Matematická analýza 1

2024/2025

2. Číselné postupnosti
Riešené príklady

Pre správne zobrazenie, fungovanie tooltipov, 2D a 3D animácií je nevyhnutné súbor otvoriť pomocou programu Adobe Reader (zásuvný modul Adobe PDF Plug-In webového prehliadača nestačí).

Kliknutím na text pred ikonou získate nápomoc.
Kliknutím na skratku v modrej lište vpravo hore sa dostanete na príslušný slajd, druhým kliknutím sa dostanete na koniec tohto slajdu.

Obsah

Zoznam riešených limít – príklady 01–80

- 01. $\lim\limits_{n\to\infty} \frac{n^2+n}{n^2-2}$.
- 02. $\lim\limits_{n\to\infty} n\Big(\sqrt[n]{3}-\sqrt[n]{2}\Big)$.
- 03. $\lim\limits_{n\to\infty} \frac{n^3-2}{n^2+n}$.
- 04. $\lim\limits_{n\to\infty} \frac{n^2+n}{n^4-n^3}$.
- 05. $\lim\limits_{n\to\infty} \big(\frac{n^2}{n+2}-\frac{n^2}{n+3}\big)$.
- 06. $\lim\limits_{n\to\infty} \big(\sqrt{n+1}-\sqrt{n}\big)$.
- 07. $\lim\limits_{n\to\infty} \frac{2^n+3^n}{2^{n+1}+3^{n+1}}$.
- 08. $\lim\limits_{n\to\infty} \frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$.
- 09. $\lim\limits_{n\to\infty} \big(1+\frac{1}{n}\big)^{2n}$.
- 10. $\lim\limits_{n\to\infty} \big(1+\frac{1}{n}\big)^{n^2+1}$.
- 11. $\lim\limits_{n\to\infty} \big(1+\frac{1}{n}\big)^{\sqrt{n+1}}$.
- 12. $\lim\limits_{n\to\infty} \big(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\big)$.
- 13. $\lim\limits_{n\to\infty} \sqrt[n]{3^n-2^n}$.
- 14. $\lim\limits_{n\to\infty} \sqrt[n]{2^n+1}$.
- 15. $\lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$.
- 16. $\lim\limits_{n\to\infty} n\big(\sqrt[n]{2}-\sqrt[n]{3}\big)$.
- 17. $\lim\limits_{n\to\infty} \big(\sqrt{n+1}-n\big)$.
- 18. $\lim\limits_{n\to\infty} \frac{n+3^n}{n-3^n}$.
- 19. $\lim\limits_{n\to\infty} \frac{n+3^{-n}}{n-3^{-n}}$.
- 20. $\lim\limits_{n\to\infty} \big(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\big)$.
- 21. $\lim\limits_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$.
- 22. $\lim\limits_{n\to\infty} \big(\frac{2n-1}{2n+3}\big)^6$.
- 23. $\lim\limits_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$.
- 24. $\lim\limits_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$.
- 25. $\lim\limits_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$.
- 26. $\lim\limits_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$.
- 27. $\lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$.
- 28. $\lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$.
- 29. $\lim\limits_{n\to\infty} n\big(\ln n-\ln(n+2)\big)$.
- 30. $\lim\limits_{n\to\infty} n\big(\ln(n+3)-\ln n\big)$.
- 31. $\lim\limits_{n\to\infty} \big(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\big)$.
- 32. $\lim\limits_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}}$.
- 33. $\lim\limits_{n\to\infty} \big(\sqrt{n^2-n+1}-\sqrt{n^2-3n+2}\big)$.
- 34. $\lim\limits_{n\to\infty} \big(\sqrt[3]{n+1}-\sqrt[3]{n+2}\big)$.
- 35. $\lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$.
- 36. $\lim\limits_{n\to\infty} \big(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\big)$.
- 37. $\lim\limits_{n\to\infty} \big(\sqrt{n^2+4n+1}-n+1\big)$.
- 38. $\lim\limits_{n\to\infty} \big(\sqrt{n^2+n+1}-\sqrt{n-1}\big)$.
- 39. $\lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$.
- 40. $\lim\limits_{n\to\infty} \big(\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\big)$.
- 41. $\lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$.
- 42. $\lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$.
- 43. $\lim\limits_{n\to\infty} \big(\sqrt[3]{n^3+1}-n+1\big)$.
- 44. $\lim\limits_{n\to\infty} \big(\sqrt[3]{n^3+1}-n+1\big)$.
- 45. $\lim\limits_{n\to\infty} \big(\frac{2n-1}{2n+3}\big)^{n+6}$.
- 46. $\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}$.
- 47. $\lim\limits_{n\to\infty} \big(\frac{2n-1}{2n+3}\big)^{n^6+6}$.
- 48. $\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}$.
- 49. $\lim\limits_{n\to\infty} \big(\frac{2n-1}{2n-3}\big)^{n^6+6}$.
- 50. $\lim\limits_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$.
- 51. $\lim\limits_{n\to\infty} \big(\frac{2n^6-1}{2n^6+3}\big)^{n+6}$.
- 52. $\lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$.
- 53. $\lim\limits_{n\to\infty} \big(\frac{2n^6-1}{2n^6+3}\big)^{n^6+6}$.
- 54. $\lim\limits_{n\to\infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$.
- 55. $\lim\limits_{n\to\infty} \big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\big)^{\sqrt[6]{n}+6}$.
- 56. $\lim\limits_{n\to\infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$.
- 57. $\lim\limits_{n\to\infty} \big(\frac{2n-1}{2n+3}\big)^{\sqrt[6]{n}+6}$.
- 58. $\lim\limits_{n\to\infty} \frac{2^n n!}{n^n}$.
- 59. $\lim\limits_{n\to\infty} \big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\big)^{n+6}$.
- 60. $\lim\limits_{n\to\infty} \frac{3^n n!}{n^n}$.
- 61. $\lim\limits_{n\to\infty} \big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\big)^{n+6}$.
- 62. $\lim\limits_{n\to\infty} \frac{8^n n!}{(3n)^n}$.
- 63. $\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$.
- 64. $\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$.
- 65. $\lim\limits_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$.
- 66. $\lim\limits_{n\to\infty} \frac{(\frac{1}{3})^n}{(\frac{1}{5})^n-(\frac{1}{4})^n}$.
- 67. $\lim\limits_{n\to\infty} \frac{(\frac{1}{4})^n}{(\frac{1}{5})^n-(\frac{1}{3})^n}$.
- 68. $\lim\limits_{n\to\infty} \frac{(\frac{1}{5})^n}{(\frac{1}{4})^n-(\frac{1}{3})^n}$.
- 69. $\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$.
- 70. $\lim\limits_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}$.
- 71. $\lim\limits_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n+3}}$.
- 72. $\lim\limits_{n\to\infty} \big(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\big)^n$.
- 73. $\lim\limits_{n\to\infty} n\big(\sqrt[n]{2}-\sqrt[n+1]{2}\big)$.
- 74. $\lim\limits_{n\to\infty} n^2\big(\sqrt[n]{2}-\sqrt[n+1]{2}\big)$.
- 75. $\lim\limits_{n\to\infty} \frac{1}{n}\ln\big(2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\big)$.
- 76. $\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$.
- 77. $\lim\limits_{n\to\infty} a_n$, ak $\{a_n\}_{n=1}^{\infty}=\Big\{\sqrt{2},\sqrt{\sqrt{2}},\sqrt{\sqrt{\sqrt{2}}},\ldots\Big\}$.
- 78. $\lim\limits_{n\to\infty} a_n$, ak $\{a_n\}_{n=1}^{\infty}=\Big\{\sqrt{2},\sqrt{2+\sqrt{2}},\sqrt{2+\sqrt{2+\sqrt{2}}},\ldots\Big\}$.
- 79. $\lim\limits_{n\to\infty} a_n$, ak rekurentne $a_1=2$, $a_{n+1}=\sqrt{2a_n+3}$, $n\in N$.
- 80. Číslo $32{,}17\overline{71}$ vyjadrite ako zlomok.

Riešené limity – 01, 02

$$\lim_{n\to\infty} \frac{n^2+n}{n^2-2}$$

$$\lim_{n\to\infty} n\left(\sqrt[n]{3}-\sqrt[n]{2}\right)$$

Riešené limity – 01, 02

$\lim\limits_{n\to\infty} \frac{n^2+n}{n^2-2}$

- $= \lim\limits_{n\to\infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})}$

- $= \lim\limits_{n\to\infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$

$\lim\limits_{n\to\infty} n\left(\sqrt[n]{3} - \sqrt[n]{2}\right)$

- $= \lim\limits_{n\to\infty} n\left(\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2}\right)$

- $= \lim\limits_{n\to\infty} n\left(\sqrt[n]{\frac{3}{2}} - 1\right)\sqrt[n]{2}$

Riešené limity – 01, 02

$$\lim_{n\to\infty} \frac{n^2+n}{n^2-2}$$

- $= \lim_{n\to\infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n\to\infty} \frac{n^2}{n^2} \cdot \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}}$

- $= \lim_{n\to\infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}}$

$$\lim_{n\to\infty} n\left(\sqrt[n]{3} - \sqrt[n]{2}\right)$$

- $= \lim_{n\to\infty} n\left(\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2}\right) = \lim_{n\to\infty} \left(n(\sqrt[n]{3} - 1) - n(\sqrt[n]{2} - 1)\right)$

- $= \lim_{n\to\infty} n\left(\sqrt[n]{\frac{3}{2}} - 1\right)\sqrt[n]{2} = \lim_{n\to\infty} n\left(\sqrt[n]{\frac{3}{2}} - 1\right) \cdot \lim_{n\to\infty} \sqrt[n]{2}$

Riešené limity – 01, 02

$\lim_{n\to\infty} \frac{n^2+n}{n^2-2}$

- $= \lim_{n\to\infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n\to\infty} \frac{n^2}{n^2} \cdot \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = 1 \cdot \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}}$

- $= \lim_{n\to\infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}}$

$\lim_{n\to\infty} n\left(\sqrt[n]{3} - \sqrt[n]{2}\right)$

- $= \lim_{n\to\infty} n\left(\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2}\right) = \lim_{n\to\infty} \left(n(\sqrt[n]{3} - 1) - n(\sqrt[n]{2} - 1)\right)$
  $= \lim_{n\to\infty} n(\sqrt[n]{3} - 1) - \lim_{n\to\infty} n(\sqrt[n]{2} - 1)$

- $= \lim_{n\to\infty} n\left(\sqrt[n]{\frac{3}{2}} - 1\right)\sqrt[n]{2} = \lim_{n\to\infty} n\left(\sqrt[n]{\frac{3}{2}} - 1\right) \cdot \lim_{n\to\infty} \sqrt[n]{2} = \ln\frac{3}{2} \cdot 1$

Riešené limity – 01, 02

$\lim\limits_{n\to\infty} \frac{n^2+n}{n^2-2}$

- $= \lim\limits_{n\to\infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim\limits_{n\to\infty} \frac{n^2}{n^2} \cdot \lim\limits_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = 1 \cdot \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}} = \frac{1+0}{1-0}$

- $= \lim\limits_{n\to\infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim\limits_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}} = \frac{1+0}{1-0}$

$\lim\limits_{n\to\infty} n\left(\sqrt[n]{3} - \sqrt[n]{2}\right) = \ln 3 - \ln 2 = \ln \frac{3}{2}$

- $= \lim\limits_{n\to\infty} n\left(\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2}\right) = \lim\limits_{n\to\infty} \left(n(\sqrt[n]{3} - 1) - n(\sqrt[n]{2} - 1)\right)$
  $= \lim\limits_{n\to\infty} n(\sqrt[n]{3} - 1) - \lim\limits_{n\to\infty} n(\sqrt[n]{2} - 1) = \ln 3 - \ln 2 = \ln \frac{3}{2}.$

- $= \lim\limits_{n\to\infty} n\left(\sqrt[n]{\frac{3}{2}} - 1\right)\sqrt[n]{2} = \lim\limits_{n\to\infty} n\left(\sqrt[n]{\frac{3}{2}} - 1\right) \cdot \lim\limits_{n\to\infty} \sqrt[n]{2} = \ln \frac{3}{2} \cdot 1 = \ln \frac{3}{2}.$

Riešené limity – 01, 02

$\lim_{n\to\infty} \frac{n^2+n}{n^2-2} = 1$

- $= \lim_{n\to\infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n\to\infty} \frac{n^2}{n^2} \cdot \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = 1 \cdot \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}} = \frac{1+0}{1-0} = 1.$

- $= \lim_{n\to\infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}} = \frac{1+0}{1-0} = 1.$

$\lim_{n\to\infty} n\left(\sqrt[n]{3} - \sqrt[n]{2}\right) = \ln 3 - \ln 2 = \ln \frac{3}{2}$

- $= \lim_{n\to\infty} n\left(\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2}\right) = \lim_{n\to\infty} \left(n(\sqrt[n]{3} - 1) - n(\sqrt[n]{2} - 1)\right)$

$= \lim_{n\to\infty} n(\sqrt[n]{3} - 1) - \lim_{n\to\infty} n(\sqrt[n]{2} - 1) = \ln 3 - \ln 2 = \ln \frac{3}{2}.$

- $= \lim_{n\to\infty} n\left(\sqrt[n]{\frac{3}{2}} - 1\right)\sqrt[n]{2} = \lim_{n\to\infty} n\left(\sqrt[n]{\frac{3}{2}} - 1\right) \cdot \lim_{n\to\infty} \sqrt[n]{2} = \ln \frac{3}{2} \cdot 1 = \ln \frac{3}{2}.$

Riešené limity – 03, 04, 05

$$\lim_{n\to\infty} \frac{n^3-2}{n^2+n}$$

$$\lim_{n\to\infty} \frac{n^2+n}{n^4-n^3}$$

$$\lim_{n\to\infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3}\right)$$

Riešené limity – 03, 04, 05

$$\lim_{n\to\infty} \frac{n^3-2}{n^2+n}$$

- $= \lim_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$

$$\lim_{n\to\infty} \frac{n^2+n}{n^4-n^3}$$

- $= \lim_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}}$

$$\lim_{n\to\infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3}\right)$$

- $= \lim_{n\to\infty} \frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)}$

Riešené limity – 03, 04, 05

$\lim\limits_{n\to\infty} \frac{n^3-2}{n^2+n}$

- $= \lim\limits_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim\limits_{n\to\infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}}$

$\lim\limits_{n\to\infty} \frac{n^2+n}{n^4-n^3}$

- $= \lim\limits_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim\limits_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}}$

$\lim\limits_{n\to\infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3}\right)$

- $= \lim\limits_{n\to\infty} \frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)} = \lim\limits_{n\to\infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6}$

Riešené limity – 03, 04, 05

$$\lim_{n\to\infty} \frac{n^3-2}{n^2+n}$$

- $= \lim_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}} = \frac{\infty-\frac{2}{\infty}}{1+\frac{1}{\infty}}$

$$\lim_{n\to\infty} \frac{n^2+n}{n^4-n^3}$$

- $= \lim_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}} = \frac{\frac{1}{\infty}+\frac{1}{\infty}}{1-\frac{1}{\infty}}$

$$\lim_{n\to\infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3}\right)$$

- $= \lim_{n\to\infty} \frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)} = \lim_{n\to\infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim_{n\to\infty} \frac{n^2}{n^2+5n+6}$

Riešené limity – 03, 04, 05

$\lim\limits_{n\to\infty} \frac{n^3-2}{n^2+n}$

- $= \lim\limits_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim\limits_{n\to\infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}} = \frac{\infty-\frac{2}{\infty}}{1+\frac{1}{\infty}} = \frac{\infty-0}{1+0}$

$\lim\limits_{n\to\infty} \frac{n^2+n}{n^4-n^3}$

- $= \lim\limits_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim\limits_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}} = \frac{\frac{1}{\infty}+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{0+0}{1-0}$

$\lim\limits_{n\to\infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3}\right)$

- $= \lim\limits_{n\to\infty} \frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)} = \lim\limits_{n\to\infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim\limits_{n\to\infty} \frac{n^2}{n^2+5n+6}$

$$= \lim\limits_{n\to\infty} \frac{n^2}{n^2+5n+6} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

Riešené limity – 03, 04, 05

$$\lim_{n\to\infty} \frac{n^3-2}{n^2+n} = \infty$$

- $= \lim_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}} = \frac{\infty-\frac{2}{\infty}}{1+\frac{1}{\infty}} = \frac{\infty-0}{1+0} = \infty.$

$$\lim_{n\to\infty} \frac{n^2+n}{n^4-n^3} = 0$$

- $= \lim_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}} = \frac{\frac{1}{\infty}+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{0+0}{1-0} = 0.$

$$\lim_{n\to\infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3}\right)$$

- $= \lim_{n\to\infty} \frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)} = \lim_{n\to\infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim_{n\to\infty} \frac{n^2}{n^2+5n+6}$

$$= \lim_{n\to\infty} \frac{n^2}{n^2+5n+6} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1}{1+\frac{5}{n}+\frac{6}{n^2}}$$

Riešené limity – 03, 04, 05

$\lim\limits_{n\to\infty} \frac{n^3-2}{n^2+n} = \infty$

- $= \lim\limits_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim\limits_{n\to\infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}} = \frac{\infty-\frac{2}{\infty}}{1+\frac{1}{\infty}} = \frac{\infty-0}{1+0} = \infty.$

$\lim\limits_{n\to\infty} \frac{n^2+n}{n^4-n^3} = 0$

- $= \lim\limits_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim\limits_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}} = \frac{\frac{1}{\infty}+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{0+0}{1-0} = 0.$

$\lim\limits_{n\to\infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3}\right)$

- $= \lim\limits_{n\to\infty} \frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)} = \lim\limits_{n\to\infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim\limits_{n\to\infty} \frac{n^2}{n^2+5n+6}$

$$= \lim\limits_{n\to\infty} \frac{n^2}{n^2+5n+6} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim\limits_{n\to\infty} \frac{1}{1+\frac{5}{n}+\frac{6}{n^2}} = \frac{1}{1+\frac{5}{\infty}+\frac{6}{\infty}}$$

Riešené limity – 03, 04, 05

$$\lim_{n\to\infty} \frac{n^3-2}{n^2+n} = \infty$$

- $= \lim_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}} = \frac{\infty-\frac{2}{\infty}}{1+\frac{1}{\infty}} = \frac{\infty-0}{1+0} = \infty.$

$$\lim_{n\to\infty} \frac{n^2+n}{n^4-n^3} = 0$$

- $= \lim_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}} = \frac{\frac{1}{\infty}+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{0+0}{1-0} = 0.$

$$\lim_{n\to\infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3}\right)$$

- $= \lim_{n\to\infty} \frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)} = \lim_{n\to\infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim_{n\to\infty} \frac{n^2}{n^2+5n+6}$

  $= \lim_{n\to\infty} \frac{n^2}{n^2+5n+6} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1}{1+\frac{5}{n}+\frac{6}{n^2}} = \frac{1}{1+\frac{5}{\infty}+\frac{6}{\infty}} = \frac{1}{1+0+0}$

Riešené limity – 03, 04, 05

$$\lim_{n\to\infty} \frac{n^3-2}{n^2+n} = \infty$$

- $= \lim\limits_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim\limits_{n\to\infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}} = \frac{\infty-\frac{2}{\infty}}{1+\frac{1}{\infty}} = \frac{\infty-0}{1+0} = \infty.$

$$\lim_{n\to\infty} \frac{n^2+n}{n^4-n^3} = 0$$

- $= \lim\limits_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim\limits_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}} = \frac{\frac{1}{\infty}+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{0+0}{1-0} = 0.$

$$\lim_{n\to\infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3}\right) = 1$$

- $= \lim\limits_{n\to\infty} \frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)} = \lim\limits_{n\to\infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim\limits_{n\to\infty} \frac{n^2}{n^2+5n+6}$

  $= \lim\limits_{n\to\infty} \frac{n^2}{n^2+5n+6} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim\limits_{n\to\infty} \frac{1}{1+\frac{5}{n}+\frac{6}{n^2}} = \frac{1}{1+\frac{5}{\infty}+\frac{6}{\infty}} = \frac{1}{1+0+0} = 1.$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$$

$$\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}$$

$$\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$$

- $= \lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$

$$\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}$$

- $= \lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}$

$$\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$$

- $= \lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}\cdot\frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}}$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \lim\limits_{n\to\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}}$

$$\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}$$

- $= \lim\limits_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim\limits_{n\to\infty}\frac{\frac{2^n}{3^n}+\frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n}+\frac{3^{n+1}}{3^n}}$

$$\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$$

- $= \lim\limits_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}\cdot\frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}} = \lim\limits_{n\to\infty}\frac{\frac{(-2)^n}{(-3)^n}+\frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n}+\frac{(-3)^{n+1}}{(-3)^n}}$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$$

- $= \lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$

$$\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}$$

- $= \lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n\to\infty}\frac{\frac{2^n}{3^n}+\frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n}+\frac{3^{n+1}}{3^n}} = \lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{2(\frac{2}{3})^n+3}$

$$\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$$

- $= \lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}\cdot\frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}} = \lim_{n\to\infty}\frac{\frac{(-2)^n}{(-3)^n}+\frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n}+\frac{(-3)^{n+1}}{(-3)^n}} = \lim_{n\to\infty}\frac{(\frac{-2}{-3})^n+1}{-2\cdot(\frac{-2}{-3})^n-3\cdot(\frac{-3}{-3})^n}$

$$= \lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{-2(\frac{2}{3})^n-3}$$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$$

- $= \lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$
  $= \frac{1}{\infty+\infty}$

$$\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}$$

- $= \lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n\to\infty}\frac{\frac{2^n}{3^n}+\frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n}+\frac{3^{n+1}}{3^n}} = \lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{2(\frac{2}{3})^n+3} = \frac{0+1}{2\cdot0+3}$

$$\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$$

- $= \lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}\cdot\frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}} = \lim_{n\to\infty}\frac{\frac{(-2)^n}{(-3)^n}+\frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n}+\frac{(-3)^{n+1}}{(-3)^n}} = \lim_{n\to\infty}\frac{(\frac{-2}{-3})^n+1}{-2\cdot(\frac{-2}{-3})^n-3\cdot(\frac{-3}{-3})^n}$
  $= \lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{-2(\frac{2}{3})^n-3} = \frac{0+1}{-2\cdot0-3}$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$$

- $= \lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$
$= \frac{1}{\infty+\infty} = \frac{1}{\infty}$

$$\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}} = \frac{1}{3}$$

- $= \lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n\to\infty}\frac{\frac{2^n}{3^n}+\frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n}+\frac{3^{n+1}}{3^n}} = \lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{2(\frac{2}{3})^n+3} = \frac{0+1}{2\cdot 0+3} = \frac{1}{3}.$

$$\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}} = -\frac{1}{3}$$

- $= \lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}\cdot\frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}} = \lim_{n\to\infty}\frac{\frac{(-2)^n}{(-3)^n}+\frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n}+\frac{(-3)^{n+1}}{(-3)^n}} = \lim_{n\to\infty}\frac{(\frac{-2}{-3})^n+1}{-2\cdot(\frac{-2}{-3})^n-3\cdot(\frac{-3}{-3})^n}$
$= \lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{-2(\frac{2}{3})^n-3} = \frac{0+1}{-2\cdot 0-3} = -\frac{1}{3}.$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)=0$$

- $= \lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$
  $= \frac{1}{\infty+\infty} = \frac{1}{\infty} = 0.$

$$\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}=\frac{1}{3}$$

- $= \lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n\to\infty}\frac{\frac{2^n}{3^n}+\frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n}+\frac{3^{n+1}}{3^n}} = \lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{2(\frac{2}{3})^n+3} = \frac{0+1}{2\cdot 0+3} = \frac{1}{3}.$

$$\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}=-\frac{1}{3}$$

- $= \lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}\cdot\frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}} = \lim_{n\to\infty}\frac{\frac{(-2)^n}{(-3)^n}+\frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n}+\frac{(-3)^{n+1}}{(-3)^n}} = \lim_{n\to\infty}\frac{(\frac{-2}{-3})^n+1}{-2\cdot(\frac{-2}{-3})^n-3\cdot(\frac{-3}{-3})^n}$
  $= \lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{-2(\frac{2}{3})^n-3} = \frac{0+1}{-2\cdot 0-3} = -\frac{1}{3}.$

Riešené limity – 09, 10, 11

$\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{2n}$

$\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{n^2+1}$

$\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{\sqrt{n}+1}$

Riešené limity – 09, 10, 11

$\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$

- $= \lim\limits_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right)^{2}$

$\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{n^2+1}$

- $= \lim\limits_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{n^2+1}{n}}$

$\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{\sqrt{n}+1}$

- $= \lim\limits_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{\sqrt{n}+1}{n}}$

Riešené limity – 09, 10, 11

$\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{2n}$

- $= \lim\limits_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^{2} = \left(\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{n}\right)^{2}$

$\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{n^2+1}$

- $= \lim\limits_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{n^2+1}{n}} = \lim\limits_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^{n+\frac{1}{n}}$

$\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{\sqrt{n}+1}$

- $= \lim\limits_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{\sqrt{n}+1}{n}} = \lim\limits_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{1}{\sqrt{n}}+\frac{1}{n}}$

Riešené limity – 09, 10, 11

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n} = e^2$$

- $= \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^2 = \left(\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\right)^2 = e^2.$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n^2+1}$$

- $= \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{\frac{n^2+1}{n}} = \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{n+\frac{1}{n}} = e^\infty$
  - $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e.$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\sqrt{n}+1}$$

- $= \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{\frac{\sqrt{n}+1}{n}} = \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{\frac{1}{\sqrt{n}}+\frac{1}{n}} = e^0$
  - $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e.$

Riešené limity – 09, 10, 11

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}=e^2$$

- $= \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right)^2 = \left(\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right)^2 = e^2.$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n^2+1}$$

- $= \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{n^2+1}{n}} = \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right)^{n+\frac{1}{n}} = e^{\infty}$

- $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=e.$
- $\lim_{n\to\infty}\left(n+\frac{1}{n}\right)=\infty+0=\infty.$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\sqrt{n}+1}$$

- $= \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{\sqrt{n}+1}{n}} = \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{1}{\sqrt{n}}+\frac{1}{n}} = e^{0}$

- $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=e.$
- $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n}}+\frac{1}{n}\right)=0+0=0.$

Riešené limity – 09, 10, 11

$\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{2n} = e^2$

- $= \lim\limits_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^2 = \left(\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{n}\right)^2 = e^2.$

$\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{n^2+1} = \infty$

- $= \lim\limits_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{n^2+1}{n}} = \lim\limits_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^{n+\frac{1}{n}} = e^{\infty} = \infty.$

  - $\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{n} = e.$
  - $\lim\limits_{n\to\infty} \left(n+\frac{1}{n}\right) = \infty + 0 = \infty.$

$\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{\sqrt{n}+1} = 1$

- $= \lim\limits_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{\sqrt{n}+1}{n}} = \lim\limits_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{1}{\sqrt{n}}+\frac{1}{n}} = e^{0} = 1.$

  - $\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{n} = e.$
  - $\lim\limits_{n\to\infty} \left(\frac{1}{\sqrt{n}}+\frac{1}{n}\right) = 0 + 0 = 0.$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right)$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right)$$

- $= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+\cdots+n=\frac{(1+n)n}{2}. \end{array} \right]$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

- $= \lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)}$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right)$$

- $= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}.\end{array}\right] = \lim\limits_{n\to\infty}\left(\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right)$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

- $= \lim\limits_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)} = \lim\limits_{n\to\infty} 3\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$

Riešené limity – 12, 13

$\lim\limits_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right)$

- $= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}.\end{array}\right] = \lim\limits_{n\to\infty}\left(\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right) = \lim\limits_{n\to\infty}\left(\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right)$

$\lim\limits_{n\to\infty}\sqrt[n]{3^n-2^n}$

- $= \lim\limits_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)} = \lim\limits_{n\to\infty} 3\sqrt[n]{1-\left(\frac{2}{3}\right)^n} = 3\lim\limits_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right)$$

- $= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}.\end{array}\right] = \lim\limits_{n\to\infty}\left(\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right) = \lim\limits_{n\to\infty}\left(\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right) = \lim\limits_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

- $= \lim\limits_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)} = \lim\limits_{n\to\infty} 3\sqrt[n]{1-\left(\frac{2}{3}\right)^n} = 3\lim\limits_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$

Pre všetky $n\in N$ platí:

- $\frac{2}{3}\geq\left(\frac{2}{3}\right)^n$.

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right)$$

- $= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}.\end{array}\right] = \lim_{n\to\infty}\left(\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right) = \lim_{n\to\infty}\left(\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right) = \lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$

$$= \lim_{n\to\infty}\frac{n^2+2n-n^2-n}{2(n+2)} = \lim_{n\to\infty}\frac{n}{2n+4}$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

- $= \lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)} = \lim_{n\to\infty} 3\sqrt[n]{1-\left(\frac{2}{3}\right)^n} = 3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$

Pre všetky $n\in N$ platí:

- $\frac{2}{3}\geq\left(\frac{2}{3}\right)^n.$ $\Rightarrow$ • $-\frac{2}{3}\leq-\left(\frac{2}{3}\right)^n.$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right)$$

- $= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}.\end{array}\right] = \lim_{n\to\infty}\left(\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right) = \lim_{n\to\infty}\left(\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right) = \lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$

$= \lim_{n\to\infty}\frac{n^2+2n-n^2-n}{2(n+2)} = \lim_{n\to\infty}\frac{n}{2n+4} = \lim_{n\to\infty}\frac{n}{2n+4}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

- $= \lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)} = \lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n} = 3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$

Pre všetky $n\in N$ platí:

- $\frac{2}{3}\geq\left(\frac{2}{3}\right)^n$. $\Rightarrow$
- $-\frac{2}{3}\leq-\left(\frac{2}{3}\right)^n$. $\Rightarrow$
- $\frac{1}{3}=1-\frac{2}{3}\leq1-\left(\frac{2}{3}\right)^n\leq1$.

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right)$$

- $= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}.\end{array}\right] = \lim_{n\to\infty}\left(\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right) = \lim_{n\to\infty}\left(\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right) = \lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$

$= \lim_{n\to\infty}\frac{n^2+2n-n^2-n}{2(n+2)} = \lim_{n\to\infty}\frac{n}{2n+4} = \lim_{n\to\infty}\frac{n}{2n+4}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{1}{2+\frac{4}{n}}$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

- $= \lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)} = \lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n} = 3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$

Pre všetky $n\in N$ platí:

- $\frac{2}{3}\geq\left(\frac{2}{3}\right)^n.$ $\Rightarrow$ • $-\frac{2}{3}\leq-\left(\frac{2}{3}\right)^n.$ $\Rightarrow$ • $\frac{1}{3}=1-\frac{2}{3}\leq 1-\left(\frac{2}{3}\right)^n\leq 1.$

$\Rightarrow$ • $\sqrt[n]{\frac{1}{3}}\leq\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\sqrt[n]{1}.$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right)$$

- $= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}.\end{array}\right] = \lim_{n\to\infty}\left(\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right) = \lim_{n\to\infty}\left(\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right) = \lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$

$$= \lim_{n\to\infty}\frac{n^2+2n-n^2-n}{2(n+2)} = \lim_{n\to\infty}\frac{n}{2n+4} = \lim_{n\to\infty}\frac{n}{2n+4}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{1}{2+\frac{4}{n}} = \frac{1}{2+\frac{4}{\infty}}$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

- $= \lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)} = \lim_{n\to\infty} 3\sqrt[n]{1-\left(\frac{2}{3}\right)^n} = 3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$

Pre všetky $n\in N$ platí:

- $\frac{2}{3}\geq\left(\frac{2}{3}\right)^n.\Rightarrow$ • $-\frac{2}{3}\leq-\left(\frac{2}{3}\right)^n.\Rightarrow$ • $\frac{1}{3}=1-\frac{2}{3}\leq 1-\left(\frac{2}{3}\right)^n\leq 1.$

$\Rightarrow$ • $\sqrt[n]{\frac{1}{3}}\leq\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\sqrt[n]{1}.$

$\Rightarrow$ • $1=\lim_{n\to\infty}\sqrt[n]{\frac{1}{3}}\leq\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\lim_{n\to\infty}\sqrt[n]{1}=1.$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right)$$

- $= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}.\end{array}\right] = \lim_{n\to\infty}\left(\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right) = \lim_{n\to\infty}\left(\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right) = \lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$

$$= \lim_{n\to\infty}\frac{n^2+2n-n^2-n}{2(n+2)} = \lim_{n\to\infty}\frac{n}{2n+4} = \lim_{n\to\infty}\frac{n}{2n+4}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{1}{2+\frac{4}{n}} = \frac{1}{2+\frac{4}{\infty}} = \frac{1}{2+0}$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

- $= \lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)} = \lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n} = 3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$

Pre všetky $n\in N$ platí:

- $\frac{2}{3}\geq\left(\frac{2}{3}\right)^n$. $\Rightarrow$ • $-\frac{2}{3}\leq-\left(\frac{2}{3}\right)^n$. $\Rightarrow$ • $\frac{1}{3}=1-\frac{2}{3}\leq1-\left(\frac{2}{3}\right)^n\leq1$.

$\Rightarrow$ • $\sqrt[n]{\frac{1}{3}}\leq\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\sqrt[n]{1}$.

$\Rightarrow$ • $1=\lim_{n\to\infty}\sqrt[n]{\frac{1}{3}}\leq\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\lim_{n\to\infty}\sqrt[n]{1}=1$. $\Rightarrow$ • $\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=1$.

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right)=\frac{1}{2}$$

- $= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}.\end{array}\right] = \lim_{n\to\infty}\left(\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right) = \lim_{n\to\infty}\left(\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right) = \lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$

$$= \lim_{n\to\infty}\frac{n^2+2n-n^2-n}{2(n+2)} = \lim_{n\to\infty}\frac{n}{2n+4} = \lim_{n\to\infty}\frac{n}{2n+4}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{1}{2+\frac{4}{n}} = \frac{1}{2+\frac{4}{\infty}} = \frac{1}{2+0} = \frac{1}{2}.$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

- $= \lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)} = \lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n} = 3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n} = 3\cdot 1$

Pre všetky $n\in N$ platí:

- $\frac{2}{3}\geq\left(\frac{2}{3}\right)^n. \Rightarrow$ • $-\frac{2}{3}\leq-\left(\frac{2}{3}\right)^n. \Rightarrow$ • $\frac{1}{3}=1-\frac{2}{3}\leq 1-\left(\frac{2}{3}\right)^n\leq 1.$

$\Rightarrow$ • $\sqrt[n]{\frac{1}{3}}\leq\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\sqrt[n]{1}.$

$\Rightarrow$ • $1=\lim_{n\to\infty}\sqrt[n]{\frac{1}{3}}\leq\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\lim_{n\to\infty}\sqrt[n]{1}=1. \Rightarrow$ • $\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=1.$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right)=\frac{1}{2}$$

- $= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}.\end{array}\right] = \lim\limits_{n\to\infty}\left(\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right) = \lim\limits_{n\to\infty}\left(\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right) = \lim\limits_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$

$$=\lim_{n\to\infty}\frac{n^2+2n-n^2-n}{2(n+2)}=\lim_{n\to\infty}\frac{n}{2n+4}=\lim_{n\to\infty}\frac{n}{2n+4}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1}{2+\frac{4}{n}}=\frac{1}{2+\frac{4}{\infty}}=\frac{1}{2+0}=\frac{1}{2}.$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}=3$$

- $= \lim\limits_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)} = \lim\limits_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n} = 3\lim\limits_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n} = 3\cdot 1 = 3.$

Pre všetky $n\in N$ platí:

- $\frac{2}{3}\geq\left(\frac{2}{3}\right)^n.$ $\Rightarrow$ • $-\frac{2}{3}\leq-\left(\frac{2}{3}\right)^n.$ $\Rightarrow$ • $\frac{1}{3}=1-\frac{2}{3}\leq 1-\left(\frac{2}{3}\right)^n\leq 1.$

$\Rightarrow$ • $\sqrt[n]{\frac{1}{3}}\leq\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\sqrt[n]{1}.$

$\Rightarrow$ • $1=\lim\limits_{n\to\infty}\sqrt[n]{\frac{1}{3}}\leq\lim\limits_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\lim\limits_{n\to\infty}\sqrt[n]{1}=1.$ $\Rightarrow$ • $\lim\limits_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=1.$

Riešené limity – 14, 15, 16

$$\lim_{n\to\infty} \sqrt[n]{2^n+1}$$

$$\lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$$

$$\lim_{n\to\infty} n\left(\sqrt[n]{2}-\sqrt[n]{3}\right)$$

Riešené limity – 14, 15, 16

$\lim\limits_{n\to\infty} \sqrt[n]{2^n+1}$

Pre všetky $n \in N$ platí:

- $2^n < 2^n + 1 < 2^n + 2^n = 2 \cdot 2^n$.

$\lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$

- $= \lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}$

$\lim\limits_{n\to\infty} n\left(\sqrt[n]{2} - \sqrt[n]{3}\right)$

- $= \lim\limits_{n\to\infty} n\left(\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1\right)$

Riešené limity – 14, 15, 16

$$\lim_{n\to\infty} \sqrt[n]{2^n+1}$$

Pre všetky $n \in N$ platí:

- $2^n < 2^n+1 < 2^n+2^n = 2\cdot 2^n. \Rightarrow$
- $2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n+1} \leq \sqrt[n]{2\cdot 2^n} = 2\cdot\sqrt[n]{2}.$

$$\lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$$

- $= \lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim\limits_{n\to\infty} \frac{\frac{1}{\sqrt{n}}-1}{\frac{1}{\sqrt{n}}+1}$

$$\lim_{n\to\infty} n\left(\sqrt[n]{2}-\sqrt[n]{3}\right)$$

- $= \lim\limits_{n\to\infty} n\left(\sqrt[n]{2}-1-\sqrt[n]{3}+1\right) = \lim\limits_{n\to\infty} n\left(\sqrt[n]{2}-1\right) - \lim\limits_{n\to\infty} n\left(\sqrt[n]{3}-1\right)$

Riešené limity – 14, 15, 16

$\lim\limits_{n\to\infty} \sqrt[n]{2^n+1}$

Pre všetky $n \in N$ platí:

- $2^n < 2^n+1 < 2^n+2^n = 2\cdot 2^n. \Rightarrow$ • $2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n+1} \leq \sqrt[n]{2\cdot 2^n} = 2\cdot\sqrt[n]{2}.$

$\Rightarrow$ • $2 = \lim\limits_{n\to\infty} 2 \leq \lim\limits_{n\to\infty} \sqrt[n]{2^n+1} \leq \lim\limits_{n\to\infty} 2\cdot\sqrt[n]{2} = 2\cdot 1 = 2.$

$\lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$

- $= \lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim\limits_{n\to\infty} \frac{\frac{1}{\sqrt{n}}-1}{\frac{1}{\sqrt{n}}+1} = \frac{\frac{1}{\sqrt{\infty}}-1}{\frac{1}{\sqrt{\infty}}+1}$

$\lim\limits_{n\to\infty} n\left(\sqrt[n]{2}-\sqrt[n]{3}\right)$

- $= \lim\limits_{n\to\infty} n\left(\sqrt[n]{2}-1-\sqrt[n]{3}+1\right) = \lim\limits_{n\to\infty} n\left(\sqrt[n]{2}-1\right) - \lim\limits_{n\to\infty} n\left(\sqrt[n]{3}-1\right)$

$= \left[\text{Pre všetky } a>0 \text{ platí } \lim\limits_{n\to\infty} n(\sqrt[n]{a}-1) = \ln a.\right]$

Riešené limity – 14, 15, 16

$\lim\limits_{n\to\infty} \sqrt[n]{2^n+1} = 2$

Pre všetky $n \in N$ platí:

- $2^n < 2^n + 1 < 2^n + 2^n = 2 \cdot 2^n. \Rightarrow$ • $2 = \sqrt[n]{2^n} \le \sqrt[n]{2^n+1} \le \sqrt[n]{2 \cdot 2^n} = 2 \cdot \sqrt[n]{2}.$

$\Rightarrow$ • $2 = \lim\limits_{n\to\infty} 2 \le \lim\limits_{n\to\infty} \sqrt[n]{2^n+1} \le \lim\limits_{n\to\infty} 2 \cdot \sqrt[n]{2} = 2 \cdot 1 = 2. \Rightarrow$ • $\lim\limits_{n\to\infty} \sqrt[n]{2^n+1} = 2.$

$\lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$

- $= \lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim\limits_{n\to\infty} \frac{\frac{1}{\sqrt{n}}-1}{\frac{1}{\sqrt{n}}+1} = \frac{\frac{1}{\sqrt{\infty}}-1}{\frac{1}{\sqrt{\infty}}+1} = \frac{\frac{1}{\infty}-1}{\frac{1}{\infty}+1}$

$\lim\limits_{n\to\infty} n\left(\sqrt[n]{2} - \sqrt[n]{3}\right)$

- $= \lim\limits_{n\to\infty} n\left(\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1\right) = \lim\limits_{n\to\infty} n\left(\sqrt[n]{2} - 1\right) - \lim\limits_{n\to\infty} n\left(\sqrt[n]{3} - 1\right)$

$= \left[\text{Pre všetky } a > 0 \text{ platí } \lim\limits_{n\to\infty} n(\sqrt[n]{a} - 1) = \ln a.\right] = \ln 2 - \ln 3$

Riešené limity – 14, 15, 16

$\lim_{n\to\infty} \sqrt[n]{2^n+1} = 2$

Pre všetky $n \in N$ platí:

- $2^n < 2^n + 1 < 2^n + 2^n = 2 \cdot 2^n. \Rightarrow$ • $2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n+1} \leq \sqrt[n]{2 \cdot 2^n} = 2 \cdot \sqrt[n]{2}.$

$\Rightarrow$ • $2 = \lim_{n\to\infty} 2 \leq \lim_{n\to\infty} \sqrt[n]{2^n+1} \leq \lim_{n\to\infty} 2 \cdot \sqrt[n]{2} = 2 \cdot 1 = 2. \Rightarrow$ • $\lim_{n\to\infty} \sqrt[n]{2^n+1} = 2.$

$\lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$

- $= \lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}-1}{\frac{1}{\sqrt{n}}+1} = \frac{\frac{1}{\sqrt{\infty}}-1}{\frac{1}{\sqrt{\infty}}+1} = \frac{\frac{1}{\infty}-1}{\frac{1}{\infty}+1} = \frac{0-1}{0+1}$

$\lim_{n\to\infty} n\left(\sqrt[n]{2} - \sqrt[n]{3}\right) = \ln\frac{2}{3}$

- $= \lim_{n\to\infty} n\left(\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1\right) = \lim_{n\to\infty} n\left(\sqrt[n]{2} - 1\right) - \lim_{n\to\infty} n\left(\sqrt[n]{3} - 1\right)$

$= \left[\text{Pre všetky } a > 0 \text{ platí } \lim_{n\to\infty} n(\sqrt[n]{a} - 1) = \ln a.\right] = \ln 2 - \ln 3 = \ln\frac{2}{3}.$

Riešené limity – 14, 15, 16

$\lim\limits_{n\to\infty} \sqrt[n]{2^n+1} = 2$

Pre všetky $n \in N$ platí:

- $2^n < 2^n + 1 < 2^n + 2^n = 2\cdot 2^n.$ $\Rightarrow$ • $2 = \sqrt[n]{2^n} \le \sqrt[n]{2^n+1} \le \sqrt[n]{2\cdot 2^n} = 2\cdot\sqrt[n]{2}.$

$\Rightarrow$ • $2 = \lim\limits_{n\to\infty} 2 \le \lim\limits_{n\to\infty} \sqrt[n]{2^n+1} \le \lim\limits_{n\to\infty} 2\cdot\sqrt[n]{2} = 2\cdot 1 = 2.$ $\Rightarrow$ • $\lim\limits_{n\to\infty} \sqrt[n]{2^n+1} = 2.$

$\lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} = -1$

- $= \lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim\limits_{n\to\infty} \frac{\frac{1}{\sqrt{n}}-1}{\frac{1}{\sqrt{n}}+1} = \frac{\frac{1}{\sqrt{\infty}}-1}{\frac{1}{\sqrt{\infty}}+1} = \frac{\frac{1}{\infty}-1}{\frac{1}{\infty}+1} = \frac{0-1}{0+1} = -1.$

$\lim\limits_{n\to\infty} n\left(\sqrt[n]{2} - \sqrt[n]{3}\right) = \ln\frac{2}{3}$

- $= \lim\limits_{n\to\infty} n\left(\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1\right) = \lim\limits_{n\to\infty} n\left(\sqrt[n]{2} - 1\right) - \lim\limits_{n\to\infty} n\left(\sqrt[n]{3} - 1\right)$

$= \left[\text{Pre všetky } a > 0 \text{ platí } \lim\limits_{n\to\infty} n(\sqrt[n]{a} - 1) = \ln a.\right] = \ln 2 - \ln 3 = \ln\frac{2}{3}.$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty} \left(\sqrt{n+1} - n\right)$$

$$\lim_{n\to\infty} \frac{n+3^n}{n-3^n}$$

$$\lim_{n\to\infty} \frac{n+3^{-n}}{n-3^{-n}}$$

Riešené limity – 17, 18, 19

$\lim_{n\to\infty} \left(\sqrt{n+1} - n\right)$

- $= \lim_{n\to\infty} \sqrt{n}\Big(\sqrt{1+\frac{1}{n}} - \sqrt{n}\Big)$

$\lim_{n\to\infty} \frac{n+3^n}{n-3^n}$

- $= \lim_{n\to\infty} \frac{n+3^n}{n-3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}}$

$\lim_{n\to\infty} \frac{n+3^{-n}}{n-3^{-n}}$

- $= \lim_{n\to\infty} \frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$

Riešené limity – 17, 18, 19

$\lim_{n\to\infty} \left(\sqrt{n+1} - n\right)$

- $= \lim_{n\to\infty} \sqrt{n}\left(\sqrt{1+\frac{1}{n}} - \sqrt{n}\right) = \infty \cdot \left(\sqrt{1+\frac{1}{\infty}} - \infty\right)$

$\lim_{n\to\infty} \frac{n+3^n}{n-3^n}$

- $= \lim_{n\to\infty} \frac{n+3^n}{n-3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n\to\infty} \frac{\frac{n}{3^n}+1}{\frac{n}{3^n}-1}$

$\lim_{n\to\infty} \frac{n+3^{-n}}{n-3^{-n}}$

- $= \lim_{n\to\infty} \frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1+\frac{1}{n\cdot 3^n}}{1-\frac{1}{n\cdot 3^n}}$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty}\left(\sqrt{n+1}-n\right)$$

- $= \lim_{n\to\infty} \sqrt{n}\left(\sqrt{1+\frac{1}{n}}-\sqrt{n}\right) = \infty\cdot\left(\sqrt{1+\frac{1}{\infty}}-\infty\right) = \infty\cdot(\sqrt{1+0}-\infty)$

$$\lim_{n\to\infty}\frac{n+3^n}{n-3^n}$$

- $= \lim_{n\to\infty}\frac{n+3^n}{n-3^n}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n\to\infty}\frac{\frac{n}{3^n}+1}{\frac{n}{3^n}-1}$

Označme $a_n = \frac{n}{3^n}$, $n\in N$.

- $\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\frac{n}{3^n}} = \lim_{n\to\infty}\frac{\sqrt[n]{n}}{3} = \frac{1}{3} < 1.$

Resp.

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}} = \lim_{n\to\infty}\frac{n+1}{3n} = \frac{1}{3} < 1.$

$\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n}{3^n} = 0.$

$$\lim_{n\to\infty}\frac{n+3^{-n}}{n-3^{-n}}$$

- $= \lim_{n\to\infty}\frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{1+\frac{1}{n\cdot 3^n}}{1-\frac{1}{n\cdot 3^n}} = \frac{1+\frac{1}{\infty\cdot\infty}}{1-\frac{1}{\infty\cdot\infty}}$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty}\left(\sqrt{n+1}-n\right)$$

- $= \lim_{n\to\infty} \sqrt{n}\Big(\sqrt{1+\frac{1}{n}}-\sqrt{n}\Big) = \infty\cdot\Big(\sqrt{1+\frac{1}{\infty}}-\infty\Big) = \infty\cdot(\sqrt{1+0}-\infty)$
  $= \infty\cdot(1-\infty)$

$$\lim_{n\to\infty}\frac{n+3^n}{n-3^n}$$

- $= \lim_{n\to\infty}\frac{n+3^n}{n-3^n}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n\to\infty}\frac{\frac{n}{3^n}+1}{\frac{n}{3^n}-1} = \frac{0+1}{0-1}$

Označme $a_n = \frac{n}{3^n}$, $n\in N$.

- $\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\frac{n}{3^n}} = \lim_{n\to\infty}\frac{\sqrt[n]{n}}{3} = \frac{1}{3} < 1.$

Resp.

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}} = \lim_{n\to\infty}\frac{n+1}{3n} = \frac{1}{3} < 1.$

$\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n}{3^n} = 0.$

$$\lim_{n\to\infty}\frac{n+3^{-n}}{n-3^{-n}}$$

- $= \lim_{n\to\infty}\frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{1+\frac{1}{n\cdot 3^n}}{1-\frac{1}{n\cdot 3^n}} = \frac{1+\frac{1}{\infty\cdot\infty}}{1-\frac{1}{\infty\cdot\infty}} = \frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}}$

Riešené limity – 17, 18, 19

$\lim_{n\to\infty}\left(\sqrt{n+1}-n\right)=-\infty$

- $=\lim_{n\to\infty}\sqrt{n}\left(\sqrt{1+\frac{1}{n}}-\sqrt{n}\right)=\infty\cdot\left(\sqrt{1+\frac{1}{\infty}}-\infty\right)=\infty\cdot\left(\sqrt{1+0}-\infty\right)$
  $=\infty\cdot(1-\infty)=\infty\cdot(-\infty)=-\infty.$

$\lim_{n\to\infty}\frac{n+3^n}{n-3^n}=-1$

- $=\lim_{n\to\infty}\frac{n+3^n}{n-3^n}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}=\lim_{n\to\infty}\frac{\frac{n}{3^n}+1}{\frac{n}{3^n}-1}=\frac{0+1}{0-1}=-1.$

Označme $a_n=\frac{n}{3^n}$, $n\in N$.

- $\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n}{3^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n}}{3}=\frac{1}{3}<1.$

Resp.

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}}=\lim_{n\to\infty}\frac{n+1}{3n}=\frac{1}{3}<1.$

$\Rightarrow \lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n}{3^n}=0.$

$\lim_{n\to\infty}\frac{n+3^{-n}}{n-3^{-n}}$

- $=\lim_{n\to\infty}\frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1+\frac{1}{n\cdot 3^n}}{1-\frac{1}{n\cdot 3^n}}=\frac{1+\frac{1}{\infty\cdot\infty}}{1-\frac{1}{\infty\cdot\infty}}=\frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}}=\frac{1+0}{1-0}$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty}\left(\sqrt{n+1}-n\right)=-\infty$$

- $= \lim_{n\to\infty}\sqrt{n}\left(\sqrt{1+\frac{1}{n}}-\sqrt{n}\right) = \infty\cdot\left(\sqrt{1+\frac{1}{\infty}}-\infty\right) = \infty\cdot\left(\sqrt{1+0}-\infty\right)$
  $= \infty\cdot(1-\infty) = \infty\cdot(-\infty) = -\infty.$

$$\lim_{n\to\infty}\frac{n+3^n}{n-3^n}=-1$$

- $= \lim_{n\to\infty}\frac{n+3^n}{n-3^n}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n\to\infty}\frac{\frac{n}{3^n}+1}{\frac{n}{3^n}-1} = \frac{0+1}{0-1} = -1.$

Označme $a_n=\frac{n}{3^n}$, $n\in N$.

- $\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\frac{n}{3^n}} = \lim_{n\to\infty}\frac{\sqrt[n]{n}}{3} = \frac{1}{3} < 1.$

Resp.

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}} = \lim_{n\to\infty}\frac{n+1}{3n} = \frac{1}{3} < 1.$

$\Rightarrow \lim_{n\to\infty}a_n = \lim_{n\to\infty}\frac{n}{3^n} = 0.$

$$\lim_{n\to\infty}\frac{n+3^{-n}}{n-3^{-n}}=1$$

- $= \lim_{n\to\infty}\frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{1+\frac{1}{n\cdot 3^n}}{1-\frac{1}{n\cdot 3^n}} = \frac{1+\frac{1}{\infty\cdot\infty}}{1-\frac{1}{\infty\cdot\infty}} = \frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{1+0}{1-0} = 1.$

Riešené limity – 20

$$\lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)$$

Riešené limity – 20

$$\lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$

Riešené limity – 20

$$\lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$

  $= \lim\limits_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$

Riešené limity – 20

$$\lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)$$

- $= \lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$

$$= \lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$

  $= \lim\limits_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$

  $= \lim\limits_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$

  $= \lim\limits_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)}$

Riešené limity – 20

$$\lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)} = \left[\begin{array}{l}\text{Pre všetky } n\in N \text{ platí}\\ \frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}.\end{array}\right]$$

Riešené limity – 20

$$\lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)} =$$

[Pre všetky $n\in N$ platí $\frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}$.]

$$= \lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)} = \left[\begin{array}{l}\text{Pre všetky } n\in N \text{ platí} \\ \frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}.\end{array}\right]$$

$$= \lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)}$$

$$= \frac{2-\frac{1}{\infty}}{\sqrt[3]{\infty}\cdot\left(\sqrt[3]{(1+\frac{3}{\infty}+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{3}{\infty}+\frac{1}{\infty}}\,\sqrt[3]{1+\frac{1}{\infty}+\frac{2}{\infty}}+\sqrt[3]{(1+\frac{1}{\infty}+\frac{2}{\infty})^2}\right)}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)} = \left[\text{Pre všetky } n\in N \text{ platí } \frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}.\right]$$

$$= \lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)}$$

$$= \frac{2-\frac{1}{\infty}}{\sqrt[3]{\infty}\cdot\left(\sqrt[3]{(1+\frac{3}{\infty}+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{3}{\infty}+\frac{1}{\infty}}\,\sqrt[3]{1+\frac{1}{\infty}+\frac{2}{\infty}}+\sqrt[3]{(1+\frac{1}{\infty}+\frac{2}{\infty})^2}\right)}$$

$$= \frac{2-0}{\infty\cdot\left(\sqrt[3]{(1+0+0)^2}+\sqrt[3]{1+0+0}\,\sqrt[3]{1+0+0}+\sqrt[3]{(1+0+0)^2}\right)}$$

Riešené limity – 20

$\lim\limits_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)$

- $= \lim\limits_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$

$$= \lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)} = \left[\text{Pre všetky } n\in N \text{ platí } \frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3}\cdot n}=\frac{1}{\sqrt[3]{n}}.\right]$$

$$= \lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)}$$

$$= \frac{2-\frac{1}{\infty}}{\sqrt[3]{\infty}\cdot\left(\sqrt[3]{(1+\frac{3}{\infty}+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{3}{\infty}+\frac{1}{\infty}}\,\sqrt[3]{1+\frac{1}{\infty}+\frac{2}{\infty}}+\sqrt[3]{(1+\frac{1}{\infty}+\frac{2}{\infty})^2}\right)}$$

$$= \frac{2-0}{\infty\cdot\left(\sqrt[3]{(1+0+0)^2}+\sqrt[3]{1+0+0}\,\sqrt[3]{1+0+0}+\sqrt[3]{(1+0+0)^2}\right)} = \frac{2}{\infty\cdot(1+1\cdot1+1)}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)\cdot\dfrac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$

$$= \lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)} = \left[\begin{array}{l}\text{Pre všetky } n\in N \text{ platí}\\ \frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3}\cdot n}=\frac{1}{\sqrt[3]{n}}.\end{array}\right]$$

$$= \lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)}$$

$$= \frac{2-\frac{1}{\infty}}{\sqrt[3]{\infty}\cdot\left(\sqrt[3]{(1+\frac{3}{\infty}+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{3}{\infty}+\frac{1}{\infty}}\,\sqrt[3]{1+\frac{1}{\infty}+\frac{2}{\infty}}+\sqrt[3]{(1+\frac{1}{\infty}+\frac{2}{\infty})^2}\right)}$$

$$= \frac{2-0}{\infty\cdot\left(\sqrt[3]{(1+0+0)^2}+\sqrt[3]{1+0+0}\,\sqrt[3]{1+0+0}+\sqrt[3]{(1+0+0)^2}\right)} = \frac{2}{\infty\cdot(1+1\cdot1+1)} = \frac{2}{\infty\cdot3}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)$$

- $= \lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$

$$= \lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)} = \left[\text{Pre všetky } n\in N \text{ platí } \frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}.\right]$$

$$= \lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)}$$

$$= \frac{2-\frac{1}{\infty}}{\sqrt[3]{\infty}\cdot\left(\sqrt[3]{(1+\frac{3}{\infty}+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{3}{\infty}+\frac{1}{\infty}}\,\sqrt[3]{1+\frac{1}{\infty}+\frac{2}{\infty}}+\sqrt[3]{(1+\frac{1}{\infty}+\frac{2}{\infty})^2}\right)}$$

$$= \frac{2-0}{\infty\cdot\left(\sqrt[3]{(1+0+0)^2}+\sqrt[3]{1+0+0}\,\sqrt[3]{1+0+0}+\sqrt[3]{(1+0+0)^2}\right)} = \frac{2}{\infty\cdot(1+1\cdot1+1)} = \frac{2}{\infty\cdot3} = \frac{2}{\infty}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)=0$$

- $= \lim_{n\to\infty}\left(\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right)\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$

$$= \lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)} = \left[\text{Pre všetky } n\in N \text{ platí } \frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}.\right]$$

$$= \lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\cdot\left(\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right)}$$

$$= \frac{2-\frac{1}{\infty}}{\sqrt[3]{\infty}\cdot\left(\sqrt[3]{(1+\frac{3}{\infty}+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{3}{\infty}+\frac{1}{\infty}}\,\sqrt[3]{1+\frac{1}{\infty}+\frac{2}{\infty}}+\sqrt[3]{(1+\frac{1}{\infty}+\frac{2}{\infty})^2}\right)}$$

$$= \frac{2-0}{\infty\cdot\left(\sqrt[3]{(1+0+0)^2}+\sqrt[3]{1+0+0}\,\sqrt[3]{1+0+0}+\sqrt[3]{(1+0+0)^2}\right)} = \frac{2}{\infty\cdot(1+1\cdot 1+1)} = \frac{2}{\infty\cdot 3} = \frac{2}{\infty} = 0.$$

Riešené limity – 21, 22

$\lim\limits_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6$

Riešené limity – 21, 22

$$\lim_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$$

- $= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}}$

- $= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}}$

  - $\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}$

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6$$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^6$

- $= \lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}\right)^6$

Riešené limity – 21, 22

$\lim\limits_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$

- $= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}} = 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}}$

- $= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}}$

- $\lim\limits_{n\to\infty} \frac{8n+1}{4n-3} = \lim\limits_{n\to\infty} \frac{8n+1}{4n-3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^6 = \lim\limits_{n\to\infty} \left(1 - \frac{4}{2n+3}\right)^6$

- $= \lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}\right)^6 = \lim\limits_{n\to\infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}}\right)^6$

Riešené limity – 21, 22

$\lim\limits_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$

- $= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}} = 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}} = 4^{\lim\limits_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}}$

- $= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}}$
  - $\lim\limits_{n\to\infty} \frac{8n+1}{4n-3} = \lim\limits_{n\to\infty} \frac{8n+1}{4n-3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim\limits_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}$

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^6 = \lim\limits_{n\to\infty} \left(1 - \frac{4}{2n+3}\right)^6 = \left(1 - \frac{4}{2\cdot\infty+3}\right)^6$

- $= \lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}\right)^6 = \lim\limits_{n\to\infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}}\right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}}\right)^6$

Riešené limity – 21, 22

$\lim\limits_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$

- $= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}} = 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}} = 4^{\lim\limits_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}}$

- $= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}}$
  - $\lim\limits_{n\to\infty} \frac{8n+1}{4n-3} = \lim\limits_{n\to\infty} \frac{8n+1}{4n-3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim\limits_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}} = \frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}$

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^6 = \lim\limits_{n\to\infty} \left(1 - \frac{4}{2n+3}\right)^6 = \left(1 - \frac{4}{2\cdot\infty+3}\right)^6 = \left(1 - \frac{4}{\infty}\right)^6$

- $= \lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}\right)^6 = \lim\limits_{n\to\infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}}\right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}}\right)^6 = \left(\frac{2-0}{2+0}\right)^6$

Riešené limity – 21, 22

$\lim\limits_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$

- $= 4^{\lim\limits_{n\to\infty}\frac{8n+1}{4n-3}} = 4^{\lim\limits_{n\to\infty}\frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}} = 4^{\lim\limits_{n\to\infty}\frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}} = 4^{\frac{8+0}{4-0}}$

- $= 4^{\lim\limits_{n\to\infty}\frac{8n+1}{4n-3}}$
  - $\lim\limits_{n\to\infty}\frac{8n+1}{4n-3} = \lim\limits_{n\to\infty}\frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim\limits_{n\to\infty}\frac{8+\frac{1}{n}}{4-\frac{3}{n}} = \frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}} = \frac{8+0}{4-0}$

$\lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^6$

- $= \lim\limits_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^6 = \lim\limits_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^6 = \left(1-\frac{4}{2\cdot\infty+3}\right)^6 = \left(1-\frac{4}{\infty}\right)^6$
  $= (1-0)^6 = 1^6$

- $= \lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}\right)^6 = \lim\limits_{n\to\infty}\left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}}\right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}}\right)^6 = \left(\frac{2-0}{2+0}\right)^6 = 1^6$

Riešené limity – 21, 22

$\lim\limits_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$

- $= 4^{\lim\limits_{n\to\infty}\frac{8n+1}{4n-3}} = 4^{\lim\limits_{n\to\infty}\frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}} = 4^{\lim\limits_{n\to\infty}\frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}} = 4^{\frac{8+0}{4-0}} = 4^2$

- $= 4^{\lim\limits_{n\to\infty}\frac{8n+1}{4n-3}} = 4^2$
  - $\lim\limits_{n\to\infty}\frac{8n+1}{4n-3} = \lim\limits_{n\to\infty}\frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim\limits_{n\to\infty}\frac{8+\frac{1}{n}}{4-\frac{3}{n}} = \frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}} = \frac{8+0}{4-0} = 2.$

$\lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^6 = 1$

- $= \lim\limits_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^6 = \lim\limits_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^6 = \left(1-\frac{4}{2\cdot\infty+3}\right)^6 = \left(1-\frac{4}{\infty}\right)^6$
  $= (1-0)^6 = 1^6 = 1.$

- $= \lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}\right)^6 = \lim\limits_{n\to\infty}\left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}}\right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}}\right)^6 = \left(\frac{2-0}{2+0}\right)^6 = 1^6 = 1.$

Riešené limity – 21, 22

$\lim\limits_{n\to\infty} 4^{\frac{8n+1}{4n-3}} = 16$

- $= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}} = 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}} = 4^{\lim\limits_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}} = 4^{\frac{8+0}{4-0}} = 4^2 = 16.$

- $= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}} = 4^2 = 16.$
  - $\lim\limits_{n\to\infty} \frac{8n+1}{4n-3} = \lim\limits_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim\limits_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}} = \frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}} = \frac{8+0}{4-0} = 2.$

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6 = 1$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^6 = \lim\limits_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^6 = \left(1-\frac{4}{2\cdot\infty+3}\right)^6 = \left(1-\frac{4}{\infty}\right)^6$
  $= (1-0)^6 = 1^6 = 1.$

- $= \lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}\right)^6 = \lim\limits_{n\to\infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}}\right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}}\right)^6 = \left(\frac{2-0}{2+0}\right)^6 = 1^6 = 1.$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

- $= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+\cdots+n = \frac{(1+n)n}{2}. \end{array} \right]$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

- $= \lim_{n\to\infty} \frac{n^2}{}$

Pre konečné aritmetické rady platí:

- $1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2.$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

- $= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+\cdots+n = \frac{(1+n)n}{2}. \end{array} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}}$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

- $= \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}}$

Pre konečné aritmetické rady platí:

- $1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1+2+3+\cdots+n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

- $= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+\cdots+n = \frac{(1+n)n}{2}. \end{array} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}}$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

- $= \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n\to\infty} \frac{2n}{n+1}$

Pre konečné aritmetické rady platí:

- $1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1+2+3+\cdots+n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

- $= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+\cdots+n=\frac{(1+n)n}{2}. \end{array} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ n^2=n\cdot n=\sqrt{n^4}. \end{array} \right]$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

- $= \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{n\cdot 2}{n\cdot(1+\frac{1}{n})}$

Pre konečné aritmetické rady platí:

- $1+3+5+\cdots+(2n-1)=(2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1)=\frac{(1+2n-1)n}{2}=n^2.$
- $1+2+3+\cdots+n=\frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

- $= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+\cdots+n = \frac{(1+n)n}{2}. \end{array} \right] = \lim\limits_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim\limits_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ n^2 = n\cdot n = \sqrt{n^4}. \end{array} \right]$

  $= \lim\limits_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}}$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

- $= \lim\limits_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim\limits_{n\to\infty} \frac{2n}{n+1} = \lim\limits_{n\to\infty} \frac{n\cdot 2}{n\cdot(1+\frac{1}{n})} = \lim\limits_{n\to\infty} \frac{2}{1+\frac{1}{n}}$

Pre konečné aritmetické rady platí:

- $1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1+2+3+\cdots+n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

- $= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+\cdots+n=\frac{(1+n)n}{2}. \end{array} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ n^2=n\cdot n=\sqrt{n^4}. \end{array} \right]$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

- $= \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{n\cdot 2}{n\cdot(1+\frac{1}{n})} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}}$

Pre konečné aritmetické rady platí:

- $1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1+2+3+\cdots+n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$\lim\limits_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$

- $= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+\cdots+n = \frac{(1+n)n}{2}. \end{array} \right] = \lim\limits_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim\limits_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ n^2 = n\cdot n = \sqrt{n^4}. \end{array} \right]$

  $= \lim\limits_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim\limits_{n\to\infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}} = \lim\limits_{n\to\infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}}$

$\lim\limits_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$

- $= \lim\limits_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim\limits_{n\to\infty} \frac{2n}{n+1} = \lim\limits_{n\to\infty} \frac{n\cdot 2}{n\cdot(1+\frac{1}{n})} = \lim\limits_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0}$

Pre konečné aritmetické rady platí:

- $1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1+2+3+\cdots+n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

- $= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+\cdots+n = \frac{(1+n)n}{2}. \end{array} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ n^2 = n\cdot n = \sqrt{n^4}. \end{array} \right]$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n} = 2$$

- $= \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{n\cdot 2}{n\cdot(1+\frac{1}{n})} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$

Pre konečné aritmetické rady platí:

- $1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1+2+3+\cdots+n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

- $= \left[\begin{array}{c}\text{Aritmetický rad} \\ 1+2+\cdots+n=\frac{(1+n)n}{2}.\end{array}\right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí} \\ n^2=n\cdot n=\sqrt{n^4}.\end{array}\right]$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}}$$

$$= \frac{1+0}{2\sqrt{9+0}}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n} = 2$$

- $= \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{n\cdot 2}{n\cdot(1+\frac{1}{n})} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$

Pre konečné aritmetické rady platí:

- $1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1+2+3+\cdots+n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

- $= \left[\begin{array}{c}\text{Aritmetický rad} \\ 1+2+\cdots+n=\frac{(1+n)n}{2}.\end{array}\right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí} \\ n^2=n\cdot n=\sqrt{n^4}.\end{array}\right]$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}}$$

$$= \frac{1+0}{2\sqrt{9+0}} = \frac{1+0}{2\cdot 3}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n} = 2$$

- $= \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{n\cdot 2}{n\cdot(1+\frac{1}{n})} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$

Pre konečné aritmetické rady platí:

- $1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1+2+3+\cdots+n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}} = \frac{1}{6}$$

- $= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+\cdots+n = \frac{(1+n)n}{2}. \end{array} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ n^2 = n\cdot n = \sqrt{n^4}. \end{array} \right]$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}}$$

$$= \frac{1+0}{2\sqrt{9+0}} = \frac{1+0}{2\cdot 3} = \frac{1}{6}.$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n} = 2$$

- $= \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{n\cdot 2}{n\cdot(1+\frac{1}{n})} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$

Pre konečné aritmetické rady platí:

- $1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1+2+3+\cdots+n = \frac{(1+n)n}{2}.$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

- $= \lim_{n\to\infty} \dfrac{\frac{n(n+1)(2n+1)}{6}}{}$

Pre konečný číselný rad platí:

- $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

- $= \lim_{n\to\infty} \dfrac{\frac{n(4n^2-1)}{3}}{}$

Pre konečný číselný rad platí:

- $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \dfrac{n(2n+1)(2n-1)}{3} = \dfrac{n(4n^2-1)}{3}$.

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

- $= \lim\limits_{n\to\infty} \dfrac{\frac{n(n+1)(2n+1)}{6}}{n^3}$

Pre konečný číselný rad platí:

- $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}.$

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

- $= \lim\limits_{n\to\infty} \dfrac{\frac{n(4n^2-1)}{3}}{n^3}$

Pre konečný číselný rad platí:

- $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \dfrac{n(2n+1)(2n-1)}{3} = \dfrac{n(4n^2-1)}{3}.$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

- $= \lim\limits_{n\to\infty} \dfrac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim\limits_{n\to\infty} \dfrac{(n+1)(2n+1)}{6n^2} = \lim\limits_{n\to\infty} \dfrac{(n+1)(2n+1)}{6n\cdot n}$

Pre konečný číselný rad platí:

- $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

- $= \lim\limits_{n\to\infty} \dfrac{\frac{n(4n^2-1)}{3}}{n^3} = \lim\limits_{n\to\infty} \dfrac{4n^2-1}{3n^2}$

Pre konečný číselný rad platí:

- $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \dfrac{n(2n+1)(2n-1)}{3} = \dfrac{n(4n^2-1)}{3}$.

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

- $= \lim\limits_{n\to\infty} \dfrac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim\limits_{n\to\infty} \dfrac{(n+1)(2n+1)}{6n^2} = \lim\limits_{n\to\infty} \dfrac{(n+1)(2n+1)}{6n\cdot n}$

  $= \frac{1}{6} \lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)\cdot\left(2+\frac{1}{n}\right)$

Pre konečný číselný rad platí:

- $1^2+2^2+3^2+\cdots+n^2 = \dfrac{n(n+1)(2n+1)}{6}.$

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

- $= \lim\limits_{n\to\infty} \dfrac{\frac{n(4n^2-1)}{3}}{n^3} = \lim\limits_{n\to\infty} \dfrac{4n^2-1}{3n^2} = \lim\limits_{n\to\infty} \left(\frac{4}{3} - \frac{1}{3n^2}\right)$

Pre konečný číselný rad platí:

- $1^2+3^2+5^2+\cdots+(2n-1)^2 = \dfrac{n(2n+1)(2n-1)}{3} = \dfrac{n(4n^2-1)}{3}.$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

- $= \lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n\cdot n}$
  $= \frac{1}{6} \lim_{n\to\infty} (1+\frac{1}{n})\cdot(2+\frac{1}{n}) = \frac{1}{6}(1+\frac{1}{\infty})\cdot(2+\frac{1}{\infty})$

Pre konečný číselný rad platí:

- $1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$.

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

- $= \lim_{n\to\infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n\to\infty} \frac{4n^2-1}{3n^2} = \lim_{n\to\infty} (\frac{4}{3} - \frac{1}{3n^2}) = \frac{4}{3} - \frac{1}{\infty}$

Pre konečný číselný rad platí:

- $1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}$.

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

- $= \lim\limits_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim\limits_{n\to\infty} \frac{(n+1)(2n+1)}{6n^2} = \lim\limits_{n\to\infty} \frac{(n+1)(2n+1)}{6n\cdot n}$

  $= \frac{1}{6} \lim\limits_{n\to\infty} (1+\frac{1}{n})\cdot(2+\frac{1}{n}) = \frac{1}{6}(1+\frac{1}{\infty})\cdot(2+\frac{1}{\infty}) = \frac{1}{6}(1+0)\cdot(2+0)$

Pre konečný číselný rad platí:

- $1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$.

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

- $= \lim\limits_{n\to\infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim\limits_{n\to\infty} \frac{4n^2-1}{3n^2} = \lim\limits_{n\to\infty} (\frac{4}{3} - \frac{1}{3n^2}) = \frac{4}{3} - \frac{1}{\infty} = \frac{4}{3} - 0$

Pre konečný číselný rad platí:

- $1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}$.

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

- $= \lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n\cdot n}$
  $= \frac{1}{6} \lim_{n\to\infty} (1+\frac{1}{n})\cdot(2+\frac{1}{n}) = \frac{1}{6}(1+\frac{1}{\infty})\cdot(2+\frac{1}{\infty}) = \frac{1}{6}(1+0)\cdot(2+0) = \frac{1}{6}\cdot 1\cdot 2$

Pre konečný číselný rad platí:

- $1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}.$

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3} = \frac{4}{3}$$

- $= \lim_{n\to\infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n\to\infty} \frac{4n^2-1}{3n^2} = \lim_{n\to\infty} \left(\frac{4}{3} - \frac{1}{3n^2}\right) = \frac{4}{3} - \frac{1}{\infty} = \frac{4}{3} - 0 = \frac{4}{3}.$

Pre konečný číselný rad platí:

- $1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3} = \frac{1}{3}$$

- $= \lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n\cdot n}$

  $= \frac{1}{6} \lim_{n\to\infty} (1+\frac{1}{n})\cdot(2+\frac{1}{n}) = \frac{1}{6}(1+\frac{1}{\infty})\cdot(2+\frac{1}{\infty}) = \frac{1}{6}(1+0)\cdot(2+0) = \frac{1}{6}\cdot 1\cdot 2 = \frac{1}{3}.$

Pre konečný číselný rad platí:

- $1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}.$

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3} = \frac{4}{3}$$

- $= \lim_{n\to\infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n\to\infty} \frac{4n^2-1}{3n^2} = \lim_{n\to\infty} (\frac{4}{3} - \frac{1}{3n^2}) = \frac{4}{3} - \frac{1}{\infty} = \frac{4}{3} - 0 = \frac{4}{3}.$

Pre konečný číselný rad platí:

- $1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

- $= \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

- $= \lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

- $= \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

- $= \lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

- $= \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$

  $= \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1}$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

- $= \lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$

  $= \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2}$

Riešené limity – 27, 28

$\lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$

  $= \lim\limits_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1}$

$\lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim\limits_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$

  $= \lim\limits_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim\limits_{n\to\infty} \sqrt{n^2} \cdot \left(\sqrt{1-\frac{1}{n}+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^2}}\right)$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

- $= \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$

  $= \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1}$

  $= -\sqrt{\infty} - \sqrt{\infty}$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

- $= \lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$

  $= \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim_{n\to\infty} \sqrt{n^2} \cdot \left(\sqrt{1-\frac{1}{n}+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^2}}\right)$

- $\lim_{n\to\infty} \sqrt{n^2} = \lim_{n\to\infty} n = \infty.$
- $\lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0.$
- $\lim_{n\to\infty} \frac{1}{n^2} = \frac{1}{\infty^2} = \frac{1}{\infty} = 0.$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$

  $= \lim\limits_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1}$

  $= -\sqrt{\infty} - \sqrt{\infty} = -\infty - \infty$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim\limits_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$

  $= \lim\limits_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim\limits_{n\to\infty} \sqrt{n^2} \cdot \left(\sqrt{1-\frac{1}{n}+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^2}}\right)$

  $= \frac{1}{2} \cdot \infty \cdot \left(\sqrt{1-0+0} + \sqrt{1-0-0}\right)$

- $\lim\limits_{n\to\infty} \sqrt{n^2} = \lim\limits_{n\to\infty} n = \infty.$
- $\lim\limits_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0.$
- $\lim\limits_{n\to\infty} \frac{1}{n^2} = \frac{1}{\infty^2} = \frac{1}{\infty} = 0.$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} = -\infty$$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$

$= \lim\limits_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1}$

$= -\sqrt{\infty} - \sqrt{\infty} = -\infty - \infty = -\infty.$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim\limits_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$

$= \lim\limits_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim\limits_{n\to\infty} \sqrt{n^2} \cdot \left(\sqrt{1-\frac{1}{n}+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^2}}\right)$

$= \frac{1}{2} \cdot \infty \cdot \left(\sqrt{1-0+0} + \sqrt{1-0-0}\right) = \frac{1}{2} \cdot \infty \cdot 2$

- $\lim\limits_{n\to\infty} \sqrt{n^2} = \lim\limits_{n\to\infty} n = \infty.$
- $\lim\limits_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0.$
- $\lim\limits_{n\to\infty} \frac{1}{n^2} = \frac{1}{\infty^2} = \frac{1}{\infty} = 0.$

Riešené limity – 27, 28

$\lim\limits_{n\to\infty} \dfrac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} = -\infty$

- $= \lim\limits_{n\to\infty} \dfrac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \dfrac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty} \dfrac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$

$= \lim\limits_{n\to\infty} \dfrac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \dfrac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1}$

$= -\sqrt{\infty} - \sqrt{\infty} = -\infty - \infty = -\infty.$

$\lim\limits_{n\to\infty} \dfrac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} = \infty$

- $= \lim\limits_{n\to\infty} \dfrac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \dfrac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim\limits_{n\to\infty} \dfrac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$

$= \lim\limits_{n\to\infty} \dfrac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \dfrac{1}{2} \lim\limits_{n\to\infty} \sqrt{n^2} \cdot \left(\sqrt{1-\frac{1}{n}+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^2}}\right)$

$= \frac{1}{2} \cdot \infty \cdot \left(\sqrt{1-0+0} + \sqrt{1-0-0}\right) = \frac{1}{2} \cdot \infty \cdot 2 = \infty.$

- $\lim\limits_{n\to\infty} \sqrt{n^2} = \lim\limits_{n\to\infty} n = \infty.$
- $\lim\limits_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0.$
- $\lim\limits_{n\to\infty} \frac{1}{n^2} = \frac{1}{\infty^2} = \frac{1}{\infty} = 0.$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim\limits_{n\to\infty} n \cdot \ln \frac{n}{n+2}$

- $= \lim\limits_{n\to\infty} n \cdot \ln \frac{n}{n+2}$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim\limits_{n\to\infty} n \cdot \ln \frac{n}{n+2} = \lim\limits_{n\to\infty} \ln \left(\frac{n}{n+2}\right)^n$

- $= \lim\limits_{n\to\infty} n \cdot \ln \frac{n}{n+2} = \lim\limits_{n\to\infty} \ln \left(\frac{n}{n+2}\right)^n$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim\limits_{n\to\infty} n \cdot \ln \frac{n}{n+2} = \lim\limits_{n\to\infty} \ln \left(\frac{n}{n+2}\right)^n = \lim\limits_{n\to\infty} \ln \left(\frac{n+2}{n}\right)^{-n}$

- $= \lim\limits_{n\to\infty} n \cdot \ln \frac{n}{n+2} = \lim\limits_{n\to\infty} \ln \left(\frac{n}{n+2}\right)^n = \lim\limits_{n\to\infty} \ln \left(\frac{n+2-2}{n+2}\right)^n$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim_{n\to\infty} n \cdot \ln \frac{n}{n+2} = \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2}{n}\right)^{-n} = \lim_{n\to\infty} \ln\left(1+\frac{2}{n}\right)^{-n}$

- $= \lim_{n\to\infty} n \cdot \ln \frac{n}{n+2} = \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2-2}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(1+\frac{-2}{n+2}\right)^n$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2}{n}\right)^{-n} = \lim_{n\to\infty} \ln\left(1+\frac{2}{n}\right)^{-n}$

$$= \lim_{n\to\infty} \ln\Big(\left(1+\tfrac{2}{n}\right)^n\Big)^{-1}$$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2-2}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(1+\frac{-2}{n+2}\right)^n$

$$= \lim_{n\to\infty} \ln\Big(\left(1+\tfrac{-2}{n+2}\right)^{n+2}\Big)^{\frac{n}{n+2}}$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2}{n}\right)^{-n} = \lim_{n\to\infty} \ln\left(1+\frac{2}{n}\right)^{-n}$

$$= \lim_{n\to\infty} \ln\left(\left(1+\frac{2}{n}\right)^n\right)^{-1} = \ln\left(e^2\right)^{-1}$$

Pre všetky $a\in R$ platí:

- $\lim_{n\to\infty} \left(1+\frac{a}{n}\right)^n = e^a.$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2-2}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(1+\frac{-2}{n+2}\right)^n$

$$= \lim_{n\to\infty} \ln\left(\left(1+\frac{-2}{n+2}\right)^{n+2}\right)^{\frac{n}{n+2}} = \ln\left(e^{-2}\right)^1$$

- $\lim_{n\to\infty} \left(1+\frac{-2}{n+2}\right)^{n+2} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty \end{array}\right] = \lim_{m\to\infty} \left(1+\frac{-2}{m}\right)^m = e^{-2}.$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2}{n}\right)^{-n} = \lim_{n\to\infty} \ln\left(1+\frac{2}{n}\right)^{-n}$

$$= \lim_{n\to\infty} \ln\Big(\left(1+\tfrac{2}{n}\right)^n\Big)^{-1} = \ln\left(e^2\right)^{-1} = \ln e^{-2}$$

Pre všetky $a\in R$ platí:

- $\lim_{n\to\infty} \left(1+\frac{a}{n}\right)^n = e^a.$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2-2}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(1+\frac{-2}{n+2}\right)^n$

$$= \lim_{n\to\infty} \ln\Big(\left(1+\tfrac{-2}{n+2}\right)^{n+2}\Big)^{\frac{n}{n+2}} = \ln\left(e^{-2}\right)^1$$

- $\lim_{n\to\infty} \left(1+\frac{-2}{n+2}\right)^{n+2} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty \end{array}\right] = \lim_{m\to\infty} \left(1+\frac{-2}{m}\right)^m = e^{-2}.$
- $\lim_{n\to\infty} \frac{n}{n+2} = \lim_{n\to\infty} \frac{n}{n\cdot(1+\frac{2}{n})} = \lim_{n\to\infty} \frac{1}{1+\frac{2}{n}} = \frac{1}{1+\frac{2}{\infty}} = \frac{1}{1+0} = 1.$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim\limits_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim\limits_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim\limits_{n\to\infty} \ln\left(\frac{n+2}{n}\right)^{-n} = \lim\limits_{n\to\infty} \ln\left(1+\frac{2}{n}\right)^{-n}$
  $= \lim\limits_{n\to\infty} \ln\left(\left(1+\frac{2}{n}\right)^n\right)^{-1} = \ln\left(e^2\right)^{-1} = \ln e^{-2} = -2.$

Pre všetky $a\in R$ platí:

- $\lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a.$

- $= \lim\limits_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim\limits_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim\limits_{n\to\infty} \ln\left(\frac{n+2-2}{n+2}\right)^n = \lim\limits_{n\to\infty} \ln\left(1+\frac{-2}{n+2}\right)^n$
  $= \lim\limits_{n\to\infty} \ln\left(\left(1+\frac{-2}{n+2}\right)^{n+2}\right)^{\frac{n}{n+2}} = \ln\left(e^{-2}\right)^1 = \ln e^{-2}$

- $\lim\limits_{n\to\infty}\left(1+\frac{-2}{n+2}\right)^{n+2} = \left[\begin{array}{c|c}\text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty\end{array}\right] = \lim\limits_{m\to\infty}\left(1+\frac{-2}{m}\right)^m = e^{-2}.$
- $\lim\limits_{n\to\infty}\frac{n}{n+2} = \lim\limits_{n\to\infty}\frac{n}{n\cdot(1+\frac{2}{n})} = \lim\limits_{n\to\infty}\frac{1}{1+\frac{2}{n}} = \frac{1}{1+\frac{2}{\infty}} = \frac{1}{1+0} = 1.$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty}\ln\left(\frac{n+2}{n}\right)^{-n} = \lim_{n\to\infty}\ln\left(1+\frac{2}{n}\right)^{-n}$
  $= \lim_{n\to\infty}\ln\left(\left(1+\frac{2}{n}\right)^n\right)^{-1} = \ln\left(e^2\right)^{-1} = \ln e^{-2} = -2.$

Pre všetky $a\in R$ platí:

- $\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a.$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty}\ln\left(\frac{n+2-2}{n+2}\right)^n = \lim_{n\to\infty}\ln\left(1+\frac{-2}{n+2}\right)^n$
  $= \lim_{n\to\infty}\ln\left(\left(1+\frac{-2}{n+2}\right)^{n+2}\right)^{\frac{n}{n+2}} = \ln\left(e^{-2}\right)^1 = \ln e^{-2} = -2.$

- $\lim_{n\to\infty}\left(1+\frac{-2}{n+2}\right)^{n+2} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\left(1+\frac{-2}{m}\right)^m = e^{-2}.$
- $\lim_{n\to\infty}\frac{n}{n+2} = \lim_{n\to\infty}\frac{n}{n\cdot(1+\frac{2}{n})} = \lim_{n\to\infty}\frac{1}{1+\frac{2}{n}} = \frac{1}{1+\frac{2}{\infty}} = \frac{1}{1+0} = 1.$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim\limits_{n\to\infty} n \cdot \ln \frac{n}{n+2}$

- $= \lim\limits_{n\to\infty} n \cdot \ln \frac{n}{n+2}$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim_{n\to\infty} n \cdot \ln \frac{n}{n+2} = \lim_{n\to\infty} \left(-n \cdot \ln \frac{n+2}{n}\right)$

- $= \lim_{n\to\infty} n \cdot \ln \frac{n}{n+2} = \lim_{n\to\infty} \ln \left(\frac{n}{n+2}\right)^n$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\left(-n\cdot\ln\frac{n+2}{n}\right) = -\lim_{n\to\infty} n\cdot\ln\frac{n+2}{n}$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty}\ln\left(\frac{n+2-2}{n+2}\right)^n$

Riešené limity – 29

$\lim\limits_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$

- $= \lim\limits_{n\to\infty} n \cdot \ln \frac{n}{n+2} = \lim\limits_{n\to\infty} \Big(-n \cdot \ln \frac{n+2}{n}\Big) = -\lim\limits_{n\to\infty} n \cdot \ln \frac{n+2}{n} = -\lim\limits_{n\to\infty} \ln\big(1+\frac{2}{n}\big)^n$

- $= \lim\limits_{n\to\infty} n \cdot \ln \frac{n}{n+2} = \lim\limits_{n\to\infty} \ln\big(\frac{n}{n+2}\big)^n = \lim\limits_{n\to\infty} \ln\big(\frac{n+2-2}{n+2}\big)^n = \lim\limits_{n\to\infty} \ln\big(1+\frac{-2}{n+2}\big)^n$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim_{n\to\infty} n \cdot \ln \frac{n}{n+2} = \lim_{n\to\infty} \Big(-n \cdot \ln \frac{n+2}{n}\Big) = -\lim_{n\to\infty} n \cdot \ln \frac{n+2}{n} = -\lim_{n\to\infty} \ln\big(1 + \frac{2}{n}\big)^n$
  $= -\ln e^2$

Pre všetky $a \in R$ platí:

- $\lim_{n\to\infty} \big(1 + \frac{a}{n}\big)^n = e^a$.

- $= \lim_{n\to\infty} n \cdot \ln \frac{n}{n+2} = \lim_{n\to\infty} \ln\big(\frac{n}{n+2}\big)^n = \lim_{n\to\infty} \ln\big(\frac{n+2-2}{n+2}\big)^n = \lim_{n\to\infty} \ln\big(1 + \frac{-2}{n+2}\big)^n$
  $= \lim_{n\to\infty} \ln\big(1 + \frac{-2}{n+2}\big)^{n+2-2}$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\left(-n\cdot\ln\frac{n+2}{n}\right) = -\lim_{n\to\infty} n\cdot\ln\frac{n+2}{n} = -\lim_{n\to\infty}\ln\left(1+\frac{2}{n}\right)^n$
  $= -\ln e^2 = -2.$

Pre všetky $a\in R$ platí:

- $\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a.$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty}\ln\left(\frac{n+2-2}{n+2}\right)^n = \lim_{n\to\infty}\ln\left(1+\frac{-2}{n+2}\right)^n$
  $= \lim_{n\to\infty}\ln\left(1+\frac{-2}{n+2}\right)^{n+2-2} = \lim_{n\to\infty}\ln\Big(\left(1+\frac{-2}{n+2}\right)^{n+2}\cdot\left(1+\frac{-2}{n+2}\right)^{-2}\Big)$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\Big(-n\cdot\ln\frac{n+2}{n}\Big) = -\lim_{n\to\infty} n\cdot\ln\frac{n+2}{n} = -\lim_{n\to\infty}\ln\big(1+\frac{2}{n}\big)^n$
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  $= \lim_{n\to\infty}\ln\big(1+\frac{-2}{n+2}\big)^{n+2-2} = \lim_{n\to\infty}\ln\Big(\big(1+\frac{-2}{n+2}\big)^{n+2}\cdot\big(1+\frac{-2}{n+2}\big)^{-2}\Big)$
  $= \ln\big(e^{-2}\cdot 1\big)$

- $\lim_{n\to\infty}\big(1+\frac{-2}{n+2}\big)^{n+2} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\big(1+\frac{-2}{m}\big)^m = e^{-2}.$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$$

- $= \lim\limits_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim\limits_{n\to\infty}\Big(-n\cdot\ln\frac{n+2}{n}\Big) = -\lim\limits_{n\to\infty} n\cdot\ln\frac{n+2}{n} = -\lim\limits_{n\to\infty}\ln\big(1+\frac{2}{n}\big)^n$
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- $\lim\limits_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a.$

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  $= \lim\limits_{n\to\infty}\ln\big(1+\frac{-2}{n+2}\big)^{n+2-2} = \lim\limits_{n\to\infty}\ln\Big(\big(1+\frac{-2}{n+2}\big)^{n+2}\cdot\big(1+\frac{-2}{n+2}\big)^{-2}\Big)$
  $= \ln\big(e^{-2}\cdot 1\big)$

- $\lim\limits_{n\to\infty}\big(1+\frac{-2}{n+2}\big)^{n+2} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty\end{array}\right] = \lim\limits_{m\to\infty}\big(1+\frac{-2}{m}\big)^m = e^{-2}.$
- $\lim\limits_{n\to\infty}\big(1+\frac{-2}{n+2}\big)^{-2} = \big(1+\frac{-2}{\infty}\big)^{-2} = (1+0)^{-2} = 1^{-2} = 1.$

Riešené limity – 29

$\lim\limits_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big)$

- $$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\Big(-n\cdot\ln\frac{n+2}{n}\Big) = -\lim_{n\to\infty} n\cdot\ln\frac{n+2}{n} = -\lim_{n\to\infty}\ln\big(1+\tfrac{2}{n}\big)^n$$
$$= -\ln e^2 = -2.$$

Pre všetky $a\in R$ platí:

- $\lim\limits_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a.$

- $$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\ln\big(\tfrac{n}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(\tfrac{n+2-2}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^n$$
$$= \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^{n+2-2} = \lim_{n\to\infty}\ln\Big(\big(1+\tfrac{-2}{n+2}\big)^{n+2}\cdot\big(1+\tfrac{-2}{n+2}\big)^{-2}\Big)$$
$$= \ln\big(e^{-2}\cdot 1\big) = \ln e^{-2}$$

- $\lim\limits_{n\to\infty}\big(1+\frac{-2}{n+2}\big)^{n+2} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty\end{array}\right] = \lim\limits_{m\to\infty}\big(1+\frac{-2}{m}\big)^m = e^{-2}.$
- $\lim\limits_{n\to\infty}\big(1+\frac{-2}{n+2}\big)^{-2} = \big(1+\frac{-2}{\infty}\big)^{-2} = (1+0)^{-2} = 1^{-2} = 1.$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big(\ln n - \ln(n+2)\Big) = -2$$

- $= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\Big(-n\cdot\ln\frac{n+2}{n}\Big) = -\lim_{n\to\infty} n\cdot\ln\frac{n+2}{n} = -\lim_{n\to\infty}\ln\big(1+\frac{2}{n}\big)^n$
  $= -\ln e^2 = -2.$

Pre všetky $a\in R$ platí:

- $\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a.$

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  $= \lim_{n\to\infty}\ln\big(1+\frac{-2}{n+2}\big)^{n+2-2} = \lim_{n\to\infty}\ln\Big(\big(1+\frac{-2}{n+2}\big)^{n+2}\cdot\big(1+\frac{-2}{n+2}\big)^{-2}\Big)$
  $= \ln\big(e^{-2}\cdot 1\big) = \ln e^{-2} = -2.$

- $\lim_{n\to\infty}\big(1+\frac{-2}{n+2}\big)^{n+2} = \left[\begin{array}{c|c}\text{Subst.} & n\to\infty\\ m=n+2 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\big(1+\frac{-2}{m}\big)^m = e^{-2}.$
- $\lim_{n\to\infty}\big(1+\frac{-2}{n+2}\big)^{-2} = \big(1+\frac{-2}{\infty}\big)^{-2} = (1+0)^{-2} = 1^{-2} = 1.$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big(\ln(n+3) - \ln n\Big) = \lim_{n\to\infty} n \cdot \ln \tfrac{n+3}{n}$$

$$\lim_{n\to\infty} \left(\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right)$$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big(\ln(n+3) - \ln n\Big) = \lim_{n\to\infty} n \cdot \ln \tfrac{n+3}{n}$$

- $= \lim\limits_{n\to\infty} n \cdot \ln\left(1 + \frac{3}{n}\right)$

$$\lim_{n\to\infty} \left(\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right)$$

- $= \lim\limits_{n\to\infty} \left(\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right) \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

- $= \lim\limits_{n\to\infty} \left(\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right) \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big(\ln(n+3)-\ln n\Big) = \lim_{n\to\infty} n\cdot\ln\frac{n+3}{n}$$

- $= \lim_{n\to\infty} n\cdot\ln(1+\frac{3}{n}) = \lim_{n\to\infty} \ln(1+\frac{3}{n})^n$

$$\lim_{n\to\infty}\Big(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\Big)$$

- $= \lim_{n\to\infty}(\sqrt{n^2+n+1}-\sqrt{n^2+n+2})\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

- $= \lim_{n\to\infty}(\sqrt{n^2+n+1}-\sqrt{n^2+n+2})\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big(\ln(n+3) - \ln n\Big) = \lim_{n\to\infty} n \cdot \ln \frac{n+3}{n}$$

- $= \lim_{n\to\infty} n \cdot \ln\left(1+\frac{3}{n}\right) = \lim_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n =$ [Pre všetky $a \in R$ platí $\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a$.]

$$\lim_{n\to\infty} \left(\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right)$$

- $= \lim_{n\to\infty} \left(\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right) \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty} \frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

  $= \lim_{n\to\infty} \frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

- $= \lim_{n\to\infty} \left(\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right) \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty} \frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

  $= \lim_{n\to\infty} \frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big(\ln(n+3)-\ln n\Big) = \lim_{n\to\infty} n\cdot\ln\frac{n+3}{n}$$

- $= \lim_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right) = \lim_{n\to\infty}\ln\left(1+\frac{3}{n}\right)^n = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí}\\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a\,.\end{array}\right] = \ln e^3$

$$\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right)$$

- $= \lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right)\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$
  $= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \frac{-1}{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}$

- $= \lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right)\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$
  $= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{-1}{\sqrt{n^2}\cdot\left(\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}}\right)}$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big(\ln(n+3)-\ln n\Big) = \lim_{n\to\infty} n\cdot\ln\tfrac{n+3}{n} = 3$$

- $= \lim_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right) = \lim_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí}\\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a\,.\end{array}\right] = \ln e^3 = 3.$

$$\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right)$$

- $= \lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right)\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

  $= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \frac{-1}{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}} = \frac{-1}{\sqrt{\infty}+\sqrt{\infty}}$

- $= \lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right)\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

  $= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{-1}{\sqrt{n^2}\cdot\left(\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}}\right)}$

- $\lim_{n\to\infty}\sqrt{n^2} = \lim_{n\to\infty} n = \infty.$
- $\lim_{n\to\infty}\frac{1}{n} = \frac{1}{\infty} = 0.$
- $\lim_{n\to\infty}\frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0$ pre $a\in R$.

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big(\ln(n+3)-\ln n\Big) = \lim_{n\to\infty} n\cdot\ln\tfrac{n+3}{n} = 3$$

- $= \lim\limits_{n\to\infty} n\cdot\ln\big(1+\frac{3}{n}\big) = \lim\limits_{n\to\infty} \ln\big(1+\frac{3}{n}\big)^n = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a\,.\end{array}\right] = \ln e^3 = 3.$

$$\lim_{n\to\infty}\Big(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\Big)$$

- $= \lim\limits_{n\to\infty}\big(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\big)\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

  $= \lim\limits_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \frac{-1}{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}} = \frac{-1}{\sqrt{\infty}+\sqrt{\infty}} = \frac{-1}{\infty}$

- $= \lim\limits_{n\to\infty}\big(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\big)\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

  $= \lim\limits_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty}\frac{-1}{\sqrt{n^2}\cdot\Big(\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}}\Big)}$

  $= \frac{-1}{\infty\cdot(\sqrt{1-0+0}+\sqrt{1-0+0})}$

- $\lim\limits_{n\to\infty}\sqrt{n^2} = \lim\limits_{n\to\infty} n = \infty.$
- $\lim\limits_{n\to\infty}\frac{1}{n} = \frac{1}{\infty} = 0.$
- $\lim\limits_{n\to\infty}\frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0$ pre $a\in R$.

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big(\ln(n+3)-\ln n\Big) = \lim_{n\to\infty} n\cdot\ln\tfrac{n+3}{n} = 3$$

- $= \lim_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right) = \lim_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a\,.\end{array}\right] = \ln e^3 = 3.$

$$\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right) = 0$$

- $= \lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right)\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

  $= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \frac{-1}{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}} = \frac{-1}{\sqrt{\infty}+\sqrt{\infty}} = \frac{-1}{\infty} = 0.$

- $= \lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right)\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

  $= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{-1}{\sqrt{n^2}\cdot\left(\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}}\right)}$

  $= \frac{-1}{\infty\cdot(\sqrt{1-0+0}+\sqrt{1-0+0})} = \frac{-1}{\infty\cdot 2}$

- $\lim_{n\to\infty}\sqrt{n^2} = \lim_{n\to\infty} n = \infty.$
- $\lim_{n\to\infty}\frac{1}{n} = \frac{1}{\infty} = 0.$
- $\lim_{n\to\infty}\frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0$ pre $a\in R.$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big(\ln(n+3)-\ln n\Big) = \lim_{n\to\infty} n\cdot\ln\tfrac{n+3}{n} = 3$$

- $= \lim\limits_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right) = \lim\limits_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a .\end{array}\right] = \ln e^3 = 3.$

$$\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right) = 0$$

- $= \lim\limits_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right)\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

  $= \lim\limits_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \frac{-1}{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}} = \frac{-1}{\sqrt{\infty}+\sqrt{\infty}} = \frac{-1}{\infty} = 0.$

- $= \lim\limits_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right)\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$

  $= \lim\limits_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty}\frac{-1}{\sqrt{n^2}\cdot\left(\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}}\right)}$

  $= \frac{-1}{\infty\cdot(\sqrt{1-0+0}+\sqrt{1-0+0})} = \frac{-1}{\infty\cdot 2} = \frac{-1}{\infty}$

- $\lim\limits_{n\to\infty}\sqrt{n^2} = \lim\limits_{n\to\infty} n = \infty.$
- $\lim\limits_{n\to\infty}\frac{1}{n} = \frac{1}{\infty} = 0.$
- $\lim\limits_{n\to\infty}\frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0$ pre $a\in R.$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big(\ln(n+3)-\ln n\Big) = \lim_{n\to\infty} n\cdot\ln\tfrac{n+3}{n} = 3$$

- $= \lim\limits_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right) = \lim\limits_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí}\\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a\,.\end{array}\right] = \ln e^3 = 3.$

$$\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right) = 0$$

- $= \lim\limits_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right)\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$
  $= \lim\limits_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \frac{-1}{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}} = \frac{-1}{\sqrt{\infty}+\sqrt{\infty}} = \frac{-1}{\infty} = 0.$

- $= \lim\limits_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right)\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$
  $= \lim\limits_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim\limits_{n\to\infty}\frac{-1}{\sqrt{n^2}\cdot\left(\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}}\right)}$
  $= \frac{-1}{\infty\cdot(\sqrt{1-0+0}+\sqrt{1-0+0})} = \frac{-1}{\infty\cdot 2} = \frac{-1}{\infty} = 0.$

- $\lim\limits_{n\to\infty}\sqrt{n^2} = \lim\limits_{n\to\infty} n = \infty.$
- $\lim\limits_{n\to\infty}\frac{1}{n} = \frac{1}{\infty} = 0.$
- $\lim\limits_{n\to\infty}\frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0$ pre $a\in R.$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

$$\lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right)$$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

- $= \frac{}{\quad -0}$

Označme $a_n = \frac{n}{2^n}$, $n \in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$

Resp.

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0.$

$$\lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right)$$

- $= \lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right) \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

- $= \dfrac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0}$

Označme $a_n = \frac{n}{2^n}$, $n \in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$

Resp.

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0.$

$$\lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right)$$

- $= \lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right) \cdot \dfrac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \dfrac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

- $= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \frac{8+0+0}{\infty+2-0}$

Označme $a_n = \frac{n}{2^n}$, $n\in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$

Resp.

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0.$

$$\lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right)$$

- $= \lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right) \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$

$= \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

- $= \dfrac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \dfrac{8+0+0}{\infty+2-0} = \dfrac{8}{\infty}$

Označme $a_n = \frac{n}{2^n}$, $n \in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$

Resp.

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0.$

$$\lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right)$$

- $= \lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right) \cdot \dfrac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \dfrac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$

$= \lim_{n\to\infty} \dfrac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ n = \sqrt{n^2}. \end{array} \right]$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

- $= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty} = 0.$

Označme $a_n = \frac{n}{2^n}$, $n \in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$

Resp.

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0.$

$$\lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right)$$

- $= \lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right) \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$

$= \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí} \\ n = \sqrt{n^2}.\end{array}\right] = \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} \cdot \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}}$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

- $= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty} = 0.$

Označme $a_n = \frac{n}{2^n}$, $n \in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$

Resp.

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0.$

$$\lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right)$$

- $= \lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right) \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$

$= \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\text{Pre všetky } n\in N \text{ platí } n=\sqrt{n^2}.\right] = \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} \cdot \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}}$

$= \lim_{n\to\infty} \frac{2-\frac{1}{n}}{\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1-\frac{3}{n}+\frac{2}{n^2}}}$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

- $= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty} = 0.$

Označme $a_n = \frac{n}{2^n}$, $n \in N$.

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Resp.

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

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$$\lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right)$$

- $= \lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right) \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$

$= \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí} \\ n=\sqrt{n^2}.\end{array}\right] = \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} \cdot \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}}$

$= \lim_{n\to\infty} \frac{2-\frac{1}{n}}{\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1-\frac{3}{n}+\frac{2}{n^2}}} = \frac{2-\frac{1}{\infty}}{\sqrt{1-\frac{1}{\infty}+\frac{1}{\infty}}+\sqrt{1-\frac{3}{\infty}+\frac{2}{\infty}}}$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

- $= \dfrac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \dfrac{8+0+0}{\infty+2-0} = \dfrac{8}{\infty} = 0.$

Označme $a_n = \frac{n}{2^n}$, $n \in N$.

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Resp.

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$\Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{n}{2^n} = 0.$

$$\lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right)$$

- $= \lim\limits_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right) \cdot \dfrac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim\limits_{n\to\infty} \dfrac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$

$= \lim\limits_{n\to\infty} \dfrac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\begin{array}{c}\text{Pre všetky } n \in N \text{ platí} \\ n = \sqrt{n^2}.\end{array}\right] = \lim\limits_{n\to\infty} \dfrac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} \cdot \dfrac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}}$

$= \lim\limits_{n\to\infty} \dfrac{2-\frac{1}{n}}{\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1-\frac{3}{n}+\frac{2}{n^2}}} = \dfrac{2-\frac{1}{\infty}}{\sqrt{1-\frac{1}{\infty}+\frac{1}{\infty}}+\sqrt{1-\frac{3}{\infty}+\frac{2}{\infty}}} = \dfrac{2-0}{\sqrt{1-0+0}+\sqrt{1-0+0}}$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

- $= \dfrac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \dfrac{8+0+0}{\infty+2-0} = \dfrac{8}{\infty} = 0.$

Označme $a_n = \frac{n}{2^n}$, $n\in N$.

- $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$

Resp.

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$\Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{n}{2^n} = 0.$

$$\lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right)$$

- $= \lim\limits_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right) \cdot \dfrac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim\limits_{n\to\infty} \dfrac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$

$$= \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí} \\ n=\sqrt{n^2}.\end{array}\right] = \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} \cdot \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}}$$

$$= \lim_{n\to\infty} \frac{2-\frac{1}{n}}{\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1-\frac{3}{n}+\frac{2}{n^2}}} = \frac{2-\frac{1}{\infty}}{\sqrt{1-\frac{1}{\infty}+\frac{1}{\infty}}+\sqrt{1-\frac{3}{\infty}+\frac{2}{\infty}}} = \frac{2-0}{\sqrt{1-0+0}+\sqrt{1-0+0}}$$

$$= \frac{2}{1+1}$$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

- $= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty} = 0.$

Označme $a_n = \frac{n}{2^n}$, $n\in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$

Resp.

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0.$

$$\lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right) = 1$$

- $= \lim_{n\to\infty} \left(\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right) \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$

$= \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí} \\ n=\sqrt{n^2}.\end{array}\right] = \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} \cdot \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}}$

$= \lim_{n\to\infty} \frac{2-\frac{1}{n}}{\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1-\frac{3}{n}+\frac{2}{n^2}}} = \frac{2-\frac{1}{\infty}}{\sqrt{1-\frac{1}{\infty}+\frac{1}{\infty}}+\sqrt{1-\frac{3}{\infty}+\frac{2}{\infty}}} = \frac{2-0}{\sqrt{1-0+0}+\sqrt{1-0+0}}$

$= \frac{2}{1+1} = 1.$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$$

Riešené limity – 34, 35

$\lim_{n\to\infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2}\right)$

- $= \lim_{n\to\infty} (\sqrt[3]{n+1} - \sqrt[3]{n+2}) \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1}\,\sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1}\,\sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}$

$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}}$

- $= \lim_{n\to\infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1}\,\sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1}\,\sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)$$

- $= \lim\limits_{n\to\infty} \left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$

  $= \lim\limits_{n\to\infty} \frac{(n+1)-(n+2)}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$

  $= \lim\limits_{n\to\infty} \frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{(n-1)-(n-2)}$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)$$

- $= \lim_{n\to\infty} \left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)\cdot \frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$
  $= \lim_{n\to\infty} \frac{(n+1)-(n+2)}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}} = \lim_{n\to\infty} \frac{-1}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$$

- $= \lim_{n\to\infty} \frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot \frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$
  $= \lim_{n\to\infty} \frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{(n-1)-(n-2)} = \lim_{n\to\infty} \frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{1}$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)$$

- $= \lim_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$

  $= \lim_{n\to\infty}\frac{(n+1)-(n+2)}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}} = \lim_{n\to\infty}\frac{-1}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$

  $= \frac{-1}{\sqrt[3]{(\infty+1)^2}+\sqrt[3]{\infty+1}\,\sqrt[3]{\infty+2}+\sqrt[3]{(\infty+2)^2}}$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$$

- $= \lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$

  $= \lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{(n-1)-(n-2)} = \lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{1}$

  $= \frac{\sqrt[3]{(\infty-1)^2}+\sqrt[3]{\infty-1}\,\sqrt[3]{\infty-2}+\sqrt[3]{(\infty-2)^2}}{1}$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$
$$= \lim_{n\to\infty}\frac{(n+1)-(n+2)}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}} = \lim_{n\to\infty}\frac{-1}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$
$$= \frac{-1}{\sqrt[3]{(\infty+1)^2}+\sqrt[3]{\infty+1}\,\sqrt[3]{\infty+2}+\sqrt[3]{(\infty+2)^2}} = \frac{-1}{\infty+\infty\cdot\infty+\infty}$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$$

- $$= \lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$$
$$= \lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{(n-1)-(n-2)} = \lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{1}$$
$$= \frac{\sqrt[3]{(\infty-1)^2}+\sqrt[3]{\infty-1}\,\sqrt[3]{\infty-2}+\sqrt[3]{(\infty-2)^2}}{1} = \infty+\infty\cdot\infty+\infty$$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$

$$= \lim_{n\to\infty}\frac{(n+1)-(n+2)}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}} = \lim_{n\to\infty}\frac{-1}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$= \frac{-1}{\sqrt[3]{(\infty+1)^2}+\sqrt[3]{\infty+1}\,\sqrt[3]{\infty+2}+\sqrt[3]{(\infty+2)^2}} = \frac{-1}{\infty+\infty\cdot\infty+\infty} = \frac{-1}{\infty}$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}} = \infty$$

- $= \lim\limits_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$

$$= \lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{(n-1)-(n-2)} = \lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{1}$$

$$= \frac{\sqrt[3]{(\infty-1)^2}+\sqrt[3]{\infty-1}\,\sqrt[3]{\infty-2}+\sqrt[3]{(\infty-2)^2}}{1} = \infty+\infty\cdot\infty+\infty = \infty.$$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)=0$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n+2}\right)\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$
  $$= \lim_{n\to\infty}\frac{(n+1)-(n+2)}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}} = \lim_{n\to\infty}\frac{-1}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$
  $$= \frac{-1}{\sqrt[3]{(\infty+1)^2}+\sqrt[3]{\infty+1}\,\sqrt[3]{\infty+2}+\sqrt[3]{(\infty+2)^2}} = \frac{-1}{\infty+\infty\cdot\infty+\infty} = \frac{-1}{\infty} = 0.$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}=\infty$$

- $$= \lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$$
  $$= \lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{(n-1)-(n-2)} = \lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{1}$$
  $$= \frac{\sqrt[3]{(\infty-1)^2}+\sqrt[3]{\infty-1}\,\sqrt[3]{\infty-2}+\sqrt[3]{(\infty-2)^2}}{1} = \infty+\infty\cdot\infty+\infty = \infty.$$

Riešené limity – 36

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)$$

Riešené limity – 36

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)$$

- $= \lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$

- $= \lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$

Riešené limity – 36

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$

  $= \lim\limits_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$

- $= \lim\limits_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim\limits_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$

Riešené limity – 36

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)$$

- $= \lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$
  $= \lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$
  $= \lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$

- $= \lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$
  $= \lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$

Riešené limity – 36

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)$$

- $= \lim\limits_{n\to\infty} \left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\dfrac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$

  $= \lim\limits_{n\to\infty}\dfrac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$

  $= \lim\limits_{n\to\infty}\dfrac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{c}\text{Pre } k\in N \text{ platí}\\ \lim\limits_{n\to\infty}(n^4\pm 1)^k=\infty.\end{array}\right]$

- $= \lim\limits_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\dfrac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim\limits_{n\to\infty}\dfrac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$

  $= \lim\limits_{n\to\infty}\dfrac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim\limits_{n\to\infty}\dfrac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\dfrac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$

Riešené limity – 36

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\dfrac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$

$$= \lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$= \lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{c}\text{Pre } k\in N \text{ platí}\\ \lim\limits_{n\to\infty}(n^4\pm 1)^k=\infty.\end{array}\right]$$

$$= \frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}}$$

- $= \lim\limits_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\dfrac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim\limits_{n\to\infty}\dfrac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$

$$= \lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$

$$= \lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$= \lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$= \lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{c}\text{Pre } k\in N \text{ platí}\\ \lim_{n\to\infty}(n^4\pm 1)^k=\infty.\end{array}\right]$$

$$= \frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\,\sqrt[4]{\infty}+\sqrt[4]{\infty}\,\sqrt[4]{\infty}+\sqrt[4]{\infty}} = \frac{1}{\infty+\infty\cdot\infty+\infty\cdot\infty+\infty}$$

- $$= \lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

$$= \lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$

$$= \lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})} = \lim_{n\to\infty}\frac{1}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$
$$= \lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$
$$= \lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{c}\text{Pre } k\in N \text{ platí}\\ \lim_{n\to\infty}(n^4\pm1)^k=\infty.\end{array}\right]$$
$$= \frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}} = \frac{1}{\infty+\infty\cdot\infty+\infty\cdot\infty+\infty} = \frac{1}{\infty}$$

- $$= \lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$
$$= \lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$
$$= \lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})} = \lim_{n\to\infty}\frac{1}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$
$$= \frac{1}{(\sqrt[4]{\infty-1}+\sqrt[4]{\infty+1})\cdot(\sqrt{\infty-1}+\sqrt{\infty+1})}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$
$$= \lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$
$$= \lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{c}\text{Pre } k\in N \text{ platí}\\ \lim_{n\to\infty}(n^4\pm1)^k=\infty.\end{array}\right]$$
$$= \frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}} = \frac{1}{\infty+\infty\cdot\infty+\infty\cdot\infty+\infty} = \frac{1}{\infty} = 0.$$

- $$= \lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$
$$= \lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$
$$= \lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})} = \lim_{n\to\infty}\frac{1}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$
$$= \frac{1}{(\sqrt[4]{\infty-1}+\sqrt[4]{\infty+1})\cdot(\sqrt{\infty-1}+\sqrt{\infty+1})} = \frac{1}{(\infty+\infty)\cdot(\infty+\infty)}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\dfrac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$

  $= \lim\limits_{n\to\infty}\dfrac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$

  $= \lim\limits_{n\to\infty}\dfrac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{c}\text{Pre } k\in N \text{ platí}\\ \lim\limits_{n\to\infty}(n^4\pm1)^k=\infty.\end{array}\right]$

  $= \dfrac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}} = \dfrac{1}{\infty+\infty\cdot\infty+\infty\cdot\infty+\infty} = \dfrac{1}{\infty} = 0.$

- $= \lim\limits_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\dfrac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim\limits_{n\to\infty}\dfrac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$

  $= \lim\limits_{n\to\infty}\dfrac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim\limits_{n\to\infty}\dfrac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\dfrac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$

  $= \lim\limits_{n\to\infty}\dfrac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})} = \lim\limits_{n\to\infty}\dfrac{1}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$

  $= \dfrac{1}{(\sqrt[4]{\infty-1}+\sqrt[4]{\infty+1})\cdot(\sqrt{\infty-1}+\sqrt{\infty+1})} = \dfrac{1}{(\infty+\infty)\cdot(\infty+\infty)} = \dfrac{1}{\infty\cdot\infty}$

Riešené limity – 36

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$
$$= \lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$
$$= \lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{c}\text{Pre } k\in N \text{ platí}\\ \lim_{n\to\infty}(n^4\pm1)^k=\infty.\end{array}\right]$$
$$= \frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\,\sqrt[4]{\infty}+\sqrt[4]{\infty}\,\sqrt[4]{\infty}+\sqrt[4]{\infty}} = \frac{1}{\infty+\infty\cdot\infty+\infty\cdot\infty+\infty} = \frac{1}{\infty} = 0.$$

- $$= \lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$
$$= \lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}} = \lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$
$$= \lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})} = \lim_{n\to\infty}\frac{1}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$
$$= \frac{1}{(\sqrt[4]{\infty-1}+\sqrt[4]{\infty+1})\cdot(\sqrt{\infty-1}+\sqrt{\infty+1})} = \frac{1}{(\infty+\infty)\cdot(\infty+\infty)} = \frac{1}{\infty\cdot\infty} = \frac{1}{\infty}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)=0$$

- $$=\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$
$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$
$$=\lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}=\left[\begin{array}{c}\text{Pre } k\in N \text{ platí}\\ \lim_{n\to\infty}(n^4\pm 1)^k=\infty.\end{array}\right]$$
$$=\frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}}=\frac{1}{\infty+\infty\cdot\infty+\infty\cdot\infty+\infty}=\frac{1}{\infty}=0.$$

- $$=\lim_{n\to\infty}\left(\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right)\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$
$$=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$
$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}=\lim_{n\to\infty}\frac{1}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$
$$=\frac{1}{(\sqrt[4]{\infty-1}+\sqrt[4]{\infty+1})\cdot(\sqrt{\infty-1}+\sqrt{\infty+1})}=\frac{1}{(\infty+\infty)\cdot(\infty+\infty)}=\frac{1}{\infty\cdot\infty}=\frac{1}{\infty}=0.$$

Riešené limity – 37

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n+1\right)$$

Riešené limity – 37

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n+1\right)$$

- $= \lim\limits_{n\to\infty} \left(\sqrt{n^2+4n+1}-(n-1)\right)\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)}$

- $= 1 + \lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right)$

Riešené limity – 37

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n+1\right)$$

- $= \lim\limits_{n\to\infty} (\sqrt{n^2+4n+1}-(n-1)) \cdot \frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)} = \lim\limits_{n\to\infty} \frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$

- $= 1+\lim\limits_{n\to\infty} (\sqrt{n^2+4n+1}-n) = 1+\lim\limits_{n\to\infty} (\sqrt{n^2+4n+1}-n) \cdot \frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$

Riešené limity – 37

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n+1\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-(n-1)\right)\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)} = \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$

  $= \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1}$

- $= 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right) = 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right)\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$

  $= 1+\lim\limits_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n}$

Riešené limity – 37

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n+1\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-(n-1)\right)\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)} = \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$
  $= \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1} = \lim\limits_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}$

- $= 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right) = 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right)\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$
  $= 1+\lim\limits_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n} = 1+\lim\limits_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}$

Riešené limity – 37

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n+1\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-(n-1)\right)\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)} = \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$
  $= \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1} = \lim\limits_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1} = \lim\limits_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$

- $= 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right) = 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right)\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$
  $= 1+\lim\limits_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n} = 1+\lim\limits_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n} = 1+\lim\limits_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$

Riešené limity – 37

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n+1\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-(n-1)\right)\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)} = \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$

  $= \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1} = \lim\limits_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1} = \lim\limits_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$

  $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí} \\ n=\sqrt{n^2}.\end{array}\right]$

- $= 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right) = 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right)\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$

  $= 1+\lim\limits_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n} = 1+\lim\limits_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n} = 1+\lim\limits_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$

  $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí} \\ n=\sqrt{n^2}.\end{array}\right]$

Riešené limity – 37

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n+1\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-(n-1)\right)\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)} = \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$

  $= \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1} = \lim\limits_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1} = \lim\limits_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$

  $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n=\sqrt{n^2}.\end{array}\right] = \lim\limits_{n\to\infty}\frac{6}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1-\frac{1}{n}}$

- $= 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right) = 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right)\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$

  $= 1+\lim\limits_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n} = 1+\lim\limits_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n} = 1+\lim\limits_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$

  $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n=\sqrt{n^2}.\end{array}\right] = 1+\lim\limits_{n\to\infty}\frac{4+\frac{1}{n}}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1}$

Riešené limity – 37

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n+1\right)$$

- $= \lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-(n-1)\right)\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)} = \lim_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$

$= \lim_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1} = \lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1} = \lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$

$= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n=\sqrt{n^2}.\end{array}\right] = \lim_{n\to\infty}\frac{6}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1-\frac{1}{n}} = \frac{6}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1-\frac{1}{\infty}}$

- $= 1+\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right) = 1+\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right)\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$

$= 1+\lim_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n} = 1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n} = 1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$

$= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n=\sqrt{n^2}.\end{array}\right] = 1+\lim_{n\to\infty}\frac{4+\frac{1}{n}}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1} = 1+\frac{4+\frac{1}{\infty}}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1}$

Riešené limity – 37

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n+1\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-(n-1)\right)\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)} = \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$

  $= \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1} = \lim\limits_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1} = \lim\limits_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$

  $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n=\sqrt{n^2}.\end{array}\right] = \lim\limits_{n\to\infty}\frac{6}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1-\frac{1}{n}} = \frac{6}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1-\frac{1}{\infty}} = \frac{6}{\sqrt{1+0+0}+1-0}$

- $= 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right) = 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right)\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$

  $= 1+\lim\limits_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n} = 1+\lim\limits_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n} = 1+\lim\limits_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$

  $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n=\sqrt{n^2}.\end{array}\right] = 1+\lim\limits_{n\to\infty}\frac{4+\frac{1}{n}}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1} = 1+\frac{4+\frac{1}{\infty}}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1} = 1+\frac{4+0}{\sqrt{1+0+0}+1}$

Riešené limity – 37

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n+1\right)$$

- $= \lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-(n-1)\right)\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)} = \lim_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$
  $= \lim_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1} = \lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1} = \lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$
  $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n=\sqrt{n^2}.\end{array}\right] = \lim_{n\to\infty}\frac{6}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1-\frac{1}{n}} = \frac{6}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1-\frac{1}{\infty}} = \frac{6}{\sqrt{1+0+0}+1-0}$
  $= \frac{6}{2}$

- $= 1+\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right) = 1+\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right)\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$
  $= 1+\lim_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n} = 1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n} = 1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$
  $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n=\sqrt{n^2}.\end{array}\right] = 1+\lim_{n\to\infty}\frac{4+\frac{1}{n}}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1} = 1+\frac{4+\frac{1}{\infty}}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1} = 1+\frac{4+0}{\sqrt{1+0+0}+1}$
  $= 1+\frac{4}{2}$

Riešené limity – 37

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-n+1\right)=3$$

- $= \lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-(n-1)\right)\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)} = \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$
  $= \lim\limits_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1} = \lim\limits_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1} = \lim\limits_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$
  $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n=\sqrt{n^2}.\end{array}\right] = \lim\limits_{n\to\infty}\frac{6}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1-\frac{1}{n}} = \frac{6}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1-\frac{1}{\infty}} = \frac{6}{\sqrt{1+0+0}+1-0}$
  $= \frac{6}{2} = 3.$

- $= 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right) = 1+\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+1}-n\right)\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$
  $= 1+\lim\limits_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n} = 1+\lim\limits_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n} = 1+\lim\limits_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$
  $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n=\sqrt{n^2}.\end{array}\right] = 1+\lim\limits_{n\to\infty}\frac{4+\frac{1}{n}}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1} = 1+\frac{4+\frac{1}{\infty}}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1} = 1+\frac{4+0}{\sqrt{1+0+0}+1}$
  $= 1+\frac{4}{2} = 3.$

Riešené limity – 38, 39, 40

$$\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n-1}\right)$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$$

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right)$$

Riešené limity – 38, 39, 40

$$\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n-1}\right)$$

- $= \lim\limits_{n\to\infty}\sqrt{n}\cdot\left(\sqrt{n+1+\frac{1}{n}}-\sqrt{1-\frac{1}{n}}\right)$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$$

- $= \lim\limits_{n\to\infty}\frac{1}{\sqrt[3]{n}\cdot\left(\sqrt{n^2+1+\frac{1}{n}}-\sqrt[3]{1-\frac{1}{n}}\right)}$

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right)$$

- $= \lim\limits_{n\to\infty}\sqrt[4]{n}\cdot\left(\sqrt[4]{n^3+\frac{1}{n}}-\sqrt[4]{1+\frac{1}{n}}\right)$

Riešené limity – 38, 39, 40

$$\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n-1}\right)$$

- $= \lim_{n\to\infty}\sqrt{n}\cdot\left(\sqrt{n+1+\frac{1}{n}}-\sqrt{1-\frac{1}{n}}\right) = \infty\cdot\left(\sqrt{\infty+1+\frac{1}{\infty}}-\sqrt{1-\frac{1}{\infty}}\right)$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$$

- $= \lim_{n\to\infty}\frac{1}{\sqrt[3]{n}\cdot\left(\sqrt{n^2+1+\frac{1}{n}}-\sqrt[3]{1-\frac{1}{n}}\right)} = \frac{1}{\infty\cdot\left(\sqrt[3]{\infty+1+\frac{1}{\infty}}-\sqrt[3]{1-\frac{1}{\infty}}\right)}$

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right)$$

- $= \lim_{n\to\infty}\sqrt[4]{n}\cdot\left(\sqrt[4]{n^3+\frac{1}{n}}-\sqrt[4]{1+\frac{1}{n}}\right) = \infty\cdot\left(\sqrt[4]{\infty+\frac{1}{\infty}}-\sqrt[4]{1+\frac{1}{\infty}}\right)$

Riešené limity – 38, 39, 40

$$\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n-1}\right)$$

- $= \lim_{n\to\infty}\sqrt{n}\cdot\left(\sqrt{n+1+\frac{1}{n}}-\sqrt{1-\frac{1}{n}}\right) = \infty\cdot\left(\sqrt{\infty+1+\frac{1}{\infty}}-\sqrt{1-\frac{1}{\infty}}\right)$
  $= \infty\cdot\left(\sqrt{\infty+1+0}-\sqrt{1-0}\right)$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$$

- $= \lim_{n\to\infty}\frac{1}{\sqrt[3]{n}\cdot\left(\sqrt{n^2+1+\frac{1}{n}}-\sqrt[3]{1-\frac{1}{n}}\right)} = \frac{1}{\infty\cdot\left(\sqrt[3]{\infty+1+\frac{1}{\infty}}-\sqrt[3]{1-\frac{1}{\infty}}\right)} = \frac{1}{\infty\cdot\left(\sqrt[3]{\infty+1+0}-\sqrt[3]{1-0}\right)}$

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right)$$

- $= \lim_{n\to\infty}\sqrt[4]{n}\cdot\left(\sqrt[4]{n^3+\frac{1}{n}}-\sqrt[4]{1+\frac{1}{n}}\right) = \infty\cdot\left(\sqrt[4]{\infty+\frac{1}{\infty}}-\sqrt[4]{1+\frac{1}{\infty}}\right)$
  $= \infty\cdot\left(\sqrt[4]{\infty+0}-\sqrt[4]{1+0}\right)$

Riešené limity – 38, 39, 40

$$\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n-1}\right)$$

- $= \lim_{n\to\infty}\sqrt{n}\cdot\left(\sqrt{n+1+\frac{1}{n}}-\sqrt{1-\frac{1}{n}}\right) = \infty\cdot\left(\sqrt{\infty+1+\frac{1}{\infty}}-\sqrt{1-\frac{1}{\infty}}\right)$
  $= \infty\cdot\left(\sqrt{\infty+1+0}-\sqrt{1-0}\right) = \infty\cdot(\infty-1)$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$$

- $= \lim_{n\to\infty}\frac{1}{\sqrt[3]{n}\cdot\left(\sqrt{n^2+1+\frac{1}{n}}-\sqrt[3]{1-\frac{1}{n}}\right)} = \frac{1}{\infty\cdot\left(\sqrt[3]{\infty+1+\frac{1}{\infty}}-\sqrt[3]{1-\frac{1}{\infty}}\right)} = \frac{1}{\infty\cdot(\sqrt[3]{\infty+1+0}-\sqrt[3]{1-0})}$
  $= \frac{1}{\infty\cdot(\infty-1)}$

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right)$$

- $= \lim_{n\to\infty}\sqrt[4]{n}\cdot\left(\sqrt[4]{n^3+\frac{1}{n}}-\sqrt[4]{1+\frac{1}{n}}\right) = \infty\cdot\left(\sqrt[4]{\infty+\frac{1}{\infty}}-\sqrt[4]{1+\frac{1}{\infty}}\right)$
  $= \infty\cdot\left(\sqrt[4]{\infty+0}-\sqrt[4]{1+0}\right) = \infty\cdot(\infty-1)$

Riešené limity – 38, 39, 40

$$\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n-1}\right)=\infty$$

- $= \lim_{n\to\infty}\sqrt{n}\cdot\left(\sqrt{n+1+\frac{1}{n}}-\sqrt{1-\frac{1}{n}}\right)=\infty\cdot\left(\sqrt{\infty+1+\frac{1}{\infty}}-\sqrt{1-\frac{1}{\infty}}\right)$
  $= \infty\cdot\left(\sqrt{\infty+1+0}-\sqrt{1-0}\right)=\infty\cdot(\infty-1)=\infty.$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}=0$$

- $= \lim_{n\to\infty}\frac{1}{\sqrt[3]{n}\cdot\left(\sqrt{n^2+1+\frac{1}{n}}-\sqrt[3]{1-\frac{1}{n}}\right)}=\frac{1}{\infty\cdot\left(\sqrt[3]{\infty+1+\frac{1}{\infty}}-\sqrt[3]{1-\frac{1}{\infty}}\right)}=\frac{1}{\infty\cdot(\sqrt[3]{\infty+1+0}-\sqrt[3]{1-0})}$
  $= \frac{1}{\infty\cdot(\infty-1)}=0.$

$$\lim_{n\to\infty}\left(\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right)=\infty$$

- $= \lim_{n\to\infty}\sqrt[4]{n}\cdot\left(\sqrt[4]{n^3+\frac{1}{n}}-\sqrt[4]{1+\frac{1}{n}}\right)=\infty\cdot\left(\sqrt[4]{\infty+\frac{1}{\infty}}-\sqrt[4]{1+\frac{1}{\infty}}\right)$
  $= \infty\cdot\left(\sqrt[4]{\infty+0}-\sqrt[4]{1+0}\right)=\infty\cdot(\infty-1)=\infty.$

Riešené limity – 41, 42

$\lim\limits_{n\to\infty} \dfrac{1}{\sqrt[3]{n^3+1}-n}$

$\lim\limits_{n\to\infty} \dfrac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$

Riešené limity – 41, 42

$$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

- $= \lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$

$$\lim_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$$

- $= \lim_{n\to\infty} \frac{1}{\sqrt[5]{n^4}.\left(\sqrt{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)}$

Riešené limity – 41, 42

$\lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim\limits_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$

$\lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\cdot\left(\sqrt{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\cdot\left(\sqrt[5]{\infty+\frac{1}{\infty}}-\sqrt[5]{1+\frac{1}{\infty}}\right)}$

Riešené limity – 41, 42

$$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim\limits_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$

  $= \lim\limits_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1}$

$$\lim_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\cdot\left(\sqrt{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\cdot\left(\sqrt[5]{\infty+\frac{1}{\infty}}-\sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\cdot(\sqrt[5]{\infty+0}-\sqrt[5]{1+0})}$

Riešené limity – 41, 42

$$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim\limits_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$

  $= \lim\limits_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1} = \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3}. \end{array} \right]$

$$\lim_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\cdot\left(\sqrt{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\cdot\left(\sqrt[5]{\infty+\frac{1}{\infty}}-\sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\cdot(\sqrt[5]{\infty+0}-\sqrt[5]{1+0})}$

  $= \frac{1}{\infty\cdot(\infty-1)}$

Riešené limity – 41, 42

$$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

- $$= \lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1} = \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3}. \end{array} \right]$$

$$= \lim_{n\to\infty} n^2 \cdot \left(\sqrt[3]{\left(1+\frac{1}{n^3}\right)^2} + \sqrt[3]{1+\frac{1}{n^3}} + 1 \right)$$

$$\lim_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$$

- $$= \lim_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\cdot\left(\sqrt{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\cdot\left(\sqrt[5]{\infty+\frac{1}{\infty}}-\sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\cdot(\sqrt[5]{\infty+0}-\sqrt[5]{1+0})}$$

$$= \frac{1}{\infty\cdot(\infty-1)} = 0.$$

Riešené limity – 41, 42

$$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim\limits_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$

  $= \lim\limits_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1} = \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3}. \end{array} \right]$

  $= \lim\limits_{n\to\infty} n^2 \cdot \left(\sqrt[3]{(1+\frac{1}{n^3})^2} + \sqrt[3]{1+\frac{1}{n^3}} + 1 \right) = \infty \cdot \left(\sqrt[3]{(1+\frac{1}{\infty})^2} + \sqrt[3]{1+\frac{1}{\infty}} + 1 \right)$

$$\lim_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\cdot\left(\sqrt[5]{n+\frac{1}{n^4}} - \sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\cdot\left(\sqrt[5]{\infty+\frac{1}{\infty}} - \sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\cdot(\sqrt[5]{\infty+0}-\sqrt[5]{1+0})}$

  $= \frac{1}{\infty\cdot(\infty-1)} = 0.$

Riešené limity – 41, 42

$\lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim\limits_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$

$= \lim\limits_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1} = \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3}. \end{array} \right]$

$= \lim\limits_{n\to\infty} n^2 \cdot \left(\sqrt[3]{(1+\frac{1}{n^3})^2} + \sqrt[3]{1+\frac{1}{n^3}} + 1\right) = \infty \cdot \left(\sqrt[3]{(1+\frac{1}{\infty})^2} + \sqrt[3]{1+\frac{1}{\infty}} + 1\right)$

$= \infty \cdot (\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} + 1)$

$\lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\cdot\left(\sqrt{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\cdot\left(\sqrt[5]{\infty+\frac{1}{\infty}}-\sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\cdot(\sqrt[5]{\infty+0}-\sqrt[5]{1+0})}$

$= \frac{1}{\infty\cdot(\infty-1)} = 0.$

Riešené limity – 41, 42

$\lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim\limits_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$

$= \lim\limits_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1} = \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3}. \end{array} \right]$

$= \lim\limits_{n\to\infty} n^2 \cdot \left(\sqrt[3]{(1+\frac{1}{n^3})^2} + \sqrt[3]{1+\frac{1}{n^3}} + 1\right) = \infty \cdot \left(\sqrt[3]{(1+\frac{1}{\infty})^2} + \sqrt[3]{1+\frac{1}{\infty}} + 1\right)$

$= \infty \cdot (\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} + 1) = \infty(1+1+1)$

$\lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$

- $= \lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\cdot\left(\sqrt[5]{n+\frac{1}{n^4}} - \sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\cdot\left(\sqrt[5]{\infty+\frac{1}{\infty}} - \sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\cdot(\sqrt[5]{\infty+0}-\sqrt[5]{1+0})}$

$= \frac{1}{\infty\cdot(\infty-1)} = 0.$

Riešené limity – 41, 42

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+1}-n}=\infty$$

- $$= \lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+1}-n}\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=\lim_{n\to\infty}\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1}=\left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n^2=n\cdot n=\sqrt[3]{n^3}\cdot n=\sqrt[3]{n^3\cdot n^3}.\end{array}\right]$$

$$=\lim_{n\to\infty}n^2\cdot\left(\sqrt[3]{(1+\tfrac{1}{n^3})^2}+\sqrt[3]{1+\tfrac{1}{n^3}}+1\right)=\infty\cdot\left(\sqrt[3]{(1+\tfrac{1}{\infty})^2}+\sqrt[3]{1+\tfrac{1}{\infty}}+1\right)$$

$$=\infty\cdot\left(\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1\right)=\infty(1+1+1)=\infty.$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}=0$$

- $$= \lim_{n\to\infty}\frac{1}{\sqrt[5]{n^4}\cdot\left(\sqrt[5]{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)}=\frac{1}{\infty\cdot\left(\sqrt[5]{\infty+\frac{1}{\infty}}-\sqrt[5]{1+\frac{1}{\infty}}\right)}=\frac{1}{\infty\cdot(\sqrt[5]{\infty+0}-\sqrt[5]{1+0})}$$

$$=\frac{1}{\infty\cdot(\infty-1)}=0.$$

Riešené limity – 43

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

Riešené limity – 43

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $= \lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-(n-1)\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$

Riešené limity – 43

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-(n-1)\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $= \lim\limits_{n\to\infty}\left(\sqrt[3]{n^3+1}-(n-1)\right)\cdot\dfrac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$

$$= \lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-(n-1)\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $= \lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-(n-1)\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$

$$= \lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-(n-1)\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3}.\end{array}\right]$$

Riešené limity – 43

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-(n-1)\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3}.\end{array}\right] = \lim_{n\to\infty}\frac{3-\frac{3}{n}+\frac{2}{n^2}}{\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}\cdot(1-\frac{1}{n})+(1-\frac{1}{n})^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-(n-1)\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3}.\end{array}\right] = \lim_{n\to\infty}\frac{3-\frac{3}{n}+\frac{2}{n^2}}{\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}\cdot(1-\frac{1}{n})+(1-\frac{1}{n})^2}$$

$$= \frac{3-\frac{3}{\infty}+\frac{2}{\infty}}{\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}\cdot(1-\frac{1}{\infty})+(1-\frac{1}{\infty})^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-(n-1)\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3}.\end{array}\right] = \lim_{n\to\infty}\frac{3-\frac{3}{n}+\frac{2}{n^2}}{\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}\cdot(1-\frac{1}{n})+(1-\frac{1}{n})^2}$$

$$= \frac{3-\frac{3}{\infty}+\frac{2}{\infty}}{\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}\cdot(1-\frac{1}{\infty})+(1-\frac{1}{\infty})^2} = \frac{3-0+0}{\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}\cdot(1-0)+(1-0)^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $$= \lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-(n-1)\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n^2=n\cdot n=\sqrt[3]{n^3}\cdot n=\sqrt[3]{n^3\cdot n^3}.\end{array}\right] = \lim_{n\to\infty}\frac{3-\frac{3}{n}+\frac{2}{n^2}}{\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}\cdot(1-\frac{1}{n})+(1-\frac{1}{n})^2}$$

$$= \frac{3-\frac{3}{\infty}+\frac{2}{\infty}}{\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}\cdot(1-\frac{1}{\infty})+(1-\frac{1}{\infty})^2} = \frac{3-0+0}{\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}\cdot(1-0)+(1-0)^2} = \frac{3}{1+1\cdot1+1}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)=1$$

- $= \lim\limits_{n\to\infty}\left(\sqrt[3]{n^3+1}-(n-1)\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$

$$= \lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}$$

$$= \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2} = \lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}\cdot(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ n^2=n\cdot n=\sqrt[3]{n^3}\cdot n=\sqrt[3]{n^3\cdot n^3}.\end{array}\right] = \lim_{n\to\infty}\frac{3-\frac{3}{n}+\frac{2}{n^2}}{\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}\cdot(1-\frac{1}{n})+(1-\frac{1}{n})^2}$$

$$= \frac{3-\frac{3}{\infty}+\frac{2}{\infty}}{\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}\cdot(1-\frac{1}{\infty})+(1-\frac{1}{\infty})^2} = \frac{3-0+0}{\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}\cdot(1-0)+(1-0)^2} = \frac{3}{1+1\cdot1+1} = 1.$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}$$

Riešené limity – 43, 44

$$\lim_{n\to\infty} \left(\sqrt[3]{n^3+1} - n + 1\right)$$

- $= 1 + \lim_{n\to\infty} \left(\sqrt[3]{n^3+1} - n\right)$

$$\lim_{n\to\infty} \frac{(n+4)! - (n+2)!}{(n+3)!}$$

- $= \lim_{n\to\infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!}\right)$

Riešené limity – 43, 44

$\lim\limits_{n\to\infty} \left(\sqrt[3]{n^3+1} - n + 1\right)$

- $= 1 + \lim\limits_{n\to\infty} \left(\sqrt[3]{n^3+1} - n\right) = 1 + \lim\limits_{n\to\infty} \left(\sqrt[3]{n^3+1} - n\right) \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$

$\lim\limits_{n\to\infty} \frac{(n+4)!-(n+2)!}{(n+3)!}$

- $= \lim\limits_{n\to\infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!}\right) = \lim\limits_{n\to\infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!}\right)$

Riešené limity – 43, 44

$\lim_{n\to\infty} \left(\sqrt[3]{n^3+1} - n + 1\right)$

- $= 1 + \lim_{n\to\infty} \left(\sqrt[3]{n^3+1} - n\right) = 1 + \lim_{n\to\infty} \left(\sqrt[3]{n^3+1} - n\right) \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$

  $= 1 + \lim_{n\to\infty} \frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$

$\lim_{n\to\infty} \frac{(n+4)!-(n+2)!}{(n+3)!}$

- $= \lim_{n\to\infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!}\right) = \lim_{n\to\infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!}\right)$

  $= \lim_{n\to\infty} \left((n+4) - \frac{1}{n+3}\right)$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $= 1+\lim\limits_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right) = 1+\lim\limits_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$

  $= 1+\lim\limits_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = 1+\lim\limits_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}$$

- $= \lim\limits_{n\to\infty}\left(\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right) = \lim\limits_{n\to\infty}\left(\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right)$

  $= \lim\limits_{n\to\infty}\left((n+4)-\frac{1}{n+3}\right) = (\infty+4)-\frac{1}{\infty+3}$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $= 1+\lim\limits_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right) = 1+\lim\limits_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$

  $= 1+\lim\limits_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = 1+\lim\limits_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí} \\ n^2 = n\sqrt[3]{n^3} = \sqrt[3]{n^3\cdot n^3}.\end{array}\right]$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}$$

- $= \lim\limits_{n\to\infty}\left(\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right) = \lim\limits_{n\to\infty}\left(\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right)$

  $= \lim\limits_{n\to\infty}\left((n+4)-\frac{1}{n+3}\right) = (\infty+4)-\frac{1}{\infty+3} = \infty-0$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $= 1+\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right) = 1+\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$

$= 1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = 1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} =$ [Pre všetky $n\in N$ platí $n^2 = n\sqrt[3]{n^3} = \sqrt[3]{n^3\cdot n^3}$.]

$= 1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)}$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}=\infty$$

- $= \lim_{n\to\infty}\left(\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right) = \lim_{n\to\infty}\left(\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right)$

$= \lim_{n\to\infty}\left((n+4)-\frac{1}{n+3}\right) = (\infty+4)-\frac{1}{\infty+3} = \infty-0 = \infty.$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $= 1+\lim\limits_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right) = 1+\lim\limits_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$

$= 1+\lim\limits_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = 1+\lim\limits_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} =$ [Pre všetky $n\in N$ platí $n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}$.]

$= 1+\lim\limits_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)} = 1+\frac{1}{\infty\cdot\left(\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}+1\right)}$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}=\infty$$

- $= \lim\limits_{n\to\infty}\left(\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right) = \lim\limits_{n\to\infty}\left(\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right)$

$= \lim\limits_{n\to\infty}\left((n+4)-\frac{1}{n+3}\right) = (\infty+4)-\frac{1}{\infty+3} = \infty-0 = \infty.$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $= 1+\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right) = 1+\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$

$= 1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = 1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} =$ [Pre všetky $n\in N$ platí $n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}$.]

$= 1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)} = 1+\frac{1}{\infty\cdot\left(\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}+1\right)}$

$= 1+\frac{1}{\infty\cdot\left(\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1\right)}$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}=\infty$$

- $= \lim_{n\to\infty}\left(\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right) = \lim_{n\to\infty}\left(\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right)$

$= \lim_{n\to\infty}\left((n+4)-\frac{1}{n+3}\right) = (\infty+4)-\frac{1}{\infty+3} = \infty-0 = \infty.$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $= 1+\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right) = 1+\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$

$= 1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = 1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} =$ [Pre všetky $n\in N$ platí $n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}$.]

$= 1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)} = 1+\frac{1}{\infty\cdot\left(\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}+1\right)}$

$= 1+\frac{1}{\infty\cdot\left(\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1\right)} = 1+\frac{1}{\infty\cdot(1+1+1)}$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}=\infty$$

- $= \lim_{n\to\infty}\left(\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right) = \lim_{n\to\infty}\left(\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right)$

$= \lim_{n\to\infty}\left((n+4)-\frac{1}{n+3}\right) = (\infty+4)-\frac{1}{\infty+3} = \infty-0 = \infty.$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $= 1+\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right) = 1+\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$

$$= 1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = 1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} =$$ [Pre všetky $n\in N$ platí $n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}$.]

$$= 1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)} = 1+\frac{1}{\infty\cdot\left(\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}+1\right)}$$

$$= 1+\frac{1}{\infty\cdot\left(\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1\right)} = 1+\frac{1}{\infty\cdot(1+1+1)} = 1+\frac{1}{\infty}$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!} = \infty$$

- $= \lim_{n\to\infty}\left(\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right) = \lim_{n\to\infty}\left(\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right)$

$$= \lim_{n\to\infty}\left((n+4)-\frac{1}{n+3}\right) = (\infty+4)-\frac{1}{\infty+3} = \infty-0 = \infty.$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)$$

- $$= 1+\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right) = 1+\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$= 1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = 1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} =$$ [Pre všetky $n\in N$ platí $n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}$.]

$$= 1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)} = 1+\frac{1}{\infty\cdot\left(\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}+1\right)}$$

$$= 1+\frac{1}{\infty\cdot\left(\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1\right)} = 1+\frac{1}{\infty\cdot(1+1+1)} = 1+\frac{1}{\infty} = 1+0$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!} = \infty$$

- $$= \lim_{n\to\infty}\left(\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right) = \lim_{n\to\infty}\left(\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right)$$

$$= \lim_{n\to\infty}\left((n+4)-\frac{1}{n+3}\right) = (\infty+4)-\frac{1}{\infty+3} = \infty-0 = \infty.$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n+1\right)=1$$

- $$= 1+\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right) = 1+\lim_{n\to\infty}\left(\sqrt[3]{n^3+1}-n\right)\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$= 1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = 1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} =$$

[Pre všetky $n\in N$ platí $n^2 = n\sqrt[3]{n^3} = \sqrt[3]{n^3\cdot n^3}$.]

$$= 1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)} = 1+\frac{1}{\infty\cdot\left(\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}+1\right)}$$

$$= 1+\frac{1}{\infty\cdot\left(\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1\right)} = 1+\frac{1}{\infty\cdot(1+1+1)} = 1+\frac{1}{\infty} = 1+0 = 1.$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}=\infty$$

- $$= \lim_{n\to\infty}\left(\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right) = \lim_{n\to\infty}\left(\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right)$$

$$= \lim_{n\to\infty}\left((n+4)-\frac{1}{n+3}\right) = (\infty+4)-\frac{1}{\infty+3} = \infty-0 = \infty.$$

Riešené limity – 45, 46

$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n+6}$

$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}$

Riešené limity – 45, 46

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n+6}$$

- $= \lim\limits_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n+6}$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3+3\cdot5^{n+1}}$$

- $= \lim\limits_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3+3\cdot5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}$

Riešené limity – 45, 46

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n+6}$$

- $= \lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{n+6}$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}$$

- $= \lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}+2}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5}$

Riešené limity – 45, 46

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{n+6}{2n+3}}$

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim\limits_{n\to\infty} \frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}+2}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5}$

Označme $a_n = \frac{n^k}{5^n}$, $n\in N$ pre $k\in N$.

- $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n^k}}{5} = \lim\limits_{n\to\infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$

Resp.
- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{(n+1)^k}{5n^k} = \lim\limits_{n\to\infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1.$

$\Rightarrow \lim\limits_{n\to\infty} \frac{n^k}{5^n} = 0.$

Riešené limity – 45, 46

$\lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n+6} = \lim\limits_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{n+6} = \lim\limits_{n\to\infty}\left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{n+6}{2n+3}} = \left(e^{-4}\right)^{\frac{1}{2}}$

  - $\lim\limits_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty\end{array}\right] = \lim\limits_{m\to\infty}\left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{l}\text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^{n} = e^{a}.\end{array}\right] = e^{-4}.$

$\lim\limits_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}$

- $= \lim\limits_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim\limits_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}+2}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5} = \frac{2\cdot 0+3\cdot 0-0+2}{2\cdot 0-0+3\cdot 5}$

Označme $a_n = \frac{n^k}{5^n}$, $n\in N$ pre $k\in N$.

- $\lim\limits_{n\to\infty}\sqrt[n]{a_n} = \lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{5} = \lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$

Resp.
- $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty}\frac{(n+1)^k}{5n^k} = \lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1.$

$\Rightarrow \lim\limits_{n\to\infty}\frac{n^k}{5^n} = 0.$

Riešené limity – 45, 46

$\lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n+6} = \lim\limits_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{n+6} = \lim\limits_{n\to\infty}\left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{n+6}{2n+3}} = (e^{-4})^{\frac{1}{2}}$

  - $\lim\limits_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty}\left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{l} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^{n} = e^{a}. \end{array}\right] = e^{-4}.$
  - $\lim\limits_{n\to\infty}\frac{n+6}{2n+3} = \lim\limits_{n\to\infty}\frac{n\cdot(1+\frac{6}{n})}{n\cdot(2+\frac{3}{n})} = \lim\limits_{n\to\infty}\frac{1+\frac{6}{n}}{2+\frac{3}{n}} = \frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}.$

$\lim\limits_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3+3\cdot5^{n+1}} = \frac{2}{15}$

- $= \lim\limits_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3+3\cdot5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim\limits_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}+2}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot5} = \frac{2\cdot0+3\cdot0-0+2}{2\cdot0-0+3\cdot5} = \frac{2}{15}.$

Označme $a_n = \frac{n^k}{5^n}$, $n\in N$ pre $k\in N$.

- $\lim\limits_{n\to\infty}\sqrt[n]{a_n} = \lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{5} = \lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$

Resp.
- $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty}\frac{(n+1)^k}{5n^k} = \lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1.$

$\Rightarrow \lim\limits_{n\to\infty}\frac{n^k}{5^n} = 0.$

Riešené limity – 45, 46

$\lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n+6}=e^{-2}$

- $=\lim\limits_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n+6}=\lim\limits_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{n+6}=\lim\limits_{n\to\infty}\left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{n+6}{2n+3}}=\left(e^{-4}\right)^{\frac{1}{2}}=e^{-2}.$
  - $\lim\limits_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim\limits_{m\to\infty}\left(1+\frac{-4}{m}\right)^{m}=\left[\begin{array}{l}\text{Pre všetky } a\in R \text{ platí}\\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^{n}=e^{a}.\end{array}\right]=e^{-4}.$
  - $\lim\limits_{n\to\infty}\frac{n+6}{2n+3}=\lim\limits_{n\to\infty}\frac{n\cdot(1+\frac{6}{n})}{n\cdot(2+\frac{3}{n})}=\lim\limits_{n\to\infty}\frac{1+\frac{6}{n}}{2+\frac{3}{n}}=\frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{1+0}{2+0}=\frac{1}{2}.$

$\lim\limits_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3+3\cdot5^{n+1}}=\frac{2}{15}$

- $=\lim\limits_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3+3\cdot5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}=\lim\limits_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}+2}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot5}=\frac{2\cdot0+3\cdot0-0+2}{2\cdot0-0+3\cdot5}=\frac{2}{15}.$

Označme $a_n=\frac{n^k}{5^n}$, $n\in N$ pre $k\in N$.

- $\lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1.$

Resp.
- $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1.$

$\Rightarrow\lim\limits_{n\to\infty}\frac{n^k}{5^n}=0.$

Riešené limity – 47, 48

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n^6+6}$

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}$

Riešené limity – 47, 48

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n^6+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n^6+6}$

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}}$

Riešené limity – 47, 48

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n^6+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{n^6+6}$

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim\limits_{n\to\infty} \frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5}$

Riešené limity – 47, 48

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n^6+6}$$

- $= \lim_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{n^6+6} = \lim_{n\to\infty} \left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{n^6+6}{2n+3}}$

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5}$

Označme $a_n = \frac{n^k}{5^n}$, $n \in N$ pre $k \in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^k}}{5} = \lim_{n\to\infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$

Resp.

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{5n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1.$

$\Rightarrow \lim_{n\to\infty} \frac{n^k}{5^n} = 0.$

Riešené limity – 47, 48

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n^6+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{n^6+6}{2n+3}} = (e^{-4})^{-\infty}$

- $\lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^{-4}.$

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim\limits_{n\to\infty} \frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5} = \frac{2\cdot 0+3\cdot 0-0}{2\cdot 0-0+3\cdot 5}$

Označme $a_n = \frac{n^k}{5^n}$, $n\in N$ pre $k\in N$.

- $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n^k}}{5} = \lim\limits_{n\to\infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$

Resp.
- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{(n+1)^k}{5n^k} = \lim\limits_{n\to\infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1.$

$\Rightarrow \lim\limits_{n\to\infty} \frac{n^k}{5^n} = 0.$

Riešené limity – 47, 48

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n^6+6}$$

- $= \lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n^6+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{n^6+6} = \lim_{n\to\infty}\left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{n^6+6}{2n+3}} = (e^{-4})^{-\infty}$

- $\lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a.\end{array}\right] = e^{-4}.$
- $\lim_{n\to\infty}\frac{n^6+6}{2n+3} = \lim_{n\to\infty}\frac{n\cdot(n^5+\frac{6}{n})}{n\cdot(2+\frac{3}{n})} = \lim_{n\to\infty}\frac{n^5+\frac{6}{n}}{2+\frac{3}{n}} = \frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{\infty+0}{2+0} = \infty.$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot5^{n+1}}$$

- $= \lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot5} = \frac{2\cdot0+3\cdot0-0}{2\cdot0-0+3\cdot5} = \frac{0}{15}$

Označme $a_n = \frac{n^k}{5^n}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}} = \lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{5} = \lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$

Resp.
- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n\to\infty}\frac{(n+1)^k}{5n^k} = \lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1.$

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Riešené limity – 47, 48

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n^6+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{n^6+6}{2n+3}} = (e^{-4})^{-\infty} = \frac{1}{e^{\infty}}$

- $\lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty} \left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^{-4}.$
- $\lim\limits_{n\to\infty} \frac{n^6+6}{2n+3} = \lim\limits_{n\to\infty} \frac{n\cdot(n^5+\frac{6}{n})}{n\cdot(2+\frac{3}{n})} = \lim\limits_{n\to\infty} \frac{n^5+\frac{6}{n}}{2+\frac{3}{n}} = \frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{\infty+0}{2+0} = \infty.$

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} = 0$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim\limits_{n\to\infty} \frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5} = \frac{2\cdot 0+3\cdot 0-0}{2\cdot 0-0+3\cdot 5} = \frac{0}{15} = 0.$

Označme $a_n = \frac{n^k}{5^n}$, $n\in N$ pre $k\in N$.

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Resp.
- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{(n+1)^k}{5n^k} = \lim\limits_{n\to\infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1.$

$\Rightarrow \lim\limits_{n\to\infty} \frac{n^k}{5^n} = 0.$

Riešené limity – 47, 48

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n^6+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{n^6+6}{2n+3}} = (e^{-4})^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty}$

- $\lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty} \left(1+\frac{a}{n}\right)^{n} = e^{a}. \end{array}\right] = e^{-4}.$
- $\lim\limits_{n\to\infty} \frac{n^6+6}{2n+3} = \lim\limits_{n\to\infty} \frac{n\cdot(n^5+\frac{6}{n})}{n\cdot(2+\frac{3}{n})} = \lim\limits_{n\to\infty} \frac{n^5+\frac{6}{n}}{2+\frac{3}{n}} = \frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{\infty+0}{2+0} = \infty.$

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} = 0$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim\limits_{n\to\infty} \frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5} = \frac{2\cdot 0+3\cdot 0-0}{2\cdot 0-0+3\cdot 5} = \frac{0}{15} = 0.$

Označme $a_n = \frac{n^k}{5^n}$, $n\in N$ pre $k\in N$.

- $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n^k}}{5} = \lim\limits_{n\to\infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$

Resp.

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{(n+1)^k}{5n^k} = \lim\limits_{n\to\infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1.$

$\Rightarrow \lim\limits_{n\to\infty} \frac{n^k}{5^n} = 0.$

Riešené limity – 47, 48

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n^6+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{n^6+6}{2n+3}} = (e^{-4})^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0.$

- $\lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty} \left(1+\frac{a}{n}\right)^{n} = e^{a}. \end{array}\right] = e^{-4}.$
- $\lim\limits_{n\to\infty} \frac{n^6+6}{2n+3} = \lim\limits_{n\to\infty} \frac{n\cdot(n^5+\frac{6}{n})}{n\cdot(2+\frac{3}{n})} = \lim\limits_{n\to\infty} \frac{n^5+\frac{6}{n}}{2+\frac{3}{n}} = \frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{\infty+0}{2+0} = \infty.$

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} = 0$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim\limits_{n\to\infty} \frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5} = \frac{2\cdot 0+3\cdot 0-0}{2\cdot 0-0+3\cdot 5} = \frac{0}{15} = 0.$

Označme $a_n = \frac{n^k}{5^n}$, $n\in N$ pre $k\in N$.

- $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n^k}}{5} = \lim\limits_{n\to\infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$

Resp.
- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{(n+1)^k}{5n^k} = \lim\limits_{n\to\infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1.$

$\Rightarrow \lim\limits_{n\to\infty} \frac{n^k}{5^n} = 0.$

Riešené limity – 47, 48

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n^6+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{n^6+6}{2n+3}} = (e^{-4})^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0.$

- $\lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^{-4}.$
- $\lim\limits_{n\to\infty} \frac{n^6+6}{2n+3} = \lim\limits_{n\to\infty} \frac{n\cdot(n^5+\frac{6}{n})}{n\cdot(2+\frac{3}{n})} = \lim\limits_{n\to\infty} \frac{n^5+\frac{6}{n}}{2+\frac{3}{n}} = \frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{\infty+0}{2+0} = \infty.$

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} = 0$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim\limits_{n\to\infty} \frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5} = \frac{2\cdot 0+3\cdot 0-0}{2\cdot 0-0+3\cdot 5} = \frac{0}{15} = 0.$

Označme $a_n = \frac{n^k}{5^n}$, $n\in N$ pre $k\in N$.

- $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n^k}}{5} = \lim\limits_{n\to\infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$

Resp.
- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{(n+1)^k}{5n^k} = \lim\limits_{n\to\infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1.$

$\Rightarrow \lim\limits_{n\to\infty} \frac{n^k}{5^n} = 0.$

Riešené limity – 47, 48

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n^6+6}=0$$

- $=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{n^6+6}{2n+3}}=(e^{-4})^{-\infty}=\frac{1}{e^{\infty}}=\frac{1}{\infty}=0.$

- $\lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^{m}=\left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí}\\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n=e^a.\end{array}\right]=e^{-4}.$
- $\lim_{n\to\infty}\frac{n^6+6}{2n+3}=\lim_{n\to\infty}\frac{n\cdot(n^5+\frac{6}{n})}{n\cdot(2+\frac{3}{n})}=\lim_{n\to\infty}\frac{n^5+\frac{6}{n}}{2+\frac{3}{n}}=\frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{\infty+0}{2+0}=\infty.$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot5^{n+1}}=0$$

- $=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}=\lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot5}=\frac{2\cdot0+3\cdot0-0}{2\cdot0-0+3\cdot5}=\frac{0}{15}=0.$

Označme $a_n=\frac{n^k}{5^n}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1.$

Resp.
- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1.$

$\Rightarrow \lim_{n\to\infty}\frac{n^k}{5^n}=0.$

Riešené limity – 49, 50

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

Riešené limity – 49, 50

$\lim_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$

- $= \lim_{n\to\infty} \left(\frac{2n-3+2}{2n-3}\right)^{n^6+6}$

$\lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$

Pre všetky $n \in N$ platí:

- $4 \leq 4n.$ $\Rightarrow$
- $n + 5 \leq 5n + 1.$

Riešené limity – 49, 50

$\lim_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$

- $= \lim_{n\to\infty} \left(\frac{2n-3+2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{n^6+6}$

$\lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$

Pre všetky $n \in N$ platí:

- $4 \leq 4n. \Rightarrow$
- $n+5 \leq 5n+1. \Rightarrow$
- $5n+1 < 5n+25 = 5(n+5).$

Riešené limity – 49, 50

$\lim_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$

- $= \lim_{n\to\infty} \left(\frac{2n-3+2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left(\left(1+\frac{2}{2n-3}\right)^{2n-3}\right)^{\frac{n^6+6}{2n-3}}$

$\lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$

Pre všetky $n \in N$ platí:

- $4 \leq 4n.$ $\Rightarrow$ • $n+5 \leq 5n+1.$ $\Rightarrow$ • $5n+1 < 5n+25 = 5(n+5).$

$\Rightarrow$ • $1 = \frac{n+5}{n+5} \leq \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5$

Riešené limity – 49, 50

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n-3+2}{2n-3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{2}{2n-3}\right)^{2n-3}\right)^{\frac{n^6+6}{2n-3}} = (e^2)^{\infty}$

- $\lim\limits_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{2n-3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n-3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{2}{m}\right)^{m} = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty} \left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^2.$

$\lim\limits_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$

Pre všetky $n\in N$ platí:

- $4 \leq 4n. \Rightarrow$ • $n+5 \leq 5n+1. \Rightarrow$ • $5n+1 < 5n+25 = 5(n+5).$

$\Rightarrow$ • $1 = \frac{n+5}{n+5} \leq \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5. \Rightarrow$ • $1 = \sqrt[n]{1} \leq \sqrt[n]{\frac{5n+1}{n+5}} \leq \sqrt[n]{5}.$

Riešené limity – 49, 50

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$$

- $= \lim_{n\to\infty} \left(\frac{2n-3+2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left(\left(1+\frac{2}{2n-3}\right)^{2n-3}\right)^{\frac{n^6+6}{2n-3}} = (e^2)^{\infty}$

- $\lim_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{2n-3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n-3 & m\to\infty \end{array}\right] = \lim_{m\to\infty} \left(1+\frac{2}{m}\right)^{m} = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty} \left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^2.$
- $\lim_{n\to\infty} \frac{n^6+6}{2n-3} = \lim_{n\to\infty} \frac{n\cdot(n^5+\frac{6}{n})}{n\cdot(2-\frac{3}{n})} = \lim_{n\to\infty} \frac{n^5+\frac{6}{n}}{2-\frac{3}{n}} = \frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}} = \frac{\infty+0}{2-0} = \infty.$

$$\lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

Pre všetky $n\in N$ platí:

- $4 \le 4n. \Rightarrow$ • $n+5 \le 5n+1. \Rightarrow$ • $5n+1 < 5n+25 = 5(n+5).$

$\Rightarrow$ • $1 = \frac{n+5}{n+5} \le \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5. \Rightarrow$ • $1 = \sqrt[n]{1} \le \sqrt[n]{\frac{5n+1}{n+5}} \le \sqrt[n]{5}.$

$\Rightarrow$ • $1 = \lim_{n\to\infty} 1 \le \lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}} \le \lim_{n\to\infty} \sqrt[n]{5} = 1.$

Riešené limity – 49, 50

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n-3+2}{2n-3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{n^6+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{2}{2n-3}\right)^{2n-3}\right)^{\frac{n^6+6}{2n-3}} = (e^2)^{\infty} = e^{\infty}$

- $\lim\limits_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{2n-3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n-3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{2}{m}\right)^m = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty} \left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^2.$
- $\lim\limits_{n\to\infty} \frac{n^6+6}{2n-3} = \lim\limits_{n\to\infty} \frac{n\cdot(n^5+\frac{6}{n})}{n\cdot(2-\frac{3}{n})} = \lim\limits_{n\to\infty} \frac{n^5+\frac{6}{n}}{2-\frac{3}{n}} = \frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}} = \frac{\infty+0}{2-0} = \infty.$

$\lim\limits_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}} = 1$

Pre všetky $n\in N$ platí:

- $4\leq 4n.$ $\Rightarrow$ • $n+5\leq 5n+1.$ $\Rightarrow$ • $5n+1 < 5n+25 = 5(n+5).$

$\Rightarrow$ • $1 = \frac{n+5}{n+5} \leq \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5.$ $\Rightarrow$ • $1 = \sqrt[n]{1} \leq \sqrt[n]{\frac{5n+1}{n+5}} \leq \sqrt[n]{5}.$

$\Rightarrow$ • $1 = \lim\limits_{n\to\infty} 1 \leq \lim\limits_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}} \leq \lim\limits_{n\to\infty} \sqrt[n]{5} = 1.$ $\Rightarrow$ • $\lim\limits_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}} = 1.$

Riešené limity – 49, 50

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n-3}\right)^{n^6+6}=\infty$$

- $=\lim_{n\to\infty}\left(\frac{2n-3+2}{2n-3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1+\frac{2}{2n-3}\right)^{n^6+6}=\lim_{n\to\infty}\left(\left(1+\frac{2}{2n-3}\right)^{2n-3}\right)^{\frac{n^6+6}{2n-3}}=(e^2)^{\infty}$
$=e^{\infty}=\infty.$

- $\lim_{n\to\infty}\left(1+\frac{2}{2n-3}\right)^{2n-3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n-3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1+\frac{2}{m}\right)^{m}=\left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí}\\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n=e^a.\end{array}\right]=e^2.$
- $\lim_{n\to\infty}\frac{n^6+6}{2n-3}=\lim_{n\to\infty}\frac{n\cdot(n^5+\frac{6}{n})}{n\cdot(2-\frac{3}{n})}=\lim_{n\to\infty}\frac{n^5+\frac{6}{n}}{2-\frac{3}{n}}=\frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}}=\frac{\infty+0}{2-0}=\infty.$

$$\lim_{n\to\infty}\sqrt[n]{\frac{5n+1}{n+5}}=1$$

Pre všetky $n\in N$ platí:

- $4\leq 4n.\Rightarrow$ • $n+5\leq 5n+1.\Rightarrow$ • $5n+1<5n+25=5(n+5).$

$\Rightarrow$ • $1=\frac{n+5}{n+5}\leq\frac{5n+1}{n+5}<\frac{5(n+5)}{n+5}=5.\Rightarrow$ • $1=\sqrt[n]{1}\leq\sqrt[n]{\frac{5n+1}{n+5}}\leq\sqrt[n]{5}.$

$\Rightarrow$ • $1=\lim_{n\to\infty}1\leq\lim_{n\to\infty}\sqrt[n]{\frac{5n+1}{n+5}}\leq\lim_{n\to\infty}\sqrt[n]{5}=1.\Rightarrow$ • $\lim_{n\to\infty}\sqrt[n]{\frac{5n+1}{n+5}}=1.$

Riešené limity – 51, 52

$\lim\limits_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$

$\lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$

Riešené limity – 51, 52

$\lim\limits_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6}$

$\lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$

- $= \lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}}$

- $= \lim\limits_{n\to\infty} \frac{n^3 \cdot \left(\frac{2}{n}+3+\frac{5}{n^3}\right)}{n^4 \cdot \left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}$

Riešené limity – 51, 52

$\lim\limits_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2n^6+3}\right)^{n+6}$

$\lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$

- $= \lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim\limits_{n\to\infty} \frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}}$

- $= \lim\limits_{n\to\infty} \frac{n^3\cdot(\frac{2}{n}+3+\frac{5}{n^3})}{n^4\cdot(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})} = \lim\limits_{n\to\infty} \frac{\frac{2}{n}+3+\frac{5}{n^3}}{n(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})}$

Riešené limity – 51, 52

$\lim\limits_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6} = \lim\limits_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{n+6} = \lim\limits_{n\to\infty}\left(\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3}\right)^{\frac{n+6}{2n^6+3}}$

$\lim\limits_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$

- $= \lim\limits_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim\limits_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{1-\infty-\frac{1}{\infty}+\frac{2}{\infty}}$

- $= \lim\limits_{n\to\infty}\frac{n^3\cdot(\frac{2}{n}+3+\frac{5}{n^3})}{n^4\cdot(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})} = \lim\limits_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{n(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\infty\cdot(\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty})}$

Riešené limity – 51, 52

$\lim\limits_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2n^6+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3}\right)^{\frac{n+6}{2n^6+3}} = (e^{-4})^0$

- $\lim\limits_{n\to\infty} \left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n^6+3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^{-4}.$

$\lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$

- $= \lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim\limits_{n\to\infty} \frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{1-\infty-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{1-\infty-0+0}$

- $= \lim\limits_{n\to\infty} \frac{n^3\cdot\left(\frac{2}{n}+3+\frac{5}{n^3}\right)}{n^4\cdot\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)} = \lim\limits_{n\to\infty} \frac{\frac{2}{n}+3+\frac{5}{n^3}}{n\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\infty\cdot\left(\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}\right)}$
  $= \frac{0+3+0}{\infty\cdot(0-1-0+0)}$

Riešené limity – 51, 52

$\lim\limits_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$

- $=\lim\limits_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6}=\lim\limits_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{n+6}=\lim\limits_{n\to\infty}\left(\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3}\right)^{\frac{n+6}{2n^6+3}}=(e^{-4})^0$

- $\lim\limits_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2n^6+3 & m\to\infty\end{array}\right]=\lim\limits_{m\to\infty}\left(1+\frac{-4}{m}\right)^m=\left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n=e^a.\end{array}\right]=e^{-4}.$
- $\lim\limits_{n\to\infty}\frac{n+6}{2n^6+3}=\lim\limits_{n\to\infty}\frac{n\cdot(1+\frac{6}{n})}{n\cdot(2n^5+\frac{3}{n})}=\lim\limits_{n\to\infty}\frac{1+\frac{6}{n}}{2n^5+\frac{3}{n}}=\frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}}=\frac{1+0}{\infty+0}=\frac{1}{\infty}=0.$

$\lim\limits_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$

- $=\lim\limits_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}=\lim\limits_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{1-\infty-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{1-\infty-0+0}=\frac{3}{-\infty}$

- $=\lim\limits_{n\to\infty}\frac{n^3\cdot(\frac{2}{n}+3+\frac{5}{n^3})}{n^4\cdot(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})}=\lim\limits_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{n(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\infty\cdot(\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty})}$

  $=\frac{0+3+0}{\infty\cdot(0-1-0+0)}=\frac{3}{\infty\cdot(-1)}=\frac{3}{-\infty}$

Riešené limity – 51, 52

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$$

- $$= \lim_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{n+6} = \lim_{n\to\infty}\left(\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3}\right)^{\frac{n+6}{2n^6+3}} = (e^{-4})^0 = e^0$$

  - $$\lim_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2n^6+3 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a.\end{array}\right] = e^{-4}.$$
  - $$\lim_{n\to\infty}\frac{n+6}{2n^6+3} = \lim_{n\to\infty}\frac{n\cdot(1+\frac{6}{n})}{n\cdot(2n^5+\frac{3}{n})} = \lim_{n\to\infty}\frac{1+\frac{6}{n}}{2n^5+\frac{3}{n}} = \frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}} = \frac{1+0}{\infty+0} = \frac{1}{\infty} = 0.$$

$$\lim_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2} = 0$$

- $$= \lim_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{1-\infty-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{1-\infty-0+0} = \frac{3}{-\infty} = 0.$$

- $$= \lim_{n\to\infty}\frac{n^3\cdot(\frac{2}{n}+3+\frac{5}{n^3})}{n^4\cdot(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})} = \lim_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{n(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\infty\cdot(\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty})}$$
  $$= \frac{0+3+0}{\infty\cdot(0-1-0+0)} = \frac{3}{\infty\cdot(-1)} = \frac{3}{-\infty} = 0.$$

Riešené limity – 51, 52

$\lim\limits_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n+6} = 1$

- $= \lim\limits_{n\to\infty} \left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2n^6+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3}\right)^{\frac{n+6}{2n^6+3}} = (e^{-4})^0 = e^0 = 1.$

  - $\lim\limits_{n\to\infty} \left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n^6+3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^{-4}.$
  - $\lim\limits_{n\to\infty} \frac{n+6}{2n^6+3} = \lim\limits_{n\to\infty} \frac{n\cdot(1+\frac{6}{n})}{n\cdot(2n^5+\frac{3}{n})} = \lim\limits_{n\to\infty} \frac{1+\frac{6}{n}}{2n^5+\frac{3}{n}} = \frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}} = \frac{1+0}{\infty+0} = \frac{1}{\infty} = 0.$

$\lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2} = 0$

- $= \lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim\limits_{n\to\infty} \frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{1-\infty-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{1-\infty-0+0} = \frac{3}{-\infty} = 0.$

- $= \lim\limits_{n\to\infty} \frac{n^3\cdot(\frac{2}{n}+3+\frac{5}{n^3})}{n^4\cdot(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})} = \lim\limits_{n\to\infty} \frac{\frac{2}{n}+3+\frac{5}{n^3}}{n(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\infty\cdot(\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty})} = \frac{0+3+0}{\infty\cdot(0-1-0+0)} = \frac{3}{\infty\cdot(-1)} = \frac{3}{-\infty} = 0.$

Riešené limity – 53, 54

$$\lim_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$$

$$\lim_{n\to\infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$$

Riešené limity – 53, 54

$\lim\limits_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6}$

$\lim\limits_{n\to\infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$

- $= \lim\limits_{n\to\infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}}$

- $= \lim\limits_{n\to\infty} \frac{n^4(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{n^3(\frac{2}{n}-3+\frac{5}{n^3})}$

Riešené limity – 53, 54

$\lim\limits_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$

- $= \lim\limits_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6} = \lim\limits_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{n^6+6}$

$\lim\limits_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$

- $= \lim\limits_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim\limits_{n\to\infty}\frac{1-n+\frac{2}{n}+\frac{2}{n^3}}{\frac{2}{n}-3+\frac{5}{n^3}}$

- $= \lim\limits_{n\to\infty}\frac{n^4(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{n^3(\frac{2}{n}-3+\frac{5}{n^3})} = \lim\limits_{n\to\infty}\frac{n(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{\frac{2}{n}-3+\frac{5}{n^3}}$

Riešené limity – 53, 54

$$\lim_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$$

- $= \lim_{n\to\infty} \left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1+\frac{-4}{2n^6+3}\right)^{n^6+6} = \lim_{n\to\infty} \left(\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3}\right)^{\frac{n^6+6}{2n^6+3}}$

$$\lim_{n\to\infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$$

- $= \lim_{n\to\infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{1-n+\frac{2}{n}+\frac{2}{n^3}}{\frac{2}{n}-3+\frac{5}{n^3}} = \frac{1-\infty+\frac{2}{\infty}+\frac{2}{\infty}}{\frac{2}{\infty}-3+\frac{5}{\infty}}$

- $= \lim_{n\to\infty} \frac{n^4(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{n^3(\frac{2}{n}-3+\frac{5}{n^3})} = \lim_{n\to\infty} \frac{n(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{\frac{2}{n}-3+\frac{5}{n^3}} = \frac{\infty\cdot(\frac{1}{\infty}-1+\frac{2}{\infty}+\frac{2}{\infty})}{\frac{2}{\infty}-3+\frac{5}{\infty}}$

Riešené limity – 53, 54

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$$

- $= \lim_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{n^6+6} = \lim_{n\to\infty}\left(\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3}\right)^{\frac{n^6+6}{2n^6+3}} = \left(e^{-4}\right)^{\frac{1}{2}}$

- $\lim_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2n^6+3 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a.\end{array}\right] = e^{-4}.$

$$\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$$

- $= \lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty}\frac{1-n+\frac{2}{n}+\frac{2}{n^3}}{\frac{2}{n}-3+\frac{5}{n^3}} = \frac{1-\infty+\frac{2}{\infty}+\frac{2}{\infty}}{\frac{2}{\infty}-3+\frac{5}{\infty}} = \frac{1-\infty+0+0}{0-3+0}$

- $= \lim_{n\to\infty}\frac{n^4\left(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4}\right)}{n^3\left(\frac{2}{n}-3+\frac{5}{n^3}\right)} = \lim_{n\to\infty}\frac{n\left(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4}\right)}{\frac{2}{n}-3+\frac{5}{n^3}} = \frac{\infty\cdot\left(\frac{1}{\infty}-1+\frac{2}{\infty}+\frac{2}{\infty}\right)}{\frac{2}{\infty}-3+\frac{5}{\infty}} = \frac{\infty\cdot(0-1+0+0)}{0-3+0}$

Riešené limity – 53, 54

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$$

- $= \lim_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{n^6+6} = \lim_{n\to\infty}\left(\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3}\right)^{\frac{n^6+6}{2n^6+3}} = \left(e^{-4}\right)^{\frac{1}{2}}$

- $\lim_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2n^6+3 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a.\end{array}\right] = e^{-4}.$
- $\lim_{n\to\infty}\frac{n^6+6}{2n^6+3} = \lim_{n\to\infty}\frac{n^6\cdot\left(1+\frac{6}{n^6}\right)}{n^6\cdot\left(2+\frac{3}{n^6}\right)} = \lim_{n\to\infty}\frac{1+\frac{6}{n^6}}{2+\frac{3}{n^6}} = \frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}.$

$$\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$$

- $= \lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty}\frac{1-n+\frac{2}{n}+\frac{2}{n^3}}{\frac{2}{n}-3+\frac{5}{n^3}} = \frac{1-\infty+\frac{2}{\infty}+\frac{2}{\infty}}{\frac{2}{\infty}-3+\frac{5}{\infty}} = \frac{1-\infty+0+0}{0-3+0} = \frac{-\infty}{-3}$

- $= \lim_{n\to\infty}\frac{n^4\left(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4}\right)}{n^3\left(\frac{2}{n}-3+\frac{5}{n^3}\right)} = \lim_{n\to\infty}\frac{n\left(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4}\right)}{\frac{2}{n}-3+\frac{5}{n^3}} = \frac{\infty\cdot\left(\frac{1}{\infty}-1+\frac{2}{\infty}+\frac{2}{\infty}\right)}{\frac{2}{\infty}-3+\frac{5}{\infty}} = \frac{\infty\cdot(0-1+0+0)}{0-3+0}$
  $= \frac{\infty\cdot(-1)}{-3} = \frac{\infty}{3}$

Riešené limity – 53, 54

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$$

- $= \lim\limits_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6} = \lim\limits_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{n^6+6} = \lim\limits_{n\to\infty}\left(\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3}\right)^{\frac{n^6+6}{2n^6+3}} = \left(e^{-4}\right)^{\frac{1}{2}} = e^{-2}.$

- $\lim\limits_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2n^6+3 & m\to\infty\end{array}\right] = \lim\limits_{m\to\infty}\left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a.\end{array}\right] = e^{-4}.$
- $\lim\limits_{n\to\infty}\frac{n^6+6}{2n^6+3} = \lim\limits_{n\to\infty}\frac{n^6\cdot(1+\frac{6}{n^6})}{n^6\cdot(2+\frac{3}{n^6})} = \lim\limits_{n\to\infty}\frac{1+\frac{6}{n^6}}{2+\frac{3}{n^6}} = \frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}.$

$$\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5} = \infty$$

- $= \lim\limits_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim\limits_{n\to\infty}\frac{1-n+\frac{2}{n}+\frac{2}{n^3}}{\frac{2}{n}-3+\frac{5}{n^3}} = \frac{1-\infty+\frac{2}{\infty}+\frac{2}{\infty}}{\frac{2}{\infty}-3+\frac{5}{\infty}} = \frac{1-\infty+0+0}{0-3+0} = \frac{-\infty}{-3} = \infty.$

- $= \lim\limits_{n\to\infty}\frac{n^4(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{n^3(\frac{2}{n}-3+\frac{5}{n^3})} = \lim\limits_{n\to\infty}\frac{n(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{\frac{2}{n}-3+\frac{5}{n^3}} = \frac{\infty\cdot(\frac{1}{\infty}-1+\frac{2}{\infty}+\frac{2}{\infty})}{\frac{2}{\infty}-3+\frac{5}{\infty}} = \frac{\infty\cdot(0-1+0+0)}{0-3+0} = \frac{\infty\cdot(-1)}{-3} = \frac{\infty}{3} = \infty.$

Riešené limity – 53, 54

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}=\mathrm{e}^{-2}$$

- $=\lim\limits_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6}=\lim\limits_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{n^6+6}=\lim\limits_{n\to\infty}\left(\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3}\right)^{\frac{n^6+6}{2n^6+3}}=(\mathrm{e}^{-4})^{\frac{1}{2}}=\mathrm{e}^{-2}.$

- $\lim\limits_{n\to\infty}\left(1+\frac{-4}{2n^6+3}\right)^{2n^6+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n^6+3 & m\to\infty\end{array}\right]=\lim\limits_{m\to\infty}\left(1+\frac{-4}{m}\right)^{m}=\left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí}\\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^{n}=\mathrm{e}^{a}.\end{array}\right]=\mathrm{e}^{-4}.$
- $\lim\limits_{n\to\infty}\frac{n^6+6}{2n^6+3}=\lim\limits_{n\to\infty}\frac{n^6\cdot(1+\frac{6}{n^6})}{n^6\cdot(2+\frac{3}{n^6})}=\lim\limits_{n\to\infty}\frac{1+\frac{6}{n^6}}{2+\frac{3}{n^6}}=\frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{1+0}{2+0}=\frac{1}{2}.$

$$\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}=\infty$$

- $=\lim\limits_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}=\lim\limits_{n\to\infty}\frac{1-n+\frac{2}{n}+\frac{2}{n^3}}{\frac{2}{n}-3+\frac{5}{n^3}}=\frac{1-\infty+\frac{2}{\infty}+\frac{2}{\infty}}{\frac{2}{\infty}-3+\frac{5}{\infty}}=\frac{1-\infty+0+0}{0-3+0}=\frac{-\infty}{-3}=\infty.$

- $=\lim\limits_{n\to\infty}\frac{n^4(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{n^3(\frac{2}{n}-3+\frac{5}{n^3})}=\lim\limits_{n\to\infty}\frac{n(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{\frac{2}{n}-3+\frac{5}{n^3}}=\frac{\infty\cdot(\frac{1}{\infty}-1+\frac{2}{\infty}+\frac{2}{\infty})}{\frac{2}{\infty}-3+\frac{5}{\infty}}=\frac{\infty\cdot(0-1+0+0)}{0-3+0}$
  $=\frac{\infty\cdot(-1)}{-3}=\frac{\infty}{3}=\infty.$

Riešené limity – 55, 56

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

$$\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

Riešené limity – 55, 56

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

- $= \lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$

$$\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

- $= \lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}}$

- $= \lim_{n\to\infty}\frac{n^4\left(\frac{2}{n^2}+3+\frac{5}{n^4}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}$

Riešené limity – 55, 56

$$\lim_{n\to\infty} \Big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\Big)^{\sqrt[6]{n}+6}$$

- $= \lim_{n\to\infty} \Big(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\Big)^{\sqrt[6]{n}+6} = \lim_{n\to\infty} \Big(1+\frac{-4}{2\sqrt[6]{n}+3}\Big)^{\sqrt[6]{n}+6}$

$$\lim_{n\to\infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

- $= \lim_{n\to\infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}$

- $= \lim_{n\to\infty} \frac{n^4(\frac{2}{n^2}+3+\frac{5}{n^4})}{n^4(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})} = \lim_{n\to\infty} \frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}$

Riešené limity – 55, 56

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

- $= \lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right)^{\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}}$

$$\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

- $= \lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}$

- $= \lim_{n\to\infty}\frac{n^4\left(\frac{2}{n^2}+3+\frac{5}{n^4}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)} = \lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}$

Riešené limity – 55, 56

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

- $= \lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right)^{\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}} = \left(e^{-4}\right)^{\frac{1}{2}}$

- $\lim_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{n}=e^{a}.\end{array}\right] = e^{-4}.$

$$\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

- $= \lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{0-1-0+0}$

- $= \lim_{n\to\infty}\frac{n^4\left(\frac{2}{n^2}+3+\frac{5}{n^4}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)} = \lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{0-1-0+0}$

Riešené limity – 55, 56

$\lim\limits_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$

- $= \lim\limits_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6} = \lim\limits_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6} = \lim\limits_{n\to\infty}\left(\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right)^{\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}} = \left(e^{-4}\right)^{\frac{1}{2}}$

- $\lim\limits_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right] = \lim\limits_{m\to\infty}\left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{l}\text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a.\end{array}\right] = e^{-4}.$
- $\lim\limits_{n\to\infty}\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3} = \lim\limits_{n\to\infty}\frac{\sqrt[6]{n}\cdot\left(1+\frac{6}{\sqrt[6]{n}}\right)}{\sqrt[6]{n}\cdot\left(2+\frac{3}{\sqrt[6]{n}}\right)} = \lim\limits_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{2+\frac{3}{\sqrt[6]{n}}} = \frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}.$

$\lim\limits_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$

- $= \lim\limits_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim\limits_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{0-1-0+0} = \frac{3}{-1}$

- $= \lim\limits_{n\to\infty}\frac{n^4\left(\frac{2}{n^2}+3+\frac{5}{n^4}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)} = \lim\limits_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{0-1-0+0} = \frac{3}{-1}$

Riešené limity – 55, 56

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}=e^{-2}$$

- $= \lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right)^{\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}} = (e^{-4})^{\frac{1}{2}} = e^{-2}.$

- $\lim_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{l}\text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{n}=e^{a}.\end{array}\right] = e^{-4}.$
- $\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3} = \lim_{n\to\infty}\frac{\sqrt[6]{n}\cdot\left(1+\frac{6}{\sqrt[6]{n}}\right)}{\sqrt[6]{n}\cdot\left(2+\frac{3}{\sqrt[6]{n}}\right)} = \lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{2+\frac{3}{\sqrt[6]{n}}} = \frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}.$

$$\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}=-3$$

- $= \lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{0-1-0+0} = \frac{3}{-1} = -3.$

- $= \lim_{n\to\infty}\frac{n^4\left(\frac{2}{n^2}+3+\frac{5}{n^4}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)} = \lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{0-1-0+0} = \frac{3}{-1} = -3.$

Riešené limity – 57, 58

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2^n n!}{n^n}$$

Riešené limity – 57, 58

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6}$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{2^n n!}{n^n}$

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}}$

Riešené limity – 57, 58

$\lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$

- $= \lim\limits_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim\limits_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{\sqrt[6]{n}+6}$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}\frac{2^n n!}{n^n}$

- $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty}\frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}} = \lim\limits_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}}$

Riešené limity – 57, 58

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{\sqrt[6]{n}+6}{2n+3}}$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{2^n n!}{n^n}$

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}} = \lim\limits_{n\to\infty} \frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim\limits_{n\to\infty} \frac{2n^n}{(n+1)^n}$

Riešené limity – 57, 58

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{\sqrt[6]{n}+6}{2n+3}} = (e^{-4})^0$

- $\lim\limits_{n\to\infty} \left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^{-4}.$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{2^n n!}{n^n}$

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}} = \lim\limits_{n\to\infty} \frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim\limits_{n\to\infty} \frac{2n^n}{(n+1)^n} = 2\lim\limits_{n\to\infty} \left(\frac{n}{n+1}\right)^n$

Riešené limity – 57, 58

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$$

- $= \lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{\sqrt[6]{n}+6}{2n+3}} = (\mathrm{e}^{-4})^{0}$

- $\lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty \end{array}\right] = \lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{n}=\mathrm{e}^{a}. \end{array}\right] = \mathrm{e}^{-4}.$
- $\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3} = \lim_{n\to\infty}\frac{\sqrt[6]{n}\cdot\left(1+\frac{6}{\sqrt[6]{n}}\right)}{n\cdot\left(2+\frac{3}{n}\right)} = \lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5}\cdot\left(2+\frac{3}{n}\right)} = \frac{1+\frac{6}{\infty}}{\infty\cdot\left(2+\frac{3}{\infty}\right)} = \frac{1+0}{\infty\cdot(2+0)} = \frac{1}{\infty} = 0.$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{2^n n!}{n^n}$$

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}} = \lim_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty}\frac{2n^n}{(n+1)^n} = 2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{n}$

  $= 2\lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^{n}\right)^{-1}$

Riešené limity – 57, 58

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$$

- $= \lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{\sqrt[6]{n}+6}{2n+3}} = (e^{-4})^0 = e^0$

  - $\lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty \end{array}\right] = \lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^{-4}.$
  - $\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3} = \lim_{n\to\infty}\frac{\sqrt[6]{n}\cdot\left(1+\frac{6}{\sqrt[6]{n}}\right)}{n\cdot\left(2+\frac{3}{n}\right)} = \lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5}\cdot\left(2+\frac{3}{n}\right)} = \frac{1+\frac{6}{\infty}}{\infty\cdot\left(2+\frac{3}{\infty}\right)} = \frac{1+0}{\infty\cdot(2+0)} = \frac{1}{\infty} = 0.$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{2^n n!}{n^n}$$

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}} = \lim_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty}\frac{2n^n}{(n+1)^n} = 2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n$

  $= 2\lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^n\right)^{-1} = 2\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{-1}$

Riešené limity – 57, 58

$\lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$

- $= \lim\limits_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim\limits_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim\limits_{n\to\infty}\left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{\sqrt[6]{n}+6}{2n+3}} = (e^{-4})^0 = e^0 = 1.$

- $\lim\limits_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty\end{array}\right] = \lim\limits_{m\to\infty}\left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a.\end{array}\right] = e^{-4}.$
- $\lim\limits_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3} = \lim\limits_{n\to\infty}\frac{\sqrt[6]{n}\cdot(1+\frac{6}{\sqrt[6]{n}})}{n\cdot(2+\frac{3}{n})} = \lim\limits_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5}\cdot(2+\frac{3}{n})} = \frac{1+\frac{6}{\infty}}{\infty\cdot(2+\frac{3}{\infty})} = \frac{1+0}{\infty\cdot(2+0)} = \frac{1}{\infty} = 0.$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}\frac{2^n n!}{n^n}$

- $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty}\frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}} = \lim\limits_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim\limits_{n\to\infty}\frac{2n^n}{(n+1)^n} = 2\lim\limits_{n\to\infty}\left(\frac{n}{n+1}\right)^n$

$$= 2\lim\limits_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^n\right)^{-1} = 2\lim\limits_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{-1} = 2\,e^{-1} = \frac{2}{e}$$

Riešené limity – 57, 58

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}=1$$

- $= \lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{\sqrt[6]{n}+6}{2n+3}} = (e^{-4})^0 = e^0 = 1.$

- $\lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^m = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a.\end{array}\right] = e^{-4}.$
- $\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3} = \lim_{n\to\infty}\frac{\sqrt[6]{n}\cdot(1+\frac{6}{\sqrt[6]{n}})}{n\cdot(2+\frac{3}{n})} = \lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5}\cdot(2+\frac{3}{n})} = \frac{1+\frac{6}{\infty}}{\infty\cdot(2+\frac{3}{\infty})} = \frac{1+0}{\infty\cdot(2+0)} = \frac{1}{\infty} = 0.$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{2^n n!}{n^n}$$

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}} = \lim_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty}\frac{2n^n}{(n+1)^n} = 2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n$

  $= 2\lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^n\right)^{-1} = 2\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{-1} = 2\,e^{-1} = \frac{2}{e} < 1.$ [$2 < e \approx 2{,}718$.]

Riešené limity – 57, 58

$\lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}=1$

- $=\lim\limits_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim\limits_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim\limits_{n\to\infty}\left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{\sqrt[6]{n}+6}{2n+3}}=(e^{-4})^0$ $= e^0 = 1.$

- $\lim\limits_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim\limits_{m\to\infty}\left(1+\frac{-4}{m}\right)^m=\left[\begin{array}{l}\text{Pre všetky } a\in R \text{ platí}\\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n=e^a.\end{array}\right]=e^{-4}.$
- $\lim\limits_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3}=\lim\limits_{n\to\infty}\frac{\sqrt[6]{n}\cdot\left(1+\frac{6}{\sqrt[6]{n}}\right)}{n\cdot\left(2+\frac{3}{n}\right)}=\lim\limits_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5}\cdot\left(2+\frac{3}{n}\right)}=\frac{1+\frac{6}{\infty}}{\infty\cdot\left(2+\frac{3}{\infty}\right)}=\frac{1+0}{\infty\cdot(2+0)}=\frac{1}{\infty}=0.$

$\lim\limits_{n\to\infty}a_n=\lim\limits_{n\to\infty}\frac{2^n n!}{n^n}$

- $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}}=\lim\limits_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}}=\lim\limits_{n\to\infty}\frac{2n^n}{(n+1)^n}=2\lim\limits_{n\to\infty}\left(\frac{n}{n+1}\right)^n$

$=2\lim\limits_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^n\right)^{-1}=2\lim\limits_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{-1}=2\,e^{-1}=\frac{2}{e}<1.$ $[2<e\approx 2{,}718.]$

$\Rightarrow$ • $\lim\limits_{n\to\infty}a_n=\lim\limits_{n\to\infty}\frac{2^n n!}{n^n}=0.$

Riešené limity – 57, 58

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}=1$$

- $=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left(\left(1+\frac{-4}{2n+3}\right)^{2n+3}\right)^{\frac{\sqrt[6]{n}+6}{2n+3}}=(e^{-4})^0$
$=e^0=1.$

- $\lim_{n\to\infty}\left(1+\frac{-4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^m=\left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí}\\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n=e^a.\end{array}\right]=e^{-4}.$
- $\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3}=\lim_{n\to\infty}\frac{\sqrt[6]{n}\cdot\left(1+\frac{6}{\sqrt[6]{n}}\right)}{n\cdot\left(2+\frac{3}{n}\right)}=\lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5}\cdot\left(2+\frac{3}{n}\right)}=\frac{1+\frac{6}{\infty}}{\infty\cdot\left(2+\frac{3}{\infty}\right)}=\frac{1+0}{\infty\cdot(2+0)}=\frac{1}{\infty}=0.$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{2^n n!}{n^n}=0$$

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}}=\lim_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{2n^n}{(n+1)^n}=2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n$
$=2\lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^n\right)^{-1}=2\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{-1}=2\,e^{-1}=\frac{2}{e}<1.$ [$2<e\approx 2{,}718$.]

$\Rightarrow$ • $\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{2^n n!}{n^n}=0.$

Riešené limity – 59, 60

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n n!}{n^n}$$

Riešené limity – 59, 60

$$\lim_{n\to\infty} \Big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\Big)^{n+6}$$

- $= \lim_{n\to\infty} \Big(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\Big)^{n+6}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n n!}{n^n}$$

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}}$

Riešené limity – 59, 60

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$$

- $= \lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{n+6}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{3^n n!}{n^n}$$

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim_{n\to\infty}\frac{3(n+1)n^n}{(n+1)^{n+1}}$

Riešené limity – 59, 60

$\lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right)^{\frac{n+6}{2\sqrt[6]{n}+3}}$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{3^n n!}{n^n}$

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim\limits_{n\to\infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim\limits_{n\to\infty} \frac{3n^n}{(n+1)^n}$

Riešené limity – 59, 60

$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$

- $= \lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty} \left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty} \left(\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right)^{\frac{n+6}{2\sqrt[6]{n}+3}}$ $= (e^{-4})^{\infty}$

- $\lim_{n\to\infty} \left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m = 2\sqrt[6]{n}+3 & m\to\infty \end{array}\right] = \lim_{m\to\infty} \left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty} \left(1+\frac{a}{n}\right)^{n} = e^{a}. \end{array}\right] = e^{-4}.$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n n!}{n^n}$

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim_{n\to\infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty} \frac{3n^n}{(n+1)^n} = 3\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{n}$

Riešené limity – 59, 60

$\lim\limits_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim\limits_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim\limits_{n\to\infty}\left(\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right)^{\frac{n+6}{2\sqrt[6]{n}+3}}$
  $= (e^{-4})^{\infty}$

- $\lim\limits_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right] = \lim\limits_{m\to\infty}\left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^{n}=e^{a}.\end{array}\right] = e^{-4}.$
- $\lim\limits_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3} = \lim\limits_{n\to\infty}\frac{n\cdot(1+\frac{6}{n})}{\sqrt[6]{n}\cdot(2+\frac{3}{\sqrt[6]{n}})} = \lim\limits_{n\to\infty}\frac{\sqrt[6]{n^5}\cdot(1+\frac{6}{n})}{2+\frac{3}{\sqrt[6]{n}}} = \frac{\infty\cdot(1+\frac{6}{\infty})}{2+\frac{3}{\infty}} = \frac{\infty\cdot(1+0)}{2+0} = \frac{\infty}{2} = \infty.$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}\frac{3^n n!}{n^n}$

- $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty}\frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim\limits_{n\to\infty}\frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim\limits_{n\to\infty}\frac{3n^n}{(n+1)^n} = 3\lim\limits_{n\to\infty}\left(\frac{n}{n+1}\right)^{n}$
  $= 3\lim\limits_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^{n}\right)^{-1}$

Riešené limity – 59, 60

$\lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right)^{\frac{n+6}{2\sqrt[6]{n}+3}}$
  $= (e^{-4})^{\infty} = e^{-\infty}$

- $\lim\limits_{n\to\infty} \left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m = 2\sqrt[6]{n}+3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty} \left(1+\frac{a}{n}\right)^{n} = e^{a}. \end{array}\right] = e^{-4}.$
- $\lim\limits_{n\to\infty} \frac{n+6}{2\sqrt[6]{n}+3} = \lim\limits_{n\to\infty} \frac{n\cdot(1+\frac{6}{n})}{\sqrt[6]{n}\cdot(2+\frac{3}{\sqrt[6]{n}})} = \lim\limits_{n\to\infty} \frac{\sqrt[6]{n^5}\cdot(1+\frac{6}{n})}{2+\frac{3}{\sqrt[6]{n}}} = \frac{\infty\cdot(1+\frac{6}{\infty})}{2+\frac{3}{\infty}} = \frac{\infty\cdot(1+0)}{2+0} = \frac{\infty}{2} = \infty.$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{3^n n!}{n^n}$

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim\limits_{n\to\infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim\limits_{n\to\infty} \frac{3n^n}{(n+1)^n} = 3\lim\limits_{n\to\infty} \left(\frac{n}{n+1}\right)^{n}$
  $= 3\lim\limits_{n\to\infty} \left(\left(\frac{n+1}{n}\right)^{n}\right)^{-1} = 3\lim\limits_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^{-1}$

Riešené limity – 59, 60

$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$

- $= \lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty} \left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty} \left(\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right)^{\frac{n+6}{2\sqrt[6]{n}+3}}$
  $= (e^{-4})^{\infty} = e^{-\infty} = \frac{1}{\infty}$

- $\lim_{n\to\infty} \left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2\sqrt[6]{n}+3 & m\to\infty \end{array}\right] = \lim_{m\to\infty} \left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty} \left(1+\frac{a}{n}\right)^{n} = e^{a}. \end{array}\right] = e^{-4}.$
- $\lim_{n\to\infty} \frac{n+6}{2\sqrt[6]{n}+3} = \lim_{n\to\infty} \frac{n\cdot(1+\frac{6}{n})}{\sqrt[6]{n}\cdot(2+\frac{3}{\sqrt[6]{n}})} = \lim_{n\to\infty} \frac{\sqrt[6]{n^5}\cdot(1+\frac{6}{n})}{2+\frac{3}{\sqrt[6]{n}}} = \frac{\infty\cdot(1+\frac{6}{\infty})}{2+\frac{3}{\infty}} = \frac{\infty\cdot(1+0)}{2+0} = \frac{\infty}{2} = \infty.$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n n!}{n^n}$

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim_{n\to\infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty} \frac{3n^n}{(n+1)^n} = 3\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{n}$
  $= 3\lim_{n\to\infty} \left(\left(\frac{n+1}{n}\right)^{n}\right)^{-1} = 3\lim_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^{-1} = 3\,e^{-1} = \frac{3}{e}$

Riešené limity – 59, 60

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6} = 0$$

- $= \lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty} \left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty} \left(\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right)^{\frac{n+6}{2\sqrt[6]{n}+3}}$
  $= (e^{-4})^{\infty} = e^{-\infty} = \frac{1}{\infty} = 0.$

- $\lim_{n\to\infty} \left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m = 2\sqrt[6]{n}+3 & m\to\infty \end{array}\right] = \lim_{m\to\infty} \left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^{-4}.$
- $\lim_{n\to\infty} \frac{n+6}{2\sqrt[6]{n}+3} = \lim_{n\to\infty} \frac{n\cdot(1+\frac{6}{n})}{\sqrt[6]{n}\cdot(2+\frac{3}{\sqrt[6]{n}})} = \lim_{n\to\infty} \frac{\sqrt[6]{n^5}\cdot(1+\frac{6}{n})}{2+\frac{3}{\sqrt[6]{n}}} = \frac{\infty\cdot(1+\frac{6}{\infty})}{2+\frac{3}{\infty}} = \frac{\infty\cdot(1+0)}{2+0} = \frac{\infty}{2} = \infty.$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n n!}{n^n}$$

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim_{n\to\infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty} \frac{3n^n}{(n+1)^n} = 3\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n$
  $= 3\lim_{n\to\infty} \left(\left(\frac{n+1}{n}\right)^n\right)^{-1} = 3\lim_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^n\right)^{-1} = 3\,e^{-1} = \frac{3}{e} > 1.$ $[3 > e \approx 2{,}718.]$

Riešené limity – 59, 60

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}=0$$

- $= \lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty}\left(\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right)^{\frac{n+6}{2\sqrt[6]{n}+3}}$
  $= (e^{-4})^{\infty} = e^{-\infty} = \frac{1}{\infty} = 0.$

  - $\lim_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^{m} = \left[\begin{array}{l}\text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{n}=e^{a}.\end{array}\right] = e^{-4}.$
  - $\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3} = \lim_{n\to\infty}\frac{n\cdot(1+\frac{6}{n})}{\sqrt[6]{n}\cdot(2+\frac{3}{\sqrt[6]{n}})} = \lim_{n\to\infty}\frac{\sqrt[6]{n^5}\cdot(1+\frac{6}{n})}{2+\frac{3}{\sqrt[6]{n}}} = \frac{\infty\cdot(1+\frac{6}{\infty})}{2+\frac{3}{\infty}} = \frac{\infty\cdot(1+0)}{2+0} = \frac{\infty}{2} = \infty.$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{3^n n!}{n^n}$$

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim_{n\to\infty}\frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty}\frac{3n^n}{(n+1)^n} = 3\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n$
  $= 3\lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^n\right)^{-1} = 3\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{-1} = 3\,e^{-1} = \frac{3}{e} > 1.$ $[3 > e \approx 2{,}718.]$

  $\Rightarrow$ • $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{3^n n!}{n^n} = \infty.$

Riešené limity – 59, 60

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}=0$$

- $=\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6}=\lim_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{n+6}=\lim_{n\to\infty}\left(\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right)^{\frac{n+6}{2\sqrt[6]{n}+3}}$
  $=(\mathrm{e}^{-4})^{\infty}=\mathrm{e}^{-\infty}=\frac{1}{\infty}=0.$

- $\lim_{n\to\infty}\left(1+\frac{-4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1+\frac{-4}{m}\right)^{m}=\left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí}\\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{n}=\mathrm{e}^{a}.\end{array}\right]=\mathrm{e}^{-4}.$
- $\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3}=\lim_{n\to\infty}\frac{n\cdot(1+\frac{6}{n})}{\sqrt[6]{n}\cdot(2+\frac{3}{\sqrt[6]{n}})}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}\cdot(1+\frac{6}{n})}{2+\frac{3}{\sqrt[6]{n}}}=\frac{\infty\cdot(1+\frac{6}{\infty})}{2+\frac{3}{\infty}}=\frac{\infty\cdot(1+0)}{2+0}=\frac{\infty}{2}=\infty.$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{3^n n!}{n^n}=\infty$$

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}}=\lim_{n\to\infty}\frac{3(n+1)n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{3n^n}{(n+1)^n}=3\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{n}$
  $=3\lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^{n}\right)^{-1}=3\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right)^{-1}=3\,\mathrm{e}^{-1}=\frac{3}{\mathrm{e}}>1.$ [$3>\mathrm{e}\approx 2{,}718$.]
  $\Rightarrow$ • $\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{3^n n!}{n^n}=\infty.$

Riešené limity – 61, 62

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{8^n n!}{(3n)^n} = \lim_{n\to\infty} \frac{8^n n!}{3^n n^n}$$

Riešené limity – 61, 62

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$$

- $= \lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{8^n n!}{(3n)^n} = \lim_{n\to\infty} \frac{8^n n!}{3^n n^n}$$

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}}$

Riešené limity – 61, 62

$\lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6}$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{8^n n!}{(3n)^n} = \lim\limits_{n\to\infty} \frac{8^n n!}{3^n n^n}$

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim\limits_{n\to\infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}}$

Riešené limity – 61, 62

$\lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}\right)^{\frac{n+6}{2\sqrt[6]{n}-3}}$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{8^n n!}{(3n)^n} = \lim\limits_{n\to\infty} \frac{8^n n!}{3^n n^n}$

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim\limits_{n\to\infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim\limits_{n\to\infty} \frac{8n^n}{3(n+1)^n}$

Riešené limity – 61, 62

$\lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}\right)^{\frac{n+6}{2\sqrt[6]{n}-3}} = (e^2)^{\infty}$

  - $\lim\limits_{n\to\infty} \left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m = 2\sqrt[6]{n}-3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{2}{m}\right)^{m} = \left[\begin{array}{l} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty} \left(1+\frac{a}{n}\right)^{n} = e^{a}. \end{array}\right] = e^2.$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{8^n n!}{(3n)^n} = \lim\limits_{n\to\infty} \frac{8^n n!}{3^n n^n}$

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim\limits_{n\to\infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim\limits_{n\to\infty} \frac{8n^n}{3(n+1)^n} = \frac{8}{3} \lim\limits_{n\to\infty} \left(\frac{n}{n+1}\right)^n$

Riešené limity – 61, 62

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$$

- $= \lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim_{n\to\infty}\left(\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}\right)^{\frac{n+6}{2\sqrt[6]{n}-3}} = (e^2)^{\infty}$

- $\lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2\sqrt[6]{n}-3 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\left(1+\frac{2}{m}\right)^m = \left[\begin{array}{c}\text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a.\end{array}\right] = e^2.$
- $\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3} = \lim_{n\to\infty}\frac{n\cdot(1+\frac{6}{n})}{\sqrt[6]{n}\cdot(2-\frac{3}{\sqrt[6]{n}})} = \lim_{n\to\infty}\frac{\sqrt[6]{n^5}\cdot(1+\frac{6}{n})}{2-\frac{3}{\sqrt[6]{n}}} = \frac{\infty\cdot(1+\frac{6}{\infty})}{2-\frac{3}{\infty}} = \frac{\infty\cdot(1+0)}{2-0} = \frac{\infty}{2} = \infty.$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{8^n n!}{(3n)^n} = \lim_{n\to\infty}\frac{8^n n!}{3^n n^n}$$

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim_{n\to\infty}\frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n\to\infty}\frac{8n^n}{3(n+1)^n} = \frac{8}{3}\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n$

  $= \frac{8}{3}\lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^n\right)^{-1}$

Riešené limity – 61, 62

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$$

- $= \lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim_{n\to\infty}\left(\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}\right)^{\frac{n+6}{2\sqrt[6]{n}-3}}$
  $= (e^2)^{\infty} = e^{\infty}$

- $\lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2\sqrt[6]{n}-3 & m\to\infty \end{array}\right] = \lim_{m\to\infty}\left(1+\frac{2}{m}\right)^m = \left[\begin{array}{c} \text{Pre všetky } a\in R \text{ platí} \\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^2.$
- $\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3} = \lim_{n\to\infty}\frac{n\cdot(1+\frac{6}{n})}{\sqrt[6]{n}\cdot(2-\frac{3}{\sqrt[6]{n}})} = \lim_{n\to\infty}\frac{\sqrt[6]{n^5}\cdot(1+\frac{6}{n})}{2-\frac{3}{\sqrt[6]{n}}} = \frac{\infty\cdot(1+\frac{6}{\infty})}{2-\frac{3}{\infty}} = \frac{\infty\cdot(1+0)}{2-0} = \frac{\infty}{2} = \infty.$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{8^n n!}{(3n)^n} = \lim_{n\to\infty}\frac{8^n n!}{3^n n^n}$$

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim_{n\to\infty}\frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n\to\infty}\frac{8n^n}{3(n+1)^n} = \frac{8}{3}\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n$
  $= \frac{8}{3}\lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^n\right)^{-1} = \frac{8}{3}\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{-1}$

Riešené limity – 61, 62

$\lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$

- $= \lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}\right)^{\frac{n+6}{2\sqrt[6]{n}-3}}$
  $= (e^2)^{\infty} = e^{\infty} = \infty.$

- $\lim\limits_{n\to\infty} \left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m = 2\sqrt[6]{n}-3 & m\to\infty \end{array}\right] = \lim\limits_{m\to\infty} \left(1+\frac{2}{m}\right)^{m} = \left[\begin{array}{l} \text{Pre všetky } a\in R \text{ platí} \\ \lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a. \end{array}\right] = e^2.$
- $\lim\limits_{n\to\infty} \frac{n+6}{2\sqrt[6]{n}-3} = \lim\limits_{n\to\infty} \frac{n\cdot\left(1+\frac{6}{n}\right)}{\sqrt[6]{n}\cdot\left(2-\frac{3}{\sqrt[6]{n}}\right)} = \lim\limits_{n\to\infty} \frac{\sqrt[6]{n^5}\cdot\left(1+\frac{6}{n}\right)}{2-\frac{3}{\sqrt[6]{n}}} = \frac{\infty\cdot\left(1+\frac{6}{\infty}\right)}{2-\frac{3}{\infty}} = \frac{\infty\cdot(1+0)}{2-0} = \frac{\infty}{2} = \infty.$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{8^n n!}{(3n)^n} = \lim\limits_{n\to\infty} \frac{8^n n!}{3^n n^n}$

- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim\limits_{n\to\infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim\limits_{n\to\infty} \frac{8n^n}{3(n+1)^n} = \frac{8}{3} \lim\limits_{n\to\infty} \left(\frac{n}{n+1}\right)^n$
  $= \frac{8}{3} \lim\limits_{n\to\infty} \left(\left(\frac{n+1}{n}\right)^n\right)^{-1} = \frac{8}{3} \lim\limits_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^n\right)^{-1} = \frac{8}{3} e^{-1} = \frac{8}{3e}$

Riešené limity – 61, 62

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}=\infty$$

- $$=\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6}=\lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6}=\lim_{n\to\infty}\left(\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}\right)^{\frac{n+6}{2\sqrt[6]{n}-3}}$$ $$=(e^2)^{\infty}=e^{\infty}=\infty.$$

  - $$\lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}-3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1+\frac{2}{m}\right)^{m}=\left[\begin{array}{l}\text{Pre všetky } a\in R \text{ platí}\\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n=e^a.\end{array}\right]=e^2.$$
  - $$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}=\lim_{n\to\infty}\frac{n\cdot(1+\frac{6}{n})}{\sqrt[6]{n}\cdot(2-\frac{3}{\sqrt[6]{n}})}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}\cdot(1+\frac{6}{n})}{2-\frac{3}{\sqrt[6]{n}}}=\frac{\infty\cdot(1+\frac{6}{\infty})}{2-\frac{3}{\infty}}=\frac{\infty\cdot(1+0)}{2-0}=\frac{\infty}{2}=\infty.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n n!}{(3n)^n}=\lim_{n\to\infty}\frac{8^n n!}{3^n n^n}$$

- $$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}}=\lim_{n\to\infty}\frac{8(n+1)n^n}{3(n+1)^{n+1}}=\lim_{n\to\infty}\frac{8n^n}{3(n+1)^n}=\frac{8}{3}\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n$$ $$=\frac{8}{3}\lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^n\right)^{-1}=\frac{8}{3}\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{-1}=\frac{8}{3}e^{-1}=\frac{8}{3e}<1.$$ [$8<3e\approx 3\cdot 2{,}718=8{,}154$.]

Riešené limity – 61, 62

$$\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\Big)^{n+6}=\infty$$

- $= \lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\Big)^{n+6} = \lim_{n\to\infty}\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{n+6} = \lim_{n\to\infty}\Big(\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{2\sqrt[6]{n}-3}\Big)^{\frac{n+6}{2\sqrt[6]{n}-3}}$
  $= (e^2)^{\infty} = e^{\infty} = \infty.$

- $\lim_{n\to\infty}\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{2\sqrt[6]{n}-3} = \Big[\text{Subst. } m=2\sqrt[6]{n}-3 \;\Big|\; n\to\infty,\ m\to\infty\Big] = \lim_{m\to\infty}\Big(1+\frac{2}{m}\Big)^{m} = \Big[\text{Pre všetky } a\in R \text{ platí } \lim_{n\to\infty}\big(1+\frac{a}{n}\big)^{n}=e^{a}.\Big] = e^2.$
- $\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3} = \lim_{n\to\infty}\frac{n\cdot(1+\frac{6}{n})}{\sqrt[6]{n}\cdot(2-\frac{3}{\sqrt[6]{n}})} = \lim_{n\to\infty}\frac{\sqrt[6]{n^5}\cdot(1+\frac{6}{n})}{2-\frac{3}{\sqrt[6]{n}}} = \frac{\infty\cdot(1+\frac{6}{\infty})}{2-\frac{3}{\infty}} = \frac{\infty\cdot(1+0)}{2-0} = \frac{\infty}{2} = \infty.$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n n!}{(3n)^n}=\lim_{n\to\infty}\frac{8^n n!}{3^n n^n}$$

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim_{n\to\infty}\frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n\to\infty}\frac{8n^n}{3(n+1)^n} = \frac{8}{3}\lim_{n\to\infty}\Big(\frac{n}{n+1}\Big)^n$
  $= \frac{8}{3}\lim_{n\to\infty}\Big(\Big(\frac{n+1}{n}\Big)^n\Big)^{-1} = \frac{8}{3}\lim_{n\to\infty}\Big(\Big(1+\frac{1}{n}\Big)^n\Big)^{-1} = \frac{8}{3}e^{-1} = \frac{8}{3e} < 1.$ $[8 < 3e \approx 3\cdot 2{,}718 = 8{,}154.]$

  $\Rightarrow$ • $\lim_{n\to\infty}a_n = \lim_{n\to\infty}\frac{8^n n!}{3^n n^n} = 0.$

Riešené limity – 61, 62

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}=\infty$$

- $$=\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6}=\lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6}=\lim_{n\to\infty}\left(\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}\right)^{\frac{n+6}{2\sqrt[6]{n}-3}}$$ $$=(\mathrm{e}^2)^{\infty}=\mathrm{e}^{\infty}=\infty.$$

- $$\lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}-3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1+\frac{2}{m}\right)^m=\left[\begin{array}{l}\text{Pre všetky } a\in R \text{ platí}\\ \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n=\mathrm{e}^a.\end{array}\right]=\mathrm{e}^2.$$
- $$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}=\lim_{n\to\infty}\frac{n\cdot(1+\frac{6}{n})}{\sqrt[6]{n}\cdot(2-\frac{3}{\sqrt[6]{n}})}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}\cdot(1+\frac{6}{n})}{2-\frac{3}{\sqrt[6]{n}}}=\frac{\infty\cdot(1+\frac{6}{\infty})}{2-\frac{3}{\infty}}=\frac{\infty\cdot(1+0)}{2-0}=\frac{\infty}{2}=\infty.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n n!}{(3n)^n}=\lim_{n\to\infty}\frac{8^n n!}{3^n n^n}=0$$

- $$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}}=\lim_{n\to\infty}\frac{8(n+1)n^n}{3(n+1)^{n+1}}=\lim_{n\to\infty}\frac{8n^n}{3(n+1)^n}=\frac{8}{3}\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n$$ $$=\frac{8}{3}\lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^n\right)^{-1}=\frac{8}{3}\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{-1}=\frac{8}{3}\mathrm{e}^{-1}=\frac{8}{3\mathrm{e}}<1.$$ $[8<3\mathrm{e}\approx 3\cdot 2{,}718=8{,}154.]$

$\Rightarrow$ • $$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n n!}{3^n n^n}=0.$$

Riešené limity – 63

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

Riešené limity – 63

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

- $= \lim\limits_{n\to\infty} \dfrac{n^4\left(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4}\right)}{n^3\left(\frac{2}{n}-1\right)}$

- $= \lim\limits_{n\to\infty} \dfrac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3} \cdot \dfrac{\frac{1}{n^3}}{\frac{1}{n^3}}$

- $= \lim\limits_{n\to\infty} \dfrac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3} \cdot \dfrac{\frac{1}{n^4}}{\frac{1}{n^4}}$

Riešené limity – 63

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

- $= \lim_{n\to\infty} \frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty} \frac{n(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1}$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{2+3n-\frac{1}{n}-2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1}$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4}}{\frac{2}{n^2}-\frac{1}{n}}$

Riešené limity – 63

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

- $= \lim\limits_{n\to\infty} \dfrac{n^4(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim\limits_{n\to\infty} \dfrac{n(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1}$

- $= \lim\limits_{n\to\infty} \dfrac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3} \cdot \dfrac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim\limits_{n\to\infty} \dfrac{2+3n-\frac{1}{n}-2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1}$

- $= \lim\limits_{n\to\infty} \dfrac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3} \cdot \dfrac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim\limits_{n\to\infty} \dfrac{\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4}}{\frac{2}{n^2}-\frac{1}{n}}$

Označme $a_n = \frac{5^n}{n^k}$, $n\in N$ pre $k\in N$.

- $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\dfrac{5^n}{n^k}} = \lim\limits_{n\to\infty} \dfrac{5}{\sqrt[n]{n^k}} = \lim\limits_{n\to\infty} \dfrac{5}{(\sqrt[n]{n})^k} = \dfrac{5}{1^k} = 5 > 1.$

Resp. • $\lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \dfrac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim\limits_{n\to\infty} \dfrac{5n^k}{(n+1)^k} = \lim\limits_{n\to\infty} \dfrac{5}{(1+\frac{1}{n})^k} = \dfrac{5}{(1+0)^k} = 5 > 1.$

$\Rightarrow \lim\limits_{n\to\infty} \dfrac{5^n}{n^k} = \infty.$

Riešené limity – 63

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

- $= \lim_{n\to\infty} \frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty} \frac{n(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1} = \frac{\infty(\frac{2}{\infty}+3-\frac{1}{\infty}-2\cdot\infty)}{\frac{2}{\infty}-1}$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{2+3n-\frac{1}{n}-2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1} = \frac{2+3\cdot\infty-\frac{1}{\infty}-2\cdot\infty}{\frac{2}{\infty}-1}$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4}}{\frac{2}{n^2}-\frac{1}{n}} = \frac{\frac{2}{\infty}+3-\frac{1}{\infty}-2\cdot\infty}{\frac{2}{\infty}-\frac{1}{\infty}}$

Označme $a_n = \frac{5^n}{n^k}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n\to\infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n\to\infty} \frac{5}{(\sqrt[n]{n})^k} = \frac{5}{1^k} = 5 > 1.$

Resp.
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n\to\infty} \frac{5n^k}{(n+1)^k} = \lim_{n\to\infty} \frac{5}{(1+\frac{1}{n})^k} = \frac{5}{(1+0)^k} = 5 > 1.$

$\Rightarrow \lim_{n\to\infty} \frac{5^n}{n^k} = \infty.$

Riešené limity – 63

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

- $= \lim\limits_{n\to\infty} \frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim\limits_{n\to\infty} \frac{n(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1} = \frac{\infty(\frac{2}{\infty}+3-\frac{1}{\infty}-2\cdot\infty)}{\frac{2}{\infty}-1} = \frac{\infty(0+3-0-\infty)}{0-1}$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim\limits_{n\to\infty} \frac{2+3n-\frac{1}{n}-2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1} = \frac{2+3\cdot\infty-\frac{1}{\infty}-2\cdot\infty}{\frac{2}{\infty}-1} = \frac{2+\infty-0-\infty}{0-1}$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim\limits_{n\to\infty} \frac{\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4}}{\frac{2}{n^2}-\frac{1}{n}} = \frac{\frac{2}{\infty}+3-\frac{1}{\infty}-2\cdot\infty}{\frac{2}{\infty}-\frac{1}{\infty}} = \frac{0+3-0-\infty}{0-0}$

Označme $a_n = \frac{5^n}{n^k}$, $n\in N$ pre $k\in N$.

- $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim\limits_{n\to\infty} \frac{5}{\sqrt[n]{n^k}} = \lim\limits_{n\to\infty} \frac{5}{(\sqrt[n]{n})^k} = \frac{5}{1^k} = 5 > 1.$

Resp.
- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim\limits_{n\to\infty} \frac{5n^k}{(n+1)^k} = \lim\limits_{n\to\infty} \frac{5}{(1+\frac{1}{n})^k} = \frac{5}{(1+0)^k} = 5 > 1.$

$\Rightarrow \lim\limits_{n\to\infty} \frac{5^n}{n^k} = \infty.$

Riešené limity – 63

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

- $= \lim_{n\to\infty}\frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty}\frac{n(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1} = \frac{\infty(\frac{2}{\infty}+3-\frac{1}{\infty}-2\cdot\infty)}{\frac{2}{\infty}-1} = \frac{\infty(0+3-0-\infty)}{0-1} = \frac{\infty\cdot(-\infty)}{-1}$

- $= \lim_{n\to\infty}\frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty}\frac{2+3n-\frac{1}{n}-2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1} = \frac{2+3\cdot\infty-\frac{1}{\infty}-2\cdot\infty}{\frac{2}{\infty}-1} = \frac{2+\infty-0-\infty}{0-1}$?.

  [Problém nekonečno mínus nekonečno.]

- $= \lim_{n\to\infty}\frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty}\frac{\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4}}{\frac{2}{n^2}-\frac{1}{n}} = \frac{\frac{2}{\infty}+3-\frac{1}{\infty}-2\cdot\infty}{\frac{2}{\infty}-\frac{1}{\infty}} = \frac{0+3-0-\infty}{0-0} = \frac{-\infty}{0}$

Označme $a_n = \frac{5^n}{n^k}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\frac{5^n}{n^k}} = \lim_{n\to\infty}\frac{5}{\sqrt[n]{n^k}} = \lim_{n\to\infty}\frac{5}{(\sqrt[n]{n})^k} = \frac{5}{1^k} = 5 > 1.$

Resp.

- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n\to\infty}\frac{5n^k}{(n+1)^k} = \lim_{n\to\infty}\frac{5}{(1+\frac{1}{n})^k} = \frac{5}{(1+0)^k} = 5 > 1.$

$\Rightarrow \lim_{n\to\infty}\frac{5^n}{n^k} = \infty.$

Riešené limity – 63

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3} = \infty$$

- $= \lim\limits_{n\to\infty} \dfrac{n^4(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim\limits_{n\to\infty} \dfrac{n(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1} = \dfrac{\infty(\frac{2}{\infty}+3-\frac{1}{\infty}-2\cdot\infty)}{\frac{2}{\infty}-1} = \dfrac{\infty(0+3-0-\infty)}{0-1} = \dfrac{\infty\cdot(-\infty)}{-1} = \infty.$

- $= \lim\limits_{n\to\infty} \dfrac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}\cdot\dfrac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim\limits_{n\to\infty} \dfrac{2+3n-\frac{1}{n}-2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1} = \dfrac{2+3\cdot\infty-\frac{1}{\infty}-2\cdot\infty}{\frac{2}{\infty}-1} = \dfrac{2+\infty-0-\infty}{0-1}\ ?.$

  [Problém nekonečno mínus nekonečno.]

- $= \lim\limits_{n\to\infty} \dfrac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}\cdot\dfrac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim\limits_{n\to\infty} \dfrac{\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4}}{\frac{2}{n^2}-\frac{1}{n}} = \dfrac{\frac{2}{\infty}+3-\frac{1}{\infty}-2\cdot\infty}{\frac{2}{\infty}-\frac{1}{\infty}} = \dfrac{0+3-0-\infty}{0-0} = \dfrac{-\infty}{0}\ ?.$

  [Problém s delením nulou.]

Označme $a_n = \frac{5^n}{n^k}$, $n\in N$ pre $k\in N$.

- $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\dfrac{5^n}{n^k}} = \lim\limits_{n\to\infty} \dfrac{5}{\sqrt[n]{n^k}} = \lim\limits_{n\to\infty} \dfrac{5}{(\sqrt[n]{n})^k} = \dfrac{5}{1^k} = 5 > 1.$

Resp.
- $\lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \dfrac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim\limits_{n\to\infty} \dfrac{5n^k}{(n+1)^k} = \lim\limits_{n\to\infty} \dfrac{5}{(1+\frac{1}{n})^k} = \dfrac{5}{(1+0)^k} = 5 > 1.$

$\Rightarrow \lim\limits_{n\to\infty} \dfrac{5^n}{n^k} = \infty.$

Riešené limity – 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

Riešené limity – 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{7^n}}{\frac{1}{7^n}}$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}}$

Riešené limity – 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{7^n}}{\frac{1}{7^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{7^n}+3\cdot\frac{n^4}{7^n}-\frac{n^2}{7^n}+2\cdot\frac{5^n}{7^n}}{2\cdot\frac{n^2}{7^n}-\frac{n^3}{7^n}+6}$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{5^n}+3\cdot\frac{n^4}{5^n}-\frac{n^2}{5^n}+2}{2\cdot\frac{n^2}{5^n}-\frac{n^3}{5^n}+6\cdot\frac{7^n}{5^n}}$

Riešené limity – 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{7^n}}{\frac{1}{7^n}} = \lim\limits_{n\to\infty} \frac{2\cdot\frac{n^3}{7^n}+3\cdot\frac{n^4}{7^n}-\frac{n^2}{7^n}+2\cdot\frac{5^n}{7^n}}{2\cdot\frac{n^2}{7^n}-\frac{n^3}{7^n}+6}$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim\limits_{n\to\infty} \frac{2\cdot\frac{n^3}{5^n}+3\cdot\frac{n^4}{5^n}-\frac{n^2}{5^n}+2}{2\cdot\frac{n^2}{5^n}-\frac{n^3}{5^n}+6\cdot\frac{7^n}{5^n}}$

Označme $a_n = \frac{n^k}{\alpha^n}$, $n\in N$ pre $k\in N$, $\alpha\in R$, $\alpha > 1$ (špeciálne $\alpha = 5$ a $\alpha = 7$).

- $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^k}{\alpha^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n^k}}{\alpha} = \lim\limits_{n\to\infty} \frac{(\sqrt[n]{n})^k}{\alpha} = \frac{1^k}{\alpha} = \frac{1}{\alpha} < 1.$

Resp.
- $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)^k}{\alpha^{n+1}}}{\frac{n^k}{\alpha^n}} = \lim\limits_{n\to\infty} \frac{(n+1)^k}{\alpha n^k} = \lim\limits_{n\to\infty} \frac{(1+\frac{1}{n})^k}{\alpha} = \frac{(1+0)^k}{\alpha} = \frac{1}{\alpha} < 1.$

$\Rightarrow \lim\limits_{n\to\infty} \frac{n^k}{\alpha^n} = 0.$

Riešené limity – 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{7^n}}{\frac{1}{7^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{7^n}+3\cdot\frac{n^4}{7^n}-\frac{n^2}{7^n}+2\cdot\frac{5^n}{7^n}}{2\cdot\frac{n^2}{7^n}-\frac{n^3}{7^n}+6}$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{5^n}+3\cdot\frac{n^4}{5^n}-\frac{n^2}{5^n}+2}{2\cdot\frac{n^2}{5^n}-\frac{n^3}{5^n}+6\cdot\frac{7^n}{5^n}}$

Označme $a_n = \frac{n^k}{\alpha^n}$, $n \in N$ pre $k \in N$, $\alpha \in R$, $\alpha > 1$ (špeciálne $\alpha = 5$ a $\alpha = 7$).

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^k}{\alpha^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^k}}{\alpha} = \lim_{n\to\infty} \frac{(\sqrt[n]{n})^k}{\alpha} = \frac{1^k}{\alpha} = \frac{1}{\alpha} < 1.$

Resp.
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{\alpha^{n+1}}}{\frac{n^k}{\alpha^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{\alpha n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{\alpha} = \frac{(1+0)^k}{\alpha} = \frac{1}{\alpha} < 1.$

$\Rightarrow \lim_{n\to\infty} \frac{n^k}{\alpha^n} = 0.$

- $\lim_{n\to\infty} \frac{5^n}{7^n} = \lim_{n\to\infty} \left(\frac{5}{7}\right)^n = 0.$ [Limita geometrickej postupnosti pre $-1 < q = \frac{5}{7} < 1$.]

- $\lim_{n\to\infty} \frac{7^n}{5^n} = \lim_{n\to\infty} \left(\frac{7}{5}\right)^n = \infty.$ [Limita geometrickej postupnosti pre $q = \frac{7}{5} > 1$.]

Riešené limity – 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{7^n}}{\frac{1}{7^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{7^n}+3\cdot\frac{n^4}{7^n}-\frac{n^2}{7^n}+2\cdot\frac{5^n}{7^n}}{2\cdot\frac{n^2}{7^n}-\frac{n^3}{7^n}+6} = \frac{2\cdot 0+3\cdot 0-0+2\cdot 0}{2\cdot 0-0+6}$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{5^n}+3\cdot\frac{n^4}{5^n}-\frac{n^2}{5^n}+2}{2\cdot\frac{n^2}{5^n}-\frac{n^3}{5^n}+6\cdot\frac{7^n}{5^n}} = \frac{2\cdot 0+3\cdot 0-0+2}{2\cdot 0-0+6\cdot\infty}$

Označme $a_n = \frac{n^k}{\alpha^n}$, $n\in N$ pre $k\in N$, $\alpha\in R$, $\alpha > 1$ (špeciálne $\alpha = 5$ a $\alpha = 7$).

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^k}{\alpha^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^k}}{\alpha} = \lim_{n\to\infty} \frac{(\sqrt[n]{n})^k}{\alpha} = \frac{1^k}{\alpha} = \frac{1}{\alpha} < 1.$

Resp.
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{\alpha^{n+1}}}{\frac{n^k}{\alpha^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{\alpha n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{\alpha} = \frac{(1+0)^k}{\alpha} = \frac{1}{\alpha} < 1.$

$\Rightarrow \lim_{n\to\infty} \frac{n^k}{\alpha^n} = 0.$

- $\lim_{n\to\infty} \frac{5^n}{7^n} = \lim_{n\to\infty} \left(\frac{5}{7}\right)^n = 0.$ [Limita geometrickej postupnosti pre $-1 < q = \frac{5}{7} < 1$.]
- $\lim_{n\to\infty} \frac{7^n}{5^n} = \lim_{n\to\infty} \left(\frac{7}{5}\right)^n = \infty.$ [Limita geometrickej postupnosti pre $q = \frac{7}{5} > 1$.]

Riešené limity – 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{7^n}}{\frac{1}{7^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{7^n}+3\cdot\frac{n^4}{7^n}-\frac{n^2}{7^n}+2\cdot\frac{5^n}{7^n}}{2\cdot\frac{n^2}{7^n}-\frac{n^3}{7^n}+6} = \frac{2\cdot 0+3\cdot 0-0+2\cdot 0}{2\cdot 0-0+6} = \frac{0}{6}$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{5^n}+3\cdot\frac{n^4}{5^n}-\frac{n^2}{5^n}+2}{2\cdot\frac{n^2}{5^n}-\frac{n^3}{5^n}+6\cdot\frac{7^n}{5^n}} = \frac{2\cdot 0+3\cdot 0-0+2}{2\cdot 0-0+6\cdot\infty} = \frac{2}{\infty}$

Označme $a_n = \frac{n^k}{\alpha^n}$, $n\in N$ pre $k\in N$, $\alpha\in R$, $\alpha > 1$ (špeciálne $\alpha = 5$ a $\alpha = 7$).

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^k}{\alpha^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^k}}{\alpha} = \lim_{n\to\infty} \frac{(\sqrt[n]{n})^k}{\alpha} = \frac{1^k}{\alpha} = \frac{1}{\alpha} < 1.$

Resp.

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{\alpha^{n+1}}}{\frac{n^k}{\alpha^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{\alpha n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{\alpha} = \frac{(1+0)^k}{\alpha} = \frac{1}{\alpha} < 1.$

$\Rightarrow \lim_{n\to\infty} \frac{n^k}{\alpha^n} = 0.$

- $\lim_{n\to\infty} \frac{5^n}{7^n} = \lim_{n\to\infty} \left(\frac{5}{7}\right)^n = 0.$ [Limita geometrickej postupnosti pre $-1 < q = \frac{5}{7} < 1$.]

- $\lim_{n\to\infty} \frac{7^n}{5^n} = \lim_{n\to\infty} \left(\frac{7}{5}\right)^n = \infty.$ [Limita geometrickej postupnosti pre $q = \frac{7}{5} > 1$.]

Riešené limity – 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} = 0$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{7^n}}{\frac{1}{7^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{7^n}+3\cdot\frac{n^4}{7^n}-\frac{n^2}{7^n}+2\cdot\frac{5^n}{7^n}}{2\cdot\frac{n^2}{7^n}-\frac{n^3}{7^n}+6} = \frac{2\cdot 0+3\cdot 0-0+2\cdot 0}{2\cdot 0-0+6} = \frac{0}{6} = 0.$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{5^n}+3\cdot\frac{n^4}{5^n}-\frac{n^2}{5^n}+2}{2\cdot\frac{n^2}{5^n}-\frac{n^3}{5^n}+6\cdot\frac{7^n}{5^n}} = \frac{2\cdot 0+3\cdot 0-0+2}{2\cdot 0-0+6\cdot\infty} = \frac{2}{\infty} = 0.$

Označme $a_n = \frac{n^k}{\alpha^n}$, $n\in N$ pre $k\in N$, $\alpha\in R$, $\alpha > 1$ (špeciálne $\alpha = 5$ a $\alpha = 7$).

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^k}{\alpha^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^k}}{\alpha} = \lim_{n\to\infty} \frac{(\sqrt[n]{n})^k}{\alpha} = \frac{1^k}{\alpha} = \frac{1}{\alpha} < 1.$

Resp.
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{\alpha^{n+1}}}{\frac{n^k}{\alpha^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{\alpha n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{\alpha} = \frac{(1+0)^k}{\alpha} = \frac{1}{\alpha} < 1.$

$\Rightarrow \lim_{n\to\infty} \frac{n^k}{\alpha^n} = 0.$

- $\lim_{n\to\infty} \frac{5^n}{7^n} = \lim_{n\to\infty} \left(\frac{5}{7}\right)^n = 0.$ [Limita geometrickej postupnosti pre $-1 < q = \frac{5}{7} < 1$.]

- $\lim_{n\to\infty} \frac{7^n}{5^n} = \lim_{n\to\infty} \left(\frac{7}{5}\right)^n = \infty.$ [Limita geometrickej postupnosti pre $q = \frac{7}{5} > 1$.]

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $= \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}. \end{array} \right]$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $= \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}. \end{array} \right] = \lim\limits_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $= \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}. \end{array} \right] = \lim\limits_{n\to\infty} \dfrac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}} \cdot \dfrac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $= \left[\begin{array}{c} \text{Pre všetky } n\in N \text{ platí} \\ \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}. \end{array} \right] = \lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí} \\ \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}.\end{array}\right] = \lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\cdot(\frac{n^6-2}{n^6})^3}+3\cdot\sqrt[6]{\frac{1}{n^7}\cdot\left(\frac{n+1}{n}\right)^3}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}\cdot\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\cdot\left(\frac{n-1}{n}\right)^3}}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}.\end{array}\right] = \lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}}\cdot\dfrac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}}=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\cdot(\frac{n^6-2}{n^6})^3}+3\cdot\sqrt[6]{\frac{1}{n^7}\cdot\left(\frac{n+1}{n}\right)^3}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}\cdot\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\cdot\left(\frac{n-1}{n}\right)^3}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{1}{n}-\frac{1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\cdot\left(1-\frac{1}{n^6}\right)^3}+3\cdot\sqrt[6]{\frac{1}{n^7}\cdot\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}\cdot\left(1-\frac{1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\cdot\left(1-\frac{1}{n}\right)}}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}.\end{array}\right] = \lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}}\cdot\dfrac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}}=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\cdot(\frac{n^6-2}{n^6})^3}+3\cdot\sqrt[6]{\frac{1}{n^7}\cdot\left(\frac{n+1}{n}\right)^3}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}\cdot\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\cdot\left(\frac{n-1}{n}\right)^3}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{1}{n}-\frac{1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\cdot\left(1-\frac{1}{n^6}\right)^3}+3\cdot\sqrt[6]{\frac{1}{n^7}\cdot\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}\cdot\left(1-\frac{1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\cdot\left(1-\frac{1}{n}\right)}}=\frac{\sqrt[3]{0-0}-\sqrt[15]{0\cdot(1-0)^3}+3\cdot\sqrt[6]{0\cdot(1+0)}}{2\cdot\sqrt[3]{1+0}+3\cdot\sqrt[12]{0\cdot(1-0)^3}-\sqrt[6]{0\cdot(1-0)}}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}.\end{array}\right] = \lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}}\cdot\frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}}=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\cdot(\frac{n^6-2}{n^6})^3}+3\cdot\sqrt[6]{\frac{1}{n^7}\cdot\left(\frac{n+1}{n}\right)^3}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}\cdot\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\cdot\left(\frac{n-1}{n}\right)^3}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{1}{n}-\frac{1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\cdot\left(1-\frac{1}{n^6}\right)^3}+3\cdot\sqrt[6]{\frac{1}{n^7}\cdot\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}\cdot\left(1-\frac{1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\cdot\left(1-\frac{1}{n}\right)}}=\frac{\sqrt[3]{0-0}-\sqrt[15]{0\cdot(1-0)^3}+3\cdot\sqrt[6]{0\cdot(1+0)}}{2\cdot\sqrt[3]{1+0}+3\cdot\sqrt[12]{0\cdot(1-0)^3}-\sqrt[6]{0\cdot(1-0)}}$$

$$=\frac{0-0+3\cdot 0}{2\cdot 1+3\cdot 0-0}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí}\\ \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}.\end{array}\right] = \lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}}\cdot\dfrac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}}=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\cdot(\frac{n^6-2}{n^6})^3}+3\cdot\sqrt[6]{\frac{1}{n^7}\cdot\left(\frac{n+1}{n}\right)^3}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}\cdot\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\cdot\left(\frac{n-1}{n}\right)^3}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{1}{n}-\frac{1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\cdot\left(1-\frac{1}{n^6}\right)^3}+3\cdot\sqrt[6]{\frac{1}{n^7}\cdot\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}\cdot\left(1-\frac{1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\cdot\left(1-\frac{1}{n}\right)}}=\frac{\sqrt[3]{0-0}-\sqrt[15]{0\cdot(1-0)^3}+3\cdot\sqrt[6]{0\cdot(1+0)}}{2\cdot\sqrt[3]{1+0}+3\cdot\sqrt[12]{0\cdot(1-0)^3}-\sqrt[6]{0\cdot(1-0)}}$$

$$=\frac{0-0+3\cdot 0}{2\cdot 1+3\cdot 0-0}=\frac{0}{2}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}} = 0$$

- $= \left[\begin{array}{c}\text{Pre všetky } n\in N \text{ platí} \\ \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}.\end{array}\right] = \lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\cdot\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\cdot(\frac{n^6-2}{n^6})^3}+3\cdot\sqrt[6]{\frac{1}{n^7}\cdot\left(\frac{n+1}{n}\right)^3}}{2\cdot\sqrt[3]{\frac{n^5+1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}\cdot\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\cdot\left(\frac{n-1}{n}\right)^3}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{1}{n}-\frac{1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\cdot\left(1-\frac{1}{n^6}\right)^3}+3\cdot\sqrt[6]{\frac{1}{n^7}\cdot\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}\cdot\left(1-\frac{1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\cdot\left(1-\frac{1}{n}\right)}} = \frac{\sqrt[3]{0-0}-\sqrt[15]{0\cdot(1-0)^3}+3\cdot\sqrt[6]{0\cdot(1+0)}}{2\cdot\sqrt[3]{1+0}+3\cdot\sqrt[12]{0\cdot(1-0)^3}-\sqrt[6]{0\cdot(1-0)}}$$

$$= \frac{0-0+3\cdot 0}{2\cdot 1+3\cdot 0-0} = \frac{0}{2} = 0.$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\cdot\sqrt{n\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\cdot\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\cdot\sqrt{n\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\cdot\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\cdot\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt{n}\cdot\sqrt{1+\frac{1}{n}}}{2\cdot\sqrt[3]{n^5}\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[4]{n^6}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\cdot\sqrt{1-\frac{1}{n}}}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\cdot\sqrt{n\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\cdot\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\cdot\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt{n}\cdot\sqrt{1+\frac{1}{n}}}{2\cdot\sqrt[3]{n^5}\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[4]{n^6}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\cdot\sqrt{1-\frac{1}{n}}} = \left[\begin{array}{l} \text{Pre } n\in N \text{ platí } \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}},\quad \sqrt[5]{n^6}=\sqrt[15]{n^{18}}, \\ \sqrt{n}=\sqrt[6]{n^3},\quad \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}},\quad \sqrt[4]{n^6}=\sqrt[12]{n^{18}}. \end{array}\right]$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $$= \lim_{n\to\infty}\frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\cdot\sqrt{n\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\cdot\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$= \lim_{n\to\infty}\frac{\sqrt[3]{n^4}\cdot\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt{n}\cdot\sqrt{1+\frac{1}{n}}}{2\cdot\sqrt[3]{n^5}\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[4]{n^6}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\cdot\sqrt{1-\frac{1}{n}}} = \left[\begin{array}{l}\text{Pre } n\in N \text{ platí } \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}},\quad \sqrt[5]{n^6}=\sqrt[15]{n^{18}},\\ \sqrt{n}=\sqrt[6]{n^3},\quad \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}},\quad \sqrt[4]{n^6}=\sqrt[12]{n^{18}}.\end{array}\right]$$

$$= \lim_{n\to\infty}\frac{\sqrt[3]{n^4}\cdot\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\cdot\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\cdot\left(2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\cdot\sqrt{1-\frac{1}{n}}\right)}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\cdot\sqrt{n\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\cdot\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\cdot\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt{n}\cdot\sqrt{1+\frac{1}{n}}}{2\cdot\sqrt[3]{n^5}\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[4]{n^6}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\cdot\sqrt{1-\frac{1}{n}}} = \left[\begin{array}{l}\text{Pre } n\in N \text{ platí } \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}},\quad \sqrt[5]{n^6}=\sqrt[15]{n^{18}},\\ \sqrt{n}=\sqrt[6]{n^3},\quad \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}},\quad \sqrt[4]{n^6}=\sqrt[12]{n^{18}}.\end{array}\right]$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\cdot\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\cdot\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\cdot\left(2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\cdot\sqrt{1-\frac{1}{n}}\right)} = \left[\text{Pre } n\in N \text{ platí } \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}}=\frac{1}{\sqrt[3]{n}}.\right]$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\cdot\sqrt{n\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\cdot\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\cdot\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt{n}\cdot\sqrt{1+\frac{1}{n}}}{2\cdot\sqrt[3]{n^5}\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[4]{n^6}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\cdot\sqrt{1-\frac{1}{n}}}=$$

[Pre $n\in N$ platí $\sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}}$, $\sqrt[5]{n^6}=\sqrt[15]{n^{18}}$,
$\sqrt{n}=\sqrt[6]{n^3}$, $\sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}$, $\sqrt[4]{n^6}=\sqrt[12]{n^{18}}$.]

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\cdot\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n18}}{\sqrt[15]{n20}}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\frac{\sqrt[6]{n3}}{\sqrt[6]{n8}}\cdot\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\cdot\left(2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\frac{\sqrt[12]{n18}}{\sqrt[12]{n20}}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n3}}{\sqrt[6]{n10}}\cdot\sqrt{1-\frac{1}{n}}\right)}=$$

[Pre $n\in N$ platí $\frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}}=\frac{1}{\sqrt[3]{n}}$.]

$$=\lim_{n\to\infty}\frac{\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[15]{\frac{1}{n^2}}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt[6]{\frac{1}{n^5}}\cdot\sqrt{1+\frac{1}{n}}}{\sqrt[3]{n}\cdot\left(2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt[6]{\frac{1}{n^7}}\cdot\sqrt{1-\frac{1}{n}}\right)}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\cdot\sqrt{n\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\cdot\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\cdot\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt{n}\cdot\sqrt{1+\frac{1}{n}}}{2\cdot\sqrt[3]{n^5}\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[4]{n^6}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\cdot\sqrt{1-\frac{1}{n}}} = \left[\begin{array}{l}\text{Pre } n\in N \text{ platí } \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}},\quad \sqrt[5]{n^6}=\sqrt[15]{n^{18}},\\ \sqrt{n}=\sqrt[6]{n^3},\quad \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}},\quad \sqrt[4]{n^6}=\sqrt[12]{n^{18}}.\end{array}\right]$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\cdot\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\cdot\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\cdot\left(2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\cdot\sqrt{1-\frac{1}{n}}\right)} = \left[\text{Pre } n\in N \text{ platí } \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}}=\frac{1}{\sqrt[3]{n}}.\right]$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[15]{\frac{1}{n^2}}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt[6]{\frac{1}{n^5}}\cdot\sqrt{1+\frac{1}{n}}}{\sqrt[3]{n}\cdot\left(2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt[6]{\frac{1}{n^7}}\cdot\sqrt{1-\frac{1}{n}}\right)}$$

$$= \frac{\sqrt[3]{1-0}-\sqrt[15]{0}\cdot\sqrt[5]{1-0}+3\cdot\sqrt[6]{0}\cdot\sqrt{1+0}}{\sqrt[3]{\infty}\cdot(2\sqrt[3]{1+0}+3\cdot\sqrt[12]{0}\cdot\sqrt[4]{1-0}-\sqrt[6]{0}\cdot\sqrt{1-0})}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\cdot\sqrt{n\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\cdot\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\cdot\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt{n}\cdot\sqrt{1+\frac{1}{n}}}{2\cdot\sqrt[3]{n^5}\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[4]{n^6}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\cdot\sqrt{1-\frac{1}{n}}}=\left[\begin{array}{l}\text{Pre } n\in N \text{ platí } \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}},\quad \sqrt[5]{n^6}=\sqrt[15]{n^{18}},\\ \sqrt{n}=\sqrt[6]{n^3},\quad \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}},\quad \sqrt[4]{n^6}=\sqrt[12]{n^{18}}.\end{array}\right]$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\cdot\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\cdot\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\cdot\left(2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\cdot\sqrt{1-\frac{1}{n}}\right)}=\left[\text{Pre } n\in N \text{ platí } \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}}=\frac{1}{\sqrt[3]{n}}.\right]$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[15]{\frac{1}{n^2}}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt[6]{\frac{1}{n^5}}\cdot\sqrt{1+\frac{1}{n}}}{\sqrt[3]{n}\cdot\left(2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt[6]{\frac{1}{n^7}}\cdot\sqrt{1-\frac{1}{n}}\right)}$$

$$=\frac{\sqrt[3]{1-0}-\sqrt[15]{0}\cdot\sqrt[5]{1-0}+3\cdot\sqrt[6]{0}\cdot\sqrt{1+0}}{\sqrt[3]{\infty}\cdot(2\sqrt[3]{1+0}+3\cdot\sqrt[12]{0}\cdot\sqrt[4]{1-0}-\sqrt[6]{0}\cdot\sqrt{1-0})}=\frac{1-0\cdot1+3\cdot0\cdot1}{\infty(2\cdot1+3\cdot0\cdot1-0\cdot1)}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

- $$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\cdot\sqrt{n\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\cdot\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\cdot\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt{n}\cdot\sqrt{1+\frac{1}{n}}}{2\cdot\sqrt[3]{n^5}\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[4]{n^6}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\cdot\sqrt{1-\frac{1}{n}}}=\left[\begin{array}{l}\text{Pre } n\in N \text{ platí } \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}},\quad \sqrt[5]{n^6}=\sqrt[15]{n^{18}},\\ \sqrt{n}=\sqrt[6]{n^3},\quad \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}},\quad \sqrt[4]{n^6}=\sqrt[12]{n^{18}}.\end{array}\right]$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\cdot\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\cdot\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\cdot\left(2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\cdot\sqrt{1-\frac{1}{n}}\right)}=\left[\text{Pre } n\in N \text{ platí } \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}}=\frac{1}{\sqrt[3]{n}}.\right]$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[15]{\frac{1}{n^2}}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt[6]{\frac{1}{n^5}}\cdot\sqrt{1+\frac{1}{n}}}{\sqrt[3]{n}\cdot\left(2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt[6]{\frac{1}{n^7}}\cdot\sqrt{1-\frac{1}{n}}\right)}$$

$$=\frac{\sqrt[3]{1-0}-\sqrt[15]{0}\cdot\sqrt[5]{1-0}+3\cdot\sqrt[6]{0}\cdot\sqrt{1+0}}{\sqrt[3]{\infty}\cdot(2\sqrt[3]{1+0}+3\cdot\sqrt[12]{0}\cdot\sqrt[4]{1-0}-\sqrt[6]{0}\cdot\sqrt{1-0})}=\frac{1-0\cdot1+3\cdot0\cdot1}{\infty(2\cdot1+3\cdot0\cdot1-0\cdot1)}=\frac{1}{\infty\cdot2}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}} = 0$$

- $$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\cdot\sqrt{n\left(1+\frac{1}{n}\right)}}{2\cdot\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\cdot\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\cdot\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt{n}\cdot\sqrt{1+\frac{1}{n}}}{2\cdot\sqrt[3]{n^5}\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[4]{n^6}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\cdot\sqrt{1-\frac{1}{n}}} = \left[\begin{array}{l}\text{Pre } n\in N \text{ platí } \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}},\quad \sqrt[5]{n^6}=\sqrt[15]{n^{18}},\\ \sqrt{n}=\sqrt[6]{n^3},\quad \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}},\quad \sqrt[4]{n^6}=\sqrt[12]{n^{18}}.\end{array}\right]$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\cdot\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n18}}{\sqrt[15]{n20}}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\frac{\sqrt[6]{n3}}{\sqrt[6]{n8}}\cdot\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\cdot\left(2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\frac{\sqrt[12]{n18}}{\sqrt[12]{n20}}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n3}}{\sqrt[6]{n10}}\cdot\sqrt{1-\frac{1}{n}}\right)} = \left[\text{Pre } n\in N \text{ platí } \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}}=\frac{1}{\sqrt[3]{n}}.\right]$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[15]{\frac{1}{n^2}}\cdot\sqrt[5]{1-\frac{2}{n^6}}+3\cdot\sqrt[6]{\frac{1}{n^5}}\cdot\sqrt{1+\frac{1}{n}}}{\sqrt[3]{n}\cdot\left(2\cdot\sqrt[3]{1+\frac{1}{n^5}}+3\cdot\sqrt[12]{\frac{1}{n^2}}\cdot\sqrt[4]{1-\frac{1}{n^6}}-\sqrt[6]{\frac{1}{n^7}}\cdot\sqrt{1-\frac{1}{n}}\right)}$$

$$= \frac{\sqrt[3]{1-0}-\sqrt[15]{0}\cdot\sqrt[5]{1-0}+3\cdot\sqrt[6]{0}\cdot\sqrt{1+0}}{\sqrt[3]{\infty}\cdot(2\sqrt[3]{1+0}+3\cdot\sqrt[12]{0}\cdot\sqrt[4]{1-0}-\sqrt[6]{0}\cdot\sqrt{1-0})} = \frac{1-0\cdot1+3\cdot0\cdot1}{\infty(2\cdot1+3\cdot0\cdot1-0\cdot1)} = \frac{1}{\infty\cdot2} = 0.$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}} = \lim_{n\to\infty} \frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

- $= \lim_{n\to\infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

- $$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\cdot\left((1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}\right)}{n^{\frac{5}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

- $$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\cdot\left((1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}\right)}{n^{\frac{5}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}=$$

[Pre $n\in N$ platí $\frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}$.]

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}} = \lim_{n\to\infty} \frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

- $$= \lim_{n\to\infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{n^{\frac{4}{3}}\cdot\Big((1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}\Big)}{n^{\frac{5}{3}}\cdot\Big(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\Big)} = \left[\text{Pre } n\in N \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}} = n^{\frac{4}{3}-\frac{5}{3}} = n^{-\frac{1}{3}} = \frac{1}{n^{\frac{1}{3}}}.\right]$$

$$= \lim_{n\to\infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\Big(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\Big)}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

- $$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\cdot\left((1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}\right)}{n^{\frac{5}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}=$$ [Pre $n\in N$ platí $\frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}$.]

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

- $$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\cdot\Big((1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}\Big)}{n^{\frac{5}{3}}\cdot\Big(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\Big)}=\Bigg[\text{Pre } n\in N \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}.\Bigg]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\Big(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\Big)}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\Big(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\Big)}$$

$$=\Bigg[\lim_{n\to\infty}n^{\frac{1}{3}}=\lim_{n\to\infty}\sqrt[3]{n}=\infty.\quad\text{Pre všetky } k,m\in N \text{ platí } \lim_{n\to\infty}n^{-\frac{k}{m}}=\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0.\Bigg]$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

- $$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\cdot\left((1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}\right)}{n^{\frac{5}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}=\left[\text{Pre } n\in N \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}.\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}$$

$$=\left[\lim_{n\to\infty}n^{\frac{1}{3}}=\lim_{n\to\infty}\sqrt[3]{n}=\infty.\quad \text{Pre všetky } k,m\in N \text{ platí } \lim_{n\to\infty}n^{-\frac{k}{m}}=\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0.\right]$$

$$=\frac{(1-0)^{\frac{1}{3}}-0\cdot(1-0)^{\frac{1}{5}}+3\cdot0\cdot(1+0)^{\frac{1}{2}}}{\infty\cdot\left(2\cdot(1+0)^{\frac{1}{3}}+3\cdot0\cdot(1-0)^{\frac{1}{4}}-0\cdot(1-0)^{\frac{1}{2}}\right)}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

- $$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\cdot\left((1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}\right)}{n^{\frac{5}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}=\left[\text{Pre } n\in N \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}.\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}$$

$$=\left[\lim_{n\to\infty}n^{\frac{1}{3}}=\lim_{n\to\infty}\sqrt[3]{n}=\infty.\quad \text{Pre všetky } k,m\in N \text{ platí } \lim_{n\to\infty}n^{-\frac{k}{m}}=\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0.\right]$$

$$=\frac{(1-0)^{\frac{1}{3}}-0\cdot(1-0)^{\frac{1}{5}}+3\cdot0\cdot(1+0)^{\frac{1}{2}}}{\infty\cdot\left(2\cdot(1+0)^{\frac{1}{3}}+3\cdot0\cdot(1-0)^{\frac{1}{4}}-0\cdot(1-0)^{\frac{1}{2}}\right)}=\frac{1-0\cdot1+3\cdot0\cdot1}{\infty\cdot(2\cdot1+3\cdot0\cdot1-0\cdot1)}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

- $$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\cdot\left((1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}\right)}{n^{\frac{5}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}=\left[\text{Pre } n\in N \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}.\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}$$

$$=\left[\lim_{n\to\infty}n^{\frac{1}{3}}=\lim_{n\to\infty}\sqrt[3]{n}=\infty.\quad \text{Pre všetky } k,m\in N \text{ platí } \lim_{n\to\infty}n^{-\frac{k}{m}}=\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0.\right]$$

$$=\frac{(1-0)^{\frac{1}{3}}-0\cdot(1-0)^{\frac{1}{5}}+3\cdot 0\cdot(1+0)^{\frac{1}{2}}}{\infty\cdot\left(2\cdot(1+0)^{\frac{1}{3}}+3\cdot 0\cdot(1-0)^{\frac{1}{4}}-0\cdot(1-0)^{\frac{1}{2}}\right)}=\frac{1-0\cdot 1+3\cdot 0\cdot 1}{\infty\cdot(2\cdot 1+3\cdot 0\cdot 1-0\cdot 1)}=\frac{1}{\infty\cdot 2}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

- $$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\cdot\Big((1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}\Big)}{n^{\frac{5}{3}}\cdot\Big(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\Big)}=\left[\text{Pre } n\in N \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}.\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\Big(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\Big)}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\Big(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\Big)}$$

$$=\left[\lim_{n\to\infty}n^{\frac{1}{3}}=\lim_{n\to\infty}\sqrt[3]{n}=\infty.\quad \text{Pre všetky } k,m\in N \text{ platí } \lim_{n\to\infty}n^{-\frac{k}{m}}=\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0.\right]$$

$$=\frac{(1-0)^{\frac{1}{3}}-0\cdot(1-0)^{\frac{1}{5}}+3\cdot0\cdot(1+0)^{\frac{1}{2}}}{\infty\cdot\Big(2\cdot(1+0)^{\frac{1}{3}}+3\cdot0\cdot(1-0)^{\frac{1}{4}}-0\cdot(1-0)^{\frac{1}{2}}\Big)}=\frac{1-0\cdot1+3\cdot0\cdot1}{\infty\cdot(2\cdot1+3\cdot0\cdot1-0\cdot1)}=\frac{1}{\infty\cdot2}=\frac{1}{\infty}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}=0$$

- $$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\cdot\left((1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}\right)}{n^{\frac{5}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}=\left[\text{Pre } n\in N \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}.\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}\cdot(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}\cdot(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\cdot\left(2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}\cdot(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}\cdot(1-\frac{1}{n})^{\frac{1}{2}}\right)}$$

$$=\left[\lim_{n\to\infty}n^{\frac{1}{3}}=\lim_{n\to\infty}\sqrt[3]{n}=\infty.\quad \text{Pre všetky } k,m\in N \text{ platí } \lim_{n\to\infty}n^{-\frac{k}{m}}=\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0.\right]$$

$$=\frac{(1-0)^{\frac{1}{3}}-0\cdot(1-0)^{\frac{1}{5}}+3\cdot0\cdot(1+0)^{\frac{1}{2}}}{\infty\cdot\left(2\cdot(1+0)^{\frac{1}{3}}+3\cdot0\cdot(1-0)^{\frac{1}{4}}-0\cdot(1-0)^{\frac{1}{2}}\right)}=\frac{1-0\cdot1+3\cdot0\cdot1}{\infty\cdot(2\cdot1+3\cdot0\cdot1-0\cdot1)}=\frac{1}{\infty\cdot2}=\frac{1}{\infty}=0.$$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n} - \frac{1}{4^n}}$$

$$\lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n} - \frac{1}{3^n}}$$

$$\lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n} - \frac{1}{3^n}}$$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}} \cdot \frac{4^n}{4^n}$

$$\lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n}$
- $= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n}$

$$\lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n}$
- $= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n}$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{3})^n}{(\frac{4}{5})^n-1}$

$$\lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{1}{(\frac{4}{5})^n-(\frac{4}{3})^n}$
- $= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{4})^n}{(\frac{3}{5})^n-1}$

$$\lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{5})^n}{1-(\frac{4}{3})^n}$
- $= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{5})^n}{(\frac{3}{4})^n-1}$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{3})^n}{(\frac{4}{5})^n-1} = \left[\text{Geometrické postupnosti: } \lim_{n\to\infty} (\frac{4}{5})^n = 0,\ \lim_{n\to\infty} (\frac{4}{3})^n = \infty. \right]$

$$\lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{1}{(\frac{4}{5})^n-(\frac{4}{3})^n} = \left[\text{Geometrické postupnosti: } \lim_{n\to\infty} (\frac{4}{5})^n = 0,\ \lim_{n\to\infty} (\frac{4}{3})^n = \infty. \right]$
- $= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{4})^n}{(\frac{3}{5})^n-1} = \left[\text{Geometrické postupnosti: } \lim_{n\to\infty} (\frac{3}{4})^n = 0,\ \lim_{n\to\infty} (\frac{3}{5})^n = 0. \right]$

$$\lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{5})^n}{1-(\frac{4}{3})^n} = \left[\text{Geometrické postupnosti: } \lim_{n\to\infty} (\frac{4}{5})^n = 0,\ \lim_{n\to\infty} (\frac{4}{3})^n = \infty. \right]$
- $= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{5})^n}{(\frac{3}{4})^n-1} = \left[\text{Geometrické postupnosti: } \lim_{n\to\infty} (\frac{3}{4})^n = 0,\ \lim_{n\to\infty} (\frac{3}{5})^n = 0. \right]$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{3})^n}{(\frac{4}{5})^n-1} = \left[\begin{array}{c}\text{Geometrické postupnosti:}\\ \lim_{n\to\infty}(\frac{4}{5})^n=0,\ \lim_{n\to\infty}(\frac{4}{3})^n=\infty.\end{array}\right] = \frac{\infty}{0-1}$

$$\lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{1}{(\frac{4}{5})^n-(\frac{4}{3})^n} = \left[\begin{array}{c}\text{Geometrické postupnosti:}\\ \lim_{n\to\infty}(\frac{4}{5})^n=0,\ \lim_{n\to\infty}(\frac{4}{3})^n=\infty.\end{array}\right] = \frac{1}{0-\infty}$

- $= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{4})^n}{(\frac{3}{5})^n-1} = \left[\begin{array}{c}\text{Geometrické postupnosti:}\\ \lim_{n\to\infty}(\frac{3}{4})^n=0,\ \lim_{n\to\infty}(\frac{3}{5})^n=0.\end{array}\right] = \frac{0}{0-1}$

$$\lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{5})^n}{1-(\frac{4}{3})^n} = \left[\begin{array}{c}\text{Geometrické postupnosti:}\\ \lim_{n\to\infty}(\frac{4}{5})^n=0,\ \lim_{n\to\infty}(\frac{4}{3})^n=\infty.\end{array}\right] = \frac{0}{1-\infty}$

- $= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{5})^n}{(\frac{3}{4})^n-1} = \left[\begin{array}{c}\text{Geometrické postupnosti:}\\ \lim_{n\to\infty}(\frac{3}{4})^n=0,\ \lim_{n\to\infty}(\frac{3}{5})^n=0.\end{array}\right] = \frac{0}{0-1}$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{3})^n}{(\frac{4}{5})^n-1} = \left[\text{Geometrické postupnosti: } \lim_{n\to\infty} (\frac{4}{5})^n = 0,\ \lim_{n\to\infty} (\frac{4}{3})^n = \infty. \right] = \frac{\infty}{0-1} = \frac{\infty}{-1}$

$$\lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{1}{(\frac{4}{5})^n-(\frac{4}{3})^n} = \left[\text{Geometrické postupnosti: } \lim_{n\to\infty} (\frac{4}{5})^n = 0,\ \lim_{n\to\infty} (\frac{4}{3})^n = \infty. \right] = \frac{1}{0-\infty} = \frac{1}{-\infty}$
- $= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{4})^n}{(\frac{3}{5})^n-1} = \left[\text{Geometrické postupnosti: } \lim_{n\to\infty} (\frac{3}{4})^n = 0,\ \lim_{n\to\infty} (\frac{3}{5})^n = 0. \right] = \frac{0}{0-1} = \frac{0}{-1}$

$$\lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}$$

- $= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{5})^n}{1-(\frac{4}{3})^n} = \left[\text{Geometrické postupnosti: } \lim_{n\to\infty} (\frac{4}{5})^n = 0,\ \lim_{n\to\infty} (\frac{4}{3})^n = \infty. \right] = \frac{0}{1-\infty} = \frac{0}{-\infty}$
- $= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{5})^n}{(\frac{3}{4})^n-1} = \left[\text{Geometrické postupnosti: } \lim_{n\to\infty} (\frac{3}{4})^n = 0,\ \lim_{n\to\infty} (\frac{3}{5})^n = 0. \right] = \frac{0}{0-1} = \frac{0}{-1}$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}} = -\infty$$

- $= \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{3})^n}{(\frac{4}{5})^n-1} = \left[\begin{array}{c} \text{Geometrické postupnosti:} \\ \lim_{n\to\infty} (\frac{4}{5})^n = 0,\ \lim_{n\to\infty} (\frac{4}{3})^n = \infty. \end{array} \right] = \frac{\infty}{0-1} = \frac{\infty}{-1} = -\infty.$

$$\lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} = 0$$

- $= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{1}{(\frac{4}{5})^n-(\frac{4}{3})^n} = \left[\begin{array}{c} \text{Geometrické postupnosti:} \\ \lim_{n\to\infty} (\frac{4}{5})^n = 0,\ \lim_{n\to\infty} (\frac{4}{3})^n = \infty. \end{array} \right] = \frac{1}{0-\infty} = \frac{1}{-\infty} = 0.$
- $= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{4})^n}{(\frac{3}{5})^n-1} = \left[\begin{array}{c} \text{Geometrické postupnosti:} \\ \lim_{n\to\infty} (\frac{3}{4})^n = 0,\ \lim_{n\to\infty} (\frac{3}{5})^n = 0. \end{array} \right] = \frac{0}{0-1} = \frac{0}{-1} = 0.$

$$\lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} = 0$$

- $= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{5})^n}{1-(\frac{4}{3})^n} = \left[\begin{array}{c} \text{Geometrické postupnosti:} \\ \lim_{n\to\infty} (\frac{4}{5})^n = 0,\ \lim_{n\to\infty} (\frac{4}{3})^n = \infty. \end{array} \right] = \frac{0}{1-\infty} = \frac{0}{-\infty} = 0.$
- $= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{5})^n}{(\frac{3}{4})^n-1} = \left[\begin{array}{c} \text{Geometrické postupnosti:} \\ \lim_{n\to\infty} (\frac{3}{4})^n = 0,\ \lim_{n\to\infty} (\frac{3}{5})^n = 0. \end{array} \right] = \frac{0}{0-1} = \frac{0}{-1} = 0.$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}}$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}}$

- $= \lim\limits_{n\to\infty} \frac{n^4\left(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4}\right)}{n^3\left(\frac{2}{n}-1\right)}$

Riešené limity – 69

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim\limits_{n\to\infty} \frac{2+3n-\frac{1}{n}+2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1}$

- $= \lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim\limits_{n\to\infty} \frac{\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4}}{\frac{2}{n^2}-\frac{1}{n}}$

- $= \lim\limits_{n\to\infty} \frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim\limits_{n\to\infty} \frac{n(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1}$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{2+3n-\frac{1}{n}+2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1}$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4}}{\frac{2}{n^2}-\frac{1}{n}}$

- $= \lim_{n\to\infty} \frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty} \frac{n(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1}$

Označme $a_n = \frac{5^n}{n^k}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n\to\infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n\to\infty} \frac{5}{(\sqrt[n]{n})^k} = \frac{5}{1^k} = 5 > 1.$

Resp.

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n\to\infty} \frac{5n^k}{(n+1)^k} = \lim_{n\to\infty} \frac{5}{(1+\frac{1}{n})^k} = \frac{5}{(1+0)^k} = 5 > 1.$

$$\Rightarrow \lim_{n\to\infty} \frac{5^n}{n^k} = \infty.$$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty}\frac{2+3n-\frac{1}{n}+2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1} = \frac{2+3\cdot\infty-\frac{1}{\infty}+2\cdot\infty}{\frac{2}{\infty}-1}$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty}\frac{\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4}}{\frac{2}{n^2}-\frac{1}{n}} = \frac{\frac{2}{\infty}+3-\frac{1}{\infty}+2\cdot\infty}{\frac{2}{\infty}-\frac{1}{\infty}}$

- $= \lim_{n\to\infty} \frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty}\frac{n(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1} = \frac{\infty\cdot(\frac{2}{\infty}+3-\frac{1}{\infty}+2\cdot\infty)}{\frac{1}{\infty}-1}$

Označme $a_n = \frac{5^n}{n^k}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\frac{5^n}{n^k}} = \lim_{n\to\infty}\frac{5}{\sqrt[n]{n^k}} = \lim_{n\to\infty}\frac{5}{(\sqrt[n]{n})^k} = \frac{5}{1^k} = 5 > 1.$

Resp.
- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n\to\infty}\frac{5n^k}{(n+1)^k} = \lim_{n\to\infty}\frac{5}{(1+\frac{1}{n})^k} = \frac{5}{(1+0)^k} = 5 > 1.$

$\Rightarrow \lim_{n\to\infty}\frac{5^n}{n^k} = \infty.$

Riešené limity – 69

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3}$$

- $=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}=\lim_{n\to\infty}\frac{2+3n-\frac{1}{n}+2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1}=\frac{2+3\cdot\infty-\frac{1}{\infty}+2\cdot\infty}{\frac{2}{\infty}-1}=\frac{2+\infty-0+\infty}{-1}$

- $=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}}=\lim_{n\to\infty}\frac{\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4}}{\frac{2}{n^2}-\frac{1}{n}}=\frac{\frac{2}{\infty}+3-\frac{1}{\infty}+2\cdot\infty}{\frac{2}{\infty}-\frac{1}{\infty}}=\frac{0+3-0+\infty}{0-0}$

- $=\lim_{n\to\infty}\frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)}=\lim_{n\to\infty}\frac{n(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1}=\frac{\infty\cdot(\frac{2}{\infty}+3-\frac{1}{\infty}+2\cdot\infty)}{\frac{1}{\infty}-1}=\frac{\infty\cdot(0+3-0+\infty)}{-1}$

Označme $a_n=\frac{5^n}{n^k}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{5^n}{n^k}}=\lim_{n\to\infty}\frac{5}{\sqrt[n]{n^k}}=\lim_{n\to\infty}\frac{5}{(\sqrt[n]{n})^k}=\frac{5}{1^k}=5>1.$

Resp.
- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}}=\lim_{n\to\infty}\frac{5n^k}{(n+1)^k}=\lim_{n\to\infty}\frac{5}{(1+\frac{1}{n})^k}=\frac{5}{(1+0)^k}=5>1.$

$\Rightarrow \lim_{n\to\infty}\frac{5^n}{n^k}=\infty.$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{2+3n-\frac{1}{n}+2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1} = \frac{2+3\cdot\infty-\frac{1}{\infty}+2\cdot\infty}{\frac{2}{\infty}-1} = \frac{2+\infty-0+\infty}{-1} = \frac{\infty}{-1}$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4}}{\frac{2}{n^2}-\frac{1}{n}} = \frac{\frac{2}{\infty}+3-\frac{1}{\infty}+2\cdot\infty}{\frac{2}{\infty}-\frac{1}{\infty}} = \frac{0+3-0+\infty}{0-0} = \frac{\infty}{0}$

- $= \lim_{n\to\infty} \frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty} \frac{n(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1} = \frac{\infty\cdot(\frac{2}{\infty}+3-\frac{1}{\infty}+2\cdot\infty)}{\frac{1}{\infty}-1} = \frac{\infty\cdot(0+3-0+\infty)}{-1} = \frac{\infty}{-1}$

Označme $a_n = \frac{5^n}{n^k}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n\to\infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n\to\infty} \frac{5}{(\sqrt[n]{n})^k} = \frac{5}{1^k} = 5 > 1.$

Resp.
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n\to\infty} \frac{5n^k}{(n+1)^k} = \lim_{n\to\infty} \frac{5}{(1+\frac{1}{n})^k} = \frac{5}{(1+0)^k} = 5 > 1.$

$\Rightarrow \lim_{n\to\infty} \frac{5^n}{n^k} = \infty.$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} = -\infty$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{2+3n-\frac{1}{n}+2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1} = \frac{2+3\cdot\infty-\frac{1}{\infty}+2\cdot\infty}{\frac{2}{\infty}-1} = \frac{2+\infty-0+\infty}{-1} = \frac{\infty}{-1} = -\infty.$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4}}{\frac{2}{n^2}-\frac{1}{n}} = \frac{\frac{2}{\infty}+3-\frac{1}{\infty}+2\cdot\infty}{\frac{2}{\infty}-\frac{1}{\infty}} = \frac{0+3-0+\infty}{0-0} = \frac{\infty}{0}\,?.$

[Problém s delením nulou.]

- $= \lim_{n\to\infty} \frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty} \frac{n(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1} = \frac{\infty\cdot(\frac{2}{\infty}+3-\frac{1}{\infty}+2\cdot\infty)}{\frac{1}{\infty}-1} = \frac{\infty\cdot(0+3-0+\infty)}{-1} = \frac{\infty}{-1} = -\infty.$

Označme $a_n = \frac{5^n}{n^k}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n\to\infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n\to\infty} \frac{5}{(\sqrt[n]{n})^k} = \frac{5}{1^k} = 5 > 1.$

Resp.
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n\to\infty} \frac{5n^k}{(n+1)^k} = \lim_{n\to\infty} \frac{5}{(1+\frac{1}{n})^k} = \frac{5}{(1+0)^k} = 5 > 1.$

$\Rightarrow \lim_{n\to\infty} \frac{5^n}{n^k} = \infty.$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{\left(n+\left(n+n^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{\left(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

- $= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}$

- $= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{\left(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

- $= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}}$

- $= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}$

Riešené limity – 70

$$\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{\left(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

- $=\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}}$

- $=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}}\cdot\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+(\frac{n+n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{\left(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

- $= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}$

- $= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+(\frac{n+n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$

  $= \lim_{n\to\infty} \frac{1}{\left(1+(\frac{1}{n}+\frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{\left(n+\left(n+n^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

- $$= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}$$
  $$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}}$$

- $$= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1+\frac{\left(n+n^{\frac{1}{2}}\right)^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{\left(n+n^{\frac{1}{2}}\right)^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{n+n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$
  $$= \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+\frac{n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{\left(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

- $= \lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}$

  $= \lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}}$

- $= \lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}}\cdot\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+(\frac{n+n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$

  $= \lim_{n\to\infty}\frac{1}{\left(1+(\frac{1}{n}+\frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$

Riešené limity – 70

$$\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{\left(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

- $$=\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}$$
$$=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{\infty}+\sqrt{\frac{1}{\infty}}}}}$$

- $$=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}}\cdot\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+(\frac{n+n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$
$$=\lim_{n\to\infty}\frac{1}{\left(1+(\frac{1}{n}+\frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$
$$=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{\left(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

- $= \lim\limits_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim\limits_{n\to\infty} \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}} = \lim\limits_{n\to\infty} \frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}} = \lim\limits_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}$
  $= \lim\limits_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}} = \lim\limits_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}} = \lim\limits_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{\infty}+\sqrt{\frac{1}{\infty}}}}} = \frac{1}{\sqrt{1+\sqrt{0+\sqrt{0}}}}$

- $= \lim\limits_{n\to\infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim\limits_{n\to\infty} \frac{1}{\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim\limits_{n\to\infty} \frac{1}{\left(1+(\frac{n+n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$
  $= \lim\limits_{n\to\infty} \frac{1}{\left(1+(\frac{1}{n}+\frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim\limits_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim\limits_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$
  $= \lim\limits_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim\limits_{n\to\infty} \frac{1}{\left(1+(\frac{1}{\infty}+\frac{1}{\infty})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{\left(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

- $$= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}$$
  $$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{\infty}+\sqrt{\frac{1}{\infty}}}}} = \frac{1}{\sqrt{1+\sqrt{0+\sqrt{0}}}}$$
  $$= \frac{1}{\sqrt{1+0}}$$

- $$= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+(\frac{n+n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$
  $$= \lim_{n\to\infty} \frac{1}{\left(1+(\frac{1}{n}+\frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$
  $$= \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+(\frac{1}{\infty}+\frac{1}{\infty})^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \frac{1}{\left(1+(0+0)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{\left(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}} = 1$$

- $= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}$
$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{\infty}+\sqrt{\frac{1}{\infty}}}}} = \frac{1}{\sqrt{1+\sqrt{0+\sqrt{0}}}}$
$= \frac{1}{\sqrt{1+0}} = 1.$

- $= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}}\cdot\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+(\frac{n+n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$
$= \lim_{n\to\infty} \frac{1}{\left(1+(\frac{1}{n}+\frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$
$= \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+(\frac{1}{\infty}+\frac{1}{\infty})^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \frac{1}{\left(1+(0+0)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \frac{1}{(1+0)^{\frac{1}{2}}}$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{\left(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}} = 1$$

- $$= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}$$
$$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{\infty}+\sqrt{\frac{1}{\infty}}}}} = \frac{1}{\sqrt{1+\sqrt{0+\sqrt{0}}}}$$
$$= \frac{1}{\sqrt{1+0}} = 1.$$

- $$= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}}\cdot\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+(\frac{n+n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$
$$= \lim_{n\to\infty} \frac{1}{\left(1+(\frac{1}{n}+\frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$
$$= \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+(\frac{1}{\infty}+\frac{1}{\infty})^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \frac{1}{\left(1+(0+0)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \frac{1}{(1+0)^{\frac{1}{2}}} = \frac{1}{1}$$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{\left(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}} = 1$$

- $$= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}$$
  $$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{\infty}+\sqrt{\frac{1}{\infty}}}}} = \frac{1}{\sqrt{1+\sqrt{0+\sqrt{0}}}}$$
  $$= \frac{1}{\sqrt{1+0}} = 1.$$

- $$= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+(\frac{n+n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$
  $$= \lim_{n\to\infty} \frac{1}{\left(1+(\frac{1}{n}+\frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$
  $$= \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+(\frac{1}{\infty}+\frac{1}{\infty})^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \frac{1}{\left(1+(0+0)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \frac{1}{(1+0)^{\frac{1}{2}}} = \frac{1}{1} = 1.$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

- $= \lim\limits_{n\to\infty} \dfrac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3} \cdot \dfrac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}$

- $= \lim\limits_{n\to\infty} \dfrac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3} \cdot \dfrac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}}$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

- $= \lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}}$

- $= \lim_{n\to\infty} \frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}}$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

- $= \lim\limits_{n\to\infty} \dfrac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3} \cdot \dfrac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim\limits_{n\to\infty} \dfrac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{l} \text{Pre } n\in N \text{ platí} \quad \sqrt[3]{n}=\sqrt[6]{n^2}, \quad \sqrt[5]{n}=\sqrt[10]{n^2}, \\ \quad \sqrt{n}=\sqrt[4]{n^2}, \quad \sqrt{n}=\sqrt[6]{n^3}, \quad \sqrt{n}=\sqrt[10]{n^5}. \end{array}\right]$

- $= \lim\limits_{n\to\infty} \dfrac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3} \cdot \dfrac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim\limits_{n\to\infty} \dfrac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim\limits_{n\to\infty} \dfrac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

- $= \lim\limits_{n\to\infty} \dfrac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3} \cdot \dfrac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim\limits_{n\to\infty} \dfrac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{l} \text{Pre } n\in N \text{ platí} \quad \sqrt[3]{n}=\sqrt[6]{n^2}, \quad \sqrt[5]{n}=\sqrt[10]{n^2}, \\ \sqrt{n}=\sqrt[4]{n^2}, \quad \sqrt{n}=\sqrt[6]{n^3}, \quad \sqrt{n}=\sqrt[10]{n^5}. \end{array}\right]$

  $= \lim\limits_{n\to\infty} \dfrac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}}$

- $= \lim\limits_{n\to\infty} \dfrac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3} \cdot \dfrac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim\limits_{n\to\infty} \dfrac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim\limits_{n\to\infty} \dfrac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$

  $= \lim\limits_{n\to\infty} \dfrac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}}$

Riešené limity – 71

$$\lim_{n\to\infty}\frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

- $= \lim_{n\to\infty}\frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty}\frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{l}\text{Pre } n\in N \text{ platí } \sqrt[3]{n}=\sqrt[6]{n^2},\ \sqrt[5]{n}=\sqrt[10]{n^2},\\ \sqrt{n}=\sqrt[4]{n^2},\ \sqrt{n}=\sqrt[6]{n^3},\ \sqrt{n}=\sqrt[10]{n^5}.\end{array}\right]$

$$= \lim_{n\to\infty}\frac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty}\frac{2\cdot\sqrt[6]{\frac{n^2}{n^3}}+3\cdot\sqrt[4]{\frac{n}{n^2}}-1}{2-\sqrt[10]{\frac{n^2}{n^5}}+\frac{3}{\sqrt{n}}}$$

- $= \lim_{n\to\infty}\frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3}\cdot\frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n\to\infty}\frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty}\frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$

$$= \lim_{n\to\infty}\frac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty}\frac{\frac{2}{n^{\frac{1}{6}}}+\frac{3}{n^{\frac{1}{4}}}-1}{2-\frac{1}{n^{\frac{3}{10}}}+\frac{3}{n^{\frac{1}{2}}}}$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

- $= \lim\limits_{n\to\infty} \dfrac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}\cdot\dfrac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim\limits_{n\to\infty} \dfrac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{l}\text{Pre } n\in N \text{ platí } \sqrt[3]{n}=\sqrt[6]{n^2},\ \sqrt[5]{n}=\sqrt[10]{n^2},\\ \sqrt{n}=\sqrt[4]{n^2},\ \sqrt{n}=\sqrt[6]{n^3},\ \sqrt{n}=\sqrt[10]{n^5}.\end{array}\right]$

  $= \lim\limits_{n\to\infty} \dfrac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}} = \lim\limits_{n\to\infty} \dfrac{2\cdot\sqrt[6]{\frac{n^2}{n^3}}+3\cdot\sqrt[4]{\frac{n}{n^2}}-1}{2-\sqrt[10]{\frac{n^2}{n^5}}+\frac{3}{\sqrt{n}}} = \lim\limits_{n\to\infty} \dfrac{2\cdot\sqrt[6]{\frac{1}{n}}+3\cdot\sqrt[4]{\frac{1}{n}}-1}{2-\sqrt[10]{\frac{1}{n^3}}+3\cdot\sqrt{\frac{1}{n}}}$

- $= \lim\limits_{n\to\infty} \dfrac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3}\cdot\dfrac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim\limits_{n\to\infty} \dfrac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim\limits_{n\to\infty} \dfrac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$

  $= \lim\limits_{n\to\infty} \dfrac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}} = \lim\limits_{n\to\infty} \dfrac{\frac{2}{n^{\frac{1}{6}}}+\frac{3}{n^{\frac{1}{4}}}-1}{2-\frac{1}{n^{\frac{3}{10}}}+\frac{3}{n^{\frac{1}{2}}}} = \lim\limits_{n\to\infty} \dfrac{\frac{2}{\sqrt[6]{n}}+\frac{3}{\sqrt[4]{n}}-1}{2-\frac{1}{\sqrt[10]{n^3}}+\frac{3}{\sqrt{n}}}$

Riešené limity – 71

$$\lim_{n\to\infty}\frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

- $= \lim\limits_{n\to\infty}\frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim\limits_{n\to\infty}\frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{l}\text{Pre } n\in N \text{ platí } \sqrt[3]{n}=\sqrt[6]{n^2},\ \sqrt[5]{n}=\sqrt[10]{n^2},\\ \sqrt{n}=\sqrt[4]{n^2},\ \sqrt{n}=\sqrt[6]{n^3},\ \sqrt{n}=\sqrt[10]{n^5}.\end{array}\right]$

  $= \lim\limits_{n\to\infty}\frac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}} = \lim\limits_{n\to\infty}\frac{2\cdot\sqrt[6]{\frac{n^2}{n^3}}+3\cdot\sqrt[4]{\frac{n}{n^2}}-1}{2-\sqrt[10]{\frac{n^2}{n^5}}+\frac{3}{\sqrt{n}}} = \lim\limits_{n\to\infty}\frac{2\cdot\sqrt[6]{\frac{1}{n}}+3\cdot\sqrt[4]{\frac{1}{n}}-1}{2-\sqrt[10]{\frac{1}{n^3}}+3\cdot\sqrt{\frac{1}{n}}}$

  $= \lim\limits_{n\to\infty}\frac{2\cdot\sqrt[6]{\frac{1}{\infty}}+3\cdot\sqrt[4]{\frac{1}{\infty}}-1}{2-\sqrt[10]{\frac{1}{\infty}}+3\cdot\sqrt{\frac{1}{\infty}}}$

- $= \lim\limits_{n\to\infty}\frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3}\cdot\frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim\limits_{n\to\infty}\frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim\limits_{n\to\infty}\frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$

  $= \lim\limits_{n\to\infty}\frac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}} = \lim\limits_{n\to\infty}\frac{\frac{2}{n^{\frac{1}{6}}}+\frac{3}{n^{\frac{1}{4}}}-1}{2-\frac{1}{n^{\frac{3}{10}}}+\frac{3}{n^{\frac{1}{2}}}} = \lim\limits_{n\to\infty}\frac{\frac{2}{\sqrt[6]{n}}+\frac{3}{\sqrt[4]{n}}-1}{2-\frac{1}{\sqrt[10]{n^3}}+\frac{3}{\sqrt{n}}} = \lim\limits_{n\to\infty}\frac{\frac{2}{\sqrt[6]{\infty}}+\frac{3}{\sqrt[4]{\infty}}-1}{2-\frac{1}{\sqrt[10]{\infty}}+\frac{3}{\sqrt{\infty}}}$

Riešené limity – 71

$$\lim_{n\to\infty}\frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

- $= \lim_{n\to\infty}\frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty}\frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{l}\text{Pre } n\in N \text{ platí } \sqrt[3]{n}=\sqrt[6]{n^2},\ \sqrt[5]{n}=\sqrt[10]{n^2},\\ \sqrt{n}=\sqrt[4]{n^2},\ \sqrt{n}=\sqrt[6]{n^3},\ \sqrt{n}=\sqrt[10]{n^5}.\end{array}\right]$

$$= \lim_{n\to\infty}\frac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty}\frac{2\cdot\sqrt[6]{\frac{n^2}{n^3}}+3\cdot\sqrt[4]{\frac{n}{n^2}}-1}{2-\sqrt[10]{\frac{n^2}{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty}\frac{2\cdot\sqrt[6]{\frac{1}{n}}+3\cdot\sqrt[4]{\frac{1}{n}}-1}{2-\sqrt[10]{\frac{1}{n^3}}+3\cdot\sqrt{\frac{1}{n}}}$$

$$= \lim_{n\to\infty}\frac{2\cdot\sqrt[6]{\frac{1}{\infty}}+3\cdot\sqrt[4]{\frac{1}{\infty}}-1}{2-\sqrt[10]{\frac{1}{\infty}}+3\cdot\sqrt{\frac{1}{\infty}}} = \lim_{n\to\infty}\frac{2\cdot\sqrt[6]{0}+3\cdot\sqrt[4]{0}-1}{2-\sqrt[10]{0}+3\cdot\sqrt{0}}$$

- $= \lim_{n\to\infty}\frac{2n^{\frac13}+3n^{\frac14}-n^{\frac12}}{2n^{\frac12}-n^{\frac15}+3}\cdot\frac{n^{-\frac12}}{n^{-\frac12}} = \lim_{n\to\infty}\frac{2n^{\frac13-\frac12}+3n^{\frac14-\frac12}-1}{2-n^{\frac15-\frac12}+3n^{-\frac12}} = \lim_{n\to\infty}\frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac12}}$

$$= \lim_{n\to\infty}\frac{2n^{-\frac16}+3n^{-\frac14}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac12}} = \lim_{n\to\infty}\frac{\frac{2}{n^{\frac16}}+\frac{3}{n^{\frac14}}-1}{2-\frac{1}{n^{\frac{3}{10}}}+\frac{3}{n^{\frac12}}} = \lim_{n\to\infty}\frac{\frac{2}{\sqrt[6]{n}}+\frac{3}{\sqrt[4]{n}}-1}{2-\frac{1}{\sqrt[10]{n^3}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty}\frac{\frac{2}{\sqrt[6]{\infty}}+\frac{3}{\sqrt[4]{\infty}}-1}{2-\frac{1}{\sqrt[10]{\infty}}+\frac{3}{\sqrt{\infty}}}$$

$$= \lim_{n\to\infty}\frac{\frac{2}{\infty}+\frac{3}{\infty}-1}{2-\frac{1}{\infty}+\frac{3}{\infty}}$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

- $$= \lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{c}\text{Pre } n\in N \text{ platí} \quad \sqrt[3]{n}=\sqrt[6]{n^2},\quad \sqrt[5]{n}=\sqrt[10]{n^2},\\ \sqrt{n}=\sqrt[4]{n^2},\quad \sqrt{n}=\sqrt[6]{n^3},\quad \sqrt{n}=\sqrt[10]{n^5}.\end{array}\right]$$

  $$= \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\cdot\sqrt[6]{\frac{n^2}{n^3}}+3\cdot\sqrt[4]{\frac{n}{n^2}}-1}{2-\sqrt[10]{\frac{n^2}{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\cdot\sqrt[6]{\frac{1}{n}}+3\cdot\sqrt[4]{\frac{1}{n}}-1}{2-\sqrt[10]{\frac{1}{n^3}}+3\cdot\sqrt{\frac{1}{n}}}$$

  $$= \lim_{n\to\infty} \frac{2\cdot\sqrt[6]{\frac{1}{\infty}}+3\cdot\sqrt[4]{\frac{1}{\infty}}-1}{2-\sqrt[10]{\frac{1}{\infty}}+3\cdot\sqrt{\frac{1}{\infty}}} = \lim_{n\to\infty} \frac{2\cdot\sqrt[6]{0}+3\cdot\sqrt[4]{0}-1}{2-\sqrt[10]{0}+3\cdot\sqrt{0}} = \frac{0+0-1}{2-0+0}$$

- $$= \lim_{n\to\infty} \frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3}\cdot\frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$$

  $$= \lim_{n\to\infty} \frac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{\frac{2}{n^{\frac{1}{6}}}+\frac{3}{n^{\frac{1}{4}}}-1}{2-\frac{1}{n^{\frac{3}{10}}}+\frac{3}{n^{\frac{1}{2}}}} = \lim_{n\to\infty} \frac{\frac{2}{\sqrt[6]{n}}+\frac{3}{\sqrt[4]{n}}-1}{2-\frac{1}{\sqrt[10]{n^3}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\frac{2}{\sqrt[6]{\infty}}+\frac{3}{\sqrt[4]{\infty}}-1}{2-\frac{1}{\sqrt[10]{\infty}}+\frac{3}{\sqrt{\infty}}}$$

  $$= \lim_{n\to\infty} \frac{\frac{2}{\infty}+\frac{3}{\infty}-1}{2-\frac{1}{\infty}+\frac{3}{\infty}} = \frac{0+0-1}{2-0+0}$$

Riešené limity – 71

$$\lim_{n\to\infty}\frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}=-\frac{1}{2}$$

- $= \lim_{n\to\infty}\frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty}\frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{l}\text{Pre } n\in N \text{ platí } \sqrt[3]{n}=\sqrt[6]{n^2},\ \sqrt[5]{n}=\sqrt[10]{n^2},\\ \sqrt{n}=\sqrt[4]{n^2},\ \sqrt{n}=\sqrt[6]{n^3},\ \sqrt{n}=\sqrt[10]{n^5}.\end{array}\right]$

  $= \lim_{n\to\infty}\frac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty}\frac{2\cdot\sqrt[6]{\frac{n^2}{n^3}}+3\cdot\sqrt[4]{\frac{n}{n^2}}-1}{2-\sqrt[10]{\frac{n^2}{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty}\frac{2\cdot\sqrt[6]{\frac{1}{n}}+3\cdot\sqrt[4]{\frac{1}{n}}-1}{2-\sqrt[10]{\frac{1}{n^3}}+3\cdot\sqrt{\frac{1}{n}}}$

  $= \lim_{n\to\infty}\frac{2\cdot\sqrt[6]{\frac{1}{\infty}}+3\cdot\sqrt[4]{\frac{1}{\infty}}-1}{2-\sqrt[10]{\frac{1}{\infty}}+3\cdot\sqrt{\frac{1}{\infty}}} = \lim_{n\to\infty}\frac{2\cdot\sqrt[6]{0}+3\cdot\sqrt[4]{0}-1}{2-\sqrt[10]{0}+3\cdot\sqrt{0}} = \frac{0+0-1}{2-0+0} = -\frac{1}{2}.$

- $= \lim_{n\to\infty}\frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3}\cdot\frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n\to\infty}\frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty}\frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$

  $= \lim_{n\to\infty}\frac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty}\frac{\frac{2}{n^{\frac{1}{6}}}+\frac{3}{n^{\frac{1}{4}}}-1}{2-\frac{1}{n^{\frac{3}{10}}}+\frac{3}{n^{\frac{1}{2}}}} = \lim_{n\to\infty}\frac{\frac{2}{\sqrt[6]{n}}+\frac{3}{\sqrt[4]{n}}-1}{2-\frac{1}{\sqrt[10]{n^3}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty}\frac{\frac{2}{\sqrt[6]{\infty}}+\frac{3}{\sqrt[4]{\infty}}-1}{2-\frac{1}{\sqrt[10]{\infty}}+\frac{3}{\sqrt{\infty}}}$

  $= \lim_{n\to\infty}\frac{\frac{2}{\infty}+\frac{3}{\infty}-1}{2-\frac{1}{\infty}+\frac{3}{\infty}} = \frac{0+0-1}{2-0+0} = -\frac{1}{2}.$

Riešené limity – 72, 73

$$\lim_{n\to\infty} \left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$$

$$\lim_{n\to\infty} n\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

Riešené limity – 72, 73

$$\lim_{n\to\infty} \left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$$

- $= \lim\limits_{n\to\infty} \left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\right)^n$

$$\lim_{n\to\infty} n\left(\sqrt[n]{2} - \sqrt[n+1]{2}\right)$$

- $= \lim\limits_{n\to\infty} n\left(\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\right)$

Riešené limity – 72, 73

$\lim_{n\to\infty} \left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$

- $= \lim_{n\to\infty} \left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\right)^n = \lim_{n\to\infty} \left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^n$

$\lim_{n\to\infty} n\left(\sqrt[n]{2} - \sqrt[n+1]{2}\right)$

- $= \lim_{n\to\infty} n\left(\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\right) = \lim_{n\to\infty} n\left(\sqrt[n]{2}-1\right) - \lim_{n\to\infty} n\left(\sqrt[n+1]{2}-1\right)$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$$

- $= \lim\limits_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\right)^n = \lim\limits_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^n$
  $= \lim\limits_{n\to\infty}\left(\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\right)^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}}$

$$\lim_{n\to\infty} n\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

- $= \lim\limits_{n\to\infty} n\left(\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\right) = \lim\limits_{n\to\infty} n\left(\sqrt[n]{2}-1\right) - \lim\limits_{n\to\infty} n\left(\sqrt[n+1]{2}-1\right) = \left[\begin{matrix}\text{Pre } a>0 \text{ platí} \\ \lim\limits_{n\to\infty} n(\sqrt[n]{a}-1)=\ln a\end{matrix}\right]$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$$

- $$= \lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\right)^n = \lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^n$$
$$= \lim_{n\to\infty}\left(\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\right)^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}} = e^{\ln\sqrt{6}}$$

- $$\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}} = \left[\begin{array}{l|l|l}\text{Subst.}\quad n\to\infty & \sqrt[n]{2}>1,\ \sqrt[n]{2}\to 1 & \sqrt[n]{2}+\sqrt[n]{3}-2>0 \\ m=\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}\to\infty & \sqrt[n]{3}>1,\ \sqrt[n]{3}\to 1 & \sqrt[n]{2}+\sqrt[n]{3}-2\to 0^+\end{array}\right] = \lim_{m\to\infty}\left(1+\frac{1}{m}\right)^m = e.$$

$$\lim_{n\to\infty} n\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

- $$= \lim_{n\to\infty} n\left(\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\right) = \lim_{n\to\infty} n\left(\sqrt[n]{2}-1\right) - \lim_{n\to\infty} n\left(\sqrt[n+1]{2}-1\right) = \left[\begin{array}{c}\text{Pre } a>0 \text{ platí} \\ \lim\limits_{n\to\infty} n(\sqrt[n]{a}-1)=\ln a\end{array}\right]$$
$$= \ln 2 - \lim_{n\to\infty}(n+1-1)\cdot\left(\sqrt[n+1]{2}-1\right)$$

Riešené limity – 72, 73

$\lim\limits_{n\to\infty}\left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$

- $= \lim\limits_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\right)^n = \lim\limits_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^n$
  $= \lim\limits_{n\to\infty}\left(\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\right)^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}} = e^{\ln\sqrt{6}}$

- $\lim\limits_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}} = \left[\begin{array}{ll|l|l}\text{Subst.} & n\to\infty & \sqrt[n]{2}>1,\ \sqrt[n]{2}\to 1 & \sqrt[n]{2}+\sqrt[n]{3}-2>0 \\ m=\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2} & \to\infty & \sqrt[n]{3}>1,\ \sqrt[n]{3}\to 1 & \sqrt[n]{2}+\sqrt[n]{3}-2\to 0^+\end{array}\right] = \lim\limits_{m\to\infty}\left(1+\frac{1}{m}\right)^m = e.$
- $\lim\limits_{n\to\infty} n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2} = \lim\limits_{n\to\infty}\frac{n(\sqrt[n]{2}-1)+n(\sqrt[n]{3}-1)}{2} = \left[\begin{array}{c}\text{Pre } a>0 \text{ platí} \\ \lim\limits_{n\to\infty} n(\sqrt[n]{a}-1)=\ln a\end{array}\right] = \frac{\ln 2+\ln 3}{2} = \frac{\ln 6}{2} = \ln 6^{\frac{1}{2}} = \ln\sqrt{6}.$

$\lim\limits_{n\to\infty} n\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$

- $= \lim\limits_{n\to\infty} n\left(\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\right) = \lim\limits_{n\to\infty} n\left(\sqrt[n]{2}-1\right) - \lim\limits_{n\to\infty} n\left(\sqrt[n+1]{2}-1\right) = \left[\begin{array}{c}\text{Pre } a>0 \text{ platí} \\ \lim\limits_{n\to\infty} n(\sqrt[n]{a}-1)=\ln a\end{array}\right]$
  $= \ln 2 - \lim\limits_{n\to\infty}(n+1-1)\cdot\left(\sqrt[n+1]{2}-1\right) = \ln 2 - \lim\limits_{n\to\infty}\left((n+1)\cdot\left(\sqrt[n+1]{2}-1\right)-\left(\sqrt[n+1]{2}-1\right)\right)$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\Big(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\Big)^n=\sqrt{6}$$

- $$=\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\Big)^n=\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^n$$
$$=\lim_{n\to\infty}\bigg(\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\bigg)^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}}=e^{\ln\sqrt{6}}=\sqrt{6}.$$

- $$\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}=\left[\begin{array}{ll|l|l}\text{Subst.} & n\to\infty & \sqrt[n]{2}>1,\ \sqrt[n]{2}\to1 & \sqrt[n]{2}+\sqrt[n]{3}-2>0\\ m=\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2} & \to\infty & \sqrt[n]{3}>1,\ \sqrt[n]{3}\to1 & \sqrt[n]{2}+\sqrt[n]{3}-2\to0^+\end{array}\right]=\lim_{m\to\infty}\Big(1+\frac{1}{m}\Big)^m=e.$$
- $$\lim_{n\to\infty}n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}=\lim_{n\to\infty}\frac{n(\sqrt[n]{2}-1)+n(\sqrt[n]{3}-1)}{2}=\left[\begin{array}{c}\text{Pre } a>0 \text{ platí}\\ \lim\limits_{n\to\infty}n(\sqrt[n]{a}-1)=\ln a\end{array}\right]=\frac{\ln2+\ln3}{2}=\frac{\ln6}{2}=\ln6^{\frac12}=\ln\sqrt{6}.$$

$$\lim_{n\to\infty}n\big(\sqrt[n]{2}-\sqrt[n+1]{2}\big)$$

- $$=\lim_{n\to\infty}n\big(\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\big)=\lim_{n\to\infty}n\big(\sqrt[n]{2}-1\big)-\lim_{n\to\infty}n\big(\sqrt[n+1]{2}-1\big)=\left[\begin{array}{c}\text{Pre } a>0 \text{ platí}\\ \lim\limits_{n\to\infty}n(\sqrt[n]{a}-1)=\ln a\end{array}\right]$$
$$=\ln2-\lim_{n\to\infty}(n+1-1)\cdot\big(\sqrt[n+1]{2}-1\big)=\ln2-\lim_{n\to\infty}\Big((n+1)\cdot\big(\sqrt[n+1]{2}-1\big)-\big(\sqrt[n+1]{2}-1\big)\Big)$$
$$=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=n+1 & m\to\infty\end{array}\right]$$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\Big(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\Big)^n=\sqrt{6}$$

- $$=\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\Big)^n=\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^n$$
$$=\lim_{n\to\infty}\Bigg(\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\Bigg)^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}}=e^{\ln\sqrt{6}}=\sqrt{6}.$$

- $$\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}=\left[\begin{array}{ll|l|l}\text{Subst.} & n\to\infty & \sqrt[n]{2}>1,\ \sqrt[n]{2}\to 1 & \sqrt[n]{2}+\sqrt[n]{3}-2>0\\ m=\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2} & \to\infty & \sqrt[n]{3}>1,\ \sqrt[n]{3}\to 1 & \sqrt[n]{2}+\sqrt[n]{3}-2\to 0^+\end{array}\right]=\lim_{m\to\infty}\Big(1+\frac{1}{m}\Big)^m=e.$$
- $$\lim_{n\to\infty}n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}=\lim_{n\to\infty}\frac{n(\sqrt[n]{2}-1)+n(\sqrt[n]{3}-1)}{2}=\left[\begin{array}{c}\text{Pre } a>0 \text{ platí}\\ \lim\limits_{n\to\infty}n(\sqrt[n]{a}-1)=\ln a\end{array}\right]=\frac{\ln 2+\ln 3}{2}=\frac{\ln 6}{2}=\ln 6^{\frac{1}{2}}=\ln\sqrt{6}.$$

$$\lim_{n\to\infty}n\big(\sqrt[n]{2}-\sqrt[n+1]{2}\big)$$

- $$=\lim_{n\to\infty}n\big(\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\big)=\lim_{n\to\infty}n\big(\sqrt[n]{2}-1\big)-\lim_{n\to\infty}n\big(\sqrt[n+1]{2}-1\big)=\left[\begin{array}{c}\text{Pre } a>0 \text{ platí}\\ \lim\limits_{n\to\infty}n(\sqrt[n]{a}-1)=\ln a\end{array}\right]$$
$$=\ln 2-\lim_{n\to\infty}(n+1-1)\cdot\big(\sqrt[n+1]{2}-1\big)=\ln 2-\lim_{n\to\infty}\Big((n+1)\cdot\big(\sqrt[n+1]{2}-1\big)-\big(\sqrt[n+1]{2}-1\big)\Big)$$
$$=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=n+1 & m\to\infty\end{array}\right]=\ln 2-\lim_{m\to\infty}m\big(\sqrt[m]{2}-1\big)+\lim_{m\to\infty}\big(\sqrt[m]{2}-1\big)$$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\Big(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\Big)^n=\sqrt{6}$$

- $= \lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\Big)^n = \lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^n$
  $= \lim_{n\to\infty}\bigg(\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\bigg)^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}} = e^{\ln\sqrt{6}} = \sqrt{6}.$

- $\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}} = \left[\begin{array}{l|l|l}\text{Subst.} \quad n\to\infty & \sqrt[n]{2}>1,\ \sqrt[n]{2}\to 1 & \sqrt[n]{2}+\sqrt[n]{3}-2>0 \\ m=\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}\to\infty & \sqrt[n]{3}>1,\ \sqrt[n]{3}\to 1 & \sqrt[n]{2}+\sqrt[n]{3}-2\to 0^+\end{array}\right] = \lim_{m\to\infty}\Big(1+\frac{1}{m}\Big)^m = e.$
- $\lim_{n\to\infty} n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2} = \lim_{n\to\infty}\frac{n(\sqrt[n]{2}-1)+n(\sqrt[n]{3}-1)}{2} = \left[\begin{array}{c}\text{Pre } a>0 \text{ platí} \\ \lim_{n\to\infty} n(\sqrt[n]{a}-1)=\ln a\end{array}\right] = \frac{\ln 2+\ln 3}{2} = \frac{\ln 6}{2} = \ln 6^{\frac{1}{2}} = \ln\sqrt{6}.$

$$\lim_{n\to\infty} n\big(\sqrt[n]{2}-\sqrt[n+1]{2}\big)$$

- $= \lim_{n\to\infty} n\big(\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\big) = \lim_{n\to\infty} n\big(\sqrt[n]{2}-1\big) - \lim_{n\to\infty} n\big(\sqrt[n+1]{2}-1\big) = \left[\begin{array}{c}\text{Pre } a>0 \text{ platí} \\ \lim_{n\to\infty} n(\sqrt[n]{a}-1)=\ln a\end{array}\right]$
  $= \ln 2 - \lim_{n\to\infty}(n+1-1)\cdot\big(\sqrt[n+1]{2}-1\big) = \ln 2 - \lim_{n\to\infty}\Big((n+1)\cdot\big(\sqrt[n+1]{2}-1\big)-\big(\sqrt[n+1]{2}-1\big)\Big)$
  $= \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=n+1 & m\to\infty\end{array}\right] = \ln 2 - \lim_{m\to\infty} m\big(\sqrt[m]{2}-1\big) + \lim_{m\to\infty}\big(\sqrt[m]{2}-1\big) = \ln 2 - \ln 2 + (1-1)$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\Big(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\Big)^n=\sqrt{6}$$

- $= \lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\Big)^n = \lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^n$
  $= \lim_{n\to\infty}\Big(\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\Big)^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}} = e^{\ln\sqrt{6}} = \sqrt{6}.$

- $\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}} = \left[\begin{array}{l|l|l|l} \text{Subst.} & n\to\infty & \sqrt[n]{2}>1,\ \sqrt[n]{2}\to 1 & \sqrt[n]{2}+\sqrt[n]{3}-2>0 \\ m=\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2} & \to\infty & \sqrt[n]{3}>1,\ \sqrt[n]{3}\to 1 & \sqrt[n]{2}+\sqrt[n]{3}-2\to 0^+ \end{array}\right] = \lim_{m\to\infty}\Big(1+\frac{1}{m}\Big)^m = e.$
- $\lim_{n\to\infty} n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2} = \lim_{n\to\infty}\frac{n(\sqrt[n]{2}-1)+n(\sqrt[n]{3}-1)}{2} = \left[\begin{array}{c}\text{Pre } a>0 \text{ platí} \\ \lim_{n\to\infty} n(\sqrt[n]{a}-1)=\ln a\end{array}\right] = \frac{\ln 2+\ln 3}{2} = \frac{\ln 6}{2} = \ln 6^{\frac{1}{2}} = \ln\sqrt{6}.$

$$\lim_{n\to\infty} n\big(\sqrt[n]{2}-\sqrt[n+1]{2}\big)=0$$

- $= \lim_{n\to\infty} n\big(\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\big) = \lim_{n\to\infty} n\big(\sqrt[n]{2}-1\big) - \lim_{n\to\infty} n\big(\sqrt[n+1]{2}-1\big) = \left[\begin{array}{c}\text{Pre } a>0 \text{ platí} \\ \lim_{n\to\infty} n(\sqrt[n]{a}-1)=\ln a\end{array}\right]$
  $= \ln 2 - \lim_{n\to\infty}(n+1-1)\cdot\big(\sqrt[n+1]{2}-1\big) = \ln 2 - \lim_{n\to\infty}\Big((n+1)\cdot\big(\sqrt[n+1]{2}-1\big)-\big(\sqrt[n+1]{2}-1\big)\Big)$
  $= \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=n+1 & m\to\infty\end{array}\right] = \ln 2 - \lim_{m\to\infty} m\big(\sqrt[m]{2}-1\big) + \lim_{m\to\infty}\big(\sqrt[m]{2}-1\big) = \ln 2 - \ln 2 + (1-1) = 0.$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

- $= \lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

- $= \lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim\limits_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right)$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

- $= \lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim\limits_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

- $= \lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim\limits_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$

  $= \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right)$

Riešené limity – 74

$\lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$

- $= \lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim\limits_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$

  $= \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{n-(n+1)}{n(n+1)}}\right)$

Riešené limity – 74

$\lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$

- $= \lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim\limits_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$

  $= \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{n-(n+1)}{n(n+1)}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{-1}{n(n+1)}}\right)$

Riešené limity – 74

$\lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$

- $= \lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$

$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{n-(n+1)}{n(n+1)}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{-1}{n(n+1)}}\right)$

$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right)$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

- $$= \lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$$

$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{n-(n+1)}{n(n+1)}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{-1}{n(n+1)}}\right)$$

$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\cdot\left(2^{\frac{1}{n(n+1)}}-1\right)$$

Riešené limity – 74

$\lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$

- $= \lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim\limits_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$

$= \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{n-(n+1)}{n(n+1)}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{-1}{n(n+1)}}\right)$

$= \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\cdot\left(2^{\frac{1}{n(n+1)}}-1\right)$

$= \lim\limits_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right)$

Riešené limity – 74

$\lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$

- $= \lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim\limits_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$

$= \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{n-(n+1)}{n(n+1)}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{-1}{n(n+1)}}\right)$

$= \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right) = \lim\limits_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\cdot\left(2^{\frac{1}{n(n+1)}}-1\right)$

$= \lim\limits_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right) = \lim\limits_{n\to\infty} \frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right)$

Riešené limity – 74

$\lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$

- $$= \lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$$
$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{n-(n+1)}{n(n+1)}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{-1}{n(n+1)}}\right)$$
$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\cdot\left(2^{\frac{1}{n(n+1)}}-1\right)$$
$$= \lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right) = \lim_{n\to\infty} \frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right)$$
$$= \lim_{n\to\infty} n\cdot(n+1)\cdot\left(\sqrt[n(n+1)]{2}-1\right)\cdot\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{n\to\infty}\frac{1}{\sqrt[n(n+1)]{2}}$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

- $$= \lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$$
$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{n-(n+1)}{n(n+1)}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{-1}{n(n+1)}}\right)$$
$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\cdot\left(2^{\frac{1}{n(n+1)}}-1\right)$$
$$= \lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right) = \lim_{n\to\infty}\frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right)$$
$$= \lim_{n\to\infty} n\cdot(n+1)\cdot\left(\sqrt[n(n+1)]{2}-1\right)\cdot\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{n\to\infty}\frac{1}{\sqrt[n(n+1)]{2}}$$
$$= \left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=n\cdot(n+1) & m\to\infty\end{array}\right]$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

- $$= \lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$$

$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{n-(n+1)}{n(n+1)}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{-1}{n(n+1)}}\right)$$

$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\cdot\left(2^{\frac{1}{n(n+1)}}-1\right)$$

$$= \lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right) = \lim_{n\to\infty}\frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right)$$

$$= \lim_{n\to\infty} n\cdot(n+1)\cdot\left(\sqrt[n(n+1)]{2}-1\right)\cdot\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{n\to\infty}\frac{1}{\sqrt[n(n+1)]{2}}$$

$$= \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=n\cdot(n+1) & m\to\infty\end{array}\right] = \lim_{m\to\infty} m\left(\sqrt[m]{2}-1\right)\cdot\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{m\to\infty}\frac{1}{\sqrt[m]{2}}$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

- $$= \lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$$
$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{n-(n+1)}{n(n+1)}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{-1}{n(n+1)}}\right)$$
$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\cdot\left(2^{\frac{1}{n(n+1)}}-1\right)$$
$$= \lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right) = \lim_{n\to\infty} \frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right)$$
$$= \lim_{n\to\infty} n\cdot(n+1)\cdot\left(\sqrt[n(n+1)]{2}-1\right)\cdot\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{n\to\infty}\frac{1}{\sqrt[n(n+1)]{2}}$$
$$= \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=n\cdot(n+1) & m\to\infty\end{array}\right] = \lim_{m\to\infty} m\left(\sqrt[m]{2}-1\right)\cdot\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{m\to\infty}\frac{1}{\sqrt[m]{2}}$$
$$= \ln 2\cdot\frac{1}{1+\frac{1}{\infty}}\cdot 1\cdot 1$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)$$

- $$= \lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right) = \lim_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$$

$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{n-(n+1)}{n(n+1)}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{-1}{n(n+1)}}\right)$$

$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right) = \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\cdot\left(2^{\frac{1}{n(n+1)}}-1\right)$$

$$= \lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right) = \lim_{n\to\infty} \frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right)$$

$$= \lim_{n\to\infty} n\cdot(n+1)\cdot\left(\sqrt[n(n+1)]{2}-1\right)\cdot\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{n\to\infty}\frac{1}{\sqrt[n(n+1)]{2}}$$

$$= \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=n\cdot(n+1) & m\to\infty\end{array}\right] = \lim_{m\to\infty} m\left(\sqrt[m]{2}-1\right)\cdot\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{m\to\infty}\frac{1}{\sqrt[m]{2}}$$

$$= \ln 2\cdot\frac{1}{1+\frac{1}{\infty}}\cdot 1\cdot 1 = \ln 2\cdot\frac{1}{1+0}\cdot 1\cdot 1$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)=\ln 2$$

- $$= \lim_{n\to\infty} n^2\left(\sqrt[n]{2}-\sqrt[n+1]{2}\right)=\lim_{n\to\infty} n^2\left(2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right)=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right)$$

$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{1}{n+1}-\frac{1}{n}}\right)=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{n-(n+1)}{n(n+1)}}\right)=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(1-2^{\frac{-1}{n(n+1)}}\right)$$

$$= \lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right)=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\cdot\left(2^{\frac{1}{n(n+1)}}-1\right)$$

$$= \lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right)=\lim_{n\to\infty}\frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\cdot\left(\sqrt[n(n+1)]{2}-1\right)$$

$$= \lim_{n\to\infty} n\cdot(n+1)\cdot\left(\sqrt[n(n+1)]{2}-1\right)\cdot\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{n\to\infty}\frac{1}{\sqrt[n(n+1)]{2}}$$

$$= \left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=n\cdot(n+1) & m\to\infty\end{array}\right]=\lim_{m\to\infty} m\left(\sqrt[m]{2}-1\right)\cdot\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{m\to\infty}\frac{1}{\sqrt[m]{2}}$$

$$= \ln 2\cdot\frac{1}{1+\frac{1}{\infty}}\cdot 1\cdot 1=\ln 2\cdot\frac{1}{1+0}\cdot 1\cdot 1=\ln 2.$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$$
$$= \lim_{n\to\infty} \frac{1}{n} \ln\left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) = \lim_{n\to\infty} \frac{1}{n} \ln\left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}$.

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$$
$$= \lim_{n\to\infty} \frac{1}{n} \ln\left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$$
$$= \lim_{n\to\infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$

$\Rightarrow$ • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n.$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$$
$$= \lim_{n\to\infty} \frac{1}{n} \ln\left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$

$\Rightarrow$ • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n.$

$\Rightarrow$ • $\ln 2^n < \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$
$< \ln\left(n \cdot 2^n\right)$.

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$$
$$= \lim_{n\to\infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$

$\Rightarrow$ • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n.$

$\Rightarrow$ • $n \cdot \ln 2 = \ln 2^n < \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$
$< \ln \left(n \cdot 2^n\right) = \ln n + \ln 2^n$.

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$$
$$= \lim_{n\to\infty} \frac{1}{n} \ln\left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$

$\Rightarrow$ • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n.$

$\Rightarrow$ • $n \cdot \ln 2 = \ln 2^n < \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$
$< \ln\left(n \cdot 2^n\right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) = \lim_{n\to\infty} \frac{1}{n} \ln\left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$

$\Rightarrow$ • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n.$

$\Rightarrow$ • $n \cdot \ln 2 = \ln 2^n < \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) < \ln\left(n \cdot 2^n\right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.$

$\Rightarrow$ • $\ln 2 < \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) < \frac{1}{n} \ln n + \ln 2$.

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) = \lim_{n\to\infty} \frac{1}{n} \ln\left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$

$\Rightarrow$ • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n.$

$\Rightarrow$ • $n \cdot \ln 2 = \ln 2^n < \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) < \ln\left(n \cdot 2^n\right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.$

$\Rightarrow$ • $\ln 2 < \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) < \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 \quad .$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$$
$$= \lim_{n\to\infty} \frac{1}{n} \ln\left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$

$\Rightarrow$ • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n.$

$\Rightarrow$ • $n \cdot \ln 2 = \ln 2^n < \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$
$< \ln\left(n \cdot 2^n\right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.$

$\Rightarrow$ • $\ln 2 < \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$
$< \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln \sqrt[n]{n} + \ln 2.$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) = \lim_{n\to\infty} \frac{1}{n} \ln\left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$

$\Rightarrow$ • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n.$

$\Rightarrow$ • $n \cdot \ln 2 = \ln 2^n < \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) < \ln\left(n \cdot 2^n\right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.$

$\Rightarrow$ • $\ln 2 < \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) < \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln \sqrt[n]{n} + \ln 2.$

$\Rightarrow$ • $\lim_{n\to\infty} \ln 2 \leq \lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) \leq \lim_{n\to\infty} \left(\ln \sqrt[n]{n} + \ln 2\right)$.

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) = \lim_{n\to\infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$

$\Rightarrow$ • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n.$

$\Rightarrow$ • $n \cdot \ln 2 = \ln 2^n < \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) < \ln\left(n \cdot 2^n\right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.$

$\Rightarrow$ • $\ln 2 < \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) < \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln \sqrt[n]{n} + \ln 2.$

$\Rightarrow$ • $\ln 2 = \lim_{n\to\infty} \ln 2 \le \lim_{n\to\infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) \le \lim_{n\to\infty} \left(\ln \sqrt[n]{n} + \ln 2\right) = \ln 1 + \ln 2$.

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) = \lim_{n\to\infty} \frac{1}{n} \ln\left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right)$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n$.

$\Rightarrow$ • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n$.

$\Rightarrow$ • $n \cdot \ln 2 = \ln 2^n < \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) < \ln\left(n \cdot 2^n\right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2$.

$\Rightarrow$ • $\ln 2 < \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) < \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln \sqrt[n]{n} + \ln 2$.

$\Rightarrow$ • $\ln 2 = \lim_{n\to\infty} \ln 2 \leq \lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) \leq \lim_{n\to\infty} \left(\ln \sqrt[n]{n} + \ln 2\right) = \ln 1 + \ln 2 = 0 + \ln 2 = \ln 2$.

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$$
$$= \lim_{n\to\infty} \frac{1}{n} \ln\left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right) = \ln 2$$

Pre všetky $n \in N$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$

$\Rightarrow$ • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n.$

$\Rightarrow$ • $n \cdot \ln 2 = \ln 2^n < \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$
$< \ln\left(n \cdot 2^n\right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.$

$\Rightarrow$ • $\ln 2 < \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$
$< \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln \sqrt[n]{n} + \ln 2.$

$\Rightarrow$ • $\ln 2 = \lim_{n\to\infty} \ln 2 \leq \lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right)$
$\leq \lim_{n\to\infty} \left(\ln \sqrt[n]{n} + \ln 2\right) = \ln 1 + \ln 2 = 0 + \ln 2 = \ln 2.$

$\Rightarrow$ • $\lim_{n\to\infty} \frac{1}{n} \ln\left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right) = \ln 2.$

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}$.

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}}$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\right\}$.

- $\lim_{n\to\infty} a_n$

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6}$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}$.

- $\lim_{n\to\infty} a_n$

$\{a_n\}_{n=1}^{\infty}$ má tvar:

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6}$

Označme $a_n = \frac{n^k}{4^n}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n\to\infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1.$

Resp.
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{4n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1.$

$\Rightarrow \lim_{n\to\infty} \frac{n^k}{4^n} = 0.$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}.$

- $\lim_{n\to\infty} a_n$

$\{a_n\}_{n=1}^{\infty}$ má tvar: • $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n\in N$ (rekurentne),

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6} = \left[\begin{array}{c}\text{Geometrická postupnosť} \\ \lim_{n\to\infty}\frac{5^n}{4^n} = \lim_{n\to\infty}\left(\frac{5}{4}\right)^n = \infty.\end{array}\right]$

Označme $a_n = \frac{n^k}{4^n}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n\to\infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1.$

Resp.
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{4n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1.$

$\Rightarrow \lim_{n\to\infty} \frac{n^k}{4^n} = 0.$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}.$

- $\lim_{n\to\infty} a_n$

$\{a_n\}_{n=1}^{\infty}$ má tvar: • $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n\in N$ (rekurentne),

- $a_1 = \sqrt{2}$,

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6} = \left[\begin{array}{c}\text{Geometrická postupnosť} \\ \lim_{n\to\infty}\frac{5^n}{4^n} = \lim_{n\to\infty}\left(\frac{5}{4}\right)^n = \infty.\end{array}\right] = \frac{2\cdot 0+3\cdot 0-0+2\cdot\infty}{2\cdot 0-0+6}$

Označme $a_n = \frac{n^k}{4^n}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n\to\infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1.$

Resp.
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{4n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1.$

$\Rightarrow \lim_{n\to\infty} \frac{n^k}{4^n} = 0.$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}.$

- $\lim_{n\to\infty} a_n$

$\{a_n\}_{n=1}^{\infty}$ má tvar: • $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n\in N$ (rekurentne),

- $a_1 = \sqrt{2}$, $a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2}$,

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6} = \left[\begin{array}{c}\text{Geometrická postupnosť} \\ \lim_{n\to\infty}\frac{5^n}{4^n} = \lim_{n\to\infty}(\frac{5}{4})^n = \infty.\end{array}\right]$

  $= \frac{2\cdot 0+3\cdot 0-0+2\cdot\infty}{2\cdot 0-0+6} = \frac{\infty}{6}$

Označme $a_n = \frac{n^k}{4^n}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n\to\infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1.$

Resp.
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{4n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1.$

$\Rightarrow \lim_{n\to\infty} \frac{n^k}{4^n} = 0.$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\right\}.$

- $\lim_{n\to\infty} a_n$

$\{a_n\}_{n=1}^{\infty}$ má tvar:
- $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n\in N$ (rekurentne),
- $a_1 = \sqrt{2}$, $a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2}$, $a_3 = \sqrt{\sqrt[4]{2}} = \sqrt[8]{2} = \sqrt[2^3]{2}$, $\dots$,

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} = \infty$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6} = \left[\begin{array}{c}\text{Geometrická postupnosť} \\ \lim_{n\to\infty}\frac{5^n}{4^n} = \lim_{n\to\infty}\left(\frac{5}{4}\right)^n = \infty.\end{array}\right]$

  $= \frac{2\cdot 0+3\cdot 0-0+2\cdot\infty}{2\cdot 0-0+6} = \frac{\infty}{6} = \infty.$

Označme $a_n = \frac{n^k}{4^n}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\frac{n^k}{4^n}} = \lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{4} = \lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1.$

Resp.
- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n\to\infty}\frac{(n+1)^k}{4n^k} = \lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1.$

$\Rightarrow \lim_{n\to\infty}\frac{n^k}{4^n} = 0.$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}.$

- $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\sqrt[2^n]{2}$

$\{a_n\}_{n=1}^{\infty}$ má tvar:
- $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n\in N$ (rekurentne),
- $a_1 = \sqrt{2}$, $a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2}$, $a_3 = \sqrt{\sqrt[4]{2}} = \sqrt[8]{2} = \sqrt[2^3]{2}$, $\ldots$, $a_n = \sqrt[2^n]{2}$, $n\in N$ (explicitne).

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} = \infty$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6} = \left[\begin{array}{c}\text{Geometrická postupnosť} \\ \lim_{n\to\infty}\frac{5^n}{4^n} = \lim_{n\to\infty}\left(\frac{5}{4}\right)^n = \infty.\end{array}\right]$

  $= \frac{2\cdot 0+3\cdot 0-0+2\cdot\infty}{2\cdot 0-0+6} = \frac{\infty}{6} = \infty.$

Označme $a_n = \frac{n^k}{4^n}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n\to\infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1.$

Resp.
- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{4n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1.$

$\Rightarrow \lim_{n\to\infty} \frac{n^k}{4^n} = 0.$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}.$

- $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt[2^n]{2} = \left[\begin{array}{c|c}\text{Subst.} & n\to\infty \\ m = n^2 & m\to\infty\end{array}\right]$

$\{a_n\}_{n=1}^{\infty}$ má tvar:
- $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n\in N$ (rekurentne),
- $a_1 = \sqrt{2}$, $a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2}$, $a_3 = \sqrt{\sqrt[4]{2}} = \sqrt[8]{2} = \sqrt[2^3]{2}$, …, $a_n = \sqrt[2^n]{2}$, $n\in N$ (explicitne).

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} = \infty$$

- $= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6} = \left[\begin{matrix}\text{Geometrická postupnosť} \\ \lim_{n\to\infty}\frac{5^n}{4^n} = \lim_{n\to\infty}\left(\frac{5}{4}\right)^n = \infty.\end{matrix}\right]$
  $= \frac{2\cdot 0+3\cdot 0-0+2\cdot\infty}{2\cdot 0-0+6} = \frac{\infty}{6} = \infty.$

Označme $a_n = \frac{n^k}{4^n}$, $n\in N$ pre $k\in N$.

- $\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\frac{n^k}{4^n}} = \lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{4} = \lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1.$

Resp.
- $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n\to\infty}\frac{(n+1)^k}{4n^k} = \lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1.$

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- $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\sqrt[2^n]{2} = \left[\begin{matrix}\text{Subst.} & n\to\infty \\ m=n^2 & m\to\infty\end{matrix}\right] = \lim_{m\to\infty}\sqrt[m]{2}$

$\{a_n\}_{n=1}^{\infty}$ je vybraná z postupnosti $\{\sqrt[n]{2}\}_{n=1}^{\infty}$ a má tvar:
- $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n\in N$ (rekurentne),
- $a_1 = \sqrt{2}$, $a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2}$, $a_3 = \sqrt{\sqrt[4]{2}} = \sqrt[8]{2} = \sqrt[2^3]{2}$, $\ldots$, $a_n = \sqrt[2^n]{2}$, $n\in N$ (explicitne).

Riešené limity – 76, 77

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Označme $a_n = \frac{n^k}{4^n}$, $n\in N$ pre $k\in N$.

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- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{4n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1.$

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Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}.$

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Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \dots\right\}$.

Riešené limity – 78

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Postupnosť $\{a_n\}_{n=1}^{\infty}$ je rekurentne zadaná:

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Riešené limity – 78

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Riešené limity – 78

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Označme $\lim\limits_{n\to\infty} a_n = a$. Platí $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} a_{n+1}$, pričom $a_n > 0$ pre všetky $n \in N$.

Riešené limity – 78

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$\Rightarrow$ • $a^2 = \lim\limits_{n\to\infty} a_n^2 = \lim\limits_{n\to\infty} a_{n+1}^2 = \lim\limits_{n\to\infty} (2+a_n) = 2 + \lim\limits_{n\to\infty} a_n = 2 + a.$

Riešené limity – 78

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Riešené limity – 78

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$\Rightarrow$ • a je riešením rovnice $a^2 = 2+a$. $\Rightarrow$ • $a^2 - a - 2 = (a-2)\cdot(a+1) = 0$.

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

Postupnosť $\{a_n\}_{n=1}^{\infty}$ je rekurentne zadaná: • $a_1 = \sqrt{2},\ a_{n+1} = \sqrt{2+a_n},\ n \in N$.

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$\Rightarrow$ • a je riešením rovnice $a^2 = 2+a$. $\Rightarrow$ • $a^2 - a - 2 = (a-2)\cdot(a+1) = 0$.

$\Rightarrow$ • $a = \lim\limits_{n\to\infty} a_n = 2$ (pokiaľ limita existuje). [Koreň $a = -1 < 0$ nevyhovuje, pretože $a_n > 0$ pre všetky $n \in N$.]

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

Postupnosť $\{a_n\}_{n=1}^{\infty}$ je rekurentne zadaná: • $a_1 = \sqrt{2},\ a_{n+1} = \sqrt{2+a_n},\ n\in N$.

Označme $\lim_{n\to\infty} a_n = a$. Platí $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$, pričom $a_n > 0$ pre všetky $n\in N$.

⇒ • $a^2 = \lim_{n\to\infty} a_n^2 = \lim_{n\to\infty} a_{n+1}^2 = \lim_{n\to\infty} (2+a_n) = 2 + \lim_{n\to\infty} a_n = 2+a$.

⇒ • a je riešením rovnice $a^2 = 2+a$. ⇒ • $a^2 - a - 2 = (a-2)\cdot(a+1) = 0$.

⇒ • $a = \lim_{n\to\infty} a_n = 2$ (pokiaľ limita existuje). [Koreň $a=-1<0$ nevyhovuje, pretože $a_n > 0$ pre všetky $n\in N$.]

Ešte musíme ukázať, že existuje $\lim_{n\to\infty} a_n$, t. j. $\{a_n\}_{n=1}^{\infty} \longrightarrow 2$.

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \dots\right\}$.

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$\Rightarrow$ • a je riešením rovnice $a^2 = 2 + a$. $\Rightarrow$ • $a^2 - a - 2 = (a-2)\cdot(a+1) = 0$.

$\Rightarrow$ • $a = \lim\limits_{n\to\infty} a_n = 2$ (pokiaľ limita existuje). [Koreň $a = -1 < 0$ nevyhovuje, pretože $a_n > 0$ pre všetky $n \in N$.]

Ešte musíme ukázať, že existuje $\lim\limits_{n\to\infty} a_n$, t. j. $\{a_n\}_{n=1}^{\infty} \longrightarrow 2$.

- $\{a_n\}_{n=1}^{\infty}$ je ohraničená zhora,

- $\{a_n\}_{n=1}^{\infty}$ je rastúca,

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

Postupnosť $\{a_n\}_{n=1}^{\infty}$ je rekurentne zadaná: • $a_1 = \sqrt{2},\ a_{n+1} = \sqrt{2+a_n},\ n \in N$.

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$\Rightarrow$ • a je riešením rovnice $a^2 = 2 + a$. $\Rightarrow$ • $a^2 - a - 2 = (a-2)\cdot(a+1) = 0$.

$\Rightarrow$ • $a = \lim\limits_{n\to\infty} a_n = 2$ (pokiaľ limita existuje). [Koreň $a = -1 < 0$ nevyhovuje, pretože $a_n > 0$ pre všetky $n \in N$.]

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- $\{a_n\}_{n=1}^{\infty}$ je ohraničená zhora, t. j. $a_n < 2$, $a_n - 2 < 0$ pre všetky $n \in N$. [Dôkaz matematickou indukciou.]

- $\{a_n\}_{n=1}^{\infty}$ je rastúca,

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}.$

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Ešte musíme ukázať, že existuje $\lim\limits_{n\to\infty} a_n$, t. j. $\{a_n\}_{n=1}^{\infty} \longrightarrow 2.$

- $\{a_n\}_{n=1}^{\infty}$ je ohraničená zhora, t. j. $a_n < 2,\ a_n - 2 < 0$ pre všetky $n \in N.$ [Dôkaz matematickou indukciou.]

Krok 1. $a_1 = \sqrt{2} < 2,\ a_2 = \sqrt{2+\sqrt{2}} < \sqrt{2+2} = \sqrt{4} = 2.$

- $\{a_n\}_{n=1}^{\infty}$ je rastúca,

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

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Označme $\lim_{n\to\infty} a_n = a$. Platí $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$, pričom $a_n > 0$ pre všetky $n \in N$.

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Ešte musíme ukázať, že existuje $\lim_{n\to\infty} a_n$, t. j. $\{a_n\}_{n=1}^{\infty} \longrightarrow 2$.

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Krok 1. $a_1 = \sqrt{2} < 2$, $a_2 = \sqrt{2+\sqrt{2}} < \sqrt{2+2} = \sqrt{4} = 2$.

Krok 2. Pre $k \in N$ platí $a_k < 2$.

- $\{a_n\}_{n=1}^{\infty}$ je rastúca,

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

Postupnosť $\{a_n\}_{n=1}^{\infty}$ je rekurentne zadaná: • $a_1 = \sqrt{2},\ a_{n+1} = \sqrt{2+a_n},\ n \in N$.

Označme $\lim_{n\to\infty} a_n = a$. Platí $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$, pričom $a_n > 0$ pre všetky $n \in N$.

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Krok 1. $a_1 = \sqrt{2} < 2,\ a_2 = \sqrt{2+\sqrt{2}} < \sqrt{2+2} = \sqrt{4} = 2$.

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• $\{a_n\}_{n=1}^{\infty}$ je rastúca,

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Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \dots\right\}$.

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Označme $\lim_{n\to\infty} a_n = a$. Platí $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$, pričom $a_n > 0$ pre všetky $n \in N$.

$\Rightarrow$ • $a^2 = \lim_{n\to\infty} a_n^2 = \lim_{n\to\infty} a_{n+1}^2 = \lim_{n\to\infty} (2+a_n) = 2 + \lim_{n\to\infty} a_n = 2+a$.

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Krok 1. $a_1 = 2 < 3,\ a_2 = \sqrt{2\cdot 2 + 3} = \sqrt{7} < \sqrt{9} = 3.$

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Riešené limity – 79

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 2,\ a_{n+1} = \sqrt{2a_n + 3},\ n \in N.$

Označme $\lim\limits_{n\to\infty} a_n = a$. Platí $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} a_{n+1}$, pričom $a_n > 0$ pre všetky $n \in N$.

$\Rightarrow$ • $a^2 = \lim\limits_{n\to\infty} a_n^2 = \lim\limits_{n\to\infty} a_{n+1}^2 = \lim\limits_{n\to\infty} (2a_n + 3) = 2 \lim\limits_{n\to\infty} a_n + 3 = 2a + 3.$

$\Rightarrow$ • a je riešením rovnice $a^2 = 2a + 3.$ $\Rightarrow$ • $a^2 - 2a - 3 = (a-3)\cdot(a+1) = 0.$

$\Rightarrow$ • $a = \lim\limits_{n\to\infty} a_n = 3$ (pokiaľ limita existuje). [Koreň $a = -1 < 0$ nevyhovuje, pretože $a_n > 0$ pre všetky $n \in N$.]

Ešte musíme ukázať, že existuje $\lim\limits_{n\to\infty} a_n$, t. j. $\{a_n\}_{n=1}^{\infty} \longrightarrow 3$.

• $\{a_n\}_{n=1}^{\infty}$ je ohraničená zhora, t. j. $a_n < 3$, $a_n - 3 < 0$, pre všetky $n \in N$. [Dôkaz matematickou indukciou.]

Krok 1. $a_1 = 2 < 3,\ a_2 = \sqrt{2\cdot 2 + 3} = \sqrt{7} < \sqrt{9} = 3.$

Krok 2. Pre $k \in N$ platí $a_k < 3$. $\Rightarrow$ Pre $k+1$ platí $a_{k+1} = \sqrt{2a_k + 3} < \sqrt{2\cdot 3 + 3} = \sqrt{9} = 3.$

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Riešené limity – 79

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- $\{a_n\}_{n=1}^{\infty}$ je rastúca, t. j. $a_n < a_{n+1}$ pre všetky $n \in N.$

Riešené limity – 79

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 2,\ a_{n+1} = \sqrt{2a_n + 3},\ n \in N.$

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• $\{a_n\}_{n=1}^{\infty}$ je rastúca, t. j. $a_n < a_{n+1}$ pre všetky $n \in N.$

$$a_n^2 - a_{n+1}^2 = a_n^2 - (2a_n + 3) = a_n^2 - 2a_n - 3 = (a_n - 3)(a_n + 1) < 0.$$

Riešené limity – 79

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 2,\ a_{n+1} = \sqrt{2a_n + 3},\ n \in N.$

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$$a_n^2 - a_{n+1}^2 = a_n^2 - (2a_n + 3) = a_n^2 - 2a_n - 3 = (a_n - 3)(a_n + 1) < 0. \Rightarrow a_n^2 < a_{n+1}^2.$$

Riešené limity – 79

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Riešené limity – 79

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Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

1

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $x = 32{,}17\overline{71}$ 2

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $x = 32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti: 3

- $x = 32{,}17\overline{71} = 32{,}177171\ldots$

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Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $x = 32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti: 4

- $x = 32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$

- $x = 32{,}17\overline{71} = 32{,}177171\ldots$
- $100x = 3\,217{,}\overline{71} = 3\,217{,}717171\ldots$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $x = 32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti: 5

- $x = 32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$
  $= 32{,}17 + \frac{71}{100\cdot 100} + \frac{71}{100\cdot 100\cdot 100} + \frac{71}{100\cdot 100\cdot 100\cdot 100} + \cdots$

- $x = 32{,}17\overline{71} = 32{,}177171\ldots$
- $100x = 3\,217{,}\overline{71} = 3\,217{,}717171\ldots$
- $10\,000x = 321\,771{,}\overline{71} = 321\,771{,}717171\ldots$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $x = 32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti: 6

- $x = 32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$
  $= 32{,}17 + \frac{71}{100\cdot 100} + \frac{71}{100\cdot 100\cdot 100} + \frac{71}{100\cdot 100\cdot 100\cdot 100} + \cdots$
  $= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\left(1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots\right)$

- $x = 32{,}17\overline{71} = 32{,}177171\ldots$
- $100x = 3\,217{,}\overline{71} = 3\,217{,}717171\ldots$
- $10\,000x = 321\,771{,}\overline{71} = 321\,771{,}717171\ldots$

$\Rightarrow$ • $10\,000x - 100x$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $x = 32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti: 7

- $x = 32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$

$$= 32{,}17 + \frac{71}{100\cdot 100} + \frac{71}{100\cdot 100\cdot 100} + \frac{71}{100\cdot 100\cdot 100\cdot 100} + \cdots$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\left(1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots\right) = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q = \frac{1}{100}\\ 1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}}\end{array}\right]$$

- $x = 32{,}17\overline{71} = 32{,}177171\ldots$
- $100x = 3\,217{,}\overline{71} = 3\,217{,}717171\ldots$
- $10\,000x = 321\,771{,}\overline{71} = 321\,771{,}717171\ldots$

$\Rightarrow$ • $10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$

$$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $x = 32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti: 8

- $x = 32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$

$$= 32{,}17 + \frac{71}{100\cdot100} + \frac{71}{100\cdot100\cdot100} + \frac{71}{100\cdot100\cdot100\cdot100} + \cdots$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot100}\cdot\Big(1 + \frac{1}{100} + \frac{1}{100\cdot100} + \cdots\Big) = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q=\frac{1}{100}\\ 1+\frac{1}{100}+\frac{1}{100\cdot100}+\cdots=\frac{1}{1-q}=\frac{1}{1-\frac{1}{100}}\end{array}\right]$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot100}\cdot\frac{1}{1-\frac{1}{100}}$$

- $x = 32{,}17\overline{71} = 32{,}177171\ldots$
- $100x = 3\,217{,}\overline{71} = 3\,217{,}717171\ldots$
- $10\,000x = 321\,771{,}\overline{71} = 321\,771{,}717171\ldots$

$\Rightarrow$
- $10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$

$$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots = 321\,771 - 3\,217$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $x = 32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti: 9

- $x = 32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$

$$= 32{,}17 + \frac{71}{100\cdot 100} + \frac{71}{100\cdot 100\cdot 100} + \frac{71}{100\cdot 100\cdot 100\cdot 100} + \cdots$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\left(1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots\right) = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q = \frac{1}{100}\\ 1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}}\end{array}\right]$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100\cdot(100-1)}$$

$$\left.\begin{array}{rcl} x = & 32{,}17\overline{71} = & 32{,}177171\ldots\\ 100x = & 3\,217{,}\overline{71} = & 3\,217{,}717171\ldots\\ 10\,000x = & 321\,771{,}\overline{71} = & 321\,771{,}717171\ldots\end{array}\right\} \Rightarrow 9\,900x = 318\,554$$

$\Rightarrow$ • $9\,900x = 10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$

$$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots = 321\,771 - 3\,217 = 318\,554.$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots = \dfrac{318\,554}{9\,900}$.

Periodické číslo $x = 32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti: 10

- $x = 32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$

$$= 32{,}17 + \frac{71}{100\cdot100} + \frac{71}{100\cdot100\cdot100} + \frac{71}{100\cdot100\cdot100\cdot100} + \cdots$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot100}\cdot\left(1 + \frac{1}{100} + \frac{1}{100\cdot100} + \cdots\right) = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q = \frac{1}{100}\\ 1 + \frac{1}{100} + \frac{1}{100\cdot100} + \cdots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}}\end{array}\right]$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100\cdot(100-1)}$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot99}$$

$$\left.\begin{array}{rlrl} \bullet\; x = & 32{,}17\overline{71} & = & 32{,}177171\ldots \\ \bullet\; 100x = & 3\,217{,}\overline{71} & = & 3\,217{,}717171\ldots \\ \bullet\; 10\,000x = & 321\,771{,}\overline{71} & = & 321\,771{,}717171\ldots \end{array}\right\} \Rightarrow 9\,900x = 318\,554 \quad \Rightarrow x = \frac{318\,554}{9\,900}.$$

$\Rightarrow$ • $9\,900x = 10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$

$$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots = 321\,771 - 3\,217 = 318\,554.$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots = \frac{318\,554}{9\,900}$.

Periodické číslo $x = 32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti: 11

- $x = 32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$

$$= 32{,}17 + \frac{71}{100\cdot 100} + \frac{71}{100\cdot 100\cdot 100} + \frac{71}{100\cdot 100\cdot 100\cdot 100} + \cdots$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\Big(1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots\Big) =$$

[Geometrický rad s kvocientom $q = \frac{1}{100}$: $1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}}$]

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100\cdot(100-1)}$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 99} = \frac{3\,217\cdot 99 + 71}{100\cdot 99}$$

$$\left.\begin{aligned} x &= 32{,}17\overline{71} = 32{,}177171\ldots \\ 100x &= 3\,217{,}\overline{71} = 3\,217{,}717171\ldots \\ 10\,000x &= 321\,771{,}\overline{71} = 321\,771{,}717171\ldots \end{aligned}\right\} \Rightarrow 9\,900x = 318\,554 \Rightarrow x = \frac{318\,554}{9\,900}.$$

$\Rightarrow$ • $9\,900x = 10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$

$$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots = 321\,771 - 3\,217 = 318\,554.$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots = \frac{318\,554}{9\,900}$.

Periodické číslo $x = 32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti: 12

- $$x = 32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$$
$$= 32{,}17 + \frac{71}{100\cdot 100} + \frac{71}{100\cdot 100\cdot 100} + \frac{71}{100\cdot 100\cdot 100\cdot 100} + \cdots$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\left(1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots\right) = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q = \frac{1}{100}\\ 1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}}\end{array}\right]$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100\cdot(100-1)}$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 99} = \frac{3\,217\cdot 99 + 71}{100\cdot 99} = \frac{318\,483+71}{9\,900}$$

$$\left.\begin{array}{rcl} x = & 32{,}17\overline{71} = & 32{,}177171\ldots \\ 100x = & 3\,217{,}\overline{71} = & 3\,217{,}717171\ldots \\ 10\,000x = & 321\,771{,}\overline{71} = & 321\,771{,}717171\ldots \end{array}\right\} \Rightarrow 9\,900x = 318\,554$$

$$\Rightarrow x = \frac{318\,554}{9\,900}.$$

$$\Rightarrow \bullet\ 9\,900x = 10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$$
$$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots = 321\,771 - 3\,217 = 318\,554.$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots = \dfrac{318\,554}{9\,900}$.

Periodické číslo $x = 32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti: 13

- $x = 32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$

$$= 32{,}17 + \frac{71}{100\cdot 100} + \frac{71}{100\cdot 100\cdot 100} + \frac{71}{100\cdot 100\cdot 100\cdot 100} + \cdots$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\Big(1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots\Big) = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q = \frac{1}{100} \\ 1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}}\end{array}\right]$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100\cdot(100-1)}$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 99} = \frac{3\,217\cdot 99 + 71}{100\cdot 99} = \frac{318\,483+71}{9\,900} = \frac{318\,554}{9\,900}.$$

$$\left.\begin{array}{rcccl} x &=& 32{,}17\overline{71} &=& 32{,}177171\ldots \\ 100x &=& 3\,217{,}\overline{71} &=& 3\,217{,}717171\ldots \\ 10\,000x &=& 321\,771{,}\overline{71} &=& 321\,771{,}717171\ldots \end{array}\right\} \Rightarrow 9\,900x = 318\,554 \Rightarrow x = \frac{318\,554}{9\,900}.$$

$\Rightarrow$ • $9\,900x = 10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$

$$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots = 321\,771 - 3\,217 = 318\,554.$$

Koniec 2. časti (príklady)

Ďakujem za pozornosť.