

Matematická analýza 1

2024/2025

6. Limita funkcie Riešené príklady

Pre správne zobrazenie, fungovanie tooltipov, 2D a 3D animácií je nevyhnutné súbor otvoriť pomocou programu Adobe Reader (zásuvný modul Adobe PDF Plug-In webového prehliadača nestačí).

Kliknutím na text pred ikonou  získate nápomoc.

Kliknutím na skratku v modrej lište vpravo hore sa dostanete na príslušný slajd, druhým kliknutím sa dostanete na koniec tohto slajdu.

Obsah

- 1 Riešené limity 01–17
- 2 Riešené limity 18–29
- 3 Riešené limity 30–45

Zoznam riešených limitů – příklady 01–45

- 01. $\lim_{x \rightarrow 0} \frac{a^x - 1}{x}$. • 02. $\lim_{x \rightarrow 2} \frac{x-2}{x^2-3x+2}$. • 03. $\lim_{x \rightarrow \infty} \left(\frac{5x^2}{x^2-1} + 2^{\frac{1}{x}} \right)$. • 04. $\lim_{x \rightarrow 0} \frac{1-3^x}{x}$. • 05. $\lim_{x \rightarrow -\infty} \frac{1-3^x}{x}$.
- 06. $\lim_{x \rightarrow 1} \frac{1-3^x}{x}$. • 07. $\lim_{x \rightarrow \infty} \frac{1-3^x}{x}$. • 08. $\lim_{x \rightarrow 0} \frac{3x + \frac{2}{x}}{x + \frac{4}{x}}$. • 09. $\lim_{x \rightarrow \infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right)$.
- 10. $\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$. • 11. $\lim_{x \rightarrow \infty} \left(\frac{3x-2}{3x+1} \right)^x$. • 12. $\lim_{x \rightarrow -\infty} \left(\frac{x+1}{x} \right)^x$. • 13. $\lim_{x \rightarrow 0} (1+3 \operatorname{tg}^2 x)^{\cot^2 x}$.
- 14. $\lim_{x \rightarrow 0} (1+3 \operatorname{tg}^2 x)^{\operatorname{tg}^2 x}$. • 15. $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x} \right)^{x^2}$. • 16. $\lim_{x \rightarrow \infty} x [\ln(x+2) - \ln x]$.
- 17. $\lim_{x \rightarrow 0} \frac{x}{\ln(1+tx)}$. • 18. $\lim_{x \rightarrow 0} \frac{1-\sqrt{1-x}}{x}$. • 19. $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$. • 20. $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{x}$.
- 21. $\lim_{x \rightarrow \infty} (x - \sqrt{x^2-1})$. • 22. $\lim_{x \rightarrow a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a}$. • 23. $\lim_{x \rightarrow a} \frac{a-x}{\sqrt{a}-\sqrt{x}}$. • 24. $\lim_{x \rightarrow \infty} \frac{\sqrt{x}-6x}{3x+1}$.
- 25. $\lim_{x \rightarrow 1} \frac{x^m-1}{x^n-1}$. • 26. $\lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$. • 27. $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$. • 28. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x}$. • 29. $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$.
- 30. $\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x}$. • 31. $\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{x}$. • 32. $\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{\sin 2x}$. • 33. $\lim_{x \rightarrow \pi} \frac{\operatorname{tg} x}{\sin 2x}$. • 34. $\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx}$. • 35. $\lim_{x \rightarrow \pi} \frac{\sin mx}{\sin nx}$.
- 36. $\lim_{x \rightarrow 2\pi} \frac{\sin mx}{\sin nx}$. • 37. $\lim_{x \rightarrow 0} \frac{1-\cos 2x}{x^2}$. • 38. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$. • 39. $\lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1}$.
- 30. $\lim_{x \rightarrow 2} \frac{\arcsin(x-2)}{x^2-2x}$. • 41. $\lim_{x \rightarrow \infty} e^x(2 + \cos x)$. • 42. $\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}}$. • 43. $\lim_{x \rightarrow 0} \frac{x^n - \sin^n x}{x - \sin x}$.
- 44. $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a}$. • 45. $\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a}$.

Riešené limity – 01, 02, 03

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x}$$

pre $a > 0, a \neq 1$

[01]

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-3x+2}$$

[02]

$$\lim_{x \rightarrow \infty} \left(\frac{5x^2}{x^2-1} + 2^{\frac{1}{x}} \right)$$

[03]

Riešené limity – 01, 02, 03

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} \quad \text{pre } a > 0, a \neq 1$$

[01]

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = a^x - 1 \mid x \rightarrow 0 \\ z + 1 = a^x \mid z \rightarrow 0 \end{array} \right]$$

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-3x+2}$$

[02]

$$\bullet = \lim_{x \rightarrow 2} \frac{x-2}{(x-2) \cdot (x-1)}$$

$$\lim_{x \rightarrow \infty} \left(\frac{5x^2}{x^2-1} + 2^{\frac{1}{x}} \right)$$

[03]

$$\bullet = \lim_{x \rightarrow \infty} \frac{5}{1 - \frac{1}{x^2}} + \lim_{x \rightarrow \infty} 2^{\frac{1}{x}}$$

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{5}{1 - \frac{1}{x^2}} + 2^{\frac{1}{x}} \right)$$

Riešené limity – 01, 02, 03

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} \quad \text{pre } a > 0, a \neq 1$$

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$$\bullet = \left[\begin{array}{l} \text{Subst. } z = a^x - 1 \mid x \rightarrow 0 \mid \ln(z+1) = \ln a^x = x \ln a \\ z+1 = a^x \mid z \rightarrow 0 \end{array} \right] \Rightarrow x = \frac{\ln(z+1)}{\ln a}$$

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-3x+2}$$

[02]

$$\bullet = \lim_{x \rightarrow 2} \frac{x-2}{(x-2) \cdot (x-1)} = \lim_{x \rightarrow 2} \frac{1}{x-1}$$

$$\lim_{x \rightarrow \infty} \left(\frac{5x^2}{x^2-1} + 2^{\frac{1}{x}} \right)$$

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$$\bullet = \lim_{x \rightarrow \infty} \frac{5}{1-\frac{1}{x^2}} + \lim_{x \rightarrow \infty} 2^{\frac{1}{x}} = \frac{5}{1-\frac{1}{\infty}} + 2^{\frac{1}{\infty}}$$

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$$\bullet = \lim_{x \rightarrow 2} \frac{x-2}{(x-2) \cdot (x-1)} = \lim_{x \rightarrow 2} \frac{1}{x-1} = \frac{1}{2-1}$$

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$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-3x+2} = 1$$

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Riešené limity – 01, 02, 03

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a \quad \text{pre } a > 0, a \neq 1$$

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$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Subst. } z = a^x - 1 \mid x \rightarrow 0 \\ z + 1 = a^x \mid z \rightarrow 0 \end{array} \left| \begin{array}{l} \ln(z+1) = \ln a^x = x \ln a \\ \Rightarrow x = \frac{\ln(z+1)}{\ln a} \end{array} \right. \right] = \lim_{z \rightarrow 0} \frac{z}{\frac{\ln(z+1)}{\ln a}} = \lim_{z \rightarrow 0} \frac{z \ln a}{\ln(z+1)} = \lim_{z \rightarrow 0} \frac{\ln a}{\frac{1}{z} \ln(z+1)} \\ &= \lim_{z \rightarrow 0} \frac{\ln a}{\ln(z+1)^{\frac{1}{z}}} = \frac{\ln a}{\lim_{z \rightarrow 0} \ln(z+1)^{\frac{1}{z}}} = \left[z \rightarrow 0 \Rightarrow (z+1)^{\frac{1}{z}} \rightarrow e \right] = \frac{\ln a}{\ln e} = \frac{\ln a}{1} = \ln a. \end{aligned}$$

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-3x+2} = 1$$

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$$\bullet = \lim_{x \rightarrow 2} \frac{x-2}{(x-2) \cdot (x-1)} = \lim_{x \rightarrow 2} \frac{1}{x-1} = \frac{1}{2-1} = 1.$$

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$$\bullet = \lim_{x \rightarrow \infty} \frac{5}{1-\frac{1}{x^2}} + \lim_{x \rightarrow \infty} 2^{\frac{1}{x}} = \frac{5}{1-\frac{1}{\infty}} + 2^{\frac{1}{\infty}} = \frac{5}{1-0} + 2^0 = 5 + 1 = 6.$$

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Riešené limity – 04, 05, 06, 07

$$\lim_{x \rightarrow 0} \frac{1-3^x}{x} = - \lim_{x \rightarrow 0} \frac{3^x-1}{x}$$

[04]

$$\lim_{x \rightarrow 1} \frac{1-3^x}{x}$$

[05]

$$\lim_{x \rightarrow -\infty} \frac{1-3^x}{x}$$

[06]

$$\lim_{x \rightarrow \infty} \frac{1-3^x}{x}$$

[07]

Riešené limity – 04, 05, 06, 07

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[04]

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$$\lim_{x \rightarrow 1} \frac{1-3^x}{x}$$

[05]

$$\bullet = \frac{1-3^1}{1}$$

$$\lim_{x \rightarrow -\infty} \frac{1-3^x}{x}$$

[06]

$$\bullet = \frac{1-3^{-\infty}}{-\infty}$$

$$\lim_{x \rightarrow \infty} \frac{1-3^x}{x}$$

[07]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{3^x}{x} \right)$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x \rightarrow 0} \frac{1-3^x}{x} = - \lim_{x \rightarrow 0} \frac{3^x-1}{x}$$

[04]

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = 3^x - 1 \mid x \rightarrow 0 \\ z + 1 = 3^x \mid z \rightarrow 0 \end{array} \mid \ln(z+1) = \ln 3^x = x \ln 3 \Rightarrow x = \frac{\ln(z+1)}{\ln 3} \right]$$

$$\lim_{x \rightarrow 1} \frac{1-3^x}{x}$$

[05]

$$\bullet = \frac{1-3^1}{1} = \frac{1-3}{1}$$

$$\lim_{x \rightarrow -\infty} \frac{1-3^x}{x}$$

[06]

$$\bullet = \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^{\infty}}}{-\infty}$$

$$\lim_{x \rightarrow \infty} \frac{1-3^x}{x}$$

[07]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{3^x}{x} \right) = \left[\lim_{x \rightarrow \infty} \frac{3^x}{x} = \infty \right]$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x \rightarrow 0} \frac{1-3^x}{x} = - \lim_{x \rightarrow 0} \frac{3^x-1}{x} \quad [04]$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = 3^x - 1 \mid x \rightarrow 0 \\ z + 1 = 3^x \mid x \rightarrow 0 \end{array} \mid \ln(z+1) = \ln 3^x = x \ln 3 \Rightarrow x = \frac{\ln(z+1)}{\ln 3} \right] = - \lim_{z \rightarrow 0} \frac{z}{\frac{\ln(z+1)}{\ln 3}}$$

$$\lim_{x \rightarrow 1} \frac{1-3^x}{x} \quad [05]$$

$$\bullet = \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1}$$

$$\lim_{x \rightarrow -\infty} \frac{1-3^x}{x} \quad [06]$$

$$\bullet = \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^{\infty}}}{-\infty} = \frac{1-0}{-\infty}$$

$$\lim_{x \rightarrow \infty} \frac{1-3^x}{x} \quad [07]$$

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{3^x}{x} \right) = \left[\lim_{x \rightarrow \infty} \frac{3^x}{x} = \infty \right] = \frac{1}{\infty} - \infty$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x \rightarrow 0} \frac{1-3^x}{x} = - \lim_{x \rightarrow 0} \frac{3^x-1}{x} \quad [04]$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = 3^x - 1 \mid x \rightarrow 0 \\ z + 1 = 3^x \mid z \rightarrow 0 \end{array} \mid \ln(z+1) = \ln 3^x = x \ln 3 \Rightarrow x = \frac{\ln(z+1)}{\ln 3} \right] = - \lim_{z \rightarrow 0} \frac{z}{\frac{\ln(z+1)}{\ln 3}} = - \lim_{z \rightarrow 0} \frac{z \ln 3}{\ln(z+1)}$$

$$\lim_{x \rightarrow 1} \frac{1-3^x}{x} = -2 \quad [05]$$

$$\bullet = \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1} = -2.$$

$$\lim_{x \rightarrow -\infty} \frac{1-3^x}{x} = 0 \quad [06]$$

$$\bullet = \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^{\infty}}}{-\infty} = \frac{1-0}{-\infty} = 0.$$

$$\lim_{x \rightarrow \infty} \frac{1-3^x}{x} \quad [07]$$

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{3^x}{x} \right) = \left[\lim_{x \rightarrow \infty} \frac{3^x}{x} = \infty \right] = \frac{1}{\infty} - \infty = 0 - \infty$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x \rightarrow 0} \frac{1-3^x}{x} = - \lim_{x \rightarrow 0} \frac{3^x-1}{x} \quad [04]$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } \begin{matrix} z = 3^x - 1 & x \rightarrow 0 \\ z + 1 = 3^x & z \rightarrow 0 \end{matrix} \left| \ln(z+1) = \ln 3^x = x \ln 3 \Rightarrow x = \frac{\ln(z+1)}{\ln 3} \right. \right] = - \lim_{z \rightarrow 0} \frac{z}{\frac{\ln(z+1)}{\ln 3}} = - \lim_{z \rightarrow 0} \frac{z \ln 3}{\ln(z+1)} \\ &= - \lim_{z \rightarrow 0} \frac{\ln 3}{\frac{1}{z} \ln(z+1)} \end{aligned}$$

$$\lim_{x \rightarrow 1} \frac{1-3^x}{x} = -2 \quad [05]$$

$$\bullet = \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1} = -2.$$

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$$\bullet = \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^{\infty}}}{-\infty} = \frac{1-0}{-\infty} = 0.$$

$$\lim_{x \rightarrow \infty} \frac{1-3^x}{x} = -\infty \quad [07]$$

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{3^x}{x} \right) = \left[\lim_{x \rightarrow \infty} \frac{3^x}{x} = \infty \right] = \frac{1}{\infty} - \infty = 0 - \infty = -\infty.$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x \rightarrow 0} \frac{1-3^x}{x} = - \lim_{x \rightarrow 0} \frac{3^x-1}{x}$$

[04]

$$\begin{aligned} \bullet &= \left[\text{Subst. } \begin{matrix} z = 3^x - 1 & x \rightarrow 0 \\ z + 1 = 3^x & z \rightarrow 0 \end{matrix} \left| \ln(z+1) = \ln 3^x = x \ln 3 \Rightarrow x = \frac{\ln(z+1)}{\ln 3} \right. \right] = - \lim_{z \rightarrow 0} \frac{z}{\frac{\ln(z+1)}{\ln 3}} = - \lim_{z \rightarrow 0} \frac{z \ln 3}{\ln(z+1)} \\ &= - \lim_{z \rightarrow 0} \frac{\ln 3}{\frac{1}{z} \ln(z+1)} = - \lim_{z \rightarrow 0} \frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} \end{aligned}$$

$$\lim_{x \rightarrow 1} \frac{1-3^x}{x} = -2$$

[05]

$$\bullet = \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1} = -2.$$

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[06]

$$\bullet = \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^{\infty}}}{-\infty} = \frac{1-0}{-\infty} = 0.$$

$$\lim_{x \rightarrow \infty} \frac{1-3^x}{x} = -\infty$$

[07]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{3^x}{x} \right) = \left[\lim_{x \rightarrow \infty} \frac{3^x}{x} = \infty \right] = \frac{1}{\infty} - \infty = 0 - \infty = -\infty.$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x \rightarrow 0} \frac{1-3^x}{x} = - \lim_{x \rightarrow 0} \frac{3^x-1}{x} \quad [04]$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Subst. } z = 3^x - 1 \mid x \rightarrow 0 \\ z + 1 = 3^x \mid z \rightarrow 0 \end{array} \mid \ln(z+1) = \ln 3^x = x \ln 3 \Rightarrow x = \frac{\ln(z+1)}{\ln 3} \right] = - \lim_{z \rightarrow 0} \frac{z}{\frac{\ln(z+1)}{\ln 3}} = - \lim_{z \rightarrow 0} \frac{z \ln 3}{\ln(z+1)} \\ &= - \lim_{z \rightarrow 0} \frac{\ln 3}{\frac{1}{z} \ln(z+1)} = - \lim_{z \rightarrow 0} \frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = \left[z \rightarrow 0 \Rightarrow (z+1)^{\frac{1}{z}} \rightarrow e \right] \end{aligned}$$

$$\lim_{x \rightarrow 1} \frac{1-3^x}{x} = -2 \quad [05]$$

$$\bullet = \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1} = -2.$$

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$$\bullet = \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^{\infty}}}{-\infty} = \frac{1-0}{-\infty} = 0.$$

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$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{3^x}{x} \right) = \left[\lim_{x \rightarrow \infty} \frac{3^x}{x} = \infty \right] = \frac{1}{\infty} - \infty = 0 - \infty = -\infty.$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x \rightarrow 0} \frac{1-3^x}{x} = - \lim_{x \rightarrow 0} \frac{3^x-1}{x}$$

[04]

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Subst. } z = 3^x - 1 \mid x \rightarrow 0 \\ z + 1 = 3^x \mid z \rightarrow 0 \end{array} \mid \ln(z+1) = \ln 3^x = x \ln 3 \Rightarrow x = \frac{\ln(z+1)}{\ln 3} \right] = - \lim_{z \rightarrow 0} \frac{z}{\frac{\ln(z+1)}{\ln 3}} = - \lim_{z \rightarrow 0} \frac{z \ln 3}{\ln(z+1)} \\ &= - \lim_{z \rightarrow 0} \frac{\ln 3}{\frac{1}{z} \ln(z+1)} = - \lim_{z \rightarrow 0} \frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = \left[z \rightarrow 0 \Rightarrow (z+1)^{\frac{1}{z}} \rightarrow e \right] = - \frac{\ln 3}{\ln e} \end{aligned}$$

$$\lim_{x \rightarrow 1} \frac{1-3^x}{x} = -2$$

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$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{3^x}{x} \right) = \left[\lim_{x \rightarrow \infty} \frac{3^x}{x} = \infty \right] = \frac{1}{\infty} - \infty = 0 - \infty = -\infty.$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x \rightarrow 0} \frac{1-3^x}{x} = - \lim_{x \rightarrow 0} \frac{3^x-1}{x}$$

[04]

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = 3^x - 1 \left| \begin{array}{l} x \rightarrow 0 \\ z + 1 = 3^x \end{array} \right| \ln(z+1) = \ln 3^x = x \ln 3 \Rightarrow x = \frac{\ln(z+1)}{\ln 3} \right] = - \lim_{z \rightarrow 0} \frac{z}{\frac{\ln(z+1)}{\ln 3}} = - \lim_{z \rightarrow 0} \frac{z \ln 3}{\ln(z+1)} \\ &= - \lim_{z \rightarrow 0} \frac{\ln 3}{\frac{1}{z} \ln(z+1)} = - \lim_{z \rightarrow 0} \frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = \left[z \rightarrow 0 \Rightarrow (z+1)^{\frac{1}{z}} \rightarrow e \right] = - \frac{\ln 3}{\ln e} = - \frac{\ln 3}{1} \end{aligned}$$

$$\lim_{x \rightarrow 1} \frac{1-3^x}{x} = -2$$

[05]

$$\bullet = \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1} = -2.$$

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[06]

$$\bullet = \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^{\infty}}}{-\infty} = \frac{1-0}{-\infty} = 0.$$

$$\lim_{x \rightarrow \infty} \frac{1-3^x}{x} = -\infty$$

[07]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{3^x}{x} \right) = \left[\lim_{x \rightarrow \infty} \frac{3^x}{x} = \infty \right] = \frac{1}{\infty} - \infty = 0 - \infty = -\infty.$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x \rightarrow 0} \frac{1-3^x}{x} = - \lim_{x \rightarrow 0} \frac{3^x-1}{x} = -\ln 3$$

[04]

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Subst. } z = 3^x - 1 \mid x \rightarrow 0 \\ z + 1 = 3^x \mid z \rightarrow 0 \end{array} \mid \ln(z+1) = \ln 3^x = x \ln 3 \Rightarrow x = \frac{\ln(z+1)}{\ln 3} \right] = - \lim_{z \rightarrow 0} \frac{z}{\frac{\ln(z+1)}{\ln 3}} = - \lim_{z \rightarrow 0} \frac{z \ln 3}{\ln(z+1)} \\ &= - \lim_{z \rightarrow 0} \frac{\ln 3}{\frac{1}{z} \ln(z+1)} = - \lim_{z \rightarrow 0} \frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = \left[z \rightarrow 0 \Rightarrow (z+1)^{\frac{1}{z}} \rightarrow e \right] = - \frac{\ln 3}{\ln e} = - \frac{\ln 3}{1} = -\ln 3. \end{aligned}$$

$$\lim_{x \rightarrow 1} \frac{1-3^x}{x} = -2$$

[05]

$$\bullet = \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1} = -2.$$

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[06]

$$\bullet = \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^{\infty}}}{-\infty} = \frac{1-0}{-\infty} = 0.$$

$$\lim_{x \rightarrow \infty} \frac{1-3^x}{x} = -\infty$$

[07]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{3^x}{x} \right) = \left[\lim_{x \rightarrow \infty} \frac{3^x}{x} = \infty \right] = \frac{1}{\infty} - \infty = 0 - \infty = -\infty.$$

Riešené limity – 08, 09, 10

$$\lim_{x \rightarrow 0} \frac{3x + \frac{2}{x}}{x + \frac{4}{x}}$$

[08]

$$\lim_{x \rightarrow \infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right)$$

[09]

$$\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$$

[10]



Riešené limity – 08, 09, 10

$$\lim_{x \rightarrow 0} \frac{3x + \frac{2}{x}}{x + \frac{4}{x}}$$

[08]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{3x + \frac{2}{x}}{x + \frac{4}{x}} \cdot \frac{x}{x} \right)$$

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{1}{x} \cdot (3x^2 + 2)}{\frac{1}{x} \cdot (x^2 + 4)}$$

$$\lim_{x \rightarrow \infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right)$$

[09]

$$\bullet = \lim_{x \rightarrow \infty} \frac{1}{x^2} - \lim_{x \rightarrow \infty} \frac{1}{x} + \lim_{x \rightarrow \infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right)$$

$$\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$$

[10]

$$\bullet = \lim_{x \rightarrow 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)}$$

Riešené limity – 08, 09, 10

$$\lim_{x \rightarrow 0} \frac{3x + \frac{2}{x}}{x + \frac{4}{x}}$$

[08]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{3x + \frac{2}{x}}{x + \frac{4}{x}} \cdot \frac{x}{x} \right) = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4}$$

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{1}{x} \cdot (3x^2 + 2)}{\frac{1}{x} \cdot (x^2 + 4)} = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4}$$

$$\lim_{x \rightarrow \infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right)$$

[09]

$$\bullet = \lim_{x \rightarrow \infty} \frac{1}{x^2} - \lim_{x \rightarrow \infty} \frac{1}{x} + \lim_{x \rightarrow \infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - 2}{\frac{2}{x} - 3}$$

$$\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$$

[10]

$$\bullet = \lim_{x \rightarrow 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} = \lim_{x \rightarrow 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)}$$

Riešené limity – 08, 09, 10

$$\lim_{x \rightarrow 0} \frac{3x + \frac{2}{x}}{x + \frac{4}{x}}$$

[08]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{3x + \frac{2}{x}}{x + \frac{4}{x}} \cdot \frac{x}{x} \right) = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4} = \frac{0+2}{0+4}$$

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{1}{x} \cdot (3x^2 + 2)}{\frac{1}{x} \cdot (x^2 + 4)} = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4} = \frac{0+2}{0+4}$$

$$\lim_{x \rightarrow \infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right)$$

[09]

$$\bullet = \lim_{x \rightarrow \infty} \frac{1}{x^2} - \lim_{x \rightarrow \infty} \frac{1}{x} + \lim_{x \rightarrow \infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - 2}{\frac{2}{x} - 3} = \frac{0-2}{0-3}$$

$$\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$$

[10]

$$\bullet = \lim_{x \rightarrow 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} \stackrel{?}{=} \lim_{x \rightarrow 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)} = \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)} \stackrel{?}{=}$$

Riešené limity – 08, 09, 10

$$\lim_{x \rightarrow 0} \frac{3x + \frac{2}{x}}{x + \frac{4}{x}} = \frac{1}{2}$$

[08]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{3x + \frac{2}{x}}{x + \frac{4}{x}} \cdot \frac{x}{x} \right) = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{1}{x} \cdot (3x^2 + 2)}{\frac{1}{x} \cdot (x^2 + 4)} = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x \rightarrow \infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right) = \frac{2}{3}$$

[09]

$$\bullet = \lim_{x \rightarrow \infty} \frac{1}{x^2} - \lim_{x \rightarrow \infty} \frac{1}{x} + \lim_{x \rightarrow \infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - 2}{\frac{2}{x} - 3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

$$\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$$

[10]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} \stackrel{?}{=} \lim_{x \rightarrow 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)} = \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)} \\ &= - \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(x-1)(1+x+x^2)} \end{aligned}$$

Riešené limity – 08, 09, 10

$$\lim_{x \rightarrow 0} \frac{3x + \frac{2}{x}}{x + \frac{4}{x}} = \frac{1}{2}$$

[08]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{3x + \frac{2}{x}}{x + \frac{4}{x}} \cdot \frac{x}{x} \right) = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{1}{x} \cdot (3x^2 + 2)}{\frac{1}{x} \cdot (x^2 + 4)} = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x \rightarrow \infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right) = \frac{2}{3}$$

[09]

$$\bullet = \lim_{x \rightarrow \infty} \frac{1}{x^2} - \lim_{x \rightarrow \infty} \frac{1}{x} + \lim_{x \rightarrow \infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - 2}{\frac{2}{x} - 3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

$$\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$$

[10]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} \stackrel{?}{=} \lim_{x \rightarrow 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)} = \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)} \stackrel{?}{=} \\ &= - \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(x-1)(1+x+x^2)} = - \lim_{x \rightarrow 1} \frac{x+2}{1+x+x^2} \end{aligned}$$

Riešené limity – 08, 09, 10

$$\lim_{x \rightarrow 0} \frac{3x + \frac{2}{x}}{x + \frac{4}{x}} = \frac{1}{2}$$

[08]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{3x + \frac{2}{x}}{x + \frac{4}{x}} \cdot \frac{x}{x} \right) = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{1}{x} \cdot (3x^2 + 2)}{\frac{1}{x} \cdot (x^2 + 4)} = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x \rightarrow \infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right) = \frac{2}{3}$$

[09]

$$\bullet = \lim_{x \rightarrow \infty} \frac{1}{x^2} - \lim_{x \rightarrow \infty} \frac{1}{x} + \lim_{x \rightarrow \infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - 2}{\frac{2}{x} - 3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

$$\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$$

[10]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} \stackrel{?}{=} \lim_{x \rightarrow 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)} = \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)} \stackrel{?}{=} \\ &= - \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(x-1)(1+x+x^2)} = - \lim_{x \rightarrow 1} \frac{x+2}{1+x+x^2} = - \frac{1+2}{1+1+1} \end{aligned}$$

Riešené limity – 08, 09, 10

$$\lim_{x \rightarrow 0} \frac{3x + \frac{2}{x}}{x + \frac{4}{x}} = \frac{1}{2}$$

[08]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{3x + \frac{2}{x}}{x + \frac{4}{x}} \cdot \frac{x}{x} \right) = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{1}{x} \cdot (3x^2 + 2)}{\frac{1}{x} \cdot (x^2 + 4)} = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x \rightarrow \infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right) = \frac{2}{3}$$

[09]

$$\bullet = \lim_{x \rightarrow \infty} \frac{1}{x^2} - \lim_{x \rightarrow \infty} \frac{1}{x} + \lim_{x \rightarrow \infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - 2}{\frac{2}{x} - 3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

$$\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$$

[10]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} \stackrel{?}{=} \lim_{x \rightarrow 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)} = \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)} \stackrel{?}{=} \\ &= - \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(x-1)(1+x+x^2)} = - \lim_{x \rightarrow 1} \frac{x+2}{1+x+x^2} = - \frac{1+2}{1+1+1} = -\frac{3}{3} \end{aligned}$$

Riešené limity – 08, 09, 10

$$\lim_{x \rightarrow 0} \frac{3x + \frac{2}{x}}{x + \frac{4}{x}} = \frac{1}{2}$$

[08]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{3x + \frac{2}{x}}{x + \frac{4}{x}} \cdot \frac{x}{x} \right) = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{1}{x} \cdot (3x^2 + 2)}{\frac{1}{x} \cdot (x^2 + 4)} = \lim_{x \rightarrow 0} \frac{3x^2 + 2}{x^2 + 4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x \rightarrow \infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x} \right) = \frac{2}{3}$$

[09]

$$\bullet = \lim_{x \rightarrow \infty} \frac{1}{x^2} - \lim_{x \rightarrow \infty} \frac{1}{x} + \lim_{x \rightarrow \infty} \left(\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = 0 - 0 + \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - 2}{\frac{2}{x} - 3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

$$\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right) = -1$$

[10]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)} = \lim_{x \rightarrow 1} \frac{x^2+x-2}{(1-x)(1+x+x^2)} = \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)} \\ &= - \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(x-1)(1+x+x^2)} = - \lim_{x \rightarrow 1} \frac{x+2}{1+x+x^2} = - \frac{1+2}{1+1+1} = - \frac{3}{3} = -1. \end{aligned}$$

Riešené limity – 11

$$\lim_{x \rightarrow \infty} \left(\frac{3x-2}{3x+1} \right)^x$$

$$\frac{1}{e}$$

[11]

Riešené limity – 11

$$\lim_{x \rightarrow \infty} \left(\frac{3x-2}{3x+1} \right)^x \quad \frac{1}{e}$$

[11]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{3x+1-3}{3x+1} \right)^{\frac{3x}{3}}$$

$$\bullet = \left[\begin{array}{l|l|l} \text{Subst. } z = 3x + 1 & x \rightarrow \infty & 3x = z - 1 \\ x = \frac{z-1}{3} & z \rightarrow \infty & 3x - 2 = z - 3 \end{array} \right]$$

Riešené limity – 11

$$\lim_{x \rightarrow \infty} \left(\frac{3x-2}{3x+1} \right)^x \quad \frac{1}{e}$$

[11]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{3x+1-3}{3x+1} \right)^{\frac{3x}{3}} = \lim_{x \rightarrow \infty} \left(1 + \frac{-3}{3x+1} \right)^{\frac{3x+1-1}{3}}$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = 3x + 1 \mid x \rightarrow \infty \mid 3x = z - 1 \\ x = \frac{z-1}{3} \mid z \rightarrow \infty \mid 3x - 2 = z - 3 \end{array} \right] = \lim_{z \rightarrow \infty} \left(\frac{z-3}{z} \right)^{\frac{z-1}{3}}$$

Riešené limity – 11

$$\lim_{x \rightarrow \infty} \left(\frac{3x-2}{3x+1} \right)^x \quad \frac{1}{e}$$

[11]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{3x+1-3}{3x+1} \right)^{\frac{3x}{3}} = \lim_{x \rightarrow \infty} \left(1 + \frac{-3}{3x+1} \right)^{\frac{3x+1-1}{3}} = \left[\begin{array}{l} \text{Subst. } z = 3x+1 \mid x \rightarrow \infty \\ z \rightarrow \infty \end{array} \right]$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = 3x+1 \mid x \rightarrow \infty \mid 3x = z-1 \\ x = \frac{z-1}{3} \mid z \rightarrow \infty \mid 3x-2 = z-3 \end{array} \right] = \lim_{z \rightarrow \infty} \left(\frac{z-3}{z} \right)^{\frac{z-1}{3}} = \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z}{3} - \frac{1}{3}}$$

$$= \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^{\frac{z}{3}} \cdot \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} \right] = \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z}{3}} \cdot \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}}$$

Riešené limity – 11

$$\lim_{x \rightarrow \infty} \left(\frac{3x-2}{3x+1} \right)^x \quad \frac{1}{e}$$

[11]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{3x+1-3}{3x+1} \right)^{\frac{3x}{3}} = \lim_{x \rightarrow \infty} \left(1 + \frac{-3}{3x+1} \right)^{\frac{3x+1-1}{3}} = \left[\begin{array}{l} \text{Subst. } z = 3x + 1 \\ x \rightarrow \infty \\ z \rightarrow \infty \end{array} \right]$$

$$= \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z-1}{3}}$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = 3x + 1 \\ x = \frac{z-1}{3} \end{array} \left| \begin{array}{l} x \rightarrow \infty \\ z \rightarrow \infty \end{array} \right| \begin{array}{l} 3x = z - 1 \\ 3x - 2 = z - 3 \end{array} \right] = \lim_{z \rightarrow \infty} \left(\frac{z-3}{z} \right)^{\frac{z-1}{3}} = \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z}{3} - \frac{1}{3}}$$

$$= \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^{\frac{z}{3}} \cdot \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} \right] = \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z}{3}} \cdot \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}}$$

$$= \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^z \right]^{\frac{1}{3}} \cdot \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}}$$

Riešené limity – 11

$$\lim_{x \rightarrow \infty} \left(\frac{3x-2}{3x+1} \right)^x \quad \frac{1}{e}$$

[11]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{3x+1-3}{3x+1} \right)^{\frac{3x}{3}} = \lim_{x \rightarrow \infty} \left(1 + \frac{-3}{3x+1} \right)^{\frac{3x+1-1}{3}} = \left[\begin{array}{l} \text{Subst. } z = 3x+1 \mid x \rightarrow \infty \\ z \rightarrow \infty \end{array} \right]$$

$$= \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z-1}{3}} = \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^z \right]^{\frac{z-1}{3 \cdot z}}$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = 3x+1 \mid x \rightarrow \infty \mid 3x = z-1 \\ x = \frac{z-1}{3} \mid z \rightarrow \infty \mid 3x-2 = z-3 \end{array} \right] = \lim_{z \rightarrow \infty} \left(\frac{z-3}{z} \right)^{\frac{z-1}{3}} = \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z}{3} - \frac{1}{3}}$$

$$= \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^{\frac{z}{3}} \cdot \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} \right] = \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z}{3}} \cdot \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}}$$

$$= \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^z \right]^{\frac{1}{3}} \cdot \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} = (e^{-3})^{\frac{1}{3}} \cdot \left(1 + \frac{-3}{\infty} \right)^{-\frac{1}{3}}$$

Riešené limity – 11

$$\lim_{x \rightarrow \infty} \left(\frac{3x-2}{3x+1} \right)^x \quad \frac{1}{e}$$

[11]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left(\frac{3x+1-3}{3x+1} \right)^{\frac{3x}{3}} = \lim_{x \rightarrow \infty} \left(1 + \frac{-3}{3x+1} \right)^{\frac{3x+1-1}{3}} = \left[\begin{array}{l} \text{Subst. } z = 3x+1 \mid x \rightarrow \infty \\ z \rightarrow \infty \end{array} \right] \\ &= \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z-1}{3}} = \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^z \right]^{\frac{z-1}{3 \cdot z}} = \left[\lim_{z \rightarrow \infty} \frac{z-1}{3z} = \lim_{z \rightarrow \infty} \left(\frac{z}{3z} - \frac{1}{3z} \right) = \lim_{z \rightarrow \infty} \left(\frac{1}{3} - \frac{1}{3z} \right) = 0 - \frac{1}{3} = -\frac{1}{3} \right] \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Subst. } z = 3x+1 \mid x \rightarrow \infty \mid 3x = z-1 \\ x = \frac{z-1}{3} \mid z \rightarrow \infty \mid 3x-2 = z-3 \end{array} \right] = \lim_{z \rightarrow \infty} \left(\frac{z-3}{z} \right)^{\frac{z-1}{3}} = \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z}{3} - \frac{1}{3}} \\ &= \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^{\frac{z}{3}} \cdot \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} \right] = \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z}{3}} \cdot \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} \\ &= \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^z \right]^{\frac{1}{3}} \cdot \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} = (e^{-3})^{\frac{1}{3}} \cdot \left(1 + \frac{-3}{\infty} \right)^{-\frac{1}{3}} \\ &= (e^{-3})^{\frac{1}{3}} \cdot (1+0)^{-\frac{1}{3}} \end{aligned}$$

Riešené limity – 11

$$\lim_{x \rightarrow \infty} \left(\frac{3x-2}{3x+1} \right)^x \quad \frac{1}{e}$$

[11]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left(\frac{3x+1-3}{3x+1} \right)^{\frac{3x}{3}} = \lim_{x \rightarrow \infty} \left(1 + \frac{-3}{3x+1} \right)^{\frac{3x+1-1}{3}} = \left[\begin{array}{l} \text{Subst. } z = 3x+1 \mid x \rightarrow \infty \\ z \rightarrow \infty \end{array} \right] \\ &= \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z-1}{3}} = \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^z \right]^{\frac{z-1}{3 \cdot z}} = \left[\begin{array}{l} \lim_{z \rightarrow \infty} \frac{z-1}{3z} = \lim_{z \rightarrow \infty} \left(\frac{z}{3z} - \frac{1}{3z} \right) = \lim_{z \rightarrow \infty} \left(\frac{1}{3} - \frac{1}{3z} \right) = 0 - \frac{1}{3} = -\frac{1}{3} \end{array} \right] \\ &= (e^{-3})^{\frac{1}{3}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Subst. } z = 3x+1 \mid x \rightarrow \infty \mid 3x = z-1 \\ x = \frac{z-1}{3} \mid z \rightarrow \infty \mid 3x-2 = z-3 \end{array} \right] = \lim_{z \rightarrow \infty} \left(\frac{z-3}{z} \right)^{\frac{z-1}{3}} = \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z}{3} - \frac{1}{3}} \\ &= \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^{\frac{z}{3}} \cdot \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} \right] = \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z}{3}} \cdot \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} \\ &= \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^z \right]^{\frac{1}{3}} \cdot \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} = (e^{-3})^{\frac{1}{3}} \cdot \left(1 + \frac{-3}{\infty} \right)^{-\frac{1}{3}} \\ &= (e^{-3})^{\frac{1}{3}} \cdot (1+0)^{-\frac{1}{3}} = e^{-1} \cdot 1^{-\frac{1}{3}} \end{aligned}$$

Riešené limity – 11

$$\lim_{x \rightarrow \infty} \left(\frac{3x-2}{3x+1} \right)^x = e^{-1} = \frac{1}{e}$$

[11]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left(\frac{3x+1-3}{3x+1} \right)^{\frac{3x}{3}} = \lim_{x \rightarrow \infty} \left(1 + \frac{-3}{3x+1} \right)^{\frac{3x+1-1}{3}} = \left[\begin{array}{l} \text{Subst. } z = 3x+1 \mid x \rightarrow \infty \\ z \rightarrow \infty \end{array} \right] \\ &= \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z-1}{3}} = \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^z \right]^{\frac{z-1}{3 \cdot z}} = \left[\lim_{z \rightarrow \infty} \frac{z-1}{3z} = \lim_{z \rightarrow \infty} \left(\frac{z}{3z} - \frac{1}{3z} \right) = \lim_{z \rightarrow \infty} \left(\frac{1}{3} - \frac{1}{3z} \right) = 0 - \frac{1}{3} = -\frac{1}{3} \right] \\ &= (e^{-3})^{\frac{1}{3}} = e^{-1} = \frac{1}{e}. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Subst. } z = 3x+1 \mid x \rightarrow \infty \mid 3x = z-1 \\ x = \frac{z-1}{3} \mid z \rightarrow \infty \mid 3x-2 = z-3 \end{array} \right] = \lim_{z \rightarrow \infty} \left(\frac{z-3}{z} \right)^{\frac{z-1}{3}} = \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z}{3} - \frac{1}{3}} \\ &= \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^{\frac{z}{3}} \cdot \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} \right] = \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{\frac{z}{3}} \cdot \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} \\ &= \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-3}{z} \right)^z \right]^{\frac{1}{3}} \cdot \lim_{z \rightarrow \infty} \left(1 + \frac{-3}{z} \right)^{-\frac{1}{3}} = (e^{-3})^{\frac{1}{3}} \cdot \left(1 + \frac{-3}{\infty} \right)^{-\frac{1}{3}} \\ &= (e^{-3})^{\frac{1}{3}} \cdot (1+0)^{-\frac{1}{3}} = e^{-1} \cdot 1^{-\frac{1}{3}} = e^{-1} = \frac{1}{e}. \end{aligned}$$

Riešené limity – 12, 13, 14

$$\lim_{x \rightarrow -\infty} \left(\frac{x+1}{x} \right)^x$$

[12]

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\cot^2 x}$$

[13]

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{tg}^2 x}$$

[14]

Riešené limity – 12, 13, 14

$$\lim_{x \rightarrow -\infty} \left(\frac{x+1}{x} \right)^x$$

[12]

$$\bullet = \lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x} \right)^x$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{cotg}^2 x}$$

[13]

$$\bullet = \lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\frac{1}{\operatorname{tg}^2 x}}$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{tg}^2 x}$$

[14]

$$\bullet = (1 + 3 \cdot 0)^0$$

Riešené limity – 12, 13, 14

$$\lim_{x \rightarrow -\infty} \left(\frac{x+1}{x} \right)^x$$

[12]

$$\bullet = \lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x} \right)^x = \left[\text{Subst. } x = -z \left| \begin{array}{l} x \rightarrow -\infty \\ z \rightarrow \infty \end{array} \right. \right]$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{cotg}^2 x}$$

[13]

$$\bullet = \lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\frac{1}{\operatorname{tg}^2 x}} = \left[\text{Subst. } z = \operatorname{tg}^2 x \left| \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right. \right]$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{tg}^2 x}$$

[14]

$$\bullet = (1 + 3 \cdot 0)^0 = 1^0$$

Riešené limity – 12, 13, 14

$$\lim_{x \rightarrow -\infty} \left(\frac{x+1}{x} \right)^x$$

[12]

$$\bullet = \lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x} \right)^x = \left[\text{Subst. } x = -z \left| \begin{array}{l} x \rightarrow -\infty \\ z \rightarrow \infty \end{array} \right. \right] = \lim_{z \rightarrow \infty} \left(1 + \frac{1}{-z} \right)^{-z}$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{cotg}^2 x}$$

[13]

$$\bullet = \lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\frac{1}{\operatorname{tg}^2 x}} = \left[\text{Subst. } z = \operatorname{tg}^2 x \left| \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right. \right] = \lim_{z \rightarrow 0} (1 + 3z)^{\frac{1}{z}}$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{tg}^2 x} = 1$$

[14]

$$\bullet = (1 + 3 \cdot 0)^0 = 1^0 = 1.$$

Riešené limity – 12, 13, 14

$$\lim_{x \rightarrow -\infty} \left(\frac{x+1}{x} \right)^x$$

[12]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x} \right)^x = \left[\text{Subst. } x = -z \left| \begin{array}{l} x \rightarrow -\infty \\ z \rightarrow \infty \end{array} \right. \right] = \lim_{z \rightarrow \infty} \left(1 + \frac{1}{-z} \right)^{-z} \\ &= \lim_{z \rightarrow \infty} \left(1 + \frac{-1}{z} \right)^{-z} \end{aligned}$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{cotg}^2 x} = e^3$$

[13]

$$\bullet = \lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\frac{1}{\operatorname{tg}^2 x}} = \left[\text{Subst. } z = \operatorname{tg}^2 x \left| \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right. \right] = \lim_{z \rightarrow 0} (1 + 3z)^{\frac{1}{z}} = e^3.$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{tg}^2 x} = 1$$

[14]

$$\bullet = (1 + 3 \cdot 0)^0 = 1^0 = 1.$$

Riešené limity – 12, 13, 14

$$\lim_{x \rightarrow -\infty} \left(\frac{x+1}{x} \right)^x$$

[12]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x} \right)^x = \left[\text{Subst. } x = -z \left| \begin{array}{l} x \rightarrow -\infty \\ z \rightarrow \infty \end{array} \right. \right] = \lim_{z \rightarrow \infty} \left(1 + \frac{1}{-z} \right)^{-z} \\ &= \lim_{z \rightarrow \infty} \left(1 + \frac{-1}{z} \right)^{-z} = \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-1}{z} \right)^z \right]^{-1} \end{aligned}$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{cotg}^2 x} = e^3$$

[13]

$$\bullet = \lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\frac{1}{\operatorname{tg}^2 x}} = \left[\text{Subst. } z = \operatorname{tg}^2 x \left| \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right. \right] = \lim_{z \rightarrow 0} (1 + 3z)^{\frac{1}{z}} = e^3.$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{tg}^2 x} = 1$$

[14]

$$\bullet = (1 + 3 \cdot 0)^0 = 1^0 = 1.$$

Riešené limity – 12, 13, 14

$$\lim_{x \rightarrow -\infty} \left(\frac{x+1}{x} \right)^x$$

[12]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x} \right)^x = \left[\text{Subst. } x = -z \left| \begin{array}{l} x \rightarrow -\infty \\ z \rightarrow \infty \end{array} \right. \right] = \lim_{z \rightarrow \infty} \left(1 + \frac{1}{-z} \right)^{-z} \\ &= \lim_{z \rightarrow \infty} \left(1 + \frac{-1}{z} \right)^{-z} = \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-1}{z} \right)^z \right]^{-1} = (e^{-1})^{-1} \end{aligned}$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{cotg}^2 x} = e^3$$

[13]

$$\bullet = \lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\frac{1}{\operatorname{tg}^2 x}} = \left[\text{Subst. } z = \operatorname{tg}^2 x \left| \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right. \right] = \lim_{z \rightarrow 0} (1 + 3z)^{\frac{1}{z}} = e^3.$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{tg}^2 x} = 1$$

[14]

$$\bullet = (1 + 3 \cdot 0)^0 = 1^0 = 1.$$

Riešené limity – 12, 13, 14

$$\lim_{x \rightarrow -\infty} \left(\frac{x+1}{x} \right)^x = e$$

[12]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x} \right)^x = \left[\text{Subst. } x = -z \left| \begin{array}{l} x \rightarrow -\infty \\ z \rightarrow \infty \end{array} \right. \right] = \lim_{z \rightarrow \infty} \left(1 + \frac{1}{-z} \right)^{-z} \\ &= \lim_{z \rightarrow \infty} \left(1 + \frac{-1}{z} \right)^{-z} = \lim_{z \rightarrow \infty} \left[\left(1 + \frac{-1}{z} \right)^z \right]^{-1} = (e^{-1})^{-1} = e. \end{aligned}$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{cotg}^2 x} = e^3$$

[13]

$$\bullet = \lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\frac{1}{\operatorname{tg}^2 x}} = \left[\text{Subst. } z = \operatorname{tg}^2 x \left| \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right. \right] = \lim_{z \rightarrow 0} (1 + 3z)^{\frac{1}{z}} = e^3.$$

$$\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg}^2 x)^{\operatorname{tg}^2 x} = 1$$

[14]

$$\bullet = (1 + 3 \cdot 0)^0 = 1^0 = 1.$$

Riešené limity – 15, 16, 17

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x^2}$$

[15]

$$\lim_{x \rightarrow \infty} x \cdot [\ln(x+2) - \ln x]$$

[16]

$$\lim_{x \rightarrow 0} \frac{x}{\ln(1+tx)} \quad \text{pre } t \in \mathbb{R} - \{0\}$$

[17]

Riešené limity – 15, 16, 17

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x^2}$$

[15]

$$\bullet = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x \cdot x}$$

$$\lim_{x \rightarrow \infty} x \cdot [\ln(x+2) - \ln x]$$

[16]

$$\bullet = \lim_{x \rightarrow \infty} \left(x \cdot \ln \frac{x+2}{x}\right)$$

$$\lim_{x \rightarrow 0} \frac{x}{\ln(1+tx)} \quad \text{pre } t \in \mathbb{R} - \{0\}$$

[17]

$$\bullet = \lim_{x \rightarrow 0} \frac{1}{\frac{1}{x} \ln(1+tx)}$$

$$\bullet =$$

Riešené limity – 15, 16, 17

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x^2}$$

[15]

$$\bullet = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x \cdot x} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{-1}{x}\right)^x\right]^x$$

$$\lim_{x \rightarrow \infty} x \cdot [\ln(x+2) - \ln x]$$

[16]

$$\bullet = \lim_{x \rightarrow \infty} \left(x \cdot \ln \frac{x+2}{x}\right) = \lim_{x \rightarrow \infty} \ln \left(\frac{x+2}{x}\right)^x$$

$$\lim_{x \rightarrow 0} \frac{x}{\ln(1+tx)}$$

pre $t \in \mathbb{R} - \{0\}$

[17]

$$\bullet = \lim_{x \rightarrow 0} \frac{1}{\frac{1}{x} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{\frac{t}{tx} \ln(1+tx)}$$

$$\bullet = \lim_{z \rightarrow 0} \frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}}$$

Riešené limity – 15, 16, 17

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x^2}$$

[15]

$$\bullet = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x \cdot x} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{-1}{x}\right)^x\right]^x = [e^{-1}]^\infty$$

$$\lim_{x \rightarrow \infty} x \cdot [\ln(x+2) - \ln x]$$

[16]

$$\bullet = \lim_{x \rightarrow \infty} \left(x \cdot \ln \frac{x+2}{x}\right) = \lim_{x \rightarrow \infty} \ln \left(\frac{x+2}{x}\right)^x = \lim_{x \rightarrow \infty} \ln \left(1 + \frac{2}{x}\right)^x$$

$$\lim_{x \rightarrow 0} \frac{x}{\ln(1+tx)}$$

pre $t \in \mathbb{R} - \{0\}$

[17]

$$\bullet = \lim_{x \rightarrow 0} \frac{1}{\frac{1}{x} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{\frac{t}{tx} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{t \cdot \ln(1+tx)^{\frac{1}{tx}}}$$

$$\bullet = \lim_{z \rightarrow 0} \frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}} = \lim_{z \rightarrow 0} \frac{1}{\frac{t}{z} \ln(1+z)^{\frac{1}{z}}}$$

Riešené limity – 15, 16, 17

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x^2}$$

[15]

$$\bullet = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x \cdot x} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{-1}{x}\right)^x\right]^x = [e^{-1}]^\infty = \frac{1}{e^\infty}$$

$$\lim_{x \rightarrow \infty} x \cdot [\ln(x+2) - \ln x]$$

[16]

$$\bullet = \lim_{x \rightarrow \infty} \left(x \cdot \ln \frac{x+2}{x}\right) = \lim_{x \rightarrow \infty} \ln \left(\frac{x+2}{x}\right)^x = \lim_{x \rightarrow \infty} \ln \left(1 + \frac{2}{x}\right)^x = \ln e^2$$

$$\lim_{x \rightarrow 0} \frac{x}{\ln(1+tx)}$$

pre $t \in \mathbb{R} - \{0\}$

[17]

$$\bullet = \lim_{x \rightarrow 0} \frac{1}{\frac{1}{x} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{\frac{t}{tx} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{t \cdot \ln(1+tx)^{\frac{1}{tx}}}$$

$$= \left[\begin{array}{l} \text{Subst. } z = tx \\ x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right]$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = tx \\ x = \frac{z}{t} \\ x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}} = \lim_{z \rightarrow 0} \frac{1}{\frac{t}{z} \ln(1+z)^{\frac{1}{z}}} = \lim_{z \rightarrow 0} \frac{1}{t \cdot \ln(1+z)^{\frac{1}{z}}}$$

Riešené limity – 15, 16, 17

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x^2}$$

[15]

$$\bullet = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x \cdot x} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{-1}{x}\right)^x\right]^x = [e^{-1}]^\infty = \frac{1}{e^\infty} = \frac{1}{\infty}$$

$$\lim_{x \rightarrow \infty} x \cdot [\ln(x+2) - \ln x] = 2$$

[16]

$$\bullet = \lim_{x \rightarrow \infty} \left(x \cdot \ln \frac{x+2}{x}\right) = \lim_{x \rightarrow \infty} \ln \left(\frac{x+2}{x}\right)^x = \lim_{x \rightarrow \infty} \ln \left(1 + \frac{2}{x}\right)^x = \ln e^2 = 2.$$

$$\lim_{x \rightarrow 0} \frac{x}{\ln(1+tx)}$$

pre $t \in \mathbb{R} - \{0\}$

[17]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \frac{1}{\frac{1}{x} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{\frac{t}{tx} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{t \cdot \ln(1+tx)^{\frac{1}{tx}}} \\ &= \left[\text{Subst. } z = tx \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{1}{t \cdot \ln(1+z)^{\frac{1}{z}}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = tx \begin{array}{l} x \rightarrow 0 \\ x = \frac{z}{t} \end{array} \right] = \lim_{z \rightarrow 0} \frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}} = \lim_{z \rightarrow 0} \frac{1}{\frac{t}{z} \ln(1+z)^{\frac{1}{z}}} = \lim_{z \rightarrow 0} \frac{1}{t \cdot \ln(1+z)^{\frac{1}{z}}} \\ &= \frac{1}{t \cdot \ln e} \end{aligned}$$

Riešené limity – 15, 16, 17

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x^2} = 0$$

[15]

$$\bullet = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x \cdot x} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{-1}{x}\right)^x\right]^x = [e^{-1}]^\infty = \frac{1}{e^\infty} = \frac{1}{\infty} = 0.$$

$$\lim_{x \rightarrow \infty} x \cdot [\ln(x+2) - \ln x] = 2$$

[16]

$$\bullet = \lim_{x \rightarrow \infty} \left(x \cdot \ln \frac{x+2}{x}\right) = \lim_{x \rightarrow \infty} \ln \left(\frac{x+2}{x}\right)^x = \lim_{x \rightarrow \infty} \ln \left(1 + \frac{2}{x}\right)^x = \ln e^2 = 2.$$

$$\lim_{x \rightarrow 0} \frac{x}{\ln(1+tx)} \quad \text{pre } t \in \mathbb{R} - \{0\}$$

[17]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \frac{1}{\frac{1}{x} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{\frac{t}{tx} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{t \cdot \ln(1+tx)^{\frac{1}{tx}}} \\ &= \left[\text{Subst. } z = tx \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{1}{t \cdot \ln(1+z)^{\frac{1}{z}}} = \frac{1}{t \cdot \ln e} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = tx \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}} = \lim_{z \rightarrow 0} \frac{1}{\frac{t}{z} \ln(1+z)^{\frac{1}{z}}} = \lim_{z \rightarrow 0} \frac{1}{t \cdot \ln(1+z)^{\frac{1}{z}}} \\ &= \frac{1}{t \cdot \ln e} = \frac{1}{t \cdot 1} \end{aligned}$$

Riešené limity – 15, 16, 17

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x^2} = 0$$

[15]

$$\bullet = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x \cdot x} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{-1}{x}\right)^x\right]^x = [e^{-1}]^\infty = \frac{1}{e^\infty} = \frac{1}{\infty} = 0.$$

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[16]

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$$\lim_{x \rightarrow 0} \frac{x}{\ln(1+tx)} = \frac{1}{t} \text{ pre } t \in \mathbb{R} - \{0\}$$

[17]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \frac{1}{\frac{1}{x} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{\frac{t}{tx} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{t \cdot \ln(1+tx)^{\frac{1}{tx}}} \\ &= \left[\text{Subst. } z = tx \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{1}{t \cdot \ln(1+z)^{\frac{1}{z}}} = \frac{1}{t \cdot \ln e} = \frac{1}{t \cdot 1} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = tx \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}} = \lim_{z \rightarrow 0} \frac{1}{\frac{t}{z} \ln(1+z)^{\frac{1}{z}}} = \lim_{z \rightarrow 0} \frac{1}{t \cdot \ln(1+z)^{\frac{1}{z}}} \\ &= \frac{1}{t \cdot \ln e} = \frac{1}{t \cdot 1} = \frac{1}{t}. \end{aligned}$$

Riešené limity – 15, 16, 17

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x^2} = 0$$

[15]

$$\bullet = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{x \cdot x} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{-1}{x}\right)^x\right]^x = [e^{-1}]^\infty = \frac{1}{e^\infty} = \frac{1}{\infty} = 0.$$

$$\lim_{x \rightarrow \infty} x \cdot [\ln(x+2) - \ln x] = 2$$

[16]

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$$\lim_{x \rightarrow 0} \frac{x}{\ln(1+tx)} = \frac{1}{t} \text{ pre } t \in \mathbb{R} - \{0\}$$

[17]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \frac{1}{\frac{1}{x} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{\frac{t}{tx} \ln(1+tx)} = \lim_{x \rightarrow 0} \frac{1}{t \cdot \ln(1+tx)^{\frac{1}{tx}}} \\ &= \left[\text{Subst. } z = tx \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{1}{t \cdot \ln(1+z)^{\frac{1}{z}}} = \frac{1}{t \cdot \ln e} = \frac{1}{t \cdot 1} = \frac{1}{t}. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = tx \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\frac{z}{t}}{\ln(1+z)^{\frac{1}{z}}} = \lim_{z \rightarrow 0} \frac{1}{\frac{t}{z} \ln(1+z)^{\frac{1}{z}}} = \lim_{z \rightarrow 0} \frac{1}{t \cdot \ln(1+z)^{\frac{1}{z}}} \\ &= \frac{1}{t \cdot \ln e} = \frac{1}{t \cdot 1} = \frac{1}{t}. \end{aligned}$$

Riešené limity – 18, 19

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{x}$$

[18]



$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$$

[19]

Riešené limity – 18, 19

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{x}$$

[18]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{1 - \sqrt{1-x}}{x} \cdot \frac{1 + \sqrt{1-x}}{1 + \sqrt{1-x}} \right)$$

$$\bullet = \left[\text{Subst. } \sqrt{1-x} = t \mid \begin{array}{l} x \rightarrow 0 \\ t \rightarrow 1 \end{array} \mid \begin{array}{l} 1-x = t^2 \\ x = 1-t^2 \end{array} \right]$$

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$$

[19]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{x}{\sqrt{1+x} - \sqrt{1-x}} \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right)$$

Riešené limity – 18, 19

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{x}$$

[18]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{1 - \sqrt{1-x}}{x} \cdot \frac{1 + \sqrt{1-x}}{1 + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{1 - (\sqrt{1-x})^2}{x(1 + \sqrt{1-x})}$$

$$\bullet = \left[\text{Subst. } \sqrt{1-x} = t \mid \begin{array}{l} x \rightarrow 0 \\ t \rightarrow 1 \end{array} \mid \begin{array}{l} 1-x = t^2 \\ x = 1-t^2 \end{array} \right] = \lim_{t \rightarrow 1} \frac{1-t}{1-t^2}$$

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$$

[19]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{x}{\sqrt{1+x} - \sqrt{1-x}} \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(\sqrt{1+x})^2 - (\sqrt{1-x})^2}$$

Riešené limity – 18, 19

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{x}$$

[18]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{1 - \sqrt{1-x}}{x} \cdot \frac{1 + \sqrt{1-x}}{1 + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{1 - (\sqrt{1-x})^2}{x(1 + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{1 - (1-x)}{x(1 + \sqrt{1-x})}$$

$$\bullet = \left[\text{Subst. } \sqrt{1-x} = t \mid \begin{array}{l} x \rightarrow 0 \\ t \rightarrow 1 \end{array} \mid \begin{array}{l} 1-x = t^2 \\ x = 1-t^2 \end{array} \right] = \lim_{t \rightarrow 1} \frac{1-t}{1-t^2} = \lim_{t \rightarrow 1} \frac{1-t}{(1-t) \cdot (1+t)}$$

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$$

[19]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{x}{\sqrt{1+x} - \sqrt{1-x}} \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(\sqrt{1+x})^2 - (\sqrt{1-x})^2} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(1+x) - (1-x)}$$

Riešené limity – 18, 19

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{x}$$

[18]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{1 - \sqrt{1-x}}{x} \cdot \frac{1 + \sqrt{1-x}}{1 + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{1 - (\sqrt{1-x})^2}{x(1 + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{1 - (1-x)}{x(1 + \sqrt{1-x})} \\ &= \lim_{x \rightarrow 0} \frac{x}{x(1 + \sqrt{1-x})} \end{aligned}$$

$$\bullet = \left[\text{Subst. } \sqrt{1-x} = t \mid \begin{array}{l} x \rightarrow 0 \\ t \rightarrow 1 \end{array} \mid \begin{array}{l} 1-x = t^2 \\ x = 1-t^2 \end{array} \right] = \lim_{t \rightarrow 1} \frac{1-t}{1-t^2} = \lim_{t \rightarrow 1} \frac{1-t}{(1-t) \cdot (1+t)} = \lim_{t \rightarrow 1} \frac{1}{1+t}$$

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$$

[19]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{x}{\sqrt{1+x} - \sqrt{1-x}} \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(\sqrt{1+x})^2 - (\sqrt{1-x})^2} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(1+x) - (1-x)} \\ &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{2x} \end{aligned}$$

Riešené limity – 18, 19

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{x}$$

[18]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{1 - \sqrt{1-x}}{x} \cdot \frac{1 + \sqrt{1-x}}{1 + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{1 - (\sqrt{1-x})^2}{x(1 + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{1 - (1-x)}{x(1 + \sqrt{1-x})} \\ &= \lim_{x \rightarrow 0} \frac{x}{x(1 + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{1}{1 + \sqrt{1-x}} \end{aligned}$$

$$\bullet = \left[\text{Subst. } \sqrt{1-x} = t \mid \begin{array}{l} x \rightarrow 0 \\ t \rightarrow 1 \end{array} \mid \begin{array}{l} 1-x = t^2 \\ x = 1-t^2 \end{array} \right] = \lim_{t \rightarrow 1} \frac{1-t}{1-t^2} = \lim_{t \rightarrow 1} \frac{1-t}{(1-t) \cdot (1+t)} = \lim_{t \rightarrow 1} \frac{1}{1+t} = \frac{1}{1+1}$$

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$$

[19]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{x}{\sqrt{1+x} - \sqrt{1-x}} \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(\sqrt{1+x})^2 - (\sqrt{1-x})^2} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(1+x) - (1-x)} \\ &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{2x} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{2} \end{aligned}$$

Riešené limity – 18, 19

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{x} = \frac{1}{2}$$

[18]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{1 - \sqrt{1-x}}{x} \cdot \frac{1 + \sqrt{1-x}}{1 + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{1 - (\sqrt{1-x})^2}{x(1 + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{1 - (1-x)}{x(1 + \sqrt{1-x})} \\ &= \lim_{x \rightarrow 0} \frac{x}{x(1 + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{1}{1 + \sqrt{1-x}} = \frac{1}{1 + \sqrt{1-0}} \end{aligned}$$

$$\bullet = \left[\text{Subst. } \sqrt{1-x} = t \mid \begin{array}{l} x \rightarrow 0 \\ t \rightarrow 1 \end{array} \mid \begin{array}{l} 1-x = t^2 \\ x = 1-t^2 \end{array} \right] = \lim_{t \rightarrow 1} \frac{1-t}{1-t^2} = \lim_{t \rightarrow 1} \frac{1-t}{(1-t) \cdot (1+t)} = \lim_{t \rightarrow 1} \frac{1}{1+t} = \frac{1}{1+1} = \frac{1}{2}.$$

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$$

[19]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{x}{\sqrt{1+x} - \sqrt{1-x}} \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(\sqrt{1+x})^2 - (\sqrt{1-x})^2} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(1+x) - (1-x)} \\ &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{2x} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{2} = \frac{\sqrt{1+0} + \sqrt{1-0}}{2} \end{aligned}$$

Riešené limity – 18, 19

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{x} = \frac{1}{2}$$

[18]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{1 - \sqrt{1-x}}{x} \cdot \frac{1 + \sqrt{1-x}}{1 + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{1 - (\sqrt{1-x})^2}{x(1 + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{1 - (1-x)}{x(1 + \sqrt{1-x})} \\ &= \lim_{x \rightarrow 0} \frac{x}{x(1 + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{1}{1 + \sqrt{1-x}} = \frac{1}{1 + \sqrt{1-0}} = \frac{1}{1+1} \end{aligned}$$

$$\bullet = \left[\text{Subst. } \sqrt{1-x} = t \mid \begin{array}{l} x \rightarrow 0 \\ t \rightarrow 1 \end{array} \mid \begin{array}{l} 1-x = t^2 \\ x = 1-t^2 \end{array} \right] = \lim_{t \rightarrow 1} \frac{1-t}{1-t^2} = \lim_{t \rightarrow 1} \frac{1-t}{(1-t) \cdot (1+t)} = \lim_{t \rightarrow 1} \frac{1}{1+t} = \frac{1}{1+1} = \frac{1}{2}.$$

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$$

[19]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{x}{\sqrt{1+x} - \sqrt{1-x}} \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(\sqrt{1+x})^2 - (\sqrt{1-x})^2} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(1+x) - (1-x)} \\ &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{2x} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{2} = \frac{\sqrt{1+0} + \sqrt{1-0}}{2} = \frac{1+1}{2} \end{aligned}$$

Riešené limity – 18, 19

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{x} = \frac{1}{2}$$

[18]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{1 - \sqrt{1-x}}{x} \cdot \frac{1 + \sqrt{1-x}}{1 + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{1 - (\sqrt{1-x})^2}{x(1 + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{1 - (1-x)}{x(1 + \sqrt{1-x})} \\ &= \lim_{x \rightarrow 0} \frac{x}{x(1 + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{1}{1 + \sqrt{1-x}} = \frac{1}{1 + \sqrt{1-0}} = \frac{1}{1+1} = \frac{1}{2}. \end{aligned}$$

$$\bullet = \left[\text{Subst. } \sqrt{1-x} = t \mid \begin{array}{l} x \rightarrow 0 \\ t \rightarrow 1 \end{array} \mid \begin{array}{l} 1-x = t^2 \\ x = 1-t^2 \end{array} \right] = \lim_{t \rightarrow 1} \frac{1-t}{1-t^2} = \lim_{t \rightarrow 1} \frac{1-t}{(1-t) \cdot (1+t)} = \lim_{t \rightarrow 1} \frac{1}{1+t} = \frac{1}{1+1} = \frac{1}{2}.$$

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}} = 1$$

[19]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{x}{\sqrt{1+x} - \sqrt{1-x}} \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(\sqrt{1+x})^2 - (\sqrt{1-x})^2} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(1+x) - (1-x)} \\ &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{2x} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{2} = \frac{\sqrt{1+0} + \sqrt{1-0}}{2} = \frac{1+1}{2} = 1. \end{aligned}$$

Riešené limity – 20, 21

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{x}$$

[20]

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2-1})$$

[21]

Riešené limity – 20, 21

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{x}$$

[20]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right)$$

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2-1})$$

[21]

$$\bullet = \lim_{x \rightarrow \infty} \left((x - \sqrt{x^2-1}) \cdot \frac{x + \sqrt{x^2-1}}{x + \sqrt{x^2-1}} \right)$$

Riešené limity – 20, 21

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{x}$$

[20]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{array}{l} x \rightarrow \infty \\ x > 0 \end{array} \middle| \sqrt{x^2} = |x| = x \right]$$

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2-1})$$

[21]

$$\bullet = \lim_{x \rightarrow \infty} \left((x - \sqrt{x^2-1}) \cdot \frac{x + \sqrt{x^2-1}}{x + \sqrt{x^2-1}} \right) = \lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2-1})(x + \sqrt{x^2-1})}{x + \sqrt{x^2-1}}$$

Riešené limity – 20, 21

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{x}$$

[20]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{array}{l} x \rightarrow \infty \\ x > 0 \end{array} \middle| \sqrt{x^2} = |x| = x \right] = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right)$$

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2-1})$$

[21]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left((x - \sqrt{x^2-1}) \cdot \frac{x + \sqrt{x^2-1}}{x + \sqrt{x^2-1}} \right) = \lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2-1})(x + \sqrt{x^2-1})}{x + \sqrt{x^2-1}} \\ &= \lim_{x \rightarrow \infty} \frac{x^2 - (\sqrt{x^2-1})^2}{x + \sqrt{x^2-1}} \end{aligned}$$

Riešené limity – 20, 21

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{x}$$

[20]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{array}{l} x \rightarrow \infty \\ x > 0 \end{array} \middle| \sqrt{x^2} = |x| = x \right] = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right) \\ &= \lim_{x \rightarrow \infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right) \end{aligned}$$

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2-1})$$

[21]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left((x - \sqrt{x^2-1}) \cdot \frac{x + \sqrt{x^2-1}}{x + \sqrt{x^2-1}} \right) = \lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2-1})(x + \sqrt{x^2-1})}{x + \sqrt{x^2-1}} \\ &= \lim_{x \rightarrow \infty} \frac{x^2 - (\sqrt{x^2-1})^2}{x + \sqrt{x^2-1}} = \lim_{x \rightarrow \infty} \frac{x^2 - (x^2 - 1)}{x + \sqrt{x^2-1}} \end{aligned}$$

Riešené limity – 20, 21

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{x}$$

[20]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{array}{l} x \rightarrow \infty \\ x > 0 \end{array} \middle| \sqrt{x^2} = |x| = x \right] = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right) \\ &= \lim_{x \rightarrow \infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right) = \lim_{x \rightarrow \infty} \left(\sqrt{1 - \frac{1}{x^2}} + \sqrt{1 + \frac{1}{x^2}} \right) \end{aligned}$$

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2-1})$$

[21]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left((x - \sqrt{x^2-1}) \cdot \frac{x + \sqrt{x^2-1}}{x + \sqrt{x^2-1}} \right) = \lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2-1})(x + \sqrt{x^2-1})}{x + \sqrt{x^2-1}} \\ &= \lim_{x \rightarrow \infty} \frac{x^2 - (\sqrt{x^2-1})^2}{x + \sqrt{x^2-1}} = \lim_{x \rightarrow \infty} \frac{x^2 - (x^2 - 1)}{x + \sqrt{x^2-1}} = \lim_{x \rightarrow \infty} \frac{1}{x + \sqrt{x^2-1}} \end{aligned}$$

Riešené limity – 20, 21

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{x}$$

[20]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{array}{l} x \rightarrow \infty \\ x > 0 \end{array} \middle| \sqrt{x^2} = |x| = x \right] = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right) \\ &= \lim_{x \rightarrow \infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right) = \lim_{x \rightarrow \infty} \left(\sqrt{1 - \frac{1}{x^2}} + \sqrt{1 + \frac{1}{x^2}} \right) \\ &= \sqrt{1 - \frac{1}{\infty}} + \sqrt{1 + \frac{1}{\infty}} \end{aligned}$$

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2-1})$$

[21]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left((x - \sqrt{x^2-1}) \cdot \frac{x + \sqrt{x^2-1}}{x + \sqrt{x^2-1}} \right) = \lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2-1})(x + \sqrt{x^2-1})}{x + \sqrt{x^2-1}} \\ &= \lim_{x \rightarrow \infty} \frac{x^2 - (\sqrt{x^2-1})^2}{x + \sqrt{x^2-1}} = \lim_{x \rightarrow \infty} \frac{x^2 - (x^2 - 1)}{x + \sqrt{x^2-1}} = \lim_{x \rightarrow \infty} \frac{1}{x + \sqrt{x^2-1}} \\ &= \frac{1}{\infty + \sqrt{\infty^2 - 1}} \end{aligned}$$

Riešené limity – 20, 21

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{x}$$

[20]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{array}{l} x \rightarrow \infty \\ x > 0 \end{array} \middle| \sqrt{x^2} = |x| = x \right] = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right) \\ &= \lim_{x \rightarrow \infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right) = \lim_{x \rightarrow \infty} \left(\sqrt{1 - \frac{1}{x^2}} + \sqrt{1 + \frac{1}{x^2}} \right) \\ &= \sqrt{1 - \frac{1}{\infty}} + \sqrt{1 + \frac{1}{\infty}} = \sqrt{1 - 0} + \sqrt{1 + 0} \end{aligned}$$

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2-1})$$

[21]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left((x - \sqrt{x^2-1}) \cdot \frac{x + \sqrt{x^2-1}}{x + \sqrt{x^2-1}} \right) = \lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2-1})(x + \sqrt{x^2-1})}{x + \sqrt{x^2-1}} \\ &= \lim_{x \rightarrow \infty} \frac{x^2 - (\sqrt{x^2-1})^2}{x + \sqrt{x^2-1}} = \lim_{x \rightarrow \infty} \frac{x^2 - (x^2 - 1)}{x + \sqrt{x^2-1}} = \lim_{x \rightarrow \infty} \frac{1}{x + \sqrt{x^2-1}} \\ &= \frac{1}{\infty + \sqrt{\infty^2 - 1}} = \frac{1}{\infty + \infty} \end{aligned}$$

Riešené limity – 20, 21

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{x} = 2$$

[20]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{array}{l} x \rightarrow \infty \\ x > 0 \end{array} \middle| \sqrt{x^2} = |x| = x \right] = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right) \\ &= \lim_{x \rightarrow \infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right) = \lim_{x \rightarrow \infty} \left(\sqrt{1 - \frac{1}{x^2}} + \sqrt{1 + \frac{1}{x^2}} \right) \\ &= \sqrt{1 - \frac{1}{\infty}} + \sqrt{1 + \frac{1}{\infty}} = \sqrt{1 - 0} + \sqrt{1 + 0} = 2. \end{aligned}$$

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2-1})$$

[21]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left((x - \sqrt{x^2-1}) \cdot \frac{x + \sqrt{x^2-1}}{x + \sqrt{x^2-1}} \right) = \lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2-1})(x + \sqrt{x^2-1})}{x + \sqrt{x^2-1}} \\ &= \lim_{x \rightarrow \infty} \frac{x^2 - (\sqrt{x^2-1})^2}{x + \sqrt{x^2-1}} = \lim_{x \rightarrow \infty} \frac{x^2 - (x^2 - 1)}{x + \sqrt{x^2-1}} = \lim_{x \rightarrow \infty} \frac{1}{x + \sqrt{x^2-1}} \\ &= \frac{1}{\infty + \sqrt{\infty^2 - 1}} = \frac{1}{\infty + \infty} = \frac{1}{\infty} \end{aligned}$$

Riešené limity – 20, 21

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1} + \sqrt{x^2+1}}{x} = 2$$

[20]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{array}{l} x \rightarrow \infty \\ x > 0 \end{array} \middle| \sqrt{x^2} = |x| = x \right] = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right) \\ &= \lim_{x \rightarrow \infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right) = \lim_{x \rightarrow \infty} \left(\sqrt{1 - \frac{1}{x^2}} + \sqrt{1 + \frac{1}{x^2}} \right) \\ &= \sqrt{1 - \frac{1}{\infty}} + \sqrt{1 + \frac{1}{\infty}} = \sqrt{1 - 0} + \sqrt{1 + 0} = 2. \end{aligned}$$

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2-1}) = 0$$

[21]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \infty} \left((x - \sqrt{x^2-1}) \cdot \frac{x + \sqrt{x^2-1}}{x + \sqrt{x^2-1}} \right) = \lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2-1})(x + \sqrt{x^2-1})}{x + \sqrt{x^2-1}} \\ &= \lim_{x \rightarrow \infty} \frac{x^2 - (\sqrt{x^2-1})^2}{x + \sqrt{x^2-1}} = \lim_{x \rightarrow \infty} \frac{x^2 - (x^2 - 1)}{x + \sqrt{x^2-1}} = \lim_{x \rightarrow \infty} \frac{1}{x + \sqrt{x^2-1}} \\ &= \frac{1}{\infty + \sqrt{\infty^2 - 1}} = \frac{1}{\infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené limity – 22

$$\lim_{x \rightarrow a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a}$$

pre $a > 0$

[22]

Riešené limity – 22

$$\lim_{x \rightarrow a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a}$$

pre $a > 0$

[22]

$$\bullet = \lim_{x \rightarrow a} \left(\frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} \cdot \frac{\sqrt{ax} + a}{\sqrt{ax} + a} \right)$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = \sqrt{ax} \mid x \rightarrow a \mid z^2 = ax \\ z \rightarrow a \mid x^2 = \frac{z^4}{a^2} \end{array} \right]$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = \sqrt{x} \mid x \rightarrow a \mid x = z^2 \\ \text{Označme } \sqrt{a} = b \mid z \rightarrow \sqrt{a} = b \mid a = b^2 \end{array} \right]$$

Riešené limity – 22

$$\lim_{x \rightarrow a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a}$$

pre $a > 0$

[22]

$$\bullet = \lim_{x \rightarrow a} \left(\frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} \cdot \frac{\sqrt{ax} + a}{\sqrt{ax} + a} \right) = \lim_{x \rightarrow a} \frac{x^2\sqrt{ax} - a \cdot (\sqrt{ax})^2 + ax^2 - aa\sqrt{ax}}{(\sqrt{ax})^2 - a^2}$$

$$\bullet = \left[\text{Subst. } z = \sqrt{ax} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow a \end{array} \right. \begin{array}{l} z^2 = ax \\ x^2 = \frac{z^4}{a^2} \end{array} \right] = \lim_{z \rightarrow a} \frac{\frac{z^4}{a^2} - a \cdot z}{z - a}$$



$$\bullet = \left[\text{Subst. } z = \sqrt{ax} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow \sqrt{a} = b \end{array} \right. \begin{array}{l} x = z^2 \\ a = b^2 \end{array} \right] = \lim_{z \rightarrow b} \frac{z^4 - b^2 \cdot bz}{bz - b^2}$$



Riešené limity – 22

$$\lim_{x \rightarrow a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a}$$

pre $a > 0$

[22]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \left(\frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} \cdot \frac{\sqrt{ax} + a}{\sqrt{ax} + a} \right) = \lim_{x \rightarrow a} \frac{x^2\sqrt{ax} - a \cdot (\sqrt{ax})^2 + ax^2 - aa\sqrt{ax}}{(\sqrt{ax})^2 - a^2} \\ &= \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x^2 - a^2) + ax^2 - a^2x}{ax - a^2} \end{aligned}$$

$$\bullet = \left[\text{Subst. } z = \sqrt{ax} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow a \end{array} \right. \begin{array}{l} z^2 = ax \\ x^2 = \frac{z^4}{a^2} \end{array} \right] = \lim_{z \rightarrow a} \frac{\frac{z^4}{a^2} - a \cdot z}{z - a} = \lim_{z \rightarrow a} \frac{\frac{z^4 - a^3z}{a^2}}{z - a}$$



$$\bullet = \left[\text{Subst. } z = \sqrt{x} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow \sqrt{a} = b \end{array} \right. \begin{array}{l} x = z^2 \\ a = b^2 \end{array} \right] = \lim_{z \rightarrow b} \frac{z^4 - b^2 \cdot bz}{bz - b^2} = \lim_{z \rightarrow b} \frac{z^4 - b^3z}{bz - b^2}$$



Riešené limity – 22

$$\lim_{x \rightarrow a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a}$$

pre $a > 0$

[22]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \left(\frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} \cdot \frac{\sqrt{ax} + a}{\sqrt{ax} + a} \right) = \lim_{x \rightarrow a} \frac{x^2\sqrt{ax} - a \cdot (\sqrt{ax})^2 + ax^2 - aa\sqrt{ax}}{(\sqrt{ax})^2 - a^2} \\ &= \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x^2 - a^2) + ax^2 - a^2x}{ax - a^2} = \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x-a) \cdot (x+a) + ax \cdot (x-a)}{a \cdot (x-a)} \end{aligned}$$

$$\bullet = \left[\text{Subst. } z = \sqrt{ax} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow a \end{array} \right. \begin{array}{l} z^2 = ax \\ x^2 = \frac{z^4}{a^2} \end{array} \right] = \lim_{z \rightarrow a} \frac{\frac{z^4}{a^2} - a \cdot z}{z - a} = \lim_{z \rightarrow a} \frac{\frac{z^4 - a^3z}{a^2}}{z - a} = \lim_{z \rightarrow a} \frac{z(z^3 - a^3)}{a^2(z - a)}$$

$$\bullet = \left[\text{Subst. } z = \sqrt{x} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow \sqrt{a} = b \end{array} \right. \begin{array}{l} x = z^2 \\ a = b^2 \end{array} \right] = \lim_{z \rightarrow b} \frac{\frac{z^4 - b^2 \cdot bz}{bz - b^2}}{bz - b^2} = \lim_{z \rightarrow b} \frac{z^4 - b^3z}{bz - b^2} = \lim_{z \rightarrow b} \frac{z(z^3 - b^3)}{b(z - b)}$$

Riešené limity – 22

$$\lim_{x \rightarrow a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a}$$

pre $a > 0$

[22]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \left(\frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} \cdot \frac{\sqrt{ax} + a}{\sqrt{ax} + a} \right) = \lim_{x \rightarrow a} \frac{x^2\sqrt{ax} - a \cdot (\sqrt{ax})^2 + ax^2 - aa\sqrt{ax}}{(\sqrt{ax})^2 - a^2} \\ &= \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x^2 - a^2) + ax^2 - a^2x}{ax - a^2} = \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x - a) \cdot (x + a) + ax \cdot (x - a)}{a \cdot (x - a)} = \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x + a) + ax}{a} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = \sqrt{ax} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow a \end{array} \right. \begin{array}{l} z^2 = ax \\ x^2 = \frac{z^4}{a^2} \end{array} \right] = \lim_{z \rightarrow a} \frac{\frac{z^4}{a^2} - a \cdot z}{z - a} = \lim_{z \rightarrow a} \frac{\frac{z^4 - a^3z}{a^2}}{z - a} = \lim_{z \rightarrow a} \frac{z(z^3 - a^3)}{a^2(z - a)} \\ &= \lim_{z \rightarrow a} \frac{z(z - a)(z^2 + za + a^2)}{a^2(z - a)} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = \sqrt{x} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow \sqrt{a} = b \end{array} \right. \begin{array}{l} x = z^2 \\ a = b^2 \end{array} \right] = \lim_{z \rightarrow b} \frac{\frac{z^4 - b^2 \cdot bz}{bz - b^2}}{z - b} = \lim_{z \rightarrow b} \frac{\frac{z^4 - b^3z}{bz - b^2}}{z - b} = \lim_{z \rightarrow b} \frac{z(z^3 - b^3)}{b(z - b)} \\ &= \lim_{z \rightarrow b} \frac{z(z - b)(z^2 + zb + b^2)}{b(z - b)} \end{aligned}$$

Riešené limity – 22

$$\lim_{x \rightarrow a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a}$$

pre $a > 0$

[22]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \left(\frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} \cdot \frac{\sqrt{ax} + a}{\sqrt{ax} + a} \right) = \lim_{x \rightarrow a} \frac{x^2\sqrt{ax} - a \cdot (\sqrt{ax})^2 + ax^2 - aa\sqrt{ax}}{(\sqrt{ax})^2 - a^2} \\ &= \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x^2 - a^2) + ax^2 - a^2x}{ax - a^2} = \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x-a) \cdot (x+a) + ax \cdot (x-a)}{a \cdot (x-a)} = \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x+a) + ax}{a} \\ &= \frac{\sqrt{a \cdot a} \cdot (a+a) + a \cdot a}{a} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = \sqrt{ax} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow a \end{array} \right. \begin{array}{l} z^2 = ax \\ x^2 = \frac{z^4}{a^2} \end{array} \right] = \lim_{z \rightarrow a} \frac{\frac{z^4}{a^2} - a \cdot z}{z - a} = \lim_{z \rightarrow a} \frac{\frac{z^4 - a^3z}{a^2}}{z - a} = \lim_{z \rightarrow a} \frac{z(z^3 - a^3)}{a^2(z - a)} \\ &= \lim_{z \rightarrow a} \frac{z(z-a)(z^2 + za + a^2)}{a^2(z-a)} = \lim_{z \rightarrow a} \frac{z(z^2 + za + a^2)}{a^2} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = \sqrt{x} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow \sqrt{a} = b \end{array} \right. \begin{array}{l} x = z^2 \\ a = b^2 \end{array} \right] = \lim_{z \rightarrow b} \frac{\frac{z^4}{b^2} - b^2 \cdot bz}{bz - b^2} = \lim_{z \rightarrow b} \frac{\frac{z^4 - b^3z}{b^2}}{bz - b^2} = \lim_{z \rightarrow b} \frac{z(z^3 - b^3)}{b(z-b)} \\ &= \lim_{z \rightarrow b} \frac{z(z-b)(z^2 + zb + b^2)}{b(z-b)} = \lim_{z \rightarrow b} \frac{z(z^2 + zb + b^2)}{b} \end{aligned}$$

Riešené limity – 22

$$\lim_{x \rightarrow a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a}$$

pre $a > 0$

[22]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \left(\frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} \cdot \frac{\sqrt{ax} + a}{\sqrt{ax} + a} \right) = \lim_{x \rightarrow a} \frac{x^2\sqrt{ax} - a \cdot (\sqrt{ax})^2 + ax^2 - aa\sqrt{ax}}{(\sqrt{ax})^2 - a^2} \\ &= \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x^2 - a^2) + ax^2 - a^2x}{ax - a^2} = \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x-a) \cdot (x+a) + ax \cdot (x-a)}{a \cdot (x-a)} = \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x+a) + ax}{a} \\ &= \frac{\sqrt{a \cdot a} \cdot (a+a) + a \cdot a}{a} = \frac{a \cdot 2a + a^2}{a} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = \sqrt{ax} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow a \end{array} \right. \begin{array}{l} z^2 = ax \\ x^2 = \frac{z^4}{a^2} \end{array} \right] = \lim_{z \rightarrow a} \frac{\frac{z^4}{a^2} - a \cdot z}{z - a} = \lim_{z \rightarrow a} \frac{\frac{z^4 - a^3z}{a^2}}{z - a} = \lim_{z \rightarrow a} \frac{z(z^3 - a^3)}{a^2(z - a)} \\ &= \lim_{z \rightarrow a} \frac{z(z-a)(z^2 + za + a^2)}{a^2(z-a)} = \lim_{z \rightarrow a} \frac{z(z^2 + za + a^2)}{a^2} = \frac{a(a^2 + a \cdot a + a^2)}{a^2} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = \sqrt{x} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow \sqrt{a} = b \end{array} \right. \begin{array}{l} x = z^2 \\ a = b^2 \end{array} \right] = \lim_{z \rightarrow b} \frac{z^4 - b^2 \cdot bz}{bz - b^2} = \lim_{z \rightarrow b} \frac{z^4 - b^3z}{bz - b^2} = \lim_{z \rightarrow b} \frac{z(z^3 - b^3)}{b(z-b)} \\ &= \lim_{z \rightarrow b} \frac{z(z-b)(z^2 + zb + b^2)}{b(z-b)} = \lim_{z \rightarrow b} \frac{z(z^2 + zb + b^2)}{b} = \frac{b(b^2 + b \cdot b + b^2)}{b} \end{aligned}$$

Riešené limity – 22

$$\lim_{x \rightarrow a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a}$$

pre $a > 0$

[22]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \left(\frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} \cdot \frac{\sqrt{ax} + a}{\sqrt{ax} + a} \right) = \lim_{x \rightarrow a} \frac{x^2\sqrt{ax} - a \cdot (\sqrt{ax})^2 + ax^2 - aa\sqrt{ax}}{(\sqrt{ax})^2 - a^2} \\ &= \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x^2 - a^2) + ax^2 - a^2x}{ax - a^2} = \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x-a) \cdot (x+a) + ax \cdot (x-a)}{a \cdot (x-a)} = \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x+a) + ax}{a} \\ &= \frac{\sqrt{a \cdot a} \cdot (a+a) + a \cdot a}{a} = \frac{a \cdot 2a + a^2}{a} = \frac{3a^2}{a} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = \sqrt{ax} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow a \end{array} \right. \begin{array}{l} z^2 = ax \\ x^2 = \frac{z^4}{a^2} \end{array} \right] = \lim_{z \rightarrow a} \frac{\frac{z^4}{a^2} - a \cdot z}{z - a} = \lim_{z \rightarrow a} \frac{\frac{z^4 - a^3z}{a^2}}{z - a} = \lim_{z \rightarrow a} \frac{z(z^3 - a^3)}{a^2(z - a)} \\ &= \lim_{z \rightarrow a} \frac{z(z-a)(z^2 + za + a^2)}{a^2(z-a)} = \lim_{z \rightarrow a} \frac{z(z^2 + za + a^2)}{a^2} = \frac{a(a^2 + a \cdot a + a^2)}{a^2} = \frac{a \cdot 3a^2}{a^2} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = \sqrt{x} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow \sqrt{a} = b \end{array} \right. \begin{array}{l} x = z^2 \\ a = b^2 \end{array} \right] = \lim_{z \rightarrow b} \frac{\frac{z^4}{b^2} - b^2 \cdot bz}{bz - b^2} = \lim_{z \rightarrow b} \frac{\frac{z^4 - b^3z}{b^2}}{bz - b^2} = \lim_{z \rightarrow b} \frac{z(z^3 - b^3)}{b(z-b)} \\ &= \lim_{z \rightarrow b} \frac{z(z-b)(z^2 + zb + b^2)}{b(z-b)} = \lim_{z \rightarrow b} \frac{z(z^2 + zb + b^2)}{b} = \frac{b(b^2 + b \cdot b + b^2)}{b} = \frac{b \cdot 3b^2}{b} = 3b^2 \end{aligned}$$

Riešené limity – 22

$$\lim_{x \rightarrow a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} = 3a \text{ pre } a > 0$$

[22]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \left(\frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} \cdot \frac{\sqrt{ax} + a}{\sqrt{ax} + a} \right) = \lim_{x \rightarrow a} \frac{x^2\sqrt{ax} - a \cdot (\sqrt{ax})^2 + ax^2 - aa\sqrt{ax}}{(\sqrt{ax})^2 - a^2} \\ &= \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x^2 - a^2) + ax^2 - a^2x}{ax - a^2} = \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x-a) \cdot (x+a) + ax \cdot (x-a)}{a \cdot (x-a)} = \lim_{x \rightarrow a} \frac{\sqrt{ax} \cdot (x+a) + ax}{a} \\ &= \frac{\sqrt{a \cdot a} \cdot (a+a) + a \cdot a}{a} = \frac{a \cdot 2a + a^2}{a} = \frac{3a^2}{a} = 3a. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = \sqrt{ax} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow a \end{array} \right. \begin{array}{l} z^2 = ax \\ x^2 = \frac{z^4}{a^2} \end{array} \right] = \lim_{z \rightarrow a} \frac{\frac{z^4}{a^2} - a \cdot z}{z - a} = \lim_{z \rightarrow a} \frac{\frac{z^4 - a^3z}{a^2}}{z - a} = \lim_{z \rightarrow a} \frac{z(z^3 - a^3)}{a^2(z - a)} \\ &= \lim_{z \rightarrow a} \frac{z(z-a)(z^2 + za + a^2)}{a^2(z-a)} = \lim_{z \rightarrow a} \frac{z(z^2 + za + a^2)}{a^2} = \frac{a(a^2 + a \cdot a + a^2)}{a^2} = \frac{a \cdot 3a^2}{a^2} = 3a. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } z = \sqrt{x} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow \sqrt{a} = b \end{array} \right. \begin{array}{l} x = z^2 \\ a = b^2 \end{array} \right] = \lim_{z \rightarrow b} \frac{\frac{z^4}{b^2} - b^2 \cdot bz}{bz - b^2} = \lim_{z \rightarrow b} \frac{\frac{z^4 - b^3z}{b^2}}{bz - b^2} = \lim_{z \rightarrow b} \frac{z(z^3 - b^3)}{b(z-b)} \\ &= \lim_{z \rightarrow b} \frac{z(z-b)(z^2 + zb + b^2)}{b(z-b)} = \lim_{z \rightarrow b} \frac{z(z^2 + zb + b^2)}{b} = \frac{b(b^2 + b \cdot b + b^2)}{b} = \frac{b \cdot 3b^2}{b} = 3b^2 = 3a. \end{aligned}$$

Riešené limity – 23, 24, 25

$$\lim_{x \rightarrow a} \frac{a-x}{\sqrt{a}-\sqrt{x}}$$

pre $a \geq 0$

[23]



$$\lim_{x \rightarrow \infty} \frac{\sqrt{x}-6x}{3x+1}$$

[24]

$$\lim_{x \rightarrow 1} \frac{x^m-1}{x^n-1}$$

pre $m, n \in \mathbb{N}$

[25]



Riešené limity – 23, 24, 25

$$\lim_{x \rightarrow a} \frac{a-x}{\sqrt{a}-\sqrt{x}} \quad \text{pre } a \geq 0$$

[23]

$$\bullet = \lim_{x \rightarrow a} \frac{(\sqrt{a}-\sqrt{x}) \cdot (\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}}$$

$$\bullet = \lim_{x \rightarrow a} \left(\frac{a-x}{\sqrt{a}-\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}} \right)$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x}-6x}{3x+1}$$

[24]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right)$$

$$\lim_{x \rightarrow 1} \frac{x^m-1}{x^n-1} \quad \text{pre } m, n \in \mathbb{N}$$

[25]

$$\bullet = \lim_{x \rightarrow 1} \frac{(x-1) \cdot (x^{m-1} + x^{m-2} + \dots + x + 1)}{(x-1) \cdot (x^{n-1} + x^{n-2} + \dots + x + 1)}$$

Riešené limity – 23, 24, 25

$$\lim_{x \rightarrow a} \frac{a-x}{\sqrt{a}-\sqrt{x}}$$

pre $a \geq 0$

[23]

$$\bullet = \lim_{x \rightarrow a} \frac{(\sqrt{a}-\sqrt{x}) \cdot (\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} \stackrel{!}{=} \lim_{x \rightarrow a} (\sqrt{a} + \sqrt{x})$$

$$\bullet = \lim_{x \rightarrow a} \left(\frac{a-x}{\sqrt{a}-\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}} \right) = \lim_{x \rightarrow a} \frac{(a-x) \cdot (\sqrt{a}+\sqrt{x})}{a-x} \stackrel{!}{=}$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x}-6x}{3x+1}$$

[24]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = \lim_{x \rightarrow \infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}}$$

$$\lim_{x \rightarrow 1} \frac{x^m-1}{x^n-1}$$

pre $m, n \in \mathbb{N}$

[25]

$$\bullet = \lim_{x \rightarrow 1} \frac{(x-1) \cdot (x^{m-1} + x^{m-2} + \dots + x + 1)}{(x-1) \cdot (x^{n-1} + x^{n-2} + \dots + x + 1)} \stackrel{!}{=} \lim_{x \rightarrow 1} \frac{x^{m-1} + x^{m-2} + \dots + x + 1}{x^{n-1} + x^{n-2} + \dots + x + 1}$$

Riešené limity – 23, 24, 25

$$\lim_{x \rightarrow a} \frac{a-x}{\sqrt{a}-\sqrt{x}}$$

pre $a \geq 0$

[23]

$$\bullet = \lim_{x \rightarrow a} \frac{(\sqrt{a}-\sqrt{x}) \cdot (\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x \rightarrow a} (\sqrt{a} + \sqrt{x}) = \sqrt{a} + \sqrt{a}$$

$$\bullet = \lim_{x \rightarrow a} \left(\frac{a-x}{\sqrt{a}-\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}} \right) = \lim_{x \rightarrow a} \frac{(a-x) \cdot (\sqrt{a}+\sqrt{x})}{a-x} = \lim_{x \rightarrow a} (\sqrt{a} + \sqrt{x})$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x}-6x}{3x+1}$$

[24]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = \lim_{x \rightarrow \infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}}$$

$$\lim_{x \rightarrow 1} \frac{x^m-1}{x^n-1}$$

pre $m, n \in \mathbb{N}$

[25]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \frac{(x-1) \cdot (x^{m-1} + x^{m-2} + \dots + x + 1)}{(x-1) \cdot (x^{n-1} + x^{n-2} + \dots + x + 1)} = \lim_{x \rightarrow 1} \frac{x^{m-1} + x^{m-2} + \dots + x + 1}{x^{n-1} + x^{n-2} + \dots + x + 1} \\ &= \frac{1^{m-1} + 1^{m-2} + \dots + 1 + 1}{1^{n-1} + 1^{n-2} + \dots + 1 + 1} \end{aligned}$$

Riešené limity – 23, 24, 25

$$\lim_{x \rightarrow a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a} \text{ pre } a \geq 0$$

[23]

$$\bullet = \lim_{x \rightarrow a} \frac{(\sqrt{a}-\sqrt{x}) \cdot (\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x \rightarrow a} (\sqrt{a} + \sqrt{x}) = \sqrt{a} + \sqrt{a} = 2\sqrt{a}.$$

$$\bullet = \lim_{x \rightarrow a} \left(\frac{a-x}{\sqrt{a}-\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}} \right) = \lim_{x \rightarrow a} \frac{(a-x) \cdot (\sqrt{a}+\sqrt{x})}{a-x} = \lim_{x \rightarrow a} (\sqrt{a} + \sqrt{x}) = \sqrt{a} + \sqrt{a} = 2\sqrt{a}.$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x}-6x}{3x+1}$$

[24]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = \lim_{x \rightarrow \infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}} = \frac{0-6}{3+0}$$

$$\lim_{x \rightarrow 1} \frac{x^m-1}{x^n-1} \text{ pre } m, n \in \mathbb{N}$$

[25]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \frac{(x-1) \cdot (x^{m-1} + x^{m-2} + \dots + x + 1)}{(x-1) \cdot (x^{n-1} + x^{n-2} + \dots + x + 1)} = \lim_{x \rightarrow 1} \frac{x^{m-1} + x^{m-2} + \dots + x + 1}{x^{n-1} + x^{n-2} + \dots + x + 1} \\ &= \frac{1^{m-1} + 1^{m-2} + \dots + 1 + 1}{1^{n-1} + 1^{n-2} + \dots + 1 + 1} = \frac{1^{m-1} + 1^{m-2} + \dots + 1^1 + 1^0}{1^{n-1} + 1^{n-2} + \dots + 1^1 + 1^0} \end{aligned}$$

Riešené limity – 23, 24, 25

$$\lim_{x \rightarrow a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a} \text{ pre } a \geq 0$$

[23]

$$\bullet = \lim_{x \rightarrow a} \frac{(\sqrt{a}-\sqrt{x}) \cdot (\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x \rightarrow a} (\sqrt{a} + \sqrt{x}) = \sqrt{a} + \sqrt{a} = 2\sqrt{a}.$$

$$\bullet = \lim_{x \rightarrow a} \left(\frac{a-x}{\sqrt{a}-\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}} \right) = \lim_{x \rightarrow a} \frac{(a-x) \cdot (\sqrt{a}+\sqrt{x})}{a-x} = \lim_{x \rightarrow a} (\sqrt{a} + \sqrt{x}) = \sqrt{a} + \sqrt{a} = 2\sqrt{a}.$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x}-6x}{3x+1} = -2$$

[24]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = \lim_{x \rightarrow \infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}} = \frac{0-6}{3+0} = -2.$$

$$\lim_{x \rightarrow 1} \frac{x^m-1}{x^n-1} \text{ pre } m, n \in \mathbb{N}$$

[25]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \frac{(x-1) \cdot (x^{m-1} + x^{m-2} + \dots + x + 1)}{(x-1) \cdot (x^{n-1} + x^{n-2} + \dots + x + 1)} = \lim_{x \rightarrow 1} \frac{x^{m-1} + x^{m-2} + \dots + x + 1}{x^{n-1} + x^{n-2} + \dots + x + 1} \\ &= \frac{1^{m-1} + 1^{m-2} + \dots + 1 + 1}{1^{n-1} + 1^{n-2} + \dots + 1 + 1} = \frac{1^{m-1} + 1^{m-2} + \dots + 1^1 + 1^0}{1^{n-1} + 1^{n-2} + \dots + 1^1 + 1^0} = \frac{m \cdot 1}{n \cdot 1} \end{aligned}$$

Riešené limity – 23, 24, 25

$$\lim_{x \rightarrow a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a} \text{ pre } a \geq 0$$

[23]

$$\bullet = \lim_{x \rightarrow a} \frac{(\sqrt{a}-\sqrt{x}) \cdot (\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x \rightarrow a} (\sqrt{a} + \sqrt{x}) = \sqrt{a} + \sqrt{a} = 2\sqrt{a}.$$

$$\bullet = \lim_{x \rightarrow a} \left(\frac{a-x}{\sqrt{a}-\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}} \right) = \lim_{x \rightarrow a} \frac{(a-x) \cdot (\sqrt{a}+\sqrt{x})}{a-x} = \lim_{x \rightarrow a} (\sqrt{a} + \sqrt{x}) = \sqrt{a} + \sqrt{a} = 2\sqrt{a}.$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x}-6x}{3x+1} = -2$$

[24]

$$\bullet = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right) = \lim_{x \rightarrow \infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}} = \frac{0-6}{3+0} = -2.$$

$$\lim_{x \rightarrow 1} \frac{x^m-1}{x^n-1} = \frac{m}{n} \text{ pre } m, n \in \mathbb{N}$$

[25]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \frac{(x-1) \cdot (x^{m-1} + x^{m-2} + \dots + x + 1)}{(x-1) \cdot (x^{n-1} + x^{n-2} + \dots + x + 1)} = \lim_{x \rightarrow 1} \frac{x^{m-1} + x^{m-2} + \dots + x + 1}{x^{n-1} + x^{n-2} + \dots + x + 1} \\ &= \frac{1^{m-1} + 1^{m-2} + \dots + 1 + 1}{1^{n-1} + 1^{n-2} + \dots + 1 + 1} = \frac{1^{m-1} + 1^{m-2} + \dots + 1^1 + 1^0}{1^{n-1} + 1^{n-2} + \dots + 1^1 + 1^0} = \frac{m \cdot 1}{n \cdot 1} = \frac{m}{n}. \end{aligned}$$

Riešené limity – 26

$$\lim_{x \rightarrow 1} \frac{\sqrt[m]{x} - 1}{\sqrt[n]{x} - 1}$$

pre $m, n \in \mathbb{N}$

[26]

Riešené limity – 26

$$\lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$$

pre $m, n \in \mathbb{N}$

[26]

$$\bullet = \lim_{x \rightarrow 1} \left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \frac{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1} \right)$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } x = z^{mn} \mid x \rightarrow 1 \\ z \rightarrow 1 \end{array} \left| \begin{array}{l} \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \\ \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \end{array} \right. \right]$$



Riešené limity – 26

$$\lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$$

pre $m, n \in \mathbb{N}$

[26]

$$\begin{aligned} &= \lim_{x \rightarrow 1} \left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \frac{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{(\sqrt[m]{x}-1) \cdot [(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1]}{(\sqrt[n]{x}-1) \cdot [(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1]} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) \end{aligned}$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } x = z^{mn} \mid x \rightarrow 1 \\ z \rightarrow 1 \end{array} \left| \begin{array}{l} \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \\ \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \end{array} \right. \right]$$

$$= \lim_{z \rightarrow 1} \frac{z^n - 1}{z^m - 1}$$

Riešené limity – 26

$$\lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$$

pre $m, n \in \mathbb{N}$

[26]

$$\begin{aligned} &= \lim_{x \rightarrow 1} \left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \frac{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{(\sqrt[m]{x}-1) \cdot [(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1]}{(\sqrt[n]{x}-1) \cdot [(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1]} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } x = z^{mn} \left| \begin{array}{l} x \rightarrow 1 \\ z \rightarrow 1 \end{array} \right. \begin{array}{l} \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \\ \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \end{array} \right] \\ &= \lim_{z \rightarrow 1} \frac{z^n - 1}{z^m - 1} = \lim_{z \rightarrow 1} \frac{(z-1) \cdot (z^{n-1} + z^{n-2} + \dots + z + 1)}{(z-1) \cdot (z^{m-1} + z^{m-2} + \dots + z + 1)} \end{aligned}$$

Riešené limity – 26

$$\lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$$

pre $m, n \in \mathbb{N}$

[26]

$$\begin{aligned} &= \lim_{x \rightarrow 1} \left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \frac{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{(\sqrt[m]{x}-1) \cdot [(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1]}{(\sqrt[n]{x}-1) \cdot [(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1]} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) = \lim_{x \rightarrow 1} \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \end{aligned}$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } x = z^{mn} \mid x \rightarrow 1 \\ z \rightarrow 1 \end{array} \left| \begin{array}{l} \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \\ \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \end{array} \right. \right]$$

$$= \lim_{z \rightarrow 1} \frac{z^n - 1}{z^m - 1} = \lim_{z \rightarrow 1} \frac{(z-1) \cdot (z^{n-1} + z^{n-2} + \dots + z + 1)}{(z-1) \cdot (z^{m-1} + z^{m-2} + \dots + z + 1)} = \lim_{z \rightarrow 1} \frac{z^{n-1} + z^{n-2} + \dots + z + 1}{z^{m-1} + z^{m-2} + \dots + z + 1}$$

Riešené limity – 26

$$\lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m} \text{ pre } m, n \in \mathbb{N}$$

[26]

$$\begin{aligned} &= \lim_{x \rightarrow 1} \left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \frac{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{(\sqrt[m]{x}-1) \cdot [(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1]}{(\sqrt[n]{x}-1) \cdot [(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1]} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) = \lim_{x \rightarrow 1} \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \\ &= \frac{1^{n-1} + 1^{n-2} + \dots + 1 + 1}{1^{m-1} + 1^{m-2} + \dots + 1 + 1} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } x = z^{mn} \left| \begin{array}{l} x \rightarrow 1 \\ z \rightarrow 1 \end{array} \right. \begin{array}{l} \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \\ \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \end{array} \right] \\ &= \lim_{z \rightarrow 1} \frac{z^n - 1}{z^m - 1} = \lim_{z \rightarrow 1} \frac{(z-1) \cdot (z^{n-1} + z^{n-2} + \dots + z + 1)}{(z-1) \cdot (z^{m-1} + z^{m-2} + \dots + z + 1)} = \lim_{z \rightarrow 1} \frac{z^{n-1} + z^{n-2} + \dots + z + 1}{z^{m-1} + z^{m-2} + \dots + z + 1} \\ &= \frac{1^{n-1} + 1^{n-2} + \dots + 1 + 1}{1^{m-1} + 1^{m-2} + \dots + 1 + 1} \end{aligned}$$

Riešené limity – 26

$$\lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m} \text{ pre } m, n \in \mathbb{N}$$

[26]

$$\begin{aligned} &= \lim_{x \rightarrow 1} \left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \frac{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{(\sqrt[m]{x}-1) \cdot [(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1]}{(\sqrt[n]{x}-1) \cdot [(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1]} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) = \lim_{x \rightarrow 1} \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \\ &= \frac{1^{n-1} + 1^{n-2} + \dots + 1 + 1}{1^{m-1} + 1^{m-2} + \dots + 1 + 1} = \frac{1^{n-1} + 1^{n-2} + \dots + 1^1 + 1^0}{1^{m-1} + 1^{m-2} + \dots + 1^1 + 1^0} \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } x = z^{mn} \mid x \rightarrow 1 \\ z \rightarrow 1 \end{array} \left| \begin{array}{l} \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \\ \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \end{array} \right. \right] \\ &= \lim_{z \rightarrow 1} \frac{z^n - 1}{z^m - 1} = \lim_{z \rightarrow 1} \frac{(z-1) \cdot (z^{n-1} + z^{n-2} + \dots + z + 1)}{(z-1) \cdot (z^{m-1} + z^{m-2} + \dots + z + 1)} = \lim_{z \rightarrow 1} \frac{z^{n-1} + z^{n-2} + \dots + z + 1}{z^{m-1} + z^{m-2} + \dots + z + 1} \\ &= \frac{1^{n-1} + 1^{n-2} + \dots + 1 + 1}{1^{m-1} + 1^{m-2} + \dots + 1 + 1} = \frac{1^{n-1} + 1^{n-2} + \dots + 1^1 + 1^0}{1^{m-1} + 1^{m-2} + \dots + 1^1 + 1^0} \end{aligned}$$

Riešené limity – 26

$$\lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m} \text{ pre } m, n \in \mathbb{N}$$

[26]

$$\begin{aligned} &= \lim_{x \rightarrow 1} \left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \frac{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{(\sqrt[m]{x}-1) \cdot [(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1]}{(\sqrt[n]{x}-1) \cdot [(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1]} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) = \lim_{x \rightarrow 1} \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \\ &= \frac{1^{n-1} + 1^{n-2} + \dots + 1 + 1}{1^{m-1} + 1^{m-2} + \dots + 1 + 1} = \frac{1^{n-1} + 1^{n-2} + \dots + 1^1 + 1^0}{1^{m-1} + 1^{m-2} + \dots + 1^1 + 1^0} = \frac{n \cdot 1}{m \cdot 1} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } x = z^{mn} \left| \begin{array}{l} x \rightarrow 1 \\ z \rightarrow 1 \end{array} \right. \begin{array}{l} \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \\ \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \end{array} \right] \\ &= \lim_{z \rightarrow 1} \frac{z^n - 1}{z^m - 1} = \lim_{z \rightarrow 1} \frac{(z-1) \cdot (z^{n-1} + z^{n-2} + \dots + z + 1)}{(z-1) \cdot (z^{m-1} + z^{m-2} + \dots + z + 1)} = \lim_{z \rightarrow 1} \frac{z^{n-1} + z^{n-2} + \dots + z + 1}{z^{m-1} + z^{m-2} + \dots + z + 1} \\ &= \frac{1^{n-1} + 1^{n-2} + \dots + 1 + 1}{1^{m-1} + 1^{m-2} + \dots + 1 + 1} = \frac{1^{n-1} + 1^{n-2} + \dots + 1^1 + 1^0}{1^{m-1} + 1^{m-2} + \dots + 1^1 + 1^0} = \frac{n \cdot 1}{m \cdot 1} \end{aligned}$$

Riešené limity – 26

$$\lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m} \text{ pre } m, n \in \mathbb{N}$$

[26]

$$\begin{aligned} &= \lim_{x \rightarrow 1} \left(\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \frac{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{(\sqrt[m]{x}-1) \cdot [(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1]}{(\sqrt[n]{x}-1) \cdot [(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1]} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \right) = \lim_{x \rightarrow 1} \frac{(\sqrt[n]{x})^{n-1} + (\sqrt[n]{x})^{n-2} + \dots + \sqrt[n]{x} + 1}{(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2} + \dots + \sqrt[m]{x} + 1} \\ &= \frac{1^{n-1} + 1^{n-2} + \dots + 1 + 1}{1^{m-1} + 1^{m-2} + \dots + 1 + 1} = \frac{1^{n-1} + 1^{n-2} + \dots + 1^1 + 1^0}{1^{m-1} + 1^{m-2} + \dots + 1^1 + 1^0} = \frac{n \cdot 1}{m \cdot 1} = \frac{n}{m}. \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } x = z^{mn} \mid x \rightarrow 1 \\ z \rightarrow 1 \end{array} \left| \begin{array}{l} \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \\ \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \end{array} \right. \right] \\ &= \lim_{z \rightarrow 1} \frac{z^n - 1}{z^m - 1} = \lim_{z \rightarrow 1} \frac{(z-1) \cdot (z^{n-1} + z^{n-2} + \dots + z + 1)}{(z-1) \cdot (z^{m-1} + z^{m-2} + \dots + z + 1)} = \lim_{z \rightarrow 1} \frac{z^{n-1} + z^{n-2} + \dots + z + 1}{z^{m-1} + z^{m-2} + \dots + z + 1} \\ &= \frac{1^{n-1} + 1^{n-2} + \dots + 1 + 1}{1^{m-1} + 1^{m-2} + \dots + 1 + 1} = \frac{1^{n-1} + 1^{n-2} + \dots + 1^1 + 1^0}{1^{m-1} + 1^{m-2} + \dots + 1^1 + 1^0} = \frac{n \cdot 1}{m \cdot 1} = \frac{n}{m}. \end{aligned}$$

Riešené limity – 27

$$\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

[27]

Riešené limity – 27

$$\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

[27]

$$\bullet = \left[\text{Príklad 26 pre } m = 2, n = 3 \right]$$

$$\bullet = \lim_{x \rightarrow 1} \left(\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \cdot \frac{(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1}{(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1} \right)$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } x = z^6 \mid x \rightarrow 1 \mid \sqrt{x} = \sqrt{z^6} = z^{\frac{6}{2}} = z^3 \\ \phantom{\text{Subst. } x = z^6} \mid z \rightarrow 1 \mid \sqrt[3]{x} = \sqrt[3]{z^6} = z^{\frac{6}{3}} = z^2 \end{array} \right]$$

$$\bullet = \left[\begin{array}{l} x \rightarrow 1, x > 0 \mid \sqrt{x} = (\sqrt[6]{x})^{\frac{6}{2}} = (\sqrt[6]{x})^3 \\ \mid x = (\sqrt[6]{x})^6 \mid \sqrt[3]{x} = (\sqrt[6]{x})^{\frac{6}{3}} = (\sqrt[6]{x})^2 \end{array} \right]$$

Riešené limity – 27

$$\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

[27]

$$\bullet = \left[\text{Príklad 26 pre } m=2, n=3 \right] = \lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$$

$$\bullet = \lim_{x \rightarrow 1} \left(\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \cdot \frac{(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1}{(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1} \right) = \lim_{x \rightarrow 1} \left(\frac{(\sqrt{x}-1) \cdot (\sqrt{x}+1)}{(\sqrt[3]{x}-1) \cdot [(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1]} \cdot \frac{(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1}{\sqrt{x}+1} \right)$$

$$\bullet = \left[\text{Subst. } x = z^6 \left| \begin{array}{l} x \rightarrow 1 \\ z \rightarrow 1 \end{array} \right. \begin{array}{l} \sqrt{x} = \sqrt{z^6} = z^{\frac{6}{2}} = z^3 \\ \sqrt[3]{x} = \sqrt[3]{z^6} = z^{\frac{6}{3}} = z^2 \end{array} \right] = \lim_{z \rightarrow 1} \frac{z^3-1}{z^2-1}$$

$$\bullet = \left[\begin{array}{l} x \rightarrow 1, x > 0 \\ x = (\sqrt[6]{x})^6 \end{array} \left| \begin{array}{l} \sqrt{x} = (\sqrt[6]{x})^{\frac{6}{2}} = (\sqrt[6]{x})^3 \\ \sqrt[3]{x} = (\sqrt[6]{x})^{\frac{6}{3}} = (\sqrt[6]{x})^2 \end{array} \right. \right] = \lim_{x \rightarrow 1} \frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1}$$

Riešené limity – 27

$$\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

[27]

$$\bullet = \left[\text{Príklad 26 pre } m=2, n=3 \right] = \lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m}$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \left(\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \cdot \frac{(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1}{(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1} \right) = \lim_{x \rightarrow 1} \left(\frac{(\sqrt{x}-1) \cdot (\sqrt{x}+1)}{(\sqrt[3]{x}-1) \cdot [(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1]} \cdot \frac{(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1}{\sqrt{x}+1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1}{\sqrt{x}+1} \right) \end{aligned}$$

$$\bullet = \left[\text{Subst. } x = z^6 \left| \begin{array}{l} x \rightarrow 1 \\ x \rightarrow 1 \end{array} \right. \begin{array}{l} \sqrt{x} = \sqrt{z^6} = z^{\frac{6}{2}} = z^3 \\ \sqrt[3]{x} = \sqrt[3]{z^6} = z^{\frac{6}{3}} = z^2 \end{array} \right] = \lim_{z \rightarrow 1} \frac{z^3-1}{z^2-1} = \lim_{z \rightarrow 1} \frac{(z-1)(z^2+z+1)}{(z-1)(z+1)}$$

$$\bullet = \left[\begin{array}{l} x \rightarrow 1, x > 0 \\ x = (\sqrt[6]{x})^6 \end{array} \left| \begin{array}{l} \sqrt{x} = (\sqrt[6]{x})^{\frac{6}{2}} = (\sqrt[6]{x})^3 \\ \sqrt[3]{x} = (\sqrt[6]{x})^{\frac{6}{3}} = (\sqrt[6]{x})^2 \end{array} \right. \right] = \lim_{x \rightarrow 1} \frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1} = \lim_{x \rightarrow 1} \frac{(\sqrt[6]{x}-1) \cdot [(\sqrt[6]{x})^2 + \sqrt[6]{x} + 1]}{(\sqrt[6]{x}-1) \cdot (\sqrt[6]{x}+1)}$$

Riešené limity – 27

$$\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

[27]

$$\bullet = \left[\text{Príklad 26 pre } m=2, n=3 \right] = \lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m}$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \left(\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} \right) = \lim_{x \rightarrow 1} \left(\frac{(\sqrt{x}-1) \cdot (\sqrt{x}+1)}{(\sqrt[3]{x}-1) \cdot [(\sqrt[3]{x})^2+\sqrt[3]{x}+1]} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right) = \lim_{x \rightarrow 1} \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \end{aligned}$$

$$\bullet = \left[\text{Subst. } x = z^6 \left| \begin{array}{l} x \rightarrow 1 \\ z \rightarrow 1 \end{array} \right. \begin{array}{l} \sqrt{x} = \sqrt{z^6} = z^2 \\ \sqrt[3]{x} = \sqrt[3]{z^6} = z^2 \end{array} \right] = \lim_{z \rightarrow 1} \frac{z^3-1}{z^2-1} = \lim_{z \rightarrow 1} \frac{(z-1)(z^2+z+1)}{(z-1)(z+1)} = \lim_{z \rightarrow 1} \frac{z^2+z+1}{z+1}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} x \rightarrow 1, x > 0 \\ x = (\sqrt[6]{x})^6 \end{array} \left| \begin{array}{l} \sqrt{x} = (\sqrt[6]{x})^{\frac{6}{2}} = (\sqrt[6]{x})^3 \\ \sqrt[3]{x} = (\sqrt[6]{x})^{\frac{6}{3}} = (\sqrt[6]{x})^2 \end{array} \right. \right] = \lim_{x \rightarrow 1} \frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1} = \lim_{x \rightarrow 1} \frac{(\sqrt[6]{x}-1) \cdot [(\sqrt[6]{x})^2+\sqrt[6]{x}+1]}{(\sqrt[6]{x}-1) \cdot (\sqrt[6]{x}+1)} \\ &= \lim_{x \rightarrow 1} \frac{(\sqrt[6]{x})^2+\sqrt[6]{x}+1}{\sqrt[6]{x}+1} \end{aligned}$$

Riešené limity – 27

$$\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

[27]

$$\bullet = \left[\text{Príklad 26 pre } m=2, n=3 \right] = \lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m}$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \left(\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} \right) = \lim_{x \rightarrow 1} \left(\frac{(\sqrt{x}-1) \cdot (\sqrt{x}+1)}{(\sqrt[3]{x}-1) \cdot [(\sqrt[3]{x})^2+\sqrt[3]{x}+1]} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right) = \lim_{x \rightarrow 1} \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} = \frac{1+1+1}{1+1} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } x = z^6 \left| \begin{array}{l} x \rightarrow 1 \\ z \rightarrow 1 \end{array} \right. \begin{array}{l} \sqrt{x} = \sqrt{z^6} = z^{\frac{6}{2}} = z^3 \\ \sqrt[3]{x} = \sqrt[3]{z^6} = z^{\frac{6}{3}} = z^2 \end{array} \right] = \lim_{z \rightarrow 1} \frac{z^3-1}{z^2-1} = \lim_{z \rightarrow 1} \frac{(z-1)(z^2+z+1)}{(z-1)(z+1)} = \lim_{z \rightarrow 1} \frac{z^2+z+1}{z+1} \\ &= \frac{1+1+1}{1+1} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} x \rightarrow 1, x > 0 \\ x = (\sqrt[6]{x})^6 \end{array} \left| \begin{array}{l} \sqrt{x} = (\sqrt[6]{x})^{\frac{6}{2}} = (\sqrt[6]{x})^3 \\ \sqrt[3]{x} = (\sqrt[6]{x})^{\frac{6}{3}} = (\sqrt[6]{x})^2 \end{array} \right. \right] = \lim_{x \rightarrow 1} \frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1} = \lim_{x \rightarrow 1} \frac{(\sqrt[6]{x}-1) \cdot [(\sqrt[6]{x})^2+\sqrt[6]{x}+1]}{(\sqrt[6]{x}-1) \cdot (\sqrt[6]{x}+1)} \\ &= \lim_{x \rightarrow 1} \frac{(\sqrt[6]{x})^2+\sqrt[6]{x}+1}{\sqrt[6]{x}+1} = \frac{1+1+1}{1+1} \end{aligned}$$

Riešené limity – 27

$$\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1} = \frac{3}{2}$$

[27]

$$\bullet = \left[\text{Príklad 26 pre } m=2, n=3 \right] = \lim_{x \rightarrow 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} = \frac{n}{m} = \frac{3}{2}.$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 1} \left(\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} \right) = \lim_{x \rightarrow 1} \left(\frac{(\sqrt{x}-1) \cdot (\sqrt{x}+1)}{(\sqrt[3]{x}-1) \cdot [(\sqrt[3]{x})^2+\sqrt[3]{x}+1]} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{x-1}{x-1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right) = \lim_{x \rightarrow 1} \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} = \frac{1+1+1}{1+1} = \frac{3}{2}. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } x = z^6 \left| \begin{array}{l} x \rightarrow 1 \\ z \rightarrow 1 \end{array} \right. \begin{array}{l} \sqrt{x} = \sqrt{z^6} = z^2 \\ \sqrt[3]{x} = \sqrt[3]{z^6} = z^2 \end{array} \right] = \lim_{z \rightarrow 1} \frac{z^3-1}{z^2-1} = \lim_{z \rightarrow 1} \frac{(z-1)(z^2+z+1)}{(z-1)(z+1)} = \lim_{z \rightarrow 1} \frac{z^2+z+1}{z+1} \\ &= \frac{1+1+1}{1+1} = \frac{3}{2}. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} x \rightarrow 1, x > 0 \\ x = (\sqrt[6]{x})^6 \end{array} \left| \begin{array}{l} \sqrt{x} = (\sqrt[6]{x})^{\frac{6}{2}} = (\sqrt[6]{x})^3 \\ \sqrt[3]{x} = (\sqrt[6]{x})^{\frac{6}{3}} = (\sqrt[6]{x})^2 \end{array} \right. \right] = \lim_{x \rightarrow 1} \frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1} = \lim_{x \rightarrow 1} \frac{(\sqrt[6]{x}-1) \cdot [(\sqrt[6]{x})^2+\sqrt[6]{x}+1]}{(\sqrt[6]{x}-1) \cdot (\sqrt[6]{x}+1)} \\ &= \lim_{x \rightarrow 1} \frac{(\sqrt[6]{x})^2+\sqrt[6]{x}+1}{\sqrt[6]{x}+1} = \frac{1+1+1}{1+1} = \frac{3}{2}. \end{aligned}$$

Riešené limity – 28, 29

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x}$$

pre $m \in \mathbb{R}$

[28]

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$$

[29]

Riešené limity – 28, 29

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} \quad \text{pre } m \in \mathbb{R}$$

[28]

$$m = 0. \Rightarrow$$

$$m \neq 0. \Rightarrow$$

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$$

[29]

$$\bullet = \left[\begin{array}{l|l} \text{Subst. } z = 5x & x \rightarrow 0 \\ x = \frac{z}{5} & z \rightarrow 0 \end{array} \right]$$

$$\bullet = \lim_{x \rightarrow 0} \frac{5 \cdot \sin 5x}{5x}$$

Riešené limity – 28, 29

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} \quad \text{pre } m \in \mathbb{R}$$

[28]

$$m = 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+0 \cdot x}-1}{x}$$

$$m \neq 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x}$$

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$$

[29]

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = 5x \mid x \rightarrow 0 \\ x = \frac{z}{5} \mid z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin z}{\frac{z}{5}}$$

$$\bullet = \lim_{x \rightarrow 0} \frac{5 \cdot \sin 5x}{5x} = \left[\begin{array}{l} \text{Subst. } z = 5x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right]$$

Riešené limity – 28, 29

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} \quad \text{pre } m \in \mathbb{R}$$

[28]

$$m = 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+0 \cdot x}-1}{x} = \lim_{x \rightarrow 0} \frac{\sqrt[3]{1}-1}{x}$$

$$m \neq 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} = \left[\text{Subst. } z^3 = 1 + mx \left| \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 1 \end{array} \right. \begin{array}{l} mx = z^3 - 1 \\ x = \frac{z^3 - 1}{m} \end{array} \right]$$

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$$

[29]

$$\bullet = \left[\text{Subst. } z = 5x \left| \begin{array}{l} x \rightarrow 0 \\ x = \frac{z}{5} \end{array} \right. \begin{array}{l} z \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin z}{\frac{z}{5}} = 5 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z}$$

$$\bullet = \lim_{x \rightarrow 0} \frac{5 \cdot \sin 5x}{5x} = \left[\text{Subst. } z = 5x \left| \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right. \right] = 5 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z}$$

Riešené limity – 28, 29

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} \quad \text{pre } m \in \mathbb{R}$$

[28]

$$m = 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+0 \cdot x}-1}{x} = \lim_{x \rightarrow 0} \frac{\sqrt[3]{1}-1}{x} = \lim_{x \rightarrow 0} 0$$

$$m \neq 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} = \left[\begin{array}{l} \text{Subst. } z^3 = 1+mx \mid x \rightarrow 0 \mid mx = z^3 - 1 \\ z \rightarrow 1 \mid x = \frac{z^3-1}{m} \end{array} \right]$$

$$= \lim_{z \rightarrow 1} \frac{z-1}{\frac{z^3-1}{m}}$$

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$$

[29]

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = 5x \mid x \rightarrow 0 \\ x = \frac{z}{5} \mid z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin z}{\frac{z}{5}} = 5 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} = 5 \cdot 1$$

$$\bullet = \lim_{x \rightarrow 0} \frac{5 \cdot \sin 5x}{5x} = \left[\begin{array}{l} \text{Subst. } z = 5x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = 5 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} = 5 \cdot 1$$

Riešené limity – 28, 29

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} \quad \text{pre } m \in \mathbb{R}$$

[28]

$$m = 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+0 \cdot x}-1}{x} = \lim_{x \rightarrow 0} \frac{\sqrt[3]{1}-1}{x} = \lim_{x \rightarrow 0} 0 = 0$$

$$\begin{aligned} m \neq 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} &= \left[\begin{array}{l} \text{Subst. } z^3 = 1 + mx \\ x \rightarrow 0 \quad \left| \begin{array}{l} mx = z^3 - 1 \\ z \rightarrow 1 \quad \left| x = \frac{z^3 - 1}{m} \end{array} \right. \end{array} \right] \\ &= \lim_{z \rightarrow 1} \frac{z-1}{\frac{z^3-1}{m}} = \lim_{z \rightarrow 1} \frac{m(z-1)}{(z-1)(z^2+z+1)} \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{x} = 5$$

[29]

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = 5x \\ x \rightarrow 0 \quad \left| \begin{array}{l} z \rightarrow 0 \\ x = \frac{z}{5} \end{array} \right. \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin z}{\frac{z}{5}} = 5 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} = 5 \cdot 1 = 5.$$

$$\bullet = \lim_{x \rightarrow 0} \frac{5 \cdot \sin 5x}{5x} = \left[\begin{array}{l} \text{Subst. } z = 5x \\ x \rightarrow 0 \quad \left| \begin{array}{l} z \rightarrow 0 \end{array} \right. \end{array} \right] = 5 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} = 5 \cdot 1 = 5.$$

Riešené limity – 28, 29

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} \quad \text{pre } m \in \mathbb{R}$$

[28]

$$m = 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+0 \cdot x}-1}{x} = \lim_{x \rightarrow 0} \frac{\sqrt[3]{1}-1}{x} = \lim_{x \rightarrow 0} 0 = 0$$

$$\begin{aligned} m \neq 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} &= \left[\text{Subst. } z^3 = 1+mx \mid \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 1 \end{array} \mid \begin{array}{l} mx = z^3 - 1 \\ x = \frac{z^3 - 1}{m} \end{array} \right] \\ &= \lim_{z \rightarrow 1} \frac{z-1}{\frac{z^3-1}{m}} = \lim_{z \rightarrow 1} \frac{m(z-1)}{(z-1)(z^2+z+1)} = \lim_{z \rightarrow 1} \frac{m}{z^2+z+1} \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{x} = 5$$

[29]

$$\bullet = \left[\text{Subst. } z = 5x \mid \begin{array}{l} x \rightarrow 0 \\ x = \frac{z}{5} \end{array} \mid z \rightarrow 0 \right] = \lim_{z \rightarrow 0} \frac{\sin z}{\frac{z}{5}} = 5 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} = 5 \cdot 1 = 5.$$

$$\bullet = \lim_{x \rightarrow 0} \frac{5 \cdot \sin 5x}{5x} = \left[\text{Subst. } z = 5x \mid \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = 5 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} = 5 \cdot 1 = 5.$$

Riešené limity – 28, 29

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} \quad \text{pre } m \in \mathbb{R}$$

[28]

$$m = 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+0 \cdot x}-1}{x} = \lim_{x \rightarrow 0} \frac{\sqrt[3]{1}-1}{x} = \lim_{x \rightarrow 0} 0 = 0$$

$$\begin{aligned} m \neq 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} &= \left[\text{Subst. } z^3 = 1+mx \mid \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 1 \end{array} \mid \begin{array}{l} mx = z^3 - 1 \\ x = \frac{z^3 - 1}{m} \end{array} \right] \\ &= \lim_{z \rightarrow 1} \frac{z-1}{\frac{z^3-1}{m}} = \lim_{z \rightarrow 1} \frac{m(z-1)}{(z-1)(z^2+z+1)} = \lim_{z \rightarrow 1} \frac{m}{z^2+z+1} = \frac{m}{1^2+1+1} \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{x} = 5$$

[29]

$$\bullet = \left[\text{Subst. } z = 5x \mid \begin{array}{l} x \rightarrow 0 \\ x = \frac{z}{5} \end{array} \mid \begin{array}{l} z \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin z}{\frac{z}{5}} = 5 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} = 5 \cdot 1 = 5.$$

$$\bullet = \lim_{x \rightarrow 0} \frac{5 \cdot \sin 5x}{5x} = \left[\text{Subst. } z = 5x \mid \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = 5 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} = 5 \cdot 1 = 5.$$

Riešené limity – 28, 29

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} = \frac{m}{3} \text{ pre } m \in \mathbb{R}$$

[28]

$$m = 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+0 \cdot x}-1}{x} = \lim_{x \rightarrow 0} \frac{\sqrt[3]{1}-1}{x} = \lim_{x \rightarrow 0} 0 = 0 = \frac{0}{3}.$$

$$\begin{aligned} m \neq 0. \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x} &= \left[\text{Subst. } z^3 = 1+mx \mid \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 1 \end{array} \mid \begin{array}{l} mx = z^3 - 1 \\ x = \frac{z^3-1}{m} \end{array} \right] \\ &= \lim_{z \rightarrow 1} \frac{z-1}{\frac{z^3-1}{m}} = \lim_{z \rightarrow 1} \frac{m(z-1)}{(z-1)(z^2+z+1)} = \lim_{z \rightarrow 1} \frac{m}{z^2+z+1} = \frac{m}{1^2+1+1} = \frac{m}{3}. \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{x} = 5$$

[29]

$$\bullet = \left[\text{Subst. } z = 5x \mid \begin{array}{l} x \rightarrow 0 \\ x = \frac{z}{5} \end{array} \mid z \rightarrow 0 \right] = \lim_{z \rightarrow 0} \frac{\sin z}{\frac{z}{5}} = 5 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} = 5 \cdot 1 = 5.$$

$$\bullet = \lim_{x \rightarrow 0} \frac{5 \cdot \sin 5x}{5x} = \left[\text{Subst. } z = 5x \mid \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] = 5 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} = 5 \cdot 1 = 5.$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x}$$

[30]

$$\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{x}$$

[31]

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{\sin 2x}$$

[32]

$$\lim_{x \rightarrow \pi} \frac{\operatorname{tg} x}{\sin 2x}$$

[33]

Riešené limity – 30, 31, 32, 33

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x}$$

[30]

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{x}$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{x}$$

[31]

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = \operatorname{arctg} x \mid z \rightarrow 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} \mid x \rightarrow 0 \end{array} \right]$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{\sin 2x}$$

[32]

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cdot \cos x}$$

$$\lim_{x \rightarrow \pi} \frac{\operatorname{tg} x}{\sin 2x}$$

[33]

$$\bullet = \lim_{x \rightarrow \pi} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cos x}$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x}$$

[30]

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right)$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{x}$$

[31]

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = \operatorname{arctg} x \mid z \rightarrow 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} \mid x \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{z}{\frac{\sin z}{\cos z}}$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{\sin 2x}$$

[32]

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cdot \cos x} = \lim_{x \rightarrow 0} \frac{1}{2 \cos^2 x}$$

$$\lim_{x \rightarrow \pi} \frac{\operatorname{tg} x}{\sin 2x}$$

[33]

$$\bullet = \lim_{x \rightarrow \pi} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cos x} = \lim_{x \rightarrow \pi} \frac{1}{2 \cos^2 x}$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} = 1$$

[30]

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right) = 1 \cdot \frac{1}{1} = 1.$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{x}$$

[31]

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = \operatorname{arctg} x \mid z \rightarrow 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} \mid x \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{z}{\frac{\sin z}{\cos z}} = \lim_{z \rightarrow 0} \left(\frac{z}{\sin z} \cdot \cos z \right)$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{\sin 2x}$$

[32]

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cdot \cos x} = \lim_{x \rightarrow 0} \frac{1}{2 \cos^2 x} = \frac{1}{2 \cdot 1^2}$$

$$\lim_{x \rightarrow \pi} \frac{\operatorname{tg} x}{\sin 2x}$$

[33]

$$\bullet = \lim_{x \rightarrow \pi} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cos x} = \lim_{x \rightarrow \pi} \frac{1}{2 \cos^2 x} = \frac{1}{2 \cdot (-1)^2}$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} = 1$$

[30]

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right) = 1 \cdot \frac{1}{1} = 1.$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{x}$$

[31]

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = \operatorname{arctg} x \mid z \rightarrow 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} \mid x \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{z}{\frac{\sin z}{\cos z}} = \lim_{z \rightarrow 0} \left(\frac{z}{\sin z} \cdot \cos z \right) = 1 \cdot 1$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{\sin 2x} = \frac{1}{2}$$

[32]

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cdot \cos x} = \lim_{x \rightarrow 0} \frac{1}{2 \cos^2 x} = \frac{1}{2 \cdot 1^2} = \frac{1}{2}.$$

$$\lim_{x \rightarrow \pi} \frac{\operatorname{tg} x}{\sin 2x} = \frac{1}{2}$$

[33]

$$\bullet = \lim_{x \rightarrow \pi} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cos x} = \lim_{x \rightarrow \pi} \frac{1}{2 \cos^2 x} = \frac{1}{2 \cdot (-1)^2} = \frac{1}{2}.$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} = 1$$

[30]

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right) = 1 \cdot \frac{1}{1} = 1.$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{x} = 1$$

[31]

$$\bullet = \left[\begin{array}{l} \text{Subst. } z = \operatorname{arctg} x \mid z \rightarrow 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} \mid x \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{z}{\frac{\sin z}{\cos z}} = \lim_{z \rightarrow 0} \left(\frac{z}{\sin z} \cdot \cos z \right) = 1 \cdot 1 = 1.$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{\sin 2x} = \frac{1}{2}$$

[32]

$$\bullet = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cdot \cos x} = \lim_{x \rightarrow 0} \frac{1}{2 \cos^2 x} = \frac{1}{2 \cdot 1^2} = \frac{1}{2}.$$

$$\lim_{x \rightarrow \pi} \frac{\operatorname{tg} x}{\sin 2x} = \frac{1}{2}$$

[33]

$$\bullet = \lim_{x \rightarrow \pi} \frac{\frac{\sin x}{\cos x}}{2 \sin x \cos x} = \lim_{x \rightarrow \pi} \frac{1}{2 \cos^2 x} = \frac{1}{2 \cdot (-1)^2} = \frac{1}{2}.$$

Riešené limity – 34, 35

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx}$$

pre $m, n \in \mathbb{Z} - \{0\}$

[34]



$$\lim_{x \rightarrow \pi} \frac{\sin mx}{\sin nx}$$

pre $m, n \in \mathbb{Z} - \{0\}$

[35]



Riešené limity – 34, 35

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx} \quad \text{pre } m, n \in \mathbb{Z} - \{0\}$$

[34]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right)$$



$$\lim_{x \rightarrow \pi} \frac{\sin mx}{\sin nx} \quad \text{pre } m, n \in \mathbb{Z} - \{0\}$$

[35]

$$\bullet = \left[\begin{array}{l} \text{Subst. } x = \pi + z \mid x \rightarrow \pi \\ z = \pi - x \mid z \rightarrow 0 \end{array} \right]$$



Riešené limity – 34, 35

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx}$$

pre $m, n \in \mathbb{Z} - \{0\}$

[34]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right) = \lim_{x \rightarrow 0} \frac{mx}{nx} \cdot \lim_{x \rightarrow 0} \frac{\sin mx}{mx} \cdot \lim_{x \rightarrow 0} \frac{nx}{\sin nx}$$



$$\lim_{x \rightarrow \pi} \frac{\sin mx}{\sin nx}$$

pre $m, n \in \mathbb{Z} - \{0\}$

[35]

$$\bullet = \left[\begin{array}{l} \text{Subst. } x = \pi + z \mid x \rightarrow \pi \\ z = \pi - x \mid z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin m(\pi + z)}{\sin n(\pi + z)}$$



Riešené limity – 34, 35

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx}$$

pre $m, n \in \mathbb{Z} - \{0\}$

[34]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right) = \lim_{x \rightarrow 0} \frac{mx}{nx} \cdot \lim_{x \rightarrow 0} \frac{\sin mx}{mx} \cdot \lim_{x \rightarrow 0} \frac{nx}{\sin nx} \\ &= \left[\text{Subst. } u = mx \left| \begin{array}{l} x \rightarrow 0 \\ u \rightarrow 0 \end{array} \right. \right] \left[\text{Subst. } v = nx \left| \begin{array}{l} x \rightarrow 0 \\ v \rightarrow 0 \end{array} \right. \right] \end{aligned}$$

$$\lim_{x \rightarrow \pi} \frac{\sin mx}{\sin nx}$$

pre $m, n \in \mathbb{Z} - \{0\}$

[35]

$$\bullet = \left[\text{Subst. } \begin{array}{l} x = \pi + z \\ z = \pi - x \end{array} \left| \begin{array}{l} x \rightarrow \pi \\ z \rightarrow 0 \end{array} \right. \right] = \lim_{z \rightarrow 0} \frac{\sin m(\pi + z)}{\sin n(\pi + z)} = \lim_{z \rightarrow 0} \frac{\sin(m\pi + mz)}{\sin(n\pi + nz)}$$



Riešené limity – 34, 35

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx}$$

pre $m, n \in \mathbb{Z} - \{0\}$

[34]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right) = \lim_{x \rightarrow 0} \frac{mx}{nx} \cdot \lim_{x \rightarrow 0} \frac{\sin mx}{mx} \cdot \lim_{x \rightarrow 0} \frac{nx}{\sin nx} \\ &= \left[\text{Subst. } u = mx \left| \begin{array}{l} x \rightarrow 0 \\ u \rightarrow 0 \end{array} \right. \right] \left[\text{Subst. } v = nx \left| \begin{array}{l} x \rightarrow 0 \\ v \rightarrow 0 \end{array} \right. \right] = \frac{m}{n} \cdot \lim_{u \rightarrow 0} \frac{\sin u}{u} \cdot \lim_{v \rightarrow 0} \frac{v}{\sin v} \end{aligned}$$

$$\lim_{x \rightarrow \pi} \frac{\sin mx}{\sin nx}$$

pre $m, n \in \mathbb{Z} - \{0\}$

[35]

$$\begin{aligned} \bullet &= \left[\text{Subst. } \begin{array}{l} x = \pi + z \\ z = \pi - x \end{array} \left| \begin{array}{l} x \rightarrow \pi \\ z \rightarrow 0 \end{array} \right. \right] = \lim_{z \rightarrow 0} \frac{\sin m(\pi + z)}{\sin n(\pi + z)} = \lim_{z \rightarrow 0} \frac{\sin(m\pi + mz)}{\sin(n\pi + nz)} \\ &= \lim_{z \rightarrow 0} \frac{\sin m\pi \cdot \cos mz + \cos m\pi \cdot \sin mz}{\sin n\pi \cdot \cos nz + \cos n\pi \cdot \sin nz} \end{aligned}$$



Riešené limity – 34, 35

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx}$$

pre $m, n \in \mathbb{Z} - \{0\}$

[34]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right) = \lim_{x \rightarrow 0} \frac{mx}{nx} \cdot \lim_{x \rightarrow 0} \frac{\sin mx}{mx} \cdot \lim_{x \rightarrow 0} \frac{nx}{\sin nx} \\ &= \left[\text{Subst. } u = mx \left| \begin{array}{l} x \rightarrow 0 \\ u \rightarrow 0 \end{array} \right. \right] \left[\text{Subst. } v = nx \left| \begin{array}{l} x \rightarrow 0 \\ v \rightarrow 0 \end{array} \right. \right] = \frac{m}{n} \cdot \lim_{u \rightarrow 0} \frac{\sin u}{u} \cdot \lim_{v \rightarrow 0} \frac{v}{\sin v} = \frac{m}{n} \cdot 1 \cdot 1 \end{aligned}$$

$$\lim_{x \rightarrow \pi} \frac{\sin mx}{\sin nx}$$

pre $m, n \in \mathbb{Z} - \{0\}$

[35]

$$\begin{aligned} \bullet &= \left[\text{Subst. } \begin{array}{l} x = \pi + z \\ z = \pi - x \end{array} \left| \begin{array}{l} x \rightarrow \pi \\ z \rightarrow 0 \end{array} \right. \right] = \lim_{z \rightarrow 0} \frac{\sin m(\pi+z)}{\sin n(\pi+z)} = \lim_{z \rightarrow 0} \frac{\sin(m\pi+mz)}{\sin(n\pi+nz)} \\ &= \lim_{z \rightarrow 0} \frac{\sin m\pi \cdot \cos mz + \cos m\pi \cdot \sin mz}{\sin n\pi \cdot \cos nz + \cos n\pi \cdot \sin nz} = \lim_{z \rightarrow 0} \frac{0 \cdot \cos mz + (-1)^m \cdot \sin mz}{0 \cdot \cos nz + (-1)^n \cdot \sin nz} \end{aligned}$$



Riešené limity – 34, 35

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx} = \frac{m}{n} \text{ pre } m, n \in \mathbb{Z} - \{0\}$$

[34]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right) = \lim_{x \rightarrow 0} \frac{mx}{nx} \cdot \lim_{x \rightarrow 0} \frac{\sin mx}{mx} \cdot \lim_{x \rightarrow 0} \frac{nx}{\sin nx} \\ &= \left[\text{Subst. } u = mx \left| \begin{array}{l} x \rightarrow 0 \\ u \rightarrow 0 \end{array} \right. \right] \left[\text{Subst. } v = nx \left| \begin{array}{l} x \rightarrow 0 \\ v \rightarrow 0 \end{array} \right. \right] = \frac{m}{n} \cdot \lim_{u \rightarrow 0} \frac{\sin u}{u} \cdot \lim_{v \rightarrow 0} \frac{v}{\sin v} = \frac{m}{n} \cdot 1 \cdot 1 = \frac{m}{n}. \end{aligned}$$

$$\lim_{x \rightarrow \pi} \frac{\sin mx}{\sin nx}$$

$$\text{pre } m, n \in \mathbb{Z} - \{0\}$$

[35]

$$\begin{aligned} \bullet &= \left[\text{Subst. } \begin{array}{l} x = \pi + z \\ z = \pi - x \end{array} \left| \begin{array}{l} x \rightarrow \pi \\ z \rightarrow 0 \end{array} \right. \right] = \lim_{z \rightarrow 0} \frac{\sin m(\pi + z)}{\sin n(\pi + z)} = \lim_{z \rightarrow 0} \frac{\sin(m\pi + mz)}{\sin(n\pi + nz)} \\ &= \lim_{z \rightarrow 0} \frac{\sin m\pi \cdot \cos mz + \cos m\pi \cdot \sin mz}{\sin n\pi \cdot \cos nz + \cos n\pi \cdot \sin nz} = \lim_{z \rightarrow 0} \frac{0 \cdot \cos mz + (-1)^m \cdot \sin mz}{0 \cdot \cos nz + (-1)^n \cdot \sin nz} = \lim_{z \rightarrow 0} \frac{(-1)^m \cdot \sin mz}{(-1)^n \cdot \sin nz} \\ &= (-1)^{m-n} \cdot \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} \end{aligned}$$

Riešené limity – 34, 35

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx} = \frac{m}{n} \text{ pre } m, n \in \mathbb{Z} - \{0\}$$

[34]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right) = \lim_{x \rightarrow 0} \frac{mx}{nx} \cdot \lim_{x \rightarrow 0} \frac{\sin mx}{mx} \cdot \lim_{x \rightarrow 0} \frac{nx}{\sin nx} \\ &= \left[\text{Subst. } u = mx \left| \begin{array}{l} x \rightarrow 0 \\ u \rightarrow 0 \end{array} \right. \right] \left[\text{Subst. } v = nx \left| \begin{array}{l} x \rightarrow 0 \\ v \rightarrow 0 \end{array} \right. \right] = \frac{m}{n} \cdot \lim_{u \rightarrow 0} \frac{\sin u}{u} \cdot \lim_{v \rightarrow 0} \frac{v}{\sin v} = \frac{m}{n} \cdot 1 \cdot 1 = \frac{m}{n}. \end{aligned}$$

$$\lim_{x \rightarrow \pi} \frac{\sin mx}{\sin nx}$$

$$\text{pre } m, n \in \mathbb{Z} - \{0\}$$

[35]

$$\begin{aligned} \bullet &= \left[\text{Subst. } \begin{array}{l} x = \pi + z \\ z = \pi - x \end{array} \left| \begin{array}{l} x \rightarrow \pi \\ z \rightarrow 0 \end{array} \right. \right] = \lim_{z \rightarrow 0} \frac{\sin m(\pi + z)}{\sin n(\pi + z)} = \lim_{z \rightarrow 0} \frac{\sin(m\pi + mz)}{\sin(n\pi + nz)} \\ &= \lim_{z \rightarrow 0} \frac{\sin m\pi \cdot \cos mz + \cos m\pi \cdot \sin mz}{\sin n\pi \cdot \cos nz + \cos n\pi \cdot \sin nz} = \lim_{z \rightarrow 0} \frac{0 \cdot \cos mz + (-1)^m \cdot \sin mz}{0 \cdot \cos nz + (-1)^n \cdot \sin nz} = \lim_{z \rightarrow 0} \frac{(-1)^m \cdot \sin mz}{(-1)^n \cdot \sin nz} \\ &= (-1)^{m-n} \cdot \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} = \left[\text{Predchádzajúci príklad 34: } \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} = \lim_{z \rightarrow 0} \left(\frac{\sin mz}{mz} \cdot \frac{mz}{nz} \cdot \frac{nz}{\sin nz} \right) = 1 \cdot \frac{m}{n} \cdot 1 = \frac{m}{n} \right] \end{aligned}$$

Riešené limity – 34, 35

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx} = \frac{m}{n} \text{ pre } m, n \in \mathbb{Z} - \{0\}$$

[34]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right) = \lim_{x \rightarrow 0} \frac{mx}{nx} \cdot \lim_{x \rightarrow 0} \frac{\sin mx}{mx} \cdot \lim_{x \rightarrow 0} \frac{nx}{\sin nx} \\ &= \left[\text{Subst. } u = mx \left| \begin{array}{l} x \rightarrow 0 \\ u \rightarrow 0 \end{array} \right. \right] \left[\text{Subst. } v = nx \left| \begin{array}{l} x \rightarrow 0 \\ v \rightarrow 0 \end{array} \right. \right] = \frac{m}{n} \cdot \lim_{u \rightarrow 0} \frac{\sin u}{u} \cdot \lim_{v \rightarrow 0} \frac{v}{\sin v} = \frac{m}{n} \cdot 1 \cdot 1 = \frac{m}{n}. \end{aligned}$$

$$\lim_{x \rightarrow \pi} \frac{\sin mx}{\sin nx} = (-1)^{m-n} \cdot \frac{m}{n} \text{ pre } m, n \in \mathbb{Z} - \{0\}$$

[35]

$$\begin{aligned} \bullet &= \left[\text{Subst. } \begin{array}{l} x = \pi + z \left| \begin{array}{l} x \rightarrow \pi \\ z \rightarrow 0 \end{array} \right. \\ z = \pi - x \left| \begin{array}{l} x \rightarrow \pi \\ z \rightarrow 0 \end{array} \right. \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin m(\pi+z)}{\sin n(\pi+z)} = \lim_{z \rightarrow 0} \frac{\sin(m\pi+mz)}{\sin(n\pi+nz)} \\ &= \lim_{z \rightarrow 0} \frac{\sin m\pi \cdot \cos mz + \cos m\pi \cdot \sin mz}{\sin n\pi \cdot \cos nz + \cos n\pi \cdot \sin nz} = \lim_{z \rightarrow 0} \frac{0 \cdot \cos mz + (-1)^m \cdot \sin mz}{0 \cdot \cos nz + (-1)^n \cdot \sin nz} = \lim_{z \rightarrow 0} \frac{(-1)^m \cdot \sin mz}{(-1)^n \cdot \sin nz} \\ &= (-1)^{m-n} \cdot \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} = \left[\text{Predchádzajúci príklad 34: } \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} = \lim_{z \rightarrow 0} \left(\frac{\sin mz}{mz} \cdot \frac{mz}{nz} \cdot \frac{nz}{\sin nz} \right) = 1 \cdot \frac{m}{n} \cdot 1 = \frac{m}{n} \right] \\ &= (-1)^{m-n} \cdot \frac{m}{n}. \end{aligned}$$

Riešené limity – 36, 37

$$\lim_{x \rightarrow 2\pi} \frac{\sin mx}{\sin nx}$$

pre $m, n \in \mathbb{Z} - \{0\}$

[36]

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$$

[37]



Riešené limity – 36, 37

$$\lim_{x \rightarrow 2\pi} \frac{\sin mx}{\sin nx} \quad \text{pre } m, n \in \mathbb{Z} - \{0\}$$

[36]

$$\bullet = \left[\begin{array}{l} \text{Subst. } x = 2\pi + z \mid x \rightarrow 2\pi \\ z = \pi - x \mid z \rightarrow 0 \end{array} \right]$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$$

[37]

$$\bullet = \lim_{x \rightarrow 0} \frac{1 - (\cos^2 x - \sin^2 x)}{x^2}$$

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{x^2} \cdot \frac{1 + \cos 2x}{1 + \cos 2x} \right)$$

$$\bullet = \left[\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow 1 - \cos 2x = 2 \sin^2 x \right]$$

Riešené limity – 36, 37

$$\lim_{x \rightarrow 2\pi} \frac{\sin mx}{\sin nx} \quad \text{pre } m, n \in \mathbb{Z} - \{0\}$$

[36]

$$\bullet = \left[\begin{array}{l} \text{Subst. } x = 2\pi + z \\ z = \pi - x \end{array} \middle| \begin{array}{l} x \rightarrow 2\pi \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin m(2\pi + z)}{\sin n(2\pi + z)}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$$

[37]

$$\bullet = \lim_{x \rightarrow 0} \frac{1 - (\cos^2 x - \sin^2 x)}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 x + \sin^2 x}{x^2}$$

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{x^2} \cdot \frac{1 + \cos 2x}{1 + \cos 2x} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x^2(1 + \cos 2x)}$$

$$\bullet = \left[\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow 1 - \cos 2x = 2 \sin^2 x \right] = \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2}$$

Riešené limity – 36, 37

$$\lim_{x \rightarrow 2\pi} \frac{\sin mx}{\sin nx} \quad \text{pre } m, n \in \mathbb{Z} - \{0\} \quad [36]$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } x = 2\pi + z \mid x \rightarrow 2\pi \\ z = \pi - x \mid z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin m(2\pi + z)}{\sin n(2\pi + z)} = \lim_{z \rightarrow 0} \frac{\sin(2m\pi + mz)}{\sin(2n\pi + nz)}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} \quad [37]$$

$$\bullet = \lim_{x \rightarrow 0} \frac{1 - (\cos^2 x - \sin^2 x)}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2}$$

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{x^2} \cdot \frac{1 + \cos 2x}{1 + \cos 2x} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{\sin^2 2x}{x^2(1 + \cos 2x)}$$

$$\bullet = \left[\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow 1 - \cos 2x = 2 \sin^2 x \right] = \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2$$

Riešené limity – 36, 37

$$\lim_{x \rightarrow 2\pi} \frac{\sin mx}{\sin nx} \quad \text{pre } m, n \in \mathbb{Z} - \{0\}$$

[36]

$$\bullet = \left[\begin{array}{l} \text{Subst. } x = 2\pi + z \\ z = \pi - x \end{array} \middle| \begin{array}{l} x \rightarrow 2\pi \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin m(2\pi + z)}{\sin n(2\pi + z)} = \lim_{z \rightarrow 0} \frac{\sin(2m\pi + mz)}{\sin(2n\pi + nz)} =$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$$

[37]

$$\bullet = \lim_{x \rightarrow 0} \frac{1 - (\cos^2 x - \sin^2 x)}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2$$

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{x^2} \cdot \frac{1 + \cos 2x}{1 + \cos 2x} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{\sin^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{4\sin^2 x}{4x^2(1 + \cos 2x)}$$

$$\bullet = \left[\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow 1 - \cos 2x = 2\sin^2 x \right] = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2$$

Riešené limity – 36, 37

$$\lim_{x \rightarrow 2\pi} \frac{\sin mx}{\sin nx} \quad \text{pre } m, n \in \mathbb{Z} - \{0\} \quad [36]$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Subst. } x = 2\pi + z \\ z = \pi - x \end{array} \middle| \begin{array}{l} x \rightarrow 2\pi \\ z \rightarrow 0 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin m(2\pi + z)}{\sin n(2\pi + z)} = \lim_{z \rightarrow 0} \frac{\sin(2m\pi + mz)}{\sin(2n\pi + nz)} = \left[\begin{array}{l} \text{Funkcia sínus je periodická} \\ \text{s periódami } 2\pi, 2m\pi, 2n\pi \end{array} \right] \\ &= \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = 2 \quad [37]$$

$$\bullet = \lim_{x \rightarrow 0} \frac{1 - (\cos^2 x - \sin^2 x)}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{x^2} \cdot \frac{1 + \cos 2x}{1 + \cos 2x} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{\sin^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{4\sin^2 2x}{4x^2(1 + \cos 2x)} \\ &= \lim_{x \rightarrow 0} \frac{4}{1 + \cos 2x} \cdot \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \right)^2 \end{aligned}$$

$$\bullet = \left[\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow 1 - \cos 2x = 2\sin^2 x \right] = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2 = 2.$$

Riešené limity – 36, 37

$$\lim_{x \rightarrow 2\pi} \frac{\sin mx}{\sin nx}$$

pre $m, n \in \mathbb{Z} - \{0\}$

[36]

$$\begin{aligned} \bullet &= \left[\text{Subst. } x = 2\pi + z \mid x \rightarrow 2\pi \right. \\ &\quad \left. z = \pi - x \mid z \rightarrow 0 \right] = \lim_{z \rightarrow 0} \frac{\sin m(2\pi + z)}{\sin n(2\pi + z)} = \lim_{z \rightarrow 0} \frac{\sin(2m\pi + mz)}{\sin(2n\pi + nz)} = \left[\text{Funkcia sínus je periodická} \right. \\ &\quad \left. \text{s periódami } 2\pi, 2m\pi, 2n\pi \right] \\ &= \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} = \left[\text{Predchádzajúci príklad 34: } \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} = \lim_{z \rightarrow 0} \left(\frac{\sin mz}{mz} \cdot \frac{mz}{nz} \cdot \frac{nz}{\sin nz} \right) = 1 \cdot \frac{m}{n} \cdot 1 = \frac{m}{n} \right] \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = 2$$

[37]

$$\bullet = \lim_{x \rightarrow 0} \frac{1 - (\cos^2 x - \sin^2 x)}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2 = 2.$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{x^2} \cdot \frac{1 + \cos 2x}{1 + \cos 2x} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{\sin^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{4\sin^2 2x}{4x^2(1 + \cos 2x)} \\ &= \lim_{x \rightarrow 0} \frac{4}{1 + \cos 2x} \cdot \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \right)^2 = \left[\text{Subst. } z = 2x \mid x \rightarrow 0 \right. \\ &\quad \left. z \rightarrow 0 \right] \end{aligned}$$

$$\bullet = \left[\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow 1 - \cos 2x = 2\sin^2 x \right] = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2 = 2.$$

Riešené limity – 36, 37

$$\lim_{x \rightarrow 2\pi} \frac{\sin mx}{\sin nx} = \frac{m}{n} \text{ pre } m, n \in \mathbb{Z} - \{0\} \quad [36]$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } x = 2\pi + z \left| \begin{array}{l} x \rightarrow 2\pi \\ z = \pi - x \end{array} \right. \right] = \lim_{z \rightarrow 0} \frac{\sin m(2\pi + z)}{\sin n(2\pi + z)} = \lim_{z \rightarrow 0} \frac{\sin(2m\pi + mz)}{\sin(2n\pi + nz)} = \left[\begin{array}{l} \text{Funkcia sínus je periodická} \\ \text{s periódami } 2\pi, 2m\pi, 2n\pi \end{array} \right] \\ &= \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} = \left[\text{Predchádzajúci príklad 34: } \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} = \lim_{z \rightarrow 0} \left(\frac{\sin mz}{mz} \cdot \frac{mz}{nz} \cdot \frac{nz}{\sin nz} \right) = 1 \cdot \frac{m}{n} \cdot 1 = \frac{m}{n} \right] = \frac{m}{n}. \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = 2 \quad [37]$$

$$\bullet = \lim_{x \rightarrow 0} \frac{1 - (\cos^2 x - \sin^2 x)}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2 = 2.$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{x^2} \cdot \frac{1 + \cos 2x}{1 + \cos 2x} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{\sin^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{4\sin^2 2x}{4x^2(1 + \cos 2x)} \\ &= \lim_{x \rightarrow 0} \frac{4}{1 + \cos 2x} \cdot \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \right)^2 = \left[\text{Subst. } z = 2x \left| \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right. \right] = \frac{4}{1 + 1} \cdot \lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^2 \end{aligned}$$

$$\bullet = \left[\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow 1 - \cos 2x = 2\sin^2 x \right] = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2 = 2.$$

Riešené limity – 36, 37

$$\lim_{x \rightarrow 2\pi} \frac{\sin mx}{\sin nx} = \frac{m}{n} \text{ pre } m, n \in \mathbb{Z} - \{0\}$$

[36]

$$\begin{aligned} \bullet &= \left[\text{Subst. } x = 2\pi + z \mid x \rightarrow 2\pi \right. \\ &\quad \left. z = \pi - x \mid z \rightarrow 0 \right] = \lim_{z \rightarrow 0} \frac{\sin m(2\pi + z)}{\sin n(2\pi + z)} = \lim_{z \rightarrow 0} \frac{\sin(2m\pi + mz)}{\sin(2n\pi + nz)} = \left[\text{Funkcia sínus je periodická} \right. \\ &\quad \left. \text{s periódami } 2\pi, 2m\pi, 2n\pi \right] \\ &= \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} = \left[\text{Predchádzajúci príklad 34: } \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} = \lim_{z \rightarrow 0} \left(\frac{\sin mz}{mz} \cdot \frac{mz}{nz} \cdot \frac{nz}{\sin nz} \right) = 1 \cdot \frac{m}{n} \cdot 1 = \frac{m}{n} \right] = \frac{m}{n}. \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = 2$$

[37]

$$\bullet = \lim_{x \rightarrow 0} \frac{1 - (\cos^2 x - \sin^2 x)}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2 = 2.$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{x^2} \cdot \frac{1 + \cos 2x}{1 + \cos 2x} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{\sin^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{4\sin^2 2x}{4x^2(1 + \cos 2x)} \\ &= \lim_{x \rightarrow 0} \frac{4}{1 + \cos 2x} \cdot \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \right)^2 = \left[\text{Subst. } z = 2x \mid x \rightarrow 0 \right. \\ &\quad \left. z \rightarrow 0 \right] = \frac{4}{1 + 1} \cdot \lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^2 = 2 \cdot 1^2 \end{aligned}$$

$$\bullet = \left[\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow 1 - \cos 2x = 2\sin^2 x \right] = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2 = 2.$$

Riešené limity – 36, 37

$$\lim_{x \rightarrow 2\pi} \frac{\sin mx}{\sin nx} = \frac{m}{n} \text{ pre } m, n \in \mathbb{Z} - \{0\}$$

[36]

$$\begin{aligned} \bullet &= \left[\text{Subst. } x = 2\pi + z \mid x \rightarrow 2\pi \right. \\ &\quad \left. z = \pi - x \mid z \rightarrow 0 \right] = \lim_{z \rightarrow 0} \frac{\sin m(2\pi + z)}{\sin n(2\pi + z)} = \lim_{z \rightarrow 0} \frac{\sin(2m\pi + mz)}{\sin(2n\pi + nz)} = \left[\text{Funkcia sínus je periodická} \right. \\ &\quad \left. \text{s periódami } 2\pi, 2m\pi, 2n\pi \right] \\ &= \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} = \left[\text{Predchádzajúci príklad 34: } \lim_{z \rightarrow 0} \frac{\sin mz}{\sin nz} = \lim_{z \rightarrow 0} \left(\frac{\sin mz}{mz} \cdot \frac{mz}{nz} \cdot \frac{nz}{\sin nz} \right) = 1 \cdot \frac{m}{n} \cdot 1 = \frac{m}{n} \right] = \frac{m}{n}. \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = 2$$

[37]

$$\bullet = \lim_{x \rightarrow 0} \frac{1 - (\cos^2 x - \sin^2 x)}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2 = 2.$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{x^2} \cdot \frac{1 + \cos 2x}{1 + \cos 2x} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{\sin^2 2x}{x^2(1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{4\sin^2 2x}{4x^2(1 + \cos 2x)} \\ &= \lim_{x \rightarrow 0} \frac{4}{1 + \cos 2x} \cdot \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \right)^2 = \left[\text{Subst. } z = 2x \mid x \rightarrow 0 \right. \\ &\quad \left. z \rightarrow 0 \right] = \frac{4}{1 + 1} \cdot \lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^2 = 2 \cdot 1^2 = 2. \end{aligned}$$

$$\bullet = \left[\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow 1 - \cos 2x = 2\sin^2 x \right] = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2 \cdot 1^2 = 2.$$

Riešené limity – 38, 39

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

[38]

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

[39]

Riešené limity – 38, 39

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

[38]

$$\bullet = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

[39]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right)$$

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1} \right)$$

Riešené limity – 38, 39

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

[38]

$$\bullet = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x) \cdot (\cos x + \sin x)}{\cos x - \sin x}$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

[39]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right) = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2 - 1}$$

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1} \right) = \left[\begin{array}{l} \text{Subst. } z = 4x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] \left[\begin{array}{l} \text{Subst. } t = \sqrt{x+1} \mid x \rightarrow 0 \\ x+1 = t^2 \mid t \rightarrow 1 \end{array} \right]$$

Riešené limity – 38, 39

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

[38]

$$\bullet = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x) \cdot (\cos x + \sin x)}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} (\cos x + \sin x)$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

[39]

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right) = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2 - 1} = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(x+1)-1}$$

$$\bullet = \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1} \right) = \left[\text{Subst. } z = 4x \left| \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right. \right] \left[\text{Subst. } t = \sqrt{x+1} \left| \begin{array}{l} x \rightarrow 0 \\ x+1 = t^2 \\ t \rightarrow 1 \end{array} \right. \right] = \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{4(t^2-1)}{t-1}$$

Riešené limity – 38, 39

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

[38]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x) \cdot (\cos x + \sin x)}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} (\cos x + \sin x) \\ &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

[39]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right) = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2 - 1} = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(x+1)-1} \\ &= \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{x} \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1} \right) = \left[\text{Subst. } z = 4x \left| \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right. \right] \left[\text{Subst. } t = \sqrt{x+1} \left| \begin{array}{l} x \rightarrow 0 \\ x+1 = t^2 \\ t \rightarrow 1 \end{array} \right. \right] = \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{4(t^2-1)}{t-1} \\ &= 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{(t-1) \cdot (t+1)}{t-1} \end{aligned}$$

Riešené limity – 38, 39

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

[38]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x) \cdot (\cos x + \sin x)}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} (\cos x + \sin x) \\ &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}. \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

[39]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right) = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2 - 1} = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(x+1)-1} \\ &= \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{x} = \lim_{x \rightarrow 0} \left(4 \cdot \frac{\sin 4x}{4x} \cdot (\sqrt{x+1}+1) \right) \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1} \right) = \left[\text{Subst. } z = 4x \left| \begin{array}{l} x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right. \right] \left[\text{Subst. } t = \sqrt{x+1} \left| \begin{array}{l} x \rightarrow 0 \\ x+1 = t^2 \\ t \rightarrow 1 \end{array} \right. \right] = \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{4(t^2-1)}{t-1} \\ &= 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{(t-1) \cdot (t+1)}{t-1} = 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} (t+1) \end{aligned}$$

Riešené limity – 38, 39

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

[38]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x) \cdot (\cos x + \sin x)}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} (\cos x + \sin x) \\ &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}. \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

[39]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right) = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2 - 1} = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(x+1)-1} \\ &= \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{x} = \lim_{x \rightarrow 0} \left(4 \cdot \frac{\sin 4x}{4x} \cdot (\sqrt{x+1}+1) \right) = \left[\begin{array}{l} \text{Subst. } z = 4x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1} \right) = \left[\begin{array}{l} \text{Subst. } z = 4x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] \left[\begin{array}{l} \text{Subst. } t = \sqrt{x+1} \mid x \rightarrow 0 \\ x+1 = t^2 \mid t \rightarrow 1 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{4(t^2-1)}{t-1} \\ &= 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{(t-1) \cdot (t+1)}{t-1} = 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} (t+1) = 4 \cdot 1 \cdot (1+1) \end{aligned}$$

Riešené limity – 38, 39

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

[38]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x) \cdot (\cos x + \sin x)}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} (\cos x + \sin x) \\ &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}. \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1} = 8$$

[39]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right) = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2 - 1} = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(x+1)-1} \\ &= \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{x} = \lim_{x \rightarrow 0} \left(4 \cdot \frac{\sin 4x}{4x} \cdot (\sqrt{x+1}+1) \right) = \left[\begin{array}{l} \text{Subst. } z = 4x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] \\ &= 4 \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{x \rightarrow 0} (\sqrt{x+1}+1) \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1} \right) = \left[\begin{array}{l} \text{Subst. } z = 4x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] \left[\begin{array}{l} \text{Subst. } t = \sqrt{x+1} \mid x \rightarrow 0 \\ x+1 = t^2 \mid t \rightarrow 1 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{4(t^2-1)}{t-1} \\ &= 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{(t-1) \cdot (t+1)}{t-1} = 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} (t+1) = 4 \cdot 1 \cdot (1+1) = 8. \end{aligned}$$

Riešené limity – 38, 39

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

[38]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x) \cdot (\cos x + \sin x)}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} (\cos x + \sin x) \\ &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}. \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1} = 8$$

[39]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right) = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2 - 1} = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(x+1)-1} \\ &= \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{x} = \lim_{x \rightarrow 0} \left(4 \cdot \frac{\sin 4x}{4x} \cdot (\sqrt{x+1}+1) \right) = \left[\begin{array}{l} \text{Subst. } z = 4x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] \\ &= 4 \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{x \rightarrow 0} (\sqrt{x+1}+1) = 4 \cdot 1 \cdot (\sqrt{0+1}+1) \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1} \right) = \left[\begin{array}{l} \text{Subst. } z = 4x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] \left[\begin{array}{l} \text{Subst. } t = \sqrt{x+1} \mid x \rightarrow 0 \\ x+1 = t^2 \mid t \rightarrow 1 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{4(t^2-1)}{t-1} \\ &= 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{(t-1) \cdot (t+1)}{t-1} = 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} (t+1) = 4 \cdot 1 \cdot (1+1) = 8. \end{aligned}$$

Riešené limity – 38, 39

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

[38]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x) \cdot (\cos x + \sin x)}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} (\cos x + \sin x) \\ &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}. \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1} = 8$$

[39]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right) = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2 - 1} = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(x+1)-1} \\ &= \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{x} = \lim_{x \rightarrow 0} \left(4 \cdot \frac{\sin 4x}{4x} \cdot (\sqrt{x+1}+1) \right) = \left[\begin{array}{l} \text{Subst. } z = 4x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] \\ &= 4 \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{x \rightarrow 0} (\sqrt{x+1}+1) = 4 \cdot 1 \cdot (\sqrt{0+1}+1) = 4 \cdot 1 \cdot 2 \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1} \right) = \left[\begin{array}{l} \text{Subst. } z = 4x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] \left[\begin{array}{l} \text{Subst. } t = \sqrt{x+1} \mid x \rightarrow 0 \\ x+1 = t^2 \mid t \rightarrow 1 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{4(t^2-1)}{t-1} \\ &= 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{(t-1) \cdot (t+1)}{t-1} = 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} (t+1) = 4 \cdot 1 \cdot (1+1) = 8. \end{aligned}$$

Riešené limity – 38, 39

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

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$$\begin{aligned} \bullet &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x) \cdot (\cos x + \sin x)}{\cos x - \sin x} = \lim_{x \rightarrow \frac{\pi}{4}} (\cos x + \sin x) \\ &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}. \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1} = 8$$

[39]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right) = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2 - 1} = \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(x+1)-1} \\ &= \lim_{x \rightarrow 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{x} = \lim_{x \rightarrow 0} \left(4 \cdot \frac{\sin 4x}{4x} \cdot (\sqrt{x+1}+1) \right) = \left[\begin{array}{l} \text{Subst. } z = 4x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] \\ &= 4 \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{x \rightarrow 0} (\sqrt{x+1}+1) = 4 \cdot 1 \cdot (\sqrt{0+1}+1) = 4 \cdot 1 \cdot 2 = 8. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \cdot \frac{4x}{\sqrt{x+1}-1} \right) = \left[\begin{array}{l} \text{Subst. } z = 4x \mid x \rightarrow 0 \\ z \rightarrow 0 \end{array} \right] \left[\begin{array}{l} \text{Subst. } t = \sqrt{x+1} \mid x \rightarrow 0 \\ x+1 = t^2 \mid t \rightarrow 1 \end{array} \right] = \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{4(t^2-1)}{t-1} \\ &= 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} \frac{(t-1) \cdot (t+1)}{t-1} = 4 \cdot \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{t \rightarrow 1} (t+1) = 4 \cdot 1 \cdot (1+1) = 8. \end{aligned}$$

Riešené limity – 40, 41, 42

$$\lim_{x \rightarrow 2} \frac{\arcsin(x-2)}{x^2-2x}$$

[40]

$$\lim_{x \rightarrow \infty} e^x(2 + \cos x)$$

[41]

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

[42]

Riešené limity – 40, 41, 42

$$\lim_{x \rightarrow 2} \frac{\arcsin(x-2)}{x^2-2x}$$

[40]

$$\bullet = \lim_{x \rightarrow 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2} \right)$$

$$\lim_{x \rightarrow \infty} e^x (2 + \cos x)$$

[41]

$$\bullet x \in \mathbb{R}.$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

[42]

$$\left[\begin{array}{l} x \in (-\pi; \pi). \\ \end{array} \right.$$

$$\left. \begin{array}{l} x \in (-\pi; 0) \\ x \in (0; \pi) \end{array} \right]$$

Riešené limity – 40, 41, 42

$$\lim_{x \rightarrow 2} \frac{\arcsin(x-2)}{x^2-2x}$$

[40]

$$\bullet = \lim_{x \rightarrow 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2} \right) = \left[\begin{array}{l} \text{Subst. } z = \arcsin(x-2) \mid z \rightarrow 0 \\ x-2 = \sin z \mid x \rightarrow 2 \end{array} \right]$$

$$\lim_{x \rightarrow \infty} e^x (2 + \cos x)$$

[41]

$$\bullet x \in \mathbb{R}. \Rightarrow e^x > 0, -1 \leq \cos x \leq 1.$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

[42]

$$\left[\begin{array}{l} x \in (-\pi; \pi). \Rightarrow -1 < \cos x < 1. \\ \end{array} \right.$$

$$\left. \begin{array}{l} x \in (-\pi; 0) \\ x \in (0; \pi) \end{array} \right]$$

Riešené limity – 40, 41, 42

$$\lim_{x \rightarrow 2} \frac{\arcsin(x-2)}{x^2-2x}$$

[40]

$$\bullet = \lim_{x \rightarrow 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2} \right) = \left[\begin{array}{l} \text{Subst. } z = \arcsin(x-2) \mid z \rightarrow 0 \\ x-2 = \sin z \mid x \rightarrow 2 \end{array} \right] = \lim_{x \rightarrow 2} \frac{1}{x} \cdot \lim_{z \rightarrow 0} \frac{z}{\sin z}$$

$$\lim_{x \rightarrow \infty} e^x (2 + \cos x)$$

[41]

$$\bullet x \in \mathbb{R}. \Rightarrow e^x > 0, -1 \leq \cos x \leq 1. \Rightarrow 1 \leq 2 + \cos x \leq 3.$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

[42]

$$\left[x \in (-\pi; \pi). \Rightarrow -1 < \cos x < 1. \Rightarrow 0 < 1 - \cos x, 0 < 1 + \cos x. \right]$$

$$\left[\begin{array}{l} x \in (-\pi; 0) \\ x \in (0; \pi) \end{array} \right]$$

Riešené limity – 40, 41, 42

$$\lim_{x \rightarrow 2} \frac{\arcsin(x-2)}{x^2-2x}$$

[40]

$$\bullet = \lim_{x \rightarrow 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2} \right) = \left[\begin{array}{l} \text{Subst. } z = \arcsin(x-2) \mid z \rightarrow 0 \\ x-2 = \sin z \mid x \rightarrow 2 \end{array} \right] = \lim_{x \rightarrow 2} \frac{1}{x} \cdot \lim_{z \rightarrow 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1$$

$$\lim_{x \rightarrow \infty} e^x(2 + \cos x)$$

[41]

$$\bullet x \in \mathbb{R}. \Rightarrow e^x > 0, -1 \leq \cos x \leq 1. \Rightarrow 1 \leq 2 + \cos x \leq 3. \Rightarrow e^x \leq e^x(2 + \cos x) \leq 3e^x.$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

[42]

$$\left[\begin{array}{l} x \in (-\pi; \pi). \Rightarrow -1 < \cos x < 1. \Rightarrow 0 < 1 - \cos x, 0 < 1 + \cos x. \\ \Rightarrow \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x} \sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x}. \end{array} \right. \begin{array}{l} x \in (-\pi; 0) \\ x \in (0; \pi) \end{array} \right]$$

Riešené limity – 40, 41, 42

$$\lim_{x \rightarrow 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

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$$\bullet = \lim_{x \rightarrow 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2} \right) = \left[\begin{array}{l} \text{Subst. } z = \arcsin(x-2) \mid z \rightarrow 0 \\ x-2 = \sin z \mid x \rightarrow 2 \end{array} \right] = \lim_{x \rightarrow 2} \frac{1}{x} \cdot \lim_{z \rightarrow 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

$$\lim_{x \rightarrow \infty} e^x(2 + \cos x)$$

[41]

$$\bullet \lim_{x \rightarrow \infty} e^x \leq \lim_{x \rightarrow \infty} e^x(2 + \cos x) \leq \lim_{x \rightarrow \infty} 3e^x$$

$$\bullet x \in \mathbb{R}. \Rightarrow e^x > 0, -1 \leq \cos x \leq 1. \Rightarrow 1 \leq 2 + \cos x \leq 3. \Rightarrow e^x \leq e^x(2 + \cos x) \leq 3e^x.$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

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$$\left[\begin{array}{l} x \in (-\pi; \pi). \Rightarrow -1 < \cos x < 1. \Rightarrow 0 < 1 - \cos x, 0 < 1 + \cos x. \\ \Rightarrow \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x} \sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x}. \end{array} \right. \begin{array}{l} x \in (-\pi; 0), x \rightarrow 0^- \\ x \in (0; \pi), x \rightarrow 0^+ \end{array}$$

$$\bullet \lim_{x \rightarrow 0^-} \frac{\sin x}{\sqrt{1-\cos x}}$$

$$\bullet \lim_{x \rightarrow 0^+} \frac{\sin x}{\sqrt{1-\cos x}}$$

Riešené limity – 40, 41, 42

$$\lim_{x \rightarrow 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

[40]

$$\bullet = \lim_{x \rightarrow 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2} \right) = \left[\begin{array}{l} \text{Subst. } z = \arcsin(x-2) \mid z \rightarrow 0 \\ x-2 = \sin z \mid x \rightarrow 2 \end{array} \right] = \lim_{x \rightarrow 2} \frac{1}{x} \cdot \lim_{z \rightarrow 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

$$\lim_{x \rightarrow \infty} e^x(2 + \cos x)$$

[41]

$$\bullet \infty = \lim_{x \rightarrow \infty} e^x \leq \lim_{x \rightarrow \infty} e^x(2 + \cos x) \leq \lim_{x \rightarrow \infty} 3e^x = \infty.$$

$$\bullet x \in \mathbb{R}. \Rightarrow e^x > 0, -1 \leq \cos x \leq 1. \Rightarrow 1 \leq 2 + \cos x \leq 3. \Rightarrow e^x \leq e^x(2 + \cos x) \leq 3e^x.$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

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$$\left[\begin{array}{l} x \in (-\pi; \pi). \Rightarrow -1 < \cos x < 1. \Rightarrow 0 < 1 - \cos x, 0 < 1 + \cos x. \\ \Rightarrow \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x} \sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x}. \end{array} \right. \begin{array}{l} x \in (-\pi; 0), x \rightarrow 0^- \Rightarrow |\sin x| = -\sin x. \\ x \in (0; \pi), x \rightarrow 0^+ \Rightarrow |\sin x| = \sin x. \end{array}$$

$$\bullet \lim_{x \rightarrow 0^-} \frac{\sin x}{\sqrt{1-\cos x}} = - \lim_{x \rightarrow 0^-} \sqrt{1+\cos x}$$

$$\bullet \lim_{x \rightarrow 0^+} \frac{\sin x}{\sqrt{1-\cos x}} = \lim_{x \rightarrow 0^+} \sqrt{1+\cos x}$$

Riešené limity – 40, 41, 42

$$\lim_{x \rightarrow 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

[40]

$$\bullet = \lim_{x \rightarrow 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2} \right) = \left[\begin{array}{l} \text{Subst. } z = \arcsin(x-2) \mid z \rightarrow 0 \\ x-2 = \sin z \mid x \rightarrow 2 \end{array} \right] = \lim_{x \rightarrow 2} \frac{1}{x} \cdot \lim_{z \rightarrow 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

$$\lim_{x \rightarrow \infty} e^x(2 + \cos x) = \infty$$

[41]

$$\bullet \infty = \lim_{x \rightarrow \infty} e^x \leq \lim_{x \rightarrow \infty} e^x(2 + \cos x) \leq \lim_{x \rightarrow \infty} 3e^x = \infty. \Rightarrow \bullet \lim_{x \rightarrow \infty} e^x(2 + \cos x) = \infty.$$

$$\bullet x \in \mathbb{R}. \Rightarrow e^x > 0, -1 \leq \cos x \leq 1. \Rightarrow 1 \leq 2 + \cos x \leq 3. \Rightarrow e^x \leq e^x(2 + \cos x) \leq 3e^x.$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

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$$\left[\begin{array}{l} x \in (-\pi; \pi). \Rightarrow -1 < \cos x < 1. \Rightarrow 0 < 1 - \cos x, 0 < 1 + \cos x. \\ \Rightarrow \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x} \sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x}. \end{array} \right. \left. \begin{array}{l} x \in (-\pi; 0), x \rightarrow 0^- \Rightarrow |\sin x| = -\sin x. \\ x \in (0; \pi), x \rightarrow 0^+ \Rightarrow |\sin x| = \sin x. \end{array} \right]$$

$$\bullet \lim_{x \rightarrow 0^-} \frac{\sin x}{\sqrt{1-\cos x}} = - \lim_{x \rightarrow 0^-} \sqrt{1+\cos x}$$

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Riešené limity – 40, 41, 42

$$\lim_{x \rightarrow 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

[40]

$$\bullet = \lim_{x \rightarrow 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2} \right) = \left[\begin{array}{l} \text{Subst. } z = \arcsin(x-2) \mid z \rightarrow 0 \\ x-2 = \sin z \mid x \rightarrow 2 \end{array} \right] = \lim_{x \rightarrow 2} \frac{1}{x} \cdot \lim_{z \rightarrow 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

$$\lim_{x \rightarrow \infty} e^x(2 + \cos x) = \infty$$

[41]

$$\bullet \infty = \lim_{x \rightarrow \infty} e^x \leq \lim_{x \rightarrow \infty} e^x(2 + \cos x) \leq \lim_{x \rightarrow \infty} 3e^x = \infty. \Rightarrow \bullet \lim_{x \rightarrow \infty} e^x(2 + \cos x) = \infty.$$

$$\bullet x \in \mathbb{R}. \Rightarrow e^x > 0, -1 \leq \cos x \leq 1. \Rightarrow 1 \leq 2 + \cos x \leq 3. \Rightarrow e^x \leq e^x(2 + \cos x) \leq 3e^x.$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

[42]

$$\left[\begin{array}{l} x \in (-\pi; \pi). \Rightarrow -1 < \cos x < 1. \Rightarrow 0 < 1 - \cos x, 0 < 1 + \cos x. \\ \Rightarrow \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x} \sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x}. \end{array} \right. \left. \begin{array}{l} x \in (-\pi; 0), x \rightarrow 0^- \Rightarrow |\sin x| = -\sin x. \\ x \in (0; \pi), x \rightarrow 0^+ \Rightarrow |\sin x| = \sin x. \end{array} \right]$$

$$\bullet \lim_{x \rightarrow 0^-} \frac{\sin x}{\sqrt{1-\cos x}} = - \lim_{x \rightarrow 0^-} \sqrt{1+\cos x} = -\sqrt{1+1}$$

$$\bullet \lim_{x \rightarrow 0^+} \frac{\sin x}{\sqrt{1-\cos x}} = \lim_{x \rightarrow 0^+} \sqrt{1+\cos x} = \sqrt{1+1}$$

Riešené limity – 40, 41, 42

$$\lim_{x \rightarrow 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

[40]

$$\bullet = \lim_{x \rightarrow 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2} \right) = \left[\begin{array}{l} \text{Subst. } z = \arcsin(x-2) \mid z \rightarrow 0 \\ x-2 = \sin z \mid x \rightarrow 2 \end{array} \right] = \lim_{x \rightarrow 2} \frac{1}{x} \cdot \lim_{z \rightarrow 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

$$\lim_{x \rightarrow \infty} e^x(2 + \cos x) = \infty$$

[41]

$$\bullet \infty = \lim_{x \rightarrow \infty} e^x \leq \lim_{x \rightarrow \infty} e^x(2 + \cos x) \leq \lim_{x \rightarrow \infty} 3e^x = \infty. \Rightarrow \bullet \lim_{x \rightarrow \infty} e^x(2 + \cos x) = \infty.$$

$$\bullet x \in \mathbb{R}. \Rightarrow e^x > 0, -1 \leq \cos x \leq 1. \Rightarrow 1 \leq 2 + \cos x \leq 3. \Rightarrow e^x \leq e^x(2 + \cos x) \leq 3e^x.$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

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$$\left[\begin{array}{l} x \in (-\pi; \pi). \Rightarrow -1 < \cos x < 1. \Rightarrow 0 < 1 - \cos x, 0 < 1 + \cos x. \\ \Rightarrow \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x} \sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x}. \end{array} \right. \begin{array}{l} x \in (-\pi; 0), x \rightarrow 0^- \Rightarrow |\sin x| = -\sin x. \\ x \in (0; \pi), x \rightarrow 0^+ \Rightarrow |\sin x| = \sin x. \end{array}$$

$$\bullet \lim_{x \rightarrow 0^-} \frac{\sin x}{\sqrt{1-\cos x}} = - \lim_{x \rightarrow 0^-} \sqrt{1+\cos x} = -\sqrt{1+1} = -\sqrt{2}.$$

$$\bullet \lim_{x \rightarrow 0^+} \frac{\sin x}{\sqrt{1-\cos x}} = \lim_{x \rightarrow 0^+} \sqrt{1+\cos x} = \sqrt{1+1} = \sqrt{2}.$$

Riešené limity – 40, 41, 42

$$\lim_{x \rightarrow 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

[40]

$$\bullet = \lim_{x \rightarrow 2} \left(\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2} \right) = \left[\begin{array}{l} \text{Subst. } z = \arcsin(x-2) \mid z \rightarrow 0 \\ x-2 = \sin z \mid x \rightarrow 2 \end{array} \right] = \lim_{x \rightarrow 2} \frac{1}{x} \cdot \lim_{z \rightarrow 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

$$\lim_{x \rightarrow \infty} e^x(2 + \cos x) = \infty$$

[41]

$$\bullet \infty = \lim_{x \rightarrow \infty} e^x \leq \lim_{x \rightarrow \infty} e^x(2 + \cos x) \leq \lim_{x \rightarrow \infty} 3e^x = \infty. \Rightarrow \bullet \lim_{x \rightarrow \infty} e^x(2 + \cos x) = \infty.$$

$$\bullet x \in \mathbb{R}. \Rightarrow e^x > 0, -1 \leq \cos x \leq 1. \Rightarrow 1 \leq 2 + \cos x \leq 3. \Rightarrow e^x \leq e^x(2 + \cos x) \leq 3e^x.$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}} \text{ neexistuje.}$$

[42]

$$\left[\begin{array}{l} x \in (-\pi; \pi). \Rightarrow -1 < \cos x < 1. \Rightarrow 0 < 1 - \cos x, 0 < 1 + \cos x. \\ \Rightarrow \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x} \sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x}. \end{array} \right. \begin{array}{l} x \in (-\pi; 0), x \rightarrow 0^- \Rightarrow |\sin x| = -\sin x. \\ x \in (0; \pi), x \rightarrow 0^+ \Rightarrow |\sin x| = \sin x. \end{array}$$

$$\left. \begin{array}{l} \bullet \lim_{x \rightarrow 0^-} \frac{\sin x}{\sqrt{1-\cos x}} = - \lim_{x \rightarrow 0^-} \sqrt{1+\cos x} = -\sqrt{1+1} = -\sqrt{2}. \\ \bullet \lim_{x \rightarrow 0^+} \frac{\sin x}{\sqrt{1-\cos x}} = \lim_{x \rightarrow 0^+} \sqrt{1+\cos x} = \sqrt{1+1} = \sqrt{2}. \end{array} \right\} \Rightarrow \bullet \lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{1-\cos x}} \nexists.$$

Riešené limity – 43, 44, 45

$$\lim_{x \rightarrow 0} \frac{x^n - \sin^n x}{x - \sin x}$$

pre $n \in \mathbb{N}, n \neq 1$

[43]



$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a}$$

pre $a \in \mathbb{R}$

[44]



$$\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a}$$

pre $a \in \mathbb{R}$

[45]



Riešené limity – 43, 44, 45

$$\lim_{x \rightarrow 0} \frac{x^n - \sin^n x}{x - \sin x} \quad \text{pre } n \in \mathbb{N}, n \neq 1$$

[43]

$$\bullet = \lim_{x \rightarrow 0} \frac{(x - \sin x) \cdot (x^{n-1} + x^{n-2} \sin x + \dots + x \sin^{n-2} x + \sin^{n-1} x)}{x - \sin x}$$

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} \quad \text{pre } a \in \mathbb{R}$$

[44]

$$\bullet = \lim_{x \rightarrow a} \frac{2 \sin \frac{x-a}{2} \cos \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}}$$

$$\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a} \quad \text{pre } a \in \mathbb{R}$$

[45]

$$\bullet = \lim_{x \rightarrow a} \frac{-2 \sin \frac{x-a}{2} \sin \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}}$$

Riešené limity – 43, 44, 45

$$\lim_{x \rightarrow 0} \frac{x^n - \sin^n x}{x - \sin x} \quad \text{pre } n \in \mathbb{N}, n \neq 1$$

[43]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \frac{(x - \sin x) \cdot (x^{n-1} + x^{n-2} \cdot \sin x + \dots + x \cdot \sin^{n-2} x + \sin^{n-1} x)}{x - \sin x} \\ &= \lim_{x \rightarrow 0} (x^{n-1} + x^{n-2} \cdot \sin x + \dots + \sin^{n-1} x) \end{aligned}$$

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} \quad \text{pre } a \in \mathbb{R}$$

[44]

$$\bullet = \lim_{x \rightarrow a} \frac{2 \sin \frac{x-a}{2} \cos \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}} = \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x \rightarrow a} \cos \frac{x+a}{2}$$

$$\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a} \quad \text{pre } a \in \mathbb{R}$$

[45]

$$\bullet = \lim_{x \rightarrow a} \frac{-2 \sin \frac{x-a}{2} \sin \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}} = - \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x \rightarrow a} \sin \frac{x+a}{2}$$

Riešené limity – 43, 44, 45

$$\lim_{x \rightarrow 0} \frac{x^n - \sin^n x}{x - \sin x} \quad \text{pre } n \in \mathbb{N}, n \neq 1$$

[43]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \frac{(x - \sin x) \cdot (x^{n-1} + x^{n-2} \cdot \sin x + \dots + x \cdot \sin^{n-2} x + \sin^{n-1} x)}{x - \sin x} \\ &= \lim_{x \rightarrow 0} (x^{n-1} + x^{n-2} \cdot \sin x + \dots + \sin^{n-1} x) = 0^{n-1} + 0^{n-2} \cdot 0 + \dots + 0^{n-1} \end{aligned}$$

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} \quad \text{pre } a \in \mathbb{R}$$

[44]

$$\bullet = \lim_{x \rightarrow a} \frac{2 \sin \frac{x-a}{2} \cos \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}} = \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x \rightarrow a} \cos \frac{x+a}{2} = \left[\begin{array}{l} \text{Subst. } z = \frac{x-a}{2} \\ x \rightarrow a \\ z \rightarrow 0 \end{array} \right]$$

$$\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a} \quad \text{pre } a \in \mathbb{R}$$

[45]

$$\bullet = \lim_{x \rightarrow a} \frac{-2 \sin \frac{x-a}{2} \sin \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}} = - \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x \rightarrow a} \sin \frac{x+a}{2} = \left[\begin{array}{l} \text{Subst. } z = \frac{x-a}{2} \\ x \rightarrow a \\ z \rightarrow 0 \end{array} \right]$$

Riešené limity – 43, 44, 45

$$\lim_{x \rightarrow 0} \frac{x^n - \sin^n x}{x - \sin x} = 0 \quad \text{pre } n \in \mathbb{N}, n \neq 1$$

[43]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \frac{(x - \sin x) \cdot (x^{n-1} + x^{n-2} \cdot \sin x + \dots + x \cdot \sin^{n-2} x + \sin^{n-1} x)}{x - \sin x} \\ &= \lim_{x \rightarrow 0} (x^{n-1} + x^{n-2} \cdot \sin x + \dots + \sin^{n-1} x) = 0^{n-1} + 0^{n-2} \cdot 0 + \dots + 0^{n-1} = 0. \end{aligned}$$

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} \quad \text{pre } a \in \mathbb{R}$$

[44]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \frac{2 \sin \frac{x-a}{2} \cos \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}} = \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x \rightarrow a} \cos \frac{x+a}{2} = \left[\text{Subst. } z = \frac{x-a}{2} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow 0 \end{array} \right. \right] \\ &= \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{x \rightarrow a} \cos \frac{x+a}{2} \end{aligned}$$

$$\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a} \quad \text{pre } a \in \mathbb{R}$$

[45]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \frac{-2 \sin \frac{x-a}{2} \sin \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}} = - \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x \rightarrow a} \sin \frac{x+a}{2} = \left[\text{Subst. } z = \frac{x-a}{2} \left| \begin{array}{l} x \rightarrow a \\ z \rightarrow 0 \end{array} \right. \right] \\ &= - \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{x \rightarrow a} \sin \frac{x+a}{2} \end{aligned}$$

Riešené limity – 43, 44, 45

$$\lim_{x \rightarrow 0} \frac{x^n - \sin^n x}{x - \sin x} = 0 \quad \text{pre } n \in \mathbb{N}, n \neq 1$$

[43]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \frac{(x - \sin x) \cdot (x^{n-1} + x^{n-2} \cdot \sin x + \dots + x \cdot \sin^{n-2} x + \sin^{n-1} x)}{x - \sin x} \\ &= \lim_{x \rightarrow 0} (x^{n-1} + x^{n-2} \cdot \sin x + \dots + \sin^{n-1} x) = 0^{n-1} + 0^{n-2} \cdot 0 + \dots + 0^{n-1} = 0. \end{aligned}$$

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} \quad \text{pre } a \in \mathbb{R}$$

[44]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \frac{2 \sin \frac{x-a}{2} \cos \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}} = \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x \rightarrow a} \cos \frac{x+a}{2} = \left[\begin{array}{l} \text{Subst. } z = \frac{x-a}{2} \mid x \rightarrow a \\ z \rightarrow 0 \end{array} \right] \\ &= \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{x \rightarrow a} \cos \frac{x+a}{2} = 1 \cdot \cos \frac{a+a}{2} \end{aligned}$$

$$\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a} \quad \text{pre } a \in \mathbb{R}$$

[45]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \frac{-2 \sin \frac{x-a}{2} \sin \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}} = - \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x \rightarrow a} \sin \frac{x+a}{2} = \left[\begin{array}{l} \text{Subst. } z = \frac{x-a}{2} \mid x \rightarrow a \\ z \rightarrow 0 \end{array} \right] \\ &= - \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{x \rightarrow a} \sin \frac{x+a}{2} = -1 \cdot \sin \frac{a+a}{2} \end{aligned}$$

Riešené limity – 43, 44, 45

$$\lim_{x \rightarrow 0} \frac{x^n - \sin^n x}{x - \sin x} = 0 \quad \text{pre } n \in \mathbb{N}, n \neq 1$$

[43]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \frac{(x - \sin x) \cdot (x^{n-1} + x^{n-2} \cdot \sin x + \dots + x \cdot \sin^{n-2} x + \sin^{n-1} x)}{x - \sin x} \\ &= \lim_{x \rightarrow 0} (x^{n-1} + x^{n-2} \cdot \sin x + \dots + \sin^{n-1} x) = 0^{n-1} + 0^{n-2} \cdot 0 + \dots + 0^{n-1} = 0. \end{aligned}$$

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} \quad \text{pre } a \in \mathbb{R}$$

[44]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \frac{2 \sin \frac{x-a}{2} \cos \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}} = \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x \rightarrow a} \cos \frac{x+a}{2} = \left[\begin{array}{l} \text{Subst. } z = \frac{x-a}{2} \mid x \rightarrow a \\ z \rightarrow 0 \end{array} \right] \\ &= \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{x \rightarrow a} \cos \frac{x+a}{2} = 1 \cdot \cos \frac{a+a}{2} = 1 \cdot \cos a \end{aligned}$$

$$\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a} \quad \text{pre } a \in \mathbb{R}$$

[45]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \frac{-2 \sin \frac{x-a}{2} \sin \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}} = - \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x \rightarrow a} \sin \frac{x+a}{2} = \left[\begin{array}{l} \text{Subst. } z = \frac{x-a}{2} \mid x \rightarrow a \\ z \rightarrow 0 \end{array} \right] \\ &= - \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{x \rightarrow a} \sin \frac{x+a}{2} = -1 \cdot \sin \frac{a+a}{2} = -1 \cdot \sin a \end{aligned}$$

Riešené limity – 43, 44, 45

$$\lim_{x \rightarrow 0} \frac{x^n - \sin^n x}{x - \sin x} = 0 \text{ pre } n \in \mathbb{N}, n \neq 1$$

[43]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow 0} \frac{(x - \sin x) \cdot (x^{n-1} + x^{n-2} \cdot \sin x + \dots + x \cdot \sin^{n-2} x + \sin^{n-1} x)}{x - \sin x} \\ &= \lim_{x \rightarrow 0} (x^{n-1} + x^{n-2} \cdot \sin x + \dots + \sin^{n-1} x) = 0^{n-1} + 0^{n-2} \cdot 0 + \dots + 0^{n-1} = 0. \end{aligned}$$

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} = \cos a \text{ pre } a \in \mathbb{R}$$

[44]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \frac{2 \sin \frac{x-a}{2} \cos \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}} = \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x \rightarrow a} \cos \frac{x+a}{2} = \left[\begin{array}{l} \text{Subst. } z = \frac{x-a}{2} \\ x \rightarrow a \\ z \rightarrow 0 \end{array} \right] \\ &= \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{x \rightarrow a} \cos \frac{x+a}{2} = 1 \cdot \cos \frac{a+a}{2} = 1 \cdot \cos a = \cos a. \end{aligned}$$

$$\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a} = -\sin a \text{ pre } a \in \mathbb{R}$$

[45]

$$\begin{aligned} \bullet &= \lim_{x \rightarrow a} \frac{-2 \sin \frac{x-a}{2} \sin \frac{x+a}{2}}{2 \cdot \frac{x-a}{2}} = - \lim_{x \rightarrow a} \frac{\sin \frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x \rightarrow a} \sin \frac{x+a}{2} = \left[\begin{array}{l} \text{Subst. } z = \frac{x-a}{2} \\ x \rightarrow a \\ z \rightarrow 0 \end{array} \right] \\ &= - \lim_{z \rightarrow 0} \frac{\sin z}{z} \cdot \lim_{x \rightarrow a} \sin \frac{x+a}{2} = -1 \cdot \sin \frac{a+a}{2} = -1 \cdot \sin a = -\sin a. \end{aligned}$$

Koniec 6. časti (príklady)

Ďakujem za pozornosť.