4} > 0 & t+1 \geq 1 > 0 \end{array}\right]$$

$$= \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{(t^2-t+1)'}{t^2-t+1}\,dt + \frac{1}{4}\int \frac{du}{u^2+\frac{3}{4}}$$

Riešené príklady – 146

$$\int \frac{x\,dx}{x^6+1}$$

[Iné riešenie 146]

- $= \left[\begin{array}{r|l} \text{Subst. } t = x^2 & x\in(-\infty;0\rangle \Rightarrow t\in\langle 0;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{dt}{t^3+1}$

$$= \left[\begin{array}{c} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array} \right] = \frac{1}{2}\int \left[\frac{\frac{1}{3}}{t+1} - \frac{\frac{t}{3}-\frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{6}\int \frac{t-2}{t^2-t+1}\,dt$$

$$= \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-4}{t^2-t+1}\,dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-1-3}{t^2-t+1}\,dt$$

$$= \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-1}{t^2-t+1}\,dt + \frac{1}{12}\int \frac{3\,dt}{t^2-t+1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u = t-\frac{1}{2} & t\in\langle 0;\infty) \Rightarrow u\in\langle -\frac{1}{2};\infty) & (t^2-t+1)' = 2t-1 \\ du = dt & t^2-t+1 = (t-\frac{1}{2})^2+1-\frac{1}{4} = (t-\frac{1}{2})^2+\frac{3}{4} = u^2+\frac{3}{4} > 0 & t+1 \geq 1 > 0 \end{array} \right]$$

$$= \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{(t^2-t+1)'}{t^2-t+1}\,dt + \frac{1}{4}\int \frac{du}{u^2+\frac{3}{4}}$$

$$= \frac{1}{6}\ln(t+1) - \frac{1}{12}\ln(t^2-t+1) + \frac{1}{4}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arctg}\frac{u}{\frac{\sqrt{3}}{2}} + c$$

$$= \frac{1}{12}\ln(t+1)^2 - \frac{1}{12}\ln(t^2-t+1) + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2u}{\sqrt{3}} + c$$

Riešené príklady – 146

$$\int \frac{x\,dx}{x^6+1} = \frac{1}{12}\ln\frac{(x^2+1)^2}{x^4-x^2+1} + \frac{1}{2\sqrt{3}}\,\mathrm{arctg}\,\frac{2x^2-1}{\sqrt{3}} + c$$

[Iné riešenie 146]

- $= \left[\begin{array}{r|l} \text{Subst. } t = x^2 & x\in(-\infty;0\rangle \Rightarrow t\in\langle 0;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int\frac{dt}{t^3+1}$

$= \left[\text{Rozklad na parciálne zlomky}\right] = \frac{1}{2}\int\left[\frac{\frac{1}{3}}{t+1} - \frac{\frac{t}{3}-\frac{2}{3}}{t^2-t+1}\right]dt = \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{6}\int\frac{t-2}{t^2-t+1}\,dt$

$= \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{2t-4}{t^2-t+1}\,dt = \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{2t-1-3}{t^2-t+1}\,dt$

$= \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{2t-1}{t^2-t+1}\,dt + \frac{1}{12}\int\frac{3\,dt}{t^2-t+1}$

$= \left[\begin{array}{r|l|l} \text{Subst. } u = t-\frac{1}{2} & t\in\langle 0;\infty)\Rightarrow u\in\langle -\frac{1}{2};\infty) & (t^2-t+1)' = 2t-1 \\ du = dt & t^2-t+1 = (t-\frac{1}{2})^2+1-\frac{1}{4} = (t-\frac{1}{2})^2+\frac{3}{4} = u^2+\frac{3}{4} > 0 & t+1 \geq 1 > 0 \end{array}\right]$

$= \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{(t^2-t+1)'}{t^2-t+1}\,dt + \frac{1}{4}\int\frac{du}{u^2+\frac{3}{4}}$

$= \frac{1}{6}\ln(t+1) - \frac{1}{12}\ln(t^2-t+1) + \frac{1}{4}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\,\mathrm{arctg}\,\frac{u}{\frac{\sqrt{3}}{2}} + c$

$= \frac{1}{12}\ln(t+1)^2 - \frac{1}{12}\ln(t^2-t+1) + \frac{1}{2\sqrt{3}}\,\mathrm{arctg}\,\frac{2u}{\sqrt{3}} + c$

$= \left[2u = 2t-1 = 2x^2-1\right] = \frac{1}{12}\ln\frac{(x^2+1)^2}{x^4-x^2+1} + \frac{1}{2\sqrt{3}}\,\mathrm{arctg}\,\frac{2x^2-1}{\sqrt{3}} + c,\ x\in R,\ c\in R.$

Riešené príklady – 147

$$\int \frac{x^2\,dx}{x^6+1}$$

[147]

Riešené príklady – 147

$\int \frac{x^2\,dx}{x^6+1}$ [147]

- $= \left[\begin{array}{c|c} \text{Subst. } t = x^3 & x \in R \\ dt = 3x^2\,dx & t \in R \end{array} \right] = \frac{1}{3} \int \frac{dt}{t^2+1}$

- $= \left[\begin{array}{l|l} \text{Korene rovnice } x^6+1=0 & (x \pm \frac{\sqrt{3}}{2} - \frac{i}{2})(x \pm \frac{\sqrt{3}}{2} + \frac{i}{2}) = (x \pm \frac{\sqrt{3}}{2})^2 - \frac{i^2}{4} = (x^2 \pm \sqrt{3}x + \frac{3}{4}) + \frac{1}{4} = x^2 \pm \sqrt{3}x + 1 > 0, \\ \text{sú: } -\frac{\sqrt{3}}{2} \pm \frac{i}{2},\ \frac{\sqrt{3}}{2} \pm \frac{i}{2},\ \pm i\,. & (x-i)(x+i) = x^2+1 > 0. \quad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

Riešené príklady – 147

$$\int \frac{x^2\,dx}{x^6+1}$$

[147]

- $= \left[\begin{array}{c|c} \text{Subst. } t = x^3 & x \in R \\ dt = 3x^2\,dx & t \in R \end{array} \right] = \dfrac{1}{3}\displaystyle\int \frac{dt}{t^2+1} = \frac{1}{3}\,\mathrm{arctg}\, t + c_1$

- $= \left[\begin{array}{c|l} \text{Korene rovnice } x^6+1=0 & (x \pm \frac{\sqrt{3}}{2} - \frac{i}{2})(x \pm \frac{\sqrt{3}}{2} + \frac{i}{2}) = (x \pm \frac{\sqrt{3}}{2})^2 - \frac{i^2}{4} = (x^2 \pm \sqrt{3}x + \frac{3}{4}) + \frac{1}{4} = x^2 \pm \sqrt{3}x + 1 > 0, \\ \text{sú: } -\frac{\sqrt{3}}{2} \pm \frac{i}{2},\ \frac{\sqrt{3}}{2} \pm \frac{i}{2},\ \pm i. & (x-i)(x+i) = x^2+1 > 0. \quad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

$= \left[\begin{array}{c} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array} \right] = \displaystyle\int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$

Riešené príklady – 147

$$\int \frac{x^2\,dx}{x^6+1} = \frac{1}{3}\operatorname{arctg} x^3 + c_1$$ [147]

- $= \left[\begin{array}{l|l} \text{Subst. } t = x^3 & x\in R \\ dt = 3x^2\,dx & t\in R \end{array} \right] = \frac{1}{3}\int \frac{dt}{t^2+1} = \frac{1}{3}\operatorname{arctg} t + c_1 = \frac{1}{3}\operatorname{arctg} x^3 + c_1,\ x\in R,\ c_1\in R.$

- $= \left[\begin{array}{l|l} \text{Korene rovnice } x^6+1=0 & (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. & (x-i)(x+i) = x^2+1>0. \quad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

$$= \left[\begin{array}{c} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array} \right] = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} & du = dx & x^2+\sqrt{3}x+1 = (x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x+\frac{\sqrt{3}}{2})^2+\frac{1}{4} = u^2+\frac{1}{4}>0 & x\in R,\ u\in R \\ v = x-\frac{\sqrt{3}}{2} & dv = dx & x^2-\sqrt{3}x+1 = (x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x-\frac{\sqrt{3}}{2})^2+\frac{1}{4} = v^2+\frac{1}{4}>0 & x\in R,\ v\in R \end{array} \right]$$

$$= -\frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{6}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{6}\int \frac{dv}{v^2+\frac{1}{4}}$$

Riešené príklady – 147

$$\int \frac{x^2\,dx}{x^6+1} = \frac{1}{3}\operatorname{arctg} x^3 + c_1$$ [147]

- $= \left[\begin{array}{r|l} \text{Subst. } t = x^3 & x\in R \\ dt = 3x^2\,dx & t\in R \end{array}\right] = \frac{1}{3}\int \frac{dt}{t^2+1} = \frac{1}{3}\operatorname{arctg} t + c_1 = \frac{1}{3}\operatorname{arctg} x^3 + c_1,\ x\in R,\ c_1\in R.$

- $= \left[\begin{array}{l|l} \text{Korene rovnice } x^6+1=0 & (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. & (x-i)(x+i) = x^2+1>0. \quad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array}\right]$

$$= \left[\begin{array}{c} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array}\right] = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \left[\begin{array}{r|l|l|l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} & du = dx & x^2+\sqrt{3}x+1 = (x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x+\frac{\sqrt{3}}{2})^2+\frac{1}{4} = u^2+\frac{1}{4}>0 & x\in R,\ u\in R \\ v = x-\frac{\sqrt{3}}{2} & dv = dx & x^2-\sqrt{3}x+1 = (x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x-\frac{\sqrt{3}}{2})^2+\frac{1}{4} = v^2+\frac{1}{4}>0 & x\in R,\ v\in R \end{array}\right]$$

$$= -\frac{1}{3}\int\frac{dx}{x^2+1} + \frac{1}{6}\int\frac{du}{u^2+\frac{1}{4}} + \frac{1}{6}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$= -\frac{1}{3}\operatorname{arctg} x + \frac{1}{6}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}} + \frac{1}{6}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}} + c_2$$

$$= -\frac{1}{3}\operatorname{arctg} x + \frac{1}{3}\operatorname{arctg} 2u + \frac{1}{3}\operatorname{arctg} 2v + c_2$$

Riešené príklady – 147

$$\int \frac{x^2\,dx}{x^6+1} = \frac{1}{3}\operatorname{arctg} x^3 + c_1 = \frac{1}{3}\operatorname{arctg}(2x+\sqrt{3}) + \frac{1}{3}\operatorname{arctg}(2x-\sqrt{3}) - \frac{1}{3}\operatorname{arctg} x + c_2$$ [147]

- $= \left[\begin{array}{c|c} \text{Subst. } t = x^3 & x\in R \\ dt = 3x^2\,dx & t\in R \end{array}\right] = \frac{1}{3}\int \frac{dt}{t^2+1} = \frac{1}{3}\operatorname{arctg} t + c_1 = \frac{1}{3}\operatorname{arctg} x^3 + c_1,\ x\in R,\ c_1\in R.$

- $= \left[\begin{array}{l|l} \text{Korene rovnice } x^6+1=0 & (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. & (x-i)(x+i) = x^2+1>0. \quad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array}\right]$

$$= \left[\begin{array}{c} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array}\right] = \int\left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} & du = dx & x^2+\sqrt{3}x+1 = (x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x+\frac{\sqrt{3}}{2})^2+\frac{1}{4} = u^2+\frac{1}{4}>0 & x\in R,\ u\in R \\ \phantom{\text{Subst. }} v = x-\frac{\sqrt{3}}{2} & dv = dx & x^2-\sqrt{3}x+1 = (x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x-\frac{\sqrt{3}}{2})^2+\frac{1}{4} = v^2+\frac{1}{4}>0 & x\in R,\ v\in R \end{array}\right]$$

$$= -\frac{1}{3}\int\frac{dx}{x^2+1} + \frac{1}{6}\int\frac{du}{u^2+\frac{1}{4}} + \frac{1}{6}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$= -\frac{1}{3}\operatorname{arctg} x + \frac{1}{6}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}} + \frac{1}{6}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}} + c_2$$

$$= -\frac{1}{3}\operatorname{arctg} x + \frac{1}{3}\operatorname{arctg} 2u + \frac{1}{3}\operatorname{arctg} 2v + c_2$$

$$= -\frac{1}{3}\operatorname{arctg} x + \frac{1}{3}\operatorname{arctg}(2x+\sqrt{3}) + \frac{1}{3}\operatorname{arctg}(2x-\sqrt{3}) + c_2,\ x\in R,\ c_2\in R.$$

Riešené príklady – 148

$$\int \frac{x^3\,dx}{x^6+1}$$

[148]

Riešené príklady – 148

$\int \frac{x^3\,dx}{x^6+1}$ [148]

- $=$ [Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\pm i$. | $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1 > 0,$
  $(x-i)(x+i) = x^2+1 > 0.$ $\Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1).$]

Riešené príklady – 148

$\int \frac{x^3\,dx}{x^6+1}$ [148]

- $=$ [Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\pm i$. | $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2})=(x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4}=(x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4}=x^2\pm\sqrt{3}x+1>0$, $(x-i)(x+i)=x^2+1>0$. $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)$.]

$= \left[\text{Rozklad na parciálne zlomky}\right] = \int\left[\frac{-\frac{x}{3}}{x^2+1}+\frac{\frac{x}{6}}{x^2+\sqrt{3}x+1}+\frac{\frac{x}{6}}{x^2-\sqrt{3}x+1}\right]dx$

Riešené príklady – 148

$\int \frac{x^3\,dx}{x^6+1}$ [148]

- $= \Big[$ Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i$. $\Big|$ $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0,$ $(x-i)(x+i)=x^2+1>0.$ $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1).\Big]$

$= \Big[$ Rozklad na parciálne zlomky $\Big]$ $= \displaystyle\int\left[\frac{-\frac{x}{3}}{x^2+1}+\frac{\frac{x}{6}}{x^2+\sqrt{3}x+1}+\frac{\frac{x}{6}}{x^2-\sqrt{3}x+1}\right]dx$

$$= \left[\begin{array}{l|l} (x^2+\sqrt{3}x+1)'=2x+\sqrt{3} & \frac{x}{6}=\frac{1}{12}\cdot 2x=\frac{1}{12}(2x+\sqrt{3}-\sqrt{3})=\frac{1}{12}(2x+\sqrt{3})-\frac{\sqrt{3}}{12} \\ (x^2-\sqrt{3}x+1)'=2x-\sqrt{3} & \frac{x}{6}=\frac{1}{12}\cdot 2x=\frac{1}{12}(2x-\sqrt{3}+\sqrt{3})=\frac{1}{12}(2x-\sqrt{3})+\frac{\sqrt{3}}{12} \end{array}\right]$$

$$= -\frac{1}{6}\int\frac{2x\,dx}{x^2+1}+\frac{1}{12}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1}-\frac{\sqrt{3}}{12}\int\frac{dx}{x^2+\sqrt{3}x+1}+\frac{1}{12}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1}+\frac{\sqrt{3}}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$$

Riešené príklady – 148

$\int \frac{x^3\,dx}{x^6+1}$ [148]

- $= \left[\text{Korene rovnice } x^6+1=0 \text{ sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. \;\middle|\; \begin{array}{l} (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ (x-i)(x+i) = x^2+1>0. \qquad \Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

$$= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l|l} (x^2+\sqrt{3}x+1)' = 2x+\sqrt{3} & \frac{x}{6} = \frac{1}{12}\cdot 2x = \frac{1}{12}(2x+\sqrt{3}-\sqrt{3}) = \frac{1}{12}(2x+\sqrt{3})-\frac{\sqrt{3}}{12} \\ (x^2-\sqrt{3}x+1)' = 2x-\sqrt{3} & \frac{x}{6} = \frac{1}{12}\cdot 2x = \frac{1}{12}(2x-\sqrt{3}+\sqrt{3}) = \frac{1}{12}(2x-\sqrt{3})+\frac{\sqrt{3}}{12} \end{array} \right]$$

$$= -\frac{1}{6}\int \frac{2x\,dx}{x^2+1} + \frac{1}{12}\int \frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{12}\int \frac{dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int \frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{12}\int \frac{dx}{x^2-\sqrt{3}x+1}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} & du = dx & x^2+\sqrt{3}x+1 = (x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x+\frac{\sqrt{3}}{2})^2+\frac{1}{4} = u^2+\frac{1}{4}>0 & x\in R,\ u\in R \\ \phantom{\text{Subst. }} v = x-\frac{\sqrt{3}}{2} & dv = dx & x^2-\sqrt{3}x+1 = (x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x-\frac{\sqrt{3}}{2})^2+\frac{1}{4} = v^2+\frac{1}{4}>0 & x\in R,\ v\in R \end{array} \right]$$

$$= -\frac{1}{6}\int \frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{12}\int \frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12}\int \frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

Riešené príklady – 148

$\int \frac{x^3\,dx}{x^6+1}$ [148]

- $= \left[\begin{array}{l} \text{Korene rovnice } x^6+1=0 \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. \end{array} \middle| \begin{array}{l} (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ (x-i)(x+i) = x^2+1>0. \qquad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

$= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx$

$= \left[\begin{array}{l} (x^2+\sqrt{3}x+1)' = 2x+\sqrt{3} \\ (x^2-\sqrt{3}x+1)' = 2x-\sqrt{3} \end{array} \middle| \begin{array}{l} \frac{x}{6} = \frac{1}{12}\cdot 2x = \frac{1}{12}(2x+\sqrt{3}-\sqrt{3}) = \frac{1}{12}(2x+\sqrt{3})-\frac{\sqrt{3}}{12} \\ \frac{x}{6} = \frac{1}{12}\cdot 2x = \frac{1}{12}(2x-\sqrt{3}+\sqrt{3}) = \frac{1}{12}(2x-\sqrt{3})+\frac{\sqrt{3}}{12} \end{array} \right]$

$= -\frac{1}{6}\int \frac{2x\,dx}{x^2+1} + \frac{1}{12}\int \frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{12}\int \frac{dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int \frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{12}\int \frac{dx}{x^2-\sqrt{3}x+1}$

$= \left[\begin{array}{l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} \\ \qquad\ \ v = x-\frac{\sqrt{3}}{2} \end{array} \middle| \begin{array}{l} du = dx \\ dv = dx \end{array} \middle| \begin{array}{l} x^2+\sqrt{3}x+1 = (x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x+\frac{\sqrt{3}}{2})^2+\frac{1}{4} = u^2+\frac{1}{4}>0 \\ x^2-\sqrt{3}x+1 = (x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x-\frac{\sqrt{3}}{2})^2+\frac{1}{4} = v^2+\frac{1}{4}>0 \end{array} \middle| \begin{array}{l} x\in R,\ u\in R \\ x\in R,\ v\in R \end{array} \right]$

$= -\frac{1}{6}\int \frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{12}\int \frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12}\int \frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{12}\int \frac{dv}{v^2+\frac{1}{4}}$

$= -\frac{1}{6}\ln\left(x^2+1\right) + \frac{\ln(x^2+\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}} + \frac{\ln(x^2-\sqrt{3}x+1)}{12} + \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}} + c$

$= -\frac{\ln(x^2+1)^2}{12} + \frac{\ln(x^2+\sqrt{3}x+1)+\ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{6}\operatorname{arctg}2u + \frac{\sqrt{3}}{6}\operatorname{arctg}2v + c$

Riešené príklady – 148

$$\int \frac{x^3\,dx}{x^6+1} = \frac{1}{12}\ln\frac{x^4-x^2+1}{(x^2+1)^2} - \frac{\sqrt{3}\,\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\sqrt{3}\,\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c \quad [148]$$

- $= \left[\begin{array}{l|l} \text{Korene rovnice } x^6+1=0 & (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. & (x-i)(x+i) = x^2+1>0. \qquad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array}\right]$

$$= \left[\begin{array}{l} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array}\right] = \int\left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \left[\begin{array}{l|l} (x^2+\sqrt{3}x+1)' = 2x+\sqrt{3} & \frac{x}{6} = \frac{1}{12}\cdot 2x = \frac{1}{12}(2x+\sqrt{3}-\sqrt{3}) = \frac{1}{12}(2x+\sqrt{3})-\frac{\sqrt{3}}{12} \\ (x^2-\sqrt{3}x+1)' = 2x-\sqrt{3} & \frac{x}{6} = \frac{1}{12}\cdot 2x = \frac{1}{12}(2x-\sqrt{3}+\sqrt{3}) = \frac{1}{12}(2x-\sqrt{3})+\frac{\sqrt{3}}{12} \end{array}\right]$$

$$= -\frac{1}{6}\int\frac{2x\,dx}{x^2+1} + \frac{1}{12}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{12}\int\frac{dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} & du = dx & x^2+\sqrt{3}x+1 = (x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x+\frac{\sqrt{3}}{2})^2+\frac{1}{4} = u^2+\frac{1}{4}>0 & x\in R,\ u\in R \\ \qquad\quad v = x-\frac{\sqrt{3}}{2} & dv = dx & x^2-\sqrt{3}x+1 = (x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x-\frac{\sqrt{3}}{2})^2+\frac{1}{4} = v^2+\frac{1}{4}>0 & x\in R,\ v\in R \end{array}\right]$$

$$= -\frac{1}{6}\int\frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{12}\int\frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{12}\int\frac{du}{u^2+\frac{1}{4}} + \frac{1}{12}\int\frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$= -\frac{1}{6}\ln(x^2+1) + \frac{\ln(x^2+\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{u}{\frac{1}{2}} + \frac{\ln(x^2-\sqrt{3}x+1)}{12} + \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{v}{\frac{1}{2}} + c$$

$$= -\frac{\ln(x^2+1)^2}{12} + \frac{\ln(x^2+\sqrt{3}x+1)+\ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{6}\,\mathrm{arctg}\,2u + \frac{\sqrt{3}}{6}\,\mathrm{arctg}\,2v + c$$

$$= \left[\begin{array}{l} (x^2+1+\sqrt{3}x)(x^2+1-\sqrt{3}x) \\ \quad = (x^2+1)^2-3x^2 = x^4-x^2+1 \end{array}\right] = \frac{1}{12}\ln\frac{x^4-x^2+1}{(x^2+1)^2} - \frac{\sqrt{3}\,\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\sqrt{3}\,\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c,$$

$$x\in R,\ c\in R.$$

Riešené príklady – 148

$$\int \frac{x^3\,dx}{x^6+1}$$

[Iné riešenie 148]

Riešené príklady – 148

$$\int \frac{x^3\,dx}{x^6+1}$$

[Iné riešenie 148]

- $= \displaystyle\int \frac{x\cdot x^2\,dx}{x^6+1} = \left[\begin{array}{r|l} \text{Subst. } t = x^2 & x\in(-\infty;0\rangle \Rightarrow t\in\langle 0;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1}$

Riešené príklady – 148

$\int \frac{x^3\,dx}{x^6+1}$

[Iné riešenie 148]

- $= \int \frac{x \cdot x^2\,dx}{x^6+1} = \left[\begin{array}{c|l} \text{Subst. } t = x^2 & x \in (-\infty; 0\rangle \Rightarrow t \in \langle 0; \infty) \\ dt = 2x\,dx & x \in \langle 0; \infty) \Rightarrow t \in \langle 0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{t\,dt}{t^3+1}$

  $= \left[\text{Rozklad na parciálne zlomky} \right] = \frac{1}{2} \int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1} \right] dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{6} \int \frac{t+1}{t^2-t+1}\,dt$

Riešené príklady – 148

$\int \frac{x^3\,dx}{x^6+1}$

[Iné riešenie 148]

- $$= \int \frac{x\cdot x^2\,dx}{x^6+1} = \left[\begin{array}{c|c} \text{Subst. } t = x^2 & x\in(-\infty;0) \Rightarrow t\in\langle 0;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1}$$
$$= \left[\text{Rozklad na parciálne zlomky}\right] = \frac{1}{2}\int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1}\right]dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{6}\int \frac{t+1}{t^2-t+1}\,dt$$
$$= -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t+2}{t^2-t+1}\,dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t-1+3}{t^2-t+1}\,dt$$
$$= -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t-1}{t^2-t+1}\,dt + \frac{1}{12}\int \frac{3\,dt}{t^2-t+1}$$

Riešené príklady – 148

$$\int \frac{x^3\,dx}{x^6+1}$$

[Iné riešenie 148]

- $= \int \frac{x\cdot x^2\,dx}{x^6+1} = \left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle \Rightarrow t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1}$
  $= \left[\begin{array}{c} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array}\right] = \frac{1}{2}\int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1}\right]dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{6}\int \frac{t+1}{t^2-t+1}\,dt$
  $= -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t+2}{t^2-t+1}\,dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t-1+3}{t^2-t+1}\,dt$
  $= -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t-1}{t^2-t+1}\,dt + \frac{1}{12}\int \frac{3\,dt}{t^2-t+1}$
  $= \left[\begin{array}{r|l|l} \text{Subst. } u=t-\frac{1}{2} & t\in\langle 0;\infty) \Rightarrow u\in\langle -\frac{1}{2};\infty) & (t^2-t+1)'=2t-1 \\ du=dt & t^2-t+1=(t-\frac{1}{2})^2+1-\frac{1}{4}=(t-\frac{1}{2})^2+\frac{3}{4}=u^2+\frac{3}{4}>0 & t+1\geq 1>0 \end{array}\right]$
  $= -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{(t^2-t+1)'}{t^2-t+1}\,dt + \frac{1}{4}\int \frac{du}{u^2+\frac{3}{4}}$

Riešené príklady – 148

$$\int \frac{x^3\,dx}{x^6+1}$$

[Iné riešenie 148]

- $$= \int \frac{x \cdot x^2\,dx}{x^6+1} = \left[\begin{array}{r|l} \text{Subst. } t = x^2 & x \in (-\infty; 0\rangle \Rightarrow t \in \langle 0; \infty) \\ dt = 2x\,dx & x \in \langle 0; \infty) \Rightarrow t \in \langle 0; \infty) \end{array}\right] = \frac{1}{2} \int \frac{t\,dt}{t^3+1}$$

$$= \left[\begin{array}{c} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array}\right] = \frac{1}{2} \int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1} \right] dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{6} \int \frac{t+1}{t^2-t+1}\,dt$$

$$= -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t+2}{t^2-t+1}\,dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t-1+3}{t^2-t+1}\,dt$$

$$= -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t-1}{t^2-t+1}\,dt + \frac{1}{12} \int \frac{3\,dt}{t^2-t+1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u = t - \frac{1}{2} & t \in \langle 0; \infty) \Rightarrow u \in \langle -\frac{1}{2}; \infty) & (t^2 - t + 1)' = 2t - 1 \\ du = dt & t^2 - t + 1 = (t - \frac{1}{2})^2 + 1 - \frac{1}{4} = (t - \frac{1}{2})^2 + \frac{3}{4} = u^2 + \frac{3}{4} > 0 & t + 1 \geq 1 > 0 \end{array}\right]$$

$$= -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{(t^2-t+1)'}{t^2-t+1}\,dt + \frac{1}{4} \int \frac{du}{u^2+\frac{3}{4}}$$

$$= -\frac{1}{6} \ln(t+1) + \frac{1}{12} \ln\left(t^2-t+1\right) + \frac{1}{4} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arctg} \frac{u}{\frac{\sqrt{3}}{2}} + c$$

$$= -\frac{1}{12} \ln(t+1)^2 + \frac{1}{12} \ln\left(t^2-t+1\right) + \frac{1}{2\sqrt{3}} \operatorname{arctg} \frac{2u}{\sqrt{3}} + c$$

Riešené príklady – 148

$$\int \frac{x^3\,dx}{x^6+1} = \frac{1}{12}\ln\frac{x^4-x^2+1}{(x^2+1)^2} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2x^2-1}{\sqrt{3}} + c$$

[Iné riešenie 148]

- $$= \int \frac{x\cdot x^2\,dx}{x^6+1} = \left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle \Rightarrow t\in\langle 0;\infty) \\ dt = 2x\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1}$$

$$= \left[\begin{array}{c} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array}\right] = \frac{1}{2}\int\left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1}\right]dt = -\frac{1}{6}\int\frac{dt}{t+1} + \frac{1}{6}\int\frac{t+1}{t^2-t+1}\,dt$$

$$= -\frac{1}{6}\int\frac{dt}{t+1} + \frac{1}{12}\int\frac{2t+2}{t^2-t+1}\,dt = -\frac{1}{6}\int\frac{dt}{t+1} + \frac{1}{12}\int\frac{2t-1+3}{t^2-t+1}\,dt$$

$$= -\frac{1}{6}\int\frac{dt}{t+1} + \frac{1}{12}\int\frac{2t-1}{t^2-t+1}\,dt + \frac{1}{12}\int\frac{3\,dt}{t^2-t+1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u = t-\frac{1}{2} & t\in\langle 0;\infty) \Rightarrow u\in\langle -\frac{1}{2};\infty) & (t^2-t+1)' = 2t-1 \\ du = dt & t^2-t+1 = (t-\frac{1}{2})^2+1-\frac{1}{4} = (t-\frac{1}{2})^2+\frac{3}{4} = u^2+\frac{3}{4} > 0 & t+1\geq 1 > 0 \end{array}\right]$$

$$= -\frac{1}{6}\int\frac{dt}{t+1} + \frac{1}{12}\int\frac{(t^2-t+1)'}{t^2-t+1}\,dt + \frac{1}{4}\int\frac{du}{u^2+\frac{3}{4}}$$

$$= -\frac{1}{6}\ln(t+1) + \frac{1}{12}\ln\left(t^2-t+1\right) + \frac{1}{4}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arctg}\frac{u}{\frac{\sqrt{3}}{2}} + c$$

$$= -\frac{1}{12}\ln(t+1)^2 + \frac{1}{12}\ln\left(t^2-t+1\right) + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2u}{\sqrt{3}} + c$$

$$= \left[\, 2u = 2t-1 = 2x^2-1 \,\right] = \frac{1}{12}\ln\frac{x^4-x^2+1}{(x^2+1)^2} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2x^2-1}{\sqrt{3}} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 149

$$\int \frac{x^4\,dx}{x^6+1}$$ [149]

Riešené príklady – 149

$$\int \frac{x^4\,dx}{x^6+1}$$ [149]

- $=$ [Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\pm i$. | $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2})=(x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4}=(x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4}=x^2\pm\sqrt{3}x+1>0$, $(x-i)(x+i)=x^2+1>0$. $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)$.]

Riešené príklady – 149

$$\int \frac{x^4\,dx}{x^6+1}$$ [149]

- $=$ [Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i$. | $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0,$ $(x-i)(x+i)=x^2+1>0.$ $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1).$]

$=$ [Rozklad na parciálne zlomky] $$= \int\left[\frac{\frac{1}{3}}{x^2+1} - \frac{\frac{x}{2\sqrt{3}}+\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

Riešené príklady – 149

$\displaystyle\int \frac{x^4\,dx}{x^6+1}$ [149]

- $= \left[\begin{array}{l} \text{Korene rovnice } x^6+1=0 \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. \end{array} \middle| \begin{array}{l} (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ (x-i)(x+i) = x^2+1>0. \qquad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

$= \left[\text{Rozklad na parciálne zlomky} \right] = \displaystyle\int \left[\frac{\frac{1}{3}}{x^2+1} - \frac{\frac{x}{2\sqrt{3}}+\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$

$= \left[\begin{array}{l} (x^2+\sqrt{3}x+1)' = 2x+\sqrt{3} \\ (x^2-\sqrt{3}x+1)' = 2x-\sqrt{3} \end{array} \middle| \begin{array}{l} \frac{x}{2\sqrt{3}}+\frac{1}{6} = \frac{1}{4\sqrt{3}}(2x+\frac{4\sqrt{3}}{6}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{4\sqrt{3}}{6}-\sqrt{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{2\sqrt{3}}{3}-\frac{3\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}) - \frac{1}{12} \\ \frac{x}{2\sqrt{3}}-\frac{1}{6} = \frac{1}{4\sqrt{3}}(2x-\frac{4\sqrt{3}}{6}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{4\sqrt{3}}{6}+\sqrt{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{2\sqrt{3}}{3}+\frac{3\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}) + \frac{1}{12} \end{array} \right]$

$= \displaystyle\frac{1}{3}\int\frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int\frac{dx}{x^2+\sqrt{3}x+1} + \frac{1}{4\sqrt{3}}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1} + \frac{1}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$

Riešené príklady – 149

$$\int \frac{x^4\,dx}{x^6+1}$$

[149]

- $= \left[\text{Korene rovnice } x^6+1=0 \text{ sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. \;\middle|\; \begin{array}{l} (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ (x-i)(x+i) = x^2+1>0. \qquad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

$$= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{\frac{1}{3}}{x^2+1} - \frac{\frac{x}{2\sqrt{3}}+\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l|l} (x^2+\sqrt{3}x+1)' = 2x+\sqrt{3} & \frac{x}{2\sqrt{3}}+\frac{1}{6} = \frac{1}{4\sqrt{3}}(2x+\frac{4\sqrt{3}}{6}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{4\sqrt{3}}{6}-\sqrt{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{2\sqrt{3}}{3}-\frac{3\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}) - \frac{1}{12} \\ (x^2-\sqrt{3}x+1)' = 2x-\sqrt{3} & \frac{x}{2\sqrt{3}}-\frac{1}{6} = \frac{1}{4\sqrt{3}}(2x-\frac{4\sqrt{3}}{6}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{4\sqrt{3}}{6}+\sqrt{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{2\sqrt{3}}{3}+\frac{3\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}) + \frac{1}{12} \end{array} \right]$$

$$= \frac{1}{3}\int\frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int\frac{dx}{x^2+\sqrt{3}x+1} + \frac{1}{4\sqrt{3}}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1} + \frac{1}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} & du = dx & x^2+\sqrt{3}x+1 = (x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x+\frac{\sqrt{3}}{2})^2+\frac{1}{4} = u^2+\frac{1}{4}>0 & x\in R,\ u\in R \\ \phantom{\text{Subst. }} v = x-\frac{\sqrt{3}}{2} & dv = dx & x^2-\sqrt{3}x+1 = (x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x-\frac{\sqrt{3}}{2})^2+\frac{1}{4} = v^2+\frac{1}{4}>0 & x\in R,\ v\in R \end{array} \right]$$

$$= \frac{1}{3}\int\frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}}\int\frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int\frac{du}{u^2+\frac{1}{4}} + \frac{1}{4\sqrt{3}}\int\frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1} + \frac{1}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

Riešené príklady – 149

$$\int \frac{x^4\,dx}{x^6+1}$$ [149]

- $= \left[\begin{array}{l} \text{Korene rovnice } x^6+1=0 \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. \end{array} \middle| \begin{array}{l} (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ (x-i)(x+i) = x^2+1>0. \qquad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

$$= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{\frac{1}{3}}{x^2+1} - \frac{\frac{x}{2\sqrt{3}}+\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l|l} (x^2+\sqrt{3}x+1)' = 2x+\sqrt{3} & \frac{x}{2\sqrt{3}}+\frac{1}{6} = \frac{1}{4\sqrt{3}}(2x+\frac{4\sqrt{3}}{6}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{4\sqrt{3}}{6}-\sqrt{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{2\sqrt{3}}{3}-\frac{3\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}) - \frac{1}{12} \\ (x^2-\sqrt{3}x+1)' = 2x-\sqrt{3} & \frac{x}{2\sqrt{3}}-\frac{1}{6} = \frac{1}{4\sqrt{3}}(2x-\frac{4\sqrt{3}}{6}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{4\sqrt{3}}{6}+\sqrt{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{2\sqrt{3}}{3}+\frac{3\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}) + \frac{1}{12} \end{array} \right]$$

$$= \frac{1}{3}\int\frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int\frac{dx}{x^2+\sqrt{3}x+1} + \frac{1}{4\sqrt{3}}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1} + \frac{1}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} & du = dx & x^2+\sqrt{3}x+1 = (x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x+\frac{\sqrt{3}}{2})^2+\frac{1}{4} = u^2+\frac{1}{4} > 0 & x\in R,\ u\in R \\ \phantom{\text{Subst. }} v = x-\frac{\sqrt{3}}{2} & dv = dx & x^2-\sqrt{3}x+1 = (x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4} = (x-\frac{\sqrt{3}}{2})^2+\frac{1}{4} = v^2+\frac{1}{4} > 0 & x\in R,\ v\in R \end{array} \right]$$

$$= \frac{1}{3}\int\frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}}\int\frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int\frac{du}{u^2+\frac{1}{4}} + \frac{1}{4\sqrt{3}}\int\frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1} + \frac{1}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{1}{3}\operatorname{arctg} x - \frac{\ln(x^2+\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}} + \frac{\ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}} + c$$

$$= \frac{1}{3}\operatorname{arctg} x + \frac{\ln(x^2-\sqrt{3}x+1)-\ln(x^2+\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{6}\operatorname{arctg} 2u + \frac{1}{6}\operatorname{arctg} 2v + c$$

Riešené príklady – 149

$$\int \frac{x^4\,dx}{x^6+1} = \frac{1}{3}\operatorname{arctg} x + \frac{1}{4\sqrt{3}} \ln \frac{x^2-\sqrt{3}x+1}{x^2+\sqrt{3}x+1} + \frac{\operatorname{arctg}(2x+\sqrt{3})}{6} + \frac{\operatorname{arctg}(2x-\sqrt{3})}{6} + c$$ [149]

- $=$ [Korene rovnice $x^6+1=0$ sú: $-\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i$. | $(x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0$, $(x-i)(x+i)=x^2+1>0$. $\Rightarrow x^6+1=(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)$.]

$= $ [Rozklad na parciálne zlomky] $\displaystyle = \int\left[\frac{\frac{1}{3}}{x^2+1} - \frac{\frac{x}{2\sqrt{3}}+\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1}\right]dx$

$=$ [$(x^2+\sqrt{3}x+1)'=2x+\sqrt{3}$ | $\frac{x}{2\sqrt{3}}+\frac{1}{6} = \frac{1}{4\sqrt{3}}(2x+\frac{4\sqrt{3}}{6}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{4\sqrt{3}}{6}-\sqrt{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3}+\frac{2\sqrt{3}}{3}-\frac{3\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x+\sqrt{3})-\frac{1}{12}$
$(x^2-\sqrt{3}x+1)'=2x-\sqrt{3}$ | $\frac{x}{2\sqrt{3}}-\frac{1}{6} = \frac{1}{4\sqrt{3}}(2x-\frac{4\sqrt{3}}{6}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{4\sqrt{3}}{6}+\sqrt{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3}-\frac{2\sqrt{3}}{3}+\frac{3\sqrt{3}}{3}) = \frac{1}{4\sqrt{3}}(2x-\sqrt{3})+\frac{1}{12}$]

$$= \frac{1}{3}\int\frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int\frac{dx}{x^2+\sqrt{3}x+1} + \frac{1}{4\sqrt{3}}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1} + \frac{1}{12}\int\frac{dx}{x^2-\sqrt{3}x+1}$$

$=$ [Subst. $u=x+\frac{\sqrt{3}}{2}$ | $du=dx$ | $x^2+\sqrt{3}x+1=(x+\frac{\sqrt{3}}{2})^2+1-\frac{3}{4}=(x+\frac{\sqrt{3}}{2})^2+\frac{1}{4}=u^2+\frac{1}{4}>0$ | $x\in R,\ u\in R$
$v=x-\frac{\sqrt{3}}{2}$ | $dv=dx$ | $x^2-\sqrt{3}x+1=(x-\frac{\sqrt{3}}{2})^2+1-\frac{3}{4}=(x-\frac{\sqrt{3}}{2})^2+\frac{1}{4}=v^2+\frac{1}{4}>0$ | $x\in R,\ v\in R$]

$$= \frac{1}{3}\int\frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}}\int\frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{12}\int\frac{du}{u^2+\frac{1}{4}} + \frac{1}{4\sqrt{3}}\int\frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1} + \frac{1}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{1}{3}\operatorname{arctg} x - \frac{\ln(x^2+\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}} + \frac{\ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}} + c$$

$$= \frac{1}{3}\operatorname{arctg} x + \frac{\ln(x^2-\sqrt{3}x+1)-\ln(x^2+\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{6}\operatorname{arctg} 2u + \frac{1}{6}\operatorname{arctg} 2v + c$$

$$= \frac{1}{3}\operatorname{arctg} x + \frac{1}{4\sqrt{3}} \ln \frac{x^2-\sqrt{3}x+1}{x^2+\sqrt{3}x+1} + \frac{\operatorname{arctg}(2x+\sqrt{3})}{6} + \frac{\operatorname{arctg}(2x-\sqrt{3})}{6} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 150

$$\int \frac{x^5\,dx}{x^6+1}$$

[150]

Riešené príklady – 150

$$\int \frac{x^5\,dx}{x^6+1}$$ [150]

- $= \left[\begin{array}{r|l} \text{Subst. } t = x^6+1 & x\in(-\infty;0) \Rightarrow t\in\langle 1;\infty) \\ dt = 6x^5\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 1;\infty) \end{array} \right] = \frac{1}{6}\int \frac{dt}{t}$

- $= \left[\begin{array}{l|l} \text{Korene rovnice } x^6+1=0 & (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. & (x-i)(x+i) = x^2+1>0. \quad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

Riešené príklady – 150

$\int \frac{x^5\,dx}{x^6+1}$ [150]

- $= \left[\begin{array}{r|l} \text{Subst. } t = x^6+1 & x\in(-\infty;0) \Rightarrow t\in\langle 1;\infty) \\ dt = 6x^5\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 1;\infty) \end{array} \right] = \frac{1}{6}\int \frac{dt}{t} = \frac{1}{6}\ln t + c$

- $= \left[\begin{array}{l|l} \text{Korene rovnice } x^6+1=0 & (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. & (x-i)(x+i) = x^2+1>0. \quad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

  $= \left[\begin{array}{c} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array} \right] = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3}+\frac{1}{2\sqrt{3}}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3}-\frac{1}{2\sqrt{3}}}{x^2-\sqrt{3}x+1} \right] dx$

Riešené príklady – 150

$$\int \frac{x^5\,dx}{x^6+1} = \frac{1}{6}\ln(x^6+1)+c$$ [150]

- $= \left[\begin{array}{r|l} \text{Subst. } t = x^6+1 & x\in(-\infty;0\rangle \Rightarrow t\in\langle 1;\infty) \\ dt = 6x^5\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 1;\infty) \end{array} \right] = \frac{1}{6}\int\frac{dt}{t} = \frac{1}{6}\ln t + c = \frac{1}{6}\ln(x^6+1)+c,\ x\in R,\ c\in R.$

- $= \left[\begin{array}{l|l} \text{Korene rovnice } x^6+1=0 & (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1>0, \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. & (x-i)(x+i) = x^2+1>0. \qquad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

$$= \left[\begin{array}{c} \text{Rozklad na} \\ \text{parciálne zlomky} \end{array} \right] = \int\left[\frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3}+\frac{1}{2\sqrt{3}}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3}-\frac{1}{2\sqrt{3}}}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \left[\begin{array}{l|l} (x^2+\sqrt{3}x+1)' = 2x+\sqrt{3} & \frac{x}{3}+\frac{1}{2\sqrt{3}} = \frac{1}{6}(2x+\frac{6}{2\sqrt{3}}) = \frac{1}{6}(2x+\frac{3}{\sqrt{3}}) = \frac{1}{6}(2x+\sqrt{3}) \\ (x^2-\sqrt{3}x+1)' = 2x-\sqrt{3} & \frac{x}{3}-\frac{1}{2\sqrt{3}} = \frac{1}{6}(2x-\frac{6}{2\sqrt{3}}) = \frac{1}{6}(2x-\frac{3}{\sqrt{3}}) = \frac{1}{6}(2x-\sqrt{3}) \end{array} \right]$$

$$= \frac{1}{6}\int\frac{2x\,dx}{x^2+1} + \frac{1}{6}\int\frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{6}\int\frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1}$$

$$= \frac{1}{6}\int\frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{6}\int\frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{6}\int\frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1}$$

Riešené príklady – 150

$$\int \frac{x^5\,dx}{x^6+1} = \frac{1}{6}\ln\left(x^6+1\right) + c$$ [150]

- $= \left[\begin{array}{r|l} \text{Subst. } t = x^6+1 & x\in(-\infty;0\rangle \Rightarrow t\in\langle 1;\infty) \\ dt = 6x^5\,dx & x\in\langle 0;\infty) \Rightarrow t\in\langle 1;\infty) \end{array} \right] = \frac{1}{6}\int \frac{dt}{t} = \frac{1}{6}\ln t + c = \frac{1}{6}\ln\left(x^6+1\right) + c$, $x\in R$, $c\in R$.

- $= \left[\begin{array}{l|l} \text{Korene rovnice } x^6+1=0 & (x\pm\frac{\sqrt{3}}{2}-\frac{i}{2})(x\pm\frac{\sqrt{3}}{2}+\frac{i}{2}) = (x\pm\frac{\sqrt{3}}{2})^2-\frac{i^2}{4} = (x^2\pm\sqrt{3}x+\frac{3}{4})+\frac{1}{4} = x^2\pm\sqrt{3}x+1 > 0, \\ \text{sú: } -\frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \frac{\sqrt{3}}{2}\pm\frac{i}{2},\ \pm i. & (x-i)(x+i) = x^2+1 > 0. \quad \Rightarrow x^6+1 = (x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1). \end{array} \right]$

$$= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3}+\frac{1}{2\sqrt{3}}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3}-\frac{1}{2\sqrt{3}}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[\begin{array}{l|l} (x^2+\sqrt{3}x+1)' = 2x+\sqrt{3} & \frac{x}{3}+\frac{1}{2\sqrt{3}} = \frac{1}{6}(2x+\frac{6}{2\sqrt{3}}) = \frac{1}{6}(2x+\frac{3}{\sqrt{3}}) = \frac{1}{6}(2x+\sqrt{3}) \\ (x^2-\sqrt{3}x+1)' = 2x-\sqrt{3} & \frac{x}{3}-\frac{1}{2\sqrt{3}} = \frac{1}{6}(2x-\frac{6}{2\sqrt{3}}) = \frac{1}{6}(2x-\frac{3}{\sqrt{3}}) = \frac{1}{6}(2x-\sqrt{3}) \end{array} \right]$$

$$= \frac{1}{6}\int \frac{2x\,dx}{x^2+1} + \frac{1}{6}\int \frac{(2x+\sqrt{3})\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{6}\int \frac{(2x-\sqrt{3})\,dx}{x^2-\sqrt{3}x+1}$$

$$= \frac{1}{6}\int \frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{6}\int \frac{(x^2+\sqrt{3}x+1)'\,dx}{x^2+\sqrt{3}x+1} + \frac{1}{6}\int \frac{(x^2-\sqrt{3}x+1)'\,dx}{x^2-\sqrt{3}x+1}$$

$$= \frac{1}{6}\ln\left(x^2+1\right) + \frac{1}{6}\ln\left(x^2+\sqrt{3}x+1\right) + \frac{1}{6}\ln\left(x^2-\sqrt{3}x+1\right) + c$$

$$= \frac{1}{6}\ln\left[(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)\right] + c = \ln\left(x^6+1\right) + c,\ x\in R,\ c\in R.$$

Riešené príklady – 151, 152

$$\int \frac{x^6\,dx}{x^6+1}$$ [151]

$$\int \frac{dx}{x^6(x^2+1)}$$ [152]

Riešené príklady – 151, 152

$\int \frac{x^6\,dx}{x^6+1}$ [151]

- $\int \frac{(x^6+1-1)\,dx}{x^6+1} = \int \left[1 - \frac{1}{x^6+1}\right] dx = \int dx - \int \frac{dx}{x^6+1}$

$\int \frac{dx}{x^6(x^2+1)}$ [152]

- $= \left[\text{Rozklad na parciálne zlomky} \right] = \int \left[\frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6} - \frac{1}{x^2+1}\right] dx = \int \left[x^{-2} - x^{-4} + x^{-6} - \frac{1}{x^2+1}\right] dx$

- $= \left[\begin{array}{l|l|l} \text{Pre } x^2 = t \text{ platí} & \text{Rozklad na} & \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t+1} \\ \frac{1}{x^6(x^2+1)} = \frac{1}{t^3(t+1)}. & \text{parciálne zlomky} & = \frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6} - \frac{1}{x^2+1} \end{array} \right] = \int \left[\frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6} - \frac{1}{x^2+1}\right] dx$

  $= \int \left[x^{-2} - x^{-4} + x^{-6} - \frac{1}{x^2+1}\right] dx$

Riešené príklady – 151, 152

$$\int \frac{x^6\,dx}{x^6+1} = x - \frac{1}{3}\operatorname{arctg} x - \frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} - \frac{\operatorname{arctg}(2x+\sqrt{3})}{6} - \frac{\operatorname{arctg}(2x-\sqrt{3})}{6} + c$$ [151]

- $\displaystyle\int \frac{(x^6+1-1)\,dx}{x^6+1} = \int\Big[1 - \frac{1}{x^6+1}\Big]dx = \int dx - \int\frac{dx}{x^6+1} =$ [Pr. 145.]
  $\displaystyle= x - \frac{1}{3}\operatorname{arctg} x - \frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} - \frac{\operatorname{arctg}(2x+\sqrt{3})}{6} - \frac{\operatorname{arctg}(2x-\sqrt{3})}{6} + c,\ x\in R,\ c\in R.$

$$\int \frac{dx}{x^6(x^2+1)} = -\frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} - \operatorname{arctg} x + c$$ [152]

- $=$ [Rozklad na parciálne zlomky] $\displaystyle= \int\Big[\frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6} - \frac{1}{x^2+1}\Big]dx = \int\Big[x^{-2} - x^{-4} + x^{-6} - \frac{1}{x^2+1}\Big]dx$
  $\displaystyle= \frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + \frac{x^{-5}}{-5} - \operatorname{arctg} x + c = -\frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} - \operatorname{arctg} x + c,$
  $x\in R,\ x\neq 0,\ c\in R.$

- $=$ [Pre $x^2=t$ platí $\frac{1}{x^6(x^2+1)} = \frac{1}{t^3(t+1)}$. | Rozklad na parciálne zlomky | $\frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{1}{t+1} = \frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6} - \frac{1}{x^2+1}$] $\displaystyle= \int\Big[\frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6} - \frac{1}{x^2+1}\Big]dx$
  $\displaystyle= \int\Big[x^{-2} - x^{-4} + x^{-6} - \frac{1}{x^2+1}\Big]dx = \frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + \frac{x^{-5}}{-5} - \operatorname{arctg} x + c$
  $\displaystyle= -\frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} - \operatorname{arctg} x + c,\ x\in R,\ x\neq 0,\ c\in R.$

Riešené príklady – 153

$$\int \frac{dx}{\sin x}$$ [153]

Riešené príklady – 153

$\int \frac{dx}{\sin x}$ [153]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq 0 & x \in (-\pi + 2k\pi; 0 + 2k\pi),\ \operatorname{tg} \frac{x}{2} < 0 \Rightarrow t \in (-\infty; 0) \\ dx = \frac{2\,dt}{t^2+1} & x \neq k\pi,\ k \in Z & x \in (0 + 2k\pi; \pi + 2k\pi),\ \operatorname{tg} \frac{x}{2} > 0 \Rightarrow t \in (0; \infty) \end{array} \right]$

- $= \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$

- $= \int \frac{\sin x \, dx}{\sin^2 x} = \int \frac{\sin x \, dx}{1 - \cos^2 x}$

Riešené príklady – 153

$$\int \frac{dx}{\sin x}$$ [153]

- $= \left[\begin{array}{r|l|l} \text{UGS:} \quad t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq 0 & x \in (-\pi + 2k\pi; 0 + 2k\pi),\ \operatorname{tg}\frac{x}{2} < 0 \Rightarrow t \in (-\infty; 0) \\ dx = \frac{2\,dt}{t^2+1} & x \neq k\pi,\ k \in Z & x \in (0 + 2k\pi; \pi + 2k\pi),\ \operatorname{tg}\frac{x}{2} > 0 \Rightarrow t \in (0; \infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t}$

- $= \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \left[\begin{array}{r|l} \text{Subst.} \quad t = \frac{x}{2} & x \in (0 + k\pi; \pi + k\pi),\ k \in Z \\ dt = \frac{dx}{2} & t \in \left(0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}\right),\ k \in Z \end{array} \right] = \int \frac{dt}{\sin t \cos t} = \int \frac{(\cos^2 t + \sin^2 t)\,dt}{\sin t \cos t}$
  $= \int \frac{\cos t\,dt}{\sin t} + \int \frac{\sin t\,dt}{\cos t} = \int \frac{\cos t\,dt}{\sin t} - \int \frac{-\sin t\,dt}{\cos t}$

- $= \int \frac{\sin x\,dx}{\sin^2 x} = \int \frac{\sin x\,dx}{1 - \cos^2 x} = \left[\begin{array}{r|l} \text{Subst.} \quad t = \cos x & x \in (0 + k\pi; \pi + k\pi),\ k \in Z \\ dt = -\sin x\,dx & t \in (-1; 1) \end{array} \right] = \int \frac{-dt}{1 - t^2} = \int \frac{dt}{t^2 - 1}$

Riešené príklady – 153

$\int \frac{dx}{\sin x}$ [153]

- $= \left[\begin{array}{r|l|l} \text{UGS:} \quad t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq 0 & x \in (-\pi + 2k\pi; 0 + 2k\pi),\ \operatorname{tg}\frac{x}{2} < 0 \Rightarrow t \in (-\infty; 0) \\ dx = \frac{2\,dt}{t^2+1} & x \neq k\pi,\ k \in Z & x \in (0 + 2k\pi; \pi + 2k\pi),\ \operatorname{tg}\frac{x}{2} > 0 \Rightarrow t \in (0; \infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t}$

  $= \ln|t| + c_1$

- $= \int \frac{dx}{2 \sin\frac{x}{2} \cos\frac{x}{2}} = \left[\begin{array}{r|l} \text{Subst.} \quad t = \frac{x}{2} & x \in (0 + k\pi; \pi + k\pi),\ k \in Z \\ dt = \frac{dx}{2} & t \in \left(0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}\right),\ k \in Z \end{array} \right] = \int \frac{dt}{\sin t \cos t} = \int \frac{(\cos^2 t + \sin^2 t)\,dt}{\sin t \cos t}$

  $= \int \frac{\cos t\,dt}{\sin t} + \int \frac{\sin t\,dt}{\cos t} = \int \frac{\cos t\,dt}{\sin t} - \int \frac{-\sin t\,dt}{\cos t} = \ln|\sin t| - \ln|\cos t| + c_1$

  $= \ln\left|\frac{\sin t}{\cos t}\right| + c_1 = \ln|\operatorname{tg} t| + c_1$

- $= \int \frac{\sin x\,dx}{\sin^2 x} = \int \frac{\sin x\,dx}{1 - \cos^2 x} = \left[\begin{array}{r|l} \text{Subst.} \quad t = \cos x & x \in (0 + k\pi; \pi + k\pi),\ k \in Z \\ dt = -\sin x\,dx & t \in (-1; 1) \end{array} \right] = \int \frac{-dt}{1 - t^2} = \int \frac{dt}{t^2 - 1}$

  $= \frac{1}{2} \ln\left|\frac{t-1}{t+1}\right| + c_2 = \left[t \in (-1; 1) \Rightarrow \begin{cases} |t+1| = t+1 \\ |t-1| = 1-t \end{cases} \right] = \frac{1}{2} \ln\frac{1-t}{1+t} + c_2$

Riešené príklady – 153

$$\int \frac{dx}{\sin x} = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c_1 = \frac{1}{2}\ln\frac{1-\cos x}{1+\cos x} + c_2 = \ln\sqrt{\frac{1-\cos x}{1+\cos x}} + c_2$$ [153]

- $= \left[\begin{array}{c|c|c} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq 0 & x\in(-\pi+2k\pi;0+2k\pi),\ \operatorname{tg}\frac{x}{2}<0 \Rightarrow t\in(-\infty;0) \\ dx = \frac{2\,dt}{t^2+1} & x\neq k\pi,\ k\in Z & x\in(0+2k\pi;\pi+2k\pi),\ \operatorname{tg}\frac{x}{2}>0 \Rightarrow t\in(0;\infty) \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t}$

  $= \ln|t| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c_1,\ x\in R,\ x\neq k\pi,\ k\in Z,\ c_1\in R.$

- $= \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \left[\begin{array}{c|c} \text{Subst. } t=\frac{x}{2} & x\in(0+k\pi;\pi+k\pi),\ k\in Z \\ dt=\frac{dx}{2} & t\in\left(0+\frac{k\pi}{2};\frac{\pi}{2}+\frac{k\pi}{2}\right),\ k\in Z \end{array}\right] = \int \frac{dt}{\sin t\cos t} = \int \frac{(\cos^2 t+\sin^2 t)\,dt}{\sin t\cos t}$

  $= \int \frac{\cos t\,dt}{\sin t} + \int \frac{\sin t\,dt}{\cos t} = \int \frac{\cos t\,dt}{\sin t} - \int \frac{-\sin t\,dt}{\cos t} = \ln|\sin t| - \ln|\cos t| + c_1$

  $= \ln\left|\frac{\sin t}{\cos t}\right| + c_1 = \ln|\operatorname{tg} t| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c_1,\ x\in R,\ x\neq k\pi,\ k\in Z,\ c_1\in R.$

- $= \int \frac{\sin x\,dx}{\sin^2 x} = \int \frac{\sin x\,dx}{1-\cos^2 x} = \left[\begin{array}{c|c} \text{Subst. } t=\cos x & x\in(0+k\pi;\pi+k\pi),\ k\in Z \\ dt=-\sin x\,dx & t\in(-1;1) \end{array}\right] = \int \frac{-dt}{1-t^2} = \int \frac{dt}{t^2-1}$

  $= \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \left[t\in(-1;1) \Rightarrow \left\{\begin{array}{l} |t+1| = t+1 \\ |t-1| = 1-t \end{array}\right.\right] = \frac{1}{2}\ln\frac{1-t}{1+t} + c_2$

  $= \frac{1}{2}\ln\frac{1-\cos x}{1+\cos x} + c_2 = \ln\sqrt{\frac{1-\cos x}{1+\cos x}} + c_2,\ x\in R,\ x\neq k\pi,\ k\in Z,\ c_2\in R.$

Riešené príklady – 154

$$\int \frac{dx}{1+\sin x}$$

[154]

Riešené príklady – 154

$$\int \frac{dx}{1+\sin x}$$ [154]

- $$= \left[\begin{array}{l|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq -1 & x\in(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi) \Rightarrow t\in(-\infty; -1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq -\frac{\pi}{2}+2k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \pi+2k\pi) \Rightarrow t\in(-1; \infty) \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}}$$
  $$= \int \frac{2\,dt}{(t+1)^2}$$

- $$= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x\,dx}{\cos^2 x}$$

Riešené príklady – 154

$$\int \frac{dx}{1+\sin x}$$ [154]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq -1 & x \in (-\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi) \Rightarrow t \in (-\infty; -1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq -\frac{\pi}{2} + 2k\pi,\, k \in Z & x \in (-\frac{\pi}{2} + 2k\pi; \pi + 2k\pi) \Rightarrow t \in (-1; \infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}}$

  $= \int \frac{2\,dt}{(t+1)^2} = \left[\begin{array}{r|l} \text{Subst. } u = t+1 & t \in (-\infty; -1) \Rightarrow u \in (-\infty; 0) \\ du = dt & t \in (-1; \infty) \Rightarrow u \in (0; \infty) \end{array} \right] = 2\int \frac{du}{u^2} = 2\int u^{-2}\,du$

- $= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x\,dx}{\cos^2 x}$

  $= \left[\begin{array}{r|l|l|l} \text{Subst. } t = \cos x & \cos x \neq 0 & x \in \langle -\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi) \Rightarrow t \in \langle -1; 0) & x \in \langle 0 + 2k\pi; \frac{\pi}{2} + 2k\pi) \Rightarrow t \in (0; 1\rangle \\ dt = -\sin x\,dx & x \neq \frac{\pi}{2} + k\pi,\, k \in Z & x \in (-\frac{\pi}{2} + 2k\pi; 0 + 2k\pi\rangle \Rightarrow t \in (0; 1\rangle & x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi\rangle \Rightarrow t \in \langle -1; 0) \end{array} \right]$

  $= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2}\,dt$

Riešené príklady – 154

$\int \frac{dx}{1+\sin x}$ [154]

- $= \left[\begin{array}{l|l|l} \text{UGS: } t=\mathrm{tg}\,\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq -1 & x\in(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi) \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x\neq -\frac{\pi}{2}+2k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \pi+2k\pi) \Rightarrow t\in(-1;\infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}}$

$= \int \frac{2\,dt}{(t+1)^2} = \left[\begin{array}{l|l} \text{Subst. } u=t+1 & t\in(-\infty;-1) \Rightarrow u\in(-\infty;0) \\ du = dt & t\in(-1;\infty) \Rightarrow u\in(0;\infty) \end{array} \right] = 2\int \frac{du}{u^2} = 2\int u^{-2}\,du$

$= 2\frac{u^{-1}}{-1} + c_1 = -\frac{2}{u} + c_1 = -\frac{2}{t+1} + c_1$

- $= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x\,dx}{\cos^2 x}$

$= \left[\begin{array}{l|l|l|l} \text{Subst. } t=\cos x & \cos x \neq 0 & x\in\langle -\pi+2k\pi; -\frac{\pi}{2}+2k\pi) \Rightarrow t\in\langle -1;0) & x\in\langle 0+2k\pi; \frac{\pi}{2}+2k\pi) \Rightarrow t\in(0;1\rangle \\ dt = -\sin x\,dx & x\neq \frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; 0+2k\pi\rangle \Rightarrow t\in(0;1\rangle & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi\rangle \Rightarrow t\in\langle -1;0) \end{array} \right]$

$= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2}\,dt = \mathrm{tg}\,x + \frac{t^{-1}}{-1} + c_2 = \mathrm{tg}\,x - \frac{1}{t} + c_2$

Riešené príklady – 154

$$\int \frac{dx}{1+\sin x} = c_1 - \frac{2}{\operatorname{tg}\frac{x}{2}+1} = \frac{\sin x - 1}{\cos x} + c_2$$ [154]

- $= \left[\begin{array}{l|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq -1 & x\in(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi) \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq -\frac{\pi}{2}+2k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \pi+2k\pi) \Rightarrow t\in(-1;\infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}}$

  $= \int \frac{2\,dt}{(t+1)^2} = \left[\begin{array}{l|l} \text{Subst. } u=t+1 & t\in(-\infty;-1) \Rightarrow u\in(-\infty;0) \\ du = dt & t\in(-1;\infty) \Rightarrow u\in(0;\infty) \end{array} \right] = 2\int \frac{du}{u^2} = 2\int u^{-2}\,du$

  $= 2\frac{u^{-1}}{-1} + c_1 = -\frac{2}{u} + c_1 = -\frac{2}{t+1} + c_1 = c_1 - \frac{2}{t+1} = c_1 - \frac{2}{\operatorname{tg}\frac{x}{2}+1},$

  $x\in R,\ x\neq -\frac{\pi}{2}+2k\pi,\ x\neq \pi+2k\pi,\ k\in Z,\ c_1\in R.$

- $= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x\,dx}{\cos^2 x}$

  $= \left[\begin{array}{l|l|l|l} \text{Subst. } t=\cos x & \cos x \neq 0 & x\in\langle -\pi+2k\pi; -\frac{\pi}{2}+2k\pi) \Rightarrow t\in\langle -1;0) & x\in\langle 0+2k\pi; \frac{\pi}{2}+2k\pi) \Rightarrow t\in(0;1\rangle \\ dt = -\sin x\,dx & x\neq \frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; 0+2k\pi\rangle \Rightarrow t\in(0;1\rangle & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi\rangle \Rightarrow t\in\langle -1;0) \end{array} \right]$

  $= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2}\,dt = \operatorname{tg} x + \frac{t^{-1}}{-1} + c_2 = \operatorname{tg} x - \frac{1}{t} + c_2$

  $= \frac{\sin x}{\cos x} - \frac{1}{\cos x} + c_2 = \frac{\sin x - 1}{\cos x} + c_2,\ x\in R,\ x\neq \frac{\pi}{2}+k\pi,\ k\in Z,\ c_2\in R.$

Riešené príklady – 155

$\int \frac{dx}{1-\sin x}$ [155]

Riešené príklady – 155

$\displaystyle\int \frac{dx}{1-\sin x}$ [155]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq 1 & x \in (-\pi + 2k\pi; \frac{\pi}{2} + 2k\pi) \Rightarrow t \in (-\infty; -1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \frac{\pi}{2} + 2k\pi, k \in Z & x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi) \Rightarrow t \in (-1; \infty) \end{array} \right] = \displaystyle\int \frac{\frac{2\,dt}{t^2+1}}{1-\frac{2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1-2t}{t^2+1}} = \int \frac{2\,dt}{(t-1)^2}$

- $= \displaystyle\int \frac{(1+\sin x)\,dx}{(1+\sin x)(1-\sin x)} = \int \frac{(1+\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1+\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} - \int \frac{-\sin x\,dx}{\cos^2 x}$

- $= \left[\begin{array}{c} \text{Nepárna} \\ \text{funkcia} \end{array} \right] = \displaystyle\int \frac{dx}{1+\sin(-x)}$

Riešené príklady – 155

$$\int \frac{dx}{1-\sin x}$$ [155]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq 1 & x\in(-\pi+2k\pi;\frac{\pi}{2}+2k\pi) \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x\neq\frac{\pi}{2}+2k\pi,\, k\in Z & x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi) \Rightarrow t\in(-1;\infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1-\frac{2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1-2t}{t^2+1}} = \int \frac{2\,dt}{(t-1)^2}$

  $= \left[\begin{array}{r|l} \text{Subst. } u=t-1 & t\in(-\infty;1) \Rightarrow u\in(-\infty;0) \\ du=dt & t\in(1;\infty) \Rightarrow u\in(0;\infty) \end{array} \right] = 2\int \frac{du}{u^2} = 2\int u^{-2}\,du$

- $= \int \frac{(1+\sin x)\,dx}{(1+\sin x)(1-\sin x)} = \int \frac{(1+\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1+\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} - \int \frac{-\sin x\,dx}{\cos^2 x}$

  $= \left[\begin{array}{r|l|l|l} \text{Subst. } t=\cos x & \cos x\neq 0 & x\in\langle-\pi+2k\pi;-\frac{\pi}{2}+2k\pi) \Rightarrow t\in\langle-1;0) & x\in\langle 0+2k\pi;\frac{\pi}{2}+2k\pi) \Rightarrow t\in(0;1\rangle \\ dt=-\sin x\,dx & x\neq\frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi;0+2k\pi\rangle \Rightarrow t\in(0;1\rangle & x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi\rangle \Rightarrow t\in\langle-1;0) \end{array} \right]$

  $= \int \frac{dx}{\cos^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} - \int t^{-2}\,dt$

- $= \left[\begin{array}{c} \text{Nepárna} \\ \text{funkcia} \end{array} \right] = \int \frac{dx}{1+\sin(-x)} = \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in R,\, x\neq\frac{\pi}{2}+2k\pi,\, k\in Z \\ dt=-dx & t\in R,\, t\neq-\frac{\pi}{2}+2k\pi,\, k\in Z \end{array} \right] = -\int \frac{dt}{1+\sin t}$

Riešené príklady – 155

$$\int \frac{dx}{1-\sin x}$$ [155]

- $= \left[\begin{array}{r|l|l} \text{UGS:} \quad t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq 1 & x \in (-\pi+2k\pi; \frac{\pi}{2}+2k\pi) \Rightarrow t \in (-\infty; -1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \frac{\pi}{2}+2k\pi, k \in Z & x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi) \Rightarrow t \in (-1; \infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1-\frac{2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1-2t}{t^2+1}} = \int \frac{2\,dt}{(t-1)^2}$

  $= \left[\begin{array}{r|l} \text{Subst. } u = t-1 & t \in (-\infty; 1) \Rightarrow u \in (-\infty; 0) \\ du = dt & t \in (1; \infty) \Rightarrow u \in (0; \infty) \end{array} \right] = 2\int \frac{du}{u^2} = 2\int u^{-2}\,du = 2\frac{u^{-1}}{-1} + c_1 = c_1 - \frac{2}{u}$

  $= c_1 - \frac{2}{t-1}$

- $= \int \frac{(1+\sin x)\,dx}{(1+\sin x)(1-\sin x)} = \int \frac{(1+\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1+\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} - \int \frac{-\sin x\,dx}{\cos^2 x}$

  $= \left[\begin{array}{r|l|l|l} \text{Subst. } t = \cos x & \cos x \neq 0 & x \in \langle -\pi+2k\pi; -\frac{\pi}{2}+2k\pi) \Rightarrow t \in \langle -1; 0) & x \in \langle 0+2k\pi; \frac{\pi}{2}+2k\pi) \Rightarrow t \in (0; 1\rangle \\ dt = -\sin x\,dx & x \neq \frac{\pi}{2}+k\pi, k \in Z & x \in (-\frac{\pi}{2}+2k\pi; 0+2k\pi\rangle \Rightarrow t \in (0; 1\rangle & x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi\rangle \Rightarrow t \in \langle -1; 0) \end{array} \right]$

  $= \int \frac{dx}{\cos^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} - \int t^{-2}\,dt = \operatorname{tg} x - \frac{t^{-1}}{-1} + c_2 = \operatorname{tg} x + \frac{1}{t} + c_2$

- $= \left[\begin{array}{c} \text{Nepárna} \\ \text{funkcia} \end{array} \right] = \int \frac{dx}{1+\sin(-x)} = \left[\begin{array}{r|l} \text{Subst. } t = -x & x \in R, x \neq \frac{\pi}{2}+2k\pi, k \in Z \\ dt = -dx & t \in R, t \neq -\frac{\pi}{2}+2k\pi, k \in Z \end{array} \right] = -\int \frac{dt}{1+\sin t} = \left[\text{Pr. 154.} \right]$

  $= c_1 + \frac{2}{\operatorname{tg}\frac{t}{2}+1} = -\frac{\sin t - 1}{\cos t} + c_2$

Riešené príklady – 155

$$\int \frac{dx}{1-\sin x} = c_1 - \frac{2}{\operatorname{tg}\frac{x}{2}-1} = \frac{\sin x+1}{\cos x} + c_2$$ [155]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq 1 & x\in(-\pi+2k\pi;\frac{\pi}{2}+2k\pi) \Rightarrow t\in(-\infty;-1) \\ dx=\frac{2\,dt}{t^2+1} & x\neq\frac{\pi}{2}+2k\pi, k\in Z & x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi) \Rightarrow t\in(-1;\infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1-\frac{2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1-2t}{t^2+1}} = \int \frac{2\,dt}{(t-1)^2}$

  $= \left[\begin{array}{r|l} \text{Subst. } u=t-1 & t\in(-\infty;1) \Rightarrow u\in(-\infty;0) \\ du=dt & t\in(1;\infty) \Rightarrow u\in(0;\infty) \end{array} \right] = 2\int\frac{du}{u^2} = 2\int u^{-2}\,du = 2\frac{u^{-1}}{-1} + c_1 = c_1 - \frac{2}{u}$

  $= c_1 - \frac{2}{t-1} = c_1 - \frac{2}{\operatorname{tg}\frac{x}{2}-1},\ x\in R,\ x\neq\frac{\pi}{2}+2k\pi,\ x\neq\pi+2k\pi,\ k\in Z,\ c_1\in R.$

- $= \int \frac{(1+\sin x)\,dx}{(1+\sin x)(1-\sin x)} = \int \frac{(1+\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1+\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} - \int \frac{-\sin x\,dx}{\cos^2 x}$

  $= \left[\begin{array}{r|l|l|l} \text{Subst. } t=\cos x & \cos x\neq 0 & x\in\langle-\pi+2k\pi;-\frac{\pi}{2}+2k\pi) \Rightarrow t\in\langle-1;0) & x\in\langle 0+2k\pi;\frac{\pi}{2}+2k\pi) \Rightarrow t\in(0;1\rangle \\ dt=-\sin x\,dx & x\neq\frac{\pi}{2}+k\pi, k\in Z & x\in(-\frac{\pi}{2}+2k\pi;0+2k\pi\rangle \Rightarrow t\in(0;1\rangle & x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi\rangle \Rightarrow t\in\langle-1;0) \end{array} \right]$

  $= \int\frac{dx}{\cos^2 x} - \int\frac{dt}{t^2} = \int\frac{dx}{\cos^2 x} - \int t^{-2}\,dt = \operatorname{tg} x - \frac{t^{-1}}{-1} + c_2 = \operatorname{tg} x + \frac{1}{t} + c_2$

  $= \frac{\sin x}{\cos x} + \frac{1}{\cos x} + c_2 = \frac{\sin x+1}{\cos x} + c_2,\ x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ k\in Z,\ c_2\in R.$

- $= \left[\begin{array}{c} \text{Nepárna} \\ \text{funkcia} \end{array} \right] = \int \frac{dx}{1+\sin(-x)} = \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in R, x\neq\frac{\pi}{2}+2k\pi, k\in Z \\ dt=-dx & t\in R, t\neq-\frac{\pi}{2}+2k\pi, k\in Z \end{array} \right] = -\int\frac{dt}{1+\sin t} = [\text{Pr. } 154.]$

  $= c_1 + \frac{2}{\operatorname{tg}\frac{t}{2}+1} = -\frac{\sin t-1}{\cos t} + c_2 = c_1 + \frac{2}{\operatorname{tg}\frac{-x}{2}+1} = -\frac{\sin(-x)-1}{\cos(-x)} + c_2$

Riešené príklady – 155

$$\int \frac{dx}{1-\sin x} = c_1 - \frac{2}{\operatorname{tg}\frac{x}{2}-1} = \frac{\sin x+1}{\cos x} + c_2$$ [155]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \neq 1 & x\in(-\pi+2k\pi;\frac{\pi}{2}+2k\pi) \Rightarrow t\in(-\infty;-1) \\ dx=\frac{2\,dt}{t^2+1} & x\neq\frac{\pi}{2}+2k\pi, k\in Z & x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi) \Rightarrow t\in(-1;\infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1-\frac{2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1-2t}{t^2+1}} = \int \frac{2\,dt}{(t-1)^2}$

  $= \left[\begin{array}{r|l} \text{Subst. } u=t-1 & t\in(-\infty;1) \Rightarrow u\in(-\infty;0) \\ du=dt & t\in(1;\infty) \Rightarrow u\in(0;\infty) \end{array} \right] = 2\int \frac{du}{u^2} = 2\int u^{-2}\,du = 2\frac{u^{-1}}{-1} + c_1 = c_1 - \frac{2}{u}$

  $= c_1 - \frac{2}{t-1} = c_1 - \frac{2}{\operatorname{tg}\frac{x}{2}-1},\ x\in R,\ x\neq\frac{\pi}{2}+2k\pi,\ x\neq\pi+2k\pi,\ k\in Z,\ c_1\in R.$

- $= \int \frac{(1+\sin x)\,dx}{(1+\sin x)(1-\sin x)} = \int \frac{(1+\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1+\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} - \int \frac{-\sin x\,dx}{\cos^2 x}$

  $= \left[\begin{array}{r|l|l|l} \text{Subst. } t=\cos x & \cos x\neq 0 & x\in\langle-\pi+2k\pi;-\frac{\pi}{2}+2k\pi) \Rightarrow t\in\langle-1;0) & x\in\langle 0+2k\pi;\frac{\pi}{2}+2k\pi) \Rightarrow t\in(0;1\rangle \\ dt=-\sin x\,dx & x\neq\frac{\pi}{2}+k\pi, k\in Z & x\in(-\frac{\pi}{2}+2k\pi;0+2k\pi\rangle \Rightarrow t\in(0;1\rangle & x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi\rangle \Rightarrow t\in\langle-1;0) \end{array} \right]$

  $= \int \frac{dx}{\cos^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} - \int t^{-2}\,dt = \operatorname{tg} x - \frac{t^{-1}}{-1} + c_2 = \operatorname{tg} x + \frac{1}{t} + c_2$

  $= \frac{\sin x}{\cos x} + \frac{1}{\cos x} + c_2 = \frac{\sin x+1}{\cos x} + c_2,\ x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ k\in Z,\ c_2\in R.$

- $= \left[\begin{array}{c} \text{Nepárna} \\ \text{funkcia} \end{array} \right] = \int \frac{dx}{1+\sin(-x)} = \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in R, x\neq\frac{\pi}{2}+2k\pi, k\in Z \\ dt=-dx & t\in R, t\neq-\frac{\pi}{2}+2k\pi, k\in Z \end{array} \right] = -\int \frac{dt}{1+\sin t} = \left[\text{Pr. 154.} \right]$

  $= c_1 + \frac{2}{\operatorname{tg}\frac{t}{2}+1} = -\frac{\sin t-1}{\cos t} + c_2 = c_1 + \frac{2}{\operatorname{tg}\frac{-x}{2}+1} = -\frac{\sin(-x)-1}{\cos(-x)} + c_2 = \left[\begin{array}{c} \text{Nepárne a párna} \\ \text{funkcie} \end{array} \right]$

  $= c_1 - \left[-\frac{2}{-\operatorname{tg}\frac{x}{2}+1} \right] = -\frac{-\sin x-1}{\cos x} + c_2 = c_1 - \frac{2}{\operatorname{tg}\frac{x}{2}-1} = \frac{\sin x+1}{\cos x} + c_2.$

Riešené príklady – 156, 157

$$\int \frac{dx}{5+4\sin x}$$ [156]

$$\int \frac{dx}{5-4\sin x}$$ [157]

Riešené príklady – 156, 157

$$\int \frac{dx}{5+4\sin x}$$ [156]

- $= \left[\begin{array}{r|l|l} \text{UGS:}\;\; t=\operatorname{tg}\frac{x}{2} & \sin x=\frac{2t}{t^2+1}\in\langle -1;1\rangle & x\in(-\pi+2k\pi;\pi+2k\pi) \\ dx=\frac{2\,dt}{t^2+1} & 5+4\sin x\geq 5-4=1>0 & t\in(-\infty;\infty),\qquad k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{5+\frac{4\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5+8t}{t^2+1}} = \int \frac{2\,dt}{5t^2+8t+5}$

$$\int \frac{dx}{5-4\sin x}$$ [157]

- $= \left[\begin{array}{r|l|l} \text{UGS:}\;\; t=\operatorname{tg}\frac{x}{2} & \sin x=\frac{2t}{t^2+1}\in\langle -1;1\rangle & x\in(-\pi+2k\pi;\pi+2k\pi) \\ dx=\frac{2\,dt}{t^2+1} & 5-4\sin x\geq 5-4=1>0 & t\in(-\infty;\infty),\qquad k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{5-\frac{4\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5-8t}{t^2+1}} = \int \frac{2\,dt}{5t^2-8t+5}$

Riešené príklady – 156, 157

$$\int \frac{dx}{5+4\sin x}$$ [156]

- $= \left[\begin{array}{r|l|l} \text{UGS:} \;\; t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \in \langle -1; 1\rangle & x \in (-\pi+2k\pi; \pi+2k\pi) \\ dx = \frac{2\,dt}{t^2+1} & 5+4\sin x \geq 5-4 = 1 > 0 & t \in (-\infty; \infty), \qquad k \in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{5+\frac{4\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5+8t}{t^2+1}} = \int \frac{2\,dt}{5t^2+8t+5}$

  $= \frac{2}{5}\int \frac{dt}{t^2+\frac{8t}{5}+5} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+5-\frac{16}{25}} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+\frac{25-16}{25}} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+\frac{9}{25}}$

$$\int \frac{dx}{5-4\sin x}$$ [157]

- $= \left[\begin{array}{r|l|l} \text{UGS:} \;\; t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \in \langle -1; 1\rangle & x \in (-\pi+2k\pi; \pi+2k\pi) \\ dx = \frac{2\,dt}{t^2+1} & 5-4\sin x \geq 5-4 = 1 > 0 & t \in (-\infty; \infty), \qquad k \in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{5-\frac{4\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5-8t}{t^2+1}} = \int \frac{2\,dt}{5t^2-8t+5}$

  $= \frac{2}{5}\int \frac{dt}{t^2-\frac{8t}{5}+5} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+5-\frac{16}{25}} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+\frac{25-16}{25}} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+\frac{9}{25}}$

Riešené príklady – 156, 157

$$\int \frac{dx}{5+4\sin x}$$ [156]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \in \langle -1;1\rangle & x\in(-\pi+2k\pi;\pi+2k\pi) \\ dx=\frac{2\,dt}{t^2+1} & 5+4\sin x \geq 5-4=1>0 & t\in(-\infty;\infty), \quad k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{5+\frac{4\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5+8t}{t^2+1}} = \int \frac{2\,dt}{5t^2+8t+5}$

  $= \frac{2}{5}\int \frac{dt}{t^2+\frac{8t}{5}+5} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+5-\frac{16}{25}} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+\frac{25-16}{25}} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+\frac{9}{25}} = \left[\begin{array}{r|l} \text{Subst. } u=t+\frac{4}{5} & t\in R \\ du=dt & u\in R \end{array} \right]$

  $= \frac{2}{5}\int \frac{du}{u^2+\frac{9}{25}}$

$$\int \frac{dx}{5-4\sin x}$$ [157]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \in \langle -1;1\rangle & x\in(-\pi+2k\pi;\pi+2k\pi) \\ dx=\frac{2\,dt}{t^2+1} & 5-4\sin x \geq 5-4=1>0 & t\in(-\infty;\infty), \quad k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{5-\frac{4\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5-8t}{t^2+1}} = \int \frac{2\,dt}{5t^2-8t+5}$

  $= \frac{2}{5}\int \frac{dt}{t^2-\frac{8t}{5}+5} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+5-\frac{16}{25}} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+\frac{25-16}{25}} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+\frac{9}{25}} = \left[\begin{array}{r|l} \text{Subst. } u=t-\frac{4}{5} & t\in R \\ du=dt & u\in R \end{array} \right]$

  $= \frac{2}{5}\int \frac{du}{u^2+\frac{9}{25}}$

Riešené príklady – 156, 157

$\int \frac{dx}{5+4\sin x}$ [156]

- $= \left[\begin{array}{l|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \in \langle -1;1\rangle & x\in(-\pi+2k\pi;\pi+2k\pi) \\ dx = \frac{2\,dt}{t^2+1} & 5+4\sin x \geq 5-4=1>0 & t\in(-\infty;\infty), \quad k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{5+\frac{4\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5+8t}{t^2+1}} = \int \frac{2\,dt}{5t^2+8t+5}$

$= \frac{2}{5}\int \frac{dt}{t^2+\frac{8t}{5}+5} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+5-\frac{16}{25}} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+\frac{25-16}{25}} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+\frac{9}{25}} = \left[\begin{array}{l|l} \text{Subst. } u=t+\frac{4}{5} & t\in R \\ du=dt & u\in R \end{array} \right]$

$= \frac{2}{5}\int \frac{du}{u^2+\frac{9}{25}} = \frac{2}{5\cdot\frac{3}{5}} \operatorname{arctg} \frac{u}{\frac{3}{5}} + c = \frac{2}{3}\operatorname{arctg}\frac{5u}{3} + c$

$\int \frac{dx}{5-4\sin x}$ [157]

- $= \left[\begin{array}{l|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \in \langle -1;1\rangle & x\in(-\pi+2k\pi;\pi+2k\pi) \\ dx = \frac{2\,dt}{t^2+1} & 5-4\sin x \geq 5-4=1>0 & t\in(-\infty;\infty), \quad k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{5-\frac{4\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5-8t}{t^2+1}} = \int \frac{2\,dt}{5t^2-8t+5}$

$= \frac{2}{5}\int \frac{dt}{t^2-\frac{8t}{5}+5} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+5-\frac{16}{25}} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+\frac{25-16}{25}} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+\frac{9}{25}} = \left[\begin{array}{l|l} \text{Subst. } u=t-\frac{4}{5} & t\in R \\ du=dt & u\in R \end{array} \right]$

$= \frac{2}{5}\int \frac{du}{u^2+\frac{9}{25}} = \frac{2}{5\cdot\frac{3}{5}} \operatorname{arctg} \frac{u}{\frac{3}{5}} + c = \frac{2}{3}\operatorname{arctg}\frac{5u}{3} + c$

Riešené príklady – 156, 157

$$\int \frac{dx}{5+4\sin x}$$ [156]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \in \langle -1; 1\rangle & x \in (-\pi+2k\pi; \pi+2k\pi) \\ dx = \frac{2\,dt}{t^2+1} & 5+4\sin x \geq 5-4 = 1 > 0 & t \in (-\infty; \infty), \quad k \in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{5+\frac{4\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5+8t}{t^2+1}} = \int \frac{2\,dt}{5t^2+8t+5}$

$$= \frac{2}{5}\int \frac{dt}{t^2+\frac{8t}{5}+5} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+5-\frac{16}{25}} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+\frac{25-16}{25}} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+\frac{9}{25}} = \left[\begin{array}{r|l} \text{Subst. } u = t+\frac{4}{5} & t \in R \\ du = dt & u \in R \end{array} \right]$$

$$= \frac{2}{5}\int \frac{du}{u^2+\frac{9}{25}} = \frac{2}{5\cdot\frac{3}{5}} \operatorname{arctg} \frac{u}{\frac{3}{5}} + c = \frac{2}{3} \operatorname{arctg} \frac{5u}{3} + c = \frac{2}{3} \operatorname{arctg} \frac{5t+4}{3} + c$$

$$\int \frac{dx}{5-4\sin x}$$ [157]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \in \langle -1; 1\rangle & x \in (-\pi+2k\pi; \pi+2k\pi) \\ dx = \frac{2\,dt}{t^2+1} & 5-4\sin x \geq 5-4 = 1 > 0 & t \in (-\infty; \infty), \quad k \in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{5-\frac{4\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5-8t}{t^2+1}} = \int \frac{2\,dt}{5t^2-8t+5}$

$$= \frac{2}{5}\int \frac{dt}{t^2-\frac{8t}{5}+5} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+5-\frac{16}{25}} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+\frac{25-16}{25}} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+\frac{9}{25}} = \left[\begin{array}{r|l} \text{Subst. } u = t-\frac{4}{5} & t \in R \\ du = dt & u \in R \end{array} \right]$$

$$= \frac{2}{5}\int \frac{du}{u^2+\frac{9}{25}} = \frac{2}{5\cdot\frac{3}{5}} \operatorname{arctg} \frac{u}{\frac{3}{5}} + c = \frac{2}{3} \operatorname{arctg} \frac{5u}{3} + c = \frac{2}{3} \operatorname{arctg} \frac{5t-4}{3} + c$$

Riešené príklady – 156, 157

$$\int \frac{dx}{5+4\sin x} = \frac{2}{3}\,\mathrm{arctg}\,\frac{5\,\mathrm{tg}\,\frac{x}{2}+4}{3} + c$$ [156]

- $= \left[\begin{array}{r|l|l} \text{UGS:}\ \ t=\mathrm{tg}\,\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \in \langle -1;1\rangle & x\in(-\pi+2k\pi;\pi+2k\pi) \\ dx=\frac{2\,dt}{t^2+1} & 5+4\sin x \geq 5-4=1>0 & t\in(-\infty;\infty),\quad k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{5+\frac{4\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5+8t}{t^2+1}} = \int \frac{2\,dt}{5t^2+8t+5}$

$= \frac{2}{5}\int \frac{dt}{t^2+\frac{8t}{5}+5} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+5-\frac{16}{25}} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+\frac{25-16}{25}} = \frac{2}{5}\int \frac{dt}{(t+\frac{4}{5})^2+\frac{9}{25}} = \left[\begin{array}{r|l} \text{Subst.}\ u=t+\frac{4}{5} & t\in R \\ du=dt & u\in R \end{array}\right]$

$= \frac{2}{5}\int \frac{du}{u^2+\frac{9}{25}} = \frac{2}{5\cdot\frac{3}{5}}\,\mathrm{arctg}\,\frac{u}{\frac{3}{5}} + c = \frac{2}{3}\,\mathrm{arctg}\,\frac{5u}{3} + c = \frac{2}{3}\,\mathrm{arctg}\,\frac{5t+4}{3} + c = \frac{2}{3}\,\mathrm{arctg}\,\frac{5\,\mathrm{tg}\,\frac{x}{2}+4}{3} + c,$

$x\in R,\ x\neq \pi+2k\pi,\ k\in Z,\ c\in R.$

$$\int \frac{dx}{5-4\sin x} = \frac{2}{3}\,\mathrm{arctg}\,\frac{5\,\mathrm{tg}\,\frac{x}{2}-4}{3} + c$$ [157]

- $= \left[\begin{array}{r|l|l} \text{UGS:}\ \ t=\mathrm{tg}\,\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \in \langle -1;1\rangle & x\in(-\pi+2k\pi;\pi+2k\pi) \\ dx=\frac{2\,dt}{t^2+1} & 5-4\sin x \geq 5-4=1>0 & t\in(-\infty;\infty),\quad k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{5-\frac{4\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5-8t}{t^2+1}} = \int \frac{2\,dt}{5t^2-8t+5}$

$= \frac{2}{5}\int \frac{dt}{t^2-\frac{8t}{5}+5} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+5-\frac{16}{25}} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+\frac{25-16}{25}} = \frac{2}{5}\int \frac{dt}{(t-\frac{4}{5})^2+\frac{9}{25}} = \left[\begin{array}{r|l} \text{Subst.}\ u=t-\frac{4}{5} & t\in R \\ du=dt & u\in R \end{array}\right]$

$= \frac{2}{5}\int \frac{du}{u^2+\frac{9}{25}} = \frac{2}{5\cdot\frac{3}{5}}\,\mathrm{arctg}\,\frac{u}{\frac{3}{5}} + c = \frac{2}{3}\,\mathrm{arctg}\,\frac{5u}{3} + c = \frac{2}{3}\,\mathrm{arctg}\,\frac{5t-4}{3} + c = \frac{2}{3}\,\mathrm{arctg}\,\frac{5\,\mathrm{tg}\,\frac{x}{2}-4}{3} + c,$

$x\in R,\ x\neq \pi+2k\pi,\ k\in Z,\ c\in R.$

Riešené príklady – 158, 159

$$\int \frac{dx}{4+5\sin x}$$ [158]

$$\int \frac{dx}{4-5\sin x}$$ [159]

Riešené príklady – 158, 159

$$\int \frac{dx}{4+5\sin x}$$ [158]

- $= \left[\begin{array}{c|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\, k\in Z,\ \sin x \neq -\frac{4}{5} \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} \Rightarrow t\in R,\ t\neq -\frac{1}{2},\ t\neq -2 \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{4+\frac{5\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{4t^2+4+10t}{t^2+1}} = \frac{2}{4}\int \frac{dt}{t^2+\frac{5}{2}t+1}$

$$\int \frac{dx}{4-5\sin x}$$ [159]

- $= \left[\begin{array}{c|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\, k\in Z,\ \sin x \neq \frac{4}{5} \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} \Rightarrow t\in R,\ t\neq \frac{1}{2},\ t\neq 2 \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{4-\frac{5\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{4t^2+4-10t}{t^2+1}} = \frac{2}{4}\int \frac{dt}{t^2-\frac{5}{2}t+1}$

Riešené príklady – 158, 159

$$\int \frac{dx}{4+5\sin x}$$ [158]

- $= \left[\begin{array}{r|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & x \in (-\pi + 2k\pi; \pi + 2k\pi), k \in Z, \ \sin x \neq -\frac{4}{5} \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} \Rightarrow t \in R, \ t \neq -\frac{1}{2}, \ t \neq -2 \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{4+\frac{5\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{4t^2+4+10t}{t^2+1}} = \frac{2}{4}\int \frac{dt}{t^2+\frac{5}{2}t+1}$

$$= \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2+1-\frac{25}{16}} = \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2+\frac{16-25}{16}} = \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2-\frac{9}{16}}$$

$$\int \frac{dx}{4-5\sin x}$$ [159]

- $= \left[\begin{array}{r|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & x \in (-\pi + 2k\pi; \pi + 2k\pi), k \in Z, \ \sin x \neq \frac{4}{5} \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} \Rightarrow t \in R, \ t \neq \frac{1}{2}, \ t \neq 2 \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{4-\frac{5\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{4t^2+4-10t}{t^2+1}} = \frac{2}{4}\int \frac{dt}{t^2-\frac{5}{2}t+1}$

$$= \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2+1-\frac{25}{16}} = \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2+\frac{16-25}{16}} = \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2-\frac{9}{16}}$$

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$$\int \frac{dx}{4+5\sin x}$$ [158]

- $= \left[\begin{array}{c|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\, k\in Z,\ \sin x\neq -\frac{4}{5} \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2t}{t^2+1} \Rightarrow t\in R,\ t\neq -\frac{1}{2},\ t\neq -2 \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{4+\frac{5\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{4t^2+4+10t}{t^2+1}} = \frac{2}{4}\int \frac{dt}{t^2+\frac{5}{2}t+1}$

$$= \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2+1-\frac{25}{16}} = \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2+\frac{16-25}{16}} = \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2-\frac{9}{16}} = \left[\begin{array}{c|l} \text{Subst. } u=t+\frac{5}{4} & t\in R,\ t\neq -\frac{1}{2},\ t\neq -2 \\ du=dt & u\in R,\ u\neq \pm\frac{3}{4} \end{array} \right]$$

$$= \frac{1}{2}\int \frac{du}{u^2-\frac{9}{16}}$$

$$\int \frac{dx}{4-5\sin x}$$ [159]

- $= \left[\begin{array}{c|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\, k\in Z,\ \sin x\neq \frac{4}{5} \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2t}{t^2+1} \Rightarrow t\in R,\ t\neq \frac{1}{2},\ t\neq 2 \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{4-\frac{5\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{4t^2+4-10t}{t^2+1}} = \frac{2}{4}\int \frac{dt}{t^2-\frac{5}{2}t+1}$

$$= \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2+1-\frac{25}{16}} = \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2+\frac{16-25}{16}} = \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2-\frac{9}{16}} = \left[\begin{array}{c|l} \text{Subst. } u=t-\frac{5}{4} & t\in R,\ t\neq \frac{1}{2},\ t\neq 2 \\ du=dt & u\in R,\ u\neq \pm\frac{3}{4} \end{array} \right]$$

$$= \frac{1}{2}\int \frac{du}{u^2-\frac{9}{16}}$$

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$$\int \frac{dx}{4+5\sin x}$$ [158]

- $= \left[\begin{array}{r|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi), k\in Z,\ \sin x \neq -\frac{4}{5} \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} \Rightarrow t\in R,\ t\neq -\frac{1}{2},\ t\neq -2 \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{4+\frac{5\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{4t^2+4+10t}{t^2+1}} = \frac{2}{4}\int \frac{dt}{t^2+\frac{5}{2}t+1}$

$= \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2+1-\frac{25}{16}} = \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2+\frac{16-25}{16}} = \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2-\frac{9}{16}} = \left[\begin{array}{r|l} \text{Subst. } u = t+\frac{5}{4} & t\in R,\ t\neq -\frac{1}{2},\ t\neq -2 \\ du = dt & u\in R,\ u\neq \pm\frac{3}{4} \end{array} \right]$

$= \frac{1}{2}\int \frac{du}{u^2-\frac{9}{16}} = \frac{1}{2\cdot 2\cdot \frac{3}{4}} \ln\left|\frac{u-\frac{3}{4}}{u+\frac{3}{4}}\right| + c = \frac{1}{3}\ln\left|\frac{4u-3}{4u+3}\right| + c$

$$\int \frac{dx}{4-5\sin x}$$ [159]

- $= \left[\begin{array}{r|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi), k\in Z,\ \sin x \neq \frac{4}{5} \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} \Rightarrow t\in R,\ t\neq \frac{1}{2},\ t\neq 2 \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{4-\frac{5\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{4t^2+4-10t}{t^2+1}} = \frac{2}{4}\int \frac{dt}{t^2-\frac{5}{2}t+1}$

$= \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2+1-\frac{25}{16}} = \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2+\frac{16-25}{16}} = \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2-\frac{9}{16}} = \left[\begin{array}{r|l} \text{Subst. } u = t-\frac{5}{4} & t\in R,\ t\neq \frac{1}{2},\ t\neq 2 \\ du = dt & u\in R,\ u\neq \pm\frac{3}{4} \end{array} \right]$

$= \frac{1}{2}\int \frac{du}{u^2-\frac{9}{16}} = \frac{1}{2\cdot 2\cdot \frac{3}{4}} \ln\left|\frac{u-\frac{3}{4}}{u+\frac{3}{4}}\right| + c = \frac{1}{3}\ln\left|\frac{4u-3}{4u+3}\right| + c$

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$$\int \frac{dx}{4+5\sin x}$$ [158]

- $= \left[\begin{array}{r|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\, k\in Z,\ \sin x\neq -\frac{4}{5} \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2t}{t^2+1} \Rightarrow t\in R,\ t\neq -\frac{1}{2},\ t\neq -2 \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{4+\frac{5\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{4t^2+4+10t}{t^2+1}} = \frac{2}{4}\int \frac{dt}{t^2+\frac{5}{2}t+1}$
$= \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2+1-\frac{25}{16}} = \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2+\frac{16-25}{16}} = \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2-\frac{9}{16}} = \left[\begin{array}{r|l} \text{Subst. } u=t+\frac{5}{4} & t\in R,\ t\neq -\frac{1}{2},\ t\neq -2 \\ du=dt & u\in R,\ u\neq \pm\frac{3}{4} \end{array} \right]$
$= \frac{1}{2}\int \frac{du}{u^2-\frac{9}{16}} = \frac{1}{2\cdot 2\cdot \frac{3}{4}} \ln\left|\frac{u-\frac{3}{4}}{u+\frac{3}{4}}\right| + c = \frac{1}{3}\ln\left|\frac{4u-3}{4u+3}\right| + c = \frac{1}{3}\ln\left|\frac{4t+5-3}{4t+5+3}\right| + c$

$$\int \frac{dx}{4-5\sin x}$$ [159]

- $= \left[\begin{array}{r|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\, k\in Z,\ \sin x\neq \frac{4}{5} \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2t}{t^2+1} \Rightarrow t\in R,\ t\neq \frac{1}{2},\ t\neq 2 \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{4-\frac{5\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{4t^2+4-10t}{t^2+1}} = \frac{2}{4}\int \frac{dt}{t^2-\frac{5}{2}t+1}$
$= \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2+1-\frac{25}{16}} = \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2+\frac{16-25}{16}} = \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2-\frac{9}{16}} = \left[\begin{array}{r|l} \text{Subst. } u=t-\frac{5}{4} & t\in R,\ t\neq \frac{1}{2},\ t\neq 2 \\ du=dt & u\in R,\ u\neq \pm\frac{3}{4} \end{array} \right]$
$= \frac{1}{2}\int \frac{du}{u^2-\frac{9}{16}} = \frac{1}{2\cdot 2\cdot \frac{3}{4}} \ln\left|\frac{u-\frac{3}{4}}{u+\frac{3}{4}}\right| + c = \frac{1}{3}\ln\left|\frac{4u-3}{4u+3}\right| + c = \frac{1}{3}\ln\left|\frac{4t-5-3}{4t-5+3}\right| + c$

Riešené príklady – 158, 159

$$\int \frac{dx}{4+5\sin x} = \frac{1}{3}\ln\left|\frac{4\,\mathrm{tg}\,\frac{x}{2}+2}{4\,\mathrm{tg}\,\frac{x}{2}+8}\right| + c = \frac{1}{3}\ln\left|\frac{2\,\mathrm{tg}\,\frac{x}{2}+1}{2\,\mathrm{tg}\,\frac{x}{2}+4}\right| + c$$ [158]

- $= \left[\begin{array}{r|l} \text{UGS: } t = \mathrm{tg}\,\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\, k\in Z,\ \sin x \neq -\frac{4}{5} \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} \Rightarrow t\in R,\ t\neq -\frac{1}{2},\ t\neq -2 \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{4+\frac{5\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{4t^2+4+10t}{t^2+1}} = \frac{2}{4}\int \frac{dt}{t^2+\frac{5}{2}t+1}$

$= \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2+1-\frac{25}{16}} = \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2+\frac{16-25}{16}} = \frac{1}{2}\int \frac{dt}{(t+\frac{5}{4})^2-\frac{9}{16}} = \left[\begin{array}{r|l} \text{Subst. } u = t+\frac{5}{4} & t\in R,\ t\neq -\frac{1}{2},\ t\neq -2 \\ du = dt & u\in R,\ u\neq \pm\frac{3}{4} \end{array}\right]$

$= \frac{1}{2}\int \frac{du}{u^2-\frac{9}{16}} = \frac{1}{2\cdot 2\cdot\frac{3}{4}}\ln\left|\frac{u-\frac{3}{4}}{u+\frac{3}{4}}\right| + c = \frac{1}{3}\ln\left|\frac{4u-3}{4u+3}\right| + c = \frac{1}{3}\ln\left|\frac{4t+5-3}{4t+5+3}\right| + c = \frac{1}{3}\ln\left|\frac{4\,\mathrm{tg}\,\frac{x}{2}+2}{4\,\mathrm{tg}\,\frac{x}{2}+8}\right| + c,$

$x\in R,\ \sin x \neq -\frac{4}{5},\ c\in R.$

$$\int \frac{dx}{4-5\sin x} = \frac{1}{3}\ln\left|\frac{4\,\mathrm{tg}\,\frac{x}{2}-8}{4\,\mathrm{tg}\,\frac{x}{2}-2}\right| + c = \frac{1}{3}\ln\left|\frac{2\,\mathrm{tg}\,\frac{x}{2}-4}{2\,\mathrm{tg}\,\frac{x}{2}-1}\right| + c$$ [159]

- $= \left[\begin{array}{r|l} \text{UGS: } t = \mathrm{tg}\,\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\, k\in Z,\ \sin x \neq \frac{4}{5} \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} \Rightarrow t\in R,\ t\neq \frac{1}{2},\ t\neq 2 \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{4-\frac{5\cdot 2t}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{4t^2+4-10t}{t^2+1}} = \frac{2}{4}\int \frac{dt}{t^2-\frac{5}{2}t+1}$

$= \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2+1-\frac{25}{16}} = \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2+\frac{16-25}{16}} = \frac{1}{2}\int \frac{dt}{(t-\frac{5}{4})^2-\frac{9}{16}} = \left[\begin{array}{r|l} \text{Subst. } u = t-\frac{5}{4} & t\in R,\ t\neq \frac{1}{2},\ t\neq 2 \\ du = dt & u\in R,\ u\neq \pm\frac{3}{4} \end{array}\right]$

$= \frac{1}{2}\int \frac{du}{u^2-\frac{9}{16}} = \frac{1}{2\cdot 2\cdot\frac{3}{4}}\ln\left|\frac{u-\frac{3}{4}}{u+\frac{3}{4}}\right| + c = \frac{1}{3}\ln\left|\frac{4u-3}{4u+3}\right| + c = \frac{1}{3}\ln\left|\frac{4t-5-3}{4t-5+3}\right| + c = \frac{1}{3}\ln\left|\frac{4\,\mathrm{tg}\,\frac{x}{2}-8}{4\,\mathrm{tg}\,\frac{x}{2}-2}\right| + c,$

$x\in R,\ \sin x \neq \frac{4}{5},\ c\in R.$

Riešené príklady – 160

$$\int \frac{dx}{\cos x}$$ [160]

Riešené príklady – 160

$$\int \frac{dx}{\cos x}$$ [160]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x \in (-\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi),\, k \in Z \Rightarrow t \in (-\infty; -1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \frac{\pi}{2} + k\pi,\, k \in Z & x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi),\, k \in Z \Rightarrow t \in (-1; 1) \\ x \neq \pi + k\pi,\, k \in Z & t \neq \pm 1 & x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi),\, k \in Z \Rightarrow t \in (1; \infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$

  $= \int \frac{2\,dt}{1-t^2} = -\int \frac{2\,dt}{t^2-1}$

- $= \int \frac{\cos x\,dx}{\cos^2 x} = \left[\begin{array}{r|l} \text{Subst. } t = \sin x & x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi),\, k \in Z \\ dt = \cos x\,dx & t \in (-1; 1) \end{array} \right]$

Riešené príklady – 160

$\int \frac{dx}{\cos x}$ [160]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \text{tg}\,\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x \in (-\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi),\, k \in Z \Rightarrow t \in (-\infty; -1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \frac{\pi}{2} + k\pi,\, k \in Z & x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi),\, k \in Z \Rightarrow t \in (-1; 1) \\ x \neq \pi + k\pi,\, k \in Z & t \neq \pm 1 & x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi),\, k \in Z \Rightarrow t \in (1; \infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$

$$= \int \frac{2\,dt}{1-t^2} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c_1 = \ln \left| \frac{t+1}{t-1} \right| + c_1$$

- $= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t = \sin x & x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi),\, k \in Z \\ dt = \cos x\,dx & t \in (-1; 1) \end{array} \right] = \int \frac{-\,dt}{1-t^2} = -\int \frac{dt}{t^2-1}$

Riešené príklady – 160

$$\int \frac{dx}{\cos x}$$ [160]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x \in (-\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi),\, k \in Z \Rightarrow t \in (-\infty; -1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \frac{\pi}{2} + k\pi,\, k \in Z & x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi),\, k \in Z \Rightarrow t \in (-1; 1) \\ x \neq \pi + k\pi,\, k \in Z & t \neq \pm 1 & x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi),\, k \in Z \Rightarrow t \in (1; \infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$

$= \int \frac{2\,dt}{1-t^2} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c_1 = \ln \left| \frac{t+1}{t-1} \right| + c_1$

$= \ln \left| \frac{\operatorname{tg}\frac{x}{2} - 1}{\operatorname{tg}\frac{x}{2} + 1} \right| + c_1 = \ln \left| \operatorname{tg}\frac{x}{2} - 1 \right| - \ln \left| \operatorname{tg}\frac{x}{2} + 1 \right| + c_1,$

$x \in R,\ x \neq \frac{\pi}{2} + k\pi,\ x \neq \pi + 2k\pi,\ k \in Z,\ c_1 \in R.$

- $= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1 - \sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t = \sin x & x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi),\, k \in Z \\ dt = \cos x\,dx & t \in (-1; 1) \end{array} \right] = \int \frac{-\,dt}{1-t^2} = -\int \frac{dt}{t^2-1}$

$= -\frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_2 = \frac{1}{2} \ln \left| \frac{t+1}{t-1} \right| + c_2 = \left[t \in (-1; 1) \Rightarrow \begin{cases} |t+1| = t+1 \\ |t-1| = 1-t \end{cases} \right] = \frac{1}{2} \ln \frac{1+t}{1-t} + c_2$

Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2$$ [160]

- $$= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x\in(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x\neq \frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-1;1) \\ x\neq \pi+k\pi,\, k\in Z & t\neq \pm 1 & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi),\, k\in Z \Rightarrow t\in(1;\infty) \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$$
$$= \int \frac{2\,dt}{1-t^2} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1$$
$$= \ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}-1\right| - \ln\left|\operatorname{tg}\frac{x}{2}+1\right| + c_1,$$
$$x\in R,\ x\neq \frac{\pi}{2}+k\pi,\ x\neq \pi+2k\pi,\ k\in Z,\ c_1\in R.$$

- $$= \int \frac{\cos x\, dx}{\cos^2 x} = \int \frac{\cos x\, dx}{1-\sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t=\sin x & x\in(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi),\, k\in Z \\ dt=\cos x\, dx & t\in(-1;1) \end{array}\right] = \int \frac{-\,dt}{1-t^2} = -\int \frac{dt}{t^2-1}$$
$$= -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\left|\frac{t+1}{t-1}\right| + c_2 = \left[t\in(-1;1) \Rightarrow \begin{cases} |t+1| = t+1 \\ |t-1| = 1-t \end{cases} \right] = \frac{1}{2}\ln\frac{1+t}{1-t} + c_2$$
$$= \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \frac{1}{2}\ln(1+\sin x) - \frac{1}{2}\ln(1-\sin x) + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2,$$
$$x\in R,\ x\neq \frac{\pi}{2}+k\pi,\ k\in Z,\ c_2\in R.$$

Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2$$ [160]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x\in(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x\neq\frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-1;1) \\ x\neq\pi+k\pi,\, k\in Z & t\neq\pm1 & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi),\, k\in Z \Rightarrow t\in(1;\infty) \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$

$$= \int \frac{2\,dt}{1-t^2} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1$$

$$= \ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}-1\right| - \ln\left|\operatorname{tg}\frac{x}{2}+1\right| + c_1,$$

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- $= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t=\sin x & x\in(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi),\, k\in Z \\ dt=\cos x\,dx & t\in(-1;1) \end{array}\right] = \int \frac{-\,dt}{1-t^2} = -\int \frac{dt}{t^2-1}$

$$= -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\left|\frac{t+1}{t-1}\right| + c_2 = \left[t\in(-1;1) \Rightarrow \begin{cases} |t+1| = t+1 \\ |t-1| = 1-t \end{cases} \right] = \frac{1}{2}\ln\frac{1+t}{1-t} + c_2$$

$$= \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \frac{1}{2}\ln(1+\sin x) - \frac{1}{2}\ln(1-\sin x) + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2,$$

$$x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ k\in Z,\ c_2\in R.$$

Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2$$ [160]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x\in(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x\neq\frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-1;1) \\ x\neq\pi+k\pi,\, k\in Z & t\neq\pm1 & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi),\, k\in Z \Rightarrow t\in(1;\infty) \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$

$$= \int \frac{2\,dt}{1-t^2} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1$$

$$= \ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}-1\right| - \ln\left|\operatorname{tg}\frac{x}{2}+1\right| + c_1,$$

$$x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ x\neq\pi+2k\pi,\ k\in Z,\ c_1\in R.$$

- $= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t=\sin x & x\in(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi),\, k\in Z \\ dt=\cos x\,dx & t\in(-1;1) \end{array}\right] = \int \frac{-\,dt}{1-t^2} = -\int \frac{dt}{t^2-1}$

$$= -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\left|\frac{t+1}{t-1}\right| + c_2 = \left[t\in(-1;1) \Rightarrow \begin{cases} |t+1| = t+1 \\ |t-1| = 1-t \end{cases} \right] = \frac{1}{2}\ln\frac{1+t}{1-t} + c_2$$

$$= \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \frac{1}{2}\ln(1+\sin x) - \frac{1}{2}\ln(1-\sin x) + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2,$$

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Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2$$ [160]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x\in(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-1;1) \\ x\neq \pi+k\pi,\, k\in Z & t\neq \pm 1 & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi),\, k\in Z \Rightarrow t\in(1;\infty) \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$

  $= \int \frac{2\,dt}{1-t^2} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1$

  $= \ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}-1\right| - \ln\left|\operatorname{tg}\frac{x}{2}+1\right| + c_1,$

  $x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ x\neq \pi+2k\pi,\ k\in Z,\ c_1\in R.$

- $= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t=\sin x & x\in(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi),\, k\in Z \\ dt = \cos x\,dx & t\in(-1;1) \end{array}\right] = \int \frac{-\,dt}{1-t^2} = -\int \frac{dt}{t^2-1}$

  $= -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\left|\frac{t+1}{t-1}\right| + c_2 = \left[t\in(-1;1) \Rightarrow \begin{cases} |t+1| = t+1 \\ |t-1| = 1-t \end{cases}\right] = \frac{1}{2}\ln\frac{1+t}{1-t} + c_2$

  $= \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \frac{1}{2}\ln(1+\sin x) - \frac{1}{2}\ln(1-\sin x) + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2,$

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Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2$$ [160]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-1;1) \\ x\neq \pi+k\pi,\, k\in Z & t\neq \pm 1 & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi),\, k\in Z \Rightarrow t\in(1;\infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$

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$$= \ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}-1\right| - \ln\left|\operatorname{tg}\frac{x}{2}+1\right| + c_1,$$

$$x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ x\neq\pi+2k\pi,\ k\in Z,\ c_1\in R.$$

- $= \int \frac{\cos x\,dx}{\cos^2 x} = \int\frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t=\sin x & x\in(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi),\, k\in Z \\ dt = \cos x\,dx & t\in(-1;1)\end{array}\right] = \int\frac{-\,dt}{1-t^2} = -\int\frac{dt}{t^2-1}$

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Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2$$ [160]

- $$= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x\in(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x\neq\frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-1;1) \\ x\neq\pi+k\pi,\, k\in Z & t\neq\pm1 & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi),\, k\in Z \Rightarrow t\in(1;\infty) \end{array}\right] = \int\frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$$

  $$= \int\frac{2\,dt}{1-t^2} = -\int\frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1$$

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- $$= \int\frac{\cos x\,dx}{\cos^2 x} = \int\frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t=\sin x & x\in(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi),\, k\in Z \\ dt=\cos x\,dx & t\in(-1;1) \end{array}\right] = \int\frac{-\,dt}{1-t^2} = -\int\frac{dt}{t^2-1}$$

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  $$= \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \frac{1}{2}\ln(1+\sin x) - \frac{1}{2}\ln(1-\sin x) + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2,$$

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Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2$$ [160]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-1;1) \\ x\neq\pi+k\pi,\, k\in Z & t\neq\pm1 & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi),\, k\in Z \Rightarrow t\in(1;\infty) \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$

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Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2$$ [160]

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$$= -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\left|\frac{t+1}{t-1}\right| + c_2 = \left[t\in(-1;1) \Rightarrow \begin{cases} |t+1| = t+1 \\ |t-1| = 1-t \end{cases}\right] = \frac{1}{2}\ln\frac{1+t}{1-t} + c_2$$

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Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2$$ [160]

- $$= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-1;1) \\ x\neq \pi+k\pi,\, k\in Z & t\neq \pm 1 & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi),\, k\in Z \Rightarrow t\in(1;\infty) \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{1-t^2} = -\int\frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1$$

$$= \ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}-1\right| - \ln\left|\operatorname{tg}\frac{x}{2}+1\right| + c_1,$$

$$x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ x\neq \pi+2k\pi,\ k\in Z,\ c_1\in R.$$

- $$= \int\frac{\cos x\,dx}{\cos^2 x} = \int\frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t=\sin x & x\in(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi),\, k\in Z \\ dt = \cos x\,dx & t\in(-1;1) \end{array}\right] = \int\frac{-\,dt}{1-t^2} = -\int\frac{dt}{t^2-1}$$

$$= -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\left|\frac{t+1}{t-1}\right| + c_2 = \left[t\in(-1;1) \Rightarrow \begin{cases} |t+1| = t+1 \\ |t-1| = 1-t\end{cases}\right] = \frac{1}{2}\ln\frac{1+t}{1-t} + c_2$$

$$= \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \frac{1}{2}\ln(1+\sin x) - \frac{1}{2}\ln(1-\sin x) + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2,$$

$$x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ k\in Z,\ c_2\in R.$$

Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2$$ [160]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x\in(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x\neq\frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-1;1) \\ x\neq\pi+k\pi,\, k\in Z & t\neq\pm1 & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi),\, k\in Z \Rightarrow t\in(1;\infty) \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$

$$= \int \frac{2\,dt}{1-t^2} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1$$

$$= \ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}-1\right| - \ln\left|\operatorname{tg}\frac{x}{2}+1\right| + c_1,$$

$$x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ x\neq\pi+2k\pi,\ k\in Z,\ c_1\in R.$$

- $= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t=\sin x & x\in(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi),\, k\in Z \\ dt=\cos x\,dx & t\in(-1;1) \end{array}\right] = \int \frac{-\,dt}{1-t^2} = -\int \frac{dt}{t^2-1}$

$$= -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\left|\frac{t+1}{t-1}\right| + c_2 = \left[t\in(-1;1) \Rightarrow \begin{cases} |t+1| = t+1 \\ |t-1| = 1-t \end{cases} \right] = \frac{1}{2}\ln\frac{1+t}{1-t} + c_2$$

$$= \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \frac{1}{2}\ln(1+\sin x) - \frac{1}{2}\ln(1-\sin x) + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2,$$

$$x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ k\in Z,\ c_2\in R.$$

Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2$$ [160]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-1;1) \\ x\neq \pi+k\pi,\, k\in Z & t\neq \pm 1 & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi),\, k\in Z \Rightarrow t\in(1;\infty) \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$

  $= \int \frac{2\,dt}{1-t^2} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1$

  $= \ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}-1\right| - \ln\left|\operatorname{tg}\frac{x}{2}+1\right| + c_1,$

  $x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ x\neq\pi+2k\pi,\ k\in Z,\ c_1\in R.$

- $= \int\frac{\cos x\,dx}{\cos^2 x} = \int\frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t=\sin x & x\in(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi),\, k\in Z \\ dt = \cos x\,dx & t\in(-1;1) \end{array}\right] = \int\frac{-\,dt}{1-t^2} = -\int\frac{dt}{t^2-1}$

  $= -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\left|\frac{t+1}{t-1}\right| + c_2 = \left[t\in(-1;1) \Rightarrow \begin{cases} |t+1| = t+1 \\ |t-1| = 1-t \end{cases}\right] = \frac{1}{2}\ln\frac{1+t}{1-t} + c_2$

  $= \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \frac{1}{2}\ln(1+\sin x) - \frac{1}{2}\ln(1-\sin x) + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2,$

  $x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ k\in Z,\ c_2\in R.$

Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2$$ [160]

- $$= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \frac{\pi}{2}+k\pi,\, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi),\, k\in Z \Rightarrow t\in(-1;1) \\ x\neq \pi+k\pi,\, k\in Z & t\neq \pm 1 & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi),\, k\in Z \Rightarrow t\in(1;\infty) \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$$
  $$= \int \frac{2\,dt}{1-t^2} = -\int\frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1$$
  $$= \ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}-1\right| - \ln\left|\operatorname{tg}\frac{x}{2}+1\right| + c_1,$$
  $$x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ x\neq\pi+2k\pi,\ k\in Z,\ c_1\in R.$$

- $$= \int\frac{\cos x\,dx}{\cos^2 x} = \int\frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t=\sin x & x\in(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi),\, k\in Z \\ dt = \cos x\,dx & t\in(-1;1) \end{array}\right] = \int\frac{-\,dt}{1-t^2} = -\int\frac{dt}{t^2-1}$$
  $$= -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\left|\frac{t+1}{t-1}\right| + c_2 = \left[t\in(-1;1) \Rightarrow \left\{\begin{array}{l} |t+1| = t+1 \\ |t-1| = 1-t \end{array}\right.\right] = \frac{1}{2}\ln\frac{1+t}{1-t} + c_2$$
  $$= \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \frac{1}{2}\ln(1+\sin x) - \frac{1}{2}\ln(1-\sin x) + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2,$$
  $$x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ k\in Z,\ c_2\in R.$$

Riešené príklady – 160

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2 \quad [160]$$

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 0 & x\in(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), k\in Z \Rightarrow t\in(-\infty;-1) \\ dx = \frac{2\,dt}{t^2+1} & x\neq\frac{\pi}{2}+k\pi, k\in Z & x\in(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi), k\in Z \Rightarrow t\in(-1;1) \\ x\neq\pi+k\pi, k\in Z & t\neq\pm 1 & x\in(\frac{\pi}{2}+2k\pi; \pi+2k\pi), k\in Z \Rightarrow t\in(1;\infty) \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$

$$= \int \frac{2\,dt}{1-t^2} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1$$

$$= \ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}-1\right| - \ln\left|\operatorname{tg}\frac{x}{2}+1\right| + c_1,$$

$$x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ x\neq\pi+2k\pi,\ k\in Z,\ c_1\in R.$$

- $= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{r|l} \text{Subst. } t=\sin x & x\in(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi), k\in Z \\ dt=\cos x\,dx & t\in(-1;1) \end{array}\right] = \int \frac{-\,dt}{1-t^2} = -\int \frac{dt}{t^2-1}$

$$= -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\left|\frac{t+1}{t-1}\right| + c_2 = \left[t\in(-1;1) \Rightarrow \begin{cases} |t+1| = t+1 \\ |t-1| = 1-t \end{cases}\right] = \frac{1}{2}\ln\frac{1+t}{1-t} + c_2$$

$$= \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2 = \frac{1}{2}\ln(1+\sin x) - \frac{1}{2}\ln(1-\sin x) + c_2 = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c_2,$$

$$x\in R,\ x\neq\frac{\pi}{2}+k\pi,\ k\in Z,\ c_2\in R.$$

Riešené príklady – 161

$$\int \frac{dx}{1+\cos x}$$ [161]

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Riešené príklady – 161

$$\int \frac{dx}{1+\cos x}$$

[161]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq -1 & x\in(-\pi+2k\pi;\pi+2k\pi) \\ dx=\frac{2\,dt}{t^2+1} & x\neq \pi+2k\pi,\, k\in Z & t\in(-\infty;\infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$

- $\int \frac{dx}{2\cos^2\frac{x}{2}}$ pr10a-11

- $= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x\,dx}{\sin^2 x}$

Riešené príklady – 161

$$\int \frac{dx}{1+\cos x}$$ [161]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \mathrm{tg}\,\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq -1 & x \in (-\pi+2k\pi; \pi+2k\pi) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \pi + 2k\pi,\, k \in Z & t \in (-\infty; \infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$

  $= t + c_1$

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- $\int \frac{dx}{2\cos^2 \frac{x}{2}} = \left[\begin{array}{r|l} \text{Subst. } t = \frac{x}{2} & x \in (-\pi+2k\pi; \pi+2k\pi),\, k \in Z \\ dt = \frac{dx}{2} & t \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi),\, k \in Z \end{array} \right] = \int \frac{dt}{\cos^2 t}$

- $= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x\,dx}{\sin^2 x}$

  $= \left[\begin{array}{r|l|l|l} \text{Subst. } t = \sin x & \sin x \neq 0 & x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\rangle \Rightarrow t \in \langle -1; 0) & x \in (0+2k\pi; \frac{\pi}{2}+2k\pi\rangle \Rightarrow t \in (0; 1\rangle \\ dt = \cos x\,dx & x \neq k\pi,\, k \in Z & x \in \langle -\frac{\pi}{2}+2k\pi; 0+2k\pi) \Rightarrow t \in \langle -1; 0) & x \in \langle \frac{\pi}{2}+2k\pi; \pi+2k\pi) \Rightarrow t \in (0; 1\rangle \end{array} \right]$

  $= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2}\,dt$

Riešené príklady – 161

$$\int \frac{dx}{1+\cos x} = \operatorname{tg}\frac{x}{2} + c_1$$ [161]

- $= \left[\begin{array}{r|l|l} \text{UGS:}\quad t = \operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq -1 & x \in (-\pi + 2k\pi; \pi + 2k\pi) \\ dx = \frac{2\,dt}{t^2+1} & x \neq \pi + 2k\pi,\, k \in Z & t \in (-\infty; \infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$

  $= t + c_1 = \operatorname{tg}\frac{x}{2} + c_1,\ x \in R,\ x \neq \pi + 2k\pi,\ k \in Z,\ c_1 \in R.$

- $\int \frac{dx}{2\cos^2\frac{x}{2}} = \left[\begin{array}{r|l} \text{Subst.}\quad t = \frac{x}{2} & x \in (-\pi + 2k\pi; \pi + 2k\pi),\, k \in Z \\ dt = \frac{dx}{2} & t \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi),\, k \in Z \end{array} \right] = \int \frac{dt}{\cos^2 t}$

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  $= \operatorname{tg} t + c_1$

- $= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x\,dx}{\sin^2 x}$

  $= \left[\begin{array}{r|l|l|l} \text{Subst.}\quad t = \sin x & \sin x \neq 0 & x \in (-\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi\rangle \Rightarrow t \in \langle -1; 0) & x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi\rangle \Rightarrow t \in (0; 1\rangle \\ dt = \cos x\,dx & x \neq k\pi,\, k \in Z & x \in \langle -\frac{\pi}{2} + 2k\pi; 0 + 2k\pi) \Rightarrow t \in \langle -1; 0) & x \in \langle \frac{\pi}{2} + 2k\pi; \pi + 2k\pi) \Rightarrow t \in (0; 1\rangle \end{array} \right]$

  $= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2}\,dt = -\operatorname{cotg} x - \frac{t^{-1}}{-1} + c_2 = \frac{1}{t} - \operatorname{cotg} x + c_2$

Riešené príklady – 161

$$\int \frac{dx}{1+\cos x} = \operatorname{tg}\frac{x}{2} + c_1 = \frac{1-\cos x}{\sin x} + c_2$$ [161]

- $= \left[\begin{array}{r|l|l} \text{UGS:}\ \ t = \operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq -1 & x\in(-\pi+2k\pi;\pi+2k\pi) \\ dx = \frac{2\,dt}{t^2+1} & x\neq \pi+2k\pi,\ k\in Z & t\in(-\infty;\infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$

  $= t + c_1 = \operatorname{tg}\frac{x}{2} + c_1,\ x\in R,\ x\neq \pi+2k\pi,\ k\in Z,\ c_1\in R.$

- $\int \frac{dx}{2\cos^2\frac{x}{2}} = \left[\begin{array}{r|l} \text{Subst.}\ \ t = \frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\ k\in Z \\ dt = \frac{dx}{2} & t\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array} \right] = \int \frac{dt}{\cos^2 t}$

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  $= \operatorname{tg} t + c_1 = \operatorname{tg}\frac{x}{2} + c_1,\ x\in R,\ x\neq \pi+2k\pi,\ k\in Z,\ c_1\in R.$

- $= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x\,dx}{\sin^2 x}$

  $= \left[\begin{array}{r|l|l|l} \text{Subst.}\ \ t = \sin x & \sin x \neq 0 & x\in(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\rangle \Rightarrow t\in\langle -1;0) & x\in(0+2k\pi;\frac{\pi}{2}+2k\pi\rangle \Rightarrow t\in(0;1\rangle \\ dt = \cos x\,dx & x\neq k\pi,\ k\in Z & x\in\langle -\frac{\pi}{2}+2k\pi;0+2k\pi) \Rightarrow t\in\langle -1;0) & x\in\langle \frac{\pi}{2}+2k\pi;\pi+2k\pi) \Rightarrow t\in(0;1\rangle \end{array} \right]$

  $= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2}\,dt = -\operatorname{cotg} x - \frac{t^{-1}}{-1} + c_2 = \frac{1}{t} - \operatorname{cotg} x + c_2$

  $= \frac{1}{\sin x} - \frac{\cos x}{\sin x} + c_2 = \frac{1-\cos x}{\sin x} + c_2,\ x\in R,\ x\neq k\pi,\ k\in Z,\ c_2\in R.$

Riešené príklady – 162

$$\int \frac{dx}{1-\cos x}$$ [162]

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Riešené príklady – 162

$\int \frac{dx}{1-\cos x}$ [162]

- $= \left[\begin{array}{c|c|c} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 1 & x \in (-\pi+2k\pi; 0+2k\pi) \Rightarrow t \in (-\infty; 0) \\ dx = \frac{2\,dt}{t^2+1} & x \neq k\pi, k \in Z & x \in (0+2k\pi; \pi+2k\pi) \Rightarrow t \in (0; \infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1-\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t^2}{t^2+1}} = \int \frac{dt}{t^2} = \int t^{-2}\,dt$

- $\int \frac{dx}{2\sin^2 \frac{x}{2}}$ pr10a-11

- $= \int \frac{(1+\cos x)\,dx}{(1+\cos x)(1-\cos x)} = \int \frac{(1+\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1+\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} + \int \frac{\cos x\,dx}{\sin^2 x}$

Riešené príklady – 162

$$\int \frac{dx}{1-\cos x}$$ [162]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 1 & x\in(-\pi+2k\pi;0+2k\pi) \Rightarrow t\in(-\infty;0) \\ dx=\frac{2\,dt}{t^2+1} & x\neq k\pi,\, k\in Z & x\in(0+2k\pi;\pi+2k\pi) \Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1-\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t^2}{t^2+1}} = \int \frac{dt}{t^2} = \int t^{-2}\,dt$

  $= \frac{t^{-1}}{-1} + c_1 = c_1 - \frac{1}{t}$

- $\int \frac{dx}{2\sin^2\frac{x}{2}} = \left[\begin{array}{r|l} \text{Subst. } t=\frac{x}{2} & x\in(0+2k\pi;2\pi+2k\pi),\, k\in Z \\ dt=\frac{dx}{2} & t\in(0+k\pi;\pi+k\pi),\, k\in Z \end{array} \right] = \int \frac{dt}{\sin^2 t}$ (pr10a-11)

- $= \int \frac{(1+\cos x)\,dx}{(1+\cos x)(1-\cos x)} = \int \frac{(1+\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1+\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} + \int \frac{\cos x\,dx}{\sin^2 x}$

  $= \left[\begin{array}{r|l|l|l} \text{Subst. } t=\sin x & \sin x\neq 0 & x\in(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\rangle \Rightarrow t\in\langle-1;0) & x\in(0+2k\pi;\frac{\pi}{2}+2k\pi\rangle \Rightarrow t\in(0;1\rangle \\ dt=\cos x\,dx & x\neq k\pi,\, k\in Z & x\in\langle-\frac{\pi}{2}+2k\pi;0+2k\pi) \Rightarrow t\in\langle-1;0) & x\in\langle\frac{\pi}{2}+2k\pi;\pi+2k\pi) \Rightarrow t\in(0;1\rangle \end{array} \right]$

  $= \int \frac{dx}{\sin^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} + \int t^{-2}\,dt$

Riešené príklady – 162

$$\int \frac{dx}{1-\cos x} = c_1 - \operatorname{cotg}\frac{x}{2}$$ [162]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \cos x = \frac{1-t^2}{t^2+1} \neq 1 & x \in (-\pi+2k\pi; 0+2k\pi) \Rightarrow t \in (-\infty; 0) \\ dx = \frac{2\,dt}{t^2+1} & x \neq k\pi,\, k \in Z & x \in (0+2k\pi; \pi+2k\pi) \Rightarrow t \in (0; \infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1-\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t^2}{t^2+1}} = \int \frac{dt}{t^2} = \int t^{-2}\,dt$

  $= \frac{t^{-1}}{-1} + c_1 = c_1 - \frac{1}{t} = c_1 - \frac{1}{\operatorname{tg}\frac{x}{2}} = c_1 - \operatorname{cotg}\frac{x}{2}$, $x \in R$, $x \neq k\pi$, $k \in Z$, $c_1 \in R$.

- $\int \frac{dx}{2\sin^2\frac{x}{2}} = \left[\begin{array}{r|l} \text{Subst. } t = \frac{x}{2} & x \in (0+2k\pi; 2\pi+2k\pi),\, k \in Z \\ dt = \frac{dx}{2} & t \in (0+k\pi; \pi+k\pi),\, k \in Z \end{array} \right] = \int \frac{dt}{\sin^2 t}$

  pr10a-11

  $= -\operatorname{cotg} t + c_1$

- $= \int \frac{(1+\cos x)\,dx}{(1+\cos x)(1-\cos x)} = \int \frac{(1+\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1+\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} + \int \frac{\cos x\,dx}{\sin^2 x}$

  $= \left[\begin{array}{r|l|l|l} \text{Subst. } t = \sin x & \sin x \neq 0 & x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\rangle \Rightarrow t \in \langle -1; 0) & x \in (0+2k\pi; \frac{\pi}{2}+2k\pi\rangle \Rightarrow t \in (0; 1\rangle \\ dt = \cos x\,dx & x \neq k\pi,\, k \in Z & x \in \langle -\frac{\pi}{2}+2k\pi; 0+2k\pi) \Rightarrow t \in \langle -1; 0) & x \in \langle \frac{\pi}{2}+2k\pi; \pi+2k\pi) \Rightarrow t \in (0; 1\rangle \end{array} \right]$

  $= \int \frac{dx}{\sin^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} + \int t^{-2}\,dt = -\operatorname{cotg} x + \frac{t^{-1}}{-1} + c_2 = c_2 - \frac{1}{t} - \operatorname{cotg} x$

Riešené príklady – 162

$$\int \frac{dx}{1-\cos x} = c_1 - \operatorname{cotg}\frac{x}{2} = c_2 - \frac{1+\sin x}{\cos x}$$ [162]

- $= \left[\begin{array}{r|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \cos x=\frac{1-t^2}{t^2+1}\neq 1 & x\in(-\pi+2k\pi;0+2k\pi)\Rightarrow t\in(-\infty;0) \\ dx=\frac{2\,dt}{t^2+1} & x\neq k\pi,\,k\in Z & x\in(0+2k\pi;\pi+2k\pi)\Rightarrow t\in(0;\infty) \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1-\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t^2}{t^2+1}} = \int \frac{dt}{t^2} = \int t^{-2}\,dt$

  $= \frac{t^{-1}}{-1} + c_1 = c_1 - \frac{1}{t} = c_1 - \frac{1}{\operatorname{tg}\frac{x}{2}} = c_1 - \operatorname{cotg}\frac{x}{2},\ x\in R,\ x\neq k\pi,\ k\in Z,\ c_1\in R.$

- $\int \frac{dx}{2\sin^2\frac{x}{2}} \overset{\text{pr10a-11}}{=} \left[\begin{array}{r|l} \text{Subst. } t=\frac{x}{2} & x\in(0+2k\pi;2\pi+2k\pi),\,k\in Z \\ dt=\frac{dx}{2} & t\in(0+k\pi;\pi+k\pi),\,k\in Z \end{array} \right] = \int \frac{dt}{\sin^2 t}$

  $= -\operatorname{cotg} t + c_1 = c_1 - \operatorname{cotg}\frac{x}{2},\ x\in R,\ x\neq 2k\pi,\ k\in Z,\ c_1\in R.$

- $= \int \frac{(1+\cos x)\,dx}{(1+\cos x)(1-\cos x)} = \int \frac{(1+\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1+\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} + \int \frac{\cos x\,dx}{\sin^2 x}$

  $= \left[\begin{array}{r|l|l|l} \text{Subst. } t=\sin x & \sin x\neq 0 & x\in(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\rangle\Rightarrow t\in\langle -1;0) & x\in(0+2k\pi;\frac{\pi}{2}+2k\pi\rangle\Rightarrow t\in(0;1\rangle \\ dt=\cos x\,dx & x\neq k\pi,\,k\in Z & x\in\langle -\frac{\pi}{2}+2k\pi;0+2k\pi)\Rightarrow t\in\langle -1;0) & x\in\langle\frac{\pi}{2}+2k\pi;\pi+2k\pi)\Rightarrow t\in(0;1\rangle \end{array} \right]$

  $= \int \frac{dx}{\sin^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} + \int t^{-2}\,dt = -\operatorname{cotg} x + \frac{t^{-1}}{-1} + c_2 = c_2 - \frac{1}{t} - \operatorname{cotg} x$

  $= c_2 - \frac{1}{\sin x} - \frac{\cos x}{\sin x} = c_2 - \frac{1+\cos x}{\sin x},\ x\in R,\ x\neq k\pi,\ k\in Z,\ c_2\in R.$