# Matematická analýza 1

## 2023/2024

## 10. Neurčitý integrál
## Riešené príklady II

Pre správne zobrazenie, fungovanie tooltipov, 2D a 3D animácií je nevyhnutné súbor otvoriť pomocou programu Adobe Reader (zásuvný modul Adobe PDF Plug-In webového prehliadača nestačí).

Kliknutím na text pred ikonou získate nápomoc.
Kliknutím na skratku v modrej lište vpravo hore sa dostanete na príslušný slajd, druhým kliknutím sa dostanete na koniec tohto slajdu.

beerb@frcatel.fri.uniza.sk http://frcatel.fri.uniza.sk/~beerb

# Obsah – príklady 101–200

# Zoznam integrálov – príklady 101–200

101. $\int \frac{dx}{x^2+4x+5}$ 102. $\int \frac{dx}{x^2+4x+3}$ 103. $\int \frac{dx}{x^2-4x+6}$ 104. $\int \frac{dx}{x^2-4x+2}$ 105. $\int \frac{dx}{x^2+4x+4}$ 106. $\int \frac{dx}{\sqrt{x^2+4x+4}}$ 107. $\int \frac{dx}{\sqrt{x^2+4x+5}}$ 108. $\int \frac{dx}{\sqrt{x^2+4x+3}}$

109. $\int \frac{dx}{\sqrt{x^2-4x+6}}$ 110. $\int \frac{dx}{\sqrt{x^2-4x+2}}$ 111. $\int \frac{dx}{\sqrt{-x^2+4x-5}}$ 112. $\int \frac{dx}{\sqrt{-x^2+4x-3}}$ 113. $\int \frac{dx}{\sqrt{x(x-1)}}$ 114. $\int \frac{dx}{\sqrt{x(2-x)}}$ 115. $\int \sqrt{x^2+4x+5}\,dx$

116. $\int \sqrt{x^2+4x+3}\,dx$ 117. $\int \sqrt{x^2-4x+6}\,dx$ 118. $\int \sqrt{x^2-4x+2}\,dx$ 119. $\int \sqrt{-x^2+4x-3}\,dx$ 120. $\int \sqrt{x(2-x)}\,dx$ 121. $\int \frac{dx}{(x^2+a^2)^n}$ 122. $\int \frac{dx}{(x^2+a^2)^2}$

123. $\int \frac{dx}{(x^2+a^2)^3}$ 124. $\int \frac{dx}{(x^2+a^2)^4}$ 125. $\int \frac{dx}{(x^2-a^2)^n}$ 126. $\int \frac{dx}{(x^2-a^2)^2}$ 127. $\int \frac{dx}{(x^2-a^2)^3}$ 128. $\int \frac{dx}{(x^2-a^2)^4}$ 129. $\int \frac{dx}{(x^2+4x+5)^2}$ 130. $\int \frac{dx}{(x^2+4x+3)^2}$ 131. $\int \frac{x^2\,dx}{(x^2+a^2)^2}$

132. $\int \frac{x^2\,dx}{(x^2-a^2)^2}$ 133. $\int \frac{x\,dx}{x^2+a^2}$ 134. $\int \frac{x\,dx}{(x^2+a^2)^n}$ 135. $\int \frac{x\,dx}{x^2-a^2}$ 136. $\int \frac{x\,dx}{(x^2-a^2)^n}$ 137. $\int \frac{x^n\,dx}{x-1}$ 138. $\int \frac{x\,dx}{x-1}$ 139. $\int \frac{x^2\,dx}{x-1}$ 140. $\int \frac{x^9\,dx}{x-1}$ 141. $\int \frac{dx}{(1-x)x^2}$

142. $\int \frac{dx}{x^6(1+x^2)}$ 143. $\int \frac{(x-2)^4\,dx}{(x-1)^2}$ 144. $\int \frac{(x-1)^4\,dx}{(x-2)^2}$ 145. $\int \frac{dx}{x^3-7x-6}$ 146. $\int \frac{dx}{x^3-2x^2-x+2}$ 147. $\int \frac{dx}{x^3-3x-2}$ 148. $\int \frac{dx}{x^3+x^2-x-1}$ 149. $\int \frac{dx}{x^3-3x^2+4x-2}$

150. $\int \frac{dx}{x^3-3x^2+3x-1}$ 151. $\int \frac{dx}{x^3-2x-4}$ 152. $\int \frac{dx}{x^6+1}$ 153. $\int \frac{x\,dx}{x^6+1}$ 154. $\int \frac{x^2\,dx}{x^6+1}$ 155. $\int \frac{x^3\,dx}{x^6+1}$ 156. $\int \frac{x^4\,dx}{x^6+1}$ 157. $\int \frac{x^5\,dx}{x^6+1}$ 158. $\int \frac{x^6\,dx}{x^6+1}$ 159. $\int \frac{dx}{2^x+1}$ 160. $\int \frac{dx}{\sqrt{2^x+1}}$

161. $\int \frac{(1+x)\,dx}{\sqrt{1-x^2}}$ 162. $\int \sqrt{\frac{1+x}{1-x}}\,dx$ 163. $\int \sqrt{\frac{1-x}{1+x}}\,dx$ 164. $\int \sqrt{\frac{x+1}{x-1}}\,dx$ 165. $\int \sqrt{\frac{x-1}{x+1}}\,dx$ 166. $\int \sqrt{\left(\frac{1+x}{1-x}\right)^3}\,dx$ 167. $\int \sqrt{\left(\frac{1-x}{1+x}\right)^3}\,dx$

168. $\int \sqrt{\left(\frac{x+1}{x-1}\right)^3}\,dx$ 169. $\int \sqrt{\left(\frac{x-1}{x+1}\right)^3}\,dx$ 170. $\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$ 171. $\int \frac{dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$ 172. $\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$ 173. $\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$

174. $\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$ 175. $\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$ 176. $\int \frac{dx}{\sqrt{x-3}+\sqrt{x-5}}$ 177. $\int \frac{dx}{\sqrt{x-3}-\sqrt{x-5}}$ 178. $\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$ 179. $\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,dx$

180. $\int \sqrt{\frac{x}{1-x\sqrt{x}}}\,dx$ 181. $\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$ 182. $\int \frac{dx}{\sqrt[6]{x}+\sqrt[4]{x}}$ 183. $\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}}$ 184. $\int \frac{dx}{\sqrt[3]{x}+1}$ 185. $\int \left[\sqrt{x^3}-\frac{1}{\sqrt{x}}\right]dx$ 186. $\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,dx$ 187. $\int \frac{dx}{(x-1)\sqrt{x-2}}$

188. $\int \frac{dx}{(x+1)\sqrt{1-x}}$ 189. $\int (x-1)\sqrt{x-2}\,dx$ 190. $\int \frac{\sqrt{1-x}}{x+1}\,dx$ 191. $\int \frac{dx}{\left(x-\sqrt{x^2-1}\right)^2}$ 192. $\int \frac{x^5\,dx}{\sqrt{x^2+1}}$ 193. $\int \frac{x^5\,dx}{\sqrt{x^3+1}}$ 194. $\int \frac{x^5\,dx}{\sqrt{x^2-1}}$ 195. $\int \frac{x^5\,dx}{\sqrt{x^3-1}}$

196. $\int \frac{x^5\,dx}{\sqrt{1-x^2}}$ 197. $\int \frac{x^5\,dx}{\sqrt{1-x^3}}$ 198. $\int \frac{\sqrt{1-x^2}}{x^2}\,dx$ 199. $\int \frac{dx}{x\sqrt{x^4+x^2+1}}$ 200. $\int \frac{dx}{x\sqrt{x^6+x^3+1}}$

# Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5}$$

$$\int \frac{dx}{x^2+4x+3}$$

# Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5}$$

$$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1}$$

$$\int \frac{dx}{x^2+4x+3}$$

$$= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1}$$

# Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5}$$

$$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R \\ dt=dx & t\in R \end{array} \right]$$

$$\int \frac{dx}{x^2+4x+3}$$

$$= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R-\{-1,-3\} \\ dt=dx & t\in R-\{\pm 1\} \end{array} \right]$$

# Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5}$$

$$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R \\ dt=dx & t\in R \end{array} \right]$$

$$= \int \frac{dt}{t^2+1}$$

$$\int \frac{dx}{x^2+4x+3}$$

$$= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R-\{-1,-3\} \\ dt=dx & t\in R-\{\pm 1\} \end{array} \right]$$

$$= \int \frac{dt}{t^2-1}$$

# Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5}$$

$$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1} = \left[ \begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right]$$

$$= \int \frac{dt}{t^2+1} = \operatorname{arctg} t + c$$

$$\int \frac{dx}{x^2+4x+3}$$

$$= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[ \begin{array}{r|l} \text{Subst. } t = x+2 & x \in R - \{-1, -3\} \\ dt = dx & t \in R - \{\pm 1\} \end{array} \right]$$

$$= \int \frac{dt}{t^2-1} = \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c$$

# Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5} = \operatorname{arctg}(x+2) + c$$

$$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1} = \left[ \begin{array}{r|l} \text{Subst. } t = x+2 & x \in R \\ dt = dx & t \in R \end{array} \right]$$

$$= \int \frac{dt}{t^2+1} = \operatorname{arctg} t + c = \operatorname{arctg}(x+2) + c, \; x \in R, \; c \in R.$$

$$\int \frac{dx}{x^2+4x+3}$$

$$= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[ \begin{array}{r|l} \text{Subst. } t = x+2 & x \in R - \{-1, -3\} \\ dt = dx & t \in R - \{\pm 1\} \end{array} \right]$$

$$= \int \frac{dt}{t^2-1} = \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c = \frac{1}{2} \ln \left| \frac{x+2-1}{x+2+1} \right| + c$$

# Riešené príklady – 101, 102

$$\int \frac{dx}{x^2+4x+5} = \operatorname{arctg}(x+2) + c$$

$$= \int \frac{dx}{x^2+4x+4+1} = \int \frac{dx}{(x+2)^2+1} = \left[\begin{array}{r|l} \text{Subst. } t=x+2 & x\in R \\ dt=dx & t\in R \end{array}\right]$$

$$= \int \frac{dt}{t^2+1} = \operatorname{arctg} t + c = \operatorname{arctg}(x+2) + c,\ x\in R,\ c\in R.$$

$$\int \frac{dx}{x^2+4x+3} = \frac{1}{2} \ln\left|\frac{x+1}{x+3}\right| + c$$

$$= \int \frac{dx}{x^2+4x+4-1} = \int \frac{dx}{(x+2)^2-1} = \left[\begin{array}{r|l} \text{Subst. } t=x+2 & x\in R-\{-1,-3\} \\ dt=dx & t\in R-\{\pm 1\} \end{array}\right]$$

$$= \int \frac{dt}{t^2-1} = \frac{1}{2} \ln\left|\frac{t-1}{t+1}\right| + c = \frac{1}{2} \ln\left|\frac{x+2-1}{x+2+1}\right| + c = \frac{1}{2} \ln\left|\frac{x+1}{x+3}\right| + c,$$

$$x\in R-\{-1,-3\},\ c\in R.$$

# Riešené príklady – 103, 104

$$\int \frac{dx}{x^2-4x+6}$$

$$\int \frac{dx}{x^2-4x+2}$$

# Riešené príklady – 103, 104

$$\int \frac{dx}{x^2-4x+6}$$

$$= \int \frac{dx}{x^2-4x+4+2} = \int \frac{dx}{(x-2)^2+(\sqrt{2})^2}$$

$$\int \frac{dx}{x^2-4x+2}$$

$$= \int \frac{dx}{x^2-4x+4-2} = \int \frac{dx}{(x-2)^2-(\sqrt{2})^2}$$

# Riešené príklady – 103, 104

$$\int \frac{dx}{x^2-4x+6}$$

$$= \int \frac{dx}{x^2-4x+4+2} = \int \frac{dx}{(x-2)^2+(\sqrt{2})^2} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in R \\ dt=dx & t\in R \end{array} \right]$$

$$\int \frac{dx}{x^2-4x+2}$$

$$= \int \frac{dx}{x^2-4x+4-2} = \int \frac{dx}{(x-2)^2-(\sqrt{2})^2} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in R-\{2\pm\sqrt{2}\} \\ dt=dx & t\in R-\{\pm\sqrt{2}\} \end{array} \right]$$

# Riešené príklady – 103, 104

$$\int \frac{dx}{x^2-4x+6}$$

$$= \int \frac{dx}{x^2-4x+4+2} = \int \frac{dx}{(x-2)^2+(\sqrt{2})^2} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in R \\ dt=dx & t\in R \end{array} \right]$$

$$= \int \frac{dt}{t^2+(\sqrt{2})^2}$$

$$\int \frac{dx}{x^2-4x+2}$$

$$= \int \frac{dx}{x^2-4x+4-2} = \int \frac{dx}{(x-2)^2-(\sqrt{2})^2} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in R-\{2\pm\sqrt{2}\} \\ dt=dx & t\in R-\{\pm\sqrt{2}\} \end{array} \right]$$

$$= \int \frac{dt}{t^2-(\sqrt{2})^2}$$

# Riešené príklady – 103, 104

$$\int \frac{dx}{x^2-4x+6}$$

$$= \int \frac{dx}{x^2-4x+4+2} = \int \frac{dx}{(x-2)^2+(\sqrt{2})^2} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in R \\ dt=dx & t\in R \end{array} \right]$$

$$= \int \frac{dt}{t^2+(\sqrt{2})^2} = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + c$$

$$\int \frac{dx}{x^2-4x+2}$$

$$= \int \frac{dx}{x^2-4x+4-2} = \int \frac{dx}{(x-2)^2-(\sqrt{2})^2} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in R-\{2\pm\sqrt{2}\} \\ dt=dx & t\in R-\{\pm\sqrt{2}\} \end{array} \right]$$

$$= \int \frac{dt}{t^2-(\sqrt{2})^2} = \frac{1}{2\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c$$

# Riešené príklady – 103, 104

$$\int \frac{dx}{x^2-4x+6} = \operatorname{arctg}(x+2) + c$$

$$= \int \frac{dx}{x^2-4x+4+2} = \int \frac{dx}{(x-2)^2+(\sqrt{2})^2} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in R \\ dt=dx & t\in R \end{array} \right]$$

$$= \int \frac{dt}{t^2+(\sqrt{2})^2} = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + c = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x-2}{\sqrt{2}} + c,\ x\in R,\ c\in R.$$

$$\int \frac{dx}{x^2-4x+2} = \frac{1}{2} \ln \left| \frac{x-2-\sqrt{2}}{x-2+\sqrt{2}} \right| + c$$

$$= \int \frac{dx}{x^2-4x+4-2} = \int \frac{dx}{(x-2)^2-(\sqrt{2})^2} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in R-\{2\pm\sqrt{2}\} \\ dt=dx & t\in R-\{\pm\sqrt{2}\} \end{array} \right]$$

$$= \int \frac{dt}{t^2-(\sqrt{2})^2} = \frac{1}{2\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c = \frac{1}{2\sqrt{2}} \ln \left| \frac{x-2-\sqrt{2}}{x-2+\sqrt{2}} \right| + c,$$

$$x\in R-\left\{2\pm\sqrt{2}\right\},\ c\in R.$$

# Riešené príklady – 105, 106

$\int \frac{dx}{x^2+4x+4}$

$\int \frac{dx}{\sqrt{x^2+4x+4}}$

# Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4}$$

$$=\int \frac{dx}{(x+2)^2}$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}}$$

$$=\int \frac{dx}{\sqrt{(x+2)^2}}=\int \frac{dx}{|x+2|}$$

# Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4}$$

$$= \int \frac{dx}{(x+2)^2} = \left[ \begin{array}{r|l} \text{Subst. } t = x+2 & x \in R - \{-2\} \\ dt = dx & t \in R - \{0\} \end{array} \right]$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}}$$

$$= \int \frac{dx}{\sqrt{(x+2)^2}} = \int \frac{dx}{|x+2|} = \left[ \begin{array}{r|l} \text{Subst. } t = x+2 & x \in R - \{-2\} \\ dt = dx & t \in R - \{0\} \end{array} \right]$$

# Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4}$$

$$= \int \frac{dx}{(x+2)^2} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R-\{-2\} \\ dt=dx & t\in R-\{0\} \end{array} \right]$$

$$= \int \frac{dt}{t^2} = \int t^{-2}\, dt$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}}$$

$$= \int \frac{dx}{\sqrt{(x+2)^2}} = \int \frac{dx}{|x+2|} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R-\{-2\} \\ dt=dx & t\in R-\{0\} \end{array} \right] = \int \frac{dt}{|t|}$$

# Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4}$$

$$= \int \frac{dx}{(x+2)^2} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R-\{-2\} \\ dt=dx & t\in R-\{0\} \end{array} \right]$$

$$= \int \frac{dt}{t^2} = \int t^{-2}\, dt = \frac{t^{-2+1}}{-2+1} + c$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}}$$

$$= \int \frac{dx}{\sqrt{(x+2)^2}} = \int \frac{dx}{|x+2|} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R-\{-2\} \\ dt=dx & t\in R-\{0\} \end{array} \right] = \int \frac{dt}{|t|}$$

$t>0$ $$= \int \frac{dt}{t}$$

$t<0$ $$= \int \frac{dt}{-t} = -\int \frac{dt}{t}$$

# Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4}$$

$$= \int \frac{dx}{(x+2)^2} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R-\{-2\} \\ dt=dx & t\in R-\{0\} \end{array} \right]$$

$$= \int \frac{dt}{t^2} = \int t^{-2}\, dt = \frac{t^{-2+1}}{-2+1} + c = -\frac{1}{t} + c$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}}$$

$$= \int \frac{dx}{\sqrt{(x+2)^2}} = \int \frac{dx}{|x+2|} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R-\{-2\} \\ dt=dx & t\in R-\{0\} \end{array} \right] = \int \frac{dt}{|t|}$$

$t>0$ $$= \int \frac{dt}{t} = \ln|t| + c$$

$t<0$ $$= \int \frac{dt}{-t} = -\int \frac{dt}{t} = -\ln|t| + c$$

# Riešené príklady – 105, 106

$$\int \frac{dx}{x^2+4x+4} = -\frac{1}{x+2} + c$$

$$= \int \frac{dx}{(x+2)^2} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R-\{-2\} \\ dt=dx & t\in R-\{0\} \end{array} \right]$$

$$= \int \frac{dt}{t^2} = \int t^{-2}\,dt = \frac{t^{-2+1}}{-2+1} + c = -\frac{1}{t} + c = -\frac{1}{x+2} + c,\ x\in R-\{-2\},\ c\in R.$$

$$\int \frac{dx}{\sqrt{x^2+4x+4}} = \operatorname{sgn}(x+2)\cdot \ln|x+2| + c$$

$$= \int \frac{dx}{\sqrt{(x+2)^2}} = \int \frac{dx}{|x+2|} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R-\{-2\} \\ dt=dx & t\in R-\{0\} \end{array} \right] = \int \frac{dt}{|t|}$$

$t>0$: $$= \int \frac{dt}{t} = \ln|t| + c = \ln|x+2| + c,\ x\in(-2;\infty),\ c\in R,$$

$t<0$: $$= \int \frac{dt}{-t} = -\int \frac{dt}{t} = -\ln|t| + c = -\ln|x+2| + c,\ x\in(-\infty;-2),\ c\in R.$$

# Riešené príklady – 107, 108

$$\int \frac{\mathrm{d}x}{\sqrt{x^2+4x+5}}$$

$$\int \frac{\mathrm{d}x}{\sqrt{x^2+4x+3}}$$

# Riešené príklady – 107, 108

$$\int \frac{dx}{\sqrt{x^2+4x+5}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4+1}} = \int \frac{dx}{\sqrt{(x+2)^2+1}}$$

$$\int \frac{dx}{\sqrt{x^2+4x+3}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4-1}} = \int \frac{dx}{\sqrt{(x+2)^2-1}}$$

# Riešené príklady – 107, 108

$$\int \frac{dx}{\sqrt{x^2+4x+5}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4+1}} = \int \frac{dx}{\sqrt{(x+2)^2+1}} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R \\ dt=dx & t\in R \end{array} \right]$$

$$\int \frac{dx}{\sqrt{x^2+4x+3}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4-1}} = \int \frac{dx}{\sqrt{(x+2)^2-1}} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in(-\infty;-3)\,,\ t\in(-\infty;-1) \\ dt=dx & x\in(-1;\infty)\,,\ t\in(1;\infty) \end{array} \right]$$

# Riešené príklady – 107, 108

$$\int \frac{dx}{\sqrt{x^2+4x+5}}$$

$$=\int \frac{dx}{\sqrt{x^2+4x+4+1}}=\int \frac{dx}{\sqrt{(x+2)^2+1}}=\left[\begin{array}{r|l}\text{Subst. } t=x+2 & x\in R \\ dt=dx & t\in R\end{array}\right]$$

$$=\int \frac{dt}{\sqrt{t^2+1}}$$

$$\int \frac{dx}{\sqrt{x^2+4x+3}}$$

$$=\int \frac{dx}{\sqrt{x^2+4x+4-1}}=\int \frac{dx}{\sqrt{(x+2)^2-1}}=\left[\begin{array}{r|l}\text{Subst. } t=x+2 & x\in(-\infty;-3),\ t\in(-\infty;-1) \\ dt=dx & x\in(-1;\infty),\ t\in(1;\infty)\end{array}\right]$$

$$=\int \frac{dt}{\sqrt{t^2-1}}$$

# Riešené príklady – 107, 108

$$\int \frac{dx}{\sqrt{x^2+4x+5}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4+1}} = \int \frac{dx}{\sqrt{(x+2)^2+1}} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in R \\ dt=dx & t\in R \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2+1}} = \ln\left(t+\sqrt{t^2+1}\right)+c$$

$$\int \frac{dx}{\sqrt{x^2+4x+3}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4-1}} = \int \frac{dx}{\sqrt{(x+2)^2-1}} = \left[ \begin{array}{r|l} \text{Subst. } t=x+2 & x\in(-\infty;-3),\ t\in(-\infty;-1) \\ dt=dx & x\in(-1;\infty),\ t\in(1;\infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2-1}} = \ln\left|t+\sqrt{t^2-1}\right|+c$$

# Riešené príklady – 107, 108

$$\int\frac{dx}{\sqrt{x^2+4x+5}}$$

$$=\int\frac{dx}{\sqrt{x^2+4x+4+1}}=\int\frac{dx}{\sqrt{(x+2)^2+1}}=\left[\begin{array}{r|l}\text{Subst. } t=x+2 & x\in R\\ dt=dx & t\in R\end{array}\right]$$

$$=\int\frac{dt}{\sqrt{t^2+1}}=\ln\left(t+\sqrt{t^2+1}\right)+c=\ln\left(x+2+\sqrt{(x+2)^2+1}\right)+c$$

$$\int\frac{dx}{\sqrt{x^2+4x+3}}$$

$$=\int\frac{dx}{\sqrt{x^2+4x+4-1}}=\int\frac{dx}{\sqrt{(x+2)^2-1}}=\left[\begin{array}{r|l}\text{Subst. } t=x+2 & x\in(-\infty;-3)\,,\ t\in(-\infty;-1)\\ dt=dx & x\in(-1;\infty)\,,\ t\in(1;\infty)\end{array}\right]$$

$$=\int\frac{dt}{\sqrt{t^2-1}}=\ln\left|t+\sqrt{t^2-1}\right|+c=\ln\left|x+2+\sqrt{(x+2)^2-1}\right|+c$$

# Riešené príklady – 107, 108

$$\int \frac{dx}{\sqrt{x^2+4x+5}} = \ln\left(x+2+\sqrt{x^2+4x+5}\right)+c$$

$$=\int \frac{dx}{\sqrt{x^2+4x+4+1}} = \int \frac{dx}{\sqrt{(x+2)^2+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x+2 & x\in R \\ dt=dx & t\in R \end{array}\right]$$

$$=\int \frac{dt}{\sqrt{t^2+1}} = \ln\left(t+\sqrt{t^2+1}\right)+c = \ln\left(x+2+\sqrt{(x+2)^2+1}\right)+c$$

$$=\ln\left(x+2+\sqrt{x^2+4x+5}\right)+c,\ x\in R,\ c\in R.$$

$$\int \frac{dx}{\sqrt{x^2+4x+3}} = \ln\left|x+2+\sqrt{x^2+4x+3}\right|+c$$

$$=\int \frac{dx}{\sqrt{x^2+4x+4-1}} = \int \frac{dx}{\sqrt{(x+2)^2-1}} = \left[\begin{array}{r|l} \text{Subst. } t=x+2 & x\in(-\infty;-3),\ t\in(-\infty;-1) \\ dt=dx & x\in(-1;\infty),\ t\in(1;\infty) \end{array}\right]$$

$$=\int \frac{dt}{\sqrt{t^2-1}} = \ln\left|t+\sqrt{t^2-1}\right|+c = \ln\left|x+2+\sqrt{(x+2)^2-1}\right|+c$$

$$=\ln\left|x+2+\sqrt{x^2+4x+3}\right|+c,\ x\in(-\infty;-3)\cup(-1;\infty),\ c\in R.$$

# Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2-4x+6}}$$

$$\int \frac{dx}{\sqrt{x^2-4x+2}}$$

# Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2-4x+6}}$$

$$= \int \frac{dx}{\sqrt{x^2-4x+4+2}} = \int \frac{dx}{\sqrt{(x-2)^2+(\sqrt{2})^2}}$$

$$\int \frac{dx}{\sqrt{x^2-4x+2}}$$

$$= \int \frac{dx}{\sqrt{x^2-4x+4-2}} = \int \frac{dx}{\sqrt{(x-2)^2-(\sqrt{2})^2}}$$

# Riešené príklady – 109, 110

$$\int \frac{\mathrm{d}x}{\sqrt{x^2-4x+6}}$$

$$= \int \frac{\mathrm{d}x}{\sqrt{x^2-4x+4+2}} = \int \frac{\mathrm{d}x}{\sqrt{(x-2)^2+(\sqrt{2})^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in R \\ \mathrm{d}t=\mathrm{d}x & t\in R \end{array} \right]$$

$$\int \frac{\mathrm{d}x}{\sqrt{x^2-4x+2}}$$

$$= \int \frac{\mathrm{d}x}{\sqrt{x^2-4x+4-2}} = \int \frac{\mathrm{d}x}{\sqrt{(x-2)^2-(\sqrt{2})^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in(-\infty;2-\sqrt{2}),\ t\in(-\infty;-\sqrt{2}) \\ \mathrm{d}t=\mathrm{d}x & x\in(2+\sqrt{2};\infty),\ t\in(\sqrt{2};\infty) \end{array} \right]$$

# Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2-4x+6}}$$

$$= \int \frac{dx}{\sqrt{x^2-4x+4+2}} = \int \frac{dx}{\sqrt{(x-2)^2+(\sqrt{2})^2}} = \left[\begin{array}{r|l} \text{Subst. } t=x-2 & x\in R \\ dt=dx & t\in R \end{array}\right]$$

$$= \int \frac{dt}{\sqrt{t^2+(\sqrt{2})^2}}$$

$$\int \frac{dx}{\sqrt{x^2-4x+2}}$$

$$= \int \frac{dx}{\sqrt{x^2-4x+4-2}} = \int \frac{dx}{\sqrt{(x-2)^2-(\sqrt{2})^2}} = \left[\begin{array}{r|l} \text{Subst. } t=x-2 & x\in(-\infty;2-\sqrt{2}),\ t\in(-\infty;-\sqrt{2}) \\ dt=dx & x\in(2+\sqrt{2};\infty),\ t\in(\sqrt{2};\infty) \end{array}\right]$$

$$= \int \frac{dt}{\sqrt{t^2-(\sqrt{2})^2}}$$

# Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2-4x+6}}$$

$$= \int \frac{dx}{\sqrt{x^2-4x+4+2}} = \int \frac{dx}{\sqrt{(x-2)^2+(\sqrt{2})^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in R \\ dt=dx & t\in R \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2+(\sqrt{2})^2}} = \ln\left(t+\sqrt{t^2+(\sqrt{2})^2}\right)+c$$

$$\int \frac{dx}{\sqrt{x^2-4x+2}}$$

$$= \int \frac{dx}{\sqrt{x^2-4x+4-2}} = \int \frac{dx}{\sqrt{(x-2)^2-(\sqrt{2})^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in(-\infty;2-\sqrt{2}),\ t\in(-\infty;-\sqrt{2}) \\ dt=dx & x\in(2+\sqrt{2};\infty),\ t\in(\sqrt{2};\infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2-(\sqrt{2})^2}} = \ln\left|t+\sqrt{t^2-(\sqrt{2})^2}\right|+c$$

# Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2-4x+6}}$$

$$= \int \frac{dx}{\sqrt{x^2-4x+4+2}} = \int \frac{dx}{\sqrt{(x-2)^2+(\sqrt{2})^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in R \\ dt=dx & t\in R \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2+(\sqrt{2})^2}} = \ln\left(t+\sqrt{t^2+(\sqrt{2})^2}\right)+c = \ln\left(x+2+\sqrt{(x-2)^2+4}\right)+c$$

$$\int \frac{dx}{\sqrt{x^2-4x+2}}$$

$$= \int \frac{dx}{\sqrt{x^2-4x+4-2}} = \int \frac{dx}{\sqrt{(x-2)^2-(\sqrt{2})^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in(-\infty;2-\sqrt{2}),\ t\in(-\infty;-\sqrt{2}) \\ dt=dx & x\in(2+\sqrt{2};\infty),\ t\in(\sqrt{2};\infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2-(\sqrt{2})^2}} = \ln\left|t+\sqrt{t^2-(\sqrt{2})^2}\right|+c = \ln\left|x-2+\sqrt{(x+2)^2-2}\right|+c$$

# Riešené príklady – 109, 110

$$\int \frac{dx}{\sqrt{x^2-4x+6}} = \ln\left(x-2+\sqrt{x^2-4x+6}\right)+c$$

$$= \int \frac{dx}{\sqrt{x^2-4x+4+2}} = \int \frac{dx}{\sqrt{(x-2)^2+(\sqrt{2})^2}} = \left[\begin{array}{r|l} \text{Subst. } t=x-2 & x\in R \\ dt=dx & t\in R \end{array}\right]$$

$$= \int \frac{dt}{\sqrt{t^2+(\sqrt{2})^2}} = \ln\left(t+\sqrt{t^2+(\sqrt{2})^2}\right)+c = \ln\left(x+2+\sqrt{(x-2)^2+4}\right)+c$$

$$= \ln\left(x-2+\sqrt{x^2-4x+6}\right)+c,\ x\in R,\ c\in R.$$

$$\int \frac{dx}{\sqrt{x^2-4x+2}} = \ln\left|x-2+\sqrt{x^2-4x+2}\right|+c$$

$$= \int \frac{dx}{\sqrt{x^2-4x+4-2}} = \int \frac{dx}{\sqrt{(x-2)^2-(\sqrt{2})^2}} = \left[\begin{array}{r|l} \text{Subst. } t=x-2 & x\in(-\infty;2-\sqrt{2}),\ t\in(-\infty;-\sqrt{2}) \\ dt=dx & x\in(2+\sqrt{2};\infty),\ t\in(\sqrt{2};\infty) \end{array}\right]$$

$$= \int \frac{dt}{\sqrt{t^2-(\sqrt{2})^2}} = \ln\left|t+\sqrt{t^2-(\sqrt{2})^2}\right|+c = \ln\left|x-2+\sqrt{(x+2)^2-2}\right|+c$$

$$= \ln\left|x-2+\sqrt{x^2-4x+2}\right|+c,\ x\in(-\infty;2-\sqrt{2})\cup(2+\sqrt{2};\infty),\ c\in R.$$

# Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}}$$

$$\int \frac{dx}{\sqrt{-x^2+4x-3}}$$

# Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+5)}}$$

$$\int \frac{dx}{\sqrt{-x^2+4x-3}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+3)}}$$

# Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+5)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4+1)}}$$

$$\int \frac{dx}{\sqrt{-x^2+4x-3}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+3)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4-1)}}$$

# Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+5)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4+1)}} = \int \frac{dx}{\sqrt{-(x-2)^2-1}}$$

$$\int \frac{dx}{\sqrt{-x^2+4x-3}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-4x+3)}} = \int \frac{dx}{\sqrt{-(x^2-4x+4-1)}} = \int \frac{dx}{\sqrt{-(x-2)^2+1}} = \int \frac{dx}{\sqrt{1-(x-2)^2}}$$

# Riešené príklady – 111, 112

$$\int \frac{dx}{\sqrt{-x^2+4x-5}}$$

$$=\int \frac{dx}{\sqrt{-(x^2-4x+5)}}=\int \frac{dx}{\sqrt{-(x^2-4x+4+1)}}=\int \frac{dx}{\sqrt{-(x-2)^2-1}}=\left[\begin{array}{c}-(x-2)^2-1\geq 1>0\\ \text{pre všetky } x\in R\end{array}\right]$$

$$\int \frac{dx}{\sqrt{-x^2+4x-3}}$$

$$=\int \frac{dx}{\sqrt{-(x^2-4x+3)}}=\int \frac{dx}{\sqrt{-(x^2-4x+4-1)}}=\int \frac{dx}{\sqrt{-(x-2)^2+1}}=\int \frac{dx}{\sqrt{1-(x-2)^2}}$$

$$=\left[\begin{array}{r|l}\text{Subst. } t=x-2 & x\in(1;3)\\ dt=dx & t\in(-1;1)\end{array}\right]$$

# Riešené príklady – 111, 112

$\int \frac{dx}{\sqrt{-x^2+4x-5}}$ nemá riešenie

$$=\int \frac{dx}{\sqrt{-(x^2-4x+5)}}=\int \frac{dx}{\sqrt{-(x^2-4x+4+1)}}=\int \frac{dx}{\sqrt{-(x-2)^2-1}}=\left[\begin{matrix} -(x-2)^2-1\geq 1>0 \\ \text{pre všetky } x\in R \end{matrix}\right].$$

Neexistuje riešenie pre žiadne $x\in R$.

$\int \frac{dx}{\sqrt{-x^2+4x-3}}$

$$=\int \frac{dx}{\sqrt{-(x^2-4x+3)}}=\int \frac{dx}{\sqrt{-(x^2-4x+4-1)}}=\int \frac{dx}{\sqrt{-(x-2)^2+1}}=\int \frac{dx}{\sqrt{1-(x-2)^2}}$$

$$=\left[\begin{matrix} \text{Subst. } t=x-2 \\ dt=dx \end{matrix}\middle| \begin{matrix} x\in(1;3) \\ t\in(-1;1) \end{matrix}\right]=\int \frac{dt}{\sqrt{1-t^2}}$$

# Riešené príklady – 111, 112

$\int \frac{dx}{\sqrt{-x^2+4x-5}}$ nemá riešenie

$$=\int \frac{dx}{\sqrt{-(x^2-4x+5)}}=\int \frac{dx}{\sqrt{-(x^2-4x+4+1)}}=\int \frac{dx}{\sqrt{-(x-2)^2-1}}=\left[\begin{matrix} -(x-2)^2-1\geq 1>0 \\ \text{pre všetky } x\in R \end{matrix}\right].$$

Neexistuje riešenie pre žiadne $x\in R$.

$\int \frac{dx}{\sqrt{-x^2+4x-3}}$

$$=\int \frac{dx}{\sqrt{-(x^2-4x+3)}}=\int \frac{dx}{\sqrt{-(x^2-4x+4-1)}}=\int \frac{dx}{\sqrt{-(x-2)^2+1}}=\int \frac{dx}{\sqrt{1-(x-2)^2}}$$

$$=\left[\begin{matrix} \text{Subst. } t=x-2 \\ dt=dx \end{matrix}\,\middle|\,\begin{matrix} x\in(1;3) \\ t\in(-1;1) \end{matrix}\right]=\int \frac{dt}{\sqrt{1-t^2}}$$

$$=\arcsin t+c_1=-\arccos t+c_2$$

# Riešené príklady – 111, 112

$\int \frac{dx}{\sqrt{-x^2+4x-5}}$ nemá riešenie

$$=\int \frac{dx}{\sqrt{-(x^2-4x+5)}}=\int \frac{dx}{\sqrt{-(x^2-4x+4+1)}}=\int \frac{dx}{\sqrt{-(x-2)^2-1}}=\left[\begin{matrix}-(x-2)^2-1\geq 1>0\\ \text{pre všetky } x\in R\end{matrix}\right].$$

Neexistuje riešenie pre žiadne $x\in R$.

$\int \frac{dx}{\sqrt{-x^2+4x-3}} = \arcsin(x-2)+c_1 = -\arccos(x-2)+c_2$

$$=\int \frac{dx}{\sqrt{-(x^2-4x+3)}}=\int \frac{dx}{\sqrt{-(x^2-4x+4-1)}}=\int \frac{dx}{\sqrt{-(x-2)^2+1}}=\int \frac{dx}{\sqrt{1-(x-2)^2}}$$

$$=\left[\begin{matrix}\text{Subst. } t=x-2 \\ dt=dx\end{matrix}\,\middle|\,\begin{matrix}x\in(1;3)\\ t\in(-1;1)\end{matrix}\right]=\int \frac{dt}{\sqrt{1-t^2}}$$

$$=\arcsin t+c_1=-\arccos t+c_2$$

$$=\arcsin(x-2)+c_1=-\arccos(x-2)+c_2,\ x\in(1;3),\ c_1,\,c_2\in R.$$

# Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}}$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}}$$

# Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}}$$

$$= \int \frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}} = \int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}}$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}}$$

# Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}}$$

$$= \int \frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}} = \int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-\frac{1}{2} & x\in(-\infty;0),\ t\in(-\infty;-\frac{1}{2}) \\ dt=dx & x\in(1;\infty),\ t\in(\frac{1}{2};\infty) \end{array} \right]$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}}$$

# Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}}$$

$$= \int \frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}} = \int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-\frac{1}{2} & x\in(-\infty;0),\ t\in(-\infty;-\frac{1}{2}) \\ dt=dx & x\in(1;\infty),\ t\in(\frac{1}{2};\infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2-(\frac{1}{2})^2}}$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-1 & x\in(0;2) \\ dt=dx & t\in(-1;1) \end{array} \right]$$

# Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}}$$

$$= \int \frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}} = \int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-\frac{1}{2} & x\in(-\infty;0)\,,\ t\in(-\infty;-\frac{1}{2}) \\ dt=dx & x\in(1;\infty)\,,\ t\in(\frac{1}{2};\infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2-(\frac{1}{2})^2}} = \ln\left|t+\sqrt{t^2-\left(\frac{1}{2}\right)^2}\right| + c_1$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-1 & x\in(0;2) \\ dt=dx & t\in(-1;1) \end{array} \right] = \int \frac{dt}{\sqrt{1-t^2}}$$

# Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}}$$

$$= \int \frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}} = \int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-\frac{1}{2} & x\in(-\infty;0),\ t\in(-\infty;-\frac{1}{2}) \\ dt=dx & x\in(1;\infty),\ t\in(\frac{1}{2};\infty) \end{array} \right]$$

$$= \int \frac{dt}{\sqrt{t^2-(\frac{1}{2})^2}} = \ln\left|t+\sqrt{t^2-\left(\tfrac{1}{2}\right)^2}\right|+c_1 = \ln\left|x-\frac{1}{2}+\sqrt{x^2-x}\right|+c_1$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=x-1 & x\in(0;2) \\ dt=dx & t\in(-1;1) \end{array} \right] = \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \arcsin t + c_1 = -\arccos t + c_2$$

# Riešené príklady – 113, 114

$$\int\frac{dx}{\sqrt{x(x-1)}}=\int\frac{dx}{\sqrt{x^2-x}}=\ln\left|x-\tfrac{1}{2}+\sqrt{x^2-x}\right|+c_1$$

$$=\int\frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac14-\frac14}}=\int\frac{dx}{\sqrt{(x-\frac12)^2-(\frac12)^2}}=\left[\begin{array}{r|l}\text{Subst. } t=x-\frac12 & x\in(-\infty;0),\ t\in(-\infty;-\frac12)\\ dt=dx & x\in(1;\infty),\ t\in(\frac12;\infty)\end{array}\right]$$

$$=\int\frac{dt}{\sqrt{t^2-(\frac12)^2}}=\ln\left|t+\sqrt{t^2-\left(\tfrac12\right)^2}\right|+c_1=\ln\left|x-\tfrac12+\sqrt{x^2-x}\right|+c_1$$

$$=\ln\left|\frac{2x-1+\sqrt{x^2-x}}{2}\right|+c_1$$

$$\int\frac{dx}{\sqrt{x(2-x)}}=\int\frac{dx}{\sqrt{-x^2+2x}}=\arcsin(x-1)+c_1=-\arccos(x-1)+c_2$$

$$=\int\frac{dx}{\sqrt{-(x^2-2x+1)+1}}=\int\frac{dx}{\sqrt{1-(x-1)^2}}=\left[\begin{array}{r|l}\text{Subst. } t=x-1 & x\in(0;2)\\ dt=dx & t\in(-1;1)\end{array}\right]=\int\frac{dt}{\sqrt{1-t^2}}$$

$$=\arcsin t+c_1=-\arccos t+c_2=\arcsin(x-1)+c_1=-\arccos(x-1)+c_2,$$

$$x\in(0;2),\ c_1,c_2\in R.$$

# Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}} = \ln\left|x-\tfrac{1}{2}+\sqrt{x^2-x}\right|+c_1$$

$$=\int \frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}}=\int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}}=\left[\begin{array}{r|l}\text{Subst. } t=x-\frac{1}{2} & x\in(-\infty;0),\ t\in(-\infty;-\frac{1}{2})\\ dt=dx & x\in(1;\infty),\ t\in(\frac{1}{2};\infty)\end{array}\right]$$

$$=\int \frac{dt}{\sqrt{t^2-(\frac{1}{2})^2}}=\ln\left|t+\sqrt{t^2-\left(\tfrac{1}{2}\right)^2}\right|+c_1=\ln\left|x-\tfrac{1}{2}+\sqrt{x^2-x}\right|+c_1$$

$$=\ln\left|\frac{2x-1+\sqrt{x^2-x}}{2}\right|+c_1=\ln\left|2x-1+\sqrt{x^2-x}\right|-\ln 2+c_1$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}} = \arcsin(x-1)+c_1 = -\arccos(x-1)+c_2$$

$$=\int \frac{dx}{\sqrt{-(x^2-2x+1)+1}}=\int \frac{dx}{\sqrt{1-(x-1)^2}}=\left[\begin{array}{r|l}\text{Subst. } t=x-1 & x\in(0;2)\\ dt=dx & t\in(-1;1)\end{array}\right]=\int \frac{dt}{\sqrt{1-t^2}}$$

$$=\arcsin t+c_1=-\arccos t+c_2=\arcsin(x-1)+c_1=-\arccos(x-1)+c_2,$$

$$x\in(0;2),\ c_1,c_2\in R.$$

# Riešené príklady – 113, 114

$$\int\frac{dx}{\sqrt{x(x-1)}}=\int\frac{dx}{\sqrt{x^2-x}}=\ln\left|x-\tfrac{1}{2}+\sqrt{x^2-x}\right|+c_1$$

$$=\int\frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}}=\int\frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}}=\left[\begin{array}{r|l}\text{Subst. } t=x-\frac{1}{2} & x\in(-\infty;0),\ t\in(-\infty;-\frac{1}{2})\\ dt=dx & x\in(1;\infty),\ t\in(\frac{1}{2};\infty)\end{array}\right]$$

$$=\int\frac{dt}{\sqrt{t^2-(\frac{1}{2})^2}}=\ln\left|t+\sqrt{t^2-\left(\tfrac{1}{2}\right)^2}\right|+c_1=\ln\left|x-\tfrac{1}{2}+\sqrt{x^2-x}\right|+c_1$$

$$=\ln\left|\frac{2x-1+\sqrt{x^2-x}}{2}\right|+c_1=\ln\left|2x-1+\sqrt{x^2-x}\right|-\ln 2+c_1=\left[\begin{array}{c}c_2=c_1-\ln 2\\ c_1\in R,\ c_2\in R\end{array}\right]$$

$$\int\frac{dx}{\sqrt{x(2-x)}}=\int\frac{dx}{\sqrt{-x^2+2x}}=\arcsin(x-1)+c_1=-\arccos(x-1)+c_2$$

$$=\int\frac{dx}{\sqrt{-(x^2-2x+1)+1}}=\int\frac{dx}{\sqrt{1-(x-1)^2}}=\left[\begin{array}{r|l}\text{Subst. } t=x-1 & x\in(0;2)\\ dt=dx & t\in(-1;1)\end{array}\right]=\int\frac{dt}{\sqrt{1-t^2}}$$

$$=\arcsin t+c_1=-\arccos t+c_2=\arcsin(x-1)+c_1=-\arccos(x-1)+c_2,$$

$$x\in(0;2),\ c_1,c_2\in R.$$

# Riešené príklady – 113, 114

$$\int \frac{dx}{\sqrt{x(x-1)}} = \int \frac{dx}{\sqrt{x^2-x}} = \ln\left|x-\tfrac{1}{2}+\sqrt{x^2-x}\right|+c_1 = \ln\left|2x-1+\sqrt{x^2-x}\right|+c_2$$

$$= \int \frac{dx}{\sqrt{x^2-2\frac{x}{2}+\frac{1}{4}-\frac{1}{4}}} = \int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}} = \left[\begin{array}{c|c} \text{Subst. } t=x-\frac{1}{2} & x\in(-\infty;0),\ t\in(-\infty;-\frac{1}{2}) \\ dt=dx & x\in(1;\infty),\ t\in(\frac{1}{2};\infty) \end{array}\right]$$

$$= \int \frac{dt}{\sqrt{t^2-(\frac{1}{2})^2}} = \ln\left|t+\sqrt{t^2-(\tfrac{1}{2})^2}\right|+c_1 = \ln\left|x-\tfrac{1}{2}+\sqrt{x^2-x}\right|+c_1$$

$$= \ln\left|\frac{2x-1+\sqrt{x^2-x}}{2}\right|+c_1 = \ln\left|2x-1+\sqrt{x^2-x}\right|-\ln 2+c_1 = \left[\begin{array}{c} c_2=c_1-\ln 2 \\ c_1\in R,\ c_2\in R \end{array}\right]$$

$$= \ln\left|2x-1+\sqrt{x^2-x}\right|+c_2,\ x\in(-\infty;0)\cup(1;\infty),\ c\in R.$$

$$\int \frac{dx}{\sqrt{x(2-x)}} = \int \frac{dx}{\sqrt{-x^2+2x}} = \arcsin(x-1)+c_1 = -\arccos(x-1)+c_2$$

$$= \int \frac{dx}{\sqrt{-(x^2-2x+1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} = \left[\begin{array}{c|c} \text{Subst. } t=x-1 & x\in(0;2) \\ dt=dx & t\in(-1;1) \end{array}\right] = \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \arcsin t+c_1 = -\arccos t+c_2 = \arcsin(x-1)+c_1 = -\arccos(x-1)+c_2,$$

$$x\in(0;2),\ c_1,c_2\in R.$$

# Riešené príklady – 115, 116

$$\int \sqrt{x^2+4x+5}\,dx$$

$$\int \sqrt{x^2+4x+3}\,dx$$

# Riešené príklady – 115, 116

$$\int \sqrt{x^2+4x+5}\, dx$$

$$=\left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array}\right]$$

$$\int \sqrt{x^2+4x+3}\, dx$$

$$=\left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in(-\infty;-3)\,,\ t\in(-\infty;-1) \\ dt=dx & =(x+2)^2-1=t^2-1 & x\in(-1;\infty)\,,\ t\in(1;\infty) \end{array}\right]$$

# Riešené príklady – 115, 116

$$\int \sqrt{x^2+4x+5}\,\mathrm{d}x$$

$$=\left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ \mathrm{d}t=\mathrm{d}x & =(x+2)^2+1=t^2+1 & t\in R \end{array}\right]=\int \sqrt{t^2+1}\,\mathrm{d}t$$

$$\int \sqrt{x^2+4x+3}\,\mathrm{d}x$$

$$=\left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in(-\infty;-3)\,,\ t\in(-\infty;-1) \\ \mathrm{d}t=\mathrm{d}x & =(x+2)^2-1=t^2-1 & x\in(-1;\infty)\,,\ t\in(1;\infty) \end{array}\right]=\int \sqrt{t^2-1}\,\mathrm{d}t$$

# Riešené príklady – 115, 116

$$\int\sqrt{x^2+4x+5}\,dx$$

$$=\left[\begin{array}{r|r|l}\text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R\\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R\end{array}\right]=\int\sqrt{t^2+1}\,dt=\frac{t\sqrt{t^2+1}}{2}+\frac{1}{2}\int\frac{dt}{\sqrt{t^2+1}}$$

$$\int\sqrt{x^2+4x+3}\,dx$$

$$=\left[\begin{array}{r|r|l}\text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in(-\infty;-3)\,,\ t\in(-\infty;-1)\\ dt=dx & =(x+2)^2-1=t^2-1 & x\in(-1;\infty)\,,\ t\in(1;\infty)\end{array}\right]=\int\sqrt{t^2-1}\,dt$$

$$=\frac{t\sqrt{t^2-1}}{2}-\frac{1}{2}\int\frac{dt}{\sqrt{t^2-1}}$$

# Riešené príklady – 115, 116

$$\int \sqrt{x^2+4x+5}\,dx$$

$$=\left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array}\right]=\int\sqrt{t^2+1}\,dt=\frac{t\sqrt{t^2+1}}{2}+\frac{1}{2}\int\frac{dt}{\sqrt{t^2+1}}$$

$$=\frac{t\sqrt{t^2+1}}{2}+\frac{\ln\left(t+\sqrt{t^2+1}\right)}{2}+c$$

$$\int \sqrt{x^2+4x+3}\,dx$$

$$=\left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in(-\infty;-3),\ t\in(-\infty;-1) \\ dt=dx & =(x+2)^2-1=t^2-1 & x\in(-1;\infty),\ t\in(1;\infty) \end{array}\right]=\int\sqrt{t^2-1}\,dt$$

$$=\frac{t\sqrt{t^2-1}}{2}-\frac{1}{2}\int\frac{dt}{\sqrt{t^2-1}}=\frac{t\sqrt{t^2-1}}{2}-\frac{\ln\left|t+\sqrt{t^2-1}\right|}{2}+c$$

# Riešené príklady – 115, 116

$$\int\sqrt{x^2+4x+5}\,dx = \frac{(x+2)\sqrt{x^2+4x+5}}{2}+\frac{\ln\left(x+2+\sqrt{x^2+4x+5}\right)}{2}+c$$

$$=\begin{bmatrix}\text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R\\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R\end{bmatrix}=\int\sqrt{t^2+1}\,dt=\frac{t\sqrt{t^2+1}}{2}+\frac{1}{2}\int\frac{dt}{\sqrt{t^2+1}}$$

$$=\frac{t\sqrt{t^2+1}}{2}+\frac{\ln\left(t+\sqrt{t^2+1}\right)}{2}+c=\frac{(x+2)\sqrt{(x+2)^2+1}}{2}+\frac{\ln\left(x+2+\sqrt{(x+2)^2+1}\right)}{2}+c$$

$$=\frac{(x+2)\sqrt{x^2+4x+5}}{2}+\frac{\ln\left(x+2+\sqrt{x^2+4x+5}\right)}{2}+c,\ x\in R,\ c\in R.$$

$$\int\sqrt{x^2+4x+3}\,dx = \frac{(x+2)\sqrt{x^2+4x+3}}{2}-\frac{\ln\left|x+2+\sqrt{x^2+4x+3}\right|}{2}+c$$

$$=\begin{bmatrix}\text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in(-\infty;-3),\ t\in(-\infty;-1)\\ dt=dx & =(x+2)^2-1=t^2-1 & x\in(-1;\infty),\ t\in(1;\infty)\end{bmatrix}=\int\sqrt{t^2-1}\,dt$$

$$=\frac{t\sqrt{t^2-1}}{2}-\frac{1}{2}\int\frac{dt}{\sqrt{t^2-1}}=\frac{t\sqrt{t^2-1}}{2}-\frac{\ln\left|t+\sqrt{t^2-1}\right|}{2}+c$$

$$=\frac{(x+2)\sqrt{x^2+4x+3}}{2}-\frac{\ln\left|x+2+\sqrt{x^2+4x+3}\right|}{2}+c,\ x\in(-\infty;-3)\cup(-1;\infty),\ c\in R.$$

# Riešené príklady – 117, 118

$$\int \sqrt{x^2-4x+6}\,dx$$

$$\int \sqrt{x^2-4x+2}\,dx$$

# Riešené príklady – 117, 118

$$\int \sqrt{x^2-4x+6}\,dx$$

$$=\left[\begin{array}{r|l|l} \text{Subst. } t=x-2 & x^2-4x+6=x^2+4x+4+2 & x\in R \\ dt=dx & \quad =(x-2)^2+2=t^2+(\sqrt{2})^2 & t\in R \end{array}\right]$$

$$\int \sqrt{x^2-4x+2}\,dx$$

$$=\left[\begin{array}{r|l|l} \text{Subst. } t=x-2 & x^2-4x+2=x^2-4x+4-2 & x\in(-\infty;-3)\,,\ t\in(-\infty;-1) \\ dt=dx & \quad =(x-2)^2-2=t^2-(\sqrt{2})^2 & x\in(-1;\infty)\,,\ t\in(1;\infty) \end{array}\right]$$

# Riešené príklady – 117, 118

$$\int \sqrt{x^2-4x+6}\,dx$$

$$=\left[\begin{array}{r|l|l} \text{Subst. } t=x-2 & x^2-4x+6=x^2+4x+4+2 & x\in R \\ dt=dx & \quad =(x-2)^2+2=t^2+(\sqrt{2})^2 & t\in R \end{array}\right]=\int \sqrt{t^2+(\sqrt{2})^2}\,dt$$

$$\int \sqrt{x^2-4x+2}\,dx$$

$$=\left[\begin{array}{r|l|l} \text{Subst. } t=x-2 & x^2-4x+2=x^2-4x+4-2 & x\in(-\infty;-3)\,,\ t\in(-\infty;-1) \\ dt=dx & \quad =(x-2)^2-2=t^2-(\sqrt{2})^2 & x\in(-1;\infty)\,,\ t\in(1;\infty) \end{array}\right]=\int \sqrt{t^2-(\sqrt{2})^2}\,dt$$

# Riešené príklady – 117, 118

$$\int \sqrt{x^2-4x+6}\,\mathrm{d}x$$

$$=\left[\begin{array}{r|l|l} \text{Subst. } t=x-2 & x^2-4x+6=x^2+4x+4+2 & x\in R \\ \mathrm{d}t=\mathrm{d}x & \quad =(x-2)^2+2=t^2+(\sqrt{2})^2 & t\in R \end{array}\right]=\int \sqrt{t^2+(\sqrt{2})^2}\,\mathrm{d}t$$

$$=\frac{t\sqrt{t^2+(\sqrt{2})^2}}{2}+\frac{1}{2}\int \frac{\mathrm{d}t}{\sqrt{t^2+(\sqrt{2})^2}}$$

$$\int \sqrt{x^2-4x+2}\,\mathrm{d}x$$

$$=\left[\begin{array}{r|l|l} \text{Subst. } t=x-2 & x^2-4x+2=x^2-4x+4-2 & x\in(-\infty;-3)\,,\ t\in(-\infty;-1) \\ \mathrm{d}t=\mathrm{d}x & \quad =(x-2)^2-2=t^2-(\sqrt{2})^2 & x\in(-1;\infty)\,,\ t\in(1;\infty) \end{array}\right]=\int \sqrt{t^2-(\sqrt{2})^2}\,\mathrm{d}t$$

$$=\frac{t\sqrt{t^2-(\sqrt{2})^2}}{2}-\frac{1}{2}\int \frac{\mathrm{d}t}{\sqrt{t^2-(\sqrt{2})^2}}$$

# Riešené príklady – 117, 118

$$\int\sqrt{x^2-4x+6}\,dx$$

$$=\left[\begin{array}{r|l|l}\text{Subst. } t=x-2 & x^2-4x+6=x^2+4x+4+2 & x\in R\\ dt=dx & \quad=(x-2)^2+2=t^2+(\sqrt{2})^2 & t\in R\end{array}\right]=\int\sqrt{t^2+(\sqrt{2})^2}\,dt$$

$$=\frac{t\sqrt{t^2+(\sqrt{2})^2}}{2}+\frac{1}{2}\int\frac{dt}{\sqrt{t^2+(\sqrt{2})^2}}=\frac{t\sqrt{t^2+(\sqrt{2})^2}}{2}+\frac{\ln\left(t+\sqrt{t^2+(\sqrt{2})^2}\right)}{2}+c$$

$$\int\sqrt{x^2-4x+2}\,dx$$

$$=\left[\begin{array}{r|l|l}\text{Subst. } t=x-2 & x^2-4x+2=x^2-4x+4-2 & x\in(-\infty;-3),\ t\in(-\infty;-1)\\ dt=dx & \quad=(x-2)^2-2=t^2-(\sqrt{2})^2 & x\in(-1;\infty),\ t\in(1;\infty)\end{array}\right]=\int\sqrt{t^2-(\sqrt{2})^2}\,dt$$

$$=\frac{t\sqrt{t^2-(\sqrt{2})^2}}{2}-\frac{1}{2}\int\frac{dt}{\sqrt{t^2-(\sqrt{2})^2}}=\frac{t\sqrt{t^2-(\sqrt{2})^2}}{2}-\frac{\ln\left|t+\sqrt{t^2-(\sqrt{2})^2}\right|}{2}+c$$

# Riešené príklady – 117, 118

$$\int \sqrt{x^2-4x+6}\,dx = \frac{(x-2)\sqrt{x^2-4x+6}}{2} + \frac{\ln\left(x-2+\sqrt{x^2-4x+6}\right)}{2} + c$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } t=x-2 & x^2-4x+6=x^2+4x+4+2 & x\in R \\ dt=dx & \quad =(x-2)^2+2=t^2+(\sqrt{2})^2 & t\in R \end{array}\right] = \int \sqrt{t^2+(\sqrt{2})^2}\,dt$$

$$= \frac{t\sqrt{t^2+(\sqrt{2})^2}}{2} + \frac{1}{2}\int \frac{dt}{\sqrt{t^2+(\sqrt{2})^2}} = \frac{t\sqrt{t^2+(\sqrt{2})^2}}{2} + \frac{\ln\left(t+\sqrt{t^2+(\sqrt{2})^2}\right)}{2} + c$$

$$= \frac{(x-2)\sqrt{x^2-4x+6}}{2} + \frac{\ln\left(x-2+\sqrt{x^2-4x+6}\right)}{2} + c,\ x\in R,\ c\in R.$$

$$\int \sqrt{x^2-4x+2}\,dx = \frac{(x-2)\sqrt{x^2+4x+2}}{2} - \frac{\ln\left|x-2+\sqrt{x^2-4x+2}\right|}{2} + c$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } t=x-2 & x^2-4x+2=x^2-4x+4-2 & x\in(-\infty;-3),\ t\in(-\infty;-1) \\ dt=dx & \quad =(x-2)^2-2=t^2-(\sqrt{2})^2 & x\in(-1;\infty),\ t\in(1;\infty) \end{array}\right] = \int \sqrt{t^2-(\sqrt{2})^2}\,dt$$

$$= \frac{t\sqrt{t^2-(\sqrt{2})^2}}{2} - \frac{1}{2}\int \frac{dt}{\sqrt{t^2-(\sqrt{2})^2}} = \frac{t\sqrt{t^2-(\sqrt{2})^2}}{2} - \frac{\ln\left|t+\sqrt{t^2-(\sqrt{2})^2}\right|}{2} + c$$

$$= \frac{(x-2)\sqrt{x^2+4x+2}}{2} - \frac{\ln\left|x-2+\sqrt{x^2-4x+2}\right|}{2} + c,\ x\in R-\left\langle 2-\sqrt{2}; 2+\sqrt{2}\right\rangle,\ c\in R.$$

# Riešené príklady – 119, 120

$$\int \sqrt{-x^2+4x-3}\,\mathrm{d}x$$

$$\int \sqrt{x(2-x)}\,\mathrm{d}x = \int \sqrt{-x^2+2x}\,\mathrm{d}x$$

# Riešené príklady – 119, 120

$$\int \sqrt{-x^2+4x-3}\,\mathrm{d}x$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x-2 & -x^2+4x-3=-(x^2-4x+4-1) & x\in(1;3) \\ \mathrm{d}t=\mathrm{d}x & =-(x-2)^2+1=1-t^2 & t\in(-1;1) \end{array} \right]$$

$$\int \sqrt{x(2-x)}\,\mathrm{d}x = \int \sqrt{-x^2+2x}\,\mathrm{d}x$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x-1 & -x^2+2x=-(x^2-2x+1-1) & x\in(0;2) \\ \mathrm{d}t=\mathrm{d}x & =-(x-1)^2+1=1-t^2 & t\in(-1;1) \end{array} \right]$$

# Riešené príklady – 119, 120

$$\int \sqrt{-x^2+4x-3}\,dx$$

$$=\left[\begin{array}{r|r|l} \text{Subst. } t=x-2 & -x^2+4x-3=-(x^2-4x+4-1) & x\in(1;3) \\ dt=dx & =-(x-2)^2+1=1-t^2 & t\in(-1;1) \end{array}\right]=\int\sqrt{1-t^2}\,dt$$

$$\int \sqrt{x(2-x)}\,dx = \int \sqrt{-x^2+2x}\,dx$$

$$=\left[\begin{array}{r|r|l} \text{Subst. } t=x-1 & -x^2+2x=-(x^2-2x+1-1) & x\in(0;2) \\ dt=dx & =-(x-1)^2+1=1-t^2 & t\in(-1;1) \end{array}\right]=\int\sqrt{1-t^2}\,dt$$

# Riešené príklady – 119, 120

$$\int \sqrt{-x^2+4x-3}\,dx$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x-2 & -x^2+4x-3=-(x^2-4x+4-1) & x\in(1;3) \\ dt=dx & =-(x-2)^2+1=1-t^2 & t\in(-1;1) \end{array} \right] = \int \sqrt{1-t^2}\,dt$$

$$= \frac{t\sqrt{1-t^2}}{2} + \frac{1}{2}\int \frac{dt}{\sqrt{1-t^2}}$$

$$\int \sqrt{x(2-x)}\,dx = \int \sqrt{-x^2+2x}\,dx$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x-1 & -x^2+2x=-(x^2-2x+1-1) & x\in(0;2) \\ dt=dx & =-(x-1)^2+1=1-t^2 & t\in(-1;1) \end{array} \right] = \int \sqrt{1-t^2}\,dt = \frac{t\sqrt{1-t^2}}{2} + \frac{1}{2}\int \frac{dt}{\sqrt{1-t^2}}$$

# Riešené príklady – 119, 120

$$\int\sqrt{-x^2+4x-3}\,dx$$

$$=\left[\begin{array}{r|l|l}\text{Subst. } t=x-2 & -x^2+4x-3=-(x^2-4x+4-1) & x\in(1;3)\\ dt=dx & \quad =-(x-2)^2+1=1-t^2 & t\in(-1;1)\end{array}\right]=\int\sqrt{1-t^2}\,dt$$

$$=\frac{t\sqrt{1-t^2}}{2}+\frac{1}{2}\int\frac{dt}{\sqrt{1-t^2}}=\frac{t\sqrt{1-t^2}}{2}+\frac{\arcsin t}{2}+c_1=\frac{t\sqrt{1-t^2}}{2}-\frac{\arccos t}{2}+c_2$$

$$\int\sqrt{x(2-x)}\,dx=\int\sqrt{-x^2+2x}\,dx$$

$$=\left[\begin{array}{r|l|l}\text{Subst. } t=x-1 & -x^2+2x=-(x^2-2x+1-1) & x\in(0;2)\\ dt=dx & \quad =-(x-1)^2+1=1-t^2 & t\in(-1;1)\end{array}\right]=\int\sqrt{1-t^2}\,dt=\frac{t\sqrt{1-t^2}}{2}+\frac{1}{2}\int\frac{dt}{\sqrt{1-t^2}}$$

$$=\frac{t\sqrt{1-t^2}}{2}+\frac{\arcsin t}{2}+c_1=\frac{t\sqrt{1-t^2}}{2}-\frac{\arccos t}{2}+c_2$$

# Riešené príklady – 119, 120

$$\int\sqrt{-x^2+4x-3}\,dx=\frac{(x-2)\sqrt{-x^2+4x-3}}{2}+\frac{\arcsin(x-2)}{2}+c_1$$

$$=\left[\begin{array}{l|l|l}\text{Subst. } t=x-2 & -x^2+4x-3=-(x^2-4x+4-1) & x\in(1;3)\\ \quad dt=dx & \quad =-(x-2)^2+1=1-t^2 & t\in(-1;1)\end{array}\right]=\int\sqrt{1-t^2}\,dt$$

$$=\frac{t\sqrt{1-t^2}}{2}+\frac{1}{2}\int\frac{dt}{\sqrt{1-t^2}}=\frac{t\sqrt{1-t^2}}{2}+\frac{\arcsin t}{2}+c_1=\frac{t\sqrt{1-t^2}}{2}-\frac{\arccos t}{2}+c_2$$

$$=\frac{(x-2)\sqrt{-x^2+4x-3}}{2}+\frac{\arcsin(x-2)}{2}+c_1=\frac{(x-2)\sqrt{-x^2+4x-3}}{2}-\frac{\arccos(x-2)}{2}+c_2,$$

$$x\in(1;3),\ c_1,c_2\in R.$$

$$\int\sqrt{x(2-x)}\,dx=\int\sqrt{-x^2+2x}\,dx=\frac{(x-2)\sqrt{-x^2+2x}}{2}+\frac{\arcsin(x-2)}{2}+c_1$$

$$=\left[\begin{array}{l|l|l}\text{Subst. } t=x-1 & -x^2+2x=-(x^2-2x+1-1) & x\in(0;2)\\ \quad dt=dx & \quad =-(x-1)^2+1=1-t^2 & t\in(-1;1)\end{array}\right]=\int\sqrt{1-t^2}\,dt=\frac{t\sqrt{1-t^2}}{2}+\frac{1}{2}\int\frac{dt}{\sqrt{1-t^2}}$$

$$=\frac{t\sqrt{1-t^2}}{2}+\frac{\arcsin t}{2}+c_1=\frac{t\sqrt{1-t^2}}{2}-\frac{\arccos t}{2}+c_2=\frac{(x-2)\sqrt{-x^2+2x}}{2}+\frac{\arcsin(x-2)}{2}+c_1$$

$$=\frac{(x-2)\sqrt{-x^2+2x}}{2}-\frac{\arccos(x-2)}{2}+c_2,\ x\in(0;2),\ c_1,c_2\in R.$$

# Riešené príklady – 121

$I_n = \int \frac{dx}{(x^2+a^2)^n}$ $a>0,\ n\in N$

# Riešené príklady – 121

$I_n = \int \frac{dx}{(x^2+a^2)^n}$ $a>0,\ n\in N$

$$= \frac{1}{a^2} \int \frac{a^2\,dx}{(x^2+a^2)^n}$$

# Riešené príklady – 121

$I_n = \int \frac{dx}{(x^2+a^2)^n}$ $a>0,\ n\in N$

$$= \frac{1}{a^2}\int \frac{a^2\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^n}$$

# Riešené príklady – 121

$I_n = \int \frac{dx}{(x^2+a^2)^n}$ $\qquad a>0,\ n\in N$

$$= \frac{1}{a^2}\int \frac{a^2\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2)\,dx}{(x^2+a^2)^n} - \frac{1}{a^2}\int \frac{x^2\,dx}{(x^2+a^2)^n}$$

# Riešené príklady – 121

$I_n = \int \frac{dx}{(x^2+a^2)^n}$ $\quad a>0,\ n\in N$

$$= \frac{1}{a^2}\int \frac{a^2\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2)\,dx}{(x^2+a^2)^n} - \frac{1}{a^2}\int \frac{x^2\,dx}{(x^2+a^2)^n}$$

$$= \frac{1}{a^2}\int \frac{dx}{(x^2+a^2)^{n-1}} - \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2+a^2)^n}$$

# Riešené príklady – 121

$I_n = \int \frac{dx}{(x^2+a^2)^n}$ $\qquad a>0,\ n\in N$

$$= \frac{1}{a^2}\int \frac{a^2\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2)\,dx}{(x^2+a^2)^n} - \frac{1}{a^2}\int \frac{x^2\,dx}{(x^2+a^2)^n}$$

$$= \frac{1}{a^2}\int \frac{dx}{(x^2+a^2)^{n-1}} - \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2+a^2)^n} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^n} & v=\frac{(x^2+a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2+a^2)^{n-1}} \end{array}\right]$$

# Riešené príklady – 121

$$I_n=\int\frac{dx}{(x^2+a^2)^n} \qquad a>0,\ n\in N$$

$$=\frac{1}{a^2}\int\frac{a^2\,dx}{(x^2+a^2)^n}=\frac{1}{a^2}\int\frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^n}=\frac{1}{a^2}\int\frac{(x^2+a^2)\,dx}{(x^2+a^2)^n}-\frac{1}{a^2}\int\frac{x^2\,dx}{(x^2+a^2)^n}$$

$$=\frac{1}{a^2}\int\frac{dx}{(x^2+a^2)^{n-1}}-\frac{1}{2a^2}\int\frac{x\cdot 2x\,dx}{(x^2+a^2)^n}=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^n} & v=\frac{(x^2+a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2+a^2)^{n-1}}\end{array}\right]$$

$$=\frac{1}{a^2}I_{n-1}-\frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2+a^2)^{n-1}}-\frac{1}{1-n}\int\frac{dx}{(x^2+a^2)^{n-1}}\right]$$

# Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2+a^2)^n} \qquad a>0,\ n\in N$$

$$= \frac{1}{a^2}\int \frac{a^2\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2)\,dx}{(x^2+a^2)^n} - \frac{1}{a^2}\int \frac{x^2\,dx}{(x^2+a^2)^n}$$

$$= \frac{1}{a^2}\int \frac{dx}{(x^2+a^2)^{n-1}} - \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2+a^2)^n} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^n} & v=\frac{(x^2+a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2+a^2)^{n-1}} \end{array}\right]$$

$$= \frac{1}{a^2}I_{n-1} - \frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2+a^2)^{n-1}} - \frac{1}{1-n}\int \frac{dx}{(x^2+a^2)^{n-1}}\right]$$

$$= \frac{1}{a^2}I_{n-1} - \frac{x}{2a^2(1-n)(x^2+a^2)^{n-1}} + \frac{1}{2a^2(1-n)}I_{n-1}$$

# Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2+a^2)^n} \qquad a>0,\ n\in N$$

$$= \frac{1}{a^2}\int \frac{a^2\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2)\,dx}{(x^2+a^2)^n} - \frac{1}{a^2}\int \frac{x^2\,dx}{(x^2+a^2)^n}$$

$$= \frac{1}{a^2}\int \frac{dx}{(x^2+a^2)^{n-1}} - \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2+a^2)^n} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^n} & v=\frac{(x^2+a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2+a^2)^{n-1}} \end{array}\right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2+a^2)^{n-1}} - \frac{1}{1-n}\int \frac{dx}{(x^2+a^2)^{n-1}}\right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{x}{2a^2(1-n)(x^2+a^2)^{n-1}} + \frac{1}{2a^2(1-n)} I_{n-1}$$

$$= \frac{1}{2a^2}\left(2+\frac{1}{1-n}\right) I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$$

# Riešené príklady – 121

$$I_n = \int \frac{dx}{(x^2+a^2)^n} \qquad a>0,\ n\in N$$

$$= \frac{1}{a^2}\int \frac{a^2\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{(x^2+a^2)\,dx}{(x^2+a^2)^n} - \frac{1}{a^2}\int \frac{x^2\,dx}{(x^2+a^2)^n}$$

$$= \frac{1}{a^2}\int \frac{dx}{(x^2+a^2)^{n-1}} - \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2+a^2)^n} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^n} & v=\frac{(x^2+a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2+a^2)^{n-1}} \end{array}\right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2+a^2)^{n-1}} - \frac{1}{1-n}\int \frac{dx}{(x^2+a^2)^{n-1}}\right]$$

$$= \frac{1}{a^2} I_{n-1} - \frac{x}{2a^2(1-n)(x^2+a^2)^{n-1}} + \frac{1}{2a^2(1-n)} I_{n-1}$$

$$= \frac{1}{2a^2}\left(2+\frac{1}{1-n}\right) I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} = \frac{3-2n}{2a^2(1-n)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$$

# Riešené príklady – 121

$$I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}\,I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}},\qquad a>0,\ n\in N$$

$$=\frac{1}{a^2}\int\frac{a^2\,dx}{(x^2+a^2)^n}=\frac{1}{a^2}\int\frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^n}=\frac{1}{a^2}\int\frac{(x^2+a^2)\,dx}{(x^2+a^2)^n}-\frac{1}{a^2}\int\frac{x^2\,dx}{(x^2+a^2)^n}$$

$$=\frac{1}{a^2}\int\frac{dx}{(x^2+a^2)^{n-1}}-\frac{1}{2a^2}\int\frac{x\cdot 2x\,dx}{(x^2+a^2)^n}=\left[\begin{array}{l|l} u=x & u'=1\\ v'=\frac{2x}{(x^2+a^2)^n} & v=\frac{(x^2+a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2+a^2)^{n-1}}\end{array}\right]$$

$$=\frac{1}{a^2}\,I_{n-1}-\frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2+a^2)^{n-1}}-\frac{1}{1-n}\int\frac{dx}{(x^2+a^2)^{n-1}}\right]$$

$$=\frac{1}{a^2}\,I_{n-1}-\frac{x}{2a^2(1-n)(x^2+a^2)^{n-1}}+\frac{1}{2a^2(1-n)}\,I_{n-1}$$

$$=\frac{1}{2a^2}\Big(2+\frac{1}{1-n}\Big)I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}=\frac{3-2n}{2a^2(1-n)}\,I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$$

$$=\frac{2n-3}{2a^2(n-1)}\,I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}},\ x\in R,\ c\in R,\ n=2,3,4,\ldots.$$

# Riešené príklady – 121

$$I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}},\quad I_1=\frac{1}{a}\operatorname{arctg}\frac{x}{a}+c,\quad a>0,\ n\in N$$

$$=\frac{1}{a^2}\int\frac{a^2\,dx}{(x^2+a^2)^n}=\frac{1}{a^2}\int\frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^n}=\frac{1}{a^2}\int\frac{(x^2+a^2)\,dx}{(x^2+a^2)^n}-\frac{1}{a^2}\int\frac{x^2\,dx}{(x^2+a^2)^n}$$

$$=\frac{1}{a^2}\int\frac{dx}{(x^2+a^2)^{n-1}}-\frac{1}{2a^2}\int\frac{x\cdot 2x\,dx}{(x^2+a^2)^n}=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^n} & v=\frac{(x^2+a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2+a^2)^{n-1}}\end{array}\right]$$

$$=\frac{1}{a^2}I_{n-1}-\frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2+a^2)^{n-1}}-\frac{1}{1-n}\int\frac{dx}{(x^2+a^2)^{n-1}}\right]$$

$$=\frac{1}{a^2}I_{n-1}-\frac{x}{2a^2(1-n)(x^2+a^2)^{n-1}}+\frac{1}{2a^2(1-n)}I_{n-1}$$

$$=\frac{1}{2a^2}\left(2+\frac{1}{1-n}\right)I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}=\frac{3-2n}{2a^2(1-n)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}$$

$$=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}},\ x\in R,\ c\in R,\ n=2,3,4,\ldots.$$

$$I_1=\int\frac{dx}{x^2+a^2}=\frac{1}{a}\operatorname{arctg}\frac{x}{a}+c,\ x\in R,\ c\in R,\ n=1.$$

# Riešené príklady – 122

$I_2 = \int \frac{dx}{(x^2+a^2)^2}$ $a > 0$

# Riešené príklady – 122

$$I_2=\int\frac{dx}{(x^2+a^2)^2}$$ $a>0$

$$=\frac{1}{a^2}\int\frac{a^2\,dx}{(x^2+a^2)^2}$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]$$

# Riešené príklady – 122

$I_2 = \int \frac{dx}{(x^2+a^2)^2}$ $a > 0$

$$= \frac{1}{a^2} \int \frac{a^2\, dx}{(x^2+a^2)^2} = \frac{1}{a^2} \int \frac{(x^2+a^2-x^2)\, dx}{(x^2+a^2)^2}$$

$$= \left[ \text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2 \cdot 2-3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)}$$

# Riešené príklady – 122

$I_2 = \int \frac{dx}{(x^2+a^2)^2}$ $\qquad a>0$

$$= \frac{1}{a^2}\int \frac{a^2\,dx}{(x^2+a^2)^2} = \frac{1}{a^2}\int \frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^2} = \frac{1}{a^2}\int \frac{dx}{x^2+a^2} - \frac{1}{2a^2}\int \frac{2x^2\,dx}{(x^2+a^2)^2}$$

$$= \left[ \text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2\cdot 2-3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)}$$

$$= \frac{1}{2a^2}\int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)}$$

# Riešené príklady – 122

$I_2=\int\frac{dx}{(x^2+a^2)^2}$ $a>0$

$$=\frac{1}{a^2}\int\frac{a^2\,dx}{(x^2+a^2)^2}=\frac{1}{a^2}\int\frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^2}=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}-\frac{1}{2a^2}\int\frac{2x^2\,dx}{(x^2+a^2)^2}$$

$$=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^2} & v=-\frac{1}{x^2+a^2}\end{array}\right]$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot2-3}{2a^2(2-1)}I_1+\frac{x}{2a^2(2-1)(x^2+a^2)}$$

$$=\frac{1}{2a^2}\int\frac{dx}{x^2+a^2}+\frac{x}{2a^2(x^2+a^2)}=\frac{1}{2a^2}\cdot\frac{1}{a}\operatorname{arctg}\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c$$

# Riešené príklady – 122

$$I_2 = \int \frac{dx}{(x^2+a^2)^2} = \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c \qquad a>0$$

$$= \frac{1}{a^2}\int \frac{a^2\,dx}{(x^2+a^2)^2} = \frac{1}{a^2}\int \frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^2} = \frac{1}{a^2}\int \frac{dx}{x^2+a^2} - \frac{1}{2a^2}\int \frac{2x^2\,dx}{(x^2+a^2)^2}$$

$$= \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^2} & v=-\frac{1}{x^2+a^2}\end{array}\right] = \frac{1}{a^2}\int \frac{dx}{x^2+a^2} - \frac{1}{2a^2}\left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2}\right]$$

$$= \left[\text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right] = \frac{2\cdot 2-3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)}$$

$$= \frac{1}{2a^2}\int \frac{dx}{x^2+a^2} + \frac{x}{2a^2(x^2+a^2)} = \frac{1}{2a^2}\cdot\frac{1}{a} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c$$

$$= \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)} + c,\ x\in R,\ c\in R.$$

# Riešené príklady – 122

$$I_2=\int\frac{dx}{(x^2+a^2)^2}=\frac{1}{2a^3}\,\mathrm{arctg}\,\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c \qquad a>0$$

$$=\frac{1}{a^2}\int\frac{a^2\,dx}{(x^2+a^2)^2}=\frac{1}{a^2}\int\frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^2}=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}-\frac{1}{2a^2}\int\frac{2x^2\,dx}{(x^2+a^2)^2}$$

$$=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^2} & v=-\frac{1}{x^2+a^2}\end{array}\right]=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}-\frac{1}{2a^2}\left[-\frac{x}{x^2+a^2}+\int\frac{dx}{x^2+a^2}\right]$$

$$=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}+\frac{x}{2a^2(x^2+a^2)}-\frac{1}{2a^2}\int\frac{dx}{x^2+a^2}$$

---

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot2-3}{2a^2(2-1)}I_1+\frac{x}{2a^2(2-1)(x^2+a^2)}$$

$$=\frac{1}{2a^2}\int\frac{dx}{x^2+a^2}+\frac{x}{2a^2(x^2+a^2)}=\frac{1}{2a^2}\cdot\frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c$$

$$=\frac{1}{2a^3}\,\mathrm{arctg}\,\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c,\ x\in R,\ c\in R.$$

# Riešené príklady – 122

$$I_2=\int\frac{dx}{(x^2+a^2)^2}=\frac{1}{2a^3}\operatorname{arctg}\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c \qquad a>0$$

$$=\frac{1}{a^2}\int\frac{a^2\,dx}{(x^2+a^2)^2}=\frac{1}{a^2}\int\frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^2}=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}-\frac{1}{2a^2}\int\frac{2x^2\,dx}{(x^2+a^2)^2}$$

$$=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^2} & v=-\frac{1}{x^2+a^2}\end{array}\right]=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}-\frac{1}{2a^2}\left[-\frac{x}{x^2+a^2}+\int\frac{dx}{x^2+a^2}\right]$$

$$=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}+\frac{x}{2a^2(x^2+a^2)}-\frac{1}{2a^2}\int\frac{dx}{x^2+a^2}=\frac{1}{2a^2}\int\frac{dx}{x^2+a^2}+\frac{x}{2a^2(x^2+a^2)}$$

---

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot 2-3}{2a^2(2-1)}I_1+\frac{x}{2a^2(2-1)(x^2+a^2)}$$

$$=\frac{1}{2a^2}\int\frac{dx}{x^2+a^2}+\frac{x}{2a^2(x^2+a^2)}=\frac{1}{2a^2}\cdot\frac{1}{a}\operatorname{arctg}\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c$$

$$=\frac{1}{2a^3}\operatorname{arctg}\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c,\ x\in R,\ c\in R.$$

# Riešené príklady – 122

$$I_2=\int\frac{dx}{(x^2+a^2)^2}=\frac{1}{2a^3}\,\mathrm{arctg}\,\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c \qquad a>0$$

$$=\frac{1}{a^2}\int\frac{a^2\,dx}{(x^2+a^2)^2}=\frac{1}{a^2}\int\frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^2}=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}-\frac{1}{2a^2}\int\frac{2x^2\,dx}{(x^2+a^2)^2}$$

$$=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^2} & v=-\frac{1}{x^2+a^2}\end{array}\right]=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}-\frac{1}{2a^2}\left[-\frac{x}{x^2+a^2}+\int\frac{dx}{x^2+a^2}\right]$$

$$=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}+\frac{x}{2a^2(x^2+a^2)}-\frac{1}{2a^2}\int\frac{dx}{x^2+a^2}=\frac{1}{2a^2}\int\frac{dx}{x^2+a^2}+\frac{x}{2a^2(x^2+a^2)}$$

$$=\frac{1}{2a^2}\cdot\frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c$$

---

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot2-3}{2a^2(2-1)}I_1+\frac{x}{2a^2(2-1)(x^2+a^2)}$$

$$=\frac{1}{2a^2}\int\frac{dx}{x^2+a^2}+\frac{x}{2a^2(x^2+a^2)}=\frac{1}{2a^2}\cdot\frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c$$

$$=\frac{1}{2a^3}\,\mathrm{arctg}\,\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c,\ x\in R,\ c\in R.$$

# Riešené príklady – 122

$$I_2=\int\frac{dx}{(x^2+a^2)^2}=\frac{1}{2a^3}\,\mathrm{arctg}\,\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c \qquad a>0$$

$$=\frac{1}{a^2}\int\frac{a^2\,dx}{(x^2+a^2)^2}=\frac{1}{a^2}\int\frac{(x^2+a^2-x^2)\,dx}{(x^2+a^2)^2}=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}-\frac{1}{2a^2}\int\frac{2x^2\,dx}{(x^2+a^2)^2}$$

$$=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^2} & v=-\frac{1}{x^2+a^2}\end{array}\right]=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}-\frac{1}{2a^2}\left[-\frac{x}{x^2+a^2}+\int\frac{dx}{x^2+a^2}\right]$$

$$=\frac{1}{a^2}\int\frac{dx}{x^2+a^2}+\frac{x}{2a^2(x^2+a^2)}-\frac{1}{2a^2}\int\frac{dx}{x^2+a^2}=\frac{1}{2a^2}\int\frac{dx}{x^2+a^2}+\frac{x}{2a^2(x^2+a^2)}$$

$$=\frac{1}{2a^2}\cdot\frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c=\frac{1}{2a^3}\,\mathrm{arctg}\,\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c,\ x\in R,\ c\in R.$$

---

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot2-3}{2a^2(2-1)}I_1+\frac{x}{2a^2(2-1)(x^2+a^2)}$$

$$=\frac{1}{2a^2}\int\frac{dx}{x^2+a^2}+\frac{x}{2a^2(x^2+a^2)}=\frac{1}{2a^2}\cdot\frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c$$

$$=\frac{1}{2a^3}\,\mathrm{arctg}\,\frac{x}{a}+\frac{x}{2a^2(x^2+a^2)}+c,\ x\in R,\ c\in R.$$

# Riešené príklady – 123, 124

$I_3 = \int \frac{dx}{(x^2+a^2)^3}$ $a > 0$

$I_4 = \int \frac{dx}{(x^2+a^2)^4}$ $a > 0$

# Riešené príklady – 123, 124

$$I_3=\int\frac{dx}{(x^2+a^2)^3}\qquad a>0$$

$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]$

$$I_4=\int\frac{dx}{(x^2+a^2)^4}\qquad a>0$$

$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]$

# Riešené príklady – 123, 124

$$I_3 = \int \frac{dx}{(x^2+a^2)^3} \qquad a>0$$

$$= \left[ \text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2\cdot 3-3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^2}$$

$$I_4 = \int \frac{dx}{(x^2+a^2)^4} \qquad a>0$$

$$= \left[ \text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2\cdot 4-3}{2a^2(4-1)} I_3 + \frac{x}{2a^2(4-1)(x^2+a^2)^3}$$

# Riešené príklady – 123, 124

$I_3 = \int \frac{dx}{(x^2+a^2)^3}$ $\quad a>0$

$$= \left[ \text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2\cdot 3-3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^2} = \left[ \text{Pr. 121} \right]$$

$I_4 = \int \frac{dx}{(x^2+a^2)^4}$ $\quad a>0$

$$= \left[ \text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2\cdot 4-3}{2a^2(4-1)} I_3 + \frac{x}{2a^2(4-1)(x^2+a^2)^3} = \left[ \text{Pr. 121} \right]$$

# Riešené príklady – 123, 124

$$I_3=\int\frac{dx}{(x^2+a^2)^3} \qquad a>0$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot3-3}{2a^2(3-1)}I_2+\frac{x}{2a^2(3-1)(x^2+a^2)^2}=\left[\text{Pr. 121}\right]$$

$$=\frac{3}{4a^2}\left[\frac{2\cdot2-3}{2a^2(2-1)}I_1+\frac{x}{2a^2(2-1)(x^2+a^2)}\right]+\frac{x}{4a^2(x^2+a^2)^2}$$

$$I_4=\int\frac{dx}{(x^2+a^2)^4} \qquad a>0$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot4-3}{2a^2(4-1)}I_3+\frac{x}{2a^2(4-1)(x^2+a^2)^3}=\left[\text{Pr. 121}\right]$$

$$=\frac{5}{6a^2}\left[\frac{2\cdot3-3}{2a^2(3-1)}I_2+\frac{x}{2a^2(3-1)(x^2+a^2)^2}\right]+\frac{x}{6a^2(x^2+a^2)^3}$$

# Riešené príklady – 123, 124

$I_3 = \int \frac{dx}{(x^2+a^2)^3}$ $a>0$

$= \left[ \text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2\cdot 3-3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^2} = \left[ \text{Pr. 121} \right]$

$= \frac{3}{4a^2} \left[ \frac{2\cdot 2-3}{2a^2(2-1)} I_1 + \frac{x}{2a^2(2-1)(x^2+a^2)} \right] + \frac{x}{4a^2(x^2+a^2)^2} = \left[ \text{Pr. 121: } I_1 = \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} \right]$

$I_4 = \int \frac{dx}{(x^2+a^2)^4}$ $a>0$

$= \left[ \text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{2\cdot 4-3}{2a^2(4-1)} I_3 + \frac{x}{2a^2(4-1)(x^2+a^2)^3} = \left[ \text{Pr. 121} \right]$

$= \frac{5}{6a^2} \left[ \frac{2\cdot 3-3}{2a^2(3-1)} I_2 + \frac{x}{2a^2(3-1)(x^2+a^2)^2} \right] + \frac{x}{6a^2(x^2+a^2)^3} = \left[ \text{Pr. 121} \right]$

# Riešené príklady – 123, 124

$$I_3=\int\frac{dx}{(x^2+a^2)^3}=\frac{3}{8a^5}\,\mathrm{arctg}\,\frac{x}{a}+\frac{3x}{8a^4(x^2+a^2)}+\frac{x}{4a^2(x^2+a^2)^2}+c \qquad a>0$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot3-3}{2a^2(3-1)}I_2+\frac{x}{2a^2(3-1)(x^2+a^2)^2}=\left[\text{Pr. 121}\right]$$

$$=\frac{3}{4a^2}\left[\frac{2\cdot2-3}{2a^2(2-1)}I_1+\frac{x}{2a^2(2-1)(x^2+a^2)}\right]+\frac{x}{4a^2(x^2+a^2)^2}=\left[\text{Pr. 121: } I_1=\int\frac{dx}{x^2+a^2}=\frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a}\right]$$

$$=\frac{3}{8a^5}\,\mathrm{arctg}\,\frac{x}{a}+\frac{3x}{8a^4(x^2+a^2)}+\frac{x}{4a^2(x^2+a^2)^2}+c,\ x\in R,\ c\in R.$$

$$I_4=\int\frac{dx}{(x^2+a^2)^4} \qquad a>0$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot4-3}{2a^2(4-1)}I_3+\frac{x}{2a^2(4-1)(x^2+a^2)^3}=\left[\text{Pr. 121}\right]$$

$$=\frac{5}{6a^2}\left[\frac{2\cdot3-3}{2a^2(3-1)}I_2+\frac{x}{2a^2(3-1)(x^2+a^2)^2}\right]+\frac{x}{6a^2(x^2+a^2)^3}=\left[\text{Pr. 121}\right]$$

$$=\frac{15}{24a^4}\left[\frac{2\cdot2-3}{2a^2(2-1)}I_1+\frac{x}{2a^2(2-1)(x^2+a^2)}\right]+\frac{5x}{24a^4(x^2+a^2)^2}+\frac{x}{6a^2(x^2+a^2)^3}$$

# Riešené príklady – 123, 124

$$I_3=\int\frac{dx}{(x^2+a^2)^3} = \frac{3}{8a^5}\,\text{arctg}\,\frac{x}{a}+\frac{3x}{8a^4(x^2+a^2)}+\frac{x}{4a^2(x^2+a^2)^2}+c \qquad a>0$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot3-3}{2a^2(3-1)}I_2+\frac{x}{2a^2(3-1)(x^2+a^2)^2}=\left[\text{Pr. 121}\right]$$

$$=\frac{3}{4a^2}\left[\frac{2\cdot2-3}{2a^2(2-1)}I_1+\frac{x}{2a^2(2-1)(x^2+a^2)}\right]+\frac{x}{4a^2(x^2+a^2)^2}=\left[\text{Pr. 121: } I_1=\int\frac{dx}{x^2+a^2}=\frac{1}{a}\,\text{arctg}\,\frac{x}{a}\right]$$

$$=\frac{3}{8a^5}\,\text{arctg}\,\frac{x}{a}+\frac{3x}{8a^4(x^2+a^2)}+\frac{x}{4a^2(x^2+a^2)^2}+c,\ x\in R,\ c\in R.$$

$$I_4=\int\frac{dx}{(x^2+a^2)^4} \qquad a>0$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot4-3}{2a^2(4-1)}I_3+\frac{x}{2a^2(4-1)(x^2+a^2)^3}=\left[\text{Pr. 121}\right]$$

$$=\frac{5}{6a^2}\left[\frac{2\cdot3-3}{2a^2(3-1)}I_2+\frac{x}{2a^2(3-1)(x^2+a^2)^2}\right]+\frac{x}{6a^2(x^2+a^2)^3}=\left[\text{Pr. 121}\right]$$

$$=\frac{15}{24a^4}\left[\frac{2\cdot2-3}{2a^2(2-1)}I_1+\frac{x}{2a^2(2-1)(x^2+a^2)}\right]+\frac{5x}{24a^4(x^2+a^2)^2}+\frac{x}{6a^2(x^2+a^2)^3}=\left[\text{Pr. 121: } I_1=\frac{1}{a}\,\text{arctg}\,\frac{x}{a}\right]$$

# Riešené príklady – 123, 124

$$I_3=\int\frac{dx}{(x^2+a^2)^3} = \frac{3}{8a^5}\operatorname{arctg}\frac{x}{a}+\frac{3x}{8a^4(x^2+a^2)}+\frac{x}{4a^2(x^2+a^2)^2}+c \qquad a>0$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot3-3}{2a^2(3-1)}I_2+\frac{x}{2a^2(3-1)(x^2+a^2)^2}=\left[\text{Pr. 121}\right]$$

$$=\frac{3}{4a^2}\left[\frac{2\cdot2-3}{2a^2(2-1)}I_1+\frac{x}{2a^2(2-1)(x^2+a^2)}\right]+\frac{x}{4a^2(x^2+a^2)^2}=\left[\text{Pr. 121: } I_1=\int\frac{dx}{x^2+a^2}=\frac{1}{a}\operatorname{arctg}\frac{x}{a}\right]$$

$$=\frac{3}{8a^5}\operatorname{arctg}\frac{x}{a}+\frac{3x}{8a^4(x^2+a^2)}+\frac{x}{4a^2(x^2+a^2)^2}+c,\ x\in R,\ c\in R.$$

$$I_4=\int\frac{dx}{(x^2+a^2)^4} = \frac{5}{16a^7}\operatorname{arctg}\frac{x}{a}+\frac{5x}{16a^2(x^2+a^2)}+\frac{5x}{24a^4(x^2+a^2)^2}+\frac{x}{6a^2(x^2+a^2)^3}+c \qquad a>0$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{2\cdot4-3}{2a^2(4-1)}I_3+\frac{x}{2a^2(4-1)(x^2+a^2)^3}=\left[\text{Pr. 121}\right]$$

$$=\frac{5}{6a^2}\left[\frac{2\cdot3-3}{2a^2(3-1)}I_2+\frac{x}{2a^2(3-1)(x^2+a^2)^2}\right]+\frac{x}{6a^2(x^2+a^2)^3}=\left[\text{Pr. 121}\right]$$

$$=\frac{15}{24a^4}\left[\frac{2\cdot2-3}{2a^2(2-1)}I_1+\frac{x}{2a^2(2-1)(x^2+a^2)}\right]+\frac{5x}{24a^4(x^2+a^2)^2}+\frac{x}{6a^2(x^2+a^2)^3}=\left[\text{Pr. 121: } I_1=\frac{1}{a}\operatorname{arctg}\frac{x}{a}\right]$$

$$=\frac{5}{16a^7}\operatorname{arctg}\frac{x}{a}+\frac{5x}{16a^2(x^2+a^2)}+\frac{5x}{24a^4(x^2+a^2)^2}+\frac{x}{6a^2(x^2+a^2)^3}+c,\ x\in R,\ c\in R.$$

# Riešené príklady – 125

$I_n = \int \frac{dx}{(x^2 - a^2)^n}$ $a > 0,\ n \in N$

# Riešené príklady – 125

$I_n = \int \frac{dx}{(x^2-a^2)^n}$ $a>0,\ n\in N$

$$= \frac{1}{-a^2} \int \frac{-a^2\,dx}{(x^2-a^2)^n}$$

# Riešené príklady – 125

$I_n = \int \frac{dx}{(x^2 - a^2)^n}$ $\qquad a > 0,\ n \in N$

$$= \frac{1}{-a^2} \int \frac{-a^2\, dx}{(x^2 - a^2)^n} = \frac{1}{-a^2} \int \frac{(x^2 - a^2 - x^2)\, dx}{(x^2 - a^2)^n}$$

# Riešené príklady – 125

$I_n = \int \frac{dx}{(x^2-a^2)^n}$ $\qquad a>0,\ n\in N$

$$= \frac{1}{-a^2}\int \frac{-a^2\,dx}{(x^2-a^2)^n} = \frac{1}{-a^2}\int \frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^n} = \frac{1}{-a^2}\int \frac{(x^2-a^2)\,dx}{(x^2-a^2)^n} - \frac{1}{-a^2}\int \frac{x^2\,dx}{(x^2-a^2)^n}$$

# Riešené príklady – 125

$I_n = \int \frac{dx}{(x^2-a^2)^n}$ $a>0,\ n\in N$

$$= \frac{1}{-a^2}\int \frac{-a^2\,dx}{(x^2-a^2)^n} = \frac{1}{-a^2}\int \frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^n} = \frac{1}{-a^2}\int \frac{(x^2-a^2)\,dx}{(x^2-a^2)^n} - \frac{1}{-a^2}\int \frac{x^2\,dx}{(x^2-a^2)^n}$$

$$= -\frac{1}{a^2}\int \frac{dx}{(x^2-a^2)^{n-1}} + \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2-a^2)^n}$$

# Riešené príklady – 125

$I_n=\int\frac{dx}{(x^2-a^2)^n}$ $\qquad a>0,\ n\in N$

$$=\frac{1}{-a^2}\int\frac{-a^2\,dx}{(x^2-a^2)^n}=\frac{1}{-a^2}\int\frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^n}=\frac{1}{-a^2}\int\frac{(x^2-a^2)\,dx}{(x^2-a^2)^n}-\frac{1}{-a^2}\int\frac{x^2\,dx}{(x^2-a^2)^n}$$

$$=-\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{n-1}}+\frac{1}{2a^2}\int\frac{x\cdot 2x\,dx}{(x^2-a^2)^n}=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^n} & v=\frac{(x^2-a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2-a^2)^{n-1}}\end{array}\right]$$

# Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2-a^2)^n} \qquad a>0,\ n\in N$$

$$= \frac{1}{-a^2}\int \frac{-a^2\,dx}{(x^2-a^2)^n} = \frac{1}{-a^2}\int \frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^n} = \frac{1}{-a^2}\int \frac{(x^2-a^2)\,dx}{(x^2-a^2)^n} - \frac{1}{-a^2}\int \frac{x^2\,dx}{(x^2-a^2)^n}$$

$$= -\frac{1}{a^2}\int \frac{dx}{(x^2-a^2)^{n-1}} + \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2-a^2)^n} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^n} & v=\frac{(x^2-a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2-a^2)^{n-1}} \end{array}\right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2-a^2)^{n-1}} - \frac{1}{1-n}\int \frac{dx}{(x^2-a^2)^{n-1}}\right]$$

# Riešené príklady – 125

$$I_n = \int \frac{dx}{(x^2-a^2)^n} \qquad a>0,\ n\in N$$

$$= \frac{1}{-a^2}\int \frac{-a^2\,dx}{(x^2-a^2)^n} = \frac{1}{-a^2}\int \frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^n} = \frac{1}{-a^2}\int \frac{(x^2-a^2)\,dx}{(x^2-a^2)^n} - \frac{1}{-a^2}\int \frac{x^2\,dx}{(x^2-a^2)^n}$$

$$= -\frac{1}{a^2}\int \frac{dx}{(x^2-a^2)^{n-1}} + \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2-a^2)^n} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^n} & v=\frac{(x^2-a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2-a^2)^{n-1}} \end{array}\right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2-a^2)^{n-1}} - \frac{1}{1-n}\int \frac{dx}{(x^2-a^2)^{n-1}}\right]$$

$$= -\frac{1}{a^2} I_{n-1} + \frac{x}{2a^2(1-n)(x^2-a^2)^{n-1}} - \frac{1}{2a^2(1-n)} I_{n-1}$$

# Riešené príklady – 125

$I_n = \int \frac{dx}{(x^2-a^2)^n}$ $\qquad a>0,\ n\in N$

$$= \frac{1}{-a^2}\int \frac{-a^2\,dx}{(x^2-a^2)^n} = \frac{1}{-a^2}\int \frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^n} = \frac{1}{-a^2}\int \frac{(x^2-a^2)\,dx}{(x^2-a^2)^n} - \frac{1}{-a^2}\int \frac{x^2\,dx}{(x^2-a^2)^n}$$

$$= -\frac{1}{a^2}\int \frac{dx}{(x^2-a^2)^{n-1}} + \frac{1}{2a^2}\int \frac{x\cdot 2x\,dx}{(x^2-a^2)^n} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^n} & v=\frac{(x^2-a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2-a^2)^{n-1}} \end{array}\right]$$

$$= -\frac{1}{a^2}I_{n-1} + \frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2-a^2)^{n-1}} - \frac{1}{1-n}\int \frac{dx}{(x^2-a^2)^{n-1}}\right]$$

$$= -\frac{1}{a^2}I_{n-1} + \frac{x}{2a^2(1-n)(x^2-a^2)^{n-1}} - \frac{1}{2a^2(1-n)}I_{n-1}$$

$$= -\frac{1}{2a^2}\left(2+\frac{1}{1-n}\right)I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$$

# Riešené príklady – 125

$$I_n=\int\frac{dx}{(x^2-a^2)^n} \qquad a>0,\ n\in N$$

$$=\frac{1}{-a^2}\int\frac{-a^2\,dx}{(x^2-a^2)^n}=\frac{1}{-a^2}\int\frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^n}=\frac{1}{-a^2}\int\frac{(x^2-a^2)\,dx}{(x^2-a^2)^n}-\frac{1}{-a^2}\int\frac{x^2\,dx}{(x^2-a^2)^n}$$

$$=-\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{n-1}}+\frac{1}{2a^2}\int\frac{x\cdot 2x\,dx}{(x^2-a^2)^n}=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^n} & v=\frac{(x^2-a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2-a^2)^{n-1}}\end{array}\right]$$

$$=-\frac{1}{a^2}I_{n-1}+\frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2-a^2)^{n-1}}-\frac{1}{1-n}\int\frac{dx}{(x^2-a^2)^{n-1}}\right]$$

$$=-\frac{1}{a^2}I_{n-1}+\frac{x}{2a^2(1-n)(x^2-a^2)^{n-1}}-\frac{1}{2a^2(1-n)}I_{n-1}$$

$$=-\frac{1}{2a^2}\left(2+\frac{1}{1-n}\right)I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}=-\frac{3-2n}{2a^2(1-n)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$$

# Riešené príklady – 125

$$I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}},\qquad a>0,\ n\in N$$

$$=\frac{1}{-a^2}\int\frac{-a^2\,dx}{(x^2-a^2)^n}=\frac{1}{-a^2}\int\frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^n}=\frac{1}{-a^2}\int\frac{(x^2-a^2)\,dx}{(x^2-a^2)^n}-\frac{1}{-a^2}\int\frac{x^2\,dx}{(x^2-a^2)^n}$$

$$=-\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{n-1}}+\frac{1}{2a^2}\int\frac{x\cdot 2x\,dx}{(x^2-a^2)^n}=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^n} & v=\frac{(x^2-a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2-a^2)^{n-1}}\end{array}\right]$$

$$=-\frac{1}{a^2}I_{n-1}+\frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2-a^2)^{n-1}}-\frac{1}{1-n}\int\frac{dx}{(x^2-a^2)^{n-1}}\right]$$

$$=-\frac{1}{a^2}I_{n-1}+\frac{x}{2a^2(1-n)(x^2-a^2)^{n-1}}-\frac{1}{2a^2(1-n)}I_{n-1}$$

$$=-\frac{1}{2a^2}\left(2+\frac{1}{1-n}\right)I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}=-\frac{3-2n}{2a^2(1-n)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$$

$$=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}},\ x\in R-\{\pm a\},\ c\in R,\ n=2,3,4,\ldots.$$

# Riešené príklady – 125

$$I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}},\ I_1=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c,\ a>0,\ n\in N$$

$$=\frac{1}{-a^2}\int\frac{-a^2\,dx}{(x^2-a^2)^n}=\frac{1}{-a^2}\int\frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^n}=\frac{1}{-a^2}\int\frac{(x^2-a^2)\,dx}{(x^2-a^2)^n}-\frac{1}{-a^2}\int\frac{x^2\,dx}{(x^2-a^2)^n}$$

$$=-\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{n-1}}+\frac{1}{2a^2}\int\frac{x\cdot 2x\,dx}{(x^2-a^2)^n}=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^n} & v=\frac{(x^2-a^2)^{1-n}}{1-n}=\frac{1}{(1-n)(x^2-a^2)^{n-1}}\end{array}\right]$$

$$=-\frac{1}{a^2}I_{n-1}+\frac{1}{2a^2}\left[\frac{x}{(1-n)(x^2-a^2)^{n-1}}-\frac{1}{1-n}\int\frac{dx}{(x^2-a^2)^{n-1}}\right]$$

$$=-\frac{1}{a^2}I_{n-1}+\frac{x}{2a^2(1-n)(x^2-a^2)^{n-1}}-\frac{1}{2a^2(1-n)}I_{n-1}$$

$$=-\frac{1}{2a^2}\left(2+\frac{1}{1-n}\right)I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}=-\frac{3-2n}{2a^2(1-n)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}$$

$$=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}},\ x\in R-\{\pm a\},\ c\in R,\ n=2,3,4,\ldots.$$

$$I_1=\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c,\ x\in R-\{\pm a\},\ c\in R,\ n=1.$$

# Riešené príklady – 126

$$I_2 = \int \frac{dx}{(x^2 - a^2)^2} \qquad a > 0$$

# Riešené príklady – 126

$$I_2 = \int \frac{dx}{(x^2-a^2)^2} \qquad a>0$$

$$= \frac{1}{-a^2} \int \frac{-a^2\,dx}{(x^2-a^2)^2}$$

$$= \left[ \text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right]$$

# Riešené príklady – 126

$I_2 = \int \frac{dx}{(x^2-a^2)^2}$ $\qquad a>0$

$$= \frac{1}{-a^2} \int \frac{-a^2\,dx}{(x^2-a^2)^2} = \frac{1}{-a^2} \int \frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2}$$

$$= \left[ \text{Pr. 121: } I_n = \int \frac{dx}{(x^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} \right] = \frac{-(2\cdot 2-3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2-a^2)}$$

# Riešené príklady – 126

$$I_2=\int\frac{dx}{(x^2-a^2)^2} \qquad a>0$$

$$=\frac{1}{-a^2}\int\frac{-a^2\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{dx}{x^2-a^2}-\frac{1}{-2a^2}\int\frac{2x^2\,dx}{(x^2-a^2)^2}$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{-(2\cdot2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}$$

$$=-\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}$$

# Riešené príklady – 126

$$I_2=\int\frac{dx}{(x^2-a^2)^2} \qquad a>0$$

$$=\frac{1}{-a^2}\int\frac{-a^2\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{dx}{x^2-a^2}-\frac{1}{-2a^2}\int\frac{2x^2\,dx}{(x^2-a^2)^2}$$

$$=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^2} & v=-\frac{1}{x^2-a^2}\end{array}\right]$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{-(2\cdot 2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}$$

$$=-\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}=-\frac{1}{2a^2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c$$

# Riešené príklady – 126

$$I_2=\int\frac{dx}{(x^2-a^2)^2}=-\frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c \qquad a>0$$

$$=\frac{1}{-a^2}\int\frac{-a^2\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{dx}{x^2-a^2}-\frac{1}{-2a^2}\int\frac{2x^2\,dx}{(x^2-a^2)^2}$$

$$=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^2} & v=-\frac{1}{x^2-a^2}\end{array}\right]=-\frac{1}{a^2}\int\frac{dx}{x^2-a^2}+\frac{1}{2a^2}\left[-\frac{x}{x^2-a^2}+\int\frac{dx}{x^2-a^2}\right]$$

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{-(2\cdot2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}$$

$$=-\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}=-\frac{1}{2a^2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c$$

$$=-\frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c,\ x\in R-\{\pm a\},\ c\in R.$$

# Riešené príklady – 126

$$I_2=\int\frac{dx}{(x^2-a^2)^2}=-\frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c \qquad a>0$$

$$=\frac{1}{-a^2}\int\frac{-a^2\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{dx}{x^2-a^2}-\frac{1}{-2a^2}\int\frac{2x^2\,dx}{(x^2-a^2)^2}$$

$$=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^2} & v=-\frac{1}{x^2-a^2}\end{array}\right]=-\frac{1}{a^2}\int\frac{dx}{x^2-a^2}+\frac{1}{2a^2}\left[-\frac{x}{x^2-a^2}+\int\frac{dx}{x^2-a^2}\right]$$

$$=-\frac{1}{a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}+\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}$$

---

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{-(2\cdot 2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}$$

$$=-\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}=-\frac{1}{2a^2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c$$

$$=-\frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c,\ x\in R-\{\pm a\},\ c\in R.$$

# Riešené príklady – 126

$$I_2=\int\frac{dx}{(x^2-a^2)^2}=-\frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c \qquad a>0$$

$$=\frac{1}{-a^2}\int\frac{-a^2\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{dx}{x^2-a^2}-\frac{1}{-2a^2}\int\frac{2x^2\,dx}{(x^2-a^2)^2}$$

$$=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^2} & v=-\frac{1}{x^2-a^2}\end{array}\right]=-\frac{1}{a^2}\int\frac{dx}{x^2-a^2}+\frac{1}{2a^2}\left[-\frac{x}{x^2-a^2}+\int\frac{dx}{x^2-a^2}\right]$$

$$=-\frac{1}{a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}+\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}=-\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}$$

---

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{-(2\cdot2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}$$

$$=-\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}=-\frac{1}{2a^2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c$$

$$=-\frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c,\; x\in R-\{\pm a\},\; c\in R.$$

# Riešené príklady – 126

$$I_2=\int\frac{dx}{(x^2-a^2)^2}=-\frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c \qquad a>0$$

$$=\frac{1}{-a^2}\int\frac{-a^2\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{dx}{x^2-a^2}-\frac{1}{-2a^2}\int\frac{2x^2\,dx}{(x^2-a^2)^2}$$

$$=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^2} & v=-\frac{1}{x^2-a^2}\end{array}\right]=-\frac{1}{a^2}\int\frac{dx}{x^2-a^2}+\frac{1}{2a^2}\left[-\frac{x}{x^2-a^2}+\int\frac{dx}{x^2-a^2}\right]$$

$$=-\frac{1}{a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}+\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}=-\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}$$

$$=-\frac{1}{2a^2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c$$

---

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{-(2\cdot2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}$$

$$=-\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}=-\frac{1}{2a^2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c$$

$$=-\frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c,\ x\in R-\{\pm a\},\ c\in R.$$

# Riešené príklady – 126

$$I_2=\int\frac{dx}{(x^2-a^2)^2}=-\frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c \qquad a>0$$

$$=\frac{1}{-a^2}\int\frac{-a^2\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{(x^2-a^2-x^2)\,dx}{(x^2-a^2)^2}=\frac{1}{-a^2}\int\frac{dx}{x^2-a^2}-\frac{1}{-2a^2}\int\frac{2x^2\,dx}{(x^2-a^2)^2}$$

$$=\left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^2} & v=-\frac{1}{x^2-a^2}\end{array}\right]=-\frac{1}{a^2}\int\frac{dx}{x^2-a^2}+\frac{1}{2a^2}\left[-\frac{x}{x^2-a^2}+\int\frac{dx}{x^2-a^2}\right]$$

$$=-\frac{1}{a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}+\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}=-\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}$$

$$=-\frac{1}{2a^2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c=-\frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c,$$

$$x\in R-\{\pm a\},\ c\in R.$$

---

$$=\left[\text{Pr. 121: } I_n=\int\frac{dx}{(x^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}}\right]=\frac{-(2\cdot2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}$$

$$=-\frac{1}{2a^2}\int\frac{dx}{x^2-a^2}-\frac{x}{2a^2(x^2-a^2)}=-\frac{1}{2a^2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c$$

$$=-\frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right|-\frac{x}{2a^2(x^2-a^2)}+c,\ x\in R-\{\pm a\},\ c\in R.$$

# Riešené príklady – 127, 128

$I_3 = \int \frac{dx}{(x^2 - a^2)^3}$ $a > 0$

$I_4 = \int \frac{dx}{(x^2 - a^2)^4}$ $a > 0$

# Riešené príklady – 127, 128

$$I_3 = \int \frac{\mathrm{d}x}{(x^2-a^2)^3} \qquad a>0$$

$$= \left[ \text{Pr. 125: } I_n = \int \frac{\mathrm{d}x}{(x^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}} \right]$$

$$I_4 = \int \frac{\mathrm{d}x}{(x^2-a^2)^4} \qquad a>0$$

$$= \left[ \text{Pr. 125: } I_n = \int \frac{\mathrm{d}x}{(x^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}} \right]$$

# Riešené príklady – 127, 128

$I_3 = \int \frac{dx}{(x^2-a^2)^3}$ $\qquad a>0$

$$= \left[ \text{Pr. 125: } I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}} \right] = \frac{-(2\cdot 3-3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2-a^2)^2}$$

$I_4 = \int \frac{dx}{(x^2-a^2)^4}$ $\qquad a>0$

$$= \left[ \text{Pr. 125: } I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}} \right] = \frac{-(2\cdot 4-3)}{2a^2(4-1)} I_3 - \frac{x}{2a^2(4-1)(x^2-a^2)^3}$$

# Riešené príklady – 127, 128

$I_3 = \int \frac{dx}{(x^2-a^2)^3}$ $a>0$

$= \left[ \text{Pr. 125: } I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}} \right] = \frac{-(2\cdot 3-3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2-a^2)^2} = \left[ \text{Pr. 125} \right]$

$I_4 = \int \frac{dx}{(x^2-a^2)^4}$ $a>0$

$= \left[ \text{Pr. 125: } I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}} \right] = \frac{-(2\cdot 4-3)}{2a^2(4-1)} I_3 - \frac{x}{2a^2(4-1)(x^2-a^2)^3} = \left[ \text{Pr. 125} \right]$

# Riešené príklady – 127, 128

$$I_3 = \int \frac{dx}{(x^2-a^2)^3} \qquad a>0$$

$$= \left[ \text{Pr. 125: } I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}} \right] = \frac{-(2\cdot 3-3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2-a^2)^2} = \left[ \text{Pr. 125} \right]$$

$$= -\frac{3}{4a^2} \left[ \frac{-(2\cdot 2-3)}{2a^2(2-1)} I_1 - \frac{x}{2a^2(2-1)(x^2-a^2)} \right] - \frac{x}{4a^2(x^2-a^2)^2}$$

$$I_4 = \int \frac{dx}{(x^2-a^2)^4} \qquad a>0$$

$$= \left[ \text{Pr. 125: } I_n = \int \frac{dx}{(x^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}} \right] = \frac{-(2\cdot 4-3)}{2a^2(4-1)} I_3 - \frac{x}{2a^2(4-1)(x^2-a^2)^3} = \left[ \text{Pr. 125} \right]$$

$$= -\frac{5}{6a^2} \left[ \frac{-(2\cdot 3-3)}{2a^2(3-1)} I_2 - \frac{x}{2a^2(3-1)(x^2-a^2)^2} \right] - \frac{x}{6a^2(x^2-a^2)^3}$$

# Riešené príklady – 127, 128

$$I_3=\int\frac{dx}{(x^2-a^2)^3} \qquad a>0$$

$$=\left[\text{Pr. 125: } I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}\right]=\frac{-(2\cdot3-3)}{2a^2(3-1)}I_2-\frac{x}{2a^2(3-1)(x^2-a^2)^2}=\left[\text{Pr. 125}\right]$$

$$=-\frac{3}{4a^2}\left[\frac{-(2\cdot2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}\right]-\frac{x}{4a^2(x^2-a^2)^2}=\left[\text{Pr. 125: } I_1=\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|\right]$$

$$I_4=\int\frac{dx}{(x^2-a^2)^4} \qquad a>0$$

$$=\left[\text{Pr. 125: } I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}\right]=\frac{-(2\cdot4-3)}{2a^2(4-1)}I_3-\frac{x}{2a^2(4-1)(x^2-a^2)^3}=\left[\text{Pr. 125}\right]$$

$$=-\frac{5}{6a^2}\left[\frac{-(2\cdot3-3)}{2a^2(3-1)}I_2-\frac{x}{2a^2(3-1)(x^2-a^2)^2}\right]-\frac{x}{6a^2(x^2-a^2)^3}=\left[\text{Pr. 125}\right]$$

# Riešené príklady – 127, 128

$$I_3=\int\frac{dx}{(x^2-a^2)^3}=\frac{3}{16a^5}\ln\left|\frac{x-a}{x+a}\right|+\frac{3x}{8a^4(x^2-a^2)}-\frac{x}{4a^2(x^2-a^2)^2}+c \qquad a>0$$

$$=\left[\text{Pr. 125: } I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}\right]=\frac{-(2\cdot 3-3)}{2a^2(3-1)}I_2-\frac{x}{2a^2(3-1)(x^2-a^2)^2}=\left[\text{Pr. 125}\right]$$

$$=-\frac{3}{4a^2}\left[\frac{-(2\cdot 2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}\right]-\frac{x}{4a^2(x^2-a^2)^2}=\left[\text{Pr. 125: } I_1=\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|\right]$$

$$=\frac{3}{16a^5}\ln\left|\frac{x-a}{x+a}\right|+\frac{3x}{8a^4(x^2-a^2)}-\frac{x}{4a^2(x^2-a^2)^2}+c,\ x\in R-\{\pm a\},\ c\in R.$$

$$I_4=\int\frac{dx}{(x^2-a^2)^4} \qquad a>0$$

$$=\left[\text{Pr. 125: } I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}\right]=\frac{-(2\cdot 4-3)}{2a^2(4-1)}I_3-\frac{x}{2a^2(4-1)(x^2-a^2)^3}=\left[\text{Pr. 125}\right]$$

$$=-\frac{5}{6a^2}\left[\frac{-(2\cdot 3-3)}{2a^2(3-1)}I_2-\frac{x}{2a^2(3-1)(x^2-a^2)^2}\right]-\frac{x}{6a^2(x^2-a^2)^3}=\left[\text{Pr. 125}\right]$$

$$=\frac{15}{24a^4}\left[\frac{-(2\cdot 2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}\right]+\frac{5x}{24a^4(x^2-a^2)^2}-\frac{x}{6a^2(x^2-a^2)^3}$$

# Riešené príklady – 127, 128

$$I_3=\int\frac{dx}{(x^2-a^2)^3}=\frac{3}{16a^5}\ln\left|\frac{x-a}{x+a}\right|+\frac{3x}{8a^4(x^2-a^2)}-\frac{x}{4a^2(x^2-a^2)^2}+c \qquad a>0$$

$$=\left[\text{Pr. 125: } I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}\right]=\frac{-(2\cdot3-3)}{2a^2(3-1)}I_2-\frac{x}{2a^2(3-1)(x^2-a^2)^2}=\left[\text{Pr. 125}\right]$$

$$=-\frac{3}{4a^2}\left[\frac{-(2\cdot2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}\right]-\frac{x}{4a^2(x^2-a^2)^2}=\left[\text{Pr. 125: } I_1=\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|\right]$$

$$=\frac{3}{16a^5}\ln\left|\frac{x-a}{x+a}\right|+\frac{3x}{8a^4(x^2-a^2)}-\frac{x}{4a^2(x^2-a^2)^2}+c,\ x\in R-\{\pm a\},\ c\in R.$$

$$I_4=\int\frac{dx}{(x^2-a^2)^4} \qquad a>0$$

$$=\left[\text{Pr. 125: } I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}\right]=\frac{-(2\cdot4-3)}{2a^2(4-1)}I_3-\frac{x}{2a^2(4-1)(x^2-a^2)^3}=\left[\text{Pr. 125}\right]$$

$$=-\frac{5}{6a^2}\left[\frac{-(2\cdot3-3)}{2a^2(3-1)}I_2-\frac{x}{2a^2(3-1)(x^2-a^2)^2}\right]-\frac{x}{6a^2(x^2-a^2)^3}=\left[\text{Pr. 125}\right]$$

$$=\frac{15}{24a^4}\left[\frac{-(2\cdot2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}\right]+\frac{5x}{24a^4(x^2-a^2)^2}-\frac{x}{6a^2(x^2-a^2)^3}=\left[\text{Pr. 125: } I_1=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|\right]$$

# Riešené príklady – 127, 128

$$I_3=\int\frac{dx}{(x^2-a^2)^3}=\frac{3}{16a^5}\ln\left|\frac{x-a}{x+a}\right|+\frac{3x}{8a^4(x^2-a^2)}-\frac{x}{4a^2(x^2-a^2)^2}+c \qquad a>0$$

$$=\left[\text{Pr. 125: } I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}\right]=\frac{-(2\cdot3-3)}{2a^2(3-1)}I_2-\frac{x}{2a^2(3-1)(x^2-a^2)^2}=\left[\text{Pr. 125}\right]$$

$$=-\frac{3}{4a^2}\left[\frac{-(2\cdot2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}\right]-\frac{x}{4a^2(x^2-a^2)^2}=\left[\text{Pr. 125: } I_1=\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|\right]$$

$$=\frac{3}{16a^5}\ln\left|\frac{x-a}{x+a}\right|+\frac{3x}{8a^4(x^2-a^2)}-\frac{x}{4a^2(x^2-a^2)^2}+c,\ x\in R-\{\pm a\},\ c\in R.$$

$$I_4=\int\frac{dx}{(x^2-a^2)^4}=-\frac{5}{32a^7}\ln\left|\frac{x-a}{x+a}\right|-\frac{5x}{16a^6(x^2-a^2)}+\frac{5x}{24a^4(x^2-a^2)^2}-\frac{x}{6a^2(x^2-a^2)^3}+c \qquad a>0$$

$$=\left[\text{Pr. 125: } I_n=\int\frac{dx}{(x^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}}\right]=\frac{-(2\cdot4-3)}{2a^2(4-1)}I_3-\frac{x}{2a^2(4-1)(x^2-a^2)^3}=\left[\text{Pr. 125}\right]$$

$$=-\frac{5}{6a^2}\left[\frac{-(2\cdot3-3)}{2a^2(3-1)}I_2-\frac{x}{2a^2(3-1)(x^2-a^2)^2}\right]-\frac{x}{6a^2(x^2-a^2)^3}=\left[\text{Pr. 125}\right]$$

$$=\frac{15}{24a^4}\left[\frac{-(2\cdot2-3)}{2a^2(2-1)}I_1-\frac{x}{2a^2(2-1)(x^2-a^2)}\right]+\frac{5x}{24a^4(x^2-a^2)^2}-\frac{x}{6a^2(x^2-a^2)^3}=\left[\text{Pr. 125: } I_1=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|\right]$$

$$=-\frac{5}{32a^7}\ln\left|\frac{x-a}{x+a}\right|-\frac{5x}{16a^6(x^2-a^2)}+\frac{5x}{24a^4(x^2-a^2)^2}-\frac{x}{6a^2(x^2-a^2)^3}+c,$$

$$x\in R-\{\pm a\},\ c\in R.$$

# Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

# Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array} \right]$$

---

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array} \right]$$

# Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array} \right] = \int \frac{1\cdot dt}{(t^2+1)^2}$$

---

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2}$$

# Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

$$=\left[\begin{array}{r|r|r} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array}\right]=\int \frac{1\cdot dt}{(t^2+1)^2}=\int \frac{(t^2+1-t^2)\,dx}{(t^2+1)^2}$$

$$=\left[\begin{array}{r|r|r} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array}\right]=\int \frac{dt}{(t^2+1)^2}=\left[\begin{array}{l} \text{Pr. 121: } I_n=\int \frac{dt}{(t^2+a^2)^n} \\ I_n=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array}\right]$$

# Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

$$= \left[ \begin{array}{r|r|r} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array} \right] = \int \frac{1\cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2)\,dx}{(t^2+1)^2}$$

$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\int \frac{2t^2\,dt}{(t^2+1)^2} \quad \left[ \begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1} \end{array} \right]$$

---

$$= \left[ \begin{array}{r|r|r} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2} = \left[ \begin{array}{l} \text{Pr. 121: } I_n=\int \frac{dt}{(t^2+a^2)^n} \\ I_n=\frac{2n-3}{2a^2(n-1)}\,I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array} \right]$$

$$= \frac{2\cdot 2-3}{2(2-1)}\,I_1 + \frac{t}{2(2-1)(t^2+1)}$$

# Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

$$= \left[ \begin{array}{r|r|r} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array} \right] = \int \frac{1\cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2)\,dx}{(t^2+1)^2}$$

$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\int \frac{2t^2\,dt}{(t^2+1)^2} = \left[ \begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1} \end{array} \right]$$

---

$$= \left[ \begin{array}{r|r|r} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2} = \left[ \begin{array}{l} \text{Pr. 121: } I_n=\int \frac{dt}{(t^2+a^2)^n} \\ I_n=\frac{2n-3}{2a^2(n-1)} I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array} \right]$$

$$= \frac{2\cdot 2-3}{2(2-1)} I_1 + \frac{t}{2(2-1)(t^2+1)} = \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)}$$

# Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array} \right] = \int \frac{1\cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2)\,dx}{(t^2+1)^2}$$

$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\int \frac{2t^2\,dt}{(t^2+1)^2} = \left[ \begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1} \end{array} \right] = \int \frac{dt}{t^2+1} - \frac{1}{2}\left[ -\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right]$$

---

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array} \right] = \int \frac{dt}{(t^2+1)^2} = \left[ \begin{array}{l} \text{Pr. 121: } I_n=\int \frac{dt}{(t^2+a^2)^n} \\ I_n=\frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array} \right]$$

$$= \frac{2\cdot 2-3}{2(2-1)} I_1 + \frac{t}{2(2-1)(t^2+1)} = \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\operatorname{arctg} t + \frac{t}{2(t^2+1)} + c$$

# Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2} = \frac{1}{2}\,\mathrm{arctg}\,(x+2) + \frac{x+2}{2(x^2+4x+5)} + c$$

$$= \left[\begin{array}{r|r|r} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array}\right] = \int \frac{1\cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2)\,dx}{(t^2+1)^2}$$

$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\int \frac{2t^2\,dt}{(t^2+1)^2} = \left[\begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1} \end{array}\right] = \int \frac{dt}{t^2+1} - \frac{1}{2}\left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1}\right]$$

$$= \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} - \frac{1}{2}\int \frac{dt}{t^2+1}$$

---

$$= \left[\begin{array}{r|r|r} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array}\right] = \int \frac{dt}{(t^2+1)^2} = \left[\begin{array}{l} \text{Pr. 121: } I_n=\int \frac{dt}{(t^2+a^2)^n} \\ I_n=\frac{2n-3}{2a^2(n-1)}\,I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array}\right]$$

$$= \frac{2\cdot 2-3}{2(2-1)}\,I_1 + \frac{t}{2(2-1)(t^2+1)} = \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\,\mathrm{arctg}\,t + \frac{t}{2(t^2+1)} + c$$

$$= \frac{1}{2}\,\mathrm{arctg}\,(x+2) + \frac{x+2}{2(x^2+4x+5)} + c,\ x\in R,\ c\in R.$$

# Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2} = \frac{1}{2}\operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c$$

$$= \left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array}\right] = \int \frac{1\cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2)\,dx}{(t^2+1)^2}$$

$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\int \frac{2t^2\,dt}{(t^2+1)^2} = \left[\begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1} \end{array}\right] = \int \frac{dt}{t^2+1} - \frac{1}{2}\left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1}\right]$$

$$= \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} - \frac{1}{2}\int \frac{dt}{t^2+1} = \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)}$$

---

$$= \left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array}\right] = \int \frac{dt}{(t^2+1)^2} = \left[\begin{array}{l} \text{Pr. 121: } I_n=\int \frac{dt}{(t^2+a^2)^n} \\ I_n=\frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array}\right]$$

$$= \frac{2\cdot 2-3}{2(2-1)} I_1 + \frac{t}{2(2-1)(t^2+1)} = \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\operatorname{arctg} t + \frac{t}{2(t^2+1)} + c$$

$$= \frac{1}{2}\operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c,\ x\in R,\ c\in R.$$

# Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2} = \frac{1}{2}\,\mathrm{arctg}\,(x+2) + \frac{x+2}{2(x^2+4x+5)} + c$$

$$= \left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array}\right] = \int \frac{1\cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2)\,dx}{(t^2+1)^2}$$

$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\int \frac{2t^2\,dt}{(t^2+1)^2} = \left[\begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1} \end{array}\right] = \int \frac{dt}{t^2+1} - \frac{1}{2}\left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1}\right]$$

$$= \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} - \frac{1}{2}\int \frac{dt}{t^2+1} = \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\,\mathrm{arctg}\,t + \frac{t}{2(t^2+1)} + c$$

---

$$= \left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+5=x^2+4x+4+1 & x\in R \\ dt=dx & =(x+2)^2+1=t^2+1 & t\in R \end{array}\right] = \int \frac{dt}{(t^2+1)^2} = \left[\begin{array}{l} \text{Pr. 121: } I_n=\int\frac{dt}{(t^2+a^2)^n} \\ I_n=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{array}\right]$$

$$= \frac{2\cdot 2-3}{2(2-1)}I_1 + \frac{t}{2(2-1)(t^2+1)} = \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\,\mathrm{arctg}\,t + \frac{t}{2(t^2+1)} + c$$

$$= \frac{1}{2}\,\mathrm{arctg}\,(x+2) + \frac{x+2}{2(x^2+4x+5)} + c,\ x\in R,\ c\in R.$$

# Riešené príklady – 129

$$\int \frac{dx}{(x^2+4x+5)^2} = \frac{1}{2}\operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c$$

$$= \left[ \begin{matrix} \text{Subst. } t=x+2 \\ dt=dx \end{matrix} \middle| \begin{matrix} x^2+4x+5=x^2+4x+4+1 \\ =(x+2)^2+1=t^2+1 \end{matrix} \middle| \begin{matrix} x\in R \\ t\in R \end{matrix} \right] = \int \frac{1\cdot dt}{(t^2+1)^2} = \int \frac{(t^2+1-t^2)\,dx}{(t^2+1)^2}$$

$$= \int \frac{dt}{t^2+1} - \frac{1}{2}\int \frac{2t^2\,dt}{(t^2+1)^2} = \left[ \begin{matrix} u=t \\ v'=\frac{2t}{(t^2+1)^2} \end{matrix} \middle| \begin{matrix} u'=1 \\ v=-\frac{1}{t^2+1} \end{matrix} \right] = \int \frac{dt}{t^2+1} - \frac{1}{2}\left[ -\frac{t}{t^2+1} + \int \frac{dt}{t^2+1} \right]$$

$$= \int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} - \frac{1}{2}\int \frac{dt}{t^2+1} = \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\operatorname{arctg} t + \frac{t}{2(t^2+1)} + c$$

$$= \frac{1}{2}\operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c,\ x\in R,\ c\in R.$$

---

$$= \left[ \begin{matrix} \text{Subst. } t=x+2 \\ dt=dx \end{matrix} \middle| \begin{matrix} x^2+4x+5=x^2+4x+4+1 \\ =(x+2)^2+1=t^2+1 \end{matrix} \middle| \begin{matrix} x\in R \\ t\in R \end{matrix} \right] = \int \frac{dt}{(t^2+1)^2} = \left[ \begin{matrix} \text{Pr. 121: } I_n=\int \frac{dt}{(t^2+a^2)^n} \\ I_n=\frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \end{matrix} \right]$$

$$= \frac{2\cdot 2-3}{2(2-1)} I_1 + \frac{t}{2(2-1)(t^2+1)} = \frac{1}{2}\int \frac{dt}{t^2+1} + \frac{t}{2(t^2+1)} = \frac{1}{2}\operatorname{arctg} t + \frac{t}{2(t^2+1)} + c$$

$$= \frac{1}{2}\operatorname{arctg}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c,\ x\in R,\ c\in R.$$

# Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

# Riešené príklady – 130

$$\int \frac{\mathrm{d}x}{(x^2+4x+3)^2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in R-\{-3,-1\} \\ \mathrm{d}t=\mathrm{d}x & =(x+2)^2-1=t^2-1 & t\in R-\{\pm 1\} \end{array} \right]$$

# Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in R-\{-3,-1\} \\ dt=dx & =(x+2)^2-1=t^2-1 & t\in R-\{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = -\int \frac{-1\cdot dt}{(t^2-1)^2}$$

# Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in R-\{-3,-1\} \\ dt=dx & =(x+2)^2-1=t^2-1 & t\in R-\{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = -\int \frac{-1\cdot dt}{(t^2-1)^2}$$

$$= -\int \frac{(t^2-1-t^2)\,dt}{(t^2-1)^2}$$

# Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in R-\{-3,-1\} \\ dt=dx & =(x+2)^2-1=t^2-1 & t\in R-\{\pm1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = -\int \frac{-1\cdot dt}{(t^2-1)^2}$$

$$= -\int \frac{(t^2-1-t^2)\,dt}{(t^2-1)^2} = -\int \frac{dt}{t^2-1} + \frac{1}{2}\int \frac{2t^2\,dt}{(t^2-1)^2}$$

# Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in R-\{-3,-1\} \\ dt=dx & =(x+2)^2-1=t^2-1 & t\in R-\{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = -\int \frac{-1\cdot dt}{(t^2-1)^2}$$

$$= -\int \frac{(t^2-1-t^2)\,dt}{(t^2-1)^2} = -\int \frac{dt}{t^2-1} + \frac{1}{2}\int \frac{2t^2\,dt}{(t^2-1)^2}$$

$$= \left[ \begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1} \end{array} \right]$$

# Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in R-\{-3,-1\} \\ dt=dx & =(x+2)^2-1=t^2-1 & t\in R-\{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = -\int \frac{-1\cdot dt}{(t^2-1)^2}$$

$$= -\int \frac{(t^2-1-t^2)\,dt}{(t^2-1)^2} = -\int \frac{dt}{t^2-1} + \frac{1}{2}\int \frac{2t^2\,dt}{(t^2-1)^2}$$

$$= \left[ \begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1} \end{array} \right] = -\int \frac{dt}{t^2-1} + \frac{1}{2}\left[ -\frac{t}{t^2-1} + \int \frac{dt}{t^2-1} \right]$$

# Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in R-\{-3,-1\} \\ dt=dx & =(x+2)^2-1=t^2-1 & t\in R-\{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = -\int \frac{-1\cdot dt}{(t^2-1)^2}$$

$$= -\int \frac{(t^2-1-t^2)\,dt}{(t^2-1)^2} = -\int \frac{dt}{t^2-1} + \frac{1}{2}\int \frac{2t^2\,dt}{(t^2-1)^2}$$

$$= \left[ \begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1} \end{array} \right] = -\int \frac{dt}{t^2-1} + \frac{1}{2}\left[ -\frac{t}{t^2-1} + \int \frac{dt}{t^2-1} \right]$$

$$= -\int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} + \frac{1}{2}\int \frac{dt}{t^2-1}$$

# Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in R-\{-3,-1\} \\ dt=dx & =(x+2)^2-1=t^2-1 & t\in R-\{\pm 1\} \end{array} \right] = \int \frac{dt}{(t^2-1)^2} = -\int \frac{-1\cdot dt}{(t^2-1)^2}$$

$$= -\int \frac{(t^2-1-t^2)\,dt}{(t^2-1)^2} = -\int \frac{dt}{t^2-1} + \frac{1}{2}\int \frac{2t^2\,dt}{(t^2-1)^2}$$

$$= \left[ \begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1} \end{array} \right] = -\int \frac{dt}{t^2-1} + \frac{1}{2}\left[ -\frac{t}{t^2-1} + \int \frac{dt}{t^2-1} \right]$$

$$= -\int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} + \frac{1}{2}\int \frac{dt}{t^2-1} = -\frac{1}{2}\int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1^2)}$$

# Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$=\left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in R-\{-3,-1\} \\ dt=dx & =(x+2)^2-1=t^2-1 & t\in R-\{\pm1\}\end{array}\right]=\int\frac{dt}{(t^2-1)^2}=-\int\frac{-1\cdot dt}{(t^2-1)^2}$$

$$=-\int\frac{(t^2-1-t^2)\,dt}{(t^2-1)^2}=-\int\frac{dt}{t^2-1}+\frac{1}{2}\int\frac{2t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}\end{array}\right]=-\int\frac{dt}{t^2-1}+\frac{1}{2}\left[-\frac{t}{t^2-1}+\int\frac{dt}{t^2-1}\right]$$

$$=-\int\frac{dt}{t^2-1}-\frac{t}{2(t^2-1)}+\frac{1}{2}\int\frac{dt}{t^2-1}=-\frac{1}{2}\int\frac{dt}{t^2-1}-\frac{t}{2(t^2-1^2)}$$

$$=-\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|-\frac{t}{2(t^2-1)}+c$$

# Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2}$$

$$=\left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in R-\{-3,-1\} \\ dt=dx & =(x+2)^2-1=t^2-1 & t\in R-\{\pm 1\} \end{array}\right] = \int \frac{dt}{(t^2-1)^2} = -\int \frac{-1\cdot dt}{(t^2-1)^2}$$

$$= -\int \frac{(t^2-1-t^2)\,dt}{(t^2-1)^2} = -\int \frac{dt}{t^2-1} + \frac{1}{2}\int \frac{2t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1} \end{array}\right] = -\int \frac{dt}{t^2-1} + \frac{1}{2}\left[-\frac{t}{t^2-1} + \int \frac{dt}{t^2-1}\right]$$

$$= -\int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} + \frac{1}{2}\int \frac{dt}{t^2-1} = -\frac{1}{2}\int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1^2)}$$

$$= -\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - \frac{t}{2(t^2-1)} + c = -\frac{1}{4}\ln\left|\frac{x+2-1}{x+2+1}\right| - \frac{x+2}{2(x^2+4x+3)} + c$$

# Riešené príklady – 130

$$\int \frac{dx}{(x^2+4x+3)^2} = -\frac{1}{4}\ln\left|\frac{x+1}{x+3}\right| - \frac{x+2}{2(x^2+4x+3)} + c$$

$$= \left[\begin{array}{r|r|l} \text{Subst. } t=x+2 & x^2+4x+3=x^2+4x+4-1 & x\in R-\{-3,-1\} \\ dt=dx & =(x+2)^2-1=t^2-1 & t\in R-\{\pm 1\}\end{array}\right] = \int \frac{dt}{(t^2-1)^2} = -\int \frac{-1\cdot dt}{(t^2-1)^2}$$

$$= -\int \frac{(t^2-1-t^2)\,dt}{(t^2-1)^2} = -\int \frac{dt}{t^2-1} + \frac{1}{2}\int \frac{2t^2\,dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}\end{array}\right] = -\int \frac{dt}{t^2-1} + \frac{1}{2}\left[-\frac{t}{t^2-1} + \int \frac{dt}{t^2-1}\right]$$

$$= -\int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1)} + \frac{1}{2}\int \frac{dt}{t^2-1} = -\frac{1}{2}\int \frac{dt}{t^2-1} - \frac{t}{2(t^2-1^2)}$$

$$= -\frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - \frac{t}{2(t^2-1)} + c = -\frac{1}{4}\ln\left|\frac{x+2-1}{x+2+1}\right| - \frac{x+2}{2(x^2+4x+3)} + c$$

$$= -\frac{1}{4}\ln\left|\frac{x+1}{x+3}\right| - \frac{x+2}{2(x^2+4x+3)} + c$$

$$= \frac{1}{4}\ln|x+3| - \frac{1}{4}\ln|x+1| - \frac{x+2}{2(x^2+4x+3)} + c,\ x\in R-\{-3,-1\},\ c\in R.$$

# Riešené príklady – 131, 132

$\int \frac{x^2\,dx}{(x^2+a^2)^2}$ $a > 0$

$\int \frac{x^2\,dx}{(x^2-a^2)^2}$ $a > 0$

# Riešené príklady – 131, 132

$\int \frac{x^2\,dx}{(x^2+a^2)^2}$ $\quad a > 0$

$$= \frac{1}{2} \int \frac{2x^2\,dx}{(x^2+a^2)^2}$$

$\int \frac{x^2\,dx}{(x^2-a^2)^2}$ $\quad a > 0$

$$= \frac{1}{2} \int \frac{2x^2\,dx}{(x^2-a^2)^2}$$

# Riešené príklady – 131, 132

$$\int \frac{x^2\,dx}{(x^2+a^2)^2} \qquad a > 0$$

$$= \frac{1}{2}\int \frac{2x^2\,dx}{(x^2+a^2)^2} = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \frac{2x}{(x^2+a^2)^2} & v = -\frac{1}{x^2+a^2} \end{array}\right]$$

$$\int \frac{x^2\,dx}{(x^2-a^2)^2} \qquad a > 0$$

$$= \frac{1}{2}\int \frac{2x^2\,dx}{(x^2-a^2)^2} = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \frac{2x}{(x^2-a^2)^2} & v = -\frac{1}{x^2-a^2} \end{array}\right]$$

# Riešené príklady – 131, 132

$\int \frac{x^2\,dx}{(x^2+a^2)^2}$ $\qquad a>0$

$$= \frac{1}{2}\int \frac{2x^2\,dx}{(x^2+a^2)^2} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^2} & v=-\frac{1}{x^2+a^2}\end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2+a^2}+\int\frac{dx}{x^2+a^2}\right]$$

$\int \frac{x^2\,dx}{(x^2-a^2)^2}$ $\qquad a>0$

$$= \frac{1}{2}\int \frac{2x^2\,dx}{(x^2-a^2)^2} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^2} & v=-\frac{1}{x^2-a^2}\end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2+a^2}+\int\frac{dx}{x^2-a^2}\right]$$

# Riešené príklady – 131, 132

$$\int \frac{x^2\,dx}{(x^2+a^2)^2} \qquad a>0$$

$$= \frac{1}{2}\int \frac{2x^2\,dx}{(x^2+a^2)^2} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^2} & v=-\frac{1}{x^2+a^2} \end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2+a^2}+\int\frac{dx}{x^2+a^2}\right]$$

$$= -\frac{x}{2(x^2+a^2)} + \frac{1}{2}\cdot\frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a} + c$$

$$\int \frac{x^2\,dx}{(x^2-a^2)^2} \qquad a>0$$

$$= \frac{1}{2}\int \frac{2x^2\,dx}{(x^2-a^2)^2} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^2} & v=-\frac{1}{x^2-a^2} \end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2+a^2}+\int\frac{dx}{x^2-a^2}\right]$$

$$= -\frac{x}{2(x^2-a^2)} + \frac{1}{2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$$

# Riešené príklady – 131, 132

$$\int \frac{x^2\,dx}{(x^2+a^2)^2} = \frac{1}{2a}\operatorname{arctg}\frac{x}{a} - \frac{x}{2(x^2+a^2)} + c \qquad a>0$$

$$= \frac{1}{2}\int \frac{2x^2\,dx}{(x^2+a^2)^2} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2+a^2)^2} & v=-\frac{1}{x^2+a^2} \end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2+a^2}\right]$$

$$= -\frac{x}{2(x^2+a^2)} + \frac{1}{2}\cdot\frac{1}{a}\operatorname{arctg}\frac{x}{a} + c = \frac{1}{2a}\operatorname{arctg}\frac{x}{a} - \frac{x}{2(x^2+a^2)} + c,\ x\in R,\ c\in R.$$

$$\int \frac{x^2\,dx}{(x^2-a^2)^2} = \frac{1}{4a}\ln\left|\frac{x-a}{x+a}\right| - \frac{x}{2(x^2-a^2)} + c \qquad a>0$$

$$= \frac{1}{2}\int \frac{2x^2\,dx}{(x^2-a^2)^2} = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\frac{2x}{(x^2-a^2)^2} & v=-\frac{1}{x^2-a^2} \end{array}\right] = \frac{1}{2}\left[-\frac{x}{x^2+a^2} + \int \frac{dx}{x^2-a^2}\right]$$

$$= -\frac{x}{2(x^2-a^2)} + \frac{1}{2}\cdot\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c = \frac{1}{4a}\ln\left|\frac{x-a}{x+a}\right| - \frac{x}{2(x^2-a^2)} + c$$

$$= \frac{1}{4a}\ln|x-a| - \frac{1}{4a}\ln|x+a| - \frac{x}{2(x^2-a^2)} + c,\ x\in R-\{\pm a\},\ c\in R.$$

# Riešené príklady – 133, 134

$\int \frac{x\,dx}{x^2+a^2}$ $a > 0$

$\int \frac{x\,dx}{(x^2+a^2)^n}$ $a > 0,\ n \in N,\ n \neq 1$

# Riešené príklady – 133, 134

$\int \frac{x\,dx}{x^2+a^2}$ $\quad a > 0$

$$= \frac{1}{2}\int \frac{2x\,dx}{x^2+a^2}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t = x^2+a^2 & x \in (-\infty; 0\rangle,\ t \in \langle a^2; \infty) \\ dt = 2x\,dx & x \in \langle 0; \infty),\ t \in \langle a^2; \infty) \end{array} \right]$$

$\int \frac{x\,dx}{(x^2+a^2)^n}$ $\quad a > 0,\ n \in N,\ n \neq 1$

$$= \left[ \begin{array}{r|l} \text{Subst. } t = x^2+a^2 & x \in (-\infty; 0\rangle,\ t \in \langle a^2; \infty) \\ dt = 2x\,dx & x \in \langle 0; \infty),\ t \in \langle a^2; \infty) \end{array} \right]$$

# Riešené príklady – 133, 134

$\int \frac{x\,dx}{x^2+a^2}$ $\qquad a>0$

$$= \frac{1}{2}\int \frac{2x\,dx}{x^2+a^2} = \frac{1}{2}\int \frac{(x^2+a^2)'\,dx}{x^2+a^2}$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2+a^2 & x\in(-\infty;0),\ t\in\langle a^2;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle a^2;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t}$$

$\int \frac{x\,dx}{(x^2+a^2)^n}$ $\qquad a>0,\ n\in N,\ n\neq 1$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2+a^2 & x\in(-\infty;0),\ t\in\langle a^2;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle a^2;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^n}$$

# Riešené príklady – 133, 134

$$\int \frac{x\,dx}{x^2+a^2} \qquad a>0$$

$$= \frac{1}{2}\int \frac{2x\,dx}{x^2+a^2} = \frac{1}{2}\int \frac{(x^2+a^2)'\,dx}{x^2+a^2} = \frac{1}{2}\ln|x^2+a^2| + c$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^2+a^2 & x\in(-\infty;0),\, t\in\langle a^2;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\, t\in\langle a^2;\infty) \end{array} \right] = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln t + c$$

$$\int \frac{x\,dx}{(x^2+a^2)^n} \qquad a>0,\ n\in N,\ n\neq 1$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^2+a^2 & x\in(-\infty;0),\, t\in\langle a^2;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\, t\in\langle a^2;\infty) \end{array} \right] = \frac{1}{2}\int \frac{dt}{t^n} = \frac{1}{2}\int t^{-n}\,dt$$

# Riešené príklady – 133, 134

$$\int \frac{x\,dx}{x^2+a^2} = \frac{1}{2}\ln\left(x^2+a^2\right) + c \qquad a>0$$

$$= \frac{1}{2}\int \frac{2x\,dx}{x^2+a^2} = \frac{1}{2}\int \frac{(x^2+a^2)'\,dx}{x^2+a^2} = \frac{1}{2}\ln\left|x^2+a^2\right| + c = \frac{1}{2}\ln\left(x^2+a^2\right) + c,$$

$x\in R,\ c\in R.$

---

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2+a^2 & x\in(-\infty;0),\ t\in\langle a^2;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle a^2;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln t + c = \frac{1}{2}\ln\left(x^2+a^2\right) + c,$$

$x\in R,\ c\in R.$

$$\int \frac{x\,dx}{(x^2+a^2)^n} \qquad a>0,\ n\in N,\ n\neq 1$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2+a^2 & x\in(-\infty;0),\ t\in\langle a^2;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle a^2;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^n} = \frac{1}{2}\int t^{-n}\,dt = \frac{t^{1-n}}{2(1-n)} + c$$

# Riešené príklady – 133, 134

$$\int \frac{x\,dx}{x^2+a^2} = \frac{1}{2}\ln\left(x^2+a^2\right)+c \qquad a>0$$

$$= \frac{1}{2}\int \frac{2x\,dx}{x^2+a^2} = \frac{1}{2}\int \frac{(x^2+a^2)'\,dx}{x^2+a^2} = \frac{1}{2}\ln\left|x^2+a^2\right|+c = \frac{1}{2}\ln\left(x^2+a^2\right)+c,$$

$x \in R$, $c \in R$.

---

$$= \left[\begin{array}{c|l} \text{Subst. } t=x^2+a^2 & x\in(-\infty;0),\ t\in\langle a^2;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle a^2;\infty) \end{array}\right] = \frac{1}{2}\int\frac{dt}{t} = \frac{1}{2}\ln t + c = \frac{1}{2}\ln\left(x^2+a^2\right)+c,$$

$x \in R$, $c \in R$.

$$\int \frac{x\,dx}{(x^2+a^2)^n} = -\frac{1}{2(n-1)(x^2+a^2)^{n-1}}+c \qquad a>0,\ n\in N,\ n\neq 1$$

$$= \left[\begin{array}{c|l} \text{Subst. } t=x^2+a^2 & x\in(-\infty;0),\ t\in\langle a^2;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle a^2;\infty) \end{array}\right] = \frac{1}{2}\int\frac{dt}{t^n} = \frac{1}{2}\int t^{-n}\,dt = \frac{t^{1-n}}{2(1-n)}+c$$

$$= \frac{(x^2+a^2)^{1-n}}{2(1-n)}+c = -\frac{1}{2(n-1)(x^2+a^2)^{n-1}}+c,\ x\in R,\ c\in R.$$

# Riešené príklady – 135, 136

$\int \frac{x\,dx}{x^2-a^2}$ $a>0$

$\int \frac{x\,dx}{(x^2-a^2)^n}$ $a>0,\ n\in N,\ n\neq 1$

# Riešené príklady – 135, 136

$$\int \frac{x\,dx}{x^2-a^2} \qquad a>0$$

$$= \frac{1}{2}\int \frac{2x\,dx}{x^2-a^2}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^2-a^2 & x\in(-\infty;0)-\{-a\},\ t\in\langle -a^2;\infty)-\{0\} \\ dt=2x\,dx & x\in\langle 0;\infty)-\{a\},\ t\in\langle -a^2;\infty)-\{0\} \end{array} \right]$$

$$\int \frac{x\,dx}{(x^2-a^2)^n} \qquad a>0,\ n\in N,\ n\neq 1$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^2-a^2 & x\in(-\infty;0)-\{-a\},\ t\in\langle -a^2;\infty)-\{0\} \\ dt=2x\,dx & x\in\langle 0;\infty)-\{a\},\ t\in\langle -a^2;\infty)-\{0\} \end{array} \right]$$

# Riešené príklady – 135, 136

$$\int \frac{x\,dx}{x^2-a^2} \qquad a>0$$

$$= \frac{1}{2}\int \frac{2x\,dx}{x^2-a^2} = \frac{1}{2}\int \frac{(x^2-a^2)'\,dx}{x^2-a^2}$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2-a^2 & x\in(-\infty;0)-\{-a\},\ t\in\langle -a^2;\infty)-\{0\} \\ dt=2x\,dx & x\in\langle 0;\infty)-\{a\},\ t\in\langle -a^2;\infty)-\{0\} \end{array}\right] = \frac{1}{2}\int \frac{dt}{t}$$

$$\int \frac{x\,dx}{(x^2-a^2)^n} \qquad a>0,\ n\in N,\ n\neq 1$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2-a^2 & x\in(-\infty;0)-\{-a\},\ t\in\langle -a^2;\infty)-\{0\} \\ dt=2x\,dx & x\in\langle 0;\infty)-\{a\},\ t\in\langle -a^2;\infty)-\{0\} \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^n}$$

# Riešené príklady – 135, 136

$$\int \frac{x\,dx}{x^2-a^2} = \frac{1}{2}\ln|x^2-a^2|+c \qquad a>0$$

$$= \frac{1}{2}\int \frac{2x\,dx}{x^2-a^2} = \frac{1}{2}\int \frac{(x^2-a^2)'\,dx}{x^2-a^2} = \frac{1}{2}\ln|x^2-a^2|+c$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2-a^2 & x\in(-\infty;0)-\{-a\},\ t\in\langle -a^2;\infty)-\{0\} \\ dt=2x\,dx & x\in\langle 0;\infty)-\{a\},\ t\in\langle -a^2;\infty)-\{0\} \end{array}\right] = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t|+c$$

$$\int \frac{x\,dx}{(x^2-a^2)^n} \qquad a>0,\ n\in N,\ n\neq 1$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2-a^2 & x\in(-\infty;0)-\{-a\},\ t\in\langle -a^2;\infty)-\{0\} \\ dt=2x\,dx & x\in\langle 0;\infty)-\{a\},\ t\in\langle -a^2;\infty)-\{0\} \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^n} = \frac{1}{2}\int t^{-n}\,dt$$

# Riešené príklady – 135, 136

$$\int \frac{x\,dx}{x^2-a^2} = \frac{1}{2}\ln|x^2-a^2| + c \qquad a>0$$

$$= \frac{1}{2}\int \frac{2x\,dx}{x^2-a^2} = \frac{1}{2}\int \frac{(x^2-a^2)'\,dx}{x^2-a^2} = \frac{1}{2}\ln|x^2-a^2| + c,\ x\in R-\{\pm a\},\ c\in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2-a^2 & x\in(-\infty;0)-\{-a\},\ t\in\langle -a^2;\infty)-\{0\} \\ dt=2x\,dx & x\in\langle 0;\infty)-\{a\},\ t\in\langle -a^2;\infty)-\{0\} \end{array}\right] = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t|+c$$

$$= \frac{1}{2}\ln|x^2-a^2|+c,\ x\in R-\{\pm a\},\ c\in R.$$

$$\int \frac{x\,dx}{(x^2-a^2)^n} \qquad a>0,\ n\in N,\ n\neq 1$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2-a^2 & x\in(-\infty;0)-\{-a\},\ t\in\langle -a^2;\infty)-\{0\} \\ dt=2x\,dx & x\in\langle 0;\infty)-\{a\},\ t\in\langle -a^2;\infty)-\{0\} \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^n} = \frac{1}{2}\int t^{-n}\,dt = \frac{t^{1-n}}{2(1-n)} + c$$

# Riešené príklady – 135, 136

$$\int \frac{x\,dx}{x^2-a^2} = \frac{1}{2}\ln|x^2-a^2| + c \qquad a>0$$

$$= \frac{1}{2}\int \frac{2x\,dx}{x^2-a^2} = \frac{1}{2}\int \frac{(x^2-a^2)'\,dx}{x^2-a^2} = \frac{1}{2}\ln|x^2-a^2| + c,\ x\in R-\{\pm a\},\ c\in R.$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^2-a^2 & x\in(-\infty;0)-\{-a\},\ t\in\langle -a^2;\infty)-\{0\} \\ dt=2x\,dx & x\in\langle 0;\infty)-\{a\},\ t\in\langle -a^2;\infty)-\{0\} \end{array} \right] = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t|+c$$

$$= \frac{1}{2}\ln|x^2-a^2|+c,\ x\in R-\{\pm a\},\ c\in R.$$

$$\int \frac{x\,dx}{(x^2-a^2)^n} = -\frac{1}{2(n-1)(x^2-a^2)^{n-1}} + c \qquad a>0,\ n\in N,\ n\neq 1$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^2-a^2 & x\in(-\infty;0)-\{-a\},\ t\in\langle -a^2;\infty)-\{0\} \\ dt=2x\,dx & x\in\langle 0;\infty)-\{a\},\ t\in\langle -a^2;\infty)-\{0\} \end{array} \right] = \frac{1}{2}\int \frac{dt}{t^n} = \frac{1}{2}\int t^{-n}\,dt = \frac{t^{1-n}}{2(1-n)} + c$$

$$= \frac{(x^2-a^2)^{1-n}}{2(1-n)} + c = -\frac{1}{2(n-1)(x^2-a^2)^{n-1}} + c,\ x\in R-\{\pm a\},\ c\in R.$$

# Riešené príklady – 137, 138

$$\int \frac{x^n\,dx}{x-1} \qquad n \in N$$

$$\int \frac{x\,dx}{x-1}$$

# Riešené príklady – 137, 138

$\int \frac{x^n\,dx}{x-1}$ $\quad n \in N$

$$= \int \frac{x^n - 1 + 1}{x-1}\,dx$$

$\int \frac{x\,dx}{x-1}$

$$= \int \frac{x-1+1}{x-1}\,dx$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t = x-1 & x \in R - \{1\} \\ dt = dx & t \in R - \{0\} \end{array} \right] = \int \frac{t+1}{t}\,dt$$

# Riešené príklady – 137, 138

$\int \frac{x^n\,dx}{x-1}$ $\qquad n \in N$

$$= \int \frac{x^n-1+1}{x-1}\,dx = \int \left[\frac{x^n-1}{x-1} + \frac{1}{x-1}\right] dx$$

$\int \frac{x\,dx}{x-1}$

$$= \int \frac{x-1+1}{x-1}\,dx = \int \left[1 + \frac{1}{x-1}\right] dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\} \end{array}\right] = \int \frac{t+1}{t}\,dt = \int \left[1+\frac{1}{t}\right] dt$$

# Riešené príklady – 137, 138

$\int \frac{x^n\,dx}{x-1}$ $\quad n\in N$

$$= \int \frac{x^n-1+1}{x-1}\,dx = \int \left[\frac{x^n-1}{x-1} + \frac{1}{x-1}\right] dx = \int \left[x^{n-1} + \cdots + x + 1 + \frac{1}{x-1}\right] dx$$

$\int \frac{x\,dx}{x-1} = x + \ln|x-1| + c$

$$= \int \frac{x-1+1}{x-1}\,dx = \int \left[1 + \frac{1}{x-1}\right] dx = x + \ln|x-1| + c,\ x\in R-\{1\},\ c\in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\} \end{array}\right] = \int \frac{t+1}{t}\,dt = \int \left[1+\frac{1}{t}\right] dt = t + \ln|t| + c_1$$

# Riešené príklady – 137, 138

$$\int \frac{x^n\,dx}{x-1} = \frac{x^n}{n}+\cdots+\frac{x^2}{2}+x+\ln|x-1|+c = \ln|x-1|+\sum_{i=1}^{n}\frac{x^i}{i}+c \qquad n\in N$$

$$=\int \frac{x^n-1+1}{x-1}\,dx=\int\left[\frac{x^n-1}{x-1}+\frac{1}{x-1}\right]dx=\int\left[x^{n-1}+\cdots+x+1+\frac{1}{x-1}\right]dx$$

$$=\frac{x^n}{n}+\cdots+\frac{x^2}{2}+x+\ln|x-1|+c=\ln|x-1|+\sum_{i=1}^{n}\frac{x^i}{i}+c,\ x\in R-\{1\},\ c\in R.$$

$$\int \frac{x\,dx}{x-1} = x+\ln|x-1|+c$$

$$=\int \frac{x-1+1}{x-1}\,dx=\int\left[1+\frac{1}{x-1}\right]dx=x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

$$=\left[\begin{array}{r|l}\text{Subst. } t=x-1 & x\in R-\{1\}\\ dt=dx & t\in R-\{0\}\end{array}\right]=\int\frac{t+1}{t}\,dt=\int\left[1+\frac{1}{t}\right]dt=t+\ln|t|+c_1$$

$$=x-1+\ln|x-1|+c_1$$

# Riešené príklady – 137, 138

$$\int \frac{x^n\,dx}{x-1} = \frac{x^n}{n}+\cdots+\frac{x^2}{2}+x+\ln|x-1|+c = \ln|x-1|+\sum_{i=1}^{n}\frac{x^i}{i}+c \qquad n\in N$$

$$= \int \frac{x^n-1+1}{x-1}\,dx = \int\left[\frac{x^n-1}{x-1}+\frac{1}{x-1}\right]dx = \int\left[x^{n-1}+\cdots+x+1+\frac{1}{x-1}\right]dx$$

$$= \frac{x^n}{n}+\cdots+\frac{x^2}{2}+x+\ln|x-1|+c = \ln|x-1|+\sum_{i=1}^{n}\frac{x^i}{i}+c,\ x\in R-\{1\},\ c\in R.$$

$$\int \frac{x\,dx}{x-1} = x+\ln|x-1|+c$$

$$= \int \frac{x-1+1}{x-1}\,dx = \int\left[1+\frac{1}{x-1}\right]dx = x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\}\end{array}\right] = \int \frac{t+1}{t}\,dt = \int\left[1+\frac{1}{t}\right]dt = t+\ln|t|+c_1$$

$$= x-1+\ln|x-1|+c_1 = \left[\begin{array}{c} c=c_1-1 \\ c\in R,\ c_1\in R\end{array}\right]$$

# Riešené príklady – 137, 138

$$\int \frac{x^n\,dx}{x-1} = \frac{x^n}{n}+\cdots+\frac{x^2}{2}+x+\ln|x-1|+c = \ln|x-1|+\sum_{i=1}^{n}\frac{x^i}{i}+c \qquad n\in N$$

$$= \int \frac{x^n-1+1}{x-1}\,dx = \int\left[\frac{x^n-1}{x-1}+\frac{1}{x-1}\right]dx = \int\left[x^{n-1}+\cdots+x+1+\frac{1}{x-1}\right]dx$$

$$= \frac{x^n}{n}+\cdots+\frac{x^2}{2}+x+\ln|x-1|+c = \ln|x-1|+\sum_{i=1}^{n}\frac{x^i}{i}+c,\ x\in R-\{1\},\ c\in R.$$

$$\int \frac{x\,dx}{x-1} = x+\ln|x-1|+c$$

$$= \int \frac{x-1+1}{x-1}\,dx = \int\left[1+\frac{1}{x-1}\right]dx = x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\}\end{array}\right] = \int\frac{t+1}{t}\,dt = \int\left[1+\frac{1}{t}\right]dt = t+\ln|t|+c_1$$

$$= x-1+\ln|x-1|+c_1 = \left[\begin{array}{c} c=c_1-1 \\ c\in R,\ c_1\in R\end{array}\right] = x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

# Riešené príklady – 139, 140

$$\int \frac{x^2\,dx}{x-1}$$

---

$$\int \frac{x^9\,dx}{x-1}$$

# Riešené príklady – 139, 140

$$\int \frac{x^2\,dx}{x-1}$$

$$= \int \frac{x^2-1+1}{x-1}\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\} \end{array}\right]$$

$$\int \frac{x^9\,dx}{x-1}$$

$$= \int \frac{x^9-1+1}{x-1}\,dx$$

# Riešené príklady – 139, 140

$$\int \frac{x^2\,dx}{x-1}$$

$$= \int \frac{x^2-1+1}{x-1}\,dx = \int \left[x+1+\frac{1}{x-1}\right]dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\} \end{array}\right] = \int \frac{(t+1)^2\,dt}{t}$$

$$\int \frac{x^9\,dx}{x-1}$$

$$= \int \frac{x^9-1+1}{x-1}\,dx = \int \left[x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1+\frac{1}{x-1}\right]dx$$

# Riešené príklady – 139, 140

$$\int \frac{x^2\,dx}{x-1} = \frac{x^2}{2}+x+\ln|x-1|+c$$

$$= \int \frac{x^2-1+1}{x-1}\,dx = \int \left[x+1+\frac{1}{x-1}\right]dx = \frac{x^2}{2}+x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

$$= \left[\begin{matrix}\text{Subst. } t=x-1 \\ dt=dx\end{matrix}\ \middle|\ \begin{matrix} x\in R-\{1\} \\ t\in R-\{0\}\end{matrix}\right] = \int \frac{(t+1)^2\,dt}{t} = \int \frac{(t^2+2t+1)\,dt}{t}$$

$$\int \frac{x^9\,dx}{x-1} =$$

$$= \int \frac{x^9-1+1}{x-1}\,dx = \int \left[x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1+\frac{1}{x-1}\right]dx$$

$$= \frac{x^9}{9}+\frac{x^8}{8}+\frac{x^7}{7}+\frac{x^6}{6}+\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

# Riešené príklady – 139, 140

$$\int \frac{x^2\,dx}{x-1} = \frac{x^2}{2}+x+\ln|x-1|+c$$

$$= \int \frac{x^2-1+1}{x-1}\,dx = \int \left[x+1+\frac{1}{x-1}\right] dx = \frac{x^2}{2}+x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\} \end{array}\right] = \int \frac{(t+1)^2\,dt}{t} = \int \frac{(t^2+2t+1)\,dt}{t} = \int \left[t+2+\frac{1}{t}\right] dt$$

$$\int \frac{x^9\,dx}{x-1} =$$

$$= \int \frac{x^9-1+1}{x-1}\,dx = \int \left[x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1+\frac{1}{x-1}\right] dx$$

$$= \frac{x^9}{9}+\frac{x^8}{8}+\frac{x^7}{7}+\frac{x^6}{6}+\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

# Riešené príklady – 139, 140

$$\int \frac{x^2\,dx}{x-1} = \frac{x^2}{2} + x + \ln|x-1| + c$$

$$= \int \frac{x^2-1+1}{x-1}\,dx = \int \left[x+1+\frac{1}{x-1}\right] dx = \frac{x^2}{2} + x + \ln|x-1| + c,\ x \in R-\{1\},\ c \in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\} \end{array}\right] = \int \frac{(t+1)^2\,dt}{t} = \int \frac{(t^2+2t+1)\,dt}{t} = \int \left[t+2+\frac{1}{t}\right] dt$$

$$= \frac{t^2}{2} + 2t + \ln|t| + c_1$$

$$\int \frac{x^9\,dx}{x-1} =$$

$$= \int \frac{x^9-1+1}{x-1}\,dx = \int \left[x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1+\frac{1}{x-1}\right] dx$$

$$= \frac{x^9}{9} + \frac{x^8}{8} + \frac{x^7}{7} + \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + c,\ x \in R-\{1\},\ c \in R.$$

# Riešené príklady – 139, 140

$$\int \frac{x^2\,dx}{x-1} = \frac{x^2}{2}+x+\ln|x-1|+c$$

$$=\int \frac{x^2-1+1}{x-1}\,dx = \int \left[x+1+\frac{1}{x-1}\right]dx = \frac{x^2}{2}+x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

---

$$=\left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\} \end{array}\right] = \int \frac{(t+1)^2\,dt}{t} = \int \frac{(t^2+2t+1)\,dt}{t} = \int \left[t+2+\frac{1}{t}\right]dt$$

$$=\frac{t^2}{2}+2t+\ln|t|+c_1 = \frac{(x-1)^2}{2}+2(x-1)+\ln|x-1|+c_1$$

$$\int \frac{x^9\,dx}{x-1} =$$

$$=\int \frac{x^9-1+1}{x-1}\,dx = \int \left[x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1+\frac{1}{x-1}\right]dx$$

$$=\frac{x^9}{9}+\frac{x^8}{8}+\frac{x^7}{7}+\frac{x^6}{6}+\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

# Riešené príklady – 139, 140

$$\int \frac{x^2\,dx}{x-1} = \frac{x^2}{2}+x+\ln|x-1|+c$$

$$= \int \frac{x^2-1+1}{x-1}\,dx = \int \left[x+1+\frac{1}{x-1}\right]dx = \frac{x^2}{2}+x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

---

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\} \end{array}\right] = \int \frac{(t+1)^2\,dt}{t} = \int \frac{(t^2+2t+1)\,dt}{t} = \int \left[t+2+\frac{1}{t}\right]dt$$

$$= \frac{t^2}{2}+2t+\ln|t|+c_1 = \frac{(x-1)^2}{2}+2(x-1)+\ln|x-1|+c_1$$

$$= \left[\begin{array}{l|l} \frac{x^2-2x+1}{2}+2(x-1) & c=c_1-\frac{3}{2} \\ \quad =\frac{x^2}{2}-x+\frac{1}{2}+2x-2 & c\in R,\ c_1\in R \end{array}\right]$$

$$\int \frac{x^9\,dx}{x-1} =$$

$$= \int \frac{x^9-1+1}{x-1}\,dx = \int \left[x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1+\frac{1}{x-1}\right]dx$$

$$= \frac{x^9}{9}+\frac{x^8}{8}+\frac{x^7}{7}+\frac{x^6}{6}+\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

# Riešené príklady – 139, 140

$$\int \frac{x^2\,dx}{x-1} = \frac{x^2}{2}+x+\ln|x-1|+c$$

$$= \int \frac{x^2-1+1}{x-1}\,dx = \int \Big[x+1+\frac{1}{x-1}\Big]\,dx = \frac{x^2}{2}+x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\} \end{array}\right] = \int \frac{(t+1)^2\,dt}{t} = \int \frac{(t^2+2t+1)\,dt}{t} = \int \Big[t+2+\frac{1}{t}\Big]\,dt$$

$$= \frac{t^2}{2}+2t+\ln|t|+c_1 = \frac{(x-1)^2}{2}+2(x-1)+\ln|x-1|+c_1$$

$$= \left[\begin{array}{l|l} \frac{x^2-2x+1}{2}+2(x-1) & c=c_1-\frac{3}{2} \\ \quad =\frac{x^2}{2}-x+\frac{1}{2}+2x-2 & c\in R,\ c_1\in R \end{array}\right] = \frac{x^2}{2}+x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

$$\int \frac{x^9\,dx}{x-1} =$$

$$= \int \frac{x^9-1+1}{x-1}\,dx = \int \Big[x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1+\frac{1}{x-1}\Big]\,dx$$

$$= \frac{x^9}{9}+\frac{x^8}{8}+\frac{x^7}{7}+\frac{x^6}{6}+\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x+\ln|x-1|+c,\ x\in R-\{1\},\ c\in R.$$

# Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2}$$

$$\int \frac{dx}{x^6(1+x^2)}$$

# Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2}$$

$= \Big[$ Rozklad na parciálne zlomky: $\frac{1}{(1-x)x^2} = \frac{\alpha}{x-1} + \frac{\beta}{x} + \frac{\gamma}{x^2}$

$1 = \alpha x^2 + \beta x(x-1) + \gamma(x-1) = -\gamma + (-\beta+\gamma)x + (\alpha+\beta)x^2 \Big]$

$$\int \frac{dx}{x^6(1+x^2)}$$

$= \Big[$ Rozklad na parciálne zlomky: $\frac{1}{x^6(1+x^2)} = \frac{1}{t^3(1+t)} = \frac{\alpha}{t+1} + \frac{\beta}{t} + \frac{\gamma}{t^2} + \frac{\delta}{t^3}$

$1 = \alpha t^3 + \beta t^2(t+1) + \gamma t(t+1) + \delta(t+1) = \delta + (\gamma+\delta)t + (\beta+\gamma)t^2 + (\alpha+\beta)t^3 \Big]$

# Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2}$$

$$= \left[ \begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{(1-x)x^2} = \frac{\alpha}{x-1} + \frac{\beta}{x} + \frac{\gamma}{x^2} \\ 1 = \alpha x^2 + \beta x(x-1) + \gamma(x-1) = -\gamma + (-\beta+\gamma)x + (\alpha+\beta)x^2 \;\Rightarrow\; 1=-\gamma,\ 0=-\beta+\gamma,\ 0=\alpha+\beta \end{array} \right]$$

$$\int \frac{dx}{x^6(1+x^2)}$$

$$= \left[ \begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{x^6(1+x^2)} = \frac{1}{t^3(1+t)} = \frac{\alpha}{t+1} + \frac{\beta}{t} + \frac{\gamma}{t^2} + \frac{\delta}{t^3} \\ 1=\alpha t^3+\beta t^2(t+1)+\gamma t(t+1)+\delta(t+1)=\delta+(\gamma+\delta)t+(\beta+\gamma)t^2+(\alpha+\beta)t^3 \;\Rightarrow\; 1=\delta,\ 0=\gamma+\delta,\ 0=\beta+\gamma,\ 0=\alpha+\beta \end{array} \right]$$

# Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2}$$

$= \Big[$ Rozklad na parciálne zlomky: $\frac{1}{(1-x)x^2} = \frac{\alpha}{x-1} + \frac{\beta}{x} + \frac{\gamma}{x^2} = \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}$, $\quad \gamma=-1, \quad \beta=-1, \quad \alpha=1$

$1 = \alpha x^2 + \beta x(x-1) + \gamma(x-1) = -\gamma + (-\beta+\gamma)x + (\alpha+\beta)x^2 \Rightarrow 1=-\gamma,\ 0=-\beta+\gamma,\ 0=\alpha+\beta \Big]$

$$\int \frac{dx}{x^6(1+x^2)}$$

$= \Big[$ Rozklad na parciálne zlomky: $\frac{1}{x^6(1+x^2)} = \frac{1}{t^3(1+t)} = \frac{\alpha}{t+1} + \frac{\beta}{t} + \frac{\gamma}{t^2} + \frac{\delta}{t^3} = -\frac{1}{x^2+1} + \frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6}$, $\quad \delta=1,\ \gamma=-1,\ \beta=1,\ \alpha=-1$

$1=\alpha t^3+\beta t^2(t+1)+\gamma t(t+1)+\delta(t+1)=\delta+(\gamma+\delta)t+(\beta+\gamma)t^2+(\alpha+\beta)t^3 \Rightarrow 1=\delta,\ 0=\gamma+\delta,\ 0=\beta+\gamma,\ 0=\alpha+\beta \Big]$

# Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2}$$

$$= \left[ \begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{(1-x)x^2} = \frac{\alpha}{x-1} + \frac{\beta}{x} + \frac{\gamma}{x^2} = \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}, \quad \gamma=-1, \quad \beta=-1, \quad \alpha=1 \\ 1 = \alpha x^2 + \beta x(x-1) + \gamma(x-1) = -\gamma + (-\beta+\gamma)x + (\alpha+\beta)x^2 \Rightarrow 1=-\gamma,\ 0=-\beta+\gamma,\ 0=\alpha+\beta \end{array} \right]$$

$$= \int \left[ \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} \right] dx = \int \left[ \frac{1}{x-1} - \frac{1}{x} - x^{-2} \right] dx$$

$$\int \frac{dx}{x^6(1+x^2)}$$

$$= \left[ \begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{x^6(1+x^2)} = \frac{1}{t^3(1+t)} = \frac{\alpha}{t+1} + \frac{\beta}{t} + \frac{\gamma}{t^2} + \frac{\delta}{t^3} = -\frac{1}{x^2+1} + \frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6}, \quad \delta=1,\ \gamma=-1,\ \beta=1,\ \alpha=-1 \\ 1=\alpha t^3+\beta t^2(t+1)+\gamma t(t+1)+\delta(t+1)=\delta+(\gamma+\delta)t+(\beta+\gamma)t^2+(\alpha+\beta)t^3 \Rightarrow 1=\delta,\ 0=\gamma+\delta,\ 0=\beta+\gamma,\ 0=\alpha+\beta \end{array} \right]$$

$$= \int \left[ \frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6} - \frac{1}{x^2+1} \right] dx = \int \left[ x^{-2} - x^{-4} + x^{-6} - \frac{1}{x^2+1} \right] dx$$

# Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(1-x)x^2} = \frac{\alpha}{x-1} + \frac{\beta}{x} + \frac{\gamma}{x^2} = \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}, \quad \gamma=-1, \quad \beta=-1, \quad \alpha=1 \atop 1 = \alpha x^2 + \beta x(x-1) + \gamma(x-1) = -\gamma + (-\beta+\gamma)x + (\alpha+\beta)x^2 \Rightarrow 1=-\gamma,\ 0=-\beta+\gamma,\ 0=\alpha+\beta \right]$$

$$= \int \left[ \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} \right] dx = \int \left[ \frac{1}{x-1} - \frac{1}{x} - x^{-2} \right] dx = \ln|x-1| - \ln|x| - \frac{x^{-1}}{-1} + c$$

$$\int \frac{dx}{x^6(1+x^2)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{x^6(1+x^2)} = \frac{1}{t^3(1+t)} = \frac{\alpha}{t+1} + \frac{\beta}{t} + \frac{\gamma}{t^2} + \frac{\delta}{t^3} = -\frac{1}{x^2+1} + \frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6}, \quad \delta=1,\ \gamma=-1,\ \beta=1,\ \alpha=-1 \atop 1=\alpha t^3+\beta t^2(t+1)+\gamma t(t+1)+\delta(t+1)=\delta+(\gamma+\delta)t+(\beta+\gamma)t^2+(\alpha+\beta)t^3 \Rightarrow 1=\delta,\ 0=\gamma+\delta,\ 0=\beta+\gamma,\ 0=\alpha+\beta \right]$$

$$= \int \left[ \frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6} - \frac{1}{x^2+1} \right] dx = \int \left[ x^{-2} - x^{-4} + x^{-6} - \frac{1}{x^2+1} \right] dx$$

$$= \frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + \frac{x^{-5}}{-5} - \operatorname{arctg} x + c$$

# Riešené príklady – 141, 142

$$\int \frac{dx}{(1-x)x^2} = \ln\left|\frac{x-1}{x}\right| + \frac{1}{x} + c$$

$$= \left[ \begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{(1-x)x^2} = \frac{\alpha}{x-1} + \frac{\beta}{x} + \frac{\gamma}{x^2} = \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}, \quad \gamma=-1, \quad \beta=-1, \quad \alpha=1 \\ 1 = \alpha x^2 + \beta x(x-1) + \gamma(x-1) = -\gamma + (-\beta+\gamma)x + (\alpha+\beta)x^2 \Rightarrow 1=-\gamma,\ 0=-\beta+\gamma,\ 0=\alpha+\beta \end{array} \right]$$

$$= \int \left[\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}\right] dx = \int \left[\frac{1}{x-1} - \frac{1}{x} - x^{-2}\right] dx = \ln|x-1| - \ln|x| - \frac{x^{-1}}{-1} + c$$

$$= \ln\left|\frac{x-1}{x}\right| + \frac{1}{x} + c,\ x \in R - \{0,\ 1\},\ c \in R.$$

$$\int \frac{dx}{x^6(1+x^2)} = -\frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} - \operatorname{arctg} x + c$$

$$= \left[ \begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{x^6(1+x^2)} = \frac{1}{t^3(1+t)} = \frac{\alpha}{t+1} + \frac{\beta}{t} + \frac{\gamma}{t^2} + \frac{\delta}{t^3} = -\frac{1}{x^2+1} + \frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6}, \quad \delta=1,\ \gamma=-1,\ \beta=1,\ \alpha=-1 \\ 1=\alpha t^3+\beta t^2(t+1)+\gamma t(t+1)+\delta(t+1)=\delta+(\gamma+\delta)t+(\beta+\gamma)t^2+(\alpha+\beta)t^3 \Rightarrow 1=\delta,\ 0=\gamma+\delta,\ 0=\beta+\gamma,\ 0=\alpha+\beta \end{array} \right]$$

$$= \int \left[\frac{1}{x^2} - \frac{1}{x^4} + \frac{1}{x^6} - \frac{1}{x^2+1}\right] dx = \int \left[x^{-2} - x^{-4} + x^{-6} - \frac{1}{x^2+1}\right] dx$$

$$= \frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + \frac{x^{-5}}{-5} - \operatorname{arctg} x + c = -\frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} - \operatorname{arctg} x + c,$$

$$x \in R - \{0\},\ c \in R.$$

# Riešené príklady – 143, 144

$$\int \frac{(x-2)^4\,dx}{(x-1)^2}$$

$$\int \frac{(x-1)^4\,dx}{(x-2)^2}$$

# Riešené príklady – 143, 144

$$\int \frac{(x-2)^4\,dx}{(x-1)^2}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\} \end{array} \right]$$

$$\int \frac{(x-1)^4\,dx}{(x-2)^2}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x-2 & x\in R-\{2\} \\ dt=dx & t\in R-\{0\} \end{array} \right]$$

# Riešené príklady – 143, 144

$$\int \frac{(x-2)^4\,dx}{(x-1)^2}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t = x-1 & x \in R - \{1\} \\ dt = dx & t \in R - \{0\} \end{array} \right] = \int \frac{(t-1)^4\,dt}{t^2}$$

$$\int \frac{(x-1)^4\,dx}{(x-2)^2}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t = x-2 & x \in R - \{2\} \\ dt = dx & t \in R - \{0\} \end{array} \right] = \int \frac{(t+1)^4\,dt}{t^2}$$

# Riešené príklady – 143, 144

$$\int \frac{(x-2)^4\,dx}{(x-1)^2}$$

$$=\left[\begin{array}{r|l}\text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\}\end{array}\right]=\int \frac{(t-1)^4\,dt}{t^2}=\int \frac{(t^4-4t^3+6t^2-4t+1)\,dt}{t^2}$$

$$\int \frac{(x-1)^4\,dx}{(x-2)^2}$$

$$=\left[\begin{array}{r|l}\text{Subst. } t=x-2 & x\in R-\{2\} \\ dt=dx & t\in R-\{0\}\end{array}\right]=\int \frac{(t+1)^4\,dt}{t^2}=\int \frac{(t^4+4t^3+6t^2+4t+1)\,dt}{t^2}$$

# Riešené príklady – 143, 144

$$\int \frac{(x-2)^4\,dx}{(x-1)^2}$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\} \end{array}\right] = \int \frac{(t-1)^4\,dt}{t^2} = \int \frac{(t^4-4t^3+6t^2-4t+1)\,dt}{t^2}$$

$$= \int \left[ t^2-4t+6-\tfrac{4}{t}+t^{-2} \right] dt$$

$$\int \frac{(x-1)^4\,dx}{(x-2)^2}$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-2 & x\in R-\{2\} \\ dt=dx & t\in R-\{0\} \end{array}\right] = \int \frac{(t+1)^4\,dt}{t^2} = \int \frac{(t^4+4t^3+6t^2+4t+1)\,dt}{t^2}$$

$$= \int \left[ t^2+4t+6+\tfrac{4}{t}+t^{-2} \right] dt$$

# Riešené príklady – 143, 144

$$\int \frac{(x-2)^4\,dx}{(x-1)^2}$$

$$=\left[\begin{array}{r|l}\text{Subst. } t=x-1 & x\in R-\{1\}\\ dt=dx & t\in R-\{0\}\end{array}\right]=\int\frac{(t-1)^4\,dt}{t^2}=\int\frac{(t^4-4t^3+6t^2-4t+1)\,dt}{t^2}$$

$$=\int\left[t^2-4t+6-\frac{4}{t}+t^{-2}\right]dt=\frac{t^3}{3}-\frac{4t^2}{2}+6t-4\ln|t|+\frac{t^{-1}}{-1}+c$$

$$\int \frac{(x-1)^4\,dx}{(x-2)^2}$$

$$=\left[\begin{array}{r|l}\text{Subst. } t=x-2 & x\in R-\{2\}\\ dt=dx & t\in R-\{0\}\end{array}\right]=\int\frac{(t+1)^4\,dt}{t^2}=\int\frac{(t^4+4t^3+6t^2+4t+1)\,dt}{t^2}$$

$$=\int\left[t^2+4t+6+\frac{4}{t}+t^{-2}\right]dt=\frac{t^3}{3}+\frac{4t^2}{2}+6t+4\ln|t|+\frac{t^{-1}}{-1}+c$$

# Riešené príklady – 143, 144

$$\int \frac{(x-2)^4\,dx}{(x-1)^2} = \frac{(x-1)^3}{3} - 2(x-1)^2 + 6(x-1) - 4\ln|x-1| - \frac{1}{x-1} + c$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in R-\{1\} \\ dt=dx & t\in R-\{0\} \end{array}\right] = \int \frac{(t-1)^4\,dt}{t^2} = \int \frac{(t^4-4t^3+6t^2-4t+1)\,dt}{t^2}$$

$$= \int \left[t^2-4t+6-\frac{4}{t}+t^{-2}\right] dt = \frac{t^3}{3} - \frac{4t^2}{2} + 6t - 4\ln|t| + \frac{t^{-1}}{-1} + c$$

$$= \frac{(x-1)^3}{3} - 2(x-1)^2 + 6(x-1) - 4\ln|x-1| - \frac{1}{x-1} + c,\ x\in R-\{1\},\ c\in R.$$

$$\int \frac{(x-1)^4\,dx}{(x-2)^2} = \frac{(x-2)^3}{3} + 2(x-2)^2 + 6(x-2) + 4\ln|x-2| - \frac{1}{x-2} + c$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-2 & x\in R-\{2\} \\ dt=dx & t\in R-\{0\} \end{array}\right] = \int \frac{(t+1)^4\,dt}{t^2} = \int \frac{(t^4+4t^3+6t^2+4t+1)\,dt}{t^2}$$

$$= \int \left[t^2+4t+6+\frac{4}{t}+t^{-2}\right] dt = \frac{t^3}{3} + \frac{4t^2}{2} + 6t + 4\ln|t| + \frac{t^{-1}}{-1} + c$$

$$= \frac{(x-2)^3}{3} + 2(x-2)^2 + 6(x-2) + 4\ln|x-2| - \frac{1}{x-2} + c,\ x\in R-\{2\},\ c\in R.$$

# Riešené príklady – 145, 146

$$\int \frac{dx}{x^3-7x-6}$$

$$\int \frac{dx}{x^3-2x^2-x+2}$$

# Riešené príklady – 145, 146

$$\int \frac{dx}{x^3-7x-6} = \int \frac{dx}{(x-1)(x-2)(x+3)}$$

$$\int \frac{dx}{x^3-2x^2-x+2} = \int \frac{dx}{(x+1)(x-1)(x-2)}$$

# Riešené príklady – 145, 146

$$\int \frac{dx}{x^3-7x-6} = \int \frac{dx}{(x-1)(x-2)(x+3)}$$

$$= \left[ \begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x-2)(x+3)} = \frac{\alpha}{x-1} + \frac{\beta}{x-2} + \frac{\gamma}{x+3} \\ 1 = \alpha(x-2)(x+3) + \beta(x-1)(x+3) + \gamma(x-1)(x-2) = (-6\alpha-3\beta+2\gamma) + (\alpha+2\beta-3\gamma)x + (\alpha+\beta+\gamma)x^2 \end{array} \right]$$

$$\int \frac{dx}{x^3-2x^2-x+2} = \int \frac{dx}{(x+1)(x-1)(x-2)}$$

$$= \left[ \begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{(x+1)(x-1)(x-2)} = \frac{\alpha}{x+1} + \frac{\beta}{x-1} + \frac{\gamma}{x-2} \\ 1 = \alpha(x-1)(x-2) + \beta(x+1)(x-2) + \gamma(x+1)(x-1) = (2\alpha-2\beta-\gamma) + (-3\alpha-\beta)x + (\alpha+\beta+\gamma)x^2 \end{array} \right]$$

# Riešené príklady – 145, 146

$$\int \frac{dx}{x^3-7x-6} = \int \frac{dx}{(x-1)(x-2)(x+3)}$$

$= \Big[$ Rozklad na parciálne zlomky: $\frac{1}{(x-1)(x-2)(x+3)} = \frac{\alpha}{x-1} + \frac{\beta}{x-2} + \frac{\gamma}{x+3} = \frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{5}}{x-2} + \frac{\frac{1}{20}}{x+3}, \quad \alpha = -\frac{1}{4}, \quad \beta = \frac{1}{5}, \quad \gamma = \frac{1}{20}$

$1 = \alpha(x-2)(x+3) + \beta(x-1)(x+3) + \gamma(x-1)(x-2) = (-6\alpha-3\beta+2\gamma) + (\alpha+2\beta-3\gamma)x + (\alpha+\beta+\gamma)x^2 \Big]$

$$\int \frac{dx}{x^3-2x^2-x+2} = \int \frac{dx}{(x+1)(x-1)(x-2)}$$

$= \Big[$ Rozklad na parciálne zlomky: $\frac{1}{(x+1)(x-1)(x-2)} = \frac{\alpha}{x+1} + \frac{\beta}{x-1} + \frac{\gamma}{x-2} = \frac{\frac{1}{6}}{x+1} + \frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{3}}{x-2}, \quad \alpha = \frac{1}{6}, \quad \beta = -\frac{1}{2}, \quad \gamma = \frac{1}{3}$

$1 = \alpha(x-1)(x-2) + \beta(x+1)(x-2) + \gamma(x+1)(x-1) = (2\alpha-2\beta-\gamma) + (-3\alpha-\beta)x + (\alpha+\beta+\gamma)x^2 \Big]$

# Riešené príklady – 145, 146

$$\int \frac{dx}{x^3-7x-6} = \int \frac{dx}{(x-1)(x-2)(x+3)}$$

$$= \left[ \begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x-2)(x+3)} = \frac{\alpha}{x-1} + \frac{\beta}{x-2} + \frac{\gamma}{x+3} = \frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{5}}{x-2} + \frac{\frac{1}{20}}{x+3}, \quad \alpha = -\frac{1}{4}, \quad \beta = \frac{1}{5}, \quad \gamma = \frac{1}{20} \\ 1 = \alpha(x-2)(x+3) + \beta(x-1)(x+3) + \gamma(x-1)(x-2) = (-6\alpha-3\beta+2\gamma) + (\alpha+2\beta-3\gamma)x + (\alpha+\beta+\gamma)x^2 \end{array} \right]$$

$$= \int \left[ \frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{5}}{x-2} + \frac{\frac{1}{20}}{x+3} \right] dx$$

$$\int \frac{dx}{x^3-2x^2-x+2} = \int \frac{dx}{(x+1)(x-1)(x-2)}$$

$$= \left[ \begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{1}{(x+1)(x-1)(x-2)} = \frac{\alpha}{x+1} + \frac{\beta}{x-1} + \frac{\gamma}{x-2} = \frac{\frac{1}{6}}{x+1} + \frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{3}}{x-2}, \quad \alpha = \frac{1}{6}, \quad \beta = -\frac{1}{2}, \quad \gamma = \frac{1}{3} \\ 1 = \alpha(x-1)(x-2) + \beta(x+1)(x-2) + \gamma(x+1)(x-1) = (2\alpha-2\beta-\gamma) + (-3\alpha-\beta)x + (\alpha+\beta+\gamma)x^2 \end{array} \right]$$

$$= \int \left[ \frac{\frac{1}{6}}{x+1} + \frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{3}}{x-2} \right] dx$$

# Riešené príklady – 145, 146

$$\int \frac{dx}{x^3-7x-6} = \int \frac{dx}{(x-1)(x-2)(x+3)} = -\frac{1}{4}\ln|x-1| + \frac{1}{5}\ln|x-2| + \frac{1}{20}\ln|x+3| + c$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x-2)(x+3)} = \frac{\alpha}{x-1}+\frac{\beta}{x-2}+\frac{\gamma}{x+3} = \frac{-\frac{1}{4}}{x-1}+\frac{\frac{1}{5}}{x-2}+\frac{\frac{1}{20}}{x+3},\quad \alpha=-\frac{1}{4},\quad \beta=\frac{1}{5},\quad \gamma=\frac{1}{20} \atop 1=\alpha(x-2)(x+3)+\beta(x-1)(x+3)+\gamma(x-1)(x-2)=(-6\alpha-3\beta+2\gamma)+(\alpha+2\beta-3\gamma)x+(\alpha+\beta+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{5}}{x-2} + \frac{\frac{1}{20}}{x+3} \right] dx = -\frac{1}{4}\ln|x-1| + \frac{1}{5}\ln|x-2| + \frac{1}{20}\ln|x+3| + c,$$

$$x \in R-\{1, 2, -3\},\ c \in R.$$

$$\int \frac{dx}{x^3-2x^2-x+2} = \int \frac{dx}{(x+1)(x-1)(x-2)} = \frac{1}{6}\ln|x+1| - \frac{1}{2}\ln|x-1| + \frac{1}{3}\ln|x-2| + c$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x+1)(x-1)(x-2)} = \frac{\alpha}{x+1}+\frac{\beta}{x-1}+\frac{\gamma}{x-2} = \frac{\frac{1}{6}}{x+1}+\frac{-\frac{1}{2}}{x-1}+\frac{\frac{1}{3}}{x-2},\quad \alpha=\frac{1}{6},\quad \beta=-\frac{1}{2},\quad \gamma=\frac{1}{3} \atop 1=\alpha(x-1)(x-2)+\beta(x+1)(x-2)+\gamma(x+1)(x-1)=(2\alpha-2\beta-\gamma)+(-3\alpha-\beta)x+(\alpha+\beta+\gamma)x^2 \right]$$

$$= \int \left[ \frac{\frac{1}{6}}{x+1} + \frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{3}}{x-2} \right] dx = \frac{1}{6}\ln|x+1| - \frac{1}{2}\ln|x-1| + \frac{1}{3}\ln|x-2| + c,$$

$$x \in R-\{\pm 1, 2\},\ c \in R.$$

# Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2}$$

$$\int \frac{dx}{x^3+x^2-x-1}$$

# Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x-2)(x+1)^2}$$

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x-1)(x+1)^2}$$

# Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x-2)(x+1)^2}$$

$= \Big[$ Rozklad na parciálne zlomky: $\frac{1}{(x-2)(x+1)^2} = \frac{\alpha}{x-2} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2}$

$1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-2\beta-2\gamma) + (2\alpha-\beta+\gamma)x + (\alpha+\beta)x^2$ $\Big]$

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x-1)(x+1)^2}$$

$= \Big[$ Rozklad na parciálne zlomky: $\frac{1}{(x-1)(x+1)^2} = \frac{\alpha}{x-1} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2}$

$1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-\beta-\gamma) + (2\alpha+\gamma)x + (\alpha+\beta)x^2$ $\Big]$

# Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x-2)(x+1)^2}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x-2)(x+1)^2} = \frac{\alpha}{x-2}+\frac{\beta}{x+1}+\frac{\gamma}{(x+1)^2} = \frac{\frac{1}{9}}{x-2}+\frac{-\frac{1}{9}}{x+1}+\frac{-\frac{1}{3}}{(x+1)^2}, \quad \alpha=\frac{1}{9}, \quad \beta=-\frac{1}{9}, \quad \gamma=-\frac{1}{3} \atop 1=\alpha(x+1)^2+\beta(x-2)(x+1)+\gamma(x-2)=(\alpha-2\beta-2\gamma)+(2\alpha-\beta+\gamma)x+(\alpha+\beta)x^2 \right]$$

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x-1)(x+1)^2}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x+1)^2} = \frac{\alpha}{x-1}+\frac{\beta}{x+1}+\frac{\gamma}{(x+1)^2} = \frac{\frac{1}{4}}{x-1}+\frac{-\frac{1}{4}}{x+1}+\frac{-\frac{1}{2}}{(x+1)^2}, \quad \alpha=\frac{1}{4}, \quad \beta=-\frac{1}{4}, \quad \gamma=-\frac{1}{2} \atop 1=\alpha(x+1)^2+\beta(x-2)(x+1)+\gamma(x-2)=(\alpha-\beta-\gamma)+(2\alpha+\gamma)x+(\alpha+\beta)x^2 \right]$$

# Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x-2)(x+1)^2}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x-2)(x+1)^2} = \frac{\alpha}{x-2} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2}, \quad \alpha=\frac{1}{9}, \quad \beta=-\frac{1}{9}, \quad \gamma=-\frac{1}{3} \atop 1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-2\beta-2\gamma) + (2\alpha-\beta+\gamma)x + (\alpha+\beta)x^2 \right]$$

$$= \int \left[ \frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2} \right] dx$$

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x-1)(x+1)^2}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x+1)^2} = \frac{\alpha}{x-1} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2}, \quad \alpha=\frac{1}{4}, \quad \beta=-\frac{1}{4}, \quad \gamma=-\frac{1}{2} \atop 1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-\beta-\gamma) + (2\alpha+\gamma)x + (\alpha+\beta)x^2 \right]$$

$$= \int \left[ \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2} \right] dx$$

# Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x-2)(x+1)^2}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x-2)(x+1)^2} = \frac{\alpha}{x-2} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2}, \quad \alpha=\frac{1}{9}, \quad \beta=-\frac{1}{9}, \quad \gamma=-\frac{1}{3} \atop 1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-2\beta-2\gamma) + (2\alpha-\beta+\gamma)x + (\alpha+\beta)x^2 \right]$$

$$= \int \left[ \frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2} \right] dx = \frac{1}{9} \ln|x-2| - \frac{1}{9} \ln|x+1| - \frac{(x+1)^{-1}}{3\cdot(-1)} + c$$

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x-1)(x+1)^2}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x+1)^2} = \frac{\alpha}{x-1} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2}, \quad \alpha=\frac{1}{4}, \quad \beta=-\frac{1}{4}, \quad \gamma=-\frac{1}{2} \atop 1 = \alpha(x+1)^2 + \beta(x-2)(x+1) + \gamma(x-2) = (\alpha-\beta-\gamma) + (2\alpha+\gamma)x + (\alpha+\beta)x^2 \right]$$

$$= \int \left[ \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2} \right] dx = \frac{1}{4} \ln|x-1| - \frac{1}{4} \ln|x+1| - \frac{(x+1)^{-1}}{2\cdot(-1)} + c$$

# Riešené príklady – 147, 148

$$\int \frac{dx}{x^3-3x-2} = \int \frac{dx}{(x-2)(x+1)^2} = \frac{1}{9}\ln|x-2| - \frac{1}{9}\ln|x+1| + \frac{1}{3(x+1)} + c$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x-2)(x+1)^2} = \frac{\alpha}{x-2} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2}, \quad \alpha=\tfrac{1}{9}, \quad \beta=-\tfrac{1}{9}, \quad \gamma=-\tfrac{1}{3} \atop 1=\alpha(x+1)^2+\beta(x-2)(x+1)+\gamma(x-2)=(\alpha-2\beta-2\gamma)+(2\alpha-\beta+\gamma)x+(\alpha+\beta)x^2 \right]$$

$$= \int \left[ \frac{\frac{1}{9}}{x-2} + \frac{-\frac{1}{9}}{x+1} + \frac{-\frac{1}{3}}{(x+1)^2} \right] dx = \frac{1}{9}\ln|x-2| - \frac{1}{9}\ln|x+1| - \frac{(x+1)^{-1}}{3\cdot(-1)} + c$$

$$= \frac{1}{9}\ln|x-2| - \frac{1}{9}\ln|x+1| + \frac{1}{3(x+1)} + c, \; x \in R-\{-1,2\}, \; c \in R.$$

$$\int \frac{dx}{x^3+x^2-x-1} = \int \frac{dx}{(x-1)(x+1)^2} = \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| + \frac{1}{2(x+1)} + c$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x-1)(x+1)^2} = \frac{\alpha}{x-1} + \frac{\beta}{x+1} + \frac{\gamma}{(x+1)^2} = \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2}, \quad \alpha=\tfrac{1}{4}, \quad \beta=-\tfrac{1}{4}, \quad \gamma=-\tfrac{1}{2} \atop 1=\alpha(x+1)^2+\beta(x-2)(x+1)+\gamma(x-2)=(\alpha-\beta-\gamma)+(2\alpha+\gamma)x+(\alpha+\beta)x^2 \right]$$

$$= \int \left[ \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2} \right] dx = \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| - \frac{(x+1)^{-1}}{2\cdot(-1)} + c$$

$$= \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| + \frac{1}{2(x+1)} + c, \; x \in R-\{\pm 1\}, \; c \in R.$$

# Riešené príklady – 149, 150

$$\int \frac{dx}{x^3-3x^2+4x-2}$$

$$\int \frac{dx}{x^3-3x^2+3x-1}$$

# Riešené príklady – 149, 150

$$\int \frac{dx}{x^3-3x^2+4x-2} = \int \frac{dx}{(x^2-2x+2)(x-1)}$$

$$\int \frac{dx}{x^3-3x^2+3x-1} = \int \frac{dx}{(x-1)^3}$$

# Riešené príklady – 149, 150

$$\int \frac{dx}{x^3-3x^2+4x-2} = \int \frac{dx}{(x^2-2x+2)(x-1)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-1)} = \frac{\alpha x+\beta}{x^2-2x+2} + \frac{\gamma}{x-1} \right.$$
$$\left. 1 = (\alpha x+\beta)(x-1) + \gamma(x^2-2x+2) = (-\beta+2\gamma) + (-\alpha+\beta-2\gamma)x + (\alpha+\gamma)x^2 \right]$$

$$\int \frac{dx}{x^3-3x^2+3x-1} = \int \frac{dx}{(x-1)^3}$$

$$= \int (x-1)^{-3}\, dx$$

# Riešené príklady – 149, 150

$$\int \frac{dx}{x^3-3x^2+4x-2} = \int \frac{dx}{(x^2-2x+2)(x-1)}$$

$= \Big[$ Rozklad na parciálne zlomky: $\frac{1}{(x^2+2x+2)(x-1)} = \frac{\alpha x+\beta}{x^2-2x+2} + \frac{\gamma}{x-1} = \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1}, \quad \alpha=-1, \quad \beta=1, \quad \gamma=1$

$1 = (\alpha x+\beta)(x-1) + \gamma(x^2-2x+2) = (-\beta+2\gamma) + (-\alpha+\beta-2\gamma)x + (\alpha+\gamma)x^2 \Big]$

$$\int \frac{dx}{x^3-3x^2+3x-1} = \int \frac{dx}{(x-1)^3}$$

$$= \int (x-1)^{-3}\, dx = \frac{(x-1)^{-2}}{-2} + c$$

# Riešené príklady – 149, 150

$$\int \frac{dx}{x^3-3x^2+4x-2} = \int \frac{dx}{(x^2-2x+2)(x-1)}$$

$$= \left[ \begin{array}{c} \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-1)} = \frac{\alpha x+\beta}{x^2-2x+2} + \frac{\gamma}{x-1} = \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1}, \quad \alpha=-1, \quad \beta=1, \quad \gamma=1 \\ 1 = (\alpha x+\beta)(x-1) + \gamma(x^2-2x+2) = (-\beta+2\gamma) + (-\alpha+\beta-2\gamma)x + (\alpha+\gamma)x^2 \end{array} \right]$$

$$= \int \left[ \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1} \right] dx$$

$$\int \frac{dx}{x^3-3x^2+3x-1} = \int \frac{dx}{(x-1)^3} = -\frac{1}{2(x-1)^2} + c$$

$$= \int (x-1)^{-3}\, dx = \frac{(x-1)^{-2}}{-2} + c = -\frac{1}{2(x-1)^2} + c, \; x \in R - \{1\}, \; c \in R.$$

# Riešené príklady – 149, 150

$$\int \frac{dx}{x^3-3x^2+4x-2} = \int \frac{dx}{(x^2-2x+2)(x-1)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-1)} = \frac{\alpha x+\beta}{x^2-2x+2} + \frac{\gamma}{x-1} = \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1}, \quad \alpha=-1, \quad \beta=1, \quad \gamma=1 \atop 1 = (\alpha x+\beta)(x-1) + \gamma(x^2-2x+2) = (-\beta+2\gamma) + (-\alpha+\beta-2\gamma)x + (\alpha+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1} \right] dx = -\int \frac{(x-1)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1}$$

$$\int \frac{dx}{x^3-3x^2+3x-1} = \int \frac{dx}{(x-1)^3} = -\frac{1}{2(x-1)^2} + c$$

$$= \int (x-1)^{-3}\,dx = \frac{(x-1)^{-2}}{-2} + c = -\frac{1}{2(x-1)^2} + c, \; x \in R-\{1\}, \; c \in R.$$

# Riešené príklady – 149, 150

$$\int \frac{dx}{x^3-3x^2+4x-2} = \int \frac{dx}{(x^2-2x+2)(x-1)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-1)} = \frac{\alpha x+\beta}{x^2-2x+2} + \frac{\gamma}{x-1} = \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1}, \quad \alpha=-1, \quad \beta=1, \quad \gamma=1 \right.$$
$$\left. 1 = (\alpha x+\beta)(x-1) + \gamma(x^2-2x+2) = (-\beta+2\gamma) + (-\alpha+\beta-2\gamma)x + (\alpha+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1} \right] dx = -\int \frac{(x-1)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1} = -\frac{1}{2} \int \frac{(2x-2)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1}$$

$$\int \frac{dx}{x^3-3x^2+3x-1} = \int \frac{dx}{(x-1)^3} = -\frac{1}{2(x-1)^2} + c$$

$$= \int (x-1)^{-3}\,dx = \frac{(x-1)^{-2}}{-2} + c = -\frac{1}{2(x-1)^2} + c, \; x \in R-\{1\}, \; c \in R.$$

# Riešené príklady – 149, 150

$$\int \frac{dx}{x^3-3x^2+4x-2} = \int \frac{dx}{(x^2-2x+2)(x-1)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-1)} = \frac{\alpha x+\beta}{x^2-2x+2} + \frac{\gamma}{x-1} = \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1}, \quad \alpha=-1, \quad \beta=1, \quad \gamma=1 \atop 1 = (\alpha x+\beta)(x-1) + \gamma(x^2-2x+2) = (-\beta+2\gamma) + (-\alpha+\beta-2\gamma)x + (\alpha+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1} \right] dx = -\int \frac{(x-1)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1} = -\frac{1}{2}\int \frac{(2x-2)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1}$$

$$= -\frac{1}{2}\int \frac{(x^2-2x+2)'\,dx}{x^2-2x+2} + \int \frac{dx}{x-1}$$

$$\int \frac{dx}{x^3-3x^2+3x-1} = \int \frac{dx}{(x-1)^3} = -\frac{1}{2(x-1)^2} + c$$

$$= \int (x-1)^{-3}\,dx = \frac{(x-1)^{-2}}{-2} + c = -\frac{1}{2(x-1)^2} + c, \; x \in R-\{1\}, \; c \in R.$$

# Riešené príklady – 149, 150

$$\int \frac{dx}{x^3-3x^2+4x-2} = \int \frac{dx}{(x^2-2x+2)(x-1)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-1)} = \frac{\alpha x+\beta}{x^2-2x+2} + \frac{\gamma}{x-1} = \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1}, \quad \alpha=-1, \quad \beta=1, \quad \gamma=1 \atop 1 = (\alpha x+\beta)(x-1) + \gamma(x^2-2x+2) = (-\beta+2\gamma) + (-\alpha+\beta-2\gamma)x + (\alpha+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1} \right] dx = -\int \frac{(x-1)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1} = -\frac{1}{2} \int \frac{(2x-2)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1}$$

$$= -\frac{1}{2} \int \frac{(x^2-2x+2)'\,dx}{x^2-2x+2} + \int \frac{dx}{x-1} = -\frac{1}{2} \ln\left|x^2-2x+2\right| + \ln|x-1| + c$$

$$\int \frac{dx}{x^3-3x^2+3x-1} = \int \frac{dx}{(x-1)^3} = -\frac{1}{2(x-1)^2} + c$$

$$= \int (x-1)^{-3}\,dx = \frac{(x-1)^{-2}}{-2} + c = -\frac{1}{2(x-1)^2} + c, \ x \in R-\{1\}, \ c \in R.$$

# Riešené príklady – 149, 150

$$\int \frac{dx}{x^3-3x^2+4x-2} = \int \frac{dx}{(x^2-2x+2)(x-1)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-1)} = \frac{\alpha x+\beta}{x^2-2x+2} + \frac{\gamma}{x-1} = \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1}, \quad \alpha=-1, \quad \beta=1, \quad \gamma=1 \atop 1 = (\alpha x+\beta)(x-1) + \gamma(x^2-2x+2) = (-\beta+2\gamma) + (-\alpha+\beta-2\gamma)x + (\alpha+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1} \right] dx = -\int \frac{(x-1)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1} = -\frac{1}{2} \int \frac{(2x-2)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1}$$

$$= -\frac{1}{2} \int \frac{(x^2-2x+2)'\,dx}{x^2-2x+2} + \int \frac{dx}{x-1} = -\frac{1}{2} \ln \left| x^2-2x+2 \right| + \ln |x-1| + c$$

$$= \left[ x^2-2x+2 \atop = (x-1)^2+1 > 0 \right]$$

$$\int \frac{dx}{x^3-3x^2+3x-1} = \int \frac{dx}{(x-1)^3} = -\frac{1}{2(x-1)^2} + c$$

$$= \int (x-1)^{-3}\,dx = \frac{(x-1)^{-2}}{-2} + c = -\frac{1}{2(x-1)^2} + c, \; x \in R-\{1\}, \; c \in R.$$

# Riešené príklady – 149, 150

$$\int \frac{dx}{x^3-3x^2+4x-2} = \int \frac{dx}{(x^2-2x+2)(x-1)}$$

$$= \left[ \begin{array}{c} \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-1)} = \frac{\alpha x+\beta}{x^2-2x+2} + \frac{\gamma}{x-1} = \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1}, \quad \alpha=-1, \quad \beta=1, \quad \gamma=1 \\ 1 = (\alpha x+\beta)(x-1) + \gamma(x^2-2x+2) = (-\beta+2\gamma) + (-\alpha+\beta-2\gamma)x + (\alpha+\gamma)x^2 \end{array} \right]$$

$$= \int \left[ \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1} \right] dx = -\int \frac{(x-1)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1} = -\frac{1}{2} \int \frac{(2x-2)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1}$$

$$= -\frac{1}{2} \int \frac{(x^2-2x+2)'\,dx}{x^2-2x+2} + \int \frac{dx}{x-1} = -\frac{1}{2} \ln \left| x^2-2x+2 \right| + \ln |x-1| + c$$

$$= \left[ \begin{array}{l} x^2-2x+2 \\ \quad =(x-1)^2+1>0 \end{array} \right] = -\frac{1}{2} \ln \left( x^2-2x+2 \right) + \frac{1}{2} \ln (x-1)^2 + c$$

$$\int \frac{dx}{x^3-3x^2+3x-1} = \int \frac{dx}{(x-1)^3} = -\frac{1}{2(x-1)^2} + c$$

$$= \int (x-1)^{-3}\,dx = \frac{(x-1)^{-2}}{-2} + c = -\frac{1}{2(x-1)^2} + c, \; x \in R-\{1\}, \; c \in R.$$

# Riešené príklady – 149, 150

$$\int \frac{dx}{x^3-3x^2+4x-2} = \int \frac{dx}{(x^2-2x+2)(x-1)} = \frac{1}{2}\ln\frac{(x-1)^2}{x^2-2x+2} + c$$

$$= \left[ \begin{array}{c} \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-1)} = \frac{\alpha x+\beta}{x^2-2x+2} + \frac{\gamma}{x-1} = \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1}, \quad \alpha=-1, \quad \beta=1, \quad \gamma=1 \\ 1 = (\alpha x+\beta)(x-1) + \gamma(x^2-2x+2) = (-\beta+2\gamma) + (-\alpha+\beta-2\gamma)x + (\alpha+\gamma)x^2 \end{array} \right]$$

$$= \int \left[ \frac{-x+1}{x^2-2x+2} + \frac{1}{x-1} \right] dx = -\int \frac{(x-1)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1} = -\frac{1}{2}\int \frac{(2x-2)\,dx}{x^2-2x+2} + \int \frac{dx}{x-1}$$

$$= -\frac{1}{2}\int \frac{(x^2-2x+2)'\,dx}{x^2-2x+2} + \int \frac{dx}{x-1} = -\frac{1}{2}\ln\left|x^2-2x+2\right| + \ln|x-1| + c$$

$$= \left[ \begin{array}{l} x^2-2x+2 \\ \quad =(x-1)^2+1>0 \end{array} \right] = -\frac{1}{2}\ln\left(x^2-2x+2\right) + \frac{1}{2}\ln(x-1)^2 + c$$

$$= \frac{1}{2}\ln\frac{(x-1)^2}{x^2-2x+2} + c, \ x\in R-\{1\}, \ c\in R.$$

$$\int \frac{dx}{x^3-3x^2+3x-1} = \int \frac{dx}{(x-1)^3} = -\frac{1}{2(x-1)^2} + c$$

$$= \int (x-1)^{-3}\,dx = \frac{(x-1)^{-2}}{-2} + c = -\frac{1}{2(x-1)^2} + c, \ x\in R-\{1\}, \ c\in R.$$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4}$$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$= \Big[$ Rozklad na parciálne zlomky: $\frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x+\beta}{x^2+2x+2} + \frac{\gamma}{x-2}$

$$1 = (\alpha x+\beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$$

$\Big]$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[ \begin{array}{c} \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x+\beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha=-\frac{1}{10}, \quad \beta=-\frac{4}{10}, \quad \gamma=\frac{1}{10} \\ 1 = (\alpha x+\beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2 \end{array} \right]$$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x+\beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha=-\frac{1}{10}, \quad \beta=-\frac{4}{10}, \quad \gamma=\frac{1}{10} \atop 1=(\alpha x+\beta)(x-2)+\gamma(x^2+2x+2)=(-2\beta+2\gamma)+(-2\alpha+\beta+2\gamma)x+(\alpha+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx$$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x+\beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha=-\frac{1}{10}, \quad \beta=-\frac{4}{10}, \quad \gamma=\frac{1}{10} \right.$$
$$\left. 1 = (\alpha x+\beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10} \int \frac{(x+4)\, dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x+\beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha=-\frac{1}{10}, \quad \beta=-\frac{4}{10}, \quad \gamma=\frac{1}{10} \right.$$
$$\left. 1 = (\alpha x+\beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10}\int \frac{(x+4)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2} = -\frac{1}{20}\int \frac{2(x+4)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2}$$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x+\beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha=-\frac{1}{10}, \quad \beta=-\frac{4}{10}, \quad \gamma=\frac{1}{10} \\ 1=(\alpha x+\beta)(x-2)+\gamma(x^2+2x+2)=(-2\beta+2\gamma)+(-2\alpha+\beta+2\gamma)x+(\alpha+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10}\int \frac{(x+4)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2} = -\frac{1}{20}\int \frac{2(x+4)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2}$$

$$= -\frac{1}{20}\int \frac{(2x+2+6)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2}$$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$=\Big[$ Rozklad na parciálne zlomky: $\frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x+\beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}$, $\alpha=-\frac{1}{10}$, $\beta=-\frac{4}{10}$, $\gamma=\frac{1}{10}$

$1 = (\alpha x+\beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2 \Big]$

$$= \int \left[ \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10}\int \frac{(x+4)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2} = -\frac{1}{20}\int \frac{2(x+4)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2}$$

$$= -\frac{1}{20}\int \frac{(2x+2+6)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2} = -\frac{1}{20}\int \frac{(2x+2)\,dx}{x^2+2x+2} - \frac{6}{20}\int \frac{dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2}$$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$= \Big[$ Rozklad na parciálne zlomky: $\frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x+\beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}$, $\alpha = -\frac{1}{10}$, $\beta = -\frac{4}{10}$, $\gamma = \frac{1}{10}$
$1 = (\alpha x+\beta)(x-2) + \gamma(x^2+2x+2) = (-2\beta+2\gamma) + (-2\alpha+\beta+2\gamma)x + (\alpha+\gamma)x^2$ $\Big]$

$$= \int \left[ \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10}\int \frac{(x+4)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2} = -\frac{1}{20}\int \frac{2(x+4)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2}$$

$$= -\frac{1}{20}\int \frac{(2x+2+6)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2} = -\frac{1}{20}\int \frac{(2x+2)\,dx}{x^2+2x+2} - \frac{6}{20}\int \frac{dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2}$$

$= \Big[$ Subst. $t = x+1$, $dt = dx$ | $x^2+2x+2 = x^2+2x+1+1 = (x+1)^2+1 = t^2+1$ | $x \in R$, $t \in R$ $\Big]$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x+\beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha=-\frac{1}{10}, \quad \beta=-\frac{4}{10}, \quad \gamma=\frac{1}{10} \\ 1=(\alpha x+\beta)(x-2)+\gamma(x^2+2x+2)=(-2\beta+2\gamma)+(-2\alpha+\beta+2\gamma)x+(\alpha+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10}\int \frac{(x+4)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2} = -\frac{1}{20}\int \frac{2(x+4)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2}$$

$$= -\frac{1}{20}\int \frac{(2x+2+6)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2} = -\frac{1}{20}\int \frac{(2x+2)\,dx}{x^2+2x+2} - \frac{6}{20}\int \frac{dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+1 & x^2+2x+2=x^2+2x+1+1 & x\in R \\ dt=dx & =(x+1)^2+1=t^2+1 & t\in R \end{array} \right] = -\frac{1}{20}\int \frac{(x^2+2x+2)'\,dx}{x^2+2x+2} - \frac{6}{20}\int \frac{dt}{t^2+1} + \frac{1}{10}\int \frac{dx}{x-2}$$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)}$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x+\beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha=-\frac{1}{10}, \quad \beta=-\frac{4}{10}, \quad \gamma=\frac{1}{10} \\ 1=(\alpha x+\beta)(x-2)+\gamma(x^2+2x+2)=(-2\beta+2\gamma)+(-2\alpha+\beta+2\gamma)x+(\alpha+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10}\int \frac{(x+4)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2} = -\frac{1}{20}\int \frac{2(x+4)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2}$$

$$= -\frac{1}{20}\int \frac{(2x+2+6)\,dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2} = -\frac{1}{20}\int \frac{(2x+2)\,dx}{x^2+2x+2} - \frac{6}{20}\int \frac{dx}{x^2+2x+2} + \frac{1}{10}\int \frac{dx}{x-2}$$

$$= \left[ \begin{array}{r|r|r} \text{Subst. } t=x+1 & x^2+2x+2=x^2+2x+1+1 & x\in R \\ dt=dx & =(x+1)^2+1=t^2+1 & t\in R \end{array} \right] = -\frac{1}{20}\int \frac{(x^2+2x+2)'\,dx}{x^2+2x+2} - \frac{6}{20}\int \frac{dt}{t^2+1} + \frac{1}{10}\int \frac{dx}{x-2}$$

$$= -\frac{1}{20}\ln|x^2+2x+2| - \frac{6}{20}\operatorname{arctg} t + \frac{1}{10}\ln|x-2| + c$$

# Riešené príklady – 151

$$\int\frac{dx}{x^3-2x-4}=\int\frac{dx}{(x^2+2x+2)(x-2)}$$

$$=\left[\text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)}=\frac{\alpha x+\beta}{x^2+2x+2}+\frac{\gamma}{x-2}=\frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2}+\frac{\frac{1}{10}}{x-2},\quad \alpha=-\frac{1}{10},\quad \beta=-\frac{4}{10},\quad \gamma=\frac{1}{10} \atop 1=(\alpha x+\beta)(x-2)+\gamma(x^2+2x+2)=(-2\beta+2\gamma)+(-2\alpha+\beta+2\gamma)x+(\alpha+\gamma)x^2\right]$$

$$=\int\left[\frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2}+\frac{\frac{1}{10}}{x-2}\right]dx=-\frac{1}{10}\int\frac{(x+4)\,dx}{x^2+2x+2}+\frac{1}{10}\int\frac{dx}{x-2}=-\frac{1}{20}\int\frac{2(x+4)\,dx}{x^2+2x+2}+\frac{1}{10}\int\frac{dx}{x-2}$$

$$=-\frac{1}{20}\int\frac{(2x+2+6)\,dx}{x^2+2x+2}+\frac{1}{10}\int\frac{dx}{x-2}=-\frac{1}{20}\int\frac{(2x+2)\,dx}{x^2+2x+2}-\frac{6}{20}\int\frac{dx}{x^2+2x+2}+\frac{1}{10}\int\frac{dx}{x-2}$$

$$=\left[\begin{array}{r|r|l}\text{Subst. } t=x+1 & x^2+2x+2=x^2+2x+1+1 & x\in R\\ dt=dx & =(x+1)^2+1=t^2+1 & t\in R\end{array}\right]=-\frac{1}{20}\int\frac{(x^2+2x+2)'\,dx}{x^2+2x+2}-\frac{6}{20}\int\frac{dt}{t^2+1}+\frac{1}{10}\int\frac{dx}{x-2}$$

$$=-\frac{1}{20}\ln|x^2+2x+2|-\frac{6}{20}\operatorname{arctg} t+\frac{1}{10}\ln|x-2|+c$$

$$=\frac{1}{20}\ln(x-2)^2-\frac{1}{20}\ln(x^2+2x+2)-\frac{3}{10}\operatorname{arctg}(x+1)+c$$

# Riešené príklady – 151

$$\int \frac{dx}{x^3-2x-4} = \int \frac{dx}{(x^2+2x+2)(x-2)} = \frac{1}{20} \ln \frac{(x-2)^2}{x^2+2x+2} - \frac{3}{10} \operatorname{arctg}(x+1) + c$$

$$= \left[ \text{Rozklad na parciálne zlomky: } \frac{1}{(x^2+2x+2)(x-2)} = \frac{\alpha x+\beta}{x^2+2x+2} + \frac{\gamma}{x-2} = \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2}, \quad \alpha=-\frac{1}{10}, \quad \beta=-\frac{4}{10}, \quad \gamma=\frac{1}{10} \atop 1=(\alpha x+\beta)(x-2)+\gamma(x^2+2x+2)=(-2\beta+2\gamma)+(-2\alpha+\beta+2\gamma)x+(\alpha+\gamma)x^2 \right]$$

$$= \int \left[ \frac{-\frac{x}{10}-\frac{4}{10}}{x^2+2x+2} + \frac{\frac{1}{10}}{x-2} \right] dx = -\frac{1}{10} \int \frac{(x+4)\,dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{2(x+4)\,dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= -\frac{1}{20} \int \frac{(2x+2+6)\,dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2} = -\frac{1}{20} \int \frac{(2x+2)\,dx}{x^2+2x+2} - \frac{6}{20} \int \frac{dx}{x^2+2x+2} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= \left[ \begin{array}{r|r|l} \text{Subst. } t=x+1 & x^2+2x+2=x^2+2x+1+1 & x\in R \\ dt=dx & =(x+1)^2+1=t^2+1 & t\in R \end{array} \right] = -\frac{1}{20} \int \frac{(x^2+2x+2)'\,dx}{x^2+2x+2} - \frac{6}{20} \int \frac{dt}{t^2+1} + \frac{1}{10} \int \frac{dx}{x-2}$$

$$= -\frac{1}{20} \ln|x^2+2x+2| - \frac{6}{20} \operatorname{arctg} t + \frac{1}{10} \ln|x-2| + c$$

$$= \frac{1}{20} \ln(x-2)^2 - \frac{1}{20} \ln(x^2+2x+2) - \frac{3}{10} \operatorname{arctg}(x+1) + c$$

$$= \frac{1}{20} \ln \frac{(x-2)^2}{x^2+2x+2} - \frac{3}{10} \operatorname{arctg}(x+1) + c, \; x\in R-\{2\}, \; c\in R.$$

# Riešené príklady – 152

$$\int \frac{dx}{x^6+1}$$

# Riešené príklady – 152

$$\int \frac{dx}{x^6+1}$$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}$$

# Riešené príklady – 152

$$\int \frac{dx}{x^6+1}$$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 152

$$\int \frac{dx}{x^6+1}$$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[ \frac{2x+\frac{4\sqrt{3}}{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 152

$\int \frac{dx}{x^6+1}$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}}\int \left[ \frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 152

$$\int \frac{dx}{x^6+1}$$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[ \frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\operatorname{arctg} x + \frac{1}{4\sqrt{3}} \int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 152

$$\int \frac{dx}{x^6+1}$$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}}\int \left[ \frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\operatorname{arctg} x + \frac{1}{4\sqrt{3}}\int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{3}-\sqrt{3}\right) \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \quad = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 & \quad = \frac{1}{3}-\frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right]$$

# Riešené príklady – 152

$$\int \frac{dx}{x^6+1}$$

$$=\int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}=\int\left[\frac{\frac{1}{3}}{x^2+1}+\frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1}+\frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{3}\int\frac{dx}{x^2+1}+\frac{1}{4\sqrt{3}}\int\left[\frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1}-\frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{3}\operatorname{arctg}x+\frac{1}{4\sqrt{3}}\int\left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1}-\frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}+\frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\left[\begin{array}{l|l|l|l|l}\text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{3}-\sqrt{3}\right)\\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \quad=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2>0 & \quad=\frac{1}{3}-\frac{1}{4}=\frac{4-3}{12}=\frac{1}{12}\end{array}\right]$$

$$=\frac{1}{3}\operatorname{arctg}x+\frac{1}{4\sqrt{3}}\int\left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1}-\frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1}\right]dx+\frac{1}{12}\int\frac{du}{u^2+\frac{1}{4}}+\frac{1}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

# Riešené príklady – 152

$\int \frac{dx}{x^6+1}$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}}\int \left[ \frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\operatorname{arctg} x + \frac{1}{4\sqrt{3}}\int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{3}-\sqrt{3}\right) \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \quad = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 & \quad = \frac{1}{3}-\frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right]$$

$$= \frac{1}{3}\operatorname{arctg} x + \frac{1}{4\sqrt{3}}\int \left[ \frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{1}{3}\operatorname{arctg} x + \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}} + c$$

# Riešené príklady – 152

$$\int \frac{dx}{x^6+1}$$

$$=\int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}=\int\left[\frac{\frac{1}{3}}{x^2+1}+\frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1}+\frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{3}\int\frac{dx}{x^2+1}+\frac{1}{4\sqrt{3}}\int\left[\frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1}-\frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{3}\operatorname{arctg}x+\frac{1}{4\sqrt{3}}\int\left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1}-\frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}+\frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\left[\begin{array}{l|l|l|l|l}\text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{3}-\sqrt{3}\right) \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \quad=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2>0 & \quad=\frac{1}{3}-\frac{1}{4}=\frac{4-3}{12}=\frac{1}{12}\end{array}\right]$$

$$=\frac{1}{3}\operatorname{arctg}x+\frac{1}{4\sqrt{3}}\int\left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1}-\frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1}\right]dx+\frac{1}{12}\int\frac{du}{u^2+\frac{1}{4}}+\frac{1}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$=\frac{1}{3}\operatorname{arctg}x+\frac{\ln(x^2+\sqrt{3}x+1)-\ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}}+\frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}}+\frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}}+c$$

$$=\frac{1}{3}\operatorname{arctg}x+\frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1}+\frac{1}{6}\operatorname{arctg}2u+\frac{1}{6}\operatorname{arctg}2v+c$$

# Riešené príklady – 152

$$\int \frac{dx}{x^6+1} = \frac{1}{3}\,\mathrm{arctg}\,x + \frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{\mathrm{arctg}\,(2x+\sqrt{3})}{6} - \frac{\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c$$

$$= \int \frac{dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}}\int \left[ \frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x + \frac{1}{4\sqrt{3}}\int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{3}-\sqrt{3}\right) \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \quad = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 & \quad = \frac{1}{3}-\frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right]$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x + \frac{1}{4\sqrt{3}}\int \left[ \frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x + \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{u}{\frac{1}{2}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{v}{\frac{1}{2}} + c$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x + \frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{1}{6}\,\mathrm{arctg}\,2u + \frac{1}{6}\,\mathrm{arctg}\,2v + c$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x + \frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c,\ x\in R,\ c\in R.$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \int \frac{x\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \int \frac{x\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 153

$\int \frac{x\,dx}{x^6+1}$

$$= \int \frac{x\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6} \int \frac{2x\,dx}{x^2+1} - \frac{1}{12} \int \left[ \frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \int \frac{x\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6}\int \frac{2x\,dx}{x^2+1} - \frac{1}{12}\int \left[ \frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6}\int \frac{(x^2+1)'\,dx}{x^2+1} - \frac{1}{12}\int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \int \frac{x\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6}\int \frac{2x\,dx}{x^2+1} - \frac{1}{12}\int \left[ \frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6}\int \frac{(x^2+1)'\,dx}{x^2+1} - \frac{1}{12}\int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & (x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1) = (x^2+1)^2-(\sqrt{3}x)^2 = x^4-x^2+1 > 0 \end{array} \right]$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$=\int \frac{x\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}=\int\left[\frac{\frac{x}{3}}{x^2+1}+\frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1}+\frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{6}\int\frac{2x\,dx}{x^2+1}-\frac{1}{12}\int\left[\frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{6}\int\frac{(x^2+1)'\,dx}{x^2+1}-\frac{1}{12}\int\left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}-\frac{\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\left[\begin{array}{l|l|l|l}\text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4}=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2>0\\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & (x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)=(x^2+1)^2-(\sqrt{3}x)^2=x^4-x^2+1>0\end{array}\right]$$

$$=\frac{1}{6}\ln(x^2+1)-\frac{1}{12}\int\left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1}+\frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1}\right]dx-\frac{\sqrt{3}}{12}\int\frac{du}{u^2+\frac{1}{4}}+\frac{\sqrt{3}}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \int \frac{x\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6}\int \frac{2x\,dx}{x^2+1} - \frac{1}{12}\int \left[ \frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6}\int \frac{(x^2+1)'\,dx}{x^2+1} - \frac{1}{12}\int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & (x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1) = (x^2+1)^2-(\sqrt{3}x)^2 = x^4-x^2+1 > 0 \end{array} \right]$$

$$= \frac{1}{6}\ln(x^2+1) - \frac{1}{12}\int \left[ \frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{\sqrt{3}}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{\ln(x^2+1)^2}{12} - \frac{\ln(x^2+\sqrt{3}x+1)+\ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{u}{\frac{1}{2}} + \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{v}{\frac{1}{2}} + c_1$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$=\int \frac{x\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}=\int\left[\frac{\frac{x}{3}}{x^2+1}+\frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1}+\frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{6}\int\frac{2x\,dx}{x^2+1}-\frac{1}{12}\int\left[\frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{6}\int\frac{(x^2+1)'\,dx}{x^2+1}-\frac{1}{12}\int\left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}-\frac{\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\left[\begin{array}{l|l|l|l}\text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4}=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2>0\\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & (x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)=(x^2+1)^2-(\sqrt{3}x)^2=x^4-x^2+1>0\end{array}\right]$$

$$=\frac{1}{6}\ln(x^2+1)-\frac{1}{12}\int\left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1}+\frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1}\right]dx-\frac{\sqrt{3}}{12}\int\frac{du}{u^2+\frac{1}{4}}+\frac{\sqrt{3}}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$=\frac{\ln(x^2+1)^2}{12}-\frac{\ln(x^2+\sqrt{3}x+1)+\ln(x^2-\sqrt{3}x+1)}{12}-\frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}}+\frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}}+c_1$$

$$=\frac{1}{12}\ln\frac{(x^2+1)^2}{(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)}-\frac{\sqrt{3}}{6}\operatorname{arctg}2u+\frac{\sqrt{3}}{6}\operatorname{arctg}2v+c_1$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1} = \frac{1}{12}\ln\frac{(x^2+1)^2}{x^4-x^2+1} - \frac{\sqrt{3}\,\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\sqrt{3}\,\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c_1$$

$$= \int \frac{x\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{x}{3}}{x^2+1} + \frac{-\frac{x}{6}-\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{6}+\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6}\int \frac{2x\,dx}{x^2+1} - \frac{1}{12}\int \left[ \frac{2x+2\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-2\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6}\int \frac{(x^2+1)'\,dx}{x^2+1} - \frac{1}{12}\int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} & du = dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 \\ \text{Subst. } v = x-\frac{\sqrt{3}}{2} & dv = dx & x\in R,\ v\in R & (x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1) = (x^2+1)^2-(\sqrt{3}x)^2 = x^4-x^2+1 > 0 \end{array} \right]$$

$$= \frac{1}{6}\ln(x^2+1) - \frac{1}{12}\int \left[ \frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{\sqrt{3}}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{\ln(x^2+1)^2}{12} - \frac{\ln(x^2+\sqrt{3}x+1)+\ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{u}{\frac{1}{2}} + \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{v}{\frac{1}{2}} + c_1$$

$$= \frac{1}{12}\ln\frac{(x^2+1)^2}{(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)} - \frac{\sqrt{3}}{6}\,\mathrm{arctg}\,2u + \frac{\sqrt{3}}{6}\,\mathrm{arctg}\,2v + c_1$$

$$= \frac{1}{12}\ln\frac{(x^2+1)^2}{x^4-x^2+1} - \frac{\sqrt{3}\,\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\sqrt{3}\,\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c_1,\ x\in R,\ c_1\in R.$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t = x^2 & x \in (-\infty; 0\rangle,\ t \in \langle 0; \infty) \\ dt = 2x\,dx & x \in \langle 0; \infty),\ t \in \langle 0; \infty) \end{array} \right]$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{dt}{t^3+1}$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array}\right]=\frac{1}{2}\int\frac{dt}{t^3+1}=\frac{1}{2}\int\frac{dt}{(t+1)(t^2-t+1)}$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^3+1} = \frac{1}{2}\int \frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int \left[ \frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3}+\frac{2}{3}}{t^2-t+1} \right] dt$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{dt}{t^3+1} = \frac{1}{2}\int \frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int\left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3}+\frac{2}{3}}{t^2-t+1}\right]dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-4}{t^2-t+1}\,dt$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{dt}{t^3+1} = \frac{1}{2}\int \frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int \left[ \frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3}+\frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-4}{t^2-t+1}\,dt$$

$$= \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-1-3}{t^2-t+1}\,dt$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{dt}{t^3+1} = \frac{1}{2}\int \frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int \left[ \frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3}+\frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-4}{t^2-t+1}\,dt$$

$$= \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-1-3}{t^2-t+1}\,dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{dt}{t^2-t+1}$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array}\right]=\frac{1}{2}\int\frac{dt}{t^3+1}=\frac{1}{2}\int\frac{dt}{(t+1)(t^2-t+1)}$$

$$=\frac{1}{2}\int\left[\frac{\frac{1}{3}}{t+1}+\frac{-\frac{t}{3}+\frac{2}{3}}{t^2-t+1}\right]dt=\frac{1}{6}\int\frac{dt}{t+1}-\frac{1}{12}\int\frac{2t-4}{t^2-t+1}\,dt$$

$$=\frac{1}{6}\int\frac{dt}{t+1}-\frac{1}{12}\int\frac{2t-1-3}{t^2-t+1}\,dt=\frac{1}{6}\int\frac{dt}{t+1}-\frac{1}{12}\int\frac{(2t-1)\,dt}{t^2-t+1}+\frac{1}{4}\int\frac{dt}{t^2-t+1}$$

$$=\left[\begin{array}{r|l|l} \text{Subst. } u=t-\frac{1}{2} & t\in\langle 0;\infty) & t^2-t+1=(t-\frac{1}{2})^2+1-\frac{1}{4}=(t-\frac{1}{2})^2+\frac{3}{4}=u^2+(\frac{\sqrt{3}}{2})^2>0 \\ du=dt & u\in\langle -\frac{1}{2};\infty) & \end{array}\right]$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0),\, t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\, t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int\frac{dt}{t^3+1} = \frac{1}{2}\int\frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int\left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3}+\frac{2}{3}}{t^2-t+1}\right]dt = \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{2t-4}{t^2-t+1}\,dt$$

$$= \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{2t-1-3}{t^2-t+1}\,dt = \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4}\int\frac{dt}{t^2-t+1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u=t-\frac{1}{2} & t\in\langle 0;\infty) & t^2-t+1=(t-\frac{1}{2})^2+1-\frac{1}{4}=(t-\frac{1}{2})^2+\frac{3}{4}=u^2+(\frac{\sqrt{3}}{2})^2>0 \\ du=dt & u\in\langle -\frac{1}{2};\infty) & \end{array}\right]$$

$$= \frac{1}{6}\ln(t+1) - \frac{1}{12}\int\frac{(t^2-t+1)'\,dt}{t^2-t+1} + \frac{1}{4}\int\frac{du}{u^2+\frac{3}{4}}$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{dt}{t^3+1} = \frac{1}{2}\int \frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int \left[ \frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3}+\frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-4}{t^2-t+1}\,dt$$

$$= \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-1-3}{t^2-t+1}\,dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{dt}{t^2-t+1}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } u=t-\frac{1}{2} & t\in\langle 0;\infty) & t^2-t+1=(t-\frac{1}{2})^2+1-\frac{1}{4}=(t-\frac{1}{2})^2+\frac{3}{4}=u^2+(\frac{\sqrt{3}}{2})^2>0 \\ du=dt & u\in\langle -\frac{1}{2};\infty) & \end{array} \right]$$

$$= \frac{1}{6}\ln(t+1) - \frac{1}{12}\int \frac{(t^2-t+1)'\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{du}{u^2+\frac{3}{4}}$$

$$= \frac{1}{12}\ln(t+1)^2 - \frac{1}{12}\ln(t^2-t+1) + \frac{1}{4}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arctg}\frac{u}{\frac{\sqrt{3}}{2}} + c_2$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{dt}{t^3+1} = \frac{1}{2}\int \frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int \left[ \frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3}+\frac{2}{3}}{t^2-t+1} \right] dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-4}{t^2-t+1}\,dt$$

$$= \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{2t-1-3}{t^2-t+1}\,dt = \frac{1}{6}\int \frac{dt}{t+1} - \frac{1}{12}\int \frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{dt}{t^2-t+1}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } u=t-\frac{1}{2} & t\in\langle 0;\infty) & t^2-t+1 = (t-\frac{1}{2})^2+1-\frac{1}{4} = (t-\frac{1}{2})^2+\frac{3}{4} = u^2+(\frac{\sqrt{3}}{2})^2 > 0 \\ du=dt & u\in\langle -\frac{1}{2};\infty) & \end{array} \right]$$

$$= \frac{1}{6}\ln(t+1) - \frac{1}{12}\int \frac{(t^2-t+1)'\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{du}{u^2+\frac{3}{4}}$$

$$= \frac{1}{12}\ln(t+1)^2 - \frac{1}{12}\ln(t^2-t+1) + \frac{1}{4}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arctg}\frac{u}{\frac{\sqrt{3}}{2}} + c_2$$

$$= \frac{1}{12}\ln\frac{(t+1)^2}{t^2-t+1} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2u}{\sqrt{3}} + c_2$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array} \right] = \frac{1}{2}\int\frac{dt}{t^3+1} = \frac{1}{2}\int\frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int\left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3}+\frac{2}{3}}{t^2-t+1}\right]dt = \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{2t-4}{t^2-t+1}\,dt$$

$$= \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{2t-1-3}{t^2-t+1}\,dt = \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4}\int\frac{dt}{t^2-t+1}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } u=t-\frac{1}{2} & t\in\langle 0;\infty) & t^2-t+1=(t-\frac{1}{2})^2+1-\frac{1}{4}=(t-\frac{1}{2})^2+\frac{3}{4}=u^2+(\frac{\sqrt{3}}{2})^2>0 \\ du=dt & u\in\langle -\frac{1}{2};\infty) & \end{array} \right]$$

$$= \frac{1}{6}\ln(t+1) - \frac{1}{12}\int\frac{(t^2-t+1)'\,dt}{t^2-t+1} + \frac{1}{4}\int\frac{du}{u^2+\frac{3}{4}}$$

$$= \frac{1}{12}\ln(t+1)^2 - \frac{1}{12}\ln(t^2-t+1) + \frac{1}{4}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arctg}\frac{u}{\frac{\sqrt{3}}{2}} + c_2$$

$$= \frac{1}{12}\ln\frac{(t+1)^2}{t^2-t+1} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2u}{\sqrt{3}} + c_2 = \frac{1}{12}\ln\frac{(t+1)^2}{t^2-t+1} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2t-1}{\sqrt{3}} + c_2$$

# Riešené príklady – 153

$$\int \frac{x\,dx}{x^6+1} = \frac{1}{12}\ln\frac{(x^2+1)^2}{x^4-x^2+1} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2x^2-1}{\sqrt{3}} + c_2$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int\frac{dt}{t^3+1} = \frac{1}{2}\int\frac{dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int\left[\frac{\frac{1}{3}}{t+1} + \frac{-\frac{t}{3}+\frac{2}{3}}{t^2-t+1}\right]dt = \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{2t-4}{t^2-t+1}\,dt$$

$$= \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{2t-1-3}{t^2-t+1}\,dt = \frac{1}{6}\int\frac{dt}{t+1} - \frac{1}{12}\int\frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4}\int\frac{dt}{t^2-t+1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u=t-\frac{1}{2} & t\in\langle 0;\infty) & t^2-t+1=(t-\frac{1}{2})^2+1-\frac{1}{4}=(t-\frac{1}{2})^2+\frac{3}{4}=u^2+(\frac{\sqrt{3}}{2})^2>0 \\ du=dt & u\in\langle -\frac{1}{2};\infty) & \end{array}\right]$$

$$= \frac{1}{6}\ln(t+1) - \frac{1}{12}\int\frac{(t^2-t+1)'\,dt}{t^2-t+1} + \frac{1}{4}\int\frac{du}{u^2+\frac{3}{4}}$$

$$= \frac{1}{12}\ln(t+1)^2 - \frac{1}{12}\ln(t^2-t+1) + \frac{1}{4}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arctg}\frac{u}{\frac{\sqrt{3}}{2}} + c_2$$

$$= \frac{1}{12}\ln\frac{(t+1)^2}{t^2-t+1} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2u}{\sqrt{3}} + c_2 = \frac{1}{12}\ln\frac{(t+1)^2}{t^2-t+1} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2t-1}{\sqrt{3}} + c_2$$

$$= \frac{1}{12}\ln\frac{(x^2+1)^2}{x^4-x^2+1} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2x^2-1}{\sqrt{3}} + c_2,\ x\in R,\ c_2\in R.$$

# Riešené príklady – 154

$$\int \frac{x^2\,dx}{x^6+1}$$

# Riešené príklady – 154

$$\int \frac{x^2\,dx}{x^6+1}$$

$$=\left[\begin{array}{rl|l} \text{Subst.} & t=x^3 & x\in R \\ & dt=3x^2\,dx & t\in R \end{array}\right]$$

---

$$=\int \frac{x^2\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}$$

# Riešené príklady – 154

$$\int \frac{x^2\,dx}{x^6+1}$$

$$=\left[\begin{array}{r|l}\text{Subst. } t=x^3 & x\in R\\ dt=3x^2\,dx & t\in R\end{array}\right]=\frac{1}{3}\int\frac{dt}{t^2+1}$$

$$=\int\frac{x^2\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}=\int\left[\frac{-\frac{1}{3}}{x^2+1}+\frac{\frac{1}{6}}{x^2+\sqrt{3}x+1}+\frac{\frac{1}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

# Riešené príklady – 154

$$\int \frac{x^2\,dx}{x^6+1}$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t=x^3 & x\in R \\ dt=3x^2\,dx & t\in R \end{array}\right] = \frac{1}{3}\int \frac{dt}{t^2+1} = \frac{1}{3}\,\text{arctg}\, t + c_1$$

---

$$= \int \frac{x^2\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1}\right] dx$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & \multirow{2}{*}{$x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0$} \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \end{array}\right]$$

# Riešené príklady – 154

$$\int \frac{x^2\,dx}{x^6+1} = \frac{1}{3}\,\mathrm{arctg}\,x^3 + c_1$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t=x^3 & x\in R \\ dt=3x^2\,dx & t\in R \end{array}\right] = \frac{1}{3}\int \frac{dt}{t^2+1} = \frac{1}{3}\,\mathrm{arctg}\,t + c_1 = \frac{1}{3}\,\mathrm{arctg}\,x^3 + c_1,\ x\in R,\ c\in R.$$

---

$$= \int \frac{x^2\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1}\right] dx$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.}\ u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 \\ \text{Subst.}\ v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \end{array}\right]$$

# Riešené príklady – 154

$$\int \frac{x^2\,dx}{x^6+1} = \frac{1}{3}\,\mathrm{arctg}\,x^3 + c_1$$

$$= \left[\begin{array}{c|c} \text{Subst.}\quad t=x^3 & x\in R \\ dt=3x^2\,dx & t\in R \end{array}\right] = \frac{1}{3}\int \frac{dt}{t^2+1} = \frac{1}{3}\,\mathrm{arctg}\,t + c_1 = \frac{1}{3}\,\mathrm{arctg}\,x^3 + c_1,\ x\in R,\ c\in R.$$

---

$$= \int \frac{x^2\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1}\right] dx$$

$$= \left[\begin{array}{c|c|c|c} \text{Subst.}\ u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 \\ \text{Subst.}\ v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \end{array}\right]$$

$$= -\frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{6}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{6}\int \frac{dv}{v^2+\frac{1}{4}}$$

# Riešené príklady – 154

$$\int \frac{x^2\,dx}{x^6+1} = \frac{1}{3}\operatorname{arctg} x^3 + c_1$$

$$=\left[\begin{matrix}\text{Subst.}\quad t=x^3 \\ dt=3x^2\,dx\end{matrix}\middle|\begin{matrix}x\in R \\ t\in R\end{matrix}\right] = \frac{1}{3}\int\frac{dt}{t^2+1} = \frac{1}{3}\operatorname{arctg} t + c_1 = \frac{1}{3}\operatorname{arctg} x^3 + c_1,\ x\in R,\ c\in R.$$

---

$$=\int\frac{x^2\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int\left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\left[\begin{matrix}\text{Subst.}\ u=x+\frac{\sqrt{3}}{2} \\ \text{Subst.}\ v=x-\frac{\sqrt{3}}{2}\end{matrix}\middle|\begin{matrix}du=dx \\ dv=dx\end{matrix}\middle|\begin{matrix}x\in R,\ u\in R \\ x\in R,\ v\in R\end{matrix}\middle| x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2>0\right]$$

$$=-\frac{1}{3}\int\frac{dx}{x^2+1}+\frac{1}{6}\int\frac{du}{u^2+\frac{1}{4}}+\frac{1}{6}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$=-\frac{1}{3}\operatorname{arctg} x+\frac{1}{6}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}}+\frac{1}{6}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}}+c_2$$

# Riešené príklady – 154

$$\int \frac{x^2\,dx}{x^6+1} = \frac{1}{3}\operatorname{arctg} x^3 + c_1$$

$$= \left[\begin{array}{c|c} \text{Subst.}\quad t=x^3 & x\in R \\ dt=3x^2\,dx & t\in R \end{array}\right] = \frac{1}{3}\int \frac{dt}{t^2+1} = \frac{1}{3}\operatorname{arctg} t + c_1 = \frac{1}{3}\operatorname{arctg} x^3 + c_1,\ x\in R,\ c\in R.$$

---

$$= \int \frac{x^2\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1}\right] dx$$

$$= \left[\begin{array}{c|c|c|} \text{Subst.}\ u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R \\ \text{Subst.}\ v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R \end{array}\ x^2\pm\sqrt{3}x+1 = \left(x\pm\tfrac{\sqrt{3}}{2}\right)^2+1-\tfrac{3}{4} = \left(x\pm\tfrac{\sqrt{3}}{2}\right)^2+\tfrac{1}{4} = \left(x\pm\tfrac{\sqrt{3}}{2}\right)^2+\left(\tfrac{1}{2}\right)^2 > 0\right]$$

$$= -\frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{6}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{6}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= -\frac{1}{3}\operatorname{arctg} x + \frac{1}{6}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}} + \frac{1}{6}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}} + c_2$$

$$= -\frac{1}{3}\operatorname{arctg} x + \frac{1}{3}\operatorname{arctg} 2u + \frac{1}{3}\operatorname{arctg} 2v + c_2$$

# Riešené príklady – 154

$$\int \frac{x^2\,dx}{x^6+1} = \frac{1}{3}\operatorname{arctg} x^3 + c_1 = -\frac{1}{3}\operatorname{arctg} x + \frac{1}{3}\operatorname{arctg}(2x+\sqrt{3}) + \frac{1}{3}\operatorname{arctg}(2x-\sqrt{3}) + c_2$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t = x^3 & x \in R \\ dt = 3x^2\,dx & t \in R \end{array}\right] = \frac{1}{3}\int \frac{dt}{t^2+1} = \frac{1}{3}\operatorname{arctg} t + c_1 = \frac{1}{3}\operatorname{arctg} x^3 + c_1,\ x \in R,\ c \in R.$$

---

$$= \int \frac{x^2\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{1}{3}}{x^2+1} + \frac{\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{1}{6}}{x^2-\sqrt{3}x+1}\right] dx$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.}\ u = x + \frac{\sqrt{3}}{2} & du = dx & x \in R,\ u \in R & x^2 \pm \sqrt{3}x + 1 = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \frac{1}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 > 0 \\ \text{Subst.}\ v = x - \frac{\sqrt{3}}{2} & dv = dx & x \in R,\ v \in R & \end{array}\right]$$

$$= -\frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{6}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{6}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= -\frac{1}{3}\operatorname{arctg} x + \frac{1}{6}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}} + \frac{1}{6}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}} + c_2$$

$$= -\frac{1}{3}\operatorname{arctg} x + \frac{1}{3}\operatorname{arctg} 2u + \frac{1}{3}\operatorname{arctg} 2v + c_2$$

$$= -\frac{1}{3}\operatorname{arctg} x + \frac{1}{3}\operatorname{arctg}(2x+\sqrt{3}) + \frac{1}{3}\operatorname{arctg}(2x-\sqrt{3}) + c_2,\ x \in R,\ c_2 \in R.$$

# Riešené príklady – 155

$$\int \frac{x^3\, dx}{x^6+1}$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x^3\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x^3\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x^3\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= -\frac{1}{6}\int \frac{2x\,dx}{x^2+1} + \frac{1}{12}\int \left[ \frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x^3\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= -\frac{1}{6} \int \frac{2x\,dx}{x^2+1} + \frac{1}{12} \int \left[ \frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1} \right] dx$$

$$= -\frac{1}{6} \int \frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{12} \int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$=\int \frac{x^3\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}=\int\left[\frac{-\frac{x}{3}}{x^2+1}+\frac{\frac{x}{6}}{x^2+\sqrt{3}x+1}+\frac{\frac{x}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=-\frac{1}{6}\int\frac{2x\,dx}{x^2+1}+\frac{1}{12}\int\left[\frac{2x}{x^2+\sqrt{3}x+1}+\frac{2x}{x^2-\sqrt{3}x+1}\right]dx$$

$$=-\frac{1}{6}\int\frac{(x^2+1)'\,dx}{x^2+1}+\frac{1}{12}\int\left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}-\frac{\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}+\frac{\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\left[\begin{array}{l|l|l|l}\text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4}=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2>0\\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & (x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)=(x^2+1)^2-(\sqrt{3}x)^2=x^4-x^2+1>0\end{array}\right]$$

# Riešené príklady – 155

$\int \frac{x^3\,dx}{x^6+1}$

$$=\int \frac{x^3\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= -\frac{1}{6}\int \frac{2x\,dx}{x^2+1} + \frac{1}{12}\int \left[ \frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1} \right] dx$$

$$= -\frac{1}{6}\int \frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{12}\int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & (x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1) = (x^2+1)^2-(\sqrt{3}x)^2 = x^4-x^2+1 > 0 \end{array} \right]$$

$$= -\frac{1}{6}\ln(x^2+1) + \frac{1}{12}\int \left[ \frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{\sqrt{3}}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$=\int \frac{x^3\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1}\right] dx$$

$$= -\frac{1}{6}\int \frac{2x\,dx}{x^2+1} + \frac{1}{12}\int \left[\frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1}\right] dx$$

$$= -\frac{1}{6}\int \frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{12}\int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2-\sqrt{3}x+1}\right] dx$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & (x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1) = (x^2+1)^2-(\sqrt{3}x)^2 = x^4-x^2+1>0 \end{array}\right]$$

$$= -\frac{1}{6}\ln(x^2+1) + \frac{1}{12}\int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1}\right] dx - \frac{\sqrt{3}}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= -\frac{\ln(x^2+1)^2}{12} + \frac{\ln(x^2+\sqrt{3}x+1)+\ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{u}{\frac{1}{2}} + \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{v}{\frac{1}{2}} + c_1$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$=\int \frac{x^3\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1}\right] dx$$

$$= -\frac{1}{6}\int \frac{2x\,dx}{x^2+1} + \frac{1}{12}\int \left[\frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1}\right] dx$$

$$= -\frac{1}{6}\int \frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{12}\int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2-\sqrt{3}x+1}\right] dx$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & (x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1) = (x^2+1)^2-(\sqrt{3}x)^2 = x^4-x^2+1 > 0 \end{array}\right]$$

$$= -\frac{1}{6}\ln(x^2+1) + \frac{1}{12}\int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1}\right] dx - \frac{\sqrt{3}}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= -\frac{\ln(x^2+1)^2}{12} + \frac{\ln(x^2+\sqrt{3}x+1)+\ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}} + \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}} + c_1$$

$$= \frac{1}{12}\ln\frac{(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)}{(x^2+1)^2} - \frac{\sqrt{3}}{6}\operatorname{arctg}2u + \frac{\sqrt{3}}{6}\operatorname{arctg}2v + c_1$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1} = \frac{1}{12}\ln\frac{x^4-x^2+1}{(x^2+1)^2} - \frac{\sqrt{3}\,\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\sqrt{3}\,\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c_1$$

$$= \int \frac{x^3\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[\frac{-\frac{x}{3}}{x^2+1} + \frac{\frac{x}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{6}}{x^2-\sqrt{3}x+1}\right] dx$$

$$= -\frac{1}{6}\int \frac{2x\,dx}{x^2+1} + \frac{1}{12}\int \left[\frac{2x}{x^2+\sqrt{3}x+1} + \frac{2x}{x^2-\sqrt{3}x+1}\right] dx$$

$$= -\frac{1}{6}\int \frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{12}\int \left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} + \frac{\sqrt{3}}{x^2-\sqrt{3}x+1}\right] dx$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & (x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1) = (x^2+1)^2-(\sqrt{3}x)^2 = x^4-x^2+1 > 0 \end{array}\right]$$

$$= -\frac{1}{6}\ln(x^2+1) + \frac{1}{12}\int \left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1}\right] dx - \frac{\sqrt{3}}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{\sqrt{3}}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= -\frac{\ln(x^2+1)^2}{12} + \frac{\ln(x^2+\sqrt{3}x+1)+\ln(x^2-\sqrt{3}x+1)}{12} - \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{u}{\frac{1}{2}} + \frac{\sqrt{3}}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{v}{\frac{1}{2}} + c_1$$

$$= \frac{1}{12}\ln\frac{(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)}{(x^2+1)^2} - \frac{\sqrt{3}}{6}\,\mathrm{arctg}\,2u + \frac{\sqrt{3}}{6}\,\mathrm{arctg}\,2v + c_1$$

$$= \frac{1}{12}\ln\frac{x^4-x^2+1}{(x^2+1)^2} - \frac{\sqrt{3}\,\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\sqrt{3}\,\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c_1,\ x\in R,\ c_1\in R.$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x \cdot x^2\,dx}{x^6+1}$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x \cdot x^2\,dx}{x^6+1} = \left[ \begin{array}{r|l} \text{Subst. } t = x^2 & x \in (-\infty; 0\rangle,\ t \in \langle 0; \infty) \\ dt = 2x\,dx & x \in \langle 0; \infty),\ t \in \langle 0; \infty) \end{array} \right]$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x\cdot x^2\,dx}{x^6+1} = \left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1}$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x \cdot x^2\,dx}{x^6+1} = \left[ \begin{array}{c|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1} = \frac{1}{2}\int \frac{t\,dt}{(t+1)(t^2-t+1)}$$

# Riešené príklady – 155

$\int \frac{x^3\,dx}{x^6+1}$

$$= \int \frac{x\cdot x^2\,dx}{x^6+1} = \left[ \begin{array}{c|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1} = \frac{1}{2}\int \frac{t\,dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int \left[ \frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1} \right] dt$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x\cdot x^2\,dx}{x^6+1} = \left[\begin{array}{c|c} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1} = \frac{1}{2}\int \frac{t\,dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int\left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1}\right]dt = -\frac{1}{6}\int\frac{dt}{t+1} + \frac{1}{12}\int\frac{2t+2}{t^2-t+1}\,dt$$

# Riešené príklady – 155

$\int \frac{x^3\,dx}{x^6+1}$

$$= \int \frac{x\cdot x^2\,dx}{x^6+1} = \left[\begin{array}{c|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1} = \frac{1}{2}\int \frac{t\,dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1}\right] dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t+2}{t^2-t+1}\,dt$$

$$= -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t-1+3}{t^2-t+1}\,dt$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x\cdot x^2\,dx}{x^6+1} = \left[ \begin{array}{c|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1} = \frac{1}{2}\int \frac{t\,dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int \left[ \frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1} \right] dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t+2}{t^2-t+1}\,dt$$

$$= -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t-1+3}{t^2-t+1}\,dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{dt}{t^2-t+1}$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x \cdot x^2\,dx}{x^6+1} = \left[ \begin{array}{c|c} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1} = \frac{1}{2}\int \frac{t\,dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int \left[ \frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1} \right] dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t+2}{t^2-t+1}\,dt$$

$$= -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t-1+3}{t^2-t+1}\,dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{dt}{t^2-t+1}$$

$$= \left[ \begin{array}{c|c|c} \text{Subst. } u=t-\frac{1}{2} & t\in\langle 0;\infty) & t^2-t+1 = (t-\frac{1}{2})^2+1-\frac{1}{4} = (t-\frac{1}{2})^2+\frac{3}{4} = u^2+(\frac{\sqrt{3}}{2})^2 > 0 \\ du=dt & u\in\langle -\frac{1}{2};\infty) & \end{array} \right]$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x\cdot x^2\,dx}{x^6+1} = \left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1} = \frac{1}{2}\int \frac{t\,dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1}\right] dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t+2}{t^2-t+1}\,dt$$

$$= -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t-1+3}{t^2-t+1}\,dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{dt}{t^2-t+1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u=t-\frac{1}{2} & t\in\langle 0;\infty) & t^2-t+1 = (t-\frac{1}{2})^2+1-\frac{1}{4} = (t-\frac{1}{2})^2+\frac{3}{4} = u^2+(\frac{\sqrt{3}}{2})^2 > 0 \\ du=dt & u\in\langle -\frac{1}{2};\infty) & \end{array}\right]$$

$$= -\frac{1}{6}\ln(t+1) + \frac{1}{12}\int \frac{(t^2-t+1)'\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{du}{u^2+\frac{3}{4}}$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x \cdot x^2\,dx}{x^6+1} = \left[ \begin{array}{r|l} \text{Subst. } t = x^2 & x \in (-\infty; 0\rangle,\ t \in \langle 0; \infty) \\ dt = 2x\,dx & x \in \langle 0; \infty),\ t \in \langle 0; \infty) \end{array} \right] = \frac{1}{2} \int \frac{t\,dt}{t^3+1} = \frac{1}{2} \int \frac{t\,dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2} \int \left[ \frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1} \right] dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t+2}{t^2-t+1}\,dt$$

$$= -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{2t-1+3}{t^2-t+1}\,dt = -\frac{1}{6} \int \frac{dt}{t+1} + \frac{1}{12} \int \frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4} \int \frac{dt}{t^2-t+1}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } u = t-\frac{1}{2} & t \in \langle 0; \infty) & t^2-t+1 = (t-\frac{1}{2})^2+1-\frac{1}{4} = (t-\frac{1}{2})^2+\frac{3}{4} = u^2+(\frac{\sqrt{3}}{2})^2 > 0 \\ du = dt & u \in \langle -\frac{1}{2}; \infty) & \end{array} \right]$$

$$= -\frac{1}{6} \ln(t+1) + \frac{1}{12} \int \frac{(t^2-t+1)'\,dt}{t^2-t+1} + \frac{1}{4} \int \frac{du}{u^2+\frac{3}{4}}$$

$$= -\frac{1}{12} \ln(t+1)^2 + \frac{1}{12} \ln(t^2-t+1) + \frac{1}{4} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arctg} \frac{u}{\frac{\sqrt{3}}{2}} + c_2$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x\cdot x^2\,dx}{x^6+1} = \left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1} = \frac{1}{2}\int \frac{t\,dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1}\right] dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t+2}{t^2-t+1}\,dt$$

$$= -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t-1+3}{t^2-t+1}\,dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{dt}{t^2-t+1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u=t-\frac{1}{2} & t\in\langle 0;\infty) & t^2-t+1 = (t-\frac{1}{2})^2+1-\frac{1}{4} = (t-\frac{1}{2})^2+\frac{3}{4} = u^2+(\frac{\sqrt{3}}{2})^2 > 0 \\ du=dt & u\in\langle -\frac{1}{2};\infty) & \end{array}\right]$$

$$= -\frac{1}{6}\ln(t+1) + \frac{1}{12}\int \frac{(t^2-t+1)'\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{du}{u^2+\frac{3}{4}}$$

$$= -\frac{1}{12}\ln(t+1)^2 + \frac{1}{12}\ln(t^2-t+1) + \frac{1}{4}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arctg}\frac{u}{\frac{\sqrt{3}}{2}} + c_2$$

$$= \frac{1}{12}\ln\frac{t^2-t+1}{(t+1)^2} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2u}{\sqrt{3}} + c_2$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1}$$

$$= \int \frac{x\cdot x^2\,dx}{x^6+1} = \left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 0;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{t\,dt}{t^3+1} = \frac{1}{2}\int \frac{t\,dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int \left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1}\right] dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t+2}{t^2-t+1}\,dt$$

$$= -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{2t-1+3}{t^2-t+1}\,dt = -\frac{1}{6}\int \frac{dt}{t+1} + \frac{1}{12}\int \frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{dt}{t^2-t+1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u=t-\frac{1}{2} & t\in\langle 0;\infty) & t^2-t+1 = (t-\frac{1}{2})^2+1-\frac{1}{4} = (t-\frac{1}{2})^2+\frac{3}{4} = u^2+(\frac{\sqrt{3}}{2})^2 > 0 \\ du=dt & u\in\langle -\frac{1}{2};\infty) & \end{array}\right]$$

$$= -\frac{1}{6}\ln(t+1) + \frac{1}{12}\int \frac{(t^2-t+1)'\,dt}{t^2-t+1} + \frac{1}{4}\int \frac{du}{u^2+\frac{3}{4}}$$

$$= -\frac{1}{12}\ln(t+1)^2 + \frac{1}{12}\ln(t^2-t+1) + \frac{1}{4}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arctg}\frac{u}{\frac{\sqrt{3}}{2}} + c_2$$

$$= \frac{1}{12}\ln\frac{t^2-t+1}{(t+1)^2} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2u}{\sqrt{3}} + c_2 = \frac{1}{12}\ln\frac{t^2-t+1}{(t+1)^2} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2t-1}{\sqrt{3}} + c_2$$

# Riešené príklady – 155

$$\int \frac{x^3\,dx}{x^6+1} = \frac{1}{12}\ln\frac{x^4-x^2+1}{(x^2+1)^2} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2x^2-1}{\sqrt{3}} + c_2$$

$$= \int \frac{x\cdot x^2\,dx}{x^6+1} = \left[\begin{array}{r|l} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) \\ dt=2x\,dx & x\in(0;\infty),\ t\in(0;\infty) \end{array}\right] = \frac{1}{2}\int\frac{t\,dt}{t^3+1} = \frac{1}{2}\int\frac{t\,dt}{(t+1)(t^2-t+1)}$$

$$= \frac{1}{2}\int\left[\frac{-\frac{1}{3}}{t+1} + \frac{\frac{t}{3}+\frac{1}{3}}{t^2-t+1}\right]dt = -\frac{1}{6}\int\frac{dt}{t+1} + \frac{1}{12}\int\frac{2t+2}{t^2-t+1}\,dt$$

$$= -\frac{1}{6}\int\frac{dt}{t+1} + \frac{1}{12}\int\frac{2t-1+3}{t^2-t+1}\,dt = -\frac{1}{6}\int\frac{dt}{t+1} + \frac{1}{12}\int\frac{(2t-1)\,dt}{t^2-t+1} + \frac{1}{4}\int\frac{dt}{t^2-t+1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u=t-\frac{1}{2} & t\in(0;\infty) & t^2-t+1 = (t-\frac{1}{2})^2+1-\frac{1}{4} = (t-\frac{1}{2})^2+\frac{3}{4} = u^2+(\frac{\sqrt{3}}{2})^2 > 0 \\ du=dt & u\in(-\frac{1}{2};\infty) & \end{array}\right]$$

$$= -\frac{1}{6}\ln(t+1) + \frac{1}{12}\int\frac{(t^2-t+1)'\,dt}{t^2-t+1} + \frac{1}{4}\int\frac{du}{u^2+\frac{3}{4}}$$

$$= -\frac{1}{12}\ln(t+1)^2 + \frac{1}{12}\ln(t^2-t+1) + \frac{1}{4}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arctg}\frac{u}{\frac{\sqrt{3}}{2}} + c_2$$

$$= \frac{1}{12}\ln\frac{t^2-t+1}{(t+1)^2} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2u}{\sqrt{3}} + c_2 = \frac{1}{12}\ln\frac{t^2-t+1}{(t+1)^2} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2t-1}{\sqrt{3}} + c_2$$

$$= \frac{1}{12}\ln\frac{x^4-x^2+1}{(x^2+1)^2} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2x^2-1}{\sqrt{3}} + c_2,\ x\in R,\ c_2\in R.$$

# Riešené príklady – 156

$$\int \frac{x^4\,dx}{x^6+1}$$

# Riešené príklady – 156

$\int \frac{x^4\,dx}{x^6+1}$

$$= \int \frac{x^4\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}$$

# Riešené príklady – 156

$\int \frac{x^4\,dx}{x^6+1}$

$$= \int \frac{x^4\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 156

$$\int \frac{x^4\,dx}{x^6+1}$$

$$=\int \frac{x^4\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}=\int\left[\frac{\frac{1}{3}}{x^2+1}+\frac{-\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2+\sqrt{3}x+1}+\frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{3}\int\frac{dx}{x^2+1}+\frac{1}{4\sqrt{3}}\int\left[-\frac{2x+\frac{4\sqrt{3}}{6}}{x^2+\sqrt{3}x+1}+\frac{2x-\frac{4\sqrt{3}}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

# Riešené príklady – 156

$\int \frac{x^4\,dx}{x^6+1}$

$$=\int \frac{x^4\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[ -\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 156

$\int \frac{x^4\,dx}{x^6+1}$

$$= \int \frac{x^4\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}}\int \left[ -\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\operatorname{arctg} x + \frac{1}{4\sqrt{3}}\int \left[ -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 156

$$\int \frac{x^4\,dx}{x^6+1}$$

$$=\int \frac{x^4\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}=\int\left[\frac{\frac{1}{3}}{x^2+1}+\frac{-\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2+\sqrt{3}x+1}+\frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{3}\int\frac{dx}{x^2+1}+\frac{1}{4\sqrt{3}}\int\left[-\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{3}\operatorname{arctg}x+\frac{1}{4\sqrt{3}}\int\left[-\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}-\frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}-\frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\left[\begin{array}{l|l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{6}-\sqrt{3}\right) \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \quad=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2>0 & \quad=\frac{1}{6}-\frac{1}{4}=\frac{2-3}{12}=-\frac{1}{12} \end{array}\right]$$

# Riešené príklady – 156

$$\int \frac{x^4\,dx}{x^6+1}$$

$$= \int \frac{x^4\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}} \int \left[ -\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\operatorname{arctg} x + \frac{1}{4\sqrt{3}} \int \left[ -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{6}-\sqrt{3}\right) \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \quad = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 & \quad = \frac{1}{6}-\frac{1}{4} = \frac{2-3}{12} = -\frac{1}{12} \end{array} \right]$$

$$= \frac{1}{3}\operatorname{arctg} x - \frac{1}{4\sqrt{3}} \int \left[ \frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

# Riešené príklady – 156

$\int \frac{x^4\,dx}{x^6+1}$

$$=\int \frac{x^4\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}=\int\left[\frac{\frac{1}{3}}{x^2+1}+\frac{-\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2+\sqrt{3}x+1}+\frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{3}\int\frac{dx}{x^2+1}+\frac{1}{4\sqrt{3}}\int\left[-\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{3}\operatorname{arctg}x+\frac{1}{4\sqrt{3}}\int\left[-\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}-\frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}-\frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\left[\begin{array}{l|l|l|l|l}\text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{6}-\sqrt{3}\right)\\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \quad=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2>0 & \quad=\frac{1}{6}-\frac{1}{4}=\frac{2-3}{12}=-\frac{1}{12}\end{array}\right]$$

$$=\frac{1}{3}\operatorname{arctg}x-\frac{1}{4\sqrt{3}}\int\left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1}-\frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1}\right]dx+\frac{1}{12}\int\frac{du}{u^2+\frac{1}{4}}+\frac{1}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$=\frac{1}{3}\operatorname{arctg}x-\frac{\ln(x^2+\sqrt{3}x+1)-\ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}}+\frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}}+\frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}}+c$$

# Riešené príklady – 156

$$\int \frac{x^4\,dx}{x^6+1}$$

$$= \int \frac{x^4\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}}\int \left[ -\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\operatorname{arctg} x + \frac{1}{4\sqrt{3}}\int \left[ -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{6}-\sqrt{3}\right) \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \quad = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 & \quad = \frac{1}{6}-\frac{1}{4} = \frac{2-3}{12} = -\frac{1}{12} \end{array} \right]$$

$$= \frac{1}{3}\operatorname{arctg} x - \frac{1}{4\sqrt{3}}\int \left[ \frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{1}{3}\operatorname{arctg} x - \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}} + c$$

$$= \frac{1}{3}\operatorname{arctg} x - \frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{1}{6}\operatorname{arctg} 2u + \frac{1}{6}\operatorname{arctg} 2v + c$$

# Riešené príklady – 156

$$\int \frac{x^4\,dx}{x^6+1} = \frac{1}{3}\,\mathrm{arctg}\,x - \frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c$$

$$= \int \frac{x^4\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{-\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{2\sqrt{3}}-\frac{1}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2+1} + \frac{1}{4\sqrt{3}}\int \left[ -\frac{2x+\frac{4\sqrt{3}}{6}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\frac{4\sqrt{3}}{6}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x + \frac{1}{4\sqrt{3}}\int \left[ -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{6}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} & du = dx & x\in R,\ u\in R & x^2 \pm \sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{6}-\sqrt{3}\right) \\ \text{Subst. } v = x-\frac{\sqrt{3}}{2} & dv = dx & x\in R,\ v\in R & \quad = \left(x\pm\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 > 0 & \quad = \frac{1}{6}-\frac{1}{4} = \frac{2-3}{12} = -\frac{1}{12} \end{array} \right]$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x - \frac{1}{4\sqrt{3}}\int \left[ \frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx + \frac{1}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x - \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{u}{\frac{1}{2}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{v}{\frac{1}{2}} + c$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x - \frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{1}{6}\,\mathrm{arctg}\,2u + \frac{1}{6}\,\mathrm{arctg}\,2v + c$$

$$= \frac{1}{3}\,\mathrm{arctg}\,x - \frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + \frac{\mathrm{arctg}\,(2x+\sqrt{3})}{6} + \frac{\mathrm{arctg}\,(2x-\sqrt{3})}{6} + c,\ x\in R,\ c\in R.$$

# Riešené príklady – 157

$$\int \frac{x^5 \, dx}{x^6+1}$$

# Riešené príklady – 157

$$\int \frac{x^5\,dx}{x^6+1}$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=x^6+1 & x\in(-\infty;0\rangle,\ t\in\langle 1;\infty) \\ dt=6x^5\,dx & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \end{array}\right]$$

---

$$=\int \frac{x^5\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}$$

# Riešené príklady – 157

$$\int \frac{x^5\,dx}{x^6+1}$$

$$=\left[\begin{array}{r|l}\text{Subst. } t=x^6+1 & x\in(-\infty;0),\ t\in\langle 1;\infty)\\ dt=6x^5\,dx & x\in\langle 0;\infty),\ t\in\langle 1;\infty)\end{array}\right]=\frac{1}{6}\int\frac{dt}{t}$$

$$=\int\frac{x^5\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}=\int\left[\frac{\frac{x}{3}}{x^2+1}+\frac{\frac{x}{3}+\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1}+\frac{\frac{x}{3}-\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

# Riešené príklady – 157

$\int \frac{x^5\,dx}{x^6+1}$

$$=\left[\begin{array}{r|l} \text{Subst. } t=x^6+1 & x\in(-\infty;0),\ t\in\langle 1;\infty) \\ dt=6x^5\,dx & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \end{array}\right]=\frac{1}{6}\int\frac{dt}{t}=\frac{1}{6}\ln t+c$$

---

$$=\int\frac{x^5\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)}=\int\left[\frac{\frac{x}{3}}{x^2+1}+\frac{\frac{x}{3}+\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1}+\frac{\frac{x}{3}-\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\frac{1}{6}\int\frac{2x\,dx}{x^2+1}+\frac{1}{6}\int\left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

# Riešené príklady – 157

$$\int \frac{x^5\,dx}{x^6+1} = \frac{1}{6}\ln(x^6+1)+c$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^6+1 & x\in(-\infty;0\rangle,\ t\in\langle 1;\infty) \\ dt=6x^5\,dx & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \end{array}\right] = \frac{1}{6}\int\frac{dt}{t} = \frac{1}{6}\ln t + c = \frac{1}{6}\ln(x^6+1)+c,$$

$x\in R,\ c\in R.$

$$= \int \frac{x^5\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int\left[\frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3}+\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3}-\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \frac{1}{6}\int\frac{2x\,dx}{x^2+1} + \frac{1}{6}\int\left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \frac{1}{6}\int\frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{6}\int\left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1}\right]dx$$

# Riešené príklady – 157

$$\int \frac{x^5\,dx}{x^6+1} = \frac{1}{6}\ln(x^6+1)+c$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^6+1 & x\in(-\infty;0\rangle,\ t\in\langle 1;\infty) \\ dt=6x^5\,dx & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \end{array} \right] = \frac{1}{6}\int\frac{dt}{t} = \frac{1}{6}\ln t + c = \frac{1}{6}\ln(x^6+1)+c,$$

$x\in R$, $c\in R$.

$$= \int \frac{x^5\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int\left[\frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3}+\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3}-\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \frac{1}{6}\int\frac{2x\,dx}{x^2+1} + \frac{1}{6}\int\left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \frac{1}{6}\int\frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{6}\int\left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \frac{1}{6}\ln(x^2+1) + \frac{1}{6}\ln(x^2+\sqrt{3}x+1) + \frac{1}{6}\ln(x^2-\sqrt{3}x+1)+c$$

# Riešené príklady – 157

$$\int \frac{x^5\,dx}{x^6+1} = \frac{1}{6}\ln(x^6+1)+c$$

$$= \left[\begin{array}{c|c} \text{Subst. } t=x^6+1 & x\in(-\infty;0),\ t\in\langle 1;\infty) \\ dt=6x^5\,dx & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \end{array}\right] = \frac{1}{6}\int\frac{dt}{t} = \frac{1}{6}\ln t + c = \frac{1}{6}\ln(x^6+1)+c,$$

$x\in R$, $c\in R$.

$$= \int \frac{x^5\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int\left[\frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3}+\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3}-\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \frac{1}{6}\int\frac{2x\,dx}{x^2+1} + \frac{1}{6}\int\left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \frac{1}{6}\int\frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{6}\int\left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1}\right]dx$$

$$= \frac{1}{6}\ln(x^2+1) + \frac{1}{6}\ln(x^2+\sqrt{3}x+1) + \frac{1}{6}\ln(x^2-\sqrt{3}x+1)+c$$

$$= \frac{1}{6}\ln\left[(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)\right]+c$$

# Riešené príklady – 157

$$\int \frac{x^5\,dx}{x^6+1} = \frac{1}{6}\ln(x^6+1)+c$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x^6+1 & x\in(-\infty;0),\ t\in\langle 1;\infty) \\ dt=6x^5\,dx & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \end{array} \right] = \frac{1}{6}\int \frac{dt}{t} = \frac{1}{6}\ln t + c = \frac{1}{6}\ln(x^6+1)+c,$$

$x\in R,\ c\in R.$

$$= \int \frac{x^5\,dx}{(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)} = \int \left[ \frac{\frac{x}{3}}{x^2+1} + \frac{\frac{x}{3}+\frac{\sqrt{3}}{6}}{x^2+\sqrt{3}x+1} + \frac{\frac{x}{3}-\frac{\sqrt{3}}{6}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6}\int \frac{2x\,dx}{x^2+1} + \frac{1}{6}\int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6}\int \frac{(x^2+1)'\,dx}{x^2+1} + \frac{1}{6}\int \left[ \frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} + \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \frac{1}{6}\ln(x^2+1) + \frac{1}{6}\ln(x^2+\sqrt{3}x+1) + \frac{1}{6}\ln(x^2-\sqrt{3}x+1)+c$$

$$= \frac{1}{6}\ln\left[(x^2+1)(x^2+\sqrt{3}x+1)(x^2-\sqrt{3}x+1)\right]+c = \frac{1}{6}\ln(x^6+1)+c,$$

$x\in R,\ c\in R.$

# Riešené príklady – 158

$\int \frac{x^6\,dx}{x^6+1}$

# Riešené príklady – 158

$$\int \frac{x^6\,dx}{x^6+1}$$

$$= \int \frac{(x^6+1-1)\,dx}{x^6+1}$$

# Riešené príklady – 158

$$\int \frac{x^6\,dx}{x^6+1}$$

$$= \int \frac{(x^6+1-1)\,dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1}$$

# Riešené príklady – 158

$\int \frac{x^6\,dx}{x^6+1}$

$$= \int \frac{(x^6+1-1)\,dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 158

$$\int \frac{x^6\,dx}{x^6+1}$$

$$= \int \frac{(x^6+1-1)\,dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{1}{3}\int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}}\int \left[ \frac{2x+\frac{4\sqrt{3}}{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 158

$$\int \frac{x^6\,dx}{x^6+1}$$

$$= \int \frac{(x^6+1-1)\,dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{1}{3}\int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}} \int \left[ \frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

# Riešené príklady – 158

$$\int \frac{x^6\,dx}{x^6+1}$$

$$=\int\frac{(x^6+1-1)\,dx}{x^6+1}=\int dx-\int\frac{dx}{x^6+1}=x-\int\left[\frac{\frac{1}{3}}{x^2+1}+\frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1}+\frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=x-\frac{1}{3}\int\frac{dx}{x^2+1}-\frac{1}{4\sqrt{3}}\int\left[\frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1}-\frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=x-\frac{\operatorname{arctg} x}{3}-\frac{1}{4\sqrt{3}}\int\left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1}-\frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}-\frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

# Riešené príklady – 158

$\int \frac{x^6\,dx}{x^6+1}$

$$= \int \frac{(x^6+1-1)\,dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{1}{3}\int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}} \int \left[ \frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} & du = dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{3}-\sqrt{3}\right) \\ \text{Subst. } v = x-\frac{\sqrt{3}}{2} & dv = dx & x\in R,\ v\in R & \quad = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 & \quad = \frac{1}{3}-\frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right]$$

# Riešené príklady – 158

$$\int \frac{x^6\,dx}{x^6+1}$$

$$= \int \frac{(x^6+1-1)\,dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{1}{3}\int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}}\int \left[ \frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}}\int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{3}-\sqrt{3}\right) \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \quad = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 & \quad = \frac{1}{3}-\frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right]$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}}\int \left[ \frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{1}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

# Riešené príklady – 158

$$\int \frac{x^6\,dx}{x^6+1}$$

$$= \int \frac{(x^6+1-1)\,dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{1}{3}\int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}} \int \left[ \frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1 = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{3}-\sqrt{3}\right) \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \quad = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4} = \left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2 > 0 & \quad = \frac{1}{3}-\frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right]$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \int \left[ \frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{1}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} - \frac{1}{12}\cdot\frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{u}{\frac{1}{2}} + \frac{1}{12}\cdot\frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{v}{\frac{1}{2}} + c$$

# Riešené príklady – 158

$\int \frac{x^6\,dx}{x^6+1}$

$$=\int \frac{(x^6+1-1)\,dx}{x^6+1}=\int dx-\int \frac{dx}{x^6+1}=x-\int\left[\frac{\frac{1}{3}}{x^2+1}+\frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1}+\frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=x-\frac{1}{3}\int\frac{dx}{x^2+1}-\frac{1}{4\sqrt{3}}\int\left[\frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1}-\frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=x-\frac{\operatorname{arctg} x}{3}-\frac{1}{4\sqrt{3}}\int\left[\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}+\frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1}-\frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1}-\frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1}\right]dx$$

$$=\left[\begin{array}{l|l|l|l|l} \text{Subst. } u=x+\frac{\sqrt{3}}{2} & du=dx & x\in R,\ u\in R & x^2\pm\sqrt{3}x+1=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+1-\frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{3}-\sqrt{3}\right) \\ \text{Subst. } v=x-\frac{\sqrt{3}}{2} & dv=dx & x\in R,\ v\in R & \quad=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}=\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2>0 & \quad=\frac{1}{3}-\frac{1}{4}=\frac{4-3}{12}=\frac{1}{12} \end{array}\right]$$

$$=x-\frac{\operatorname{arctg} x}{3}-\frac{1}{4\sqrt{3}}\int\left[\frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1}-\frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1}\right]dx-\frac{1}{12}\int\frac{du}{u^2+\frac{1}{4}}+\frac{1}{12}\int\frac{dv}{v^2+\frac{1}{4}}$$

$$=x-\frac{\operatorname{arctg} x}{3}-\frac{\ln(x^2+\sqrt{3}x+1)-\ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}}-\frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{u}{\frac{1}{2}}+\frac{1}{12}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{v}{\frac{1}{2}}+c$$

$$=x-\frac{\operatorname{arctg} x}{3}-\frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1}-\frac{1}{6}\operatorname{arctg} 2u+\frac{1}{6}\operatorname{arctg} 2v+c$$

# Riešené príklady – 158

$$\int \frac{x^6\,dx}{x^6+1} = x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} - \frac{\operatorname{arctg}(2x+\sqrt{3})}{6} + \frac{\operatorname{arctg}(2x-\sqrt{3})}{6} + c$$

$$= \int \frac{(x^6+1-1)\,dx}{x^6+1} = \int dx - \int \frac{dx}{x^6+1} = x - \int \left[ \frac{\frac{1}{3}}{x^2+1} + \frac{\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2+\sqrt{3}x+1} + \frac{-\frac{x}{2\sqrt{3}}+\frac{1}{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{1}{3}\int \frac{dx}{x^2+1} - \frac{1}{4\sqrt{3}} \int \left[ \frac{2x+\frac{4\sqrt{3}}{3}+\sqrt{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\frac{4\sqrt{3}}{3}-\sqrt{3}+\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \int \left[ \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1} + \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2+\sqrt{3}x+1} - \frac{2x-\sqrt{3}}{x^2-\sqrt{3}x+1} - \frac{\frac{4\sqrt{3}}{3}-\sqrt{3}}{x^2-\sqrt{3}x+1} \right] dx$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u = x+\frac{\sqrt{3}}{2} & du = dx & x \in R,\ u \in R & x^2 \pm \sqrt{3}x+1 = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + 1 - \frac{3}{4} & \frac{1}{4\sqrt{3}}\left(\frac{4\sqrt{3}}{3}-\sqrt{3}\right) \\ \text{Subst. } v = x-\frac{\sqrt{3}}{2} & dv = dx & x \in R,\ v \in R & \quad = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \frac{1}{4} = \left(x \pm \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 > 0 & \quad = \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12} \end{array} \right]$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \int \left[ \frac{(x^2+\sqrt{3}x+1)'}{x^2+\sqrt{3}x+1} - \frac{(x^2-\sqrt{3}x+1)'}{x^2-\sqrt{3}x+1} \right] dx - \frac{1}{12}\int \frac{du}{u^2+\frac{1}{4}} + \frac{1}{12}\int \frac{dv}{v^2+\frac{1}{4}}$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{\ln(x^2+\sqrt{3}x+1) - \ln(x^2-\sqrt{3}x+1)}{4\sqrt{3}} - \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{u}{\frac{1}{2}} + \frac{1}{12} \cdot \frac{1}{\frac{1}{2}} \operatorname{arctg} \frac{v}{\frac{1}{2}} + c$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} - \frac{1}{6} \operatorname{arctg} 2u + \frac{1}{6} \operatorname{arctg} 2v + c$$

$$= x - \frac{\operatorname{arctg} x}{3} - \frac{1}{4\sqrt{3}} \ln \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} - \frac{\operatorname{arctg}(2x+\sqrt{3})}{6} + \frac{\operatorname{arctg}(2x-\sqrt{3})}{6} + c,\ x \in R,\ c \in R.$$

# Riešené príklady – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$\int \frac{dx}{\sqrt{2^x+1}}$$

# Riešené príklady – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=2^x+1,\ 2^x=t-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} & dx=\frac{dt}{(t-1)\ln 2} \end{array} \right]$$

$$\int \frac{dx}{\sqrt{2^x+1}}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=\sqrt{2^x+1},\ 2^x=t^2-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} & dx=\frac{2t\,dt}{(t^2-1)\ln 2} \end{array} \right]$$

# Riešené príklady – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=2^x+1,\ 2^x=t-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} & dx=\frac{dt}{(t-1)\ln 2} \end{array} \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t}$$

$$\int \frac{dx}{\sqrt{2^x+1}}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=\sqrt{2^x+1},\ 2^x=t^2-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} & dx=\frac{2t\,dt}{(t^2-1)\ln 2} \end{array} \right] = \int \frac{\frac{2t\,dt}{(t^2-1)\ln 2}}{t}$$

# Riešené príklady – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t = 2^x+1,\ 2^x = t-1 & x \in R,\ t \in (1;\infty) \\ x = \log_2(t-1) = \frac{\ln(t-1)}{\ln 2} & dx = \frac{dt}{(t-1)\ln 2} \end{array} \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \frac{1}{\ln 2} \int \frac{dt}{t(t-1)}$$

$$\int \frac{dx}{\sqrt{2^x+1}}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t = \sqrt{2^x+1},\ 2^x = t^2-1 & x \in R,\ t \in (1;\infty) \\ x = \log_2(t^2-1) = \frac{\ln(t^2-1)}{\ln 2} & dx = \frac{2t\,dt}{(t^2-1)\ln 2} \end{array} \right] = \int \frac{\frac{2t\,dt}{(t^2-1)\ln 2}}{t} = \frac{2}{\ln 2} \int \frac{dt}{t^2-1}$$

# Riešené príklady – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=2^x+1,\ 2^x=t-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} & dx=\frac{dt}{(t-1)\ln 2} \end{array} \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \frac{1}{\ln 2}\int \frac{dt}{t(t-1)} = \frac{1}{\ln 2}\int \frac{(t-t+1)\,dt}{t(t-1)}$$

$$\int \frac{dx}{\sqrt{2^x+1}}$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=\sqrt{2^x+1},\ 2^x=t^2-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} & dx=\frac{2t\,dt}{(t^2-1)\ln 2} \end{array} \right] = \int \frac{\frac{2t\,dt}{(t^2-1)\ln 2}}{t} = \frac{2}{\ln 2}\int \frac{dt}{t^2-1} = \frac{2}{2\ln 2}\ln \frac{t-1}{t+1} + c$$

# Riešené príklady – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$=\left[\begin{array}{c|c} \text{Subst. } t=2^x+1,\ 2^x=t-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} & dx=\frac{dt}{(t-1)\ln 2}\end{array}\right]=\int\frac{\frac{dt}{(t-1)\ln 2}}{t}=\frac{1}{\ln 2}\int\frac{dt}{t(t-1)}=\frac{1}{\ln 2}\int\frac{(t-t+1)\,dt}{t(t-1)}$$

$$=\frac{1}{\ln 2}\int\left[\frac{1}{t-1}-\frac{1}{t}\right]dt$$

$$\int \frac{dx}{\sqrt{2^x+1}}=\frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}+c$$

$$=\left[\begin{array}{c|c} \text{Subst. } t=\sqrt{2^x+1},\ 2^x=t^2-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} & dx=\frac{2t\,dt}{(t^2-1)\ln 2}\end{array}\right]=\int\frac{\frac{2t\,dt}{(t^2-1)\ln 2}}{t}=\frac{2}{\ln 2}\int\frac{dt}{t^2-1}=\frac{2}{2\ln 2}\ln\frac{t-1}{t+1}+c$$

$$=\frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}+c$$

# Riešené príklady – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=2^x+1,\ 2^x=t-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} & dx=\frac{dt}{(t-1)\ln 2} \end{array} \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \frac{1}{\ln 2}\int \frac{dt}{t(t-1)} = \frac{1}{\ln 2}\int \frac{(t-t+1)\,dt}{t(t-1)}$$

$$= \frac{1}{\ln 2}\int \left[\frac{1}{t-1}-\frac{1}{t}\right] dt = \frac{1}{\ln 2}\left[\ln(t-1)-\ln t\right]+c$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}+c$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=\sqrt{2^x+1},\ 2^x=t^2-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} & dx=\frac{2t\,dt}{(t^2-1)\ln 2} \end{array} \right] = \int \frac{\frac{2t\,dt}{(t^2-1)\ln 2}}{t} = \frac{2}{\ln 2}\int \frac{dt}{t^2-1} = \frac{2}{2\ln 2}\ln\frac{t-1}{t+1}+c$$

$$= \frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}+c = \frac{1}{\ln 2}\ln\left[\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}\,\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}-1}\right]+c$$

# Riešené príklady – 159, 160

$$\int \frac{dx}{2^x+1}$$

$$=\left[\begin{array}{c|c} \text{Subst. } t=2^x+1,\ 2^x=t-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} & dx=\frac{dt}{(t-1)\ln 2}\end{array}\right]=\int\frac{\frac{dt}{(t-1)\ln 2}}{t}=\frac{1}{\ln 2}\int\frac{dt}{t(t-1)}=\frac{1}{\ln 2}\int\frac{(t-t+1)\,dt}{t(t-1)}$$

$$=\frac{1}{\ln 2}\int\left[\frac{1}{t-1}-\frac{1}{t}\right]dt=\frac{1}{\ln 2}\Big[\ln(t-1)-\ln t\Big]+c=\frac{1}{\ln 2}\ln\frac{t-1}{t}+c$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}+c$$

$$=\left[\begin{array}{c|c} \text{Subst. } t=\sqrt{2^x+1},\ 2^x=t^2-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} & dx=\frac{2t\,dt}{(t^2-1)\ln 2}\end{array}\right]=\int\frac{\frac{2t\,dt}{(t^2-1)\ln 2}}{t}=\frac{2}{\ln 2}\int\frac{dt}{t^2-1}=\frac{2}{2\ln 2}\ln\frac{t-1}{t+1}+c$$

$$=\frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}+c=\frac{1}{\ln 2}\ln\left[\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}\,\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}-1}\right]+c=\frac{1}{\ln 2}\ln\frac{2^x+1-2\sqrt{2^x+1}+1}{2^x+1-1}+c$$

# Riešené príklady – 159, 160

$$\int \frac{dx}{2^x+1} = \frac{1}{\ln 2} \ln \frac{2^x}{2^x+1} + c$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=2^x+1,\ 2^x=t-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} & dx=\frac{dt}{(t-1)\ln 2} \end{array} \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \frac{1}{\ln 2}\int \frac{dt}{t(t-1)} = \frac{1}{\ln 2}\int \frac{(t-t+1)\,dt}{t(t-1)}$$

$$= \frac{1}{\ln 2}\int \left[\frac{1}{t-1}-\frac{1}{t}\right] dt = \frac{1}{\ln 2}\Big[\ln(t-1)-\ln t\Big]+c = \frac{1}{\ln 2}\ln\frac{t-1}{t}+c = \frac{1}{\ln 2}\ln\frac{2^x}{2^x+1}+c$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2} \ln \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=\sqrt{2^x+1},\ 2^x=t^2-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} & dx=\frac{2t\,dt}{(t^2-1)\ln 2} \end{array} \right] = \int \frac{\frac{2t\,dt}{(t^2-1)\ln 2}}{t} = \frac{2}{\ln 2}\int \frac{dt}{t^2-1} = \frac{2}{2\ln 2}\ln\frac{t-1}{t+1}+c$$

$$= \frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}+c = \frac{1}{\ln 2}\ln\left[\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}\,\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}-1}\right]+c = \frac{1}{\ln 2}\ln\frac{2^x+1-2\sqrt{2^x+1}+1}{2^x+1-1}+c$$

$$= \frac{\ln(2^x+2-2\sqrt{2^x+1})-\ln 2^x}{\ln 2}+c$$

# Riešené príklady – 159, 160

$$\int \frac{dx}{2^x+1} = \frac{1}{\ln 2} \ln \frac{2^x}{2^x+1} + c$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=2^x+1,\ 2^x=t-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} & dx=\frac{dt}{(t-1)\ln 2} \end{array} \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \frac{1}{\ln 2}\int \frac{dt}{t(t-1)} = \frac{1}{\ln 2}\int \frac{(t-t+1)\,dt}{t(t-1)}$$

$$= \frac{1}{\ln 2}\int\left[\frac{1}{t-1} - \frac{1}{t}\right] dt = \frac{1}{\ln 2}\Big[\ln(t-1) - \ln t\Big] + c = \frac{1}{\ln 2}\ln\frac{t-1}{t} + c = \frac{1}{\ln 2}\ln\frac{2^x}{2^x+1} + c$$

$$= \frac{\ln 2^x - \ln(2^x+1)}{\ln 2} + c$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=\sqrt{2^x+1},\ 2^x=t^2-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} & dx=\frac{2t\,dt}{(t^2-1)\ln 2} \end{array} \right] = \int \frac{\frac{2t\,dt}{(t^2-1)\ln 2}}{t} = \frac{2}{\ln 2}\int\frac{dt}{t^2-1} = \frac{2}{2\ln 2}\ln\frac{t-1}{t+1} + c$$

$$= \frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c = \frac{1}{\ln 2}\ln\left[\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}\,\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}-1}\right] + c = \frac{1}{\ln 2}\ln\frac{2^x+1-2\sqrt{2^x+1}+1}{2^x+1-1} + c$$

$$= \frac{\ln(2^x+2-2\sqrt{2^x+1}) - \ln 2^x}{\ln 2} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1}) - x\ln 2}{\ln 2} + c$$

# Riešené príklady – 159, 160

$$\int \frac{dx}{2^x+1} = \frac{1}{\ln 2} \ln \frac{2^x}{2^x+1} + c$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=2^x+1,\ 2^x=t-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} & dx=\frac{dt}{(t-1)\ln 2} \end{array} \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \frac{1}{\ln 2}\int \frac{dt}{t(t-1)} = \frac{1}{\ln 2}\int \frac{(t-t+1)\,dt}{t(t-1)}$$

$$= \frac{1}{\ln 2}\int \left[\frac{1}{t-1} - \frac{1}{t}\right] dt = \frac{1}{\ln 2}\Big[\ln(t-1) - \ln t\Big] + c = \frac{1}{\ln 2}\ln\frac{t-1}{t} + c = \frac{1}{\ln 2}\ln\frac{2^x}{2^x+1} + c$$

$$= \frac{\ln 2^x - \ln(2^x+1)}{\ln 2} + c = \frac{x\ln 2 - \ln(2^x+1)}{\ln 2} + c$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1})}{\ln 2} - x + c$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=\sqrt{2^x+1},\ 2^x=t^2-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} & dx=\frac{2t\,dt}{(t^2-1)\ln 2} \end{array} \right] = \int \frac{\frac{2t\,dt}{(t^2-1)\ln 2}}{t} = \frac{2}{\ln 2}\int \frac{dt}{t^2-1} = \frac{2}{2\ln 2}\ln\frac{t-1}{t+1} + c$$

$$= \frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c = \frac{1}{\ln 2}\ln\left[\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}\,\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}-1}\right] + c = \frac{1}{\ln 2}\ln\frac{2^x+1-2\sqrt{2^x+1}+1}{2^x+1-1} + c$$

$$= \frac{\ln(2^x+2-2\sqrt{2^x+1}) - \ln 2^x}{\ln 2} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1}) - x\ln 2}{\ln 2} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1})}{\ln 2} - x + c,$$

$$x\in R,\ c\in R.$$

# Riešené príklady – 159, 160

$$\int \frac{dx}{2^x+1} = \frac{1}{\ln 2} \ln \frac{2^x}{2^x+1} + c = x - \frac{\ln(2^x+1)}{\ln 2} + c$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=2^x+1,\ 2^x=t-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t-1)=\frac{\ln(t-1)}{\ln 2} & dx=\frac{dt}{(t-1)\ln 2} \end{array} \right] = \int \frac{\frac{dt}{(t-1)\ln 2}}{t} = \frac{1}{\ln 2}\int \frac{dt}{t(t-1)} = \frac{1}{\ln 2}\int \frac{(t-t+1)\,dt}{t(t-1)}$$

$$= \frac{1}{\ln 2}\int \left[\frac{1}{t-1} - \frac{1}{t}\right] dt = \frac{1}{\ln 2}\Big[\ln(t-1) - \ln t\Big] + c = \frac{1}{\ln 2}\ln\frac{t-1}{t} + c = \frac{1}{\ln 2}\ln\frac{2^x}{2^x+1} + c$$

$$= \frac{\ln 2^x - \ln(2^x+1)}{\ln 2} + c = \frac{x\ln 2 - \ln(2^x+1)}{\ln 2} + c = x - \frac{\ln(2^x+1)}{\ln 2} + c,\ x\in R,\ c\in R.$$

$$\int \frac{dx}{\sqrt{2^x+1}} = \frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1})}{\ln 2} - x + c$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=\sqrt{2^x+1},\ 2^x=t^2-1 & x\in R,\ t\in(1;\infty) \\ x=\log_2(t^2-1)=\frac{\ln(t^2-1)}{\ln 2} & dx=\frac{2t\,dt}{(t^2-1)\ln 2} \end{array} \right] = \int \frac{\frac{2t\,dt}{(t^2-1)\ln 2}}{t} = \frac{2}{\ln 2}\int \frac{dt}{t^2-1} = \frac{2}{2\ln 2}\ln\frac{t-1}{t+1} + c$$

$$= \frac{1}{\ln 2}\ln\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} + c = \frac{1}{\ln 2}\ln\left[\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1}\,\frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}-1}\right] + c = \frac{1}{\ln 2}\ln\frac{2^x+1-2\sqrt{2^x+1}+1}{2^x+1-1} + c$$

$$= \frac{\ln(2^x+2-2\sqrt{2^x+1}) - \ln 2^x}{\ln 2} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1}) - x\ln 2}{\ln 2} + c = \frac{\ln(2^x+2-2\sqrt{2^x+1})}{\ln 2} - x + c,$$

$$x\in R,\ c\in R.$$

# Riešené príklady – 161

$$\int \frac{(1+x)\,dx}{\sqrt{1-x^2}}$$

# Riešené príklady – 161

$$\int \frac{(1+x)\,\mathrm{d}x}{\sqrt{1-x^2}}$$

$$= \int \frac{\mathrm{d}x}{\sqrt{1-x^2}} + \int \frac{x\,\mathrm{d}x}{\sqrt{1-x^2}}$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t = \frac{1+x}{\sqrt{1-x^2}} = \frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x} - 1} & x \in (-1; 1) \\ t^2 = \frac{1+x}{1-x}, \ t^2(1-x) = 1+x \ \Rightarrow \ x = \frac{t^2-1}{t^2+1}, \ \mathrm{d}x = \frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,\mathrm{d}t = \frac{4t\,\mathrm{d}t}{(t^2+1)^2} & t \in (0; \infty) \end{array} \right]$$

# Riešené príklady – 161

$$\int \frac{(1+x)\,dx}{\sqrt{1-x^2}}$$

$$=\int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x\,dx}{\sqrt{1-x^2}} = \left[\begin{array}{r|l} \text{Subst. } t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle \end{array}\right]$$

---

$$=\left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{1+x}{\sqrt{1-x^2}}=\frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1} & x\in(-1;1) \\ t^2=\frac{1+x}{1-x},\ t^2(1-x)=1+x \ \Rightarrow\ x=\frac{t^2-1}{t^2+1},\ dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in(0;\infty) \end{array}\right]=\int \frac{4t^2\,dt}{(t^2+1)^2}$$

# Riešené príklady – 161

$$\int \frac{(1+x)\,dx}{\sqrt{1-x^2}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x\,dx}{\sqrt{1-x^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int t^{-\frac{1}{2}}\,dt$$

---

$$= \left[ \begin{array}{l|l} \text{Subst. } t=\frac{1+x}{\sqrt{1-x^2}} = \frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}-1} & x\in(-1;1) \\ t^2=\frac{1+x}{1-x},\ t^2(1-x)=1+x \Rightarrow x=\frac{t^2-1}{t^2+1},\ dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in(0;\infty) \end{array} \right] = \int \frac{4t^2\,dt}{(t^2+1)^2}$$

$$= \left[ \begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1} \end{array} \right]$$

# Riešené príklady – 161

$$\int \frac{(1+x)\,dx}{\sqrt{1-x^2}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x\,dx}{\sqrt{1-x^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int t^{-\frac{1}{2}}\,dt = \arcsin x - \frac{1}{2}\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

---

$$= \left[ \begin{array}{l|l} \text{Subst. } \quad t = \frac{1+x}{\sqrt{1-x^2}} = \frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}-1} & x\in(-1;1) \\ t^2=\frac{1+x}{1-x},\ t^2(1-x)=1+x \ \Rightarrow\ x=\frac{t^2-1}{t^2+1},\ dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in(0;\infty) \end{array} \right] = \int \frac{4t^2\,dt}{(t^2+1)^2}$$

$$= \left[ \begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2\,dt}{t^2+1}$$

# Riešené príklady – 161

$$\int \frac{(1+x)\,dx}{\sqrt{1-x^2}}$$

$$=\int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x\,dx}{\sqrt{1-x^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=1-x^2 & x\in(-1;0\rangle,\, t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\, t\in(0;1\rangle \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$=\int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int t^{-\frac{1}{2}}\,dt = \arcsin x - \frac{1}{2}\,\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = \arcsin x - \sqrt{t} + c$$

---

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\frac{1+x}{\sqrt{1-x^2}}=\frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1} & x\in(-1;1) \\ t^2=\frac{1+x}{1-x},\ t^2(1-x)=1+x \ \Rightarrow\ x=\frac{t^2-1}{t^2+1},\ dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in(0;\infty) \end{array} \right] = \int \frac{4t^2\,dt}{(t^2+1)^2}$$

$$= \left[ \begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2\,dt}{t^2+1} = -\frac{2t}{t^2+1} + 2\operatorname{arctg} t + c$$

# Riešené príklady – 161

$$\int \frac{(1+x)\,dx}{\sqrt{1-x^2}} = \arcsin x - \sqrt{1-x^2} + c$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x\,dx}{\sqrt{1-x^2}} = \left[\begin{array}{c|c} \text{Subst. } t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle \end{array}\right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int t^{-\frac{1}{2}}\,dt = \arcsin x - \frac{1}{2}\,\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = \arcsin x - \sqrt{t} + c$$

$$= \arcsin x - \sqrt{1-x^2} + c,\ x\in(-1;1),\ c\in R.$$

---

$$= \left[\begin{array}{l|l} \text{Subst. } t=\frac{1+x}{\sqrt{1-x^2}}=\frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1} & x\in(-1;1) \\ t^2=\frac{1+x}{1-x},\ t^2(1-x)=1+x \ \Rightarrow\ x=\frac{t^2-1}{t^2+1},\ dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in(0;\infty) \end{array}\right] = \int \frac{4t^2\,dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1} \end{array}\right] = -\frac{2t}{t^2+1} + \int \frac{2\,dt}{t^2+1} = -\frac{2t}{t^2+1} + 2\,\mathrm{arctg}\, t + c$$

$$= -\frac{2\,\frac{1+x}{\sqrt{1-x^2}}}{\frac{1+x}{1-x}+1} + 2\,\mathrm{arctg}\,\frac{1+x}{\sqrt{1-x^2}} + c$$

# Riešené príklady – 161

$$\int \frac{(1+x)\,dx}{\sqrt{1-x^2}} = \arcsin x - \sqrt{1-x^2} + c$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x\,dx}{\sqrt{1-x^2}} = \left[ \begin{array}{c|c} \text{Subst. } t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int t^{-\frac{1}{2}}\,dt = \arcsin x - \frac{1}{2}\,\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = \arcsin x - \sqrt{t} + c$$

$$= \arcsin x - \sqrt{1-x^2} + c,\ x\in(-1;1),\ c\in R.$$

---

$$= \left[ \begin{array}{c|c} \text{Subst. } \quad t=\frac{1+x}{\sqrt{1-x^2}} = \frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x}-1} & x\in(-1;1) \\ t^2=\frac{1+x}{1-x},\ t^2(1-x)=1+x \ \Rightarrow\ x=\frac{t^2-1}{t^2+1},\ dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in(0;\infty) \end{array} \right] = \int \frac{4t^2\,dt}{(t^2+1)^2}$$

$$= \left[ \begin{array}{c|c} u=2t & u'=2 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1} + \int \frac{2\,dt}{t^2+1} = -\frac{2t}{t^2+1} + 2\,\mathrm{arctg}\, t + c$$

$$= -\frac{2\frac{1+x}{\sqrt{1-x^2}}}{\frac{1+x}{1-x}+1} + 2\,\mathrm{arctg}\,\frac{1+x}{\sqrt{1-x^2}} + c = \left[ \frac{2\frac{1+x}{\sqrt{1-x^2}}}{\frac{1+x}{1-x}+1} = \frac{\frac{2(1+x)}{\sqrt{1-x^2}}}{\frac{1+x+1-x}{1-x}} = \frac{\frac{2(1+x)}{\sqrt{1-x^2}}}{\frac{2}{1-x}} = \frac{(1+x)(1-x)}{\sqrt{1-x^2}} = \frac{1-x^2}{\sqrt{1-x^2}} = \sqrt{1-x^2} \right]$$

# Riešené príklady – 161

$$\int\frac{(1+x)\,dx}{\sqrt{1-x^2}}=\arcsin x-\sqrt{1-x^2}+c=2\,\mathrm{arctg}\,\frac{1+x}{\sqrt{1-x^2}}-\sqrt{1-x^2}+c$$

$$=\int\frac{dx}{\sqrt{1-x^2}}+\int\frac{x\,dx}{\sqrt{1-x^2}}=\left[\begin{array}{c|c}\text{Subst. } t=1-x^2 & x\in(-1;0),\ t\in(0;1) \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle\end{array}\right]=\int\frac{dx}{\sqrt{1-x^2}}-\frac{1}{2}\int\frac{dt}{\sqrt{t}}$$

$$=\int\frac{dx}{\sqrt{1-x^2}}-\frac{1}{2}\int t^{-\frac{1}{2}}\,dt=\arcsin x-\frac{1}{2}\,\frac{t^{\frac{1}{2}}}{\frac{1}{2}}+c=\arcsin x-\sqrt{t}+c$$

$$=\arcsin x-\sqrt{1-x^2}+c,\ x\in(-1;1),\ c\in R.$$

---

$$=\left[\begin{array}{c|c}\text{Subst. } t=\frac{1+x}{\sqrt{1-x^2}}=\frac{\sqrt{(1+x)(1+x)}}{\sqrt{(1-x)(1+x)}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1} & x\in(-1;1) \\ t^2=\frac{1+x}{1-x},\ t^2(1-x)=1+x\ \Rightarrow\ x=\frac{t^2-1}{t^2+1},\ dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in(0;\infty)\end{array}\right]=\int\frac{4t^2\,dt}{(t^2+1)^2}$$

$$=\left[\begin{array}{c|c}u=2t & u'=2 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1}\end{array}\right]=-\frac{2t}{t^2+1}+\int\frac{2\,dt}{t^2+1}=-\frac{2t}{t^2+1}+2\,\mathrm{arctg}\,t+c$$

$$=-\frac{2\frac{1+x}{\sqrt{1-x^2}}}{\frac{1+x}{1-x}+1}+2\,\mathrm{arctg}\,\frac{1+x}{\sqrt{1-x^2}}+c=\left[\frac{2\frac{1+x}{\sqrt{1-x^2}}}{\frac{1+x}{1-x}+1}=\frac{\frac{2(1+x)}{\sqrt{1-x^2}}}{\frac{1+x+1-x}{1-x}}=\frac{\frac{2(1+x)}{\sqrt{1-x^2}}}{\frac{2}{1-x}}=\frac{(1+x)(1-x)}{\sqrt{1-x^2}}=\frac{1-x^2}{\sqrt{1-x^2}}=\sqrt{1-x^2}\right]$$

$$=2\,\mathrm{arctg}\,\frac{1+x}{\sqrt{1-x^2}}-\sqrt{1-x^2}+c,\ x\in(-1;1),\ c\in R.$$

# Riešené príklady – 162

$$\int \sqrt{\frac{1+x}{1-x}}\, dx$$

---

# Riešené príklady – 162

$$\int \sqrt{\frac{1+x}{1-x}}\,dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1}, \quad t^2=\frac{1+x}{1-x} & x\in\langle -1;1) \\ t^2(1-x)=1+x \;\Rightarrow\; x=\frac{t^2-1}{t^2+1},\; dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array} \right]$$

---

$$= \left[ \sqrt{\frac{1+x}{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}\cdot\frac{\sqrt{1+x}}{\sqrt{1+x}}=\frac{1+x}{\sqrt{1-x^2}},\; x\in(-1;1) \right]$$

# Riešené príklady – 162

$$\int\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x} & x\in\langle -1;1) \\ t^2(1-x)=1+x \;\Rightarrow\; x=\frac{t^2-1}{t^2+1},\quad dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right]=\int\frac{4t^2\,dt}{(t^2+1)^2}$$

---

$$=\left[\sqrt{\frac{1+x}{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}\cdot\frac{\sqrt{1+x}}{\sqrt{1+x}}=\frac{1+x}{\sqrt{1-x^2}},\ x\in(-1;1)\right]=\int\frac{(1+x)\,dx}{\sqrt{1-x^2}}$$

# Riešené príklady – 162

$$\int\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x} & x\in\langle -1;1)\\ t^2(1-x)=1+x\ \Rightarrow\ x=\frac{t^2-1}{t^2+1},\quad dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty)\end{array}\right]=\int\frac{4t^2\,dt}{(t^2+1)^2}$$

$$=\left[\begin{array}{l|l}u=2t & u'=2\\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1}\end{array}\right]$$

---

$$=\left[\sqrt{\frac{1+x}{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}\cdot\frac{\sqrt{1+x}}{\sqrt{1+x}}=\frac{1+x}{\sqrt{1-x^2}},\ x\in(-1;1)\right]=\int\frac{(1+x)\,dx}{\sqrt{1-x^2}}=\int\frac{dx}{\sqrt{1-x^2}}+\int\frac{x\,dx}{\sqrt{1-x^2}}$$

# Riešené príklady – 162

$$\int\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x} & x\in\langle-1;1) \\ t^2(1-x)=1+x\ \Rightarrow\ x=\frac{t^2-1}{t^2+1},\quad dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty)\end{array}\right]=\int\frac{4t^2\,dt}{(t^2+1)^2}$$

$$=\left[\begin{array}{l|l}u=2t & u'=2 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1}\end{array}\right]=-\frac{2t}{t^2+1}+\int\frac{2\,dt}{t^2+1}$$

---

$$=\left[\sqrt{\frac{1+x}{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}\cdot\frac{\sqrt{1+x}}{\sqrt{1+x}}=\frac{1+x}{\sqrt{1-x^2}},\ x\in(-1;1)\right]=\int\frac{(1+x)\,dx}{\sqrt{1-x^2}}=\int\frac{dx}{\sqrt{1-x^2}}+\int\frac{x\,dx}{\sqrt{1-x^2}}$$

$$=\left[\begin{array}{r|l}\text{Subst.}\ t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle\end{array}\right]$$

# Riešené príklady – 162

$\int \sqrt{\frac{1+x}{1-x}}\,dx$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x} & x\in\langle -1;1) \\ t^2(1-x)=1+x \;\Rightarrow\; x=\frac{t^2-1}{t^2+1},\quad dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right] = \int \frac{4t^2\,dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1} \end{array}\right] = -\frac{2t}{t^2+1}+\int\frac{2\,dt}{t^2+1} = -\frac{2t}{t^2+1}+2\,\mathrm{arctg}\,t+c_1$$

---

$$= \left[\sqrt{\frac{1+x}{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}\cdot\frac{\sqrt{1+x}}{\sqrt{1+x}}=\frac{1+x}{\sqrt{1-x^2}},\ x\in(-1;1)\right] = \int\frac{(1+x)\,dx}{\sqrt{1-x^2}} = \int\frac{dx}{\sqrt{1-x^2}}+\int\frac{x\,dx}{\sqrt{1-x^2}}$$

$$= \left[\begin{array}{l|l} \text{Subst.}\ t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle \end{array}\right] = \int\frac{dx}{\sqrt{1-x^2}}-\frac{1}{2}\int\frac{dt}{\sqrt{t}}$$

# Riešené príklady – 162

$$\int \sqrt{\frac{1+x}{1-x}}\,dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x} & x\in(-1;1) \\ t^2(1-x)=1+x \;\Rightarrow\; x=\frac{t^2-1}{t^2+1},\quad dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array} \right] = \int \frac{4t^2\,dt}{(t^2+1)^2}$$

$$= \left[ \begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1} \end{array} \right] = -\frac{2t}{t^2+1}+\int \frac{2\,dt}{t^2+1} = -\frac{2t}{t^2+1}+2\operatorname{arctg} t + c_1$$

$$= \left[ \frac{2t}{t^2+1}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x}+1}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x+1-x}{1-x}}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{2}{1-x}}=\sqrt{\frac{1+x}{1-x}\cdot(1-x)^2}=\sqrt{(1+x)(1-x)}=\sqrt{1-x^2} \right]$$

---

$$= \left[ \sqrt{\frac{1+x}{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}\cdot\frac{\sqrt{1+x}}{\sqrt{1+x}}=\frac{1+x}{\sqrt{1-x^2}},\ x\in(-1;1) \right] = \int \frac{(1+x)\,dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x\,dx}{\sqrt{1-x^2}}$$

$$= \left[ \begin{array}{l|l} \text{Subst. } t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle \end{array} \right] = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int t^{-\frac{1}{2}}\,dt$$

# Riešené príklady – 162

$$\int\sqrt{\frac{1+x}{1-x}}\,dx = 2\arctg\sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1$$

$$=\left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x} & x\in\langle-1;1) \\ t^2(1-x)=1+x \Rightarrow x=\frac{t^2-1}{t^2+1},\quad dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right]=\int\frac{4t^2\,dt}{(t^2+1)^2}$$

$$=\left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1} \end{array}\right]=-\frac{2t}{t^2+1}+\int\frac{2\,dt}{t^2+1}=-\frac{2t}{t^2+1}+2\arctg t + c_1$$

$$=\left[\frac{2t}{t^2+1}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x}+1}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x+1-x}{1-x}}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{2}{1-x}}=\sqrt{\frac{1+x}{1-x}\cdot(1-x)^2}=\sqrt{(1+x)(1-x)}=\sqrt{1-x^2}\right]$$

$$=2\arctg\sqrt{\frac{1+x}{1-x}}-\sqrt{1-x^2}+c_1,\ x\in\langle-1;1),\ c_1\in R.$$

---

$$=\left[\sqrt{\frac{1+x}{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}\cdot\frac{\sqrt{1+x}}{\sqrt{1+x}}=\frac{1+x}{\sqrt{1-x^2}},\ x\in(-1;1)\right]=\int\frac{(1+x)\,dx}{\sqrt{1-x^2}}=\int\frac{dx}{\sqrt{1-x^2}}+\int\frac{x\,dx}{\sqrt{1-x^2}}$$

$$=\left[\begin{array}{r|l} \text{Subst.}\ t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle \end{array}\right]=\int\frac{dx}{\sqrt{1-x^2}}-\frac{1}{2}\int\frac{dt}{\sqrt{t}}=\int\frac{dx}{\sqrt{1-x^2}}-\frac{1}{2}\int t^{-\frac{1}{2}}\,dt$$

$$=\arcsin x-\frac{1}{2}\,\frac{t^{\frac{1}{2}}}{\frac{1}{2}}+c_2$$

# Riešené príklady – 162

$$\int\sqrt{\frac{1+x}{1-x}}\,dx = 2\,\mathrm{arctg}\sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x} & x\in\langle -1;1) \\ t^2(1-x)=1+x\ \Rightarrow\ x=\frac{t^2-1}{t^2+1},\ \ dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right] = \int\frac{4t^2\,dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1} \end{array}\right] = -\frac{2t}{t^2+1}+\int\frac{2\,dt}{t^2+1} = -\frac{2t}{t^2+1}+2\,\mathrm{arctg}\,t+c_1$$

$$= \left[\frac{2t}{t^2+1}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x}+1}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x+1-x}{1-x}}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{2}{1-x}}=\sqrt{\frac{1+x}{1-x}\cdot(1-x)^2}=\sqrt{(1+x)(1-x)}=\sqrt{1-x^2}\right]$$

$$= 2\,\mathrm{arctg}\sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1,\ x\in\langle -1;1),\ c_1\in R.$$

---

$$= \left[\sqrt{\frac{1+x}{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}\cdot\frac{\sqrt{1+x}}{\sqrt{1+x}}=\frac{1+x}{\sqrt{1-x^2}},\ x\in(-1;1)\right] = \int\frac{(1+x)\,dx}{\sqrt{1-x^2}} = \int\frac{dx}{\sqrt{1-x^2}}+\int\frac{x\,dx}{\sqrt{1-x^2}}$$

$$= \left[\begin{array}{l|l} \text{Subst.}\ t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle \end{array}\right] = \int\frac{dx}{\sqrt{1-x^2}}-\frac{1}{2}\int\frac{dt}{\sqrt{t}} = \int\frac{dx}{\sqrt{1-x^2}}-\frac{1}{2}\int t^{-\frac{1}{2}}\,dt$$

$$= \arcsin x - \frac{1}{2}\,\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_2 = \arcsin x - \sqrt{t} + c_2$$

# Riešené príklady – 162

$$\int\sqrt{\frac{1+x}{1-x}}\,dx = 2\operatorname{arctg}\sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1 = \arcsin x - \sqrt{1-x^2} + c_2$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1}, \quad t^2=\frac{1+x}{1-x} & x\in\langle-1;1) \\ t^2(1-x)=1+x \;\Rightarrow\; x=\frac{t^2-1}{t^2+1}, \quad dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right] = \int\frac{4t^2\,dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}=-(t^2+1)^{-1} \end{array}\right] = -\frac{2t}{t^2+1}+\int\frac{2\,dt}{t^2+1} = -\frac{2t}{t^2+1}+2\operatorname{arctg} t + c_1$$

$$= \left[\frac{2t}{t^2+1}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x}+1}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x+1-x}{1-x}}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{2}{1-x}}=\sqrt{\frac{1+x}{1-x}\cdot(1-x)^2}=\sqrt{(1+x)(1-x)}=\sqrt{1-x^2}\right]$$

$$= 2\operatorname{arctg}\sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1, \; x\in\langle-1;1), \; c_1\in R.$$

---

$$= \left[\sqrt{\frac{1+x}{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}\cdot\frac{\sqrt{1+x}}{\sqrt{1+x}}=\frac{1+x}{\sqrt{1-x^2}}, \; x\in(-1;1)\right] = \int\frac{(1+x)\,dx}{\sqrt{1-x^2}} = \int\frac{dx}{\sqrt{1-x^2}} + \int\frac{x\,dx}{\sqrt{1-x^2}}$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=1-x^2 & x\in(-1;0\rangle, t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1), t\in(0;1\rangle \end{array}\right] = \int\frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int\frac{dt}{\sqrt{t}} = \int\frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int t^{-\frac{1}{2}}\,dt$$

$$= \arcsin x - \frac{1}{2}\,\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_2 = \arcsin x - \sqrt{t} + c_2 = \arcsin x - \sqrt{1-x^2} + c_2,$$

$$x\in(-1;1), \; c_2\in R.$$

# Riešené príklady – 162

$$\int \sqrt{\frac{1+x}{1-x}}\, dx$$

# Riešené príklady – 162

$$\int \sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-\,dx & t\in\langle -1;1) \end{array}\right]$$

---

$$=\left[\begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-\,dx & t\in\langle -1;1) \end{array}\right]$$

# Riešené príklady – 162

$$\int \sqrt{\frac{1+x}{1-x}}\,dx$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-\,dx & t\in\langle -1;1) \end{array} \right] = -\int \sqrt{\frac{1-t}{1+t}}\,dt$$

---

$$= \left[ \begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-\,dx & t\in\langle -1;1) \end{array} \right] = -\int \sqrt{\frac{1-t}{1+t}}\,dt$$

# Riešené príklady – 162

$$\int \sqrt{\frac{1+x}{1-x}}\,dx$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-dx & t\in\langle -1;1) \end{array} \right] = -\int \sqrt{\frac{1-t}{1+t}}\,dt$$

$$= \left[ \text{Pr. 163: } \int \sqrt{\tfrac{1-x}{1+x}}\,dx = \sqrt{1-x^2} - 2\operatorname{arctg}\sqrt{\tfrac{1-x}{1+x}} + c_1,\ x\in\langle -1;1) \right]$$

---

$$= \left[ \begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-dx & t\in\langle -1;1) \end{array} \right] = -\int \sqrt{\frac{1-t}{1+t}}\,dt$$

$$= \left[ \text{Pr. 163: } \int \sqrt{\tfrac{1-x}{1+x}}\,dx = \arcsin x + \sqrt{1-x^2} + c_2,\ x\in(-1;1) \right]$$

# Riešené príklady – 162

$$\int \sqrt{\frac{1+x}{1-x}}\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1) \\ dt=-dx & t\in\langle -1;1) \end{array}\right] = -\int \sqrt{\frac{1-t}{1+t}}\,dt$$

$$= \left[\text{Pr. 163: } \int \sqrt{\tfrac{1-x}{1+x}}\,dx = \sqrt{1-x^2} - 2\operatorname{arctg}\sqrt{\tfrac{1-x}{1+x}} + c_1,\ x\in\langle -1;1)\right]$$

$$= -\left[\sqrt{1-t^2} - 2\operatorname{arctg}\sqrt{\frac{1-t}{1+t}}\right] + c_1$$

---

$$= \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1) \\ dt=-dx & t\in\langle -1;1) \end{array}\right] = -\int \sqrt{\frac{1-t}{1+t}}\,dt$$

$$= \left[\text{Pr. 163: } \int \sqrt{\tfrac{1-x}{1+x}}\,dx = \arcsin x + \sqrt{1-x^2} + c_2,\ x\in(-1;1)\right] = -\left[\arcsin t + \sqrt{1-t^2}\right] + c_2$$

# Riešené príklady – 162

$\int \sqrt{\frac{1+x}{1-x}}\, dx$

$$=\left[\begin{array}{r|l}\text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-\,dx & t\in\langle -1;1)\end{array}\right]=-\int\sqrt{\frac{1-t}{1+t}}\, dt$$

$$=\left[\text{Pr. 163: } \int\sqrt{\tfrac{1-x}{1+x}}\, dx=\sqrt{1-x^2}-2\operatorname{arctg}\sqrt{\tfrac{1-x}{1+x}}+c_1,\ x\in\langle -1;1)\right]$$

$$=-\left[\sqrt{1-t^2}-2\operatorname{arctg}\sqrt{\frac{1-t}{1+t}}\right]+c_1=2\operatorname{arctg}\sqrt{\frac{1+x}{1-x}}-\sqrt{1-(-x)^2}+c_1$$

---

$$=\left[\begin{array}{r|l}\text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-\,dx & t\in\langle -1;1)\end{array}\right]=-\int\sqrt{\frac{1-t}{1+t}}\, dt$$

$$=\left[\text{Pr. 163: } \int\sqrt{\tfrac{1-x}{1+x}}\, dx=\arcsin x+\sqrt{1-x^2}+c_2,\ x\in(-1;1)\right]=-\left[\arcsin t+\sqrt{1-t^2}\right]+c_2$$

$$=-\arcsin(-x)-\sqrt{1-(-x)^2}+c_2$$

# Riešené príklady – 162

$$\int\sqrt{\frac{1+x}{1-x}}\,dx = 2\,\mathrm{arctg}\sqrt{\frac{1-x}{1+x}} - \sqrt{1-x^2} + c_1 = \arcsin x - \sqrt{1-x^2} + c_2$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-\,dx & t\in\langle-1;1) \end{array}\right] = -\int\sqrt{\frac{1-t}{1+t}}\,dt$$

$$= \left[\text{Pr. 163: } \int\sqrt{\tfrac{1-x}{1+x}}\,dx = \sqrt{1-x^2} - 2\,\mathrm{arctg}\sqrt{\tfrac{1-x}{1+x}} + c_1,\ x\in\langle-1;1)\right]$$

$$= -\left[\sqrt{1-t^2} - 2\,\mathrm{arctg}\sqrt{\frac{1-t}{1+t}}\right] + c_1 = 2\,\mathrm{arctg}\sqrt{\frac{1+x}{1-x}} - \sqrt{1-(-x)^2} + c_1$$

$$= 2\,\mathrm{arctg}\sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1,\ x\in(-1;1\rangle,\ c_1\in R.$$

---

$$= \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-\,dx & t\in\langle-1;1) \end{array}\right] = -\int\sqrt{\frac{1-t}{1+t}}\,dt$$

$$= \left[\text{Pr. 163: } \int\sqrt{\tfrac{1-x}{1+x}}\,dx = \arcsin x + \sqrt{1-x^2} + c_2,\ x\in(-1;1)\right] = -\left[\arcsin t + \sqrt{1-t^2}\right] + c_2$$

$$= -\arcsin(-x) - \sqrt{1-(-x)^2} + c_2 = \arcsin x - \sqrt{1-x^2} + c_2,$$

$$x\in(-1;1),\ c_2\in R.$$

# Riešené príklady – 163

$$\int \sqrt{\frac{1-x}{1+x}}\, dx$$

# Riešené príklady – 163

$$\int \sqrt{\frac{1-x}{1+x}}\,dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1}, \quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x \;\Rightarrow\; x=\frac{1-t^2}{t^2+1},\; dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array} \right]$$

---

$$= \left[ \sqrt{\frac{1-x}{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}\cdot\frac{\sqrt{1-x}}{\sqrt{1-x}}=\frac{1-x}{\sqrt{1-x^2}},\; x\in(-1;1) \right]$$

# Riešené príklady – 163

$$\int \sqrt{\frac{1-x}{1+x}}\,dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1}, \quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x \;\Rightarrow\; x=\frac{1-t^2}{t^2+1}, \quad dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array} \right] = \int \frac{-4t^2\,dt}{(t^2+1)^2}$$

---

$$= \left[ \sqrt{\frac{1-x}{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}\cdot\frac{\sqrt{1-x}}{\sqrt{1-x}}=\frac{1-x}{\sqrt{1-x^2}},\; x\in(-1;1) \right] = \int \frac{(1-x)\,dx}{\sqrt{1-x^2}}$$

# Riešené príklady – 163

$$\int \sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1},\quad t^2=\frac{1-x}{1+x} & x\in(-1;1) \\ t^2(1+x)=1-x \;\Rightarrow\; x=\frac{1-t^2}{t^2+1},\quad dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right]=\int\frac{-4t^2\,dt}{(t^2+1)^2}$$

$$=\left[\begin{array}{l|l} u=2t & u'=2 \\ v'=-\frac{2t}{(t^2+1)^2} & v=\frac{1}{t^2+1}=(t^2+1)^{-1} \end{array}\right]$$

---

$$=\left[\sqrt{\frac{1-x}{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}\cdot\frac{\sqrt{1-x}}{\sqrt{1-x}}=\frac{1-x}{\sqrt{1-x^2}},\ x\in(-1;1)\right]=\int\frac{(1-x)\,dx}{\sqrt{1-x^2}}=\int\frac{dx}{\sqrt{1-x^2}}-\int\frac{x\,dx}{\sqrt{1-x^2}}$$

# Riešené príklady – 163

$$\int \sqrt{\frac{1-x}{1+x}}\,dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1}, \quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x \;\Rightarrow\; x=\frac{1-t^2}{t^2+1}, \quad dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array} \right] = \int \frac{-4t^2\,dt}{(t^2+1)^2}$$

$$= \left[ \begin{array}{l|l} u=2t & u'=2 \\ v'=-\frac{2t}{(t^2+1)^2} & v=\frac{1}{t^2+1}=(t^2+1)^{-1} \end{array} \right] = \frac{2t}{t^2+1} - \int \frac{2\,dt}{t^2+1}$$

---

$$= \left[ \sqrt{\frac{1-x}{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}\cdot\frac{\sqrt{1-x}}{\sqrt{1-x}}=\frac{1-x}{\sqrt{1-x^2}},\; x\in(-1;1) \right] = \int \frac{(1-x)\,dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x\,dx}{\sqrt{1-x^2}}$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle \end{array} \right]$$

# Riešené príklady – 163

$$\int\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1},\quad t^2=\frac{1-x}{1+x} & x\in(-1;1) \\ t^2(1+x)=1-x\ \Rightarrow\ x=\frac{1-t^2}{t^2+1},\ dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty)\end{array}\right]=\int\frac{-4t^2\,dt}{(t^2+1)^2}$$

$$=\left[\begin{array}{l|l}u=2t & u'=2 \\ v'=-\frac{2t}{(t^2+1)^2} & v=\frac{1}{t^2+1}=(t^2+1)^{-1}\end{array}\right]=\frac{2t}{t^2+1}-\int\frac{2\,dt}{t^2+1}=\frac{2t}{t^2+1}-2\operatorname{arctg}t+c_1$$

---

$$=\left[\sqrt{\frac{1-x}{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}\cdot\frac{\sqrt{1-x}}{\sqrt{1-x}}=\frac{1-x}{\sqrt{1-x^2}},\ x\in(-1;1)\right]=\int\frac{(1-x)\,dx}{\sqrt{1-x^2}}=\int\frac{dx}{\sqrt{1-x^2}}-\int\frac{x\,dx}{\sqrt{1-x^2}}$$

$$=\left[\begin{array}{r|l}\text{Subst. } t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle\end{array}\right]=\int\frac{dx}{\sqrt{1-x^2}}+\frac{1}{2}\int\frac{dt}{\sqrt{t}}$$

# Riešené príklady – 163

$$\int\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1},\quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x\ \Rightarrow\ x=\frac{1-t^2}{t^2+1},\ \ dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty)\end{array}\right]=\int\frac{-4t^2\,dt}{(t^2+1)^2}$$

$$=\left[\begin{array}{l|l}u=2t & u'=2 \\ v'=-\frac{2t}{(t^2+1)^2} & v=\frac{1}{t^2+1}=(t^2+1)^{-1}\end{array}\right]=\frac{2t}{t^2+1}-\int\frac{2\,dt}{t^2+1}=\frac{2t}{t^2+1}-2\operatorname{arctg}t+c_1$$

$$=\left[\frac{2t}{t^2+1}=\frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x}{1+x}+1}=\frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x+1+x}{1+x}}=\frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{2}{1+x}}=\sqrt{\frac{1-x}{1+x}\cdot(1+x)^2}=\sqrt{(1-x)(1+x)}=\sqrt{1-x^2}\right]$$

---

$$=\left[\sqrt{\frac{1-x}{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}\cdot\frac{\sqrt{1-x}}{\sqrt{1-x}}=\frac{1-x}{\sqrt{1-x^2}},\ x\in(-1;1)\right]=\int\frac{(1-x)\,dx}{\sqrt{1-x^2}}=\int\frac{dx}{\sqrt{1-x^2}}-\int\frac{x\,dx}{\sqrt{1-x^2}}$$

$$=\left[\begin{array}{r|l}\text{Subst.}\ t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle\end{array}\right]=\int\frac{dx}{\sqrt{1-x^2}}+\frac{1}{2}\int\frac{dt}{\sqrt{t}}=\int\frac{dx}{\sqrt{1-x^2}}+\frac{1}{2}\int t^{-\frac{1}{2}}\,dt$$

# Riešené príklady – 163

$$\int\sqrt{\frac{1-x}{1+x}}\,dx = \sqrt{1-x^2} - 2\,\mathrm{arctg}\,\sqrt{\frac{1-x}{1+x}} + c_1$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1},\quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x \;\Rightarrow\; x=\frac{1-t^2}{t^2+1},\quad dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right] = \int\frac{-4t^2\,dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=-\frac{2t}{(t^2+1)^2} & v=\frac{1}{t^2+1}=(t^2+1)^{-1} \end{array}\right] = \frac{2t}{t^2+1} - \int\frac{2\,dt}{t^2+1} = \frac{2t}{t^2+1} - 2\,\mathrm{arctg}\,t + c_1$$

$$= \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x}{1+x}+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x+1+x}{1+x}} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{2}{1+x}} = \sqrt{\frac{1-x}{1+x}\cdot(1+x)^2} = \sqrt{(1-x)(1+x)} = \sqrt{1-x^2}\right]$$

$$= \sqrt{1-x^2} - 2\,\mathrm{arctg}\,\sqrt{\frac{1-x}{1+x}} + c_1,\; x\in(-1;1\rangle,\; c_1\in R.$$

---

$$= \left[\sqrt{\frac{1-x}{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}}\cdot\frac{\sqrt{1-x}}{\sqrt{1-x}} = \frac{1-x}{\sqrt{1-x^2}},\; x\in(-1;1)\right] = \int\frac{(1-x)\,dx}{\sqrt{1-x^2}} = \int\frac{dx}{\sqrt{1-x^2}} - \int\frac{x\,dx}{\sqrt{1-x^2}}$$

$$= \left[\begin{array}{r|l} \text{Subst.}\; t=1-x^2 & x\in(-1;0\rangle,\, t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\, t\in(0;1\rangle \end{array}\right] = \int\frac{dx}{\sqrt{1-x^2}} + \frac{1}{2}\int\frac{dt}{\sqrt{t}} = \int\frac{dx}{\sqrt{1-x^2}} + \frac{1}{2}\int t^{-\frac{1}{2}}\,dt$$

$$= \arcsin x + \frac{1}{2}\,\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_2$$

# Riešené príklady – 163

$$\int\sqrt{\frac{1-x}{1+x}}\,dx=\sqrt{1-x^2}-2\,\mathrm{arctg}\sqrt{\frac{1-x}{1+x}}+c_1$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1},\quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x\ \Rightarrow\ x=\frac{1-t^2}{t^2+1},\quad dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty)\end{array}\right]=\int\frac{-4t^2\,dt}{(t^2+1)^2}$$

$$=\left[\begin{array}{l|l}u=2t & u'=2\\ v'=-\frac{2t}{(t^2+1)^2} & v=\frac{1}{t^2+1}=(t^2+1)^{-1}\end{array}\right]=\frac{2t}{t^2+1}-\int\frac{2\,dt}{t^2+1}=\frac{2t}{t^2+1}-2\,\mathrm{arctg}\,t+c_1$$

$$=\left[\frac{2t}{t^2+1}=\frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x}{1+x}+1}=\frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x+1+x}{1+x}}=\frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{2}{1+x}}=\sqrt{\frac{1-x}{1+x}\cdot(1+x)^2}=\sqrt{(1-x)(1+x)}=\sqrt{1-x^2}\right]$$

$$=\sqrt{1-x^2}-2\,\mathrm{arctg}\sqrt{\frac{1-x}{1+x}}+c_1,\ x\in(-1;1\rangle,\ c_1\in R.$$

---

$$=\left[\sqrt{\frac{1-x}{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\frac{\sqrt{1-x}}{\sqrt{1+x}}\cdot\frac{\sqrt{1-x}}{\sqrt{1-x}}=\frac{1-x}{\sqrt{1-x^2}},\ x\in(-1;1)\right]=\int\frac{(1-x)\,dx}{\sqrt{1-x^2}}=\int\frac{dx}{\sqrt{1-x^2}}-\int\frac{x\,dx}{\sqrt{1-x^2}}$$

$$=\left[\begin{array}{r|l}\text{Subst.}\ t=1-x^2 & x\in(-1;0\rangle,\ t\in(0;1\rangle\\ dt=-2x\,dx & x\in\langle 0;1),\ t\in(0;1\rangle\end{array}\right]=\int\frac{dx}{\sqrt{1-x^2}}+\frac{1}{2}\int\frac{dt}{\sqrt{t}}=\int\frac{dx}{\sqrt{1-x^2}}+\frac{1}{2}\int t^{-\frac{1}{2}}\,dt$$

$$=\arcsin x+\frac{1}{2}\,\frac{t^{\frac{1}{2}}}{\frac{1}{2}}+c_2=\arcsin x+\sqrt{t}+c_2$$

# Riešené príklady – 163

$$\int\sqrt{\frac{1-x}{1+x}}\,dx = \sqrt{1-x^2} - 2\,\mathrm{arctg}\sqrt{\frac{1-x}{1+x}} + c_1 = \arcsin x + \sqrt{1-x^2} + c_2$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1},\quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x \;\Rightarrow\; x=\frac{1-t^2}{t^2+1},\quad dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right] = \int\frac{-4t^2\,dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=-\frac{2t}{(t^2+1)^2} & v=\frac{1}{t^2+1}=(t^2+1)^{-1} \end{array}\right] = \frac{2t}{t^2+1} - \int\frac{2\,dt}{t^2+1} = \frac{2t}{t^2+1} - 2\,\mathrm{arctg}\,t + c_1$$

$$= \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x}{1+x}+1} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x+1+x}{1+x}} = \frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{2}{1+x}} = \sqrt{\frac{1-x}{1+x}\cdot(1+x)^2} = \sqrt{(1-x)(1+x)} = \sqrt{1-x^2}\right]$$

$$= \sqrt{1-x^2} - 2\,\mathrm{arctg}\sqrt{\frac{1-x}{1+x}} + c_1,\quad x\in(-1;1\rangle,\quad c_1\in R.$$

$$= \left[\sqrt{\frac{1-x}{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}}\cdot\frac{\sqrt{1-x}}{\sqrt{1-x}} = \frac{1-x}{\sqrt{1-x^2}},\; x\in(-1;1)\right] = \int\frac{(1-x)\,dx}{\sqrt{1-x^2}} = \int\frac{dx}{\sqrt{1-x^2}} - \int\frac{x\,dx}{\sqrt{1-x^2}}$$

$$= \left[\begin{array}{l|l} \text{Subst.}\; t=1-x^2 & x\in(-1;0\rangle,\, t\in(0;1\rangle \\ dt=-2x\,dx & x\in\langle 0;1),\, t\in(0;1\rangle \end{array}\right] = \int\frac{dx}{\sqrt{1-x^2}} + \frac{1}{2}\int\frac{dt}{\sqrt{t}} = \int\frac{dx}{\sqrt{1-x^2}} + \frac{1}{2}\int t^{-\frac{1}{2}}\,dt$$

$$= \arcsin x + \frac{1}{2}\,\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_2 = \arcsin x + \sqrt{t} + c_2 = \arcsin x + \sqrt{1-x^2} + c_2,$$

$$x\in(-1;1),\quad c_2\in R.$$

# Riešené príklady – 163

$$\int \sqrt{\frac{1-x}{1+x}}\,dx$$

# Riešené príklady – 163

$$\int \sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x$$

$$= \begin{bmatrix} \text{Subst. } t=-x & x\in(-1;1\rangle \\ \mathrm{d}t=-\,\mathrm{d}x & t\in\langle -1;1) \end{bmatrix}$$

---

$$= \begin{bmatrix} \text{Subst. } t=-x & x\in(-1;1\rangle \\ \mathrm{d}t=-\,\mathrm{d}x & t\in\langle -1;1) \end{bmatrix}$$

# Riešené príklady – 163

$$\int \sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-\,dx & t\in\langle -1;1) \end{array}\right]=-\int \sqrt{\frac{1+t}{1-t}}\,dt$$

---

$$=\left[\begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-\,dx & t\in\langle -1;1) \end{array}\right]=-\int \sqrt{\frac{1+t}{1-t}}\,dt$$

# Riešené príklady – 163

$\int \sqrt{\frac{1-x}{1+x}}\, dx$

$= \left[ \begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-dx & t\in\langle -1;1) \end{array} \right] = -\int \sqrt{\frac{1+t}{1-t}}\, dt$

$= \left[ \text{Pr. 162: } \int \sqrt{\frac{1+x}{1-x}}\, dx = 2\,\text{arctg}\,\sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2} + c_1,\ x\in\langle -1;1) \right]$

---

$= \left[ \begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-dx & t\in\langle -1;1) \end{array} \right] = -\int \sqrt{\frac{1+t}{1-t}}\, dt$

$= \left[ \text{Pr. 162: } \int \sqrt{\frac{1+x}{1-x}}\, dx = \arcsin x - \sqrt{1-x^2} + c_2,\ x\in(-1;1) \right]$

# Riešené príklady – 163

$$\int \sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x$$

$$= \left[\begin{array}{c|c} \text{Subst. } t=-x & x\in(-1;1\rangle \\ \mathrm{d}t=-\mathrm{d}x & t\in\langle -1;1) \end{array}\right] = -\int \sqrt{\frac{1+t}{1-t}}\,\mathrm{d}t$$

$$= \left[\text{Pr. 162: } \int \sqrt{\tfrac{1+x}{1-x}}\,\mathrm{d}x = 2\,\mathrm{arctg}\sqrt{\tfrac{1+x}{1-x}} - \sqrt{1-x^2} + c_1,\ x\in\langle -1;1)\right]$$

$$= -\left[2\,\mathrm{arctg}\sqrt{\frac{1+t}{1-t}} - \sqrt{1-t^2}\right] + c_1$$

---

$$= \left[\begin{array}{c|c} \text{Subst. } t=-x & x\in(-1;1\rangle \\ \mathrm{d}t=-\mathrm{d}x & t\in\langle -1;1) \end{array}\right] = -\int \sqrt{\frac{1+t}{1-t}}\,\mathrm{d}t$$

$$= \left[\text{Pr. 162: } \int \sqrt{\tfrac{1+x}{1-x}}\,\mathrm{d}x = \arcsin x - \sqrt{1-x^2} + c_2,\ x\in(-1;1)\right] = -\left[\arcsin t - \sqrt{1-t^2}\right] + c_2$$

# Riešené príklady – 163

$$\int \sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{array}{r|l}\text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-dx & t\in\langle -1;1)\end{array}\right]=-\int\sqrt{\frac{1+t}{1-t}}\,dt$$

$$=\left[\text{Pr. 162: } \int\sqrt{\tfrac{1+x}{1-x}}\,dx = 2\,\text{arctg}\sqrt{\tfrac{1+x}{1-x}}-\sqrt{1-x^2}+c_1,\ x\in\langle -1;1)\right]$$

$$=-\left[2\,\text{arctg}\sqrt{\frac{1+t}{1-t}}-\sqrt{1-t^2}\right]+c_1=-2\,\text{arctg}\sqrt{\frac{1-x}{1+x}}+\sqrt{1-(-x)^2}+c_1$$

---

$$=\left[\begin{array}{r|l}\text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-dx & t\in\langle -1;1)\end{array}\right]=-\int\sqrt{\frac{1+t}{1-t}}\,dt$$

$$=\left[\text{Pr. 162: } \int\sqrt{\tfrac{1+x}{1-x}}\,dx = \arcsin x-\sqrt{1-x^2}+c_2,\ x\in(-1;1)\right]=-\left[\arcsin t-\sqrt{1-t^2}\right]+c_2$$

$$=-\arcsin(-x)+\sqrt{1-(-x)^2}+c_2$$

# Riešené príklady – 163

$$\int\sqrt{\frac{1-x}{1+x}}\,dx = -2\,\mathrm{arctg}\,\sqrt{\frac{1-x}{1+x}} + \sqrt{1-x^2} + c_1 = \arcsin x + \sqrt{1-x^2} + c_2$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-\,dx & t\in\langle -1;1) \end{array}\right] = -\int\sqrt{\frac{1+t}{1-t}}\,dt$$

$$= \left[\text{Pr. 162: } \int\sqrt{\tfrac{1+x}{1-x}}\,dx = 2\,\mathrm{arctg}\,\sqrt{\tfrac{1+x}{1-x}} - \sqrt{1-x^2} + c_1,\ x\in\langle -1;1)\right]$$

$$= -\left[2\,\mathrm{arctg}\,\sqrt{\frac{1+t}{1-t}} - \sqrt{1-t^2}\right] + c_1 = -2\,\mathrm{arctg}\,\sqrt{\frac{1-x}{1+x}} + \sqrt{1-(-x)^2} + c_1$$

$$= -2\,\mathrm{arctg}\,\sqrt{\frac{1-x}{1+x}} + \sqrt{1-x^2} + c_1,\ x\in(-1;1\rangle,\ c_1\in R.$$

---

$$= \left[\begin{array}{r|l} \text{Subst. } t=-x & x\in(-1;1\rangle \\ dt=-\,dx & t\in\langle -1;1) \end{array}\right] = -\int\sqrt{\frac{1+t}{1-t}}\,dt$$

$$= \left[\text{Pr. 162: } \int\sqrt{\tfrac{1+x}{1-x}}\,dx = \arcsin x - \sqrt{1-x^2} + c_2,\ x\in(-1;1)\right] = -\left[\arcsin t - \sqrt{1-t^2}\right] + c_2$$

$$= -\arcsin(-x) + \sqrt{1-(-x)^2} + c_2 = \arcsin x + \sqrt{1-x^2} + c_2,$$

$$x\in(-1;1),\ c_2\in R.$$

# Riešené príklady – 164

$$\int \sqrt{\frac{x+1}{x-1}}\, dx$$

# Riešené príklady – 164

$$\int \sqrt{\frac{x+1}{x-1}}\,dx$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} \quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}}, & t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle\,,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1},\ dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & & x\in(1;\infty)\,,\ t\in(1;\infty) \end{array} \right]$$

# Riešené príklady – 164

$$\int \sqrt{\frac{x+1}{x-1}}\,\mathrm{d}x$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1+\frac{2}{x-1}}, \quad t^2 = \frac{x+1}{x-1} & x \in (-\infty; -1\rangle,\ t \in \langle 0; 1) \\ t^2(x-1) = x+1,\ x = \frac{t^2+1}{t^2-1},\ \mathrm{d}x = \frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,\mathrm{d}t = \frac{-4t\,\mathrm{d}t}{(t^2-1)^2} & x \in (1; \infty),\ t \in (1; \infty) \end{array} \right] = \int \frac{-4t^2\,\mathrm{d}t}{(t^2-1)^2}$$

# Riešené príklady – 164

$$\int \sqrt{\frac{x+1}{x-1}}\,dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \qquad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}},\quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1},\ dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty) \end{array} \right] = \int \frac{-4t^2\,dt}{(t^2-1)^2}$$

$$= \left[ \begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{-2t}{(t^2-1)^2} & v=\frac{1}{t^2-1}=(t^2-1)^{-1} \end{array} \right]$$

# Riešené príklady – 164

$$\int\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{array}{ll|l}\text{Subst.} \quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}}, \quad t^2=\frac{x+1}{x-1} & & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1},\ dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & & x\in(1;\infty),\ t\in(1;\infty)\end{array}\right]=\int\frac{-4t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{-2t}{(t^2-1)^2} & v=\frac{1}{t^2-1}=(t^2-1)^{-1}\end{array}\right]=\frac{2t}{t^2-1}-\int\frac{2\,dt}{t^2-1}$$

# Riešené príklady – 164

$\int \sqrt{\frac{x+1}{x-1}}\,dx$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}}, \quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1},\ dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty) \end{array}\right] = \int \frac{-4t^2\,dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{-2t}{(t^2-1)^2} & v=\frac{1}{t^2-1}=(t^2-1)^{-1} \end{array}\right] = \frac{2t}{t^2-1} - \int \frac{2\,dt}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c$$

# Riešené príklady – 164

$$\int \sqrt{\frac{x+1}{x-1}}\,\mathrm{d}x$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}},\quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1, & x=\frac{t^2+1}{t^2-1},\quad \mathrm{d}x=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,\mathrm{d}t=\frac{-4t\,\mathrm{d}t}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty) \end{array} \right] = \int \frac{-4t^2\,\mathrm{d}t}{(t^2-1)^2}$$

$$= \left[ \begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{-2t}{(t^2-1)^2} & v=\frac{1}{t^2-1}=(t^2-1)^{-1} \end{array} \right] = \frac{2t}{t^2-1} - \int \frac{2\,\mathrm{d}t}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c$$

$$= \frac{2t}{t^2-1} + \ln|t+1| - \ln|t-1| + c$$

# Riešené príklady – 164

$$\int\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}},\quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1),\ t\in\langle 0;1)\\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1},\ dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty)\end{array}\right]=\int\frac{-4t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l}u=2t & u'=2\\ v'=\frac{-2t}{(t^2-1)^2} & v=\frac{1}{t^2-1}=(t^2-1)^{-1}\end{array}\right]=\frac{2t}{t^2-1}-\int\frac{2\,dt}{t^2-1}=\frac{2t}{t^2-1}-\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$=\frac{2t}{t^2-1}+\ln|t+1|-\ln|t-1|+c=\left[\frac{2t}{t^2-1}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}}=(x-1)\sqrt{\frac{x+1}{x-1}}\right]$$

# Riešené príklady – 164

$$\int\sqrt{\frac{x+1}{x-1}}\,dx=(x-1)\sqrt{\frac{x+1}{x-1}}+\ln\left[\sqrt{\frac{x+1}{x-1}}+1\right]-\ln\left|\sqrt{\frac{x+1}{x-1}}-1\right|+c$$

$$=\left[\begin{array}{ll|l}\text{Subst.} & t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}},\quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1, & x=\frac{t^2+1}{t^2-1},\quad dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty)\end{array}\right]=\int\frac{-4t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l}u=2t & u'=2 \\ v'=\frac{-2t}{(t^2-1)^2} & v=\frac{1}{t^2-1}=(t^2-1)^{-1}\end{array}\right]=\frac{2t}{t^2-1}-\int\frac{2\,dt}{t^2-1}=\frac{2t}{t^2-1}-\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$=\frac{2t}{t^2-1}+\ln|t+1|-\ln|t-1|+c=\left[\frac{2t}{t^2-1}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}}=(x-1)\sqrt{\frac{x+1}{x-1}}\right]$$

$$=(x-1)\sqrt{\frac{x+1}{x-1}}+\ln\left[\sqrt{\frac{x+1}{x-1}}+1\right]-\ln\left|\sqrt{\frac{x+1}{x-1}}-1\right|+c,$$

$$x\in(-\infty;-1\rangle\cup(1;\infty),\ c\in R.$$

# Riešené príklady – 164

$$\int\sqrt{\frac{x+1}{x-1}}\,dx = (x-1)\sqrt{\frac{x+1}{x-1}} + \ln\left[\sqrt{\frac{x+1}{x-1}}+1\right] - \ln\left|\sqrt{\frac{x+1}{x-1}}-1\right| + c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} \quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}}, & t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1}, & dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty) \end{array}\right] = \int\frac{-4t^2\,dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{-2t}{(t^2-1)^2} & v=\frac{1}{t^2-1}=(t^2-1)^{-1} \end{array}\right] = \frac{2t}{t^2-1} - \int\frac{2\,dt}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c$$

$$= \frac{2t}{t^2-1} + \ln|t+1| - \ln|t-1| + c = \left[\frac{2t}{t^2-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}} = \frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}} = (x-1)\sqrt{\frac{x+1}{x-1}}\right]$$

$$= (x-1)\sqrt{\frac{x+1}{x-1}} + \ln\left[\sqrt{\frac{x+1}{x-1}}+1\right] - \ln\left|\sqrt{\frac{x+1}{x-1}}-1\right| + c,$$

$$x\in(-\infty;-1\rangle\cup(1;\infty),\ c\in R.$$

$$= (x-1)\frac{\sqrt{x+1}}{\sqrt{x-1}} + \ln\frac{\frac{\sqrt{x+1}}{\sqrt{x-1}}+1}{\frac{\sqrt{x+1}}{\sqrt{x-1}}-1} + c$$

# Riešené príklady – 164

$$\int\sqrt{\frac{x+1}{x-1}}\,dx = (x-1)\sqrt{\frac{x+1}{x-1}} + \ln\left[\sqrt{\frac{x+1}{x-1}}+1\right] - \ln\left|\sqrt{\frac{x+1}{x-1}}-1\right| + c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}},\quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1, & x=\frac{t^2+1}{t^2-1},\quad dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty) \end{array}\right] = \int\frac{-4t^2\,dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{-2t}{(t^2-1)^2} & v=\frac{1}{t^2-1}=(t^2-1)^{-1} \end{array}\right] = \frac{2t}{t^2-1} - \int\frac{2\,dt}{t^2-1} = \frac{2t}{t^2-1} - \frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c$$

$$= \frac{2t}{t^2-1} + \ln|t+1| - \ln|t-1| + c = \left[\frac{2t}{t^2-1}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}}=(x-1)\sqrt{\frac{x+1}{x-1}}\right]$$

$$= (x-1)\sqrt{\frac{x+1}{x-1}} + \ln\left[\sqrt{\frac{x+1}{x-1}}+1\right] - \ln\left|\sqrt{\frac{x+1}{x-1}}-1\right| + c,$$

$$x\in(-\infty;-1\rangle\cup(1;\infty),\ c\in R.$$

---

$$= (x-1)\frac{\sqrt{x+1}}{\sqrt{x-1}} + \ln\frac{\frac{\sqrt{x+1}}{\sqrt{x-1}}+1}{\frac{\sqrt{x+1}}{\sqrt{x-1}}-1} + c = \sqrt{x^2-1} + \ln\left[\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\cdot\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\right] + c$$

# Riešené príklady – 164

$$\int\sqrt{\frac{x+1}{x-1}}\,dx=(x-1)\sqrt{\frac{x+1}{x-1}}+\ln\left[\sqrt{\frac{x+1}{x-1}}+1\right]-\ln\left|\sqrt{\frac{x+1}{x-1}}-1\right|+c$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}},\quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1)\\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1},\ dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty)\end{array}\right]=\int\frac{-4t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l}u=2t & u'=2\\ v'=\frac{-2t}{(t^2-1)^2} & v=\frac{1}{t^2-1}=(t^2-1)^{-1}\end{array}\right]=\frac{2t}{t^2-1}-\int\frac{2\,dt}{t^2-1}=\frac{2t}{t^2-1}-\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$=\frac{2t}{t^2-1}+\ln|t+1|-\ln|t-1|+c=\left[\frac{2t}{t^2-1}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}}=(x-1)\sqrt{\frac{x+1}{x-1}}\right]$$

$$=(x-1)\sqrt{\frac{x+1}{x-1}}+\ln\left[\sqrt{\frac{x+1}{x-1}}+1\right]-\ln\left|\sqrt{\frac{x+1}{x-1}}-1\right|+c,$$

$$x\in(-\infty;-1\rangle\cup(1;\infty),\ c\in R.$$

$$=(x-1)\frac{\sqrt{x+1}}{\sqrt{x-1}}+\ln\frac{\frac{\sqrt{x+1}}{\sqrt{x-1}}+1}{\frac{\sqrt{x+1}}{\sqrt{x-1}}-1}+c=\sqrt{x^2-1}+\ln\left[\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\cdot\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\right]+c$$

$$=\sqrt{x^2-1}+\ln\frac{x+1+2\sqrt{x^2-1}+x-1}{x+1-(x-1)}+c$$

# Riešené príklady – 164

$$\int\sqrt{\frac{x+1}{x-1}}\,dx=(x-1)\sqrt{\frac{x+1}{x-1}}+\ln\left[\sqrt{\frac{x+1}{x-1}}+1\right]-\ln\left|\sqrt{\frac{x+1}{x-1}}-1\right|+c$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}},\quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1)\\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1},\ dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty)\end{array}\right]=\int\frac{-4t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l}u=2t & u'=2\\ v'=\frac{-2t}{(t^2-1)^2} & v=\frac{1}{t^2-1}=(t^2-1)^{-1}\end{array}\right]=\frac{2t}{t^2-1}-\int\frac{2\,dt}{t^2-1}=\frac{2t}{t^2-1}-\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$=\frac{2t}{t^2-1}+\ln|t+1|-\ln|t-1|+c=\left[\frac{2t}{t^2-1}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}}=(x-1)\sqrt{\frac{x+1}{x-1}}\right]$$

$$=(x-1)\sqrt{\frac{x+1}{x-1}}+\ln\left[\sqrt{\frac{x+1}{x-1}}+1\right]-\ln\left|\sqrt{\frac{x+1}{x-1}}-1\right|+c,$$

$$x\in(-\infty;-1\rangle\cup(1;\infty),\ c\in R.$$

---

$$=(x-1)\frac{\sqrt{x+1}}{\sqrt{x-1}}+\ln\frac{\frac{\sqrt{x+1}}{\sqrt{x-1}}+1}{\frac{\sqrt{x+1}}{\sqrt{x-1}}-1}+c=\sqrt{x^2-1}+\ln\left[\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\cdot\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\right]+c$$

$$=\sqrt{x^2-1}+\ln\frac{x+1+2\sqrt{x^2-1}+x-1}{x+1-(x-1)}+c=\sqrt{x^2-1}+\ln\frac{2x+2\sqrt{x^2-1}}{2}+c$$

# Riešené príklady – 164

$$\int\sqrt{\frac{x+1}{x-1}}\,dx=(x-1)\sqrt{\frac{x+1}{x-1}}+\ln\left[\sqrt{\frac{x+1}{x-1}}+1\right]-\ln\left|\sqrt{\frac{x+1}{x-1}}-1\right|+c$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}},\quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1},\ dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty)\end{array}\right]=\int\frac{-4t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l}u=2t & u'=2 \\ v'=\frac{-2t}{(t^2-1)^2} & v=\frac{1}{t^2-1}=(t^2-1)^{-1}\end{array}\right]=\frac{2t}{t^2-1}-\int\frac{2\,dt}{t^2-1}=\frac{2t}{t^2-1}-\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$=\frac{2t}{t^2-1}+\ln|t+1|-\ln|t-1|+c=\left[\frac{2t}{t^2-1}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-x+1}{x-1}}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}}=(x-1)\sqrt{\frac{x+1}{x-1}}\right]$$

$$=(x-1)\sqrt{\frac{x+1}{x-1}}+\ln\left[\sqrt{\frac{x+1}{x-1}}+1\right]-\ln\left|\sqrt{\frac{x+1}{x-1}}-1\right|+c,$$

$$x\in(-\infty;-1\rangle\cup(1;\infty),\ c\in R.$$

---

$$=(x-1)\frac{\sqrt{x+1}}{\sqrt{x-1}}+\ln\frac{\frac{\sqrt{x+1}}{\sqrt{x-1}}+1}{\frac{\sqrt{x+1}}{\sqrt{x-1}}-1}+c=\sqrt{x^2-1}+\ln\left[\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\cdot\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\right]+c$$

$$=\sqrt{x^2-1}+\ln\frac{x+1+2\sqrt{x^2-1}+x-1}{x+1-(x-1)}+c=\sqrt{x^2-1}+\ln\frac{2x+2\sqrt{x^2-1}}{2}+c$$

$$=\sqrt{x^2-1}+\ln\left[x+\sqrt{x^2-1}\right]+c,\ x\in(1;\infty),\ c\in R.$$

# Riešené príklady – 165

$$\int \sqrt{\frac{x-1}{x+1}}\, dx$$

# Riešené príklady – 165

$$\int \sqrt{\frac{x-1}{x+1}}\,\mathrm{d}x$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ \mathrm{d}x=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,\mathrm{d}t=\frac{4t\,\mathrm{d}t}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array} \right]$$

# Riešené príklady – 165

$$\int \sqrt{\frac{x-1}{x+1}}\,dx$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}}, \quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right] = \int \frac{4t^2\,dt}{(t^2-1)^2}$$

# Riešené príklady – 165

$$\int \sqrt{\frac{x-1}{x+1}}\,dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \qquad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array} \right] = \int \frac{4t^2\,dt}{(t^2-1)^2}$$

$$= \left[ \begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}=-(t^2-1)^{-1} \end{array} \right]$$

# Riešené príklady – 165

$$\int\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{l|l} \text{Subst.} \qquad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right]=\int\frac{4t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}=-(t^2-1)^{-1} \end{array}\right]=-\frac{2t}{t^2-1}+\int\frac{2\,dt}{t^2-1}$$

# Riešené príklady – 165

$$\int \sqrt{\frac{x-1}{x+1}}\,dx$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} \quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}}, \quad t^2=\frac{x-1}{x+1} & & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array} \right] = \int \frac{4t^2\,dt}{(t^2-1)^2}$$

$$= \left[ \begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}=-(t^2-1)^{-1} \end{array} \right] = -\frac{2t}{t^2-1} + \int \frac{2\,dt}{t^2-1} = -\frac{2t}{t^2-1} + \frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c$$

# Riešené príklady – 165

$$\int \sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{l|l} \text{Subst.}\qquad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right]=\int\frac{4t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}=-(t^2-1)^{-1} \end{array}\right]=-\frac{2t}{t^2-1}+\int\frac{2\,dt}{t^2-1}=-\frac{2t}{t^2-1}+\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$=-\frac{2t}{t^2-1}+\ln|t-1|-\ln|t+1|+c$$

# Riešené príklady – 165

$$\int\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{l|l} \text{Subst.}\quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right]=\int\frac{4t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}=-(t^2-1)^{-1} \end{array}\right]=-\frac{2t}{t^2-1}+\int\frac{2\,dt}{t^2-1}=-\frac{2t}{t^2-1}+\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$=-\frac{2t}{t^2-1}+\ln|t-1|-\ln|t+1|+c=\left[-\frac{2t}{t^2-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x-1}{x+1}}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}}=(x+1)\sqrt{\frac{x-1}{x+1}}\right]$$

# Riešené príklady – 165

$$\int\sqrt{\frac{x-1}{x+1}}\,dx = (x+1)\sqrt{\frac{x-1}{x+1}} + \ln\left|\sqrt{\frac{x-1}{x+1}}-1\right| - \ln\left[\sqrt{\frac{x-1}{x+1}}+1\right] + c$$

$$=\left[\begin{array}{l|l} \text{Subst.} \quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right]=\int\frac{4t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}=-(t^2-1)^{-1} \end{array}\right]=-\frac{2t}{t^2-1}+\int\frac{2\,dt}{t^2-1}=-\frac{2t}{t^2-1}+\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$=-\frac{2t}{t^2-1}+\ln|t-1|-\ln|t+1|+c=\left[-\frac{2t}{t^2-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x-1}{x+1}}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}}=(x+1)\sqrt{\frac{x-1}{x+1}}\right]$$

$$=(x+1)\sqrt{\frac{x-1}{x+1}}+\ln\left|\sqrt{\frac{x-1}{x+1}}-1\right|-\ln\left[\sqrt{\frac{x-1}{x+1}}+1\right]+c,$$

$$x\in(-\infty;-1)\cup\langle 1;\infty),\ c\in R.$$

# Riešené príklady – 165

$$\int\sqrt{\frac{x-1}{x+1}}\,dx = (x+1)\sqrt{\frac{x-1}{x+1}} + \ln\left|\sqrt{\frac{x-1}{x+1}}-1\right| - \ln\left[\sqrt{\frac{x-1}{x+1}}+1\right] + c$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}}, \quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right] = \int\frac{4t^2\,dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}=-(t^2-1)^{-1} \end{array}\right] = -\frac{2t}{t^2-1}+\int\frac{2\,dt}{t^2-1} = -\frac{2t}{t^2-1}+\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$= -\frac{2t}{t^2-1}+\ln|t-1|-\ln|t+1|+c = \left[-\frac{2t}{t^2-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x-1}{x+1}}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}}=(x+1)\sqrt{\frac{x-1}{x+1}}\right]$$

$$= (x+1)\sqrt{\frac{x-1}{x+1}} + \ln\left|\sqrt{\frac{x-1}{x+1}}-1\right| - \ln\left[\sqrt{\frac{x-1}{x+1}}+1\right] + c,$$

$$x\in(-\infty;-1)\cup\langle 1;\infty),\ c\in R.$$

$$= (x+1)\frac{\sqrt{x-1}}{\sqrt{x+1}} + \ln\frac{1-\frac{\sqrt{x-1}}{\sqrt{x+1}}}{1+\frac{\sqrt{x-1}}{\sqrt{x+1}}} + c$$

# Riešené príklady – 165

$$\int\sqrt{\frac{x-1}{x+1}}\,dx = (x+1)\sqrt{\frac{x-1}{x+1}} + \ln\left|\sqrt{\frac{x-1}{x+1}}-1\right| - \ln\left[\sqrt{\frac{x-1}{x+1}}+1\right] + c$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1)\,,\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty)\,,\ t\in\langle 0;1)\end{array}\right]=\int\frac{4t^2\,dt}{(t^2-1)^2}$$

$$=\left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}=-(t^2-1)^{-1}\end{array}\right]=-\frac{2t}{t^2-1}+\int\frac{2\,dt}{t^2-1}=-\frac{2t}{t^2-1}+\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$=-\frac{2t}{t^2-1}+\ln|t-1|-\ln|t+1|+c=\left[-\frac{2t}{t^2-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x-1}{x+1}}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}}=(x+1)\sqrt{\frac{x-1}{x+1}}\right]$$

$$=(x+1)\sqrt{\frac{x-1}{x+1}}+\ln\left|\sqrt{\frac{x-1}{x+1}}-1\right|-\ln\left[\sqrt{\frac{x-1}{x+1}}+1\right]+c,$$

$$x\in(-\infty;-1)\cup\langle 1;\infty),\ c\in R.$$

---

$$=(x+1)\frac{\sqrt{x-1}}{\sqrt{x+1}}+\ln\frac{1-\frac{\sqrt{x-1}}{\sqrt{x+1}}}{1+\frac{\sqrt{x-1}}{\sqrt{x+1}}}+c=\sqrt{x^2-1}+\ln\left[\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\cdot\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\right]+c$$

# Riešené príklady – 165

$$\int\sqrt{\frac{x-1}{x+1}}\,dx = (x+1)\sqrt{\frac{x-1}{x+1}} + \ln\left|\sqrt{\frac{x-1}{x+1}}-1\right| - \ln\left[\sqrt{\frac{x-1}{x+1}}+1\right] + c$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}}, \quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right] = \int\frac{4t^2\,dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}=-(t^2-1)^{-1} \end{array}\right] = -\frac{2t}{t^2-1}+\int\frac{2\,dt}{t^2-1} = -\frac{2t}{t^2-1}+\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$= -\frac{2t}{t^2-1}+\ln|t-1|-\ln|t+1|+c = \left[-\frac{2t}{t^2-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x-1}{x+1}}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}}=(x+1)\sqrt{\frac{x-1}{x+1}}\right]$$

$$= (x+1)\sqrt{\frac{x-1}{x+1}} + \ln\left|\sqrt{\frac{x-1}{x+1}}-1\right| - \ln\left[\sqrt{\frac{x-1}{x+1}}+1\right] + c,$$

$$x\in(-\infty;-1)\cup\langle 1;\infty),\ c\in R.$$

---

$$= (x+1)\frac{\sqrt{x-1}}{\sqrt{x+1}} + \ln\frac{1-\frac{\sqrt{x-1}}{\sqrt{x+1}}}{1+\frac{\sqrt{x-1}}{\sqrt{x+1}}} + c = \sqrt{x^2-1} + \ln\left[\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\cdot\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\right] + c$$

$$= \sqrt{x^2-1} + \ln\frac{x+1-2\sqrt{x^2-1}+x-1}{x+1-(x-1)} + c$$

# Riešené príklady – 165

$$\int\sqrt{\frac{x-1}{x+1}}\,dx = (x+1)\sqrt{\frac{x-1}{x+1}} + \ln\left|\sqrt{\frac{x-1}{x+1}}-1\right| - \ln\left[\sqrt{\frac{x-1}{x+1}}+1\right] + c$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}}, \quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right] = \int\frac{4t^2\,dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}=-(t^2-1)^{-1} \end{array}\right] = -\frac{2t}{t^2-1}+\int\frac{2\,dt}{t^2-1} = -\frac{2t}{t^2-1}+\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$= -\frac{2t}{t^2-1}+\ln|t-1|-\ln|t+1|+c = \left[-\frac{2t}{t^2-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x-1}{x+1}}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}}=(x+1)\sqrt{\frac{x-1}{x+1}}\right]$$

$$= (x+1)\sqrt{\frac{x-1}{x+1}}+\ln\left|\sqrt{\frac{x-1}{x+1}}-1\right|-\ln\left[\sqrt{\frac{x-1}{x+1}}+1\right]+c,$$

$$x\in(-\infty;-1)\cup\langle 1;\infty),\ c\in R.$$

---

$$= (x+1)\frac{\sqrt{x-1}}{\sqrt{x+1}}+\ln\frac{1-\frac{\sqrt{x-1}}{\sqrt{x+1}}}{1+\frac{\sqrt{x-1}}{\sqrt{x+1}}}+c = \sqrt{x^2-1}+\ln\left[\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\cdot\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\right]+c$$

$$= \sqrt{x^2-1}+\ln\frac{x+1-2\sqrt{x^2-1}+x-1}{x+1-(x-1)}+c = \sqrt{x^2-1}+\ln\frac{2x-2\sqrt{x^2-1}}{2}+c$$

# Riešené príklady – 165

$$\int\sqrt{\frac{x-1}{x+1}}\,dx = (x+1)\sqrt{\frac{x-1}{x+1}} + \ln\left|\sqrt{\frac{x-1}{x+1}}-1\right| - \ln\left[\sqrt{\frac{x-1}{x+1}}+1\right] + c$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right] = \int\frac{4t^2\,dt}{(t^2-1)^2}$$

$$= \left[\begin{array}{l|l} u=2t & u'=2 \\ v'=\frac{2t}{(t^2-1)^2} & v=-\frac{1}{t^2-1}=-(t^2-1)^{-1} \end{array}\right] = -\frac{2t}{t^2-1}+\int\frac{2\,dt}{t^2-1} = -\frac{2t}{t^2-1}+\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$= -\frac{2t}{t^2-1}+\ln|t-1|-\ln|t+1|+c = \left[-\frac{2t}{t^2-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-x-1}{x+1}}=-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}}=(x+1)\sqrt{\frac{x-1}{x+1}}\right]$$

$$= (x+1)\sqrt{\frac{x-1}{x+1}} + \ln\left|\sqrt{\frac{x-1}{x+1}}-1\right| - \ln\left[\sqrt{\frac{x-1}{x+1}}+1\right] + c,$$

$$x\in(-\infty;-1)\cup\langle 1;\infty),\ c\in R.$$

$$= (x+1)\frac{\sqrt{x-1}}{\sqrt{x+1}} + \ln\frac{1-\frac{\sqrt{x-1}}{\sqrt{x+1}}}{1+\frac{\sqrt{x-1}}{\sqrt{x+1}}} + c = \sqrt{x^2-1} + \ln\left[\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\cdot\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\right]+c$$

$$= \sqrt{x^2-1} + \ln\frac{x+1-2\sqrt{x^2-1}+x-1}{x+1-(x-1)} + c = \sqrt{x^2-1} + \ln\frac{2x-2\sqrt{x^2-1}}{2} + c$$

$$= \sqrt{x^2-1} + \ln\left[x-\sqrt{x^2-1}\right] + c,\ x\in\langle 1;\infty),\ c\in R.$$

# Riešené príklady – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3}\,dx$$

# Riešené príklady – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3}\,dx$$

$$=\left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x} & x\in\langle -1;1) \\ t^2(1-x)=1+x \;\Rightarrow\; x=\frac{t^2-1}{t^2+1},\;\; dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right]$$

# Riešené príklady – 166

$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3}\,dx$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1}, \quad t^2=\frac{1+x}{1-x} & x\in\langle -1;1) \\ t^2(1-x)=1+x \;\Rightarrow\; x=\frac{t^2-1}{t^2+1},\;\; dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array} \right] = \int \frac{t^3\cdot 4t\,dt}{(t^2+1)^2}$$

# Riešené príklady – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3}\, dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x} - 1}, \quad t^2 = \frac{1+x}{1-x} & x \in \langle -1; 1) \\ t^2(1-x) = 1+x \;\Rightarrow\; x = \frac{t^2-1}{t^2+1}, \;\; dx = \frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\, dt = \frac{4t\, dt}{(t^2+1)^2} & t \in \langle 0; \infty) \end{array} \right] = \int \frac{t^3 \cdot 4t\, dt}{(t^2+1)^2}$$

$$= \left[ \frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right]$$

# Riešené príklady – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3}\,\mathrm{d}x$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1}, \quad t^2=\frac{1+x}{1-x} & x\in\langle -1;1) \\ t^2(1-x)=1+x \;\Rightarrow\; x=\frac{t^2-1}{t^2+1}, \quad \mathrm{d}x=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,\mathrm{d}t=\frac{4t\,\mathrm{d}t}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array} \right] = \int \frac{t^3\cdot 4t\,\mathrm{d}t}{(t^2+1)^2}$$

$$= \left[ \frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2} \right] = 4\int \left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]\mathrm{d}t$$

$$= 4\int \mathrm{d}t - 8\int \frac{\mathrm{d}t}{t^2+1} + 4\int \frac{\mathrm{d}t}{(t^2+1)^2}$$

# Riešené príklady – 166

$$\int\sqrt{\left(\frac{1+x}{1-x}\right)^3}\,dx$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x} & x\in\langle-1;1)\\ t^2(1-x)=1+x\ \Rightarrow\ x=\frac{t^2-1}{t^2+1},\quad dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty)\end{array}\right]=\int\frac{t^3\cdot 4t\,dt}{(t^2+1)^2}$$

$$=\left[\frac{t^4}{(t^2+1)^2}=\frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2}=\frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2}=1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]=4\int\left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]dt$$

$$=4\int dt-8\int\frac{dt}{t^2+1}+4\int\frac{dt}{(t^2+1)^2}=\left[\text{Pr. 121: } I_n=\int\frac{dt}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\right]$$

# Riešené príklady – 166

$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3}\,dx$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x} & x\in\langle -1;1)\\ t^2(1-x)=1+x\ \Rightarrow\ x=\frac{t^2-1}{t^2+1},\quad dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty)\end{array}\right]=\int\frac{t^3\cdot 4t\,dt}{(t^2+1)^2}$$

$$=\left[\frac{t^4}{(t^2+1)^2}=\frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2}=\frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2}=1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]=4\int\left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]dt$$

$$=4\int dt-8\int\frac{dt}{t^2+1}+4\int\frac{dt}{(t^2+1)^2}=\left[\text{Pr. 121: } I_n=\int\frac{dt}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\right]$$

$$=4\int dt-8\int\frac{dt}{t^2+1}+4\left[\frac{2\cdot 2-3}{2(2-1)}\int\frac{dt}{t^2+1}+\frac{t}{2(2-1)(t^2+1)}\right]$$

# Riešené príklady – 166

$$\int\sqrt{\left(\frac{1+x}{1-x}\right)^3}\,\mathrm{d}x$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x} & x\in\langle-1;1)\\ t^2(1-x)=1+x\;\Rightarrow\; x=\frac{t^2-1}{t^2+1},\quad \mathrm{d}x=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,\mathrm{d}t=\frac{4t\,\mathrm{d}t}{(t^2+1)^2} & t\in\langle 0;\infty)\end{array}\right]=\int\frac{t^3\cdot 4t\,\mathrm{d}t}{(t^2+1)^2}$$

$$=\left[\frac{t^4}{(t^2+1)^2}=\frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2}=\frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2}=1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]=4\int\left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]\mathrm{d}t$$

$$=4\int\mathrm{d}t-8\int\frac{\mathrm{d}t}{t^2+1}+4\int\frac{\mathrm{d}t}{(t^2+1)^2}=\left[\text{Pr. 121: } I_n=\int\frac{\mathrm{d}t}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\right]$$

$$=4\int\mathrm{d}t-8\int\frac{\mathrm{d}t}{t^2+1}+4\left[\frac{2\cdot2-3}{2(2-1)}\int\frac{\mathrm{d}t}{t^2+1}+\frac{t}{2(2-1)(t^2+1)}\right]$$

$$=4\int\mathrm{d}t-8\int\frac{\mathrm{d}t}{t^2+1}+2\int\frac{\mathrm{d}t}{t^2+1}+\frac{2t}{t^2+1}$$

# Riešené príklady – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3}\,dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1}, \quad t^2=\frac{1+x}{1-x} & x\in\langle -1;1) \\ t^2(1-x)=1+x \;\Rightarrow\; x=\frac{t^2-1}{t^2+1},\;\; dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array} \right] = \int \frac{t^3\cdot 4t\,dt}{(t^2+1)^2}$$

$$= \left[ \frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2} \right] = 4\int \left[ 1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2} \right] dt$$

$$= 4\int dt - 8\int \frac{dt}{t^2+1} + 4\int \frac{dt}{(t^2+1)^2} = \left[ \text{Pr. 121: } I_n=\int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right]$$

$$= 4\int dt - 8\int \frac{dt}{t^2+1} + 4\left[ \frac{2\cdot 2-3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)} \right]$$

$$= 4\int dt - 8\int \frac{dt}{t^2+1} + 2\int \frac{dt}{t^2+1} + \frac{2t}{t^2+1} = 4\int dt - 6\int \frac{dt}{t^2+1} + \frac{2t}{t^2+1}$$

# Riešené príklady – 166

$$\int \sqrt{\left(\frac{1+x}{1-x}\right)^3}\, dx$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{2-(1-x)}{1-x}} = \sqrt{\frac{2}{1-x} - 1}, \quad t^2 = \frac{1+x}{1-x} & x \in \langle -1; 1) \\ t^2(1-x) = 1+x \;\Rightarrow\; x = \frac{t^2-1}{t^2+1}, \quad dx = \frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\, dt = \frac{4t\, dt}{(t^2+1)^2} & t \in \langle 0; \infty) \end{array}\right] = \int \frac{t^3 \cdot 4t\, dt}{(t^2+1)^2}$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2}\right] = 4\int \left[1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2}\right] dt$$

$$= 4\int dt - 8\int \frac{dt}{t^2+1} + 4\int \frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\right]$$

$$= 4\int dt - 8\int \frac{dt}{t^2+1} + 4\left[\frac{2\cdot 2-3}{2(2-1)} \int \frac{dt}{t^2+1} + \frac{t}{2(2-1)(t^2+1)}\right]$$

$$= 4\int dt - 8\int \frac{dt}{t^2+1} + 2\int \frac{dt}{t^2+1} + \frac{2t}{t^2+1} = 4\int dt - 6\int \frac{dt}{t^2+1} + \frac{2t}{t^2+1}$$

$$= 4t - 6 \operatorname{arctg} t + \frac{2t}{t^2+1} + c$$

# Riešené príklady – 166

$$\int\sqrt{\left(\frac{1+x}{1-x}\right)^3}\,\mathrm{d}x$$

$$=\left[\begin{array}{l}\text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x}\\ t^2(1-x)=1+x\ \Rightarrow\ x=\frac{t^2-1}{t^2+1},\quad \mathrm{d}x=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,\mathrm{d}t=\frac{4t\,\mathrm{d}t}{(t^2+1)^2}\end{array}\left|\begin{array}{l}x\in\langle -1;1)\\ t\in\langle 0;\infty)\end{array}\right.\right]=\int\frac{t^3\cdot 4t\,\mathrm{d}t}{(t^2+1)^2}$$

$$=\left[\frac{t^4}{(t^2+1)^2}=\frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2}=\frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2}=1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]=4\int\left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]\mathrm{d}t$$

$$=4\int\mathrm{d}t-8\int\frac{\mathrm{d}t}{t^2+1}+4\int\frac{\mathrm{d}t}{(t^2+1)^2}=\left[\text{Pr. 121: } I_n=\int\frac{\mathrm{d}t}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\right]$$

$$=4\int\mathrm{d}t-8\int\frac{\mathrm{d}t}{t^2+1}+4\left[\frac{2\cdot 2-3}{2(2-1)}\int\frac{\mathrm{d}t}{t^2+1}+\frac{t}{2(2-1)(t^2+1)}\right]$$

$$=4\int\mathrm{d}t-8\int\frac{\mathrm{d}t}{t^2+1}+2\int\frac{\mathrm{d}t}{t^2+1}+\frac{2t}{t^2+1}=4\int\mathrm{d}t-6\int\frac{\mathrm{d}t}{t^2+1}+\frac{2t}{t^2+1}$$

$$=4t-6\,\mathrm{arctg}\,t+\frac{2t}{t^2+1}+c=\left[\frac{2t}{t^2+1}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x}+1}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x+1-x}{1-x}}=\frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{2}{1-x}}=\sqrt{\frac{1+x}{1-x}\cdot(1-x)^2}=\sqrt{1-x^2}\right]$$

# Riešené príklady – 166

$$\int\sqrt{\left(\frac{1+x}{1-x}\right)^3}\,dx = 4\sqrt{\frac{1+x}{1-x}} - 6\,\mathrm{arctg}\sqrt{\frac{1+x}{1-x}} + \sqrt{1-x^2} + c$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1+x}}{\sqrt{1-x}}=\sqrt{\frac{1+x}{1-x}}=\sqrt{\frac{2-(1-x)}{1-x}}=\sqrt{\frac{2}{1-x}-1},\quad t^2=\frac{1+x}{1-x} & x\in\langle -1;1) \\ t^2(1-x)=1+x \;\Rightarrow\; x=\frac{t^2-1}{t^2+1},\quad dx=\frac{2t(t^2+1)-(t^2-1)\cdot 2t}{(t^2+1)^2}\,dt=\frac{4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right] = \int\frac{t^3\cdot 4t\,dt}{(t^2+1)^2}$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right] = 4\int\left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]dt$$

$$= 4\int dt - 8\int\frac{dt}{t^2+1} + 4\int\frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n=\int\frac{dt}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\right]$$

$$= 4\int dt - 8\int\frac{dt}{t^2+1} + 4\left[\frac{2\cdot 2-3}{2(2-1)}\int\frac{dt}{t^2+1}+\frac{t}{2(2-1)(t^2+1)}\right]$$

$$= 4\int dt - 8\int\frac{dt}{t^2+1} + 2\int\frac{dt}{t^2+1} + \frac{2t}{t^2+1} = 4\int dt - 6\int\frac{dt}{t^2+1} + \frac{2t}{t^2+1}$$

$$= 4t - 6\,\mathrm{arctg}\,t + \frac{2t}{t^2+1} + c = \left[\frac{2t}{t^2+1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x}+1} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x+1-x}{1-x}} = \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{2}{1-x}} = \sqrt{\frac{1+x}{1-x}\cdot(1-x)^2} = \sqrt{1-x^2}\right]$$

$$= 4\sqrt{\frac{1+x}{1-x}} - 6\,\mathrm{arctg}\sqrt{\frac{1+x}{1-x}} + \sqrt{1-x^2} + c,\; x\in\langle -1;1),\; c\in R.$$

# Riešené príklady – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3}\,dx$$

# Riešené príklady – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3}\,\mathrm{d}x$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1}, \quad t^2=\frac{1-x}{1+x} & x\in(-1;1) \\ t^2(1+x)=1-x \ \Rightarrow\ x=\frac{1-t^2}{t^2+1}, \quad \mathrm{d}x=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,\mathrm{d}t=\frac{-4t\,\mathrm{d}t}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right]$$

# Riešené príklady – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3}\, dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1}, \quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x \;\Rightarrow\; x=\frac{1-t^2}{t^2+1}, \quad dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array} \right] = \int \frac{t^3\cdot(-4t)\,dt}{(t^2+1)^4}$$

# Riešené príklady – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3}\,dx$$

$$=\left[\begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1}, \quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x \;\Rightarrow\; x=\frac{1-t^2}{t^2+1}, \quad dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right]=\int \frac{t^3\cdot(-4t)\,dt}{(t^2+1)^4}$$

$$=\left[\frac{t^4}{(t^2+1)^2}=\frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2}=\frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2}=1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]$$

# Riešené príklady – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3}\,\mathrm{d}x$$

$$=\left[\begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1}, \quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x \;\Rightarrow\; x=\frac{1-t^2}{t^2+1}, \quad \mathrm{d}x=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,\mathrm{d}t=\frac{-4t\,\mathrm{d}t}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right]=\int\frac{t^3\cdot(-4t)\,\mathrm{d}t}{(t^2+1)^4}$$

$$=\left[\frac{t^4}{(t^2+1)^2}=\frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2}=\frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2}=1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]=-4\int\left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]\mathrm{d}t$$

$$=-4\int\mathrm{d}t+8\int\frac{\mathrm{d}t}{t^2+1}-4\int\frac{\mathrm{d}t}{(t^2+1)^2}$$

# Riešené príklady – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3}\, dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2-(1+x)}{1+x}} = \sqrt{\frac{2}{1+x} - 1}, \quad t^2 = \frac{1-x}{1+x} & x \in (-1;1\rangle \\ t^2(1+x) = 1-x \;\Rightarrow\; x = \frac{1-t^2}{t^2+1}, \quad dx = \frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\, dt = \frac{-4t\, dt}{(t^2+1)^2} & t \in \langle 0;\infty) \end{array} \right] = \int \frac{t^3 \cdot (-4t)\, dt}{(t^2+1)^4}$$

$$= \left[ \frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] = -4\int \left[ 1 - \frac{2}{t^2+1} + \frac{1}{(t^2+1)^2} \right] dt$$

$$= -4\int dt + 8\int \frac{dt}{t^2+1} - 4\int \frac{dt}{(t^2+1)^2} = \left[ \text{Pr. 121: } I_n = \int \frac{dt}{(t^2+a^2)^n} = \frac{2n-3}{2a^2(n-1)} I_{n-1} + \frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right]$$

# Riešené príklady – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3}\,dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1}, \quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x \;\Rightarrow\; x=\frac{1-t^2}{t^2+1}, \quad dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array} \right] = \int \frac{t^3\cdot(-4t)\,dt}{(t^2+1)^4}$$

$$= \left[ \frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2} \right] = -4\int\left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]dt$$

$$= -4\int dt + 8\int \frac{dt}{t^2+1} - 4\int\frac{dt}{(t^2+1)^2} = \left[ \text{Pr. 121: } I_n=\int\frac{dt}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}} \right]$$

$$= -4\int dt + 8\int \frac{dt}{t^2+1} - 4\left[\frac{2\cdot2-3}{2(2-1)}\int\frac{dt}{t^2+1}+\frac{t}{2(2-1)(t^2+1)}\right]$$

# Riešené príklady – 167

$$\int\sqrt{\left(\frac{1-x}{1+x}\right)^3}\,\mathrm{d}x$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1},\quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x \ \Rightarrow\ x=\frac{1-t^2}{t^2+1},\quad \mathrm{d}x=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,\mathrm{d}t=\frac{-4t\,\mathrm{d}t}{(t^2+1)^2} & t\in\langle 0;\infty)\end{array}\right]=\int\frac{t^3\cdot(-4t)\,\mathrm{d}t}{(t^2+1)^4}$$

$$=\left[\frac{t^4}{(t^2+1)^2}=\frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2}=\frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2}=1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]=-4\int\left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]\mathrm{d}t$$

$$=-4\int\mathrm{d}t+8\int\frac{\mathrm{d}t}{t^2+1}-4\int\frac{\mathrm{d}t}{(t^2+1)^2}=\left[\text{Pr. 121: } I_n=\int\frac{\mathrm{d}t}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\right]$$

$$=-4\int\mathrm{d}t+8\int\frac{\mathrm{d}t}{t^2+1}-4\left[\frac{2\cdot 2-3}{2(2-1)}\int\frac{\mathrm{d}t}{t^2+1}+\frac{t}{2(2-1)(t^2+1)}\right]$$

$$=-4\int\mathrm{d}t+8\int\frac{\mathrm{d}t}{t^2+1}-2\int\frac{\mathrm{d}t}{t^2+1}-\frac{2t}{t^2+1}$$

# Riešené príklady – 167

$$\int \sqrt{\left(\frac{1-x}{1+x}\right)^3}\,dx$$

$$=\left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1},\quad t^2=\frac{1-x}{1+x} & x\in(-1;1) \\ t^2(1+x)=1-x \;\Rightarrow\; x=\frac{1-t^2}{t^2+1},\quad dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right]=\int\frac{t^3\cdot(-4t)\,dt}{(t^2+1)^4}$$

$$=\left[\frac{t^4}{(t^2+1)^2}=\frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2}=\frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2}=1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]=-4\int\left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]dt$$

$$=-4\int dt+8\int\frac{dt}{t^2+1}-4\int\frac{dt}{(t^2+1)^2}=\left[\text{Pr. 121: } I_n=\int\frac{dt}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\right]$$

$$=-4\int dt+8\int\frac{dt}{t^2+1}-4\left[\frac{2\cdot 2-3}{2(2-1)}\int\frac{dt}{t^2+1}+\frac{t}{2(2-1)(t^2+1)}\right]$$

$$=-4\int dt+8\int\frac{dt}{t^2+1}-2\int\frac{dt}{t^2+1}-\frac{2t}{t^2+1}=-4\int dt+6\int\frac{dt}{t^2+1}-\frac{2t}{t^2+1}$$

# Riešené príklady – 167

$$\int\sqrt{\left(\frac{1-x}{1+x}\right)^3}\,dx$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1},\quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x\ \Rightarrow\ x=\frac{1-t^2}{t^2+1},\quad dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty)\end{array}\right]=\int\frac{t^3\cdot(-4t)\,dt}{(t^2+1)^4}$$

$$=\left[\frac{t^4}{(t^2+1)^2}=\frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2}=\frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2}=1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]=-4\int\left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]dt$$

$$=-4\int dt+8\int\frac{dt}{t^2+1}-4\int\frac{dt}{(t^2+1)^2}=\left[\text{Pr. 121: } I_n=\int\frac{dt}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\right]$$

$$=-4\int dt+8\int\frac{dt}{t^2+1}-4\left[\frac{2\cdot 2-3}{2(2-1)}\int\frac{dt}{t^2+1}+\frac{t}{2(2-1)(t^2+1)}\right]$$

$$=-4\int dt+8\int\frac{dt}{t^2+1}-2\int\frac{dt}{t^2+1}-\frac{2t}{t^2+1}=-4\int dt+6\int\frac{dt}{t^2+1}-\frac{2t}{t^2+1}$$

$$=-4t+6\,\mathrm{arctg}\,t-\frac{2t}{t^2+1}+c$$

# Riešené príklady – 167

$$\int\sqrt{\left(\frac{1-x}{1+x}\right)^3}\,\mathrm{d}x$$

$$=\left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1},\quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x \;\Rightarrow\; x=\frac{1-t^2}{t^2+1},\quad \mathrm{d}x=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,\mathrm{d}t=\frac{-4t\,\mathrm{d}t}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right]=\int\frac{t^3\cdot(-4t)\,\mathrm{d}t}{(t^2+1)^4}$$

$$=\left[\frac{t^4}{(t^2+1)^2}=\frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2}=\frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2}=1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]=-4\int\left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]\mathrm{d}t$$

$$=-4\int\mathrm{d}t+8\int\frac{\mathrm{d}t}{t^2+1}-4\int\frac{\mathrm{d}t}{(t^2+1)^2}=\left[\text{Pr. 121: } I_n=\int\frac{\mathrm{d}t}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\right]$$

$$=-4\int\mathrm{d}t+8\int\frac{\mathrm{d}t}{t^2+1}-4\left[\frac{2\cdot 2-3}{2(2-1)}\int\frac{\mathrm{d}t}{t^2+1}+\frac{t}{2(2-1)(t^2+1)}\right]$$

$$=-4\int\mathrm{d}t+8\int\frac{\mathrm{d}t}{t^2+1}-2\int\frac{\mathrm{d}t}{t^2+1}-\frac{2t}{t^2+1}=-4\int\mathrm{d}t+6\int\frac{\mathrm{d}t}{t^2+1}-\frac{2t}{t^2+1}$$

$$=-4t+6\operatorname{arctg}t-\frac{2t}{t^2+1}+c=\left[\frac{2t}{t^2+1}=\frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x}{1+x}+1}=\frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x+1+x}{1+x}}=\frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{2}{1+x}}=\sqrt{\frac{1-x}{1+x}\cdot(1+x)^2}=\sqrt{1-x^2}\right]$$

# Riešené príklady – 167

$$\int\sqrt{\left(\frac{1-x}{1+x}\right)^3}\,dx = -4\sqrt{\frac{1-x}{1+x}} + 6\,\mathrm{arctg}\sqrt{\frac{1-x}{1+x}} - \sqrt{1-x^2} + c$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=\frac{\sqrt{1-x}}{\sqrt{1+x}}=\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{2-(1+x)}{1+x}}=\sqrt{\frac{2}{1+x}-1},\quad t^2=\frac{1-x}{1+x} & x\in(-1;1\rangle \\ t^2(1+x)=1-x \Rightarrow x=\frac{1-t^2}{t^2+1},\quad dx=\frac{-2t(t^2+1)-(1-t^2)\cdot 2t}{(t^2+1)^2}\,dt=\frac{-4t\,dt}{(t^2+1)^2} & t\in\langle 0;\infty) \end{array}\right] = \int\frac{t^3\cdot(-4t)\,dt}{(t^2+1)^4}$$

$$= \left[\frac{t^4}{(t^2+1)^2} = \frac{t^4+2t^2+1-2t^2-2+1}{(t^2+1)^2} = \frac{(t^2+1)^2-2(t^2+1)+1}{(t^2+1)^2} = 1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right] = -4\int\left[1-\frac{2}{t^2+1}+\frac{1}{(t^2+1)^2}\right]dt$$

$$= -4\int dt + 8\int\frac{dt}{t^2+1} - 4\int\frac{dt}{(t^2+1)^2} = \left[\text{Pr. 121: } I_n=\int\frac{dt}{(t^2+a^2)^n}=\frac{2n-3}{2a^2(n-1)}I_{n-1}+\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}\right]$$

$$= -4\int dt + 8\int\frac{dt}{t^2+1} - 4\left[\frac{2\cdot2-3}{2(2-1)}\int\frac{dt}{t^2+1}+\frac{t}{2(2-1)(t^2+1)}\right]$$

$$= -4\int dt + 8\int\frac{dt}{t^2+1} - 2\int\frac{dt}{t^2+1} - \frac{2t}{t^2+1} = -4\int dt + 6\int\frac{dt}{t^2+1} - \frac{2t}{t^2+1}$$

$$= -4t + 6\,\mathrm{arctg}\,t - \frac{2t}{t^2+1} + c = \left[\frac{2t}{t^2+1}=\frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x}{1+x}+1}=\frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{1-x+1+x}{1+x}}=\frac{2\sqrt{\frac{1-x}{1+x}}}{\frac{2}{1+x}}=\sqrt{\frac{1-x}{1+x}\cdot(1+x)^2}=\sqrt{1-x^2}\right]$$

$$= -4\sqrt{\frac{1-x}{1+x}} + 6\,\mathrm{arctg}\sqrt{\frac{1-x}{1+x}} - \sqrt{1-x^2} + c,\ x\in(-1;1\rangle,\ c\in R.$$

# Riešené príklady – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3}\, \mathrm{d}x$$

# Riešené príklady – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3}\,dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}}, \quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1},\ dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty) \end{array} \right]$$

# Riešené príklady – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3}\,dx$$

$$= \left[\begin{array}{ll|l} \text{Subst.} \quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}}, & t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1},\ dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & & x\in(1;\infty),\ t\in(1;\infty) \end{array}\right] = \int \frac{t^3\cdot(-4t)\,dt}{(t^2-1)^2}$$

# Riešené príklady – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3}\, dx$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}},\quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1}, & dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty) \end{array}\right] = \int \frac{t^3\cdot(-4t)\,dt}{(t^2-1)^2}$$

$$= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2} = \frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2} = 1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]$$

# Riešené príklady – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3}\,dx$$

$$= \left[\begin{array}{ll|l} \text{Subst.} \quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}}, & t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1}, & dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty) \end{array}\right] = \int \frac{t^3\cdot(-4t)\,dt}{(t^2-1)^2}$$

$$= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2} = \frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2} = 1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right] = -4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$= -4\int dt - 8\int\frac{dt}{t^2-1} - 4\int\frac{dt}{(t^2-1)^2}$$

# Riešené príklady – 168

$$\int\sqrt{\left(\frac{x+1}{x-1}\right)^3}\,\mathrm{d}x$$

$$=\left[\begin{array}{ll|l} \text{Subst.} \quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}}, & t^2=\frac{x+1}{x-1} & x\in(-\infty;-1)\,,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1}, & \mathrm{d}x=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,\mathrm{d}t=\frac{-4t\,\mathrm{d}t}{(t^2-1)^2} & x\in(1;\infty)\,,\ t\in(1;\infty) \end{array}\right]=\int\frac{t^3\cdot(-4t)\,\mathrm{d}t}{(t^2-1)^2}$$

$$=\left[\frac{t^4}{(t^2-1)^2}=\frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2}=\frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2}=1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]=-4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]\mathrm{d}t$$

$$=-4\int\mathrm{d}t-8\int\frac{\mathrm{d}t}{t^2-1}-4\int\frac{\mathrm{d}t}{(t^2-1)^2}=\left[\text{Pr. 125: } I_n=\int\frac{\mathrm{d}t}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

# Riešené príklady – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3}\,dx$$

$$=\left[\begin{array}{ll|l} \text{Subst.} \quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}}, & t^2=\frac{x+1}{x-1} & x\in(-\infty;-1),\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1}, & dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty) \end{array}\right]=\int\frac{t^3\cdot(-4t)\,dt}{(t^2-1)^2}$$

$$=\left[\frac{t^4}{(t^2-1)^2}=\frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2}=\frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2}=1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]=-4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}-4\int\frac{dt}{(t^2-1)^2}=\left[\text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}-4\left[\frac{-(2\cdot 2-3)}{2(2-1)}\int\frac{dt}{t^2-1}-\frac{t}{2(2-1)(t^2-1)}\right]$$

# Riešené príklady – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3}\,dx$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} \quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}}, & t^2=\frac{x+1}{x-1} & x\in(-\infty;-1),\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1},\ dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & & x\in(1;\infty),\ t\in(1;\infty) \end{array} \right] = \int \frac{t^3\cdot(-4t)\,dt}{(t^2-1)^2}$$

$$= \left[ \frac{t^4}{(t^2-1)^2} = \frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2} = \frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2} = 1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2} \right] = -4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$= -4\int dt - 8\int\frac{dt}{t^2-1} - 4\int\frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

$$= -4\int dt - 8\int\frac{dt}{t^2-1} - 4\left[\frac{-(2\cdot 2-3)}{2(2-1)}\int\frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)}\right]$$

$$= -4\int dt - 8\int\frac{dt}{t^2-1} + 2\int\frac{dt}{t^2-1} + \frac{2t}{t^2-1}$$

# Riešené príklady – 168

$$\int \sqrt{\left(\frac{x+1}{x-1}\right)^3}\, dx$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} \quad t = \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{(x-1)+2}{x-1}} = \sqrt{1+\frac{2}{x-1}}, & t^2 = \frac{x+1}{x-1} & x\in(-\infty;-1),\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1}, & dx = \frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt = \frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty) \end{array} \right] = \int \frac{t^3\cdot(-4t)\,dt}{(t^2-1)^2}$$

$$= \left[ \frac{t^4}{(t^2-1)^2} = \frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2} = \frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2} = 1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2} \right] = -4\int \left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right] dt$$

$$= -4\int dt - 8\int \frac{dt}{t^2-1} - 4\int \frac{dt}{(t^2-1)^2} = \left[ \text{Pr. 125: } I_n = \int \frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)} I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right]$$

$$= -4\int dt - 8\int \frac{dt}{t^2-1} - 4\left[ \frac{-(2\cdot 2-3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right]$$

$$= -4\int dt - 8\int \frac{dt}{t^2-1} + 2\int \frac{dt}{t^2-1} + \frac{2t}{t^2-1} = -4\int dt - 6\int \frac{dt}{t^2-1} + \frac{2t}{t^2-1}$$

# Riešené príklady – 168

$$\int\sqrt{\left(\frac{x+1}{x-1}\right)^3}\,dx$$

$$=\left[\begin{array}{l|l} \text{Subst.}\quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}},\quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1),\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1},\ dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty)\end{array}\right]=\int\frac{t^3\cdot(-4t)\,dt}{(t^2-1)^2}$$

$$=\left[\frac{t^4}{(t^2-1)^2}=\frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2}=\frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2}=1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]=-4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}-4\int\frac{dt}{(t^2-1)^2}=\left[\text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}-4\left[\frac{-(2\cdot 2-3)}{2(2-1)}\int\frac{dt}{t^2-1}-\frac{t}{2(2-1)(t^2-1)}\right]$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}+2\int\frac{dt}{t^2-1}+\frac{2t}{t^2-1}=-4\int dt-6\int\frac{dt}{t^2-1}+\frac{2t}{t^2-1}$$

$$=-4t-6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|+\frac{2t}{t^2-1}+c$$

# Riešené príklady – 168

$$\int\sqrt{\left(\frac{x+1}{x-1}\right)^3}\,dx$$

$$=\left[\begin{array}{ll|l}\text{Subst.} & t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}},\quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1),\ t\in\langle 0;1)\\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1}, & dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty)\end{array}\right]=\int\frac{t^3\cdot(-4t)\,dt}{(t^2-1)^2}$$

$$=\left[\frac{t^4}{(t^2-1)^2}=\frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2}=\frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2}=1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]=-4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}-4\int\frac{dt}{(t^2-1)^2}=\left[\text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}-4\left[\frac{-(2\cdot 2-3)}{2(2-1)}\int\frac{dt}{t^2-1}-\frac{t}{2(2-1)(t^2-1)}\right]$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}+2\int\frac{dt}{t^2-1}+\frac{2t}{t^2-1}=-4\int dt-6\int\frac{dt}{t^2-1}+\frac{2t}{t^2-1}$$

$$=-4t-6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|+\frac{2t}{t^2-1}+c=\frac{2t}{t^2-1}-4t-3\ln|t-1|+3\ln|t+1|+c$$

# Riešené príklady – 168

$$\int\sqrt{\left(\frac{x+1}{x-1}\right)^3}\,dx$$

$$=\left[\begin{array}{ll|l}\text{Subst.} & t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}},\quad t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1}, & dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty)\end{array}\right]=\int\frac{t^3\cdot(-4t)\,dt}{(t^2-1)^2}$$

$$=\left[\frac{t^4}{(t^2-1)^2}=\frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2}=\frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2}=1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]=-4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}-4\int\frac{dt}{(t^2-1)^2}=\left[\text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}-4\left[\frac{-(2\cdot 2-3)}{2(2-1)}\int\frac{dt}{t^2-1}-\frac{t}{2(2-1)(t^2-1)}\right]$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}+2\int\frac{dt}{t^2-1}+\frac{2t}{t^2-1}=-4\int dt-6\int\frac{dt}{t^2-1}+\frac{2t}{t^2-1}$$

$$=-4t-6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|+\frac{2t}{t^2-1}+c=\frac{2t}{t^2-1}-4t-3\ln|t-1|+3\ln|t+1|+c$$

$$=\left[\frac{2t}{t^2-1}-4t=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}-4\sqrt{\frac{x+1}{x-1}}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-(x-1)}{x-1}}-4\sqrt{\frac{x+1}{x-1}}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}}-4\sqrt{\frac{x+1}{x-1}}=(x-1)\sqrt{\frac{x+1}{x-1}}-4\sqrt{\frac{x+1}{x-1}}=(x-5)\sqrt{\frac{x+1}{x-1}}\right]$$

# Riešené príklady – 168

$$\int\sqrt{\left(\frac{x+1}{x-1}\right)^3}\,dx = (x-5)\sqrt{\frac{x+1}{x-1}} - 3\ln\left|\sqrt{\frac{x+1}{x-1}}-1\right| + 3\ln\left[\sqrt{\frac{x+1}{x-1}}+1\right] + c$$

$$=\left[\begin{array}{ll|l} \text{Subst.} \quad t=\sqrt{\frac{x+1}{x-1}}=\sqrt{\frac{(x-1)+2}{x-1}}=\sqrt{1+\frac{2}{x-1}}, & t^2=\frac{x+1}{x-1} & x\in(-\infty;-1\rangle,\ t\in\langle 0;1) \\ t^2(x-1)=x+1,\ x=\frac{t^2+1}{t^2-1}, & dx=\frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}\,dt=\frac{-4t\,dt}{(t^2-1)^2} & x\in(1;\infty),\ t\in(1;\infty)\end{array}\right]=\int\frac{t^3\cdot(-4t)\,dt}{(t^2-1)^2}$$

$$=\left[\frac{t^4}{(t^2-1)^2}=\frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2}=\frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2}=1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]=-4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}-4\int\frac{dt}{(t^2-1)^2}=\left[\text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}-4\left[\frac{-(2\cdot 2-3)}{2(2-1)}\int\frac{dt}{t^2-1}-\frac{t}{2(2-1)(t^2-1)}\right]$$

$$=-4\int dt-8\int\frac{dt}{t^2-1}+2\int\frac{dt}{t^2-1}+\frac{2t}{t^2-1}=-4\int dt-6\int\frac{dt}{t^2-1}+\frac{2t}{t^2-1}$$

$$=-4t-6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|+\frac{2t}{t^2-1}+c=\frac{2t}{t^2-1}-4t-3\ln|t-1|+3\ln|t+1|+c$$

$$=\left[\frac{2t}{t^2-1}-4t=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}-4\sqrt{\frac{x+1}{x-1}}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{x+1-(x-1)}{x-1}}-4\sqrt{\frac{x+1}{x-1}}=\frac{2\sqrt{\frac{x+1}{x-1}}}{\frac{2}{x-1}}-4\sqrt{\frac{x+1}{x-1}}=(x-1)\sqrt{\frac{x+1}{x-1}}-4\sqrt{\frac{x+1}{x-1}}=(x-5)\sqrt{\frac{x+1}{x-1}}\right]$$

$$=(x-5)\sqrt{\frac{x+1}{x-1}}-3\ln\left|\sqrt{\frac{x+1}{x-1}}-1\right|+3\ln\left[\sqrt{\frac{x+1}{x-1}}+1\right]+c,$$

$$x\in(-\infty;-1\rangle\cup(1;\infty),\ c\in R.$$

# Riešené príklady – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3}\,\mathrm{d}x$$

# Riešené príklady – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3}\,dx$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2}, & dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array} \right]$$

# Riešené príklady – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3}\,dx$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2}, & dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array} \right] = \int \frac{t^3\cdot 4t\,dt}{(t^2-1)^2}$$

# Riešené príklady – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3}\, dx$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}}, \quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1, & x=\frac{t^2+1}{1-t^2}, \quad dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array} \right] = \int \frac{t^3\cdot 4t\,dt}{(t^2-1)^2}$$

$$= \left[ \frac{t^4}{(t^2-1)^2} = \frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2} = \frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2} = 1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2} \right]$$

# Riešené príklady – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3}\, dx$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2}, & dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array} \right] = \int \frac{t^3\cdot 4t\,dt}{(t^2-1)^2}$$

$$= \left[ \frac{t^4}{(t^2-1)^2} = \frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2} = \frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2} = 1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2} \right] = 4\int \left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right] dt$$

$$= 4\int dt + 8\int \frac{dt}{t^2-1} + 4\int \frac{dt}{(t^2-1)^2}$$

# Riešené príklady – 169

$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3}\,dx$

$$=\left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2}, & dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right]=\int\frac{t^3\cdot 4t\,dt}{(t^2-1)^2}$$

$$=\left[\frac{t^4}{(t^2-1)^2}=\frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2}=\frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2}=1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]=4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$=4\int dt+8\int\frac{dt}{t^2-1}+4\int\frac{dt}{(t^2-1)^2}=\left[\text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

# Riešené príklady – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3}\, dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}}, \quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array} \right] = \int \frac{t^3\cdot 4t\,dt}{(t^2-1)^2}$$

$$= \left[ \frac{t^4}{(t^2-1)^2} = \frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2} = \frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2} = 1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2} \right] = 4\int \left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right] dt$$

$$= 4\int dt + 8\int \frac{dt}{t^2-1} + 4\int \frac{dt}{(t^2-1)^2} = \left[ \text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right]$$

$$= 4\int dt + 8\int \frac{dt}{t^2-1} + 4\left[ \frac{-(2\cdot 2-3)}{2(2-1)} \int \frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)} \right]$$

# Riešené príklady – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3}\, dx$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}}, \quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array} \right] = \int \frac{t^3\cdot 4t\,dt}{(t^2-1)^2}$$

$$= \left[ \frac{t^4}{(t^2-1)^2} = \frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2} = \frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2} = 1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2} \right] = 4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$= 4\int dt + 8\int\frac{dt}{t^2-1} + 4\int\frac{dt}{(t^2-1)^2} = \left[ \text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n} = \frac{-(2n-3)}{2a^2(n-1)}I_{n-1} - \frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}} \right]$$

$$= 4\int dt + 8\int\frac{dt}{t^2-1} + 4\left[\frac{-(2\cdot 2-3)}{2(2-1)}\int\frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)}\right]$$

$$= 4\int dt + 8\int\frac{dt}{t^2-1} - 2\int\frac{dt}{t^2-1} - \frac{2t}{t^2-1}$$

# Riešené príklady – 169

$\int\sqrt{\left(\frac{x-1}{x+1}\right)^3}\,\mathrm{d}x$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ \mathrm{d}x=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,\mathrm{d}t=\frac{4t\,\mathrm{d}t}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1)\end{array}\right]=\int\frac{t^3\cdot 4t\,\mathrm{d}t}{(t^2-1)^2}$$

$$=\left[\frac{t^4}{(t^2-1)^2}=\frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2}=\frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2}=1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]=4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]\mathrm{d}t$$

$$=4\int\mathrm{d}t+8\int\frac{\mathrm{d}t}{t^2-1}+4\int\frac{\mathrm{d}t}{(t^2-1)^2}=\left[\text{Pr. 125: } I_n=\int\frac{\mathrm{d}t}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

$$=4\int\mathrm{d}t+8\int\frac{\mathrm{d}t}{t^2-1}+4\left[\frac{-(2\cdot 2-3)}{2(2-1)}\int\frac{\mathrm{d}t}{t^2-1}-\frac{t}{2(2-1)(t^2-1)}\right]$$

$$=4\int\mathrm{d}t+8\int\frac{\mathrm{d}t}{t^2-1}-2\int\frac{\mathrm{d}t}{t^2-1}-\frac{2t}{t^2-1}=4\int\mathrm{d}t+6\int\frac{\mathrm{d}t}{t^2-1}-\frac{2t}{t^2-1}$$

# Riešené príklady – 169

$$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3}\,dx$$

$$=\left[\begin{array}{l|l} \text{Subst.} \quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right]=\int\frac{t^3\cdot 4t\,dt}{(t^2-1)^2}$$

$$=\left[\frac{t^4}{(t^2-1)^2}=\frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2}=\frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2}=1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]=4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$=4\int dt+8\int\frac{dt}{t^2-1}+4\int\frac{dt}{(t^2-1)^2}=\left[\text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

$$=4\int dt+8\int\frac{dt}{t^2-1}+4\left[\frac{-(2\cdot 2-3)}{2(2-1)}\int\frac{dt}{t^2-1}-\frac{t}{2(2-1)(t^2-1)}\right]$$

$$=4\int dt+8\int\frac{dt}{t^2-1}-2\int\frac{dt}{t^2-1}-\frac{2t}{t^2-1}=4\int dt+6\int\frac{dt}{t^2-1}-\frac{2t}{t^2-1}$$

$$=4t+6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|-\frac{2t}{t^2-1}+c$$

# Riešené príklady – 169

$\int \sqrt{\left(\frac{x-1}{x+1}\right)^3}\, dx$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}}, \quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right] = \int \frac{t^3\cdot 4t\,dt}{(t^2-1)^2}$$

$$= \left[\frac{t^4}{(t^2-1)^2} = \frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2} = \frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2} = 1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right] = 4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$= 4\int dt + 8\int\frac{dt}{t^2-1} + 4\int\frac{dt}{(t^2-1)^2} = \left[\text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

$$= 4\int dt + 8\int\frac{dt}{t^2-1} + 4\left[\frac{-(2\cdot 2-3)}{2(2-1)}\int\frac{dt}{t^2-1} - \frac{t}{2(2-1)(t^2-1)}\right]$$

$$= 4\int dt + 8\int\frac{dt}{t^2-1} - 2\int\frac{dt}{t^2-1} - \frac{2t}{t^2-1} = 4\int dt + 6\int\frac{dt}{t^2-1} - \frac{2t}{t^2-1}$$

$$= 4t + 6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - \frac{2t}{t^2-1} + c = 4t + 3\ln|t-1| - 3\ln|t+1| - \frac{2t}{t^2-1} + c$$

# Riešené príklady – 169

$$\int\sqrt{\left(\frac{x-1}{x+1}\right)^3}\,dx$$

$$=\left[\begin{array}{l|l}\text{Subst.}\quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}},\quad t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & x\in\langle 1;\infty),\ t\in\langle 0;1)\end{array}\right]=\int\frac{t^3\cdot 4t\,dt}{(t^2-1)^2}$$

$$=\left[\frac{t^4}{(t^2-1)^2}=\frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2}=\frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2}=1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]=4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$=4\int dt+8\int\frac{dt}{t^2-1}+4\int\frac{dt}{(t^2-1)^2}=\left[\text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

$$=4\int dt+8\int\frac{dt}{t^2-1}+4\left[\frac{-(2\cdot 2-3)}{2(2-1)}\int\frac{dt}{t^2-1}-\frac{t}{2(2-1)(t^2-1)}\right]$$

$$=4\int dt+8\int\frac{dt}{t^2-1}-2\int\frac{dt}{t^2-1}-\frac{2t}{t^2-1}=4\int dt+6\int\frac{dt}{t^2-1}-\frac{2t}{t^2-1}$$

$$=4t+6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|-\frac{2t}{t^2-1}+c=4t+3\ln|t-1|-3\ln|t+1|-\frac{2t}{t^2-1}+c$$

$$=\left[4t-\frac{2t}{t^2-1}=4\sqrt{\frac{x-1}{x+1}}-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}=4\sqrt{\frac{x-1}{x+1}}-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-(x+1)}{x+1}}=4\sqrt{\frac{x-1}{x+1}}-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}}=4\sqrt{\frac{x-1}{x+1}}+(x+1)\sqrt{\frac{x-1}{x+1}}=(x+5)\sqrt{\frac{x-1}{x+1}}\right]$$

# Riešené príklady – 169

$$\int\sqrt{\left(\frac{x-1}{x+1}\right)^3}\,dx = (x+5)\sqrt{\frac{x-1}{x+1}} + 3\ln\left|\sqrt{\frac{x-1}{x+1}}-1\right| - 3\ln\left[\sqrt{\frac{x-1}{x+1}}+1\right] + c$$

$$=\left[\begin{array}{ll|l} \text{Subst.} \quad t=\sqrt{\frac{x-1}{x+1}}=\sqrt{\frac{(x+1)-2}{x+1}}=\sqrt{1-\frac{2}{x+1}}, & t^2=\frac{x-1}{x+1} & x\in(-\infty;-1),\ t\in(1;\infty) \\ t^2(x+1)=x-1,\ x=\frac{t^2+1}{1-t^2},\ dx=\frac{2t(1-t^2)-(t^2+1)\cdot(-2t)}{(1-t^2)^2}\,dt=\frac{4t\,dt}{(t^2-1)^2} & & x\in\langle 1;\infty),\ t\in\langle 0;1) \end{array}\right]=\int\frac{t^3\cdot 4t\,dt}{(t^2-1)^2}$$

$$=\left[\frac{t^4}{(t^2-1)^2}=\frac{t^4-2t^2+1+2t^2-2+1}{(t^2-1)^2}=\frac{(t^2-1)^2+2(t^2-1)+1}{(t^2-1)^2}=1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]=4\int\left[1+\frac{2}{t^2-1}+\frac{1}{(t^2-1)^2}\right]dt$$

$$=4\int dt+8\int\frac{dt}{t^2-1}+4\int\frac{dt}{(t^2-1)^2}=\left[\text{Pr. 125: } I_n=\int\frac{dt}{(t^2-a^2)^n}=\frac{-(2n-3)}{2a^2(n-1)}I_{n-1}-\frac{t}{2a^2(n-1)(t^2-a^2)^{n-1}}\right]$$

$$=4\int dt+8\int\frac{dt}{t^2-1}+4\left[\frac{-(2\cdot 2-3)}{2(2-1)}\int\frac{dt}{t^2-1}-\frac{t}{2(2-1)(t^2-1)}\right]$$

$$=4\int dt+8\int\frac{dt}{t^2-1}-2\int\frac{dt}{t^2-1}-\frac{2t}{t^2-1}=4\int dt+6\int\frac{dt}{t^2-1}-\frac{2t}{t^2-1}$$

$$=4t+6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|-\frac{2t}{t^2-1}+c=4t+3\ln|t-1|-3\ln|t+1|-\frac{2t}{t^2-1}+c$$

$$=\left[4t-\frac{2t}{t^2-1}=4\sqrt{\frac{x-1}{x+1}}-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}=4\sqrt{\frac{x-1}{x+1}}-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1-(x+1)}{x+1}}=4\sqrt{\frac{x-1}{x+1}}-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{-2}{x+1}}=4\sqrt{\frac{x-1}{x+1}}+(x+1)\sqrt{\frac{x-1}{x+1}}=(x+5)\sqrt{\frac{x-1}{x+1}}\right]$$

$$=(x+5)\sqrt{\frac{x-1}{x+1}}+3\ln\left|\sqrt{\frac{x-1}{x+1}}-1\right|-3\ln\left[\sqrt{\frac{x-1}{x+1}}+1\right]+c,$$

$$x\in(-\infty;-1)\cup\langle 1;\infty),\ c\in R.$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\,dx$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\,dx = \left[ \begin{array}{l} \sqrt{x} \in R:\ x \in \langle 0; \infty) \\ 1-\sqrt{x} > 0:\ x \in \langle 0; 1) \end{array} \middle|\ x \in \langle 0; 1) \right]$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\,dx = \left[\begin{array}{l|l} \sqrt{x}\in R:\ x\in\langle 0;\infty) & x\in\langle 0;1) \\ 1-\sqrt{x}>0:\ x\in\langle 0;1) & \end{array}\right] = \int \frac{(1-\sqrt{x})\,dx}{\sqrt{1-(\sqrt{x})^2}}$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\,dx=\left[\begin{matrix}\sqrt{x}\in R:\ x\in\langle 0;\infty)\\ 1-\sqrt{x}>0:\ x\in\langle 0;1)\end{matrix}\,\middle|\, x\in\langle 0;1)\right]=\int \frac{(1-\sqrt{x})\,dx}{\sqrt{1-(\sqrt{x})^2}}=\int \frac{(1-\sqrt{x})\,dx}{\sqrt{1-x}}$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\,dx=\left[\begin{matrix}\sqrt{x}\in R\colon\ x\in\langle 0;\infty) \\ 1-\sqrt{x}>0\colon\ x\in\langle 0;1)\end{matrix}\ \middle|\ x\in\langle 0;1)\right]=\int \frac{(1-\sqrt{x})\,dx}{\sqrt{1-(\sqrt{x})^2}}=\int \frac{(1-\sqrt{x})\,dx}{\sqrt{1-x}}$$

$$=\int \frac{dx}{\sqrt{1-x}}-\int \frac{\sqrt{x}\,dx}{\sqrt{1-x}}$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\,dx=\left[\begin{matrix}\sqrt{x}\in R\colon\ x\in\langle 0;\infty)\\ 1-\sqrt{x}>0\colon\ x\in\langle 0;1)\end{matrix}\ \middle|\ x\in\langle 0;1)\right]=\int\frac{(1-\sqrt{x})\,dx}{\sqrt{1-(\sqrt{x})^2}}=\int\frac{(1-\sqrt{x})\,dx}{\sqrt{1-x}}$$

$$=\int\frac{dx}{\sqrt{1-x}}-\int\frac{\sqrt{x}\,dx}{\sqrt{1-x}}=\int(1-x)^{-\frac{1}{2}}\,dx-\int\frac{\sqrt{x}\,dx}{\sqrt{1-x}}$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\,dx = \left[\begin{array}{l|l} \sqrt{x}\in R\colon\ x\in\langle 0;\infty) & x\in\langle 0;1) \\ 1-\sqrt{x}>0\colon\ x\in\langle 0;1) & \end{array}\right] = \int \frac{(1-\sqrt{x})\,dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x})\,dx}{\sqrt{1-x}}$$

$$= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x}\,dx}{\sqrt{1-x}} = \int (1-x)^{-\frac{1}{2}}\,dx - \int \frac{\sqrt{x}\,dx}{\sqrt{1-x}} = \left[\begin{array}{rl|l} \text{Subst.} & z=1-x & x\in\langle 0;1) \\ & dz=-\,dx & z\in(0;1\rangle \end{array}\right]$$

$$= \left[\begin{array}{rl|l|l} \text{Subst.} & t=\frac{\sqrt{x}}{\sqrt{1-x}}=\sqrt{\frac{x}{1-x}}=\sqrt{\frac{1}{1-x}-1} & t^2(1-x)=x & x\in\langle 0;1) \\ & dx=\frac{2t(t^2+1)-t^2\cdot 2t}{(t^2+1)^2}\,dt=\frac{2t\,dt}{(t^2+1)^2} & x=\frac{t^2}{t^2+1} & t\in\langle 0;\infty) \end{array}\right]$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\, dx$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\, dx = \left[ \begin{array}{l|l} \sqrt{x}\in R\colon\ x\in\langle 0;\infty) & x\in\langle 0;1) \\ 1-\sqrt{x}>0\colon\ x\in\langle 0;1) & \end{array} \right] = \int \frac{(1-\sqrt{x})\, dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x})\, dx}{\sqrt{1-x}}$$

$$= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x}\, dx}{\sqrt{1-x}} = \int (1-x)^{-\frac{1}{2}}\, dx - \int \frac{\sqrt{x}\, dx}{\sqrt{1-x}} = \left[ \begin{array}{r|l} \text{Subst.}\quad z=1-x & x\in\langle 0;1) \\ dz=-\,dx & z\in(0;1\rangle \end{array} \right]$$

$$= \left[ \begin{array}{l|l|l} \text{Subst.}\quad t=\frac{\sqrt{x}}{\sqrt{1-x}}=\sqrt{\frac{x}{1-x}}=\sqrt{\frac{1}{1-x}-1} & t^2(1-x)=x & x\in\langle 0;1) \\ dx=\frac{2t(t^2+1)-t^2\cdot 2t}{(t^2+1)^2}\, dt=\frac{2t\, dt}{(t^2+1)^2} & x=\frac{t^2}{t^2+1} & t\in\langle 0;\infty) \end{array} \right] = -\int z^{-\frac{1}{2}}\, dz - \int \frac{t\cdot 2t\, dt}{(t^2+1)^2}$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\,dx=\left[\begin{array}{l|l}\sqrt{x}\in R\colon\ x\in\langle 0;\infty) & \\ 1-\sqrt{x}>0\colon\ x\in\langle 0;1) & x\in\langle 0;1)\end{array}\right]=\int\frac{(1-\sqrt{x})\,dx}{\sqrt{1-(\sqrt{x})^2}}=\int\frac{(1-\sqrt{x})\,dx}{\sqrt{1-x}}$$

$$=\int\frac{dx}{\sqrt{1-x}}-\int\frac{\sqrt{x}\,dx}{\sqrt{1-x}}=\int(1-x)^{-\frac{1}{2}}\,dx-\int\frac{\sqrt{x}\,dx}{\sqrt{1-x}}=\left[\begin{array}{r|l}\text{Subst.}\quad z=1-x & x\in\langle 0;1)\\ dz=-\,dx & z\in(0;1\rangle\end{array}\right]$$

$$=\left[\begin{array}{l|l|l}\text{Subst.}\quad t=\frac{\sqrt{x}}{\sqrt{1-x}}=\sqrt{\frac{x}{1-x}}=\sqrt{\frac{1}{1-x}-1} & t^2(1-x)=x & x\in\langle 0;1)\\ dx=\frac{2t(t^2+1)-t^2\cdot 2t}{(t^2+1)^2}\,dt=\frac{2t\,dt}{(t^2+1)^2} & x=\frac{t^2}{t^2+1} & t\in\langle 0;\infty)\end{array}\right]=-\int z^{-\frac{1}{2}}\,dz-\int\frac{t\cdot 2t\,dt}{(t^2+1)^2}$$

$$=\left[\begin{array}{l|l}u=t & u'=1\\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1}\end{array}\right]$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\,dx=\left[\begin{matrix}\sqrt{x}\in R:\ x\in\langle 0;\infty) \\ 1-\sqrt{x}>0:\ x\in\langle 0;1)\end{matrix}\ \middle|\ x\in\langle 0;1)\right]=\int\frac{(1-\sqrt{x})\,dx}{\sqrt{1-(\sqrt{x})^2}}=\int\frac{(1-\sqrt{x})\,dx}{\sqrt{1-x}}$$

$$=\int\frac{dx}{\sqrt{1-x}}-\int\frac{\sqrt{x}\,dx}{\sqrt{1-x}}=\int(1-x)^{-\frac{1}{2}}\,dx-\int\frac{\sqrt{x}\,dx}{\sqrt{1-x}}=\left[\begin{matrix}\text{Subst.}\quad z=1-x \\ dz=-\,dx\end{matrix}\ \middle|\ \begin{matrix}x\in\langle 0;1) \\ z\in(0;1\rangle\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst.}\quad t=\frac{\sqrt{x}}{\sqrt{1-x}}=\sqrt{\frac{x}{1-x}}=\sqrt{\frac{1}{1-x}-1} \\ dx=\frac{2t(t^2+1)-t^2\cdot 2t}{(t^2+1)^2}\,dt=\frac{2t\,dt}{(t^2+1)^2}\end{matrix}\ \middle|\ \begin{matrix}t^2(1-x)=x \\ x=\frac{t^2}{t^2+1}\end{matrix}\ \middle|\ \begin{matrix}x\in\langle 0;1) \\ t\in\langle 0;\infty)\end{matrix}\right]=-\int z^{-\frac{1}{2}}\,dz-\int\frac{t\cdot 2t\,dt}{(t^2+1)^2}$$

$$=\left[\begin{matrix}u=t \\ v'=\frac{2t}{(t^2+1)^2}\end{matrix}\ \middle|\ \begin{matrix}u'=1 \\ v=-\frac{1}{t^2+1}\end{matrix}\right]=-\frac{z^{\frac{1}{2}}}{\frac{1}{2}}-\left[-\frac{t}{t^2+1}+\int\frac{dt}{t^2+1}\right]=-2z^{\frac{1}{2}}+\frac{t}{t^2+1}-\int\frac{dt}{t^2+1}$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\,dx=\left[\begin{matrix}\sqrt{x}\in R:\ x\in\langle 0;\infty) \\ 1-\sqrt{x}>0:\ x\in\langle 0;1)\end{matrix}\,\middle|\, x\in\langle 0;1)\right]=\int\frac{(1-\sqrt{x})\,dx}{\sqrt{1-(\sqrt{x})^2}}=\int\frac{(1-\sqrt{x})\,dx}{\sqrt{1-x}}$$

$$=\int\frac{dx}{\sqrt{1-x}}-\int\frac{\sqrt{x}\,dx}{\sqrt{1-x}}=\int(1-x)^{-\frac{1}{2}}\,dx-\int\frac{\sqrt{x}\,dx}{\sqrt{1-x}}=\left[\begin{matrix}\text{Subst.}\ \ z=1-x \\ dz=-\,dx\end{matrix}\,\middle|\,\begin{matrix}x\in\langle 0;1) \\ z\in(0;1\rangle\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst.}\ \ t=\frac{\sqrt{x}}{\sqrt{1-x}}=\sqrt{\frac{x}{1-x}}=\sqrt{\frac{1}{1-x}-1} \\ dx=\frac{2t(t^2+1)-t^2\cdot 2t}{(t^2+1)^2}\,dt=\frac{2t\,dt}{(t^2+1)^2}\end{matrix}\,\middle|\,\begin{matrix}t^2(1-x)=x \\ x=\frac{t^2}{t^2+1}\end{matrix}\,\middle|\,\begin{matrix}x\in\langle 0;1) \\ t\in\langle 0;\infty)\end{matrix}\right]=-\int z^{-\frac{1}{2}}\,dz-\int\frac{t\cdot 2t\,dt}{(t^2+1)^2}$$

$$=\left[\begin{matrix}u=t \\ v'=\frac{2t}{(t^2+1)^2}\end{matrix}\,\middle|\,\begin{matrix}u'=1 \\ v=-\frac{1}{t^2+1}\end{matrix}\right]=-\frac{z^{\frac{1}{2}}}{\frac{1}{2}}-\left[-\frac{t}{t^2+1}+\int\frac{dt}{t^2+1}\right]=-2z^{\frac{1}{2}}+\frac{t}{t^2+1}-\int\frac{dt}{t^2+1}$$

$$=-2\sqrt{z}+\frac{t}{t^2+1}-\operatorname{arctg} t+c_1$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\,dx=\left[\begin{matrix}\sqrt{x}\in R\colon\ x\in\langle 0;\infty)\\ 1-\sqrt{x}>0\colon\ x\in\langle 0;1)\end{matrix}\ \middle|\ x\in\langle 0;1)\right]=\int\frac{(1-\sqrt{x})\,dx}{\sqrt{1-(\sqrt{x})^2}}=\int\frac{(1-\sqrt{x})\,dx}{\sqrt{1-x}}$$

$$=\int\frac{dx}{\sqrt{1-x}}-\int\frac{\sqrt{x}\,dx}{\sqrt{1-x}}=\int(1-x)^{-\frac{1}{2}}\,dx-\int\frac{\sqrt{x}\,dx}{\sqrt{1-x}}=\left[\begin{matrix}\text{Subst.}\quad z=1-x\\ dz=-\,dx\end{matrix}\ \middle|\ \begin{matrix}x\in\langle 0;1)\\ z\in(0;1\rangle\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst.}\quad t=\frac{\sqrt{x}}{\sqrt{1-x}}=\sqrt{\frac{x}{1-x}}=\sqrt{\frac{1}{1-x}-1}\\ dx=\frac{2t(t^2+1)-t^2\cdot 2t}{(t^2+1)^2}\,dt=\frac{2t\,dt}{(t^2+1)^2}\end{matrix}\ \middle|\ \begin{matrix}t^2(1-x)=x\\ x=\frac{t^2}{t^2+1}\end{matrix}\ \middle|\ \begin{matrix}x\in\langle 0;1)\\ t\in\langle 0;\infty)\end{matrix}\right]=-\int z^{-\frac{1}{2}}\,dz-\int\frac{t\cdot 2t\,dt}{(t^2+1)^2}$$

$$=\left[\begin{matrix}u=t\\ v'=\frac{2t}{(t^2+1)^2}\end{matrix}\ \middle|\ \begin{matrix}u'=1\\ v=-\frac{1}{t^2+1}\end{matrix}\right]=-\frac{z^{\frac{1}{2}}}{\frac{1}{2}}-\left[-\frac{t}{t^2+1}+\int\frac{dt}{t^2+1}\right]=-2z^{\frac{1}{2}}+\frac{t}{t^2+1}-\int\frac{dt}{t^2+1}$$

$$=-2\sqrt{z}+\frac{t}{t^2+1}-\operatorname{arctg} t+c_1=\left[\frac{t}{t^2+1}=\frac{\frac{\sqrt{x}}{\sqrt{1-x}}}{\frac{x}{1-x}+1}=\frac{\frac{\sqrt{x}}{\sqrt{1-x}}}{\frac{x+1-x}{1-x}}=\frac{(1-x)\sqrt{x}}{\sqrt{1-x}}=\sqrt{x(1-x)}=\sqrt{x-x^2}\right]$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx = -2\sqrt{1-x} + \sqrt{x-x^2} - \operatorname{arctg}\frac{\sqrt{x}}{\sqrt{1-x}} + c_1$$

$$= \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1-\sqrt{x}}}\,dx = \left[\begin{array}{l|l} \sqrt{x}\in R\colon\ x\in\langle 0;\infty) & x\in\langle 0;1) \\ 1-\sqrt{x}>0\colon\ x\in\langle 0;1) & \end{array}\right] = \int \frac{(1-\sqrt{x})\,dx}{\sqrt{1-(\sqrt{x})^2}} = \int \frac{(1-\sqrt{x})\,dx}{\sqrt{1-x}}$$

$$= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x}\,dx}{\sqrt{1-x}} = \int (1-x)^{-\frac{1}{2}}\,dx - \int \frac{\sqrt{x}\,dx}{\sqrt{1-x}} = \left[\begin{array}{r|l} \text{Subst. } z=1-x & x\in\langle 0;1) \\ dz=-\,dx & z\in(0;1\rangle \end{array}\right]$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } t=\frac{\sqrt{x}}{\sqrt{1-x}}=\sqrt{\frac{x}{1-x}}=\sqrt{\frac{1}{1-x}-1} & t^2(1-x)=x & x\in\langle 0;1) \\ dx=\frac{2t(t^2+1)-t^2\cdot 2t}{(t^2+1)^2}\,dt=\frac{2t\,dt}{(t^2+1)^2} & x=\frac{t^2}{t^2+1} & t\in\langle 0;\infty) \end{array}\right] = -\int z^{-\frac{1}{2}}\,dz - \int \frac{t\cdot 2t\,dt}{(t^2+1)^2}$$

$$= \left[\begin{array}{l|l} u=t & u'=1 \\ v'=\frac{2t}{(t^2+1)^2} & v=-\frac{1}{t^2+1} \end{array}\right] = -\frac{z^{\frac{1}{2}}}{\frac{1}{2}} - \left[-\frac{t}{t^2+1} + \int \frac{dt}{t^2+1}\right] = -2z^{\frac{1}{2}} + \frac{t}{t^2+1} - \int \frac{dt}{t^2+1}$$

$$= -2\sqrt{z} + \frac{t}{t^2+1} - \operatorname{arctg} t + c_1 = \left[\frac{t}{t^2+1} = \frac{\frac{\sqrt{x}}{\sqrt{1-x}}}{\frac{x}{1-x}+1} = \frac{\frac{\sqrt{x}}{\sqrt{1-x}}}{\frac{x+1-x}{1-x}} = \frac{(1-x)\sqrt{x}}{\sqrt{1-x}} = \sqrt{x(1-x)} = \sqrt{x-x^2}\right]$$

$$= -2\sqrt{1-x} + \sqrt{x-x^2} - \operatorname{arctg}\frac{\sqrt{x}}{\sqrt{1-x}} + c_1,\ x\in\langle 0;1),\ c_1\in R.$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\, dx$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \left[ \begin{array}{l} \sqrt{x} \in R\colon\ x \in \langle 0; \infty) \\ 1-\sqrt{x} \geq 0\colon\ x \in \langle 0; 1\rangle \end{array} \middle|\, x \in \langle 0; 1\rangle \right]$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \left[\begin{array}{l} \sqrt{x} \in R\colon\ x \in \langle 0; \infty) \\ 1-\sqrt{x} \geq 0\colon\ x \in \langle 0; 1\rangle \end{array} \middle|\, x \in \langle 0; 1\rangle \right] = \left[\begin{array}{ll} \text{Subst.} & t = \sqrt{x} \\ x = t^2, & dx = 2t\,dt \end{array} \middle|\, \begin{array}{l} x \in \langle 0; 1\rangle \\ t \in \langle 0; 1\rangle \end{array} \right]$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\left[\begin{matrix}\sqrt{x}\in R:\ x\in\langle 0;\infty)\\ 1-\sqrt{x}\geq 0:\ x\in\langle 0;1\rangle\end{matrix}\ \middle|\ x\in\langle 0;1\rangle\right]=\left[\begin{matrix}\text{Subst.} & t=\sqrt{x}\\ x=t^2, & dx=2t\,dt\end{matrix}\ \middle|\ \begin{matrix}x\in\langle 0;1\rangle\\ t\in\langle 0;1\rangle\end{matrix}\right]=\int\frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \left[\begin{array}{l} \sqrt{x}\in R\colon\ x\in\langle 0;\infty) \\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle \end{array}\middle|\, x\in\langle 0;1\rangle\right] = \left[\begin{array}{ll} \text{Subst.} & t=\sqrt{x} \\ x=t^2, & dx=2t\,dt \end{array}\middle|\begin{array}{l} x\in\langle 0;1\rangle \\ t\in\langle 0;1\rangle \end{array}\right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$= \left[\begin{array}{l} \text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} \\ t=\frac{1-z^2}{z^2+1},\ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2} \end{array}\middle|\begin{array}{l} z^2(1+t)=1-t \\ t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle \end{array}\right]$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\left[\begin{matrix}\sqrt{x}\in R\colon\ x\in\langle 0;\infty)\\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle\end{matrix}\;\middle|\; x\in\langle 0;1\rangle\right]=\left[\begin{matrix}\text{Subst.}\quad t=\sqrt{x}\\ x=t^2,\ dx=2t\,dt\end{matrix}\;\middle|\;\begin{matrix}x\in\langle 0;1\rangle\\ t\in\langle 0;1\rangle\end{matrix}\right]=\int\frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$=\left[\begin{matrix}\text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1}\\ t=\frac{1-z^2}{z^2+1},\ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2}\end{matrix}\;\middle|\;\begin{matrix}z^2(1+t)=1-t\\ t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle\end{matrix}\right]=\int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\left[\begin{matrix}\sqrt{x}\in R\colon\ x\in\langle 0;\infty)\\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle\end{matrix}\ \middle|\ x\in\langle 0;1\rangle\right]=\left[\begin{matrix}\text{Subst.}\quad t=\sqrt{x}\\ x=t^2,\ dx=2t\,dt\end{matrix}\ \middle|\ \begin{matrix}x\in\langle 0;1\rangle\\ t\in\langle 0;1\rangle\end{matrix}\right]=\int\frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$=\left[\begin{matrix}\text{Subst.}\ z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1}\\ t=\frac{1-z^2}{z^2+1},\ \ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2}\end{matrix}\ \middle|\ \begin{matrix}z^2(1+t)=1-t\\ t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle\end{matrix}\right]=\int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$=8\int\frac{z^2(z^2-1)\,dz}{(z^2+1)^3}$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\left[\begin{array}{l}\sqrt{x}\in R:\ x\in\langle 0;\infty)\\ 1-\sqrt{x}\geq 0:\ x\in\langle 0;1\rangle\end{array}\middle|\ x\in\langle 0;1\rangle\right]=\left[\begin{array}{ll}\text{Subst.}\quad t=\sqrt{x} & x\in\langle 0;1\rangle\\ x=t^2,\ dx=2t\,dt & t\in\langle 0;1\rangle\end{array}\right]=\int\frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$=\left[\begin{array}{l|l}\text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & z^2(1+t)=1-t\\ t=\frac{1-z^2}{z^2+1},\ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2} & t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle\end{array}\right]=\int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$=8\int\frac{z^2(z^2-1)\,dz}{(z^2+1)^3}=\left[\begin{array}{l}\text{Rozklad na parciálne zlomky:}\\ \frac{z^4-z^2}{(z^2+1)^3}=\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\end{array}\right]$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \left[\begin{matrix} \sqrt{x}\in R\colon\ x\in\langle 0;\infty) \\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle \end{matrix}\ \middle|\ x\in\langle 0;1\rangle\right] = \left[\begin{matrix}\text{Subst.} & t=\sqrt{x} \\ x=t^2, & dx=2t\,dt\end{matrix}\ \middle|\ \begin{matrix} x\in\langle 0;1\rangle \\ t\in\langle 0;1\rangle\end{matrix}\right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$= \left[\begin{matrix}\text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} \\ t=\frac{1-z^2}{z^2+1},\ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2}\end{matrix}\ \middle|\ \begin{matrix} z^2(1+t)=1-t \\ t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle\end{matrix}\right] = \int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$= 8\int\frac{z^2(z^2-1)\,dz}{(z^2+1)^3} = \left[\begin{matrix}\text{Rozklad na parciálne zlomky:} \\ \frac{z^4-z^2}{(z^2+1)^3}=\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\end{matrix}\right] = 8\int\left[\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\right]dz$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\left[\begin{matrix}\sqrt{x}\in R\colon\ x\in\langle 0;\infty)\\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle\end{matrix}\ \middle|\ x\in\langle 0;1\rangle\right]=\left[\begin{matrix}\text{Subst.}\quad t=\sqrt{x}\\ x=t^2,\ dx=2t\,dt\end{matrix}\ \middle|\ \begin{matrix}x\in\langle 0;1\rangle\\ t\in\langle 0;1\rangle\end{matrix}\right]=\int\frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$=\left[\begin{matrix}\text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1}\\ t=\frac{1-z^2}{z^2+1},\ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2}\end{matrix}\ \middle|\ \begin{matrix}z^2(1+t)=1-t\\ t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle\end{matrix}\right]=\int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$=8\int\frac{z^2(z^2-1)\,dz}{(z^2+1)^3}=\left[\begin{matrix}\text{Rozklad na parciálne zlomky:}\\ \frac{z^4-z^2}{(z^2+1)^3}=\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\end{matrix}\right]=8\int\left[\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\right]dz$$

$$=\left[\begin{matrix}\text{Pr. 121: } I_n=\int\frac{dz}{(z^2+1)^n}\\ =\frac{2n-3}{2n-2}I_{n-1}+\frac{z}{(2n-2)(z^2+1)^{n-1}}\end{matrix}\right]$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \left[\begin{array}{l|l} \sqrt{x}\in R\colon\ x\in\langle 0;\infty) & \\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle & x\in\langle 0;1\rangle \end{array}\right] = \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{x} & x\in\langle 0;1\rangle \\ x=t^2, & dx=2t\,dt & t\in\langle 0;1\rangle \end{array}\right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$= \left[\begin{array}{l|l} \text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & z^2(1+t)=1-t \\ t=\frac{1-z^2}{z^2+1},\quad dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2} & t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle \end{array}\right] = \int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$= 8\int\frac{z^2(z^2-1)\,dz}{(z^2+1)^3} = \left[\begin{array}{l} \text{Rozklad na parciálne zlomky:} \\ \frac{z^4-z^2}{(z^2+1)^3}=\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1} \end{array}\right] = 8\int\left[\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\right]dz$$

$$= \left[\begin{array}{l} \text{Pr. 121: } I_n=\int\frac{dz}{(z^2+1)^n} \\ \quad =\frac{2n-3}{2n-2}I_{n-1}+\frac{z}{(2n-2)(z^2+1)^{n-1}} \end{array}\right] = 16I_3-24I_2+8I_1$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\left[\begin{array}{l|l}\sqrt{x}\in R\colon\ x\in\langle 0;\infty) & \\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle & x\in\langle 0;1\rangle\end{array}\right]=\left[\begin{array}{ll|l}\text{Subst.} & t=\sqrt{x} & x\in\langle 0;1\rangle \\ x=t^2, & dx=2t\,dt & t\in\langle 0;1\rangle\end{array}\right]=\int\frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$=\left[\begin{array}{l|l}\text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & z^2(1+t)=1-t \\ t=\frac{1-z^2}{z^2+1},\ \ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2} & t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle\end{array}\right]=\int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$=8\int\frac{z^2(z^2-1)\,dz}{(z^2+1)^3}=\left[\begin{array}{l}\text{Rozklad na parciálne zlomky:} \\ \frac{z^4-z^2}{(z^2+1)^3}=\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\end{array}\right]=8\int\left[\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\right]dz$$

$$=\left[\begin{array}{l}\text{Pr. 121: } I_n=\int\frac{dz}{(z^2+1)^n} \\ \quad=\frac{2n-3}{2n-2}I_{n-1}+\frac{z}{(2n-2)(z^2+1)^{n-1}}\end{array}\right]=16I_3-24I_2+8I_1=16\left[\frac{3I_2}{4}+\frac{z}{4(z^2+1)^2}\right]-24I_2+8I_1$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \left[\begin{matrix} \sqrt{x}\in R\colon\ x\in\langle 0;\infty) \\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle \end{matrix}\,\middle|\, x\in\langle 0;1\rangle\right] = \left[\begin{matrix}\text{Subst.}\quad t=\sqrt{x} \\ x=t^2,\ dx=2t\,dt\end{matrix}\,\middle|\,\begin{matrix} x\in\langle 0;1\rangle \\ t\in\langle 0;1\rangle\end{matrix}\right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$= \left[\begin{matrix}\text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} \\ t=\frac{1-z^2}{z^2+1},\ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2}\end{matrix}\,\middle|\,\begin{matrix} z^2(1+t)=1-t \\ t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle\end{matrix}\right] = \int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$= 8\int\frac{z^2(z^2-1)\,dz}{(z^2+1)^3} = \left[\begin{matrix}\text{Rozklad na parciálne zlomky:} \\ \frac{z^4-z^2}{(z^2+1)^3}=\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\end{matrix}\right] = 8\int\left[\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\right]dz$$

$$= \left[\begin{matrix}\text{Pr. 121: } I_n=\int\frac{dz}{(z^2+1)^n} \\ =\frac{2n-3}{2n-2}I_{n-1}+\frac{z}{(2n-2)(z^2+1)^{n-1}}\end{matrix}\right] = 16I_3-24I_2+8I_1 = 16\left[\frac{3I_2}{4}+\frac{z}{4(z^2+1)^2}\right]-24I_2+8I_1$$

$$= \frac{4z}{(z^2+1)^2}-12I_2+8I_1$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \left[\begin{array}{l} \sqrt{x}\in R\colon\ x\in\langle 0;\infty)\\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle \end{array}\,\middle|\, x\in\langle 0;1\rangle\right] = \left[\begin{array}{l} \text{Subst. } t=\sqrt{x}\\ x=t^2,\ dx=2t\,dt \end{array}\,\middle|\, \begin{array}{l} x\in\langle 0;1\rangle\\ t\in\langle 0;1\rangle\end{array}\right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$= \left[\begin{array}{l} \text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1}\\ t=\frac{1-z^2}{z^2+1},\ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2} \end{array}\,\middle|\, \begin{array}{l} z^2(1+t)=1-t\\ t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle\end{array}\right] = \int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$= 8\int \frac{z^2(z^2-1)\,dz}{(z^2+1)^3} = \left[\begin{array}{l}\text{Rozklad na parciálne zlomky:}\\ \frac{z^4-z^2}{(z^2+1)^3}=\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\end{array}\right] = 8\int\left[\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\right]dz$$

$$= \left[\begin{array}{l}\text{Pr. 121: } I_n=\int\frac{dz}{(z^2+1)^n}\\ \quad =\frac{2n-3}{2n-2}I_{n-1}+\frac{z}{(2n-2)(z^2+1)^{n-1}}\end{array}\right] = 16I_3-24I_2+8I_1 = 16\left[\frac{3I_2}{4}+\frac{z}{4(z^2+1)^2}\right]-24I_2+8I_1$$

$$= \frac{4z}{(z^2+1)^2}-12I_2+8I_1 = \frac{4z}{(z^2+1)^2}-12\left[\frac{I_1}{2}+\frac{z}{2(z^2+1)}\right]+8I_1$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \left[\begin{array}{l} \sqrt{x}\in R\colon\ x\in\langle 0;\infty) \\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle \end{array}\middle|\, x\in\langle 0;1\rangle\right] = \left[\begin{array}{l} \text{Subst.}\quad t=\sqrt{x} \\ x=t^2,\ dx=2t\,dt \end{array}\middle|\begin{array}{l} x\in\langle 0;1\rangle \\ t\in\langle 0;1\rangle\end{array}\right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$= \left[\begin{array}{l} \text{Subst.}\ z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} \\ t=\frac{1-z^2}{z^2+1},\ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2} \end{array}\middle|\begin{array}{l} z^2(1+t)=1-t \\ t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle\end{array}\right] = \int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$= 8\int \frac{z^2(z^2-1)\,dz}{(z^2+1)^3} = \left[\begin{array}{l}\text{Rozklad na parciálne zlomky:} \\ \frac{z^4-z^2}{(z^2+1)^3}=\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\end{array}\right] = 8\int\left[\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\right]dz$$

$$= \left[\begin{array}{l}\text{Pr. 121: } I_n=\int\frac{dz}{(z^2+1)^n} \\ \quad =\frac{2n-3}{2n-2}I_{n-1}+\frac{z}{(2n-2)(z^2+1)^{n-1}}\end{array}\right] = 16I_3-24I_2+8I_1 = 16\left[\frac{3I_2}{4}+\frac{z}{4(z^2+1)^2}\right]-24I_2+8I_1$$

$$= \frac{4z}{(z^2+1)^2}-12I_2+8I_1 = \frac{4z}{(z^2+1)^2}-12\left[\frac{I_1}{2}+\frac{z}{2(z^2+1)}\right]+8I_1 = \frac{4z}{(z^2+1)^2}-\frac{6z}{z^2+1}+2I_1$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \left[\begin{array}{l|l} \sqrt{x}\in R\colon\ x\in\langle 0;\infty) & \\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle & x\in\langle 0;1\rangle \end{array}\right] = \left[\begin{array}{l|l} \text{Subst.}\quad t=\sqrt{x} & x\in\langle 0;1\rangle \\ x=t^2,\ dx=2t\,dt & t\in\langle 0;1\rangle \end{array}\right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$= \left[\begin{array}{l|l} \text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & z^2(1+t)=1-t \\ t=\frac{1-z^2}{z^2+1},\ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2} & t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle \end{array}\right] = \int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$= 8\int\frac{z^2(z^2-1)\,dz}{(z^2+1)^3} = \left[\begin{array}{l} \text{Rozklad na parciálne zlomky:} \\ \frac{z^4-z^2}{(z^2+1)^3}=\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1} \end{array}\right] = 8\int\left[\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\right]dz$$

$$= \left[\begin{array}{l} \text{Pr. 121: } I_n=\int\frac{dz}{(z^2+1)^n} \\ \quad=\frac{2n-3}{2n-2}I_{n-1}+\frac{z}{(2n-2)(z^2+1)^{n-1}} \end{array}\right] = 16I_3-24I_2+8I_1 = 16\left[\frac{3I_2}{4}+\frac{z}{4(z^2+1)^2}\right]-24I_2+8I_1$$

$$= \frac{4z}{(z^2+1)^2}-12I_2+8I_1 = \frac{4z}{(z^2+1)^2}-12\left[\frac{I_1}{2}+\frac{z}{2(z^2+1)}\right]+8I_1 = \frac{4z}{(z^2+1)^2}-\frac{6z}{z^2+1}+2I_1$$

$$= \frac{4z}{(z^2+1)^2}-\frac{6z}{z^2+1}+2\operatorname{arctg} z+c$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$=\left[\begin{matrix}\sqrt{x}\in R\colon\ x\in\langle 0;\infty)\\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle\end{matrix}\ \middle|\ x\in\langle 0;1\rangle\right]=\left[\begin{matrix}\text{Subst.}\quad t=\sqrt{x}\\ x=t^2,\ dx=2t\,dt\end{matrix}\ \middle|\ \begin{matrix}x\in\langle 0;1\rangle\\ t\in\langle 0;1\rangle\end{matrix}\right]=\int\frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$=\left[\begin{matrix}\text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1}\\ t=\frac{1-z^2}{z^2+1},\ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}dz=\frac{-4z\,dz}{(z^2+1)^2}\end{matrix}\ \middle|\ \begin{matrix}z^2(1+t)=1-t\\ t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle\end{matrix}\right]=\int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$=8\int\frac{z^2(z^2-1)\,dz}{(z^2+1)^3}=\left[\begin{matrix}\text{Rozklad na parciálne zlomky:}\\ \frac{z^4-z^2}{(z^2+1)^3}=\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\end{matrix}\right]=8\int\left[\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\right]dz$$

$$=\left[\begin{matrix}\text{Pr. 121: } I_n=\int\frac{dz}{(z^2+1)^n}\\ =\frac{2n-3}{2n-2}I_{n-1}+\frac{z}{(2n-2)(z^2+1)^{n-1}}\end{matrix}\right]=16I_3-24I_2+8I_1=16\left[\frac{3I_2}{4}+\frac{z}{4(z^2+1)^2}\right]-24I_2+8I_1$$

$$=\frac{4z}{(z^2+1)^2}-12I_2+8I_1=\frac{4z}{(z^2+1)^2}-12\left[\frac{I_1}{2}+\frac{z}{2(z^2+1)}\right]+8I_1=\frac{4z}{(z^2+1)^2}-\frac{6z}{z^2+1}+2I_1$$

$$=\frac{4z}{(z^2+1)^2}-\frac{6z}{z^2+1}+2\,\mathrm{arctg}\,z+c=\left[z=\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\ \middle|\ \frac{4z}{(z^2+1)^2}=\sqrt{1-x}+\sqrt{x-x^2}\ \middle|\ \frac{6z}{z^2+1}=\sqrt{x-x^2}-2\sqrt{1-x}\right]$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx$$

$$= \left[ \begin{array}{l} \sqrt{x}\in R\colon\ x\in\langle 0;\infty) \\ 1-\sqrt{x}\geq 0\colon\ x\in\langle 0;1\rangle \end{array} \middle| x\in\langle 0;1\rangle \right] = \left[ \begin{array}{ll} \text{Subst.}\quad t=\sqrt{x} & x\in\langle 0;1\rangle \\ x=t^2,\ dx=2t\,dt & t\in\langle 0;1\rangle \end{array} \right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$= \left[ \begin{array}{l|l} \text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & z^2(1+t)=1-t \\ t=\frac{1-z^2}{z^2+1},\ dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2} & t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle \end{array} \right] = \int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$= 8\int\frac{z^2(z^2-1)\,dz}{(z^2+1)^3} = \left[ \begin{array}{l} \text{Rozklad na parciálne zlomky:} \\ \frac{z^4-z^2}{(z^2+1)^3}=\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1} \end{array} \right] = 8\int\left[\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\right]dz$$

$$= \left[ \begin{array}{l} \text{Pr. 121: } I_n=\int\frac{dz}{(z^2+1)^n} \\ \quad =\frac{2n-3}{2n-2}I_{n-1}+\frac{z}{(2n-2)(z^2+1)^{n-1}} \end{array} \right] = 16I_3-24I_2+8I_1 = 16\left[\frac{3I_2}{4}+\frac{z}{4(z^2+1)^2}\right]-24I_2+8I_1$$

$$= \frac{4z}{(z^2+1)^2}-12I_2+8I_1 = \frac{4z}{(z^2+1)^2}-12\left[\frac{I_1}{2}+\frac{z}{2(z^2+1)}\right]+8I_1 = \frac{4z}{(z^2+1)^2}-\frac{6z}{z^2+1}+2I_1$$

$$= \frac{4z}{(z^2+1)^2}-\frac{6z}{z^2+1}+2\,\mathrm{arctg}\,z+c = \left[ z=\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} \,\middle|\, \frac{4z}{(z^2+1)^2}=\sqrt{1-x}+\sqrt{x-x^2} \,\middle|\, \frac{6z}{z^2+1}=\sqrt{x-x^2}-2\sqrt{1-x} \right]$$

$$= \sqrt{1-x}+\sqrt{x-x^2}-3\sqrt{1-x}+2\,\mathrm{arctg}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}+c$$

# Riešené príklady – 170

$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\,dx = -2\sqrt{1-x} + \sqrt{x-x^2} + 2\,\mathrm{arctg}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} + c$$

$$= \left[\begin{array}{l|l} \sqrt{x}\in R:\ x\in\langle 0;\infty) & \\ 1-\sqrt{x}\geq 0:\ x\in\langle 0;1\rangle & x\in\langle 0;1\rangle \end{array}\right] = \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{x} & x\in\langle 0;1\rangle \\ x=t^2, & dx=2t\,dt & t\in\langle 0;1\rangle \end{array}\right] = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\cdot 2t\,dt$$

$$= \left[\begin{array}{l|l} \text{Subst. } z=\frac{\sqrt{1-t}}{\sqrt{1+t}}=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & z^2(1+t)=1-t \\ t=\frac{1-z^2}{z^2+1},\quad dt=\frac{-2z(z^2+1)-(1-z^2)\cdot 2z}{(z^2+1)^2}\,dz=\frac{-4z\,dz}{(z^2+1)^2} & t\in\langle 0;1\rangle,\ z\in\langle 0;1\rangle \end{array}\right] = \int z\cdot\frac{2(1-z^2)}{z^2+1}\cdot\frac{-4z\,dz}{(z^2+1)^2}$$

$$= 8\int\frac{z^2(z^2-1)\,dz}{(z^2+1)^3} = \left[\begin{array}{l} \text{Rozklad na parciálne zlomky:} \\ \frac{z^4-z^2}{(z^2+1)^3}=\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1} \end{array}\right] = 8\int\left[\frac{2}{(z^2+1)^3}-\frac{3}{(z^2+1)^2}+\frac{1}{z^2+1}\right]dz$$

$$= \left[\begin{array}{l} \text{Pr. 121: } I_n=\int\frac{dz}{(z^2+1)^n} \\ \quad =\frac{2n-3}{2n-2}I_{n-1}+\frac{z}{(2n-2)(z^2+1)^{n-1}} \end{array}\right] = 16I_3-24I_2+8I_1 = 16\left[\frac{3I_2}{4}+\frac{z}{4(z^2+1)^2}\right]-24I_2+8I_1$$

$$= \frac{4z}{(z^2+1)^2}-12I_2+8I_1 = \frac{4z}{(z^2+1)^2}-12\left[\frac{I_1}{2}+\frac{z}{2(z^2+1)}\right]+8I_1 = \frac{4z}{(z^2+1)^2}-\frac{6z}{z^2+1}+2I_1$$

$$= \frac{4z}{(z^2+1)^2}-\frac{6z}{z^2+1}+2\,\mathrm{arctg}\,z+c = \left[\begin{array}{l|l|l} z=\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}} & \frac{4z}{(z^2+1)^2}=\sqrt{1-x}+\sqrt{x-x^2} & \frac{6z}{z^2+1}=\sqrt{x-x^2}-2\sqrt{1-x} \end{array}\right]$$

$$= \sqrt{1-x}+\sqrt{x-x^2}-3\sqrt{1-x}+2\,\mathrm{arctg}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}+c$$

$$= \sqrt{x-x^2}-2\sqrt{1-x}+2\,\mathrm{arctg}\,\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}+c,\quad x\in\langle 0;1\rangle,\ c\in R.$$

# Riešené príklady – 171

$$\int \frac{dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$$

# Riešené príklady – 171

$$\int \frac{\mathrm{d}x}{\sqrt{x+1}+\sqrt[3]{x+1}}$$

$$= \left[ \begin{array}{l|l|l|l} \text{Subst.} & t=\sqrt[6]{x+1} & \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 & x\in(-1;\infty) \\ t^6=x+1 & 6t^5\,\mathrm{d}t=\mathrm{d}x & \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 & t\in(0;\infty) \end{array} \right]$$

# Riešené príklady – 171

$$\int \frac{dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$$

$$= \left[ \begin{array}{l|l|l|l} \text{Subst.} & t=\sqrt[6]{x+1} & \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 & x \in (-1;\infty) \\ t^6 = x+1 & 6t^5\,dt = dx & \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 & t \in (0;\infty) \end{array} \right] = \int \frac{6t^5\,dt}{t^3+t^2}$$

# Riešené príklady – 171

$$\int \frac{dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$$

$$= \left[ \begin{array}{l|l|l|l} \text{Subst.} & t=\sqrt[6]{x+1} & \sqrt{x+1}=(\sqrt[6]{x+1})^3=t^3 & x\in(-1;\infty) \\ t^6=x+1 & 6t^5\,dt=dx & \sqrt[3]{x+1}=(\sqrt[6]{x+1})^2=t^2 & t\in(0;\infty) \end{array} \right] = \int \frac{6t^5\,dt}{t^3+t^2} = 6\int \frac{t^3\,dt}{t+1}$$

# Riešené príklady – 171

$$\int \frac{dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$$

$$= \left[ \begin{array}{l|l|l|l} \text{Subst.} & t=\sqrt[6]{x+1} & \sqrt{x+1}=(\sqrt[6]{x+1})^3=t^3 & x\in(-1;\infty) \\ t^6=x+1 & 6t^5\,dt=dx & \sqrt[3]{x+1}=(\sqrt[6]{x+1})^2=t^2 & t\in(0;\infty) \end{array} \right] = \int \frac{6t^5\,dt}{t^3+t^2} = 6\int \frac{t^3\,dt}{t+1}$$

$$= 6\int \frac{t^3+t^2-t^2-t+t+1-1}{t+1}\,dt$$

$$= \left[ \begin{array}{l|l|l} \text{Subst.} & u=t+1=\sqrt[6]{x+1}+1 & t\in(0;\infty) \\ t=u-1 & dt=du & u\in(1;\infty) \end{array} \right]$$

# Riešené príklady – 171

$$\int \frac{dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$$

$$= \left[ \begin{array}{l|l|l|l} \text{Subst.} & t=\sqrt[6]{x+1} & \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 & x\in(-1;\infty) \\ t^6=x+1 & 6t^5\,dt=dx & \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 & t\in(0;\infty) \end{array} \right] = \int \frac{6t^5\,dt}{t^3+t^2} = 6\int \frac{t^3\,dt}{t+1}$$

$$= 6\int \frac{t^3+t^2-t^2-t+t+1-1}{t+1}\,dt = 6\int \frac{t^3+1-1}{t+1}\,dt$$

$$= \left[ \begin{array}{l|l|l} \text{Subst.} & u=t+1=\sqrt[6]{x+1}+1 & t\in(0;\infty) \\ t=u-1 & dt=du & u\in(1;\infty) \end{array} \right] = 6\int \frac{(u-1)^3\,du}{u}$$

# Riešené príklady – 171

$$\int \frac{dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$$

$$=\left[\begin{array}{l|l|l|l} \text{Subst.} & t=\sqrt[6]{x+1} & \sqrt{x+1}=(\sqrt[6]{x+1})^3=t^3 & x\in(-1;\infty) \\ t^6=x+1 & 6t^5\,dt=dx & \sqrt[3]{x+1}=(\sqrt[6]{x+1})^2=t^2 & t\in(0;\infty) \end{array}\right]=\int\frac{6t^5\,dt}{t^3+t^2}=6\int\frac{t^3\,dt}{t+1}$$

$$=6\int\frac{t^3+t^2-t^2-t+t+1-1}{t+1}\,dt=6\int\frac{t^3+1-1}{t+1}\,dt=6\int\left(t^2-t+1-\frac{1}{t+1}\right)dt$$

$$=\left[\begin{array}{l|l|l} \text{Subst.} & u=t+1=\sqrt[6]{x+1}+1 & t\in(0;\infty) \\ t=u-1 & dt=du & u\in(1;\infty) \end{array}\right]=6\int\frac{(u-1)^3\,du}{u}=6\int\frac{u^3-3u^2+3u-1}{u}\,du$$

# Riešené príklady – 171

$$\int \frac{dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$$

$$= \left[ \begin{array}{l|l|l|l} \text{Subst.} & t=\sqrt[6]{x+1} & \sqrt{x+1}=(\sqrt[6]{x+1})^3=t^3 & x\in(-1;\infty) \\ t^6=x+1 & 6t^5\,dt=dx & \sqrt[3]{x+1}=(\sqrt[6]{x+1})^2=t^2 & t\in(0;\infty) \end{array} \right] = \int \frac{6t^5\,dt}{t^3+t^2} = 6\int \frac{t^3\,dt}{t+1}$$

$$=6\int \frac{t^3+t^2-t^2-t+t+1-1}{t+1}\,dt = 6\int \frac{t^3+1-1}{t+1}\,dt = 6\int \left(t^2-t+1-\frac{1}{t+1}\right)dt$$

$$=6\left(\frac{t^3}{3}-\frac{t^2}{2}+t-\ln|t+1|\right)+c$$

$$= \left[ \begin{array}{l|l|l} \text{Subst.} & u=t+1=\sqrt[6]{x+1}+1 & t\in(0;\infty) \\ t=u-1 & dt=du & u\in(1;\infty) \end{array} \right] = 6\int \frac{(u-1)^3\,du}{u} = 6\int \frac{u^3-3u^2+3u-1}{u}\,du$$

$$=6\int \left(u^2-3u+3-\frac{1}{u}\right)du$$

# Riešené príklady – 171

$$\int \frac{dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$$

$$=\left[\begin{array}{l|l|l|l} \text{Subst.} & t=\sqrt[6]{x+1} & \sqrt{x+1}=(\sqrt[6]{x+1})^3=t^3 & x\in(-1;\infty) \\ t^6=x+1 & 6t^5\,dt=dx & \sqrt[3]{x+1}=(\sqrt[6]{x+1})^2=t^2 & t\in(0;\infty) \end{array}\right]=\int\frac{6t^5\,dt}{t^3+t^2}=6\int\frac{t^3\,dt}{t+1}$$

$$=6\int\frac{t^3+t^2-t^2-t+t+1-1}{t+1}\,dt=6\int\frac{t^3+1-1}{t+1}\,dt=6\int\left(t^2-t+1-\frac{1}{t+1}\right)dt$$

$$=6\left(\frac{t^3}{3}-\frac{t^2}{2}+t-\ln|t+1|\right)+c=2t^3-3t^2+6t-6\ln(t+1)+c$$

$$=\left[\begin{array}{l|l|l} \text{Subst.} & u=t+1=\sqrt[6]{x+1}+1 & t\in(0;\infty) \\ t=u-1 & dt=du & u\in(1;\infty) \end{array}\right]=6\int\frac{(u-1)^3\,du}{u}=6\int\frac{u^3-3u^2+3u-1}{u}\,du$$

$$=6\int\left(u^2-3u+3-\frac{1}{u}\right)du=6\left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln|u|\right)+c$$

# Riešené príklady – 171

$$\int \frac{dx}{\sqrt{x+1}+\sqrt[3]{x+1}} = 2\sqrt{x+1} - 3\sqrt[3]{x+1} + 6\sqrt[6]{x+1} - 6\ln\left(\sqrt[6]{x+1}+1\right) + c$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.} & t=\sqrt[6]{x+1} & \sqrt{x+1} = (\sqrt[6]{x+1})^3 = t^3 & x\in(-1;\infty) \\ t^6 = x+1 & 6t^5\,dt = dx & \sqrt[3]{x+1} = (\sqrt[6]{x+1})^2 = t^2 & t\in(0;\infty) \end{array}\right] = \int \frac{6t^5\,dt}{t^3+t^2} = 6\int \frac{t^3\,dt}{t+1}$$

$$= 6\int \frac{t^3+t^2-t^2-t+t+1-1}{t+1}\,dt = 6\int \frac{t^3+1-1}{t+1}\,dt = 6\int \left(t^2 - t + 1 - \frac{1}{t+1}\right) dt$$

$$= 6\left(\frac{t^3}{3} - \frac{t^2}{2} + t - \ln|t+1|\right) + c = 2t^3 - 3t^2 + 6t - 6\ln(t+1) + c$$

$$= 2\sqrt{x+1} - 3\sqrt[3]{x+1} + 6\sqrt[6]{x+1} - 6\ln\left(\sqrt[6]{x+1}+1\right) + c,\ x\in(-1;\infty),\ c\in R.$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} & u=t+1=\sqrt[6]{x+1}+1 & t\in(0;\infty) \\ t=u-1 & dt=du & u\in(1;\infty) \end{array}\right] = 6\int \frac{(u-1)^3\,du}{u} = 6\int \frac{u^3-3u^2+3u-1}{u}\,du$$

$$= 6\int \left(u^2-3u+3-\frac{1}{u}\right) du = 6\left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln|u|\right) + c$$

$$= 2(\sqrt[6]{x+1}+1)^3 - 9(\sqrt[6]{x+1}+1)^2 + (\sqrt[6]{x+1}+1) - 6\ln\left(\sqrt[6]{x+1}+1\right) + c,$$

$$x\in(-1;\infty),\ c\in R.$$

# Riešené príklady – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}}\, dx$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{1+x}{1-x}\leq 1 \\ x\neq 1\end{matrix}\,\middle|\,\begin{matrix} 1-x>0 \Leftrightarrow 1>x: & 0\leq 1+x\leq 1-x, & \text{t. j.} & -1\leq x,\ 2x\leq 0, & \text{t. j.} & x\in\langle -1;0\rangle \\ 1-x<0 \Leftrightarrow 1<x: & 0\geq 1+x\geq 1-x, & \text{t. j.} & -1\geq x,\ 2x\geq 0, & \text{t. j.} & x \text{ neexistuje}\end{matrix}\,\middle|\,\begin{matrix} x\in\langle -1;0\rangle \\ \text{t. j.}\ \ -1\leq x\leq 0\end{matrix}\right]$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{array}{c|l|c} 0\le\frac{1+x}{1-x}\le 1 & 1-x>0 \Leftrightarrow 1>x:\quad 0\le 1+x\le 1-x,\ \text{t. j. } -1\le x,\ 2x\le 0,\ \text{t. j. } x\in\langle -1;0) & x\in\langle -1;0) \\ x\neq 1 & 1-x<0 \Leftrightarrow 1<x:\quad 0\ge 1+x\ge 1-x,\ \text{t. j. } -1\ge x,\ 2x\ge 0,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } -1\le x\le 0 \end{array}\right]$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{1+x}{1-x}} & u'=\frac{1}{\sqrt{1-\frac{1+x}{1-x}}}\cdot\frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\cdot\frac{(1-x)-(1+x)\cdot(-1)}{(1-x)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{1-x-(1+x)}{1-x}}\sqrt{\frac{1+x}{1-x}}\sqrt{(1-x)^4}}=\frac{1}{\sqrt{\frac{-2x}{1-x}\cdot\frac{1+x}{1-x}\cdot(1-x)^4}} \\ v'=1 & v=x \qquad =\frac{1}{\sqrt{-2x(1+x)\cdot(1-x)^2}}=\frac{1}{\sqrt{2}\,|1-x|\,\sqrt{-x(1+x)}}=\frac{\sqrt{2}}{2(1-x)\sqrt{-x(x+1)}}=\frac{-\sqrt{2}}{2(x-1)\sqrt{-x^2-x}} \end{array}\right]$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{1+x}{1-x}\leq 1 \\ x\neq 1\end{matrix}\,\middle|\, \begin{matrix}1-x>0 \Leftrightarrow 1>x\colon\ \ 0\leq 1+x\leq 1-x,\ \text{t. j.}\ -1\leq x,\ 2x\leq 0,\ \text{t. j.}\ \ x\in\langle -1;0\rangle \\ 1-x<0 \Leftrightarrow 1<x\colon\ \ 0\geq 1+x\geq 1-x,\ \text{t. j.}\ -1\geq x,\ 2x\geq 0,\ \text{t. j.}\ \ x\ \text{neexistuje}\end{matrix}\,\middle|\,\begin{matrix} x\in\langle -1;0\rangle \\ \text{t. j.}\ -1\leq x\leq 0\end{matrix}\right]$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{\frac{1+x}{1-x}} \\ v'=1\end{matrix}\,\middle|\,\begin{matrix} u'=\frac{1}{\sqrt{1-\frac{1+x}{1-x}}}\cdot\frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\cdot\frac{(1-x)-(1+x)\cdot(-1)}{(1-x)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{1-x-(1+x)}{1-x}}\sqrt{\frac{1+x}{1-x}}\sqrt{(1-x)^4}}=\frac{1}{\sqrt{\frac{-2x}{1-x}\cdot\frac{1+x}{1-x}\cdot(1-x)^4}} \\ v=x \qquad =\frac{1}{\sqrt{-2x(1+x)\cdot(1-x)^2}}=\frac{1}{\sqrt{2}\,|1-x|\,\sqrt{-x(1+x)}}=\frac{\sqrt{2}}{2(1-x)\sqrt{-x(x+1)}}=\frac{-\sqrt{2}}{2(x-1)\sqrt{-x^2-x}}\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{x}{(x-1)\sqrt{-x^2-x}}\,dx$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{matrix} 0\le\frac{1+x}{1-x}\le 1 \\ x\neq 1\end{matrix}\,\middle|\,\begin{matrix}1-x>0 \Leftrightarrow 1>x\colon\ \ 0\le 1+x\le 1-x,\ \text{t. j. } -1\le x,\ 2x\le 0,\ \text{t. j. } x\in\langle -1;0\rangle \\ 1-x<0 \Leftrightarrow 1<x\colon\ \ 0\ge 1+x\ge 1-x,\ \text{t. j. } -1\ge x,\ 2x\ge 0,\ \text{t. j. } x \text{ neexistuje}\end{matrix}\,\middle|\,\begin{matrix} x\in\langle -1;0\rangle \\ \text{t. j. } -1\le x\le 0\end{matrix}\right]$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{\frac{1+x}{1-x}} \\ v'=1\end{matrix}\,\middle|\,\begin{matrix} u'=\frac{1}{\sqrt{1-\frac{1+x}{1-x}}}\cdot\frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\cdot\frac{(1-x)-(1+x)\cdot(-1)}{(1-x)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{1-x-(1+x)}{1-x}}\sqrt{\frac{1+x}{1-x}}\sqrt{(1-x)^4}}=\frac{1}{\sqrt{\frac{-2x}{1-x}\cdot\frac{1+x}{1-x}\cdot(1-x)^4}} \\ v=x \qquad =\frac{1}{\sqrt{-2x(1+x)\cdot(1-x)^2}}=\frac{1}{\sqrt{2}\,|1-x|\,\sqrt{-x(1+x)}}=\frac{\sqrt{2}}{2(1-x)\sqrt{-x(x+1)}}=\frac{-\sqrt{2}}{2(x-1)\sqrt{-x^2-x}}\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{x}{(x-1)\sqrt{-x^2-x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{x-1+1}{(x-1)\sqrt{-x^2-x}}\,dx$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$= \left[\begin{array}{c|l|c} 0\leq\frac{1+x}{1-x}\leq 1 & 1-x>0 \Leftrightarrow 1>x:\quad 0\leq 1+x\leq 1-x,\ \text{t. j.}\ -1\leq x,\ 2x\leq 0,\ \text{t. j.}\ x\in\langle -1;0\rangle & x\in\langle -1;0) \\ x\neq 1 & 1-x<0 \Leftrightarrow 1<x:\quad 0\geq 1+x\geq 1-x,\ \text{t. j.}\ -1\geq x,\ 2x\geq 0,\ \text{t. j.}\ x \text{ neexistuje} & \text{t. j.}\ -1\leq x\leq 0 \end{array}\right]$$

$$= \left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{1+x}{1-x}} & u'=\frac{1}{\sqrt{1-\frac{1+x}{1-x}}}\cdot\frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\cdot\frac{(1-x)-(1+x)\cdot(-1)}{(1-x)^2} = \frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{1-x-(1+x)}{1-x}}\sqrt{\frac{1+x}{1-x}}\sqrt{(1-x)^4}} = \frac{1}{\sqrt{\frac{-2x}{1-x}\cdot\frac{1+x}{1-x}\cdot(1-x)^4}} \\ v'=1 & v=x \qquad = \frac{1}{\sqrt{-2x(1+x)\cdot(1-x)^2}} = \frac{1}{\sqrt{2}\,|1-x|\,\sqrt{-x(1+x)}} = \frac{\sqrt{2}}{2(1-x)\sqrt{-x(x+1)}} = \frac{-\sqrt{2}}{2(x-1)\sqrt{-x^2-x}} \end{array}\right]$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\int\frac{x}{(x-1)\sqrt{-x^2-x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\int\frac{x-1+1}{(x-1)\sqrt{-x^2-x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\int\frac{dx}{\sqrt{-x^2-x}} + \frac{\sqrt{2}}{2}\int\frac{dx}{(x-1)\sqrt{-x^2-x}}$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{array}{c|l|l} 0\leq\frac{1+x}{1-x}\leq 1 & 1-x>0 \Leftrightarrow 1>x:\quad 0\leq 1+x\leq 1-x,\ \text{t. j. } -1\leq x,\ 2x\leq 0,\ \text{t. j. } x\in\langle -1;0\rangle & x\in\langle -1;0) \\ x\neq 1 & 1-x<0 \Leftrightarrow 1<x:\quad 0\geq 1+x\geq 1-x,\ \text{t. j. } -1\geq x,\ 2x\geq 0,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } -1\leq x\leq 0 \end{array}\right]$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{1+x}{1-x}} & u'=\frac{1}{\sqrt{1-\frac{1+x}{1-x}}}\cdot\frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\cdot\frac{(1-x)-(1+x)\cdot(-1)}{(1-x)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{1-x-(1+x)}{1-x}}\sqrt{\frac{1+x}{1-x}}\sqrt{(1-x)^4}}=\frac{1}{\sqrt{\frac{-2x}{1-x}\cdot\frac{1+x}{1-x}\cdot(1-x)^4}} \\ v'=1 & v=x \qquad =\frac{1}{\sqrt{-2x(1+x)\cdot(1-x)^2}}=\frac{1}{\sqrt{2}\,|1-x|\,\sqrt{-x(1+x)}}=\frac{\sqrt{2}}{2(1-x)\sqrt{-x(x+1)}}=\frac{-\sqrt{2}}{2(x-1)\sqrt{-x^2-x}} \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{x}{(x-1)\sqrt{-x^2-x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{x-1+1}{(x-1)\sqrt{-x^2-x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{dx}{\sqrt{-x^2-x}}+\frac{\sqrt{2}}{2}\int\frac{dx}{(x-1)\sqrt{-x^2-x}}$$

$$=\left[\begin{array}{l|l|l|l|l} \text{Subst. } t=x+\frac{1}{2} & dt=dx & x\in(-1;0) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2-x=-(x^2+x)=-(x^2+2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x+\frac{1}{2})^2 \\ \text{Subst. } z=-\frac{1}{x} & x=-z^{-1} & dx=\frac{dz}{z^2} & z\in(1;\infty) & x-1=-\frac{1}{z}-1=-\frac{1+z}{z},\quad \sqrt{-x^2-x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z} \end{array}\right]$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{1+x}{1-x}} & u'=\frac{1}{\sqrt{1-\frac{1+x}{1-x}}}\cdot\frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\cdot\frac{(1-x)-(1+x)\cdot(-1)}{(1-x)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{1-x-(1+x)}{1-x}}\sqrt{\frac{1+x}{1-x}}\sqrt{(1-x)^4}}=\frac{1}{\sqrt{\frac{-2x}{1-x}\cdot\frac{1+x}{1-x}\cdot(1-x)^4}} \\ v'=1 & v=x \qquad =\frac{1}{\sqrt{-2x(1+x)\cdot(1-x)^2}}=\frac{1}{\sqrt{2}\,|1-x|\,\sqrt{-x(1+x)}}=\frac{\sqrt{2}}{2(1-x)\sqrt{-x(x+1)}}=\frac{-\sqrt{2}}{2(x-1)\sqrt{-x^2-x}} \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{x}{(x-1)\sqrt{-x^2-x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{x-1+1}{(x-1)\sqrt{-x^2-x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{dx}{\sqrt{-x^2-x}}+\frac{\sqrt{2}}{2}\int\frac{dx}{(x-1)\sqrt{-x^2-x}}$$

$$=\left[\begin{array}{l|l|l|l|l} \text{Subst. } t=x+\frac{1}{2} & dt=dx & x\in(-1;0) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2-x=-(x^2+x)=-(x^2+2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x+\frac{1}{2})^2 \\ \text{Subst. } z=-\frac{1}{x} & x=-z^{-1} & dx=\frac{dz}{z^2} & z\in(1;\infty) & x-1=-\frac{1}{z}-1=-\frac{1+z}{z},\quad \sqrt{-x^2-x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z} \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{dt}{\sqrt{\frac{1}{4}-t^2}}+\frac{\sqrt{2}}{2}\int\frac{\frac{dz}{z^2}}{-\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\int \frac{x}{(x-1)\sqrt{-x^2-x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\int \frac{x-1+1}{(x-1)\sqrt{-x^2-x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\int \frac{dx}{\sqrt{-x^2-x}} + \frac{\sqrt{2}}{2}\int \frac{dx}{(x-1)\sqrt{-x^2-x}}$$

$$= \left[\begin{array}{l|l|l|l|l} \text{Subst. } t=x+\frac{1}{2} & dt=dx & x\in(-1;0) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2-x=-(x^2+x)=-(x^2+2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x+\frac{1}{2})^2 \\ \text{Subst. } z=-\frac{1}{x} & x=-z^{-1} & dx=\frac{dz}{z^2} & z\in(1;\infty) & x-1=-\frac{1}{z}-1=-\frac{1+z}{z},\quad \sqrt{-x^2-x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z} \end{array}\right]$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\int \frac{dt}{\sqrt{\frac{1}{4}-t^2}} + \frac{\sqrt{2}}{2}\int \frac{\frac{dz}{z^2}}{-\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}} - \frac{\sqrt{2}}{2}\int \frac{dz}{(z+1)\sqrt{z-1}}$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{x}{(x-1)\sqrt{-x^2-x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{x-1+1}{(x-1)\sqrt{-x^2-x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{dx}{\sqrt{-x^2-x}}+\frac{\sqrt{2}}{2}\int\frac{dx}{(x-1)\sqrt{-x^2-x}}$$

$$=\left[\begin{array}{l|l|l|l|l}\text{Subst. } t=x+\frac{1}{2} & dt=dx & x\in(-1;0) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2-x=-(x^2+x)=-(x^2+2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x+\frac{1}{2})^2 \\ \text{Subst. } z=-\frac{1}{x} & x=-z^{-1} & dx=\frac{dz}{z^2} & z\in(1;\infty) & x-1=-\frac{1}{z}-1=-\frac{1+z}{z},\quad \sqrt{-x^2-x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z}\end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{dt}{\sqrt{\frac{1}{4}-t^2}}+\frac{\sqrt{2}}{2}\int\frac{\frac{dz}{z^2}}{-\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}}-\frac{\sqrt{2}}{2}\int\frac{dz}{(z+1)\sqrt{z-1}}=\left[\begin{array}{r|l}\text{Subst. } u=\sqrt{z-1} & u^2=z-1 \\ z=u^2+1 & dz=2u\,du \\ z\in(1;\infty) & u\in(0;\infty)\end{array}\right]$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\int \frac{x-1+1}{(x-1)\sqrt{-x^2-x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\int \frac{dx}{\sqrt{-x^2-x}} + \frac{\sqrt{2}}{2}\int \frac{dx}{(x-1)\sqrt{-x^2-x}}$$

$$= \left[\begin{array}{l|l|l|l|l} \text{Subst. } t=x+\frac{1}{2} & dt=dx & x\in(-1;0) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2-x=-(x^2+x)=-(x^2+2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x+\frac{1}{2})^2 \\ \text{Subst. } z=-\frac{1}{x} & x=-z^{-1} & dx=\frac{dz}{z^2} & z\in(1;\infty) & x-1=-\frac{1}{z}-1=-\frac{1+z}{z},\quad \sqrt{-x^2-x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z} \end{array}\right]$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\int \frac{dt}{\sqrt{\frac{1}{4}-t^2}} + \frac{\sqrt{2}}{2}\int \frac{\frac{dz}{z^2}}{-\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}} - \frac{\sqrt{2}}{2}\int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{r|l} \text{Subst. } u=\sqrt{z-1} & u^2=z-1 \\ z=u^2+1 & dz=2u\,du \\ z\in(1;\infty) & u\in(0;\infty) \end{array}\right]$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\arcsin 2t - \frac{\sqrt{2}}{2}\int \frac{2u\,du}{(u^2+2)u}$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{dx}{\sqrt{-x^2-x}}+\frac{\sqrt{2}}{2}\int\frac{dx}{(x-1)\sqrt{-x^2-x}}$$

$$=\left[\begin{array}{l|l|l|l|l} \text{Subst. } t=x+\frac{1}{2} & dt=dx & x\in(-1;0) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2-x=-(x^2+x)=-(x^2+2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x+\frac{1}{2})^2 \\ \text{Subst. } z=-\frac{1}{x} & x=-z^{-1} & dx=\frac{dz}{z^2} & z\in(1;\infty) & x-1=-\frac{1}{z}-1=-\frac{1+z}{z},\quad \sqrt{-x^2-x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z} \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{dt}{\sqrt{\frac{1}{4}-t^2}}+\frac{\sqrt{2}}{2}\int\frac{\frac{dz}{z^2}}{-\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}}-\frac{\sqrt{2}}{2}\int\frac{dz}{(z+1)\sqrt{z-1}}=\left[\begin{array}{r|l} \text{Subst. } u=\sqrt{z-1} & u^2=z-1 \\ z=u^2+1 & dz=2u\,du \\ z\in(1;\infty) & u\in(0;\infty) \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\arcsin 2t-\frac{\sqrt{2}}{2}\int\frac{2u\,du}{(u^2+2)u}$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\arcsin 2t-\sqrt{2}\int\frac{du}{u^2+2}$$

# Riešené príklady – 172

$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$

$$=\left[\begin{array}{l|l|l|l|l} \text{Subst. } t=x+\frac{1}{2} & dt=dx & x\in(-1;0) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2-x=-(x^2+x)=-(x^2+2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x+\frac{1}{2})^2 \\ \text{Subst. } z=-\frac{1}{x} & x=-z^{-1} & dx=\frac{dz}{z^2} & z\in(1;\infty) & x-1=-\frac{1}{z}-1=-\frac{1+z}{z},\quad \sqrt{-x^2-x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z} \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\int\frac{dt}{\sqrt{\frac{1}{4}-t^2}}+\frac{\sqrt{2}}{2}\int\frac{\frac{dz}{z^2}}{-\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}}-\frac{\sqrt{2}}{2}\int\frac{dz}{(z+1)\sqrt{z-1}}=\left[\begin{array}{r|l} \text{Subst. } u=\sqrt{z-1} & u^2=z-1 \\ z=u^2+1 & dz=2u\,du \\ z\in(1;\infty) & u\in(0;\infty) \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\arcsin 2t-\frac{\sqrt{2}}{2}\int\frac{2u\,du}{(u^2+2)u}$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\arcsin 2t-\sqrt{2}\int\frac{du}{u^2+2}$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt{2}}{2}\arcsin 2t-\frac{\sqrt{2}}{\sqrt{2}}\operatorname{arctg}\frac{u}{\sqrt{2}}+c_1$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\tfrac{1+x}{1-x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\int\frac{dt}{\sqrt{\frac{1}{4}-t^2}} + \frac{\sqrt{2}}{2}\int\frac{\frac{dz}{z^2}}{-\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}} - \frac{\sqrt{2}}{2}\int\frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{r|l} \text{Subst. } u=\sqrt{z-1} & u^2=z-1 \\ z=u^2+1 & dz=2u\,du \\ z\in(1;\infty) & u\in(0;\infty) \end{array}\right]$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\arcsin 2t - \frac{\sqrt{2}}{2}\int\frac{2u\,du}{(u^2+2)u}$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\arcsin 2t - \sqrt{2}\int\frac{du}{u^2+2}$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\arcsin 2t - \frac{\sqrt{2}}{\sqrt{2}}\operatorname{arctg}\frac{u}{\sqrt{2}} + c_1$$

$$= x\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\arcsin(2x+1) - \operatorname{arctg}\frac{\sqrt{z-1}}{\sqrt{2}} + c_1$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt2}{2}\int\frac{dt}{\sqrt{\frac14-t^2}}+\frac{\sqrt2}{2}\int\frac{\frac{dz}{z^2}}{-\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt2}{2}\arcsin\frac{t}{\frac12}-\frac{\sqrt2}{2}\int\frac{dz}{(z+1)\sqrt{z-1}}=\left[\begin{array}{r|l}\text{Subst. } u=\sqrt{z-1} & u^2=z-1\\ z=u^2+1 & dz=2u\,du\\ z\in(1;\infty) & u\in(0;\infty)\end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt2}{2}\arcsin 2t-\frac{\sqrt2}{2}\int\frac{2u\,du}{(u^2+2)u}$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt2}{2}\arcsin 2t-\sqrt2\int\frac{du}{u^2+2}$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt2}{2}\arcsin 2t-\frac{\sqrt2}{\sqrt2}\operatorname{arctg}\frac{u}{\sqrt2}+c_1$$

$$=x\arcsin\sqrt{\frac{1+x}{1-x}}+\frac{\sqrt2}{2}\arcsin(2x+1)-\operatorname{arctg}\frac{\sqrt{z-1}}{\sqrt2}+c_1=\left[\begin{array}{l}\frac{\sqrt{z-1}}{\sqrt2}=\frac{1}{\sqrt2}\sqrt{-\frac1x-1}\\ \quad=\frac{1}{\sqrt2}\sqrt{-\frac{1-x}{x}}=\sqrt{\frac{-1-x}{2x}}\end{array}\right]$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\tfrac{1+x}{1-x}}\,dx = x\arcsin\sqrt{\tfrac{1+x}{1-x}} + \tfrac{\sqrt{2}}{2}\arcsin(2x+1) - \operatorname{arctg}\sqrt{\tfrac{-1-x}{2x}} + c_1$$

$$= x\arcsin\sqrt{\tfrac{1+x}{1-x}} + \tfrac{\sqrt{2}}{2}\arcsin\,\tfrac{t}{\frac{1}{2}} - \tfrac{\sqrt{2}}{2}\int\frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{r|l} \text{Subst. } u=\sqrt{z-1} & u^2=z-1 \\ z=u^2+1 & dz=2u\,du \\ z\in(1;\infty) & u\in(0;\infty) \end{array}\right]$$

$$= x\arcsin\sqrt{\tfrac{1+x}{1-x}} + \tfrac{\sqrt{2}}{2}\arcsin 2t - \tfrac{\sqrt{2}}{2}\int\frac{2u\,du}{(u^2+2)u}$$

$$= x\arcsin\sqrt{\tfrac{1+x}{1-x}} + \tfrac{\sqrt{2}}{2}\arcsin 2t - \sqrt{2}\int\frac{du}{u^2+2}$$

$$= x\arcsin\sqrt{\tfrac{1+x}{1-x}} + \tfrac{\sqrt{2}}{2}\arcsin 2t - \tfrac{\sqrt{2}}{\sqrt{2}}\operatorname{arctg}\tfrac{u}{\sqrt{2}} + c_1$$

$$= x\arcsin\sqrt{\tfrac{1+x}{1-x}} + \tfrac{\sqrt{2}}{2}\arcsin(2x+1) - \operatorname{arctg}\tfrac{\sqrt{z-1}}{\sqrt{2}} + c_1 = \left[\begin{array}{l} \frac{\sqrt{z-1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\sqrt{-\frac{1}{x}-1} \\ \quad = \frac{1}{\sqrt{2}}\sqrt{-\frac{1-x}{x}} = \sqrt{\frac{-1-x}{2x}} \end{array}\right]$$

$$= x\arcsin\sqrt{\tfrac{1+x}{1-x}} + \tfrac{\sqrt{2}}{2}\arcsin(2x+1) - \operatorname{arctg}\sqrt{-\tfrac{1+x}{2x}} + c_1,\ x\in(-1;0),\ c\in R.$$

# Riešené príklady – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}}\, dx$$

# Riešené príklady – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}}\, dx$$

$$= \left[ \begin{matrix} 0 \le \frac{1+x}{1-x} \le 1 \\ x \ne 1 \end{matrix} \;\middle|\; \begin{matrix} 1-x>0 \Leftrightarrow 1>x\colon \quad 0 \le 1+x \le 1-x, \text{ t. j. } -1 \le x,\ 2x \le 0, \text{ t. j. } x \in \langle -1; 0 \rangle \\ 1-x<0 \Leftrightarrow 1<x\colon \quad 0 \ge 1+x \ge 1-x, \text{ t. j. } -1 \ge x,\ 2x \ge 0, \text{ t. j. } x \text{ neexistuje} \end{matrix} \;\middle|\; \begin{matrix} x \in \langle -1; 0 \rangle \\ \text{t. j. } -1 \le x \le 0 \end{matrix} \right]$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{matrix}0\leq\frac{1+x}{1-x}\leq 1\\ x\neq 1\end{matrix}\;\middle|\;\begin{matrix}1-x>0\Leftrightarrow 1>x:\quad 0\leq 1+x\leq 1-x,\ \text{t. j.}\ -1\leq x,\ 2x\leq 0,\ \text{t. j.}\ x\in\langle -1;0)\\ 1-x<0\Leftrightarrow 1<x:\quad 0\geq 1+x\geq 1-x,\ \text{t. j.}\ -1\geq x,\ 2x\geq 0,\ \text{t. j.}\ x\ \text{neexistuje}\end{matrix}\;\middle|\;\begin{matrix}x\in\langle -1;0)\\ \text{t. j.}\ -1\leq x\leq 0\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst.}\quad t=1-x\\ dt=-\,dx\end{matrix}\;\middle|\;\begin{matrix}x\in\langle -1;0)\\ t\in\langle 1;2)\end{matrix}\right]$$

# Riešené príklady – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}}\, dx$$

$$= \left[ \begin{array}{c|c|c} 0 \le \frac{1+x}{1-x} \le 1 & 1-x>0 \Leftrightarrow 1>x: \quad 0 \le 1+x \le 1-x, \text{ t. j. } -1 \le x,\ 2x \le 0, \text{ t. j. } x \in \langle -1; 0) & x \in \langle -1; 0) \\ x \neq 1 & 1-x<0 \Leftrightarrow 1<x: \quad 0 \ge 1+x \ge 1-x, \text{ t. j. } -1 \ge x,\ 2x \ge 0, \text{ t. j. } x \text{ neexistuje} & \text{t. j. } -1 \le x \le 0 \end{array} \right]$$

$$= \left[ \begin{array}{c|c} \text{Subst. } t=1-x & x \in \langle -1; 0) \\ dt = -dx & t \in (1; 2\rangle \end{array} \right] = -\int \arcsin \sqrt{\frac{2-t}{t}}\, dt = -\int \arcsin \sqrt{\frac{2}{t} - 1}\, dt$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{array}{c|c|c} 0\leq\frac{1+x}{1-x}\leq 1 & 1-x>0\Leftrightarrow 1>x:\quad 0\leq 1+x\leq 1-x,\ \text{t. j.}\ -1\leq x,\ 2x\leq 0,\ \text{t. j.}\ x\in\langle -1;0) & x\in\langle -1;0) \\ x\neq 1 & 1-x<0\Leftrightarrow 1<x:\quad 0\geq 1+x\geq 1-x,\ \text{t. j.}\ -1\geq x,\ 2x\geq 0,\ \text{t. j.}\ x\ \text{neexistuje} & \text{t. j.}\ -1\leq x\leq 0 \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst.}\quad t=1-x & x\in\langle -1;0) \\ dt=-dx & t\in(1;2\rangle \end{array}\right]=-\int \arcsin\sqrt{\frac{2-t}{t}}\,dt=-\int \arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}}=\frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad =\frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}}=\frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}}=\frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}}=\frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array}\right]$$

# Riešené príklady – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}}\, dx$$

$$= \left[ \begin{array}{c|l|l} 0 \le \frac{1+x}{1-x} \le 1 & 1-x>0 \Leftrightarrow 1>x:\quad 0\le 1+x \le 1-x,\ \text{t. j.}\ -1\le x,\ 2x\le 0,\ \text{t. j.}\ x\in\langle -1;0) & x\in\langle -1;0) \\ x\neq 1 & 1-x<0 \Leftrightarrow 1<x:\quad 0\ge 1+x\ge 1-x,\ \text{t. j.}\ -1\ge x,\ 2x\ge 0,\ \text{t. j.}\ x \text{ neexistuje} & \text{t. j.}\ -1\le x\le 0 \end{array} \right]$$

$$= \left[ \begin{array}{c|c} \text{Subst.}\quad t=1-x & x\in\langle -1;0) \\ dt=-dx & t\in\langle 1;2\rangle \end{array} \right] = -\int \arcsin \sqrt{\frac{2-t}{t}}\, dt = -\int \arcsin \sqrt{\frac{2}{t}-1}\, dt$$

$$= \left[ \begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}} = \frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad = \frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}} = \frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array} \right]$$

$$= -\left[ t\arcsin\sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2}\int \frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \right]$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{array}{c|l|l} 0\leq\frac{1+x}{1-x}\leq 1 & 1-x>0 \Leftrightarrow 1>x:\quad 0\leq 1+x\leq 1-x,\ \text{t. j.}\ -1\leq x,\ 2x\leq 0,\ \text{t. j.}\ x\in\langle -1;0\rangle & x\in\langle -1;0\rangle \\ x\neq 1 & 1-x<0 \Leftrightarrow 1<x:\quad 0\geq 1+x\geq 1-x,\ \text{t. j.}\ -1\geq x,\ 2x\geq 0,\ \text{t. j.}\ x \text{ neexistuje} & \text{t. j.}\ -1\leq x\leq 0 \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst.}\quad t=1-x & x\in\langle -1;0\rangle \\ dt=-dx & t\in\langle 1;2\rangle \end{array}\right]=-\int \arcsin\sqrt{\frac{2-t}{t}}\,dt=-\int \arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}}=\frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad =\frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}}=\frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}}=\frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}}=\frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array}\right]$$

$$=-\left[t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}}\right]=\left[\begin{array}{r|l} \text{Subst.}\quad z=t-\frac{3}{2} & t\in(1;2) \\ dz=dt & z\in(-\frac{1}{2};\frac{1}{2}) \end{array}\right]$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$= \left[ \begin{array}{c|l|l} 0 \leq \frac{1+x}{1-x} \leq 1 & 1-x>0 \Leftrightarrow 1>x:\quad 0\leq 1+x \leq 1-x,\ \text{t. j.}\ -1\leq x,\ 2x\leq 0,\ \text{t. j.}\ x\in\langle -1;0) & x\in\langle -1;0) \\ x\neq 1 & 1-x<0 \Leftrightarrow 1<x:\quad 0\geq 1+x \geq 1-x,\ \text{t. j.}\ -1\geq x,\ 2x\geq 0,\ \text{t. j.}\ x \text{ neexistuje} & \text{t. j.}\ -1\leq x\leq 0 \end{array} \right]$$

$$= \left[ \begin{array}{r|l} \text{Subst.}\quad t=1-x & x\in\langle -1;0) \\ dt=-dx & t\in\langle 1;2) \end{array} \right] = -\int \arcsin\sqrt{\frac{2-t}{t}}\,dt = -\int \arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$= \left[ \begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}} = \frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad = \frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}} = \frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array} \right]$$

$$= -\left[ t\arcsin\sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2}\int \frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \right] = \left[ \begin{array}{r|l} \text{Subst.}\quad z=t-\frac{3}{2} & t\in(1;2) \\ dz=dt & z\in(-\frac{1}{2};\frac{1}{2}) \end{array} \right]$$

$$= -t\arcsin\sqrt{\frac{2-t}{t}} - \frac{\sqrt{2}}{2}\int \frac{dz}{\sqrt{(\frac{1}{2})^2-z^2}}$$

# Riešené príklady – 172

$$\int \arcsin \sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{array}{c|l|l} 0\leq\frac{1+x}{1-x}\leq 1 & 1-x>0 \Leftrightarrow 1>x:\quad 0\leq 1+x\leq 1-x,\ \text{t. j.}\ -1\leq x,\ 2x\leq 0,\ \text{t. j.}\ x\in\langle -1;0) & x\in\langle -1;0) \\ x\neq 1 & 1-x<0 \Leftrightarrow 1<x:\quad 0\geq 1+x\geq 1-x,\ \text{t. j.}\ -1\geq x,\ 2x\geq 0,\ \text{t. j.}\ x \text{ neexistuje} & \text{t. j.}\ -1\leq x\leq 0 \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst.}\quad t=1-x & x\in\langle -1;0\rangle \\ dt=-dx & t\in\langle 1;2\rangle \end{array}\right] = -\int \arcsin\sqrt{\frac{2-t}{t}}\,dt = -\int \arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}} = \frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad = \frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}} = \frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array}\right]$$

$$=-\left[t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}}\right] = \left[\begin{array}{r|l} \text{Subst.}\quad z=t-\frac{3}{2} & t\in(1;2) \\ dz=dt & z\in(-\frac{1}{2};\frac{1}{2}) \end{array}\right]$$

$$=-t\arcsin\sqrt{\frac{2-t}{t}}-\frac{\sqrt{2}}{2}\int\frac{dz}{\sqrt{(\frac{1}{2})^2-z^2}} = -t\arcsin\sqrt{\frac{2-t}{t}}-\frac{\sqrt{2}}{2}\arcsin\frac{z}{\frac{1}{2}}+c_2$$

beerb@frcatel.fri.uniza.sk http://frcatel.fri.uniza.sk/~beerb

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{array}{c|l|l} 0\leq\frac{1+x}{1-x}\leq 1 & 1-x>0 \Leftrightarrow 1>x:\quad 0\leq 1+x\leq 1-x,\ \text{t. j. } -1\leq x,\ 2x\leq 0,\ \text{t. j. } x\in\langle -1;0) & x\in\langle -1;0) \\ x\neq 1 & 1-x<0 \Leftrightarrow 1<x:\quad 0\geq 1+x\geq 1-x,\ \text{t. j. } -1\geq x,\ 2x\geq 0,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } -1\leq x\leq 0 \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst.}\quad t=1-x & x\in\langle -1;0) \\ dt=-dx & t\in\langle 1;2) \end{array}\right]=-\int \arcsin\sqrt{\frac{2-t}{t}}\,dt=-\int \arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}}=\frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad =\frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}}=\frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}}=\frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}}=\frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array}\right]$$

$$=-\left[t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}}\right]=\left[\begin{array}{r|l} \text{Subst.}\quad z=t-\frac{3}{2} & t\in(1;2) \\ dz=dt & z\in(-\frac{1}{2};\frac{1}{2}) \end{array}\right]$$

$$=-t\arcsin\sqrt{\frac{2-t}{t}}-\frac{\sqrt{2}}{2}\int\frac{dz}{\sqrt{(\frac{1}{2})^2-z^2}}=-t\arcsin\sqrt{\frac{2-t}{t}}-\frac{\sqrt{2}}{2}\arcsin\frac{z}{\frac{1}{2}}+c_2$$

$$=-(1-x)\arcsin\sqrt{\frac{1+x}{1-x}}-\frac{\sqrt{2}}{2}\arcsin 2z+c_2$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx$$

$$=\left[\begin{array}{c|l|l} 0\leq\frac{1+x}{1-x}\leq 1 & 1-x>0 \Leftrightarrow 1>x:\quad 0\leq 1+x\leq 1-x,\ \text{t. j. } -1\leq x,\ 2x\leq 0,\ \text{t. j. } x\in\langle -1;0) & x\in\langle -1;0) \\ x\neq 1 & 1-x<0 \Leftrightarrow 1<x:\quad 0\geq 1+x\geq 1-x,\ \text{t. j. } -1\geq x,\ 2x\geq 0,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } -1\leq x\leq 0 \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst.}\quad t=1-x & x\in\langle -1;0) \\ dt=-\,dx & t\in\langle 1;2) \end{array}\right]=-\int\arcsin\sqrt{\frac{2-t}{t}}\,dt=-\int\arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}}=\frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad =\frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}}=\frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}}=\frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}}=\frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array}\right]$$

$$=-\left[t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}}\right]=\left[\begin{array}{r|l} \text{Subst.}\quad z=t-\frac{3}{2} & t\in(1;2) \\ dz=dt & z\in(-\frac{1}{2};\frac{1}{2}) \end{array}\right]$$

$$=-t\arcsin\sqrt{\frac{2-t}{t}}-\frac{\sqrt{2}}{2}\int\frac{dz}{\sqrt{(\frac{1}{2})^2-z^2}}=-t\arcsin\sqrt{\frac{2-t}{t}}-\frac{\sqrt{2}}{2}\arcsin\frac{z}{\frac{1}{2}}+c_2$$

$$=-(1-x)\arcsin\sqrt{\frac{1+x}{1-x}}-\frac{\sqrt{2}}{2}\arcsin 2z+c_2=\left[\begin{array}{l} 2z=2(t-\frac{3}{2})=2(-x-\frac{1}{2}) \\ \quad=-2x-1=-(2x+1) \end{array}\right]$$

# Riešené príklady – 172

$$\int \arcsin\sqrt{\frac{1+x}{1-x}}\,dx = (x-1)\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\arcsin(2x+1) + c_2$$

$$= \left[\begin{array}{c|l|l} 0 \le \frac{1+x}{1-x} \le 1 & 1-x>0 \Leftrightarrow 1>x:\ \ 0\le 1+x \le 1-x,\ \text{t. j.}\ -1\le x,\ 2x\le 0,\ \text{t. j.}\ x\in\langle -1;0\rangle & x\in\langle -1;0\rangle \\ x\ne 1 & 1-x<0 \Leftrightarrow 1<x:\ \ 0\ge 1+x \ge 1-x,\ \text{t. j.}\ -1\ge x,\ 2x\ge 0,\ \text{t. j.}\ x\ \text{neexistuje} & \text{t. j.}\ -1\le x\le 0 \end{array}\right]$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t=1-x & x\in\langle -1;0\rangle \\ dt=-dx & t\in\langle 1;2\rangle \end{array}\right] = -\int \arcsin\sqrt{\frac{2-t}{t}}\,dt = -\int \arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$= \left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}} = \frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad = \frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}} = \frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array}\right]$$

$$= -\left[t\arcsin\sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}}\right] = \left[\begin{array}{r|l} \text{Subst.}\quad z=t-\frac{3}{2} & t\in(1;2) \\ dz=dt & z\in(-\frac{1}{2};\frac{1}{2}) \end{array}\right]$$

$$= -t\arcsin\sqrt{\frac{2-t}{t}} - \frac{\sqrt{2}}{2}\int\frac{dz}{\sqrt{(\frac{1}{2})^2-z^2}} = -t\arcsin\sqrt{\frac{2-t}{t}} - \frac{\sqrt{2}}{2}\arcsin\frac{z}{\frac{1}{2}} + c_2$$

$$= -(1-x)\arcsin\sqrt{\frac{1+x}{1-x}} - \frac{\sqrt{2}}{2}\arcsin 2z + c_2 = \left[\begin{array}{l} 2z=2(t-\frac{3}{2})=2(-x-\frac{1}{2}) \\ \quad =-2x-1=-(2x+1) \end{array}\right]$$

$$= (x-1)\arcsin\sqrt{\frac{1+x}{1-x}} + \frac{\sqrt{2}}{2}\arcsin(2x+1) + c_2,\ x\in(-1;0),\ c\in R.$$

# Riešené príklady – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}}\, dx$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{1-x}{1+x}\leq 1 \\ x\neq 1\end{matrix}\,\middle|\,\begin{matrix} 1+x>0 \Leftrightarrow x>-1\text{:}\quad 0\leq 1-x\leq 1+x,\ \text{t. j.}\ \ x\leq 1,\ 0\leq 2x,\ \text{t. j.}\ \ x\in\langle 0;1\rangle \\ 1+x<0 \Leftrightarrow x<-1\text{:}\quad 0\geq 1-x\geq 1+x,\ \text{t. j.}\ \ x\geq 1,\ 0\geq 2x,\ \text{t. j.}\ \ x\ \text{neexistuje}\end{matrix}\,\middle|\,\begin{matrix} x\in\langle 0;1\rangle \\ \text{t. j.}\ \ 0\leq x\leq 1\end{matrix}\right]$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{array}{c|l|l} 0\leq\frac{1-x}{1+x}\leq 1 & 1+x>0 \Leftrightarrow x>-1:\ \ 0\leq 1-x\leq 1+x,\ \text{t. j. } x\leq 1,\ 0\leq 2x,\ \text{t. j. } x\in\langle 0;1\rangle & x\in\langle 0;1\rangle \\ x\neq 1 & 1+x<0 \Leftrightarrow x<-1:\ \ 0\geq 1-x\geq 1+x,\ \text{t. j. } x\geq 1,\ 0\geq 2x,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 0\leq x\leq 1 \end{array}\right]$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{1-x}{1+x}} & u'=\frac{1}{\sqrt{1-\frac{1-x}{1+x}}}\cdot\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}}\cdot\frac{-(1+x)-(1-x)}{(1+x)^2}=\frac{-\frac{1}{2}\cdot 2}{\sqrt{\frac{1+x-(1-x)}{1+x}}\sqrt{\frac{1-x}{1+x}}\sqrt{(1+x)^4}}=\frac{-1}{\sqrt{\frac{2x}{1+x}\cdot\frac{1-x}{1+x}\cdot(1+x)^4}} \\ v'=1 & v=x \qquad =\frac{-1}{\sqrt{2x(1-x)\cdot(1+x)^2}}=\frac{-1}{\sqrt{2}\,|1+x|\,\sqrt{x(1-x)}}=\frac{-\sqrt{2}}{2(1+x)\sqrt{x(1-x)}}=\frac{-\sqrt{2}}{2(x+1)\sqrt{-x^2+x}} \end{array}\right]$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{1-x}{1+x}\leq 1 \\ x\neq 1\end{matrix}\,\middle|\,\begin{matrix} 1+x>0\Leftrightarrow x>-1\colon\ 0\leq 1-x\leq 1+x,\ \text{t. j. } x\leq 1,\, 0\leq 2x,\ \text{t. j. } x\in\langle 0;1\rangle \\ 1+x<0\Leftrightarrow x<-1\colon\ 0\geq 1-x\geq 1+x,\ \text{t. j. } x\geq 1,\, 0\geq 2x,\ \text{t. j. } x \text{ neexistuje}\end{matrix}\,\middle|\,\begin{matrix} x\in\langle 0;1\rangle \\ \text{t. j. } 0\leq x\leq 1\end{matrix}\right]$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{\frac{1-x}{1+x}} \\ v'=1\end{matrix}\,\middle|\,\begin{matrix} u'=\frac{1}{\sqrt{1-\frac{1-x}{1+x}}}\cdot\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}}\cdot\frac{-(1+x)-(1-x)}{(1+x)^2}=\frac{-\frac{1}{2}\cdot 2}{\sqrt{\frac{1+x-(1-x)}{1+x}}\sqrt{\frac{1-x}{1+x}}\sqrt{(1+x)^4}}=\frac{-1}{\sqrt{\frac{2x}{1+x}\cdot\frac{1-x}{1+x}\cdot(1+x)^4}} \\ v=x \qquad =\frac{-1}{\sqrt{2x(1-x)\cdot(1+x)^2}}=\frac{-1}{\sqrt{2}\,|1+x|\,\sqrt{x(1-x)}}=\frac{-\sqrt{2}}{2(1+x)\sqrt{x(1-x)}}=\frac{-\sqrt{2}}{2(x+1)\sqrt{-x^2+x}}\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{x}{(x+1)\sqrt{-x^2+x}}\,dx$$

# Riešené príklady – 173

## $\int \arcsin \sqrt{\frac{1-x}{1+x}}\, dx$

$$= \left[ \begin{array}{c|l|l} 0 \le \frac{1-x}{1+x} \le 1 & 1+x>0 \Leftrightarrow x>-1:\quad 0\le 1-x\le 1+x,\ \text{ t. j. }\ x\le 1,\ 0\le 2x,\ \text{ t. j. }\ x\in\langle 0;1\rangle & x\in\langle 0;1\rangle \\ x\neq 1 & 1+x<0 \Leftrightarrow x<-1:\quad 0\ge 1-x\ge 1+x,\ \text{ t. j. }\ x\ge 1,\ 0\ge 2x,\ \text{ t. j. }\ x \text{ neexistuje} & \text{ t. j. }\ 0\le x\le 1 \end{array} \right]$$

$$= \left[ \begin{array}{l|l} u=\arcsin\sqrt{\frac{1-x}{1+x}} & u'=\frac{1}{\sqrt{1-\frac{1-x}{1+x}}}\cdot\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}}\cdot\frac{-(1+x)-(1-x)}{(1+x)^2}=\frac{-\frac{1}{2}\cdot 2}{\sqrt{\frac{1+x-(1-x)}{1+x}}\sqrt{\frac{1-x}{1+x}}\sqrt{(1+x)^4}}=\frac{-1}{\sqrt{\frac{2x}{1+x}\cdot\frac{1-x}{1+x}\cdot(1+x)^4}} \\ v'=1 & v=x \qquad =\frac{-1}{\sqrt{2x(1-x)\cdot(1+x)^2}}=\frac{-1}{\sqrt{2}\,|1+x|\,\sqrt{x(1-x)}}=\frac{-\sqrt{2}}{2(1+x)\sqrt{x(1-x)}}=\frac{-\sqrt{2}}{2(x+1)\sqrt{-x^2+x}} \end{array} \right]$$

$$= x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{x}{(x+1)\sqrt{-x^2+x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{x+1-1}{(x+1)\sqrt{-x^2+x}}\,dx$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{1-x}{1+x}\leq 1 \\ x\neq 1\end{matrix}\middle| \begin{matrix} 1+x>0 \Leftrightarrow x>-1\colon\;\; 0\leq 1-x\leq 1+x,\;\text{t. j. } x\leq 1,\, 0\leq 2x,\;\text{t. j. } x\in\langle 0;1\rangle \\ 1+x<0 \Leftrightarrow x<-1\colon\;\; 0\geq 1-x\geq 1+x,\;\text{t. j. } x\geq 1,\, 0\geq 2x,\;\text{t. j. } x \text{ neexistuje}\end{matrix}\middle| \begin{matrix} x\in\langle 0;1\rangle \\ \text{t. j. } 0\leq x\leq 1\end{matrix}\right]$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{\frac{1-x}{1+x}} \\ v'=1\end{matrix}\middle| \begin{matrix} u'=\frac{1}{\sqrt{1-\frac{1-x}{1+x}}}\cdot\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}}\cdot\frac{-(1+x)-(1-x)}{(1+x)^2}=\frac{-\frac{1}{2}\cdot 2}{\sqrt{\frac{1+x-(1-x)}{1+x}}\sqrt{\frac{1-x}{1+x}}\sqrt{(1+x)^4}}=\frac{-1}{\sqrt{\frac{2x}{1+x}\cdot\frac{1-x}{1+x}\cdot(1+x)^4}} \\ v=x \qquad =\frac{-1}{\sqrt{2x(1-x)\cdot(1+x)^2}}=\frac{-1}{\sqrt{2}\,|1+x|\,\sqrt{x(1-x)}}=\frac{-\sqrt{2}}{2(1+x)\sqrt{x(1-x)}}=\frac{-\sqrt{2}}{2(x+1)\sqrt{-x^2+x}}\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{x}{(x+1)\sqrt{-x^2+x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{x+1-1}{(x+1)\sqrt{-x^2+x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{dx}{\sqrt{-x^2+x}}-\frac{\sqrt{2}}{2}\int\frac{dx}{(x+1)\sqrt{-x^2+x}}$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{1-x}{1+x}\leq 1 \\ x\neq 1\end{matrix}\left|\begin{matrix} 1+x>0 \Leftrightarrow x>-1\colon\ \ 0\leq 1-x\leq 1+x,\ \text{t. j.}\ \ x\leq 1,\ 0\leq 2x,\ \text{t. j.}\ \ x\in\langle 0;1\rangle \\ 1+x<0 \Leftrightarrow x<-1\colon\ \ 0\geq 1-x\geq 1+x,\ \text{t. j.}\ \ x\geq 1,\ 0\geq 2x,\ \text{t. j.}\ \ x \text{ neexistuje}\end{matrix}\right|\begin{matrix} x\in\langle 0;1\rangle \\ \text{t. j.}\ \ 0\leq x\leq 1\end{matrix}\right]$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{\frac{1-x}{1+x}} \\ v'=1\end{matrix}\left|\begin{matrix} u'=\frac{1}{\sqrt{1-\frac{1-x}{1+x}}}\cdot\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}}\cdot\frac{-(1+x)-(1-x)}{(1+x)^2}=\frac{-\frac{1}{2}\cdot 2}{\sqrt{\frac{1+x-(1-x)}{1+x}}\sqrt{\frac{1-x}{1+x}}\sqrt{(1+x)^4}}=\frac{-1}{\sqrt{\frac{2x}{1+x}\cdot\frac{1-x}{1+x}\cdot(1+x)^4}} \\ v=x \qquad =\frac{-1}{\sqrt{2x(1-x)\cdot(1+x)^2}}=\frac{-1}{\sqrt{2}\,|1+x|\,\sqrt{x(1-x)}}=\frac{-\sqrt{2}}{2(1+x)\sqrt{x(1-x)}}=\frac{-\sqrt{2}}{2(x+1)\sqrt{-x^2+x}}\end{matrix}\right.\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{x}{(x+1)\sqrt{-x^2+x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{x+1-1}{(x+1)\sqrt{-x^2+x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{dx}{\sqrt{-x^2+x}}-\frac{\sqrt{2}}{2}\int\frac{dx}{(x+1)\sqrt{-x^2+x}}$$

$$=\left[\begin{matrix} \text{Subst. } t=x-\frac{1}{2} \\ \text{Subst. } z=\frac{1}{x}\end{matrix}\left|\begin{matrix} dt=dx \\ x=z^{-1}\end{matrix}\right|\begin{matrix} x\in(0;1) \\ dx=-\frac{dz}{z^2}\end{matrix}\left|\begin{matrix} t\in(-\frac{1}{2};\frac{1}{2}) \\ z\in(1;\infty)\end{matrix}\right|\begin{matrix} -x^2+x=-(x^2-x)=-(x^2-2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x-\frac{1}{2})^2 \\ x+1=\frac{1}{z}+1=\frac{1+z}{z},\quad \sqrt{-x^2+x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z}\end{matrix}\right]$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{1-x}{1+x}} & u'=\frac{1}{\sqrt{1-\frac{1-x}{1+x}}}\cdot\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}}\cdot\frac{-(1+x)-(1-x)}{(1+x)^2}=\frac{-\frac{1}{2}\cdot 2}{\sqrt{\frac{1+x-(1-x)}{1+x}}\sqrt{\frac{1-x}{1+x}}\sqrt{(1+x)^4}}=\frac{-1}{\sqrt{\frac{2x}{1+x}\cdot\frac{1-x}{1+x}\cdot(1+x)^4}} \\ v'=1 & v=x \qquad =\frac{-1}{\sqrt{2x(1-x)\cdot(1+x)^2}}=\frac{-1}{\sqrt{2}\,|1+x|\,\sqrt{x(1-x)}}=\frac{-\sqrt{2}}{2(1+x)\sqrt{x(1-x)}}=\frac{-\sqrt{2}}{2(x+1)\sqrt{-x^2+x}} \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{x}{(x+1)\sqrt{-x^2+x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{x+1-1}{(x+1)\sqrt{-x^2+x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{dx}{\sqrt{-x^2+x}}-\frac{\sqrt{2}}{2}\int\frac{dx}{(x+1)\sqrt{-x^2+x}}$$

$$=\left[\begin{array}{l|l|l|l|l} \text{Subst. } t=x-\frac{1}{2} & dt=dx & x\in(0;1) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2+x=-(x^2-x)=-(x^2-2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x-\frac{1}{2})^2 \\ \text{Subst. } z=\frac{1}{x} & x=z^{-1} & dx=-\frac{dz}{z^2} & z\in(1;\infty) & x+1=\frac{1}{z}+1=\frac{1+z}{z},\quad \sqrt{-x^2+x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z} \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{dt}{\sqrt{\frac{1}{4}-t^2}}-\frac{\sqrt{2}}{2}\int\frac{-\frac{dz}{z^2}}{\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\int \frac{x}{(x+1)\sqrt{-x^2+x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\int \frac{x+1-1}{(x+1)\sqrt{-x^2+x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\int \frac{dx}{\sqrt{-x^2+x}} - \frac{\sqrt{2}}{2}\int \frac{dx}{(x+1)\sqrt{-x^2+x}}$$

$$= \left[\begin{array}{l|l|l|l|l} \text{Subst. } t=x-\frac{1}{2} & dt=dx & x\in(0;1) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2+x=-(x^2-x)=-(x^2-2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x-\frac{1}{2})^2 \\ \text{Subst. } z=\frac{1}{x} & x=z^{-1} & dx=-\frac{dz}{z^2} & z\in(1;\infty) & x+1=\frac{1}{z}+1=\frac{1+z}{z},\quad \sqrt{-x^2+x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z} \end{array}\right]$$

$$= x\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\int \frac{dt}{\sqrt{\frac{1}{4}-t^2}} - \frac{\sqrt{2}}{2}\int \frac{-\frac{dz}{z^2}}{\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$= x\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}} + \frac{\sqrt{2}}{2}\int \frac{dz}{(z+1)\sqrt{z-1}}$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\int \frac{x}{(x+1)\sqrt{-x^2+x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\int \frac{x+1-1}{(x+1)\sqrt{-x^2+x}}\,dx$$

$$= x\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\int \frac{dx}{\sqrt{-x^2+x}} - \frac{\sqrt{2}}{2}\int \frac{dx}{(x+1)\sqrt{-x^2+x}}$$

$$= \left[\begin{array}{l|l|l|l|l} \text{Subst. } t=x-\frac{1}{2} & dt=dx & x\in(0;1) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2+x=-(x^2-x)=-(x^2-2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x-\frac{1}{2})^2 \\ \text{Subst. } z=\frac{1}{x} & x=z^{-1} & dx=-\frac{dz}{z^2} & z\in(1;\infty) & x+1=\frac{1}{z}+1=\frac{1+z}{z},\quad \sqrt{-x^2+x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z} \end{array}\right]$$

$$= x\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\int \frac{dt}{\sqrt{\frac{1}{4}-t^2}} - \frac{\sqrt{2}}{2}\int \frac{-\frac{dz}{z^2}}{\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$= x\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}} + \frac{\sqrt{2}}{2}\int \frac{dz}{(z+1)\sqrt{z-1}} = \left[\begin{array}{r|l} \text{Subst. } u=\sqrt{z-1} & u^2=z-1 \\ z=u^2+1 & dz=2u\,du \\ z\in(1;\infty) & u\in(0;\infty) \end{array}\right]$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{x+1-1}{(x+1)\sqrt{-x^2+x}}\,\mathrm{d}x$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{\mathrm{d}x}{\sqrt{-x^2+x}}-\frac{\sqrt{2}}{2}\int\frac{\mathrm{d}x}{(x+1)\sqrt{-x^2+x}}$$

$$=\left[\begin{array}{l|l|l|l|l}\text{Subst. } t=x-\frac{1}{2} & \mathrm{d}t=\mathrm{d}x & x\in(0;1) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2+x=-(x^2-x)=-(x^2-2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x-\frac{1}{2})^2\\ \text{Subst. } z=\frac{1}{x} & x=z^{-1} & \mathrm{d}x=-\frac{\mathrm{d}z}{z^2} & z\in(1;\infty) & x+1=\frac{1}{z}+1=\frac{1+z}{z},\quad \sqrt{-x^2+x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z}\end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{\mathrm{d}t}{\sqrt{\frac{1}{4}-t^2}}-\frac{\sqrt{2}}{2}\int\frac{-\frac{\mathrm{d}z}{z^2}}{\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}}+\frac{\sqrt{2}}{2}\int\frac{\mathrm{d}z}{(z+1)\sqrt{z-1}}=\left[\begin{array}{r|l}\text{Subst. } u=\sqrt{z-1} & u^2=z-1\\ z=u^2+1 & \mathrm{d}z=2u\,\mathrm{d}u\\ z\in(1;\infty) & u\in(0;\infty)\end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2t+\frac{\sqrt{2}}{2}\int\frac{2u\,\mathrm{d}u}{(u^2+2)u}$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{dx}{\sqrt{-x^2+x}}-\frac{\sqrt{2}}{2}\int\frac{dx}{(x+1)\sqrt{-x^2+x}}$$

$$=\left[\begin{array}{l|l|l|l|l} \text{Subst. } t=x-\frac{1}{2} & dt=dx & x\in(0;1) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2+x=-(x^2-x)=-(x^2-2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x-\frac{1}{2})^2 \\ \text{Subst. } z=\frac{1}{x} & x=z^{-1} & dx=-\frac{dz}{z^2} & z\in(1;\infty) & x+1=\frac{1}{z}+1=\frac{1+z}{z},\quad \sqrt{-x^2+x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z} \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{dt}{\sqrt{\frac{1}{4}-t^2}}-\frac{\sqrt{2}}{2}\int\frac{-\frac{dz}{z^2}}{\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}}+\frac{\sqrt{2}}{2}\int\frac{dz}{(z+1)\sqrt{z-1}}=\left[\begin{array}{r|l} \text{Subst. } u=\sqrt{z-1} & u^2=z-1 \\ z=u^2+1 & dz=2u\,du \\ z\in(1;\infty) & u\in(0;\infty) \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2t+\frac{\sqrt{2}}{2}\int\frac{2u\,du}{(u^2+2)u}$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2t+\sqrt{2}\int\frac{du}{u^2+2}$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x$$

$$=\left[\begin{array}{l|l|l|l|l} \text{Subst. } t=x-\frac{1}{2} & \mathrm{d}t=\mathrm{d}x & x\in(0;1) & t\in(-\frac{1}{2};\frac{1}{2}) & -x^2+x=-(x^2-x)=-(x^2-2\cdot\frac{x}{2}+\frac{1}{4})+\frac{1}{4}=(\frac{1}{2})^2-(x-\frac{1}{2})^2 \\ \text{Subst. } z=\frac{1}{x} & x=z^{-1} & \mathrm{d}x=-\frac{\mathrm{d}z}{z^2} & z\in(1;\infty) & x+1=\frac{1}{z}+1=\frac{1+z}{z},\quad \sqrt{-x^2+x}=\sqrt{-\frac{1}{z^2}+\frac{1}{z}}=\sqrt{\frac{-1+z}{z^2}}=\frac{\sqrt{z-1}}{z} \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{\mathrm{d}t}{\sqrt{\frac{1}{4}-t^2}}-\frac{\sqrt{2}}{2}\int\frac{-\frac{\mathrm{d}z}{z^2}}{\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}}+\frac{\sqrt{2}}{2}\int\frac{\mathrm{d}z}{(z+1)\sqrt{z-1}}=\left[\begin{array}{r|l} \text{Subst. } u=\sqrt{z-1} & u^2=z-1 \\ z=u^2+1 & \mathrm{d}z=2u\,\mathrm{d}u \\ z\in(1;\infty) & u\in(0;\infty) \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2t+\frac{\sqrt{2}}{2}\int\frac{2u\,\mathrm{d}u}{(u^2+2)u}$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2t+\sqrt{2}\int\frac{\mathrm{d}u}{u^2+2}$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2t+\frac{\sqrt{2}}{\sqrt{2}}\operatorname{arctg}\frac{u}{\sqrt{2}}+c_1$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{dt}{\sqrt{\frac{1}{4}-t^2}}-\frac{\sqrt{2}}{2}\int\frac{-\frac{dz}{z^2}}{\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}}+\frac{\sqrt{2}}{2}\int\frac{dz}{(z+1)\sqrt{z-1}}=\left[\begin{array}{r|l}\text{Subst. } u=\sqrt{z-1} & u^2=z-1\\ z=u^2+1 & dz=2u\,du\\ z\in(1;\infty) & u\in(0;\infty)\end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2t+\frac{\sqrt{2}}{2}\int\frac{2u\,du}{(u^2+2)u}$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2t+\sqrt{2}\int\frac{du}{u^2+2}$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2t+\frac{\sqrt{2}}{\sqrt{2}}\operatorname{arctg}\frac{u}{\sqrt{2}}+c_1$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin(2x-1)+\operatorname{arctg}\frac{\sqrt{z-1}}{\sqrt{2}}+c_1$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\tfrac{1-x}{1+x}}\,dx$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\int\frac{dt}{\sqrt{\frac{1}{4}-t^2}}-\frac{\sqrt{2}}{2}\int\frac{-\frac{dz}{z^2}}{\frac{z+1}{z}\cdot\frac{\sqrt{z-1}}{z}}$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin\frac{t}{\frac{1}{2}}+\frac{\sqrt{2}}{2}\int\frac{dz}{(z+1)\sqrt{z-1}}=\left[\begin{array}{r|l}\text{Subst. } u=\sqrt{z-1} & u^2=z-1\\ z=u^2+1 & dz=2u\,du\\ z\in(1;\infty) & u\in(0;\infty)\end{array}\right]$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2t+\frac{\sqrt{2}}{2}\int\frac{2u\,du}{(u^2+2)u}$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2t+\sqrt{2}\int\frac{du}{u^2+2}$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2t+\frac{\sqrt{2}}{\sqrt{2}}\operatorname{arctg}\frac{u}{\sqrt{2}}+c_1$$

$$=x\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin(2x-1)+\operatorname{arctg}\frac{\sqrt{z-1}}{\sqrt{2}}+c_1=\left[\begin{array}{r}\frac{\sqrt{z-1}}{\sqrt{2}}=\frac{1}{\sqrt{2}}\sqrt{\frac{1}{x}-1}\\ =\frac{1}{\sqrt{2}}\sqrt{\frac{1-x}{x}}=\sqrt{\frac{1-x}{2x}}\end{array}\right]$$

# Riešené príklady – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}}\,dx = x \arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin(2x-1) + \operatorname{arctg} \sqrt{\frac{1-x}{2x}} + c_1$$

$$= x \arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin \frac{t}{\frac{1}{2}} + \frac{\sqrt{2}}{2} \int \frac{dz}{(z+1)\sqrt{z-1}} = \left[ \begin{array}{r|l} \text{Subst. } u=\sqrt{z-1} & u^2=z-1 \\ z=u^2+1 & dz=2u\,du \\ z\in(1;\infty) & u\in(0;\infty) \end{array} \right]$$

$$= x \arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \frac{\sqrt{2}}{2} \int \frac{2u\,du}{(u^2+2)u}$$

$$= x \arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \sqrt{2} \int \frac{du}{u^2+2}$$

$$= x \arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin 2t + \frac{\sqrt{2}}{\sqrt{2}} \operatorname{arctg} \frac{u}{\sqrt{2}} + c_1$$

$$= x \arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin(2x-1) + \operatorname{arctg} \frac{\sqrt{z-1}}{\sqrt{2}} + c_1 = \left[ \begin{array}{r} \frac{\sqrt{z-1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\sqrt{\frac{1}{x}-1} \\ = \frac{1}{\sqrt{2}}\sqrt{\frac{1-x}{x}} = \sqrt{\frac{1-x}{2x}} \end{array} \right]$$

$$= x \arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2} \arcsin(2x-1) + \operatorname{arctg} \sqrt{\frac{1-x}{2x}} + c_1,\ x\in(0;1),\ c\in R.$$

# Riešené príklady – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}}\, dx$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{1-x}{1+x}\leq 1 \\ x\neq 1\end{matrix}\;\middle|\;\begin{matrix} 1+x>0 \Leftrightarrow x>-1\colon\quad 0\leq 1-x\leq 1+x,\ \text{t. j.}\ \ x\leq 1,\, 0\leq 2x,\ \text{t. j.}\ \ x\in\langle 0;1\rangle \\ 1+x<0 \Leftrightarrow x<-1\colon\quad 0\geq 1-x\geq 1+x,\ \text{t. j.}\ \ x\geq 1,\, 0\geq 2x,\ \text{t. j.}\ \ x\ \text{neexistuje}\end{matrix}\;\middle|\;\begin{matrix} x\in\langle 0;1\rangle \\ \text{t. j.}\ \ 0\leq x\leq 1\end{matrix}\right]$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{1-x}{1+x}\leq 1 \\ x\neq 1\end{matrix}\,\middle|\,\begin{matrix} 1+x>0 \Leftrightarrow x>-1\colon & 0\leq 1-x\leq 1+x, \text{ t. j. } x\leq 1,\, 0\leq 2x, \text{ t. j. } x\in\langle 0;1\rangle \\ 1+x<0 \Leftrightarrow x<-1\colon & 0\geq 1-x\geq 1+x, \text{ t. j. } x\geq 1,\, 0\geq 2x, \text{ t. j. } x \text{ neexistuje}\end{matrix}\,\middle|\,\begin{matrix} x\in\langle 0;1\rangle \\ \text{t. j. } 0\leq x\leq 1\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst. } t=1+x \\ dt=dx\end{matrix}\,\middle|\,\begin{matrix} x\in\langle 0;1\rangle \\ t\in\langle 1;2\rangle\end{matrix}\right]$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x$$

$$=\left[\begin{matrix} 0\leq\frac{1-x}{1+x}\leq 1 \\ x\neq 1\end{matrix}\,\middle|\,\begin{matrix}1+x>0 \Leftrightarrow x>-1\colon\ \ 0\leq 1-x\leq 1+x,\ \text{t. j.}\ \ x\leq 1,\, 0\leq 2x,\ \text{t. j.}\ \ x\in\langle 0;1\rangle \\ 1+x<0 \Leftrightarrow x<-1\colon\ \ 0\geq 1-x\geq 1+x,\ \text{t. j.}\ \ x\geq 1,\, 0\geq 2x,\ \text{t. j.}\ \ x \text{ neexistuje}\end{matrix}\,\middle|\,\begin{matrix}x\in\langle 0;1\rangle \\ \text{t. j.}\ \ 0\leq x\leq 1\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst.}\ \ t=1+x \\ \mathrm{d}t=\mathrm{d}x\end{matrix}\,\middle|\,\begin{matrix}x\in\langle 0;1\rangle \\ t\in\langle 1;2\rangle\end{matrix}\right]=\int \arcsin\sqrt{\frac{2-t}{t}}\,\mathrm{d}t=\int \arcsin\sqrt{\frac{2}{t}-1}\,\mathrm{d}t$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{array}{c|l|l} 0\leq\frac{1-x}{1+x}\leq 1 & 1+x>0 \Leftrightarrow x>-1:\quad 0\leq 1-x\leq 1+x,\ \text{t. j.}\ \ x\leq 1,\ 0\leq 2x,\ \text{t. j.}\ \ x\in\langle 0;1\rangle & x\in\langle 0;1\rangle \\ x\neq 1 & 1+x<0 \Leftrightarrow x<-1:\quad 0\geq 1-x\geq 1+x,\ \text{t. j.}\ \ x\geq 1,\ 0\geq 2x,\ \text{t. j.}\ \ x\ \text{neexistuje} & \text{t. j.}\ \ 0\leq x\leq 1 \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst.}\quad t=1+x & x\in\langle 0;1\rangle \\ dt=dx & t\in\langle 1;2\rangle \end{array}\right]=\int \arcsin\sqrt{\frac{2-t}{t}}\,dt=\int \arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}}=\frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad =\frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}}=\frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}}=\frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}}=\frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array}\right]$$

# Riešené príklady – 173

## $\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$

$$= \left[\begin{array}{c|l|l} 0\leq\frac{1-x}{1+x}\leq 1 & 1+x>0 \Leftrightarrow x>-1:\ \ 0\leq 1-x\leq 1+x,\ \text{t. j.}\ \ x\leq 1,\ 0\leq 2x,\ \text{t. j.}\ \ x\in\langle 0;1\rangle & x\in\langle 0;1\rangle \\ x\neq 1 & 1+x<0 \Leftrightarrow x<-1:\ \ 0\geq 1-x\geq 1+x,\ \text{t. j.}\ \ x\geq 1,\ 0\geq 2x,\ \text{t. j.}\ \ x \text{ neexistuje} & \text{t. j.}\ \ 0\leq x\leq 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} \text{Subst.}\ \ t=1+x & x\in\langle 0;1\rangle \\ dt=dx & t\in\langle 1;2\rangle \end{array}\right] = \int \arcsin\sqrt{\frac{2-t}{t}}\,dt = \int \arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$= \left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}} = \frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad = \frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}} = \frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array}\right]$$

$$= t\arcsin\sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}}$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{array}{c|l|l} 0\le\frac{1-x}{1+x}\le 1 & 1+x>0 \Leftrightarrow x>-1:\quad 0\le 1-x\le 1+x,\ \text{t. j. } x\le 1,\ 0\le 2x,\ \text{t. j. } x\in\langle 0;1\rangle & x\in\langle 0;1\rangle \\ x\ne 1 & 1+x<0 \Leftrightarrow x<-1:\quad 0\ge 1-x\ge 1+x,\ \text{t. j. } x\ge 1,\ 0\ge 2x,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 0\le x\le 1 \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=1+x & x\in\langle 0;1\rangle \\ dt=dx & t\in\langle 1;2\rangle \end{array}\right]=\int \arcsin\sqrt{\frac{2-t}{t}}\,dt=\int \arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}}=\frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad =\frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}}=\frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}}=\frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}}=\frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}}=\left[\begin{array}{r|l} \text{Subst. } z=t-\frac{3}{2} & t\in(1;2) \\ dz=dt & z\in(-\frac{1}{2};\frac{1}{2}) \end{array}\right]$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{1-x}{1+x}\leq 1 \\ x\neq 1\end{matrix}\middle|\begin{matrix} 1+x>0 \Leftrightarrow x>-1\colon\ \ 0\leq 1-x\leq 1+x,\ \text{t. j. } x\leq 1,\, 0\leq 2x,\ \text{t. j. } x\in\langle 0;1\rangle \\ 1+x<0 \Leftrightarrow x<-1\colon\ \ 0\geq 1-x\geq 1+x,\ \text{t. j. } x\geq 1,\, 0\geq 2x,\ \text{t. j. } x \text{ neexistuje}\end{matrix}\middle|\begin{matrix} x\in\langle 0;1\rangle \\ \text{t. j. } 0\leq x\leq 1\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst.}\ \ t=1+x \\ dt=dx\end{matrix}\middle|\begin{matrix} x\in\langle 0;1\rangle \\ t\in\langle 1;2\rangle\end{matrix}\right]=\int \arcsin\sqrt{\frac{2-t}{t}}\,dt=\int \arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{\frac{2}{t}-1} \\ \\ v'=1\end{matrix}\middle|\begin{matrix} u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}}=\frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v=t \qquad\qquad =\frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}}=\frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}}=\frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}}=\frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}}\end{matrix}\right]$$

$$=t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}}=\left[\begin{matrix}\text{Subst.}\ \ z=t-\frac{3}{2} \\ dz=dt\end{matrix}\middle|\begin{matrix} t\in(1;2) \\ z\in(-\frac{1}{2};\frac{1}{2})\end{matrix}\right]$$

$$=t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\int\frac{dz}{\sqrt{(\frac{1}{2})^2-z^2}}$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{1-x}{1+x}\leq 1 \\ x\neq 1\end{matrix}\,\middle|\,\begin{matrix} 1+x>0\Leftrightarrow x>-1\colon\ 0\leq 1-x\leq 1+x,\ \text{t. j.}\ x\leq 1,\ 0\leq 2x,\ \text{t. j.}\ x\in\langle 0;1\rangle \\ 1+x<0\Leftrightarrow x<-1\colon\ 0\geq 1-x\geq 1+x,\ \text{t. j.}\ x\geq 1,\ 0\geq 2x,\ \text{t. j.}\ x\ \text{neexistuje}\end{matrix}\,\middle|\,\begin{matrix} x\in\langle 0;1\rangle \\ \text{t. j.}\ 0\leq x\leq 1\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst.}\ t=1+x \\ dt=dx\end{matrix}\,\middle|\,\begin{matrix} x\in\langle 0;1\rangle \\ t\in\langle 1;2\rangle\end{matrix}\right]=\int\arcsin\sqrt{\frac{2-t}{t}}\,dt=\int\arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{\frac{2}{t}-1} \\ v'=1\end{matrix}\,\middle|\,\begin{matrix} u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}}=\frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v=t \qquad =\frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}}=\frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}}=\frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}}=\frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}}\end{matrix}\right]$$

$$=t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}}=\left[\begin{matrix}\text{Subst.}\ z=t-\frac{3}{2} \\ dz=dt\end{matrix}\,\middle|\,\begin{matrix} t\in(1;2) \\ z\in(-\frac{1}{2};\frac{1}{2})\end{matrix}\right]$$

$$=t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\int\frac{dz}{\sqrt{(\frac{1}{2})^2-z^2}}=t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\arcsin\frac{z}{\frac{1}{2}}+c_2$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx$$

$$=\left[\begin{array}{c|l|l} 0\leq\frac{1-x}{1+x}\leq 1 & 1+x>0 \Leftrightarrow x>-1:\ \ 0\leq 1-x\leq 1+x,\ \text{t. j. } x\leq 1,\ 0\leq 2x,\ \text{t. j. } x\in\langle 0;1\rangle & x\in\langle 0;1\rangle \\ x\neq 1 & 1+x<0 \Leftrightarrow x<-1:\ \ 0\geq 1-x\geq 1+x,\ \text{t. j. } x\geq 1,\ 0\geq 2x,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 0\leq x\leq 1 \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=1+x & x\in\langle 0;1\rangle \\ dt=dx & t\in\langle 1;2\rangle \end{array}\right]=\int \arcsin\sqrt{\frac{2-t}{t}}\,dt=\int \arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}}=\frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad =\frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}}=\frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}}=\frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}}=\frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}}=\left[\begin{array}{r|l} \text{Subst. } z=t-\frac{3}{2} & t\in(1;2) \\ dz=dt & z\in(-\frac{1}{2};\frac{1}{2}) \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\int\frac{dz}{\sqrt{(\frac{1}{2})^2-z^2}}=t\arcsin\sqrt{\frac{2-t}{t}}+\frac{\sqrt{2}}{2}\arcsin\frac{z}{\frac{1}{2}}+c_2$$

$$=(1+x)\arcsin\sqrt{\frac{1-x}{1+x}}+\frac{\sqrt{2}}{2}\arcsin 2z+c_2$$

# Riešené príklady – 173

$$\int \arcsin \sqrt{\frac{1-x}{1+x}}\,dx$$

$$= \left[ \begin{array}{c|l|l} 0 \le \frac{1-x}{1+x} \le 1 & 1+x>0 \Leftrightarrow x>-1:\quad 0\le 1-x \le 1+x,\ \text{t. j.}\ \ x\le 1,\ 0\le 2x,\ \text{t. j.}\ \ x\in\langle 0;1\rangle & x\in\langle 0;1\rangle \\ x\neq 1 & 1+x<0 \Leftrightarrow x<-1:\quad 0\ge 1-x \ge 1+x,\ \text{t. j.}\ \ x\ge 1,\ 0\ge 2x,\ \text{t. j.}\ \ x\ \text{neexistuje} & \text{t. j.}\ \ 0\le x\le 1 \end{array} \right]$$

$$= \left[ \begin{array}{rr|l} \text{Subst.} & t=1+x & x\in\langle 0;1\rangle \\ & dt=dx & t\in\langle 1;2\rangle \end{array} \right] = \int \arcsin \sqrt{\frac{2-t}{t}}\,dt = \int \arcsin \sqrt{\frac{2}{t}-1}\,dt$$

$$= \left[ \begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}} = \frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad = \frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}} = \frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array} \right]$$

$$= t\arcsin\sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2}\int \frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} = \left[ \begin{array}{rr|l} \text{Subst.} & z=t-\frac{3}{2} & t\in(1;2) \\ & dz=dt & z\in(-\frac{1}{2};\frac{1}{2}) \end{array} \right]$$

$$= t\arcsin\sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2}\int \frac{dz}{\sqrt{(\frac{1}{2})^2-z^2}} = t\arcsin\sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2}\arcsin\frac{z}{\frac{1}{2}} + c_2$$

$$= (1+x)\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\arcsin 2z + c_2 = \left[\, 2z=2(t-\tfrac{3}{2})=2(x-\tfrac{1}{2})=2x-1 \,\right]$$

# Riešené príklady – 173

$$\int \arcsin\sqrt{\frac{1-x}{1+x}}\,dx = (x+1)\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\arcsin(2x-1) + c_2$$

$$= \left[\begin{array}{c|l|l} 0\leq\frac{1-x}{1+x}\leq 1 & 1+x>0 \Leftrightarrow x>-1:\ \ 0\leq 1-x\leq 1+x,\ \text{t. j. } x\leq 1,\ 0\leq 2x,\ \text{t. j. } x\in\langle 0;1\rangle & x\in\langle 0;1\rangle \\ x\neq 1 & 1+x<0 \Leftrightarrow x<-1:\ \ 0\geq 1-x\geq 1+x,\ \text{t. j. } x\geq 1,\ 0\geq 2x,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 0\leq x\leq 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} \text{Subst.}\quad t=1+x & x\in\langle 0;1\rangle \\ dt=dx & t\in\langle 1;2\rangle \end{array}\right] = \int \arcsin\sqrt{\frac{2-t}{t}}\,dt = \int \arcsin\sqrt{\frac{2}{t}-1}\,dt$$

$$= \left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{2}{t}-1} & u'=\frac{1}{\sqrt{1-(\frac{2}{t}-1)}}\cdot\frac{1}{2}\left(\frac{2}{t}-1\right)^{-\frac{1}{2}}\cdot(-2t^{-2}) = \frac{-1}{\sqrt{2-\frac{2}{t}}\sqrt{\frac{2-t}{t}}\sqrt{t^4}} = \frac{-1}{\sqrt{\frac{2t-2}{t}\cdot\frac{2-t}{t}\cdot t^2}} = \frac{-1}{\sqrt{2}\sqrt{(t-1)(2-t)t^2}} \\ v'=1 & v=t \qquad = \frac{-\sqrt{2}}{2\,|t|\,\sqrt{(t-1)(2-t)}} = \frac{-\sqrt{2}}{2t\sqrt{3t-t^2-2}} = \frac{-\sqrt{2}}{2t\sqrt{-t^2+2\cdot\frac{3t}{2}-\frac{9}{4}+\frac{9}{4}-2}} = \frac{-\sqrt{2}}{2t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} \end{array}\right]$$

$$= t\arcsin\sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{\frac{1}{4}-(t-\frac{3}{2})^2}} = \left[\begin{array}{c|c} \text{Subst.}\quad z=t-\frac{3}{2} & t\in(1;2) \\ dz=dt & z\in(-\frac{1}{2};\frac{1}{2}) \end{array}\right]$$

$$= t\arcsin\sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2}\int\frac{dz}{\sqrt{(\frac{1}{2})^2-z^2}} = t\arcsin\sqrt{\frac{2-t}{t}} + \frac{\sqrt{2}}{2}\arcsin\frac{z}{\frac{1}{2}} + c_2$$

$$= (1+x)\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\arcsin 2z + c_2 = \left[2z=2(t-\tfrac{3}{2})=2(x-\tfrac{1}{2})=2x-1\right]$$

$$= (x+1)\arcsin\sqrt{\frac{1-x}{1+x}} + \frac{\sqrt{2}}{2}\arcsin(2x-1) + c_2,\ x\in(0;1),\ c\in R.$$

# Riešené príklady – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}}\, dx$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{x+1}{x-1}\leq 1 \\ x\neq 1\end{matrix}\left| \begin{matrix} x-1>0 \Leftrightarrow x>1\colon & 0\leq x+1\leq x-1, & \text{t. j.} & -1\leq x,\ 1\leq -1, & \text{t. j.} & x \text{ neexistuje} \\ x-1<0 \Leftrightarrow x<1\colon & 0\geq x+1\geq x-1, & \text{t. j.} & -1\geq x,\ 1\geq -1, & \text{t. j.} & x\in(-\infty;-1\rangle \end{matrix}\right| \begin{matrix} x\in(-\infty;-1\rangle \\ \text{t. j. } x\leq -1\end{matrix}\right]$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$= \left[\begin{array}{c|l|c} 0\leq\frac{x+1}{x-1}\leq 1 & x-1>0 \Leftrightarrow x>1:\quad 0\leq x+1\leq x-1,\ \text{t. j.}\ -1\leq x,\ 1\leq -1,\ \text{t. j.}\ x \text{ neexistuje} & x\in(-\infty;-1\rangle \\ x\neq 1 & x-1<0 \Leftrightarrow x<1:\quad 0\geq x+1\geq x-1,\ \text{t. j.}\ -1\geq x,\ 1\geq -1,\ \text{t. j.}\ x\in(-\infty;-1\rangle & \text{t. j. } x\leq -1 \end{array}\right]$$

$$= \left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{x+1}{x-1}} & u'=\frac{1}{\sqrt{1-\frac{x+1}{x-1}}}\cdot\frac{1}{2}\left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}}\cdot\frac{(x-1)-(x+1)}{(x-1)^2}=\frac{\frac{1}{2}\cdot(-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}}\sqrt{\frac{x+1}{x-1}}\sqrt{(x-1)^4}}=\frac{-1}{\sqrt{\frac{-2}{x-1}\cdot\frac{x+1}{x-1}\cdot(x-1)^4}} \\ v'=1 & v=x \qquad =\frac{-1}{\sqrt{-2(x+1)\cdot(x-1)^2}}=\frac{-1}{\sqrt{2}\,|x-1|\,\sqrt{-(x+1)}}=\frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}}=\frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}} \end{array}\right]$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{x+1}{x-1}\leq 1 \\ x\neq 1\end{matrix}\,\middle|\,\begin{matrix} x-1>0 \Leftrightarrow x>1\colon\ \ 0\leq x+1\leq x-1,\ \text{t. j.}\ -1\leq x,\ 1\leq -1,\ \text{t. j.}\ x \text{ neexistuje} \\ x-1<0 \Leftrightarrow x<1\colon\ \ 0\geq x+1\geq x-1,\ \text{t. j.}\ -1\geq x,\ 1\geq -1,\ \text{t. j.}\ x\in(-\infty;-1\rangle \end{matrix}\,\middle|\,\begin{matrix} x\in(-\infty;-1\rangle \\ \text{t. j. } x\leq -1\end{matrix}\right]$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{\frac{x+1}{x-1}} \\ v'=1\end{matrix}\,\middle|\,\begin{matrix} u'=\frac{1}{\sqrt{1-\frac{x+1}{x-1}}}\cdot\frac{1}{2}\left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}}\cdot\frac{(x-1)-(x+1)}{(x-1)^2}=\frac{\frac{1}{2}\cdot(-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}}\sqrt{\frac{x+1}{x-1}}\sqrt{(x-1)^4}}=\frac{-1}{\sqrt{\frac{-2}{x-1}\cdot\frac{x+1}{x-1}\cdot(x-1)^4}} \\ v=x \qquad =\frac{-1}{\sqrt{-2(x+1)\cdot(x-1)^2}}=\frac{-1}{\sqrt{2}\,|x-1|\,\sqrt{-(x+1)}}=\frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}}=\frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}}\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{x+1}{x-1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x-1)\sqrt{-x-1}}$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{array}{c|l|r} 0\le\frac{x+1}{x-1}\le 1 & x-1>0 \Leftrightarrow x>1:\quad 0\le x+1\le x-1,\ \text{t. j. } -1\le x,\ 1\le -1,\ \text{t. j. } x \text{ neexistuje} & x\in(-\infty;-1\rangle \\ x\neq 1 & x-1<0 \Leftrightarrow x<1:\quad 0\ge x+1\ge x-1,\ \text{t. j. } -1\ge x,\ 1\ge -1,\ \text{t. j. } x\in(-\infty;-1\rangle & \text{t. j. } x\le -1 \end{array}\right]$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{x+1}{x-1}} & u'=\frac{1}{\sqrt{1-\frac{x+1}{x-1}}}\cdot\frac{1}{2}\left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}}\cdot\frac{(x-1)-(x+1)}{(x-1)^2}=\frac{\frac{1}{2}\cdot(-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}}\sqrt{\frac{x+1}{x-1}}\sqrt{(x-1)^4}}=\frac{-1}{\sqrt{\frac{-2}{x-1}\cdot\frac{x+1}{x-1}\cdot(x-1)^4}} \\ v'=1 & v=x \qquad =\frac{-1}{\sqrt{-2(x+1)\cdot(x-1)^2}}=\frac{-1}{\sqrt{2}\,|x-1|\,\sqrt{-(x+1)}}=\frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}}=\frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}} \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{x+1}{x-1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x-1)\sqrt{-x-1}}=\left[\begin{array}{r|l|l} \text{Subst. } t=\sqrt{-x-1} & t^2=-x-1 & x\in(-\infty;-1) \\ x=-t^2-1 & dx=-2t\,dt & t\in(0;\infty) \end{array}\right]$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{matrix}0\leq\frac{x+1}{x-1}\leq 1\\ x\neq 1\end{matrix}\;\middle|\;\begin{matrix}x-1>0\Leftrightarrow x>1\colon\;\; 0\leq x+1\leq x-1,\ \text{t. j. } -1\leq x,\ 1\leq -1,\ \text{t. j. } x \text{ neexistuje}\\ x-1<0\Leftrightarrow x<1\colon\;\; 0\geq x+1\geq x-1,\ \text{t. j. } -1\geq x,\ 1\geq -1,\ \text{t. j. } x\in(-\infty;-1\rangle\end{matrix}\;\middle|\;\begin{matrix}x\in(-\infty;-1\rangle\\ \text{t. j. } x\leq -1\end{matrix}\right]$$

$$=\left[\begin{matrix}u=\arcsin\sqrt{\frac{x+1}{x-1}}\\ v'=1\end{matrix}\;\middle|\;\begin{matrix}u'=\frac{1}{\sqrt{1-\frac{x+1}{x-1}}}\cdot\frac{1}{2}\left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}}\cdot\frac{(x-1)-(x+1)}{(x-1)^2}=\frac{\frac{1}{2}\cdot(-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}}\sqrt{\frac{x+1}{x-1}}\sqrt{(x-1)^4}}=\frac{-1}{\sqrt{\frac{-2}{x-1}\cdot\frac{x+1}{x-1}\cdot(x-1)^4}}\\ v=x \qquad =\frac{-1}{\sqrt{-2(x+1)\cdot(x-1)^2}}=\frac{-1}{\sqrt{2}\,|x-1|\,\sqrt{-(x+1)}}=\frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}}=\frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}}\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{x+1}{x-1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x-1)\sqrt{-x-1}}=\left[\begin{matrix}\text{Subst. } t=\sqrt{-x-1}\\ x=-t^2-1\end{matrix}\;\middle|\;\begin{matrix}t^2=-x-1\\ dx=-2t\,dt\end{matrix}\;\middle|\;\begin{matrix}x\in(-\infty;-1)\\ t\in(0;\infty)\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{x+1}{x-1}}-\frac{\sqrt{2}}{2}\int\frac{(-t^2-1)(-2t)\,dt}{(-t^2-2)t}$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{matrix}0\leq\frac{x+1}{x-1}\leq 1\\ x\neq 1\end{matrix}\;\middle|\;\begin{matrix}x-1>0\Leftrightarrow x>1:\quad 0\leq x+1\leq x-1,\ \text{t. j.}\ -1\leq x,\ 1\leq -1,\ \text{t. j.}\ x\ \text{neexistuje}\\ x-1<0\Leftrightarrow x<1:\quad 0\geq x+1\geq x-1,\ \text{t. j.}\ -1\geq x,\ 1\geq -1,\ \text{t. j.}\ x\in(-\infty;-1\rangle\end{matrix}\;\middle|\;\begin{matrix}x\in(-\infty;-1\rangle\\ \text{t. j. } x\leq -1\end{matrix}\right]$$

$$=\left[\begin{matrix}u=\arcsin\sqrt{\frac{x+1}{x-1}}\\ v'=1\end{matrix}\;\middle|\;\begin{matrix}u'=\frac{1}{\sqrt{1-\frac{x+1}{x-1}}}\cdot\frac{1}{2}\left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}}\cdot\frac{(x-1)-(x+1)}{(x-1)^2}=\frac{\frac{1}{2}\cdot(-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}}\sqrt{\frac{x+1}{x-1}}\sqrt{(x-1)^4}}=\frac{-1}{\sqrt{\frac{-2}{x-1}\cdot\frac{x+1}{x-1}\cdot(x-1)^4}}\\ v=x\qquad=\frac{-1}{\sqrt{-2(x+1)\cdot(x-1)^2}}=\frac{-1}{\sqrt{2}\,|x-1|\,\sqrt{-(x+1)}}=\frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}}=\frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}}\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{x+1}{x-1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x-1)\sqrt{-x-1}}=\left[\begin{matrix}\text{Subst. } t=\sqrt{-x-1}\\ x=-t^2-1\end{matrix}\;\middle|\;\begin{matrix}t^2=-x-1\\ dx=-2t\,dt\end{matrix}\;\middle|\;\begin{matrix}x\in(-\infty;-1)\\ t\in(0;\infty)\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{x+1}{x-1}}-\frac{\sqrt{2}}{2}\int\frac{(-t^2-1)(-2t)\,dt}{(-t^2-2)t}=x\arcsin\sqrt{\frac{x+1}{x-1}}+\sqrt{2}\int\frac{(t^2+1)\,dt}{t^2+2}$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{x+1}{x-1}\leq 1 \\ x\neq 1\end{matrix}\;\middle|\; \begin{matrix} x-1>0 \Leftrightarrow x>1:\quad 0\leq x+1\leq x-1,\ \text{t. j.}\ -1\leq x,\ 1\leq -1,\ \text{t. j.}\ x \text{ neexistuje} \\ x-1<0 \Leftrightarrow x<1:\quad 0\geq x+1\geq x-1,\ \text{t. j.}\ -1\geq x,\ 1\geq -1,\ \text{t. j.}\ x\in(-\infty;-1\rangle \end{matrix}\;\middle|\; \begin{matrix} x\in(-\infty;-1\rangle \\ \text{t. j. } x\leq -1\end{matrix}\right]$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{\frac{x+1}{x-1}} \\ v'=1\end{matrix}\;\middle|\; \begin{matrix} u'=\frac{1}{\sqrt{1-\frac{x+1}{x-1}}}\cdot\frac{1}{2}\left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}}\cdot\frac{(x-1)-(x+1)}{(x-1)^2}=\frac{\frac{1}{2}\cdot(-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}}\sqrt{\frac{x+1}{x-1}}\sqrt{(x-1)^4}}=\frac{-1}{\sqrt{\frac{-2}{x-1}\cdot\frac{x+1}{x-1}\cdot(x-1)^4}} \\ v=x \qquad =\frac{-1}{\sqrt{-2(x+1)\cdot(x-1)^2}}=\frac{-1}{\sqrt{2}\,|x-1|\,\sqrt{-(x+1)}}=\frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}}=\frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}}\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{x+1}{x-1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x-1)\sqrt{-x-1}}=\left[\begin{matrix}\text{Subst. } t=\sqrt{-x-1} \\ x=-t^2-1\end{matrix}\;\middle|\;\begin{matrix} t^2=-x-1 \\ dx=-2t\,dt\end{matrix}\;\middle|\;\begin{matrix} x\in(-\infty;-1) \\ t\in(0;\infty)\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{x+1}{x-1}}-\frac{\sqrt{2}}{2}\int\frac{(-t^2-1)(-2t)\,dt}{(-t^2-2)t}=x\arcsin\sqrt{\frac{x+1}{x-1}}+\sqrt{2}\int\frac{(t^2+1)\,dt}{t^2+2}$$

$$=x\arcsin\sqrt{\frac{x+1}{x-1}}+\sqrt{2}\int\frac{(t^2+2-1)\,dt}{t^2+2}$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{array}{c|l|c} 0\leq\frac{x+1}{x-1}\leq 1 & x-1>0 \Leftrightarrow x>1:\quad 0\leq x+1\leq x-1,\ \text{t. j.}\ -1\leq x,\ 1\leq -1,\ \text{t. j.}\ x \text{ neexistuje} & x\in(-\infty;-1\rangle \\ x\neq 1 & x-1<0 \Leftrightarrow x<1:\quad 0\geq x+1\geq x-1,\ \text{t. j.}\ -1\geq x,\ 1\geq -1,\ \text{t. j.}\ x\in(-\infty;-1\rangle & \text{t. j. } x\leq -1 \end{array}\right]$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{x+1}{x-1}} & u'=\frac{1}{\sqrt{1-\frac{x+1}{x-1}}}\cdot\frac{1}{2}\left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}}\cdot\frac{(x-1)-(x+1)}{(x-1)^2}=\frac{\frac{1}{2}\cdot(-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}}\sqrt{\frac{x+1}{x-1}}\sqrt{(x-1)^4}}=\frac{-1}{\sqrt{\frac{-2}{x-1}\cdot\frac{x+1}{x-1}\cdot(x-1)^4}} \\ v'=1 & v=x \qquad =\frac{-1}{\sqrt{-2(x+1)\cdot(x-1)^2}}=\frac{-1}{\sqrt{2}\,|x-1|\,\sqrt{-(x+1)}}=\frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}}=\frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}} \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{x+1}{x-1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x-1)\sqrt{-x-1}}=\left[\begin{array}{r|l|l} \text{Subst. } t=\sqrt{-x-1} & t^2=-x-1 & x\in(-\infty;-1) \\ x=-t^2-1 & dx=-2t\,dt & t\in(0;\infty) \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{x+1}{x-1}}-\frac{\sqrt{2}}{2}\int\frac{(-t^2-1)(-2t)\,dt}{(-t^2-2)t}=x\arcsin\sqrt{\frac{x+1}{x-1}}+\sqrt{2}\int\frac{(t^2+1)\,dt}{t^2+2}$$

$$=x\arcsin\sqrt{\frac{x+1}{x-1}}+\sqrt{2}\int\frac{(t^2+2-1)\,dt}{t^2+2}=x\arcsin\sqrt{\frac{x+1}{x-1}}+\sqrt{2}\int dt-\sqrt{2}\int\frac{dt}{t^2+(\sqrt{2})^2}$$

# Riešené príklady – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}}\, dx$$

$$= \left[ \begin{array}{c|l|c} 0 \leq \frac{x+1}{x-1} \leq 1 & x-1>0 \Leftrightarrow x>1:\ \ 0\leq x+1 \leq x-1, \text{ t. j. } -1\leq x,\ 1\leq -1, \text{ t. j. } x \text{ neexistuje} & x\in(-\infty;-1\rangle \\ x\neq 1 & x-1<0 \Leftrightarrow x<1:\ \ 0\geq x+1 \geq x-1, \text{ t. j. } -1\geq x,\ 1\geq -1, \text{ t. j. } x\in(-\infty;-1\rangle & \text{t. j. } x\leq -1 \end{array} \right]$$

$$= \left[ \begin{array}{l|l} u = \arcsin\sqrt{\frac{x+1}{x-1}} & u' = \frac{1}{\sqrt{1-\frac{x+1}{x-1}}}\cdot\frac{1}{2}\left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}}\cdot\frac{(x-1)-(x+1)}{(x-1)^2} = \frac{\frac{1}{2}\cdot(-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}}\sqrt{\frac{x+1}{x-1}}\sqrt{(x-1)^4}} = \frac{-1}{\sqrt{\frac{-2}{x-1}\cdot\frac{x+1}{x-1}\cdot(x-1)^4}} \\ v' = 1 & v = x \qquad = \frac{-1}{\sqrt{-2(x+1)\cdot(x-1)^2}} = \frac{-1}{\sqrt{2}\,|x-1|\,\sqrt{-(x+1)}} = \frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}} = \frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}} \end{array} \right]$$

$$= x \arcsin\sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2}\int \frac{x\,dx}{(x-1)\sqrt{-x-1}} = \left[ \begin{array}{r|l|l} \text{Subst. } t=\sqrt{-x-1} & t^2=-x-1 & x\in(-\infty;-1) \\ x=-t^2-1 & dx=-2t\,dt & t\in(0;\infty) \end{array} \right]$$

$$= x \arcsin\sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2}\int \frac{(-t^2-1)(-2t)\,dt}{(-t^2-2)t} = x \arcsin\sqrt{\frac{x+1}{x-1}} + \sqrt{2}\int \frac{(t^2+1)\,dt}{t^2+2}$$

$$= x \arcsin\sqrt{\frac{x+1}{x-1}} + \sqrt{2}\int \frac{(t^2+2-1)\,dt}{t^2+2} = x \arcsin\sqrt{\frac{x+1}{x-1}} + \sqrt{2}\int dt - \sqrt{2}\int \frac{dt}{t^2+(\sqrt{2})^2}$$

$$= x \arcsin\sqrt{\frac{x+1}{x-1}} + \sqrt{2}t - \frac{\sqrt{2}}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + c_1$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx = x\arcsin\sqrt{\frac{x+1}{x-1}} + \sqrt{2(-x-1)} - \operatorname{arctg}\sqrt{\frac{-x-1}{2}} + c_1$$

$$= \left[\begin{array}{c|l|r} 0\leq\frac{x+1}{x-1}\leq 1 & x-1>0 \Leftrightarrow x>1:\ \ 0\leq x+1\leq x-1,\ \text{t. j. } -1\leq x,\ 1\leq -1,\ \text{t. j. } x \text{ neexistuje} & x\in(-\infty;-1\rangle \\ x\neq 1 & x-1<0 \Leftrightarrow x<1:\ \ 0\geq x+1\geq x-1,\ \text{t. j. } -1\geq x,\ 1\geq -1,\ \text{t. j. } x\in(-\infty;-1\rangle & \text{t. j. } x\leq -1 \end{array}\right]$$

$$= \left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{x+1}{x-1}} & u'=\frac{1}{\sqrt{1-\frac{x+1}{x-1}}}\cdot\frac{1}{2}\left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}}\cdot\frac{(x-1)-(x+1)}{(x-1)^2} = \frac{\frac{1}{2}\cdot(-2)}{\sqrt{\frac{x-1-(x+1)}{x-1}}\sqrt{\frac{x+1}{x-1}}\sqrt{(x-1)^4}} = \frac{-1}{\sqrt{\frac{-2}{x-1}\cdot\frac{x+1}{x-1}\cdot(x-1)^4}} \\ v'=1 & v=x \qquad = \frac{-1}{\sqrt{-2(x+1)\cdot(x-1)^2}} = \frac{-1}{\sqrt{2}\,|x-1|\,\sqrt{-(x+1)}} = \frac{-\sqrt{2}}{2(1-x)\sqrt{-(x+1)}} = \frac{\sqrt{2}}{2(x-1)\sqrt{-x-1}} \end{array}\right]$$

$$= x\arcsin\sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x-1)\sqrt{-x-1}} = \left[\begin{array}{r|l|l} \text{Subst. } t=\sqrt{-x-1} & t^2=-x-1 & x\in(-\infty;-1) \\ x=-t^2-1 & dx=-2t\,dt & t\in(0;\infty) \end{array}\right]$$

$$= x\arcsin\sqrt{\frac{x+1}{x-1}} - \frac{\sqrt{2}}{2}\int\frac{(-t^2-1)(-2t)\,dt}{(-t^2-2)t} = x\arcsin\sqrt{\frac{x+1}{x-1}} + \sqrt{2}\int\frac{(t^2+1)\,dt}{t^2+2}$$

$$= x\arcsin\sqrt{\frac{x+1}{x-1}} + \sqrt{2}\int\frac{(t^2+2-1)\,dt}{t^2+2} = x\arcsin\sqrt{\frac{x+1}{x-1}} + \sqrt{2}\int dt - \sqrt{2}\int\frac{dt}{t^2+(\sqrt{2})^2}$$

$$= x\arcsin\sqrt{\frac{x+1}{x-1}} + \sqrt{2}t - \frac{\sqrt{2}}{\sqrt{2}}\operatorname{arctg}\frac{t}{\sqrt{2}} + c_1$$

$$= x\arcsin\sqrt{\frac{x+1}{x-1}} + \sqrt{2(-x-1)} - \operatorname{arctg}\sqrt{\frac{-x-1}{2}} + c_1,\ x\in(-\infty;-1),\ c_1\in R.$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

# Riešené príklady – 174

$$\int \arcsin \sqrt{\frac{x+1}{x-1}}\, dx$$

$$= \left[ \begin{matrix} 0 \leq \frac{x+1}{x-1} \leq 1 \\ x \neq 1 \end{matrix} \middle| \begin{matrix} x-1>0 \Leftrightarrow x>1\colon \;\; 0 \leq x+1 \leq x-1, \text{ t. j. } -1 \leq x,\ 1 \leq -1, \text{ t. j. } x \text{ neexistuje} \\ x-1<0 \Leftrightarrow x<1\colon \;\; 0 \geq x+1 \geq x-1, \text{ t. j. } -1 \geq x,\ 1 \geq -1, \text{ t. j. } x \in (-\infty; -1\rangle \end{matrix} \middle| \begin{matrix} x \in (-\infty; -1\rangle \\ \text{t. j. } x \leq -1 \end{matrix} \right]$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{array}{c|l|r} 0\le\frac{x+1}{x-1}\le 1 & x-1>0 \Leftrightarrow x>1:\quad 0\le x+1\le x-1,\ \text{t. j. } -1\le x,\ 1\le -1,\ \text{t. j. } x \text{ neexistuje} & x\in(-\infty;-1\rangle \\ x\neq 1 & x-1<0 \Leftrightarrow x<1:\quad 0\ge x+1\ge x-1,\ \text{t. j. } -1\ge x,\ 1\ge -1,\ \text{t. j. } x\in(-\infty;-1\rangle & \text{t. j. } x\le -1 \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst. } \quad t=x-1 & x\in(-\infty;-1\rangle \\ dt=dx & t\in(-\infty;-2\rangle \end{array}\right]$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{x+1}{x-1}\leq 1 \\ x\neq 1\end{matrix}\,\middle|\,\begin{matrix} x-1>0 \Leftrightarrow x>1\colon\ \ 0\leq x+1\leq x-1,\ \text{t. j. } -1\leq x,\ 1\leq -1,\ \text{t. j. } x \text{ neexistuje} \\ x-1<0 \Leftrightarrow x<1\colon\ \ 0\geq x+1\geq x-1,\ \text{t. j. } -1\geq x,\ 1\geq -1,\ \text{t. j. } x\in(-\infty;-1\rangle\end{matrix}\,\middle|\,\begin{matrix} x\in(-\infty;-1\rangle \\ \text{t. j. } x\leq -1\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst. } & t=x-1 \\ & dt=dx\end{matrix}\,\middle|\,\begin{matrix} x\in(-\infty;-1\rangle \\ t\in(-\infty;-2\rangle\end{matrix}\right]=\int \arcsin\sqrt{\frac{t+2}{t}}\,dt=\int \arcsin\sqrt{1+\frac{2}{t}}\,dt$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{matrix}0\leq\frac{x+1}{x-1}\leq 1\\ x\neq 1\end{matrix}\;\middle|\;\begin{matrix}x-1>0 \Leftrightarrow x>1:\; 0\leq x+1\leq x-1,\text{ t. j. } -1\leq x,\ 1\leq -1,\text{ t. j. } x\text{ neexistuje}\\ x-1<0 \Leftrightarrow x<1:\; 0\geq x+1\geq x-1,\text{ t. j. } -1\geq x,\ 1\geq -1,\text{ t. j. } x\in(-\infty;-1\rangle\end{matrix}\;\middle|\;\begin{matrix}x\in(-\infty;-1\rangle\\ \text{t. j. } x\leq -1\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst.}\; t=x-1\\ dt=dx\end{matrix}\;\middle|\;\begin{matrix}x\in(-\infty;-1\rangle\\ t\in(-\infty;-2\rangle\end{matrix}\right]=\int\arcsin\sqrt{\frac{t+2}{t}}\,dt=\int\arcsin\sqrt{1+\frac{2}{t}}\,dt$$

$$=\left[\begin{matrix}u=\arcsin\sqrt{1+\frac{2}{t}}\\ v'=1\end{matrix}\;\middle|\;\begin{matrix}u'=\frac{1}{\sqrt{1-(1+\frac{2}{t})}}\cdot\frac{1}{2}\left(1+\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{-\frac{2}{t}}\sqrt{\frac{t+2}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{-2(t+2)t^2}}\\ v=t\qquad\qquad =\frac{-1}{\sqrt{2}\,|t|\,\sqrt{-(t+2)}}=\frac{-\sqrt{2}}{2(-t)\sqrt{-t-2}}=\frac{\sqrt{2}}{2t\sqrt{-t-2}}\end{matrix}\right]$$

$$=t\arcsin\sqrt{\frac{t+2}{t}}-\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{-t-2}}$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{matrix}0\leq\frac{x+1}{x-1}\leq 1\\ x\neq 1\end{matrix}\;\middle|\;\begin{matrix}x-1>0\Leftrightarrow x>1:\;\; 0\leq x+1\leq x-1,\text{ t. j. } -1\leq x,\ 1\leq -1,\text{ t. j. } x\text{ neexistuje}\\ x-1<0\Leftrightarrow x<1:\;\; 0\geq x+1\geq x-1,\text{ t. j. } -1\geq x,\ 1\geq -1,\text{ t. j. } x\in(-\infty;-1\rangle\end{matrix}\;\middle|\;\begin{matrix}x\in(-\infty;-1\rangle\\ \text{t. j. } x\leq -1\end{matrix}\right]$$

$$=\left[\text{Subst.}\;\begin{matrix}t=x-1\\ dt=dx\end{matrix}\;\middle|\;\begin{matrix}x\in(-\infty;-1\rangle\\ t\in(-\infty;-2\rangle\end{matrix}\right]=\int\arcsin\sqrt{\frac{t+2}{t}}\,dt=\int\arcsin\sqrt{1+\frac{2}{t}}\,dt$$

$$=\left[\begin{matrix}u=\arcsin\sqrt{1+\frac{2}{t}}\\ v'=1\end{matrix}\;\middle|\;\begin{matrix}u'=\frac{1}{\sqrt{1-(1+\frac{2}{t})}}\cdot\frac{1}{2}\left(1+\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{-\frac{2}{t}}\sqrt{\frac{t+2}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{-2(t+2)t^2}}\\ v=t\qquad\qquad=\frac{-1}{\sqrt{2}\,|t|\,\sqrt{-(t+2)}}=\frac{-\sqrt{2}}{2(-t)\sqrt{-t-2}}=\frac{\sqrt{2}}{2t\sqrt{-t-2}}\end{matrix}\right]$$

$$=t\arcsin\sqrt{\frac{t+2}{t}}-\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{-t-2}}=\left[\text{Subst.}\;\begin{matrix}z=-t-2\\ dz=-dt\end{matrix}\;\middle|\;\begin{matrix}t\in(-\infty;-2)\\ z\in(0;\infty)\end{matrix}\right]$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{matrix}0\leq\frac{x+1}{x-1}\leq 1\\ x\neq 1\end{matrix}\,\middle|\,\begin{matrix}x-1>0\Leftrightarrow x>1:\quad 0\leq x+1\leq x-1,\ \text{t. j.}\ -1\leq x,\ 1\leq -1,\ \text{t. j.}\ x\ \text{neexistuje}\\ x-1<0\Leftrightarrow x<1:\quad 0\geq x+1\geq x-1,\ \text{t. j.}\ -1\geq x,\ 1\geq -1,\ \text{t. j.}\ x\in(-\infty;-1\rangle\end{matrix}\,\middle|\,\begin{matrix}x\in(-\infty;-1\rangle\\ \text{t. j.}\ x\leq -1\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst.}\ \ t=x-1\\ dt=dx\end{matrix}\,\middle|\,\begin{matrix}x\in(-\infty;-1\rangle\\ t\in(-\infty;-2\rangle\end{matrix}\right]=\int\arcsin\sqrt{\frac{t+2}{t}}\,dt=\int\arcsin\sqrt{1+\frac{2}{t}}\,dt$$

$$=\left[\begin{matrix}u=\arcsin\sqrt{1+\frac{2}{t}}\\ v'=1\end{matrix}\,\middle|\,\begin{matrix}u'=\frac{1}{\sqrt{1-(1+\frac{2}{t})}}\cdot\frac{1}{2}\left(1+\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{-\frac{2}{t}}\sqrt{\frac{t+2}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{-2(t+2)t^2}}\\ v=t\qquad\qquad =\frac{-1}{\sqrt{2}\,|t|\,\sqrt{-(t+2)}}=\frac{-\sqrt{2}}{2(-t)\sqrt{-t-2}}=\frac{\sqrt{2}}{2t\sqrt{-t-2}}\end{matrix}\right]$$

$$=t\arcsin\sqrt{\frac{t+2}{t}}-\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{-t-2}}=\left[\begin{matrix}\text{Subst.}\ \ z=-t-2\\ dz=-\,dt\end{matrix}\,\middle|\,\begin{matrix}t\in(-\infty;-2)\\ z\in(0;\infty)\end{matrix}\right]$$

$$=t\arcsin\sqrt{\frac{t+2}{t}}-\frac{\sqrt{2}}{2}\int\frac{-\,dz}{\sqrt{z}}$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{matrix} 0\le\frac{x+1}{x-1}\le 1 \\ x\neq 1\end{matrix}\middle|\begin{matrix} x-1>0\Leftrightarrow x>1\colon\ \ 0\le x+1\le x-1,\text{ t. j. } -1\le x,\ 1\le -1,\text{ t. j. } x\text{ neexistuje} \\ x-1<0\Leftrightarrow x<1\colon\ \ 0\ge x+1\ge x-1,\text{ t. j. } -1\ge x,\ 1\ge -1,\text{ t. j. } x\in(-\infty;-1\rangle\end{matrix}\middle|\begin{matrix} x\in(-\infty;-1\rangle \\ \text{t. j. } x\le -1\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst.}\ \ t=x-1 \\ dt=dx\end{matrix}\middle|\begin{matrix} x\in(-\infty;-1\rangle \\ t\in(-\infty;-2\rangle\end{matrix}\right]=\int\arcsin\sqrt{\frac{t+2}{t}}\,dt=\int\arcsin\sqrt{1+\frac{2}{t}}\,dt$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{1+\frac{2}{t}} \\ v'=1\end{matrix}\middle|\begin{matrix} u'=\frac{1}{\sqrt{1-(1+\frac{2}{t})}}\cdot\frac{1}{2}\left(1+\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{-\frac{2}{t}}\sqrt{\frac{t+2}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{-2(t+2)t^2}} \\ v=t \qquad\qquad =\frac{-1}{\sqrt{2}\,|t|\,\sqrt{-(t+2)}}=\frac{-\sqrt{2}}{2(-t)\sqrt{-t-2}}=\frac{\sqrt{2}}{2t\sqrt{-t-2}}\end{matrix}\right]$$

$$=t\arcsin\sqrt{\frac{t+2}{t}}-\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{-t-2}}=\left[\begin{matrix}\text{Subst.}\ \ z=-t-2 \\ dz=-\,dt\end{matrix}\middle|\begin{matrix} t\in(-\infty;-2) \\ z\in(0;\infty)\end{matrix}\right]$$

$$=t\arcsin\sqrt{\frac{t+2}{t}}-\frac{\sqrt{2}}{2}\int\frac{-\,dz}{\sqrt{z}}=t\arcsin\sqrt{\frac{t+2}{t}}+\frac{\sqrt{2}}{2}\int z^{-\frac{1}{2}}\,dz$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{array}{c|l|c} 0\leq\frac{x+1}{x-1}\leq 1 & x-1>0 \Leftrightarrow x>1:\ \ 0\leq x+1\leq x-1,\ \text{t. j. } -1\leq x,\ 1\leq -1,\ \text{t. j. } x \text{ neexistuje} & x\in(-\infty;-1\rangle \\ x\neq 1 & x-1<0 \Leftrightarrow x<1:\ \ 0\geq x+1\geq x-1,\ \text{t. j. } -1\geq x,\ 1\geq -1,\ \text{t. j. } x\in(-\infty;-1\rangle & \text{t. j. } x\leq -1 \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in(-\infty;-1\rangle \\ dt=dx & t\in(-\infty;-2\rangle \end{array}\right]=\int \arcsin\sqrt{\frac{t+2}{t}}\,dt=\int \arcsin\sqrt{1+\frac{2}{t}}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{1+\frac{2}{t}} & u'=\frac{1}{\sqrt{1-(1+\frac{2}{t})}}\cdot\frac{1}{2}\left(1+\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{-\frac{2}{t}}\sqrt{\frac{t+2}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{-2(t+2)t^2}} \\ v'=1 & v=t \qquad =\frac{-1}{\sqrt{2}\,|t|\,\sqrt{-(t+2)}}=\frac{-\sqrt{2}}{2(-t)\sqrt{-t-2}}=\frac{\sqrt{2}}{2t\sqrt{-t-2}} \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{t+2}{t}}-\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{-t-2}}=\left[\begin{array}{l|l} \text{Subst. } z=-t-2 & t\in(-\infty;-2) \\ dz=-dt & z\in(0;\infty) \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{t+2}{t}}-\frac{\sqrt{2}}{2}\int\frac{-dz}{\sqrt{z}}=t\arcsin\sqrt{\frac{t+2}{t}}+\frac{\sqrt{2}}{2}\int z^{-\frac{1}{2}}\,dz$$

$$=t\arcsin\sqrt{\frac{t+2}{t}}+\frac{\sqrt{2}}{2}\cdot\frac{z^{\frac{1}{2}}}{\frac{1}{2}}+c_2$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx$$

$$=\left[\begin{array}{c|l|c} 0\le\frac{x+1}{x-1}\le 1 & x-1>0 \Leftrightarrow x>1:\quad 0\le x+1\le x-1,\ \text{t. j. } -1\le x,\ 1\le -1,\ \text{t. j. } x \text{ neexistuje} & x\in(-\infty;-1\rangle \\ x\neq 1 & x-1<0 \Leftrightarrow x<1:\quad 0\ge x+1\ge x-1,\ \text{t. j. } -1\ge x,\ 1\ge -1,\ \text{t. j. } x\in(-\infty;-1\rangle & \text{t. j. } x\le -1 \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in(-\infty;-1\rangle \\ dt=dx & t\in(-\infty;-2\rangle \end{array}\right]=\int \arcsin\sqrt{\frac{t+2}{t}}\,dt=\int \arcsin\sqrt{1+\frac{2}{t}}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{1+\frac{2}{t}} & u'=\frac{1}{\sqrt{1-(1+\frac{2}{t})}}\cdot\frac{1}{2}\left(1+\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-2t^{-2})=\frac{-1}{\sqrt{-\frac{2}{t}}\sqrt{\frac{t+2}{t}}\sqrt{t^4}}=\frac{-1}{\sqrt{-2(t+2)t^2}} \\ & \qquad =\frac{-1}{\sqrt{2}\,|t|\,\sqrt{-(t+2)}}=\frac{-\sqrt{2}}{2(-t)\sqrt{-t-2}}=\frac{\sqrt{2}}{2t\sqrt{-t-2}} \\ v'=1 & v=t \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{t+2}{t}}-\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{-t-2}}=\left[\begin{array}{r|l} \text{Subst. } z=-t-2 & t\in(-\infty;-2) \\ dz=-dt & z\in(0;\infty) \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{t+2}{t}}-\frac{\sqrt{2}}{2}\int\frac{-dz}{\sqrt{z}}=t\arcsin\sqrt{\frac{t+2}{t}}+\frac{\sqrt{2}}{2}\int z^{-\frac{1}{2}}\,dz$$

$$=t\arcsin\sqrt{\frac{t+2}{t}}+\frac{\sqrt{2}}{2}\cdot\frac{z^{\frac{1}{2}}}{\frac{1}{2}}+c_2=t\arcsin\sqrt{\frac{t+2}{t}}+\sqrt{2z}+c_2$$

# Riešené príklady – 174

$$\int \arcsin\sqrt{\frac{x+1}{x-1}}\,dx = (x-1)\arcsin\sqrt{\frac{x+1}{x-1}} + \sqrt{2(-x-1)} + c_2$$

$$= \left[\begin{array}{c|l|r} 0 \le \frac{x+1}{x-1} \le 1 & x-1>0 \Leftrightarrow x>1:\ \ 0 \le x+1 \le x-1,\ \text{t. j.}\ -1 \le x,\ 1 \le -1,\ \text{t. j.}\ x\ \text{neexistuje} & x\in(-\infty;-1\rangle \\ x \ne 1 & x-1<0 \Leftrightarrow x<1:\ \ 0 \ge x+1 \ge x-1,\ \text{t. j.}\ -1 \ge x,\ 1 \ge -1,\ \text{t. j.}\ x\in(-\infty;-1\rangle & \text{t. j. } x \le -1 \end{array}\right]$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t=x-1 & x\in(-\infty;-1\rangle \\ dt=dx & t\in(-\infty;-2\rangle \end{array}\right] = \int \arcsin\sqrt{\frac{t+2}{t}}\,dt = \int \arcsin\sqrt{1+\frac{2}{t}}\,dt$$

$$= \left[\begin{array}{l|l} u = \arcsin\sqrt{1+\frac{2}{t}} & u' = \frac{1}{\sqrt{1-(1+\frac{2}{t})}}\cdot\frac{1}{2}\left(1+\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-2t^{-2}) = \frac{-1}{\sqrt{-\frac{2}{t}}\sqrt{\frac{t+2}{t}}\sqrt{t^4}} = \frac{-1}{\sqrt{-2(t+2)t^2}} \\ v' = 1 & v = t \qquad\qquad = \frac{-1}{\sqrt{2}\,|t|\,\sqrt{-(t+2)}} = \frac{-\sqrt{2}}{2(-t)\sqrt{-t-2}} = \frac{\sqrt{2}}{2t\sqrt{-t-2}} \end{array}\right]$$

$$= t\arcsin\sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{-t-2}} = \left[\begin{array}{r|l} \text{Subst.}\quad z=-t-2=-x-1 & t\in(-\infty;-2) \\ dz=-dt & z\in(0;\infty) \end{array}\right]$$

$$= t\arcsin\sqrt{\frac{t+2}{t}} - \frac{\sqrt{2}}{2}\int\frac{-dz}{\sqrt{z}} = t\arcsin\sqrt{\frac{t+2}{t}} + \frac{\sqrt{2}}{2}\int z^{-\frac{1}{2}}\,dz$$

$$= t\arcsin\sqrt{\frac{t+2}{t}} + \frac{\sqrt{2}}{2}\cdot\frac{z^{\frac{1}{2}}}{\frac{1}{2}} + c_2 = t\arcsin\sqrt{\frac{t+2}{t}} + \sqrt{2z} + c_2$$

$$= (x-1)\arcsin\sqrt{\frac{x+1}{x-1}} + \sqrt{2(-x-1)} + c_2,\ x\in(-\infty;-1),\ c_2\in R.$$

# Riešené príklady – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}}\, dx$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{x-1}{x+1}\leq 1 \\ x\neq -1 \end{matrix}\,\middle|\, \begin{matrix} x+1>0 \Leftrightarrow x>-1\colon & 0\leq x-1\leq x+1, & \text{t. j. } 1\leq x,\ -1\leq 1, & \text{t. j. } & x\in\langle 1;\infty) \\ x+1<0 \Leftrightarrow x<-1\colon & 0\geq x-1\geq x+1, & \text{t. j. } 1\geq x,\ -1\geq 1, & \text{t. j. } & x \text{ neexistuje} \end{matrix}\,\middle|\, \begin{matrix} x\in\langle 1;\infty) \\ \text{t. j. } 1\leq x \end{matrix}\right]$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{c} 0\leq\frac{x-1}{x+1}\leq 1 \\ x\neq -1 \end{array}\middle| \begin{array}{l} x+1>0 \Leftrightarrow x>-1:\quad 0\leq x-1\leq x+1,\ \text{t. j. } 1\leq x,\ -1\leq 1,\ \text{t. j. } x\in\langle 1;\infty) \\ x+1<0 \Leftrightarrow x<-1:\quad 0\geq x-1\geq x+1,\ \text{t. j. } 1\geq x,\ -1\geq 1,\ \text{t. j. } x \text{ neexistuje} \end{array}\middle| \begin{array}{c} x\in\langle 1;\infty) \\ \text{t. j. } 1\leq x \end{array}\right]$$

$$=\left[\begin{array}{l} u=\arcsin\sqrt{\frac{x-1}{x+1}} \\ v'=1 \end{array}\middle| \begin{array}{l} u'=\frac{1}{\sqrt{1-\frac{x-1}{x+1}}}\cdot\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}\cdot\frac{(x+1)-(x-1)}{(x+1)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}}\sqrt{\frac{x-1}{x+1}}\sqrt{(x+1)^4}}=\frac{1}{\sqrt{\frac{2}{x+1}\cdot\frac{x-1}{x+1}\cdot(x+1)^4}} \\ \qquad =\frac{1}{\sqrt{2(x+1)\cdot(x+1)^2}}=\frac{1}{\sqrt{2}\,|x+1|\,\sqrt{x-1}}=\frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \\ v=x \end{array}\right]$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{x-1}{x+1}\leq 1 \\ x\neq -1\end{matrix}\,\middle|\,\begin{matrix} x+1>0 \Leftrightarrow x>-1\colon\ \ 0\leq x-1\leq x+1,\ \text{t. j. } 1\leq x,\ -1\leq 1,\ \text{t. j. } x\in\langle 1;\infty) \\ x+1<0 \Leftrightarrow x<-1\colon\ \ 0\geq x-1\geq x+1,\ \text{t. j. } 1\geq x,\ -1\geq 1,\ \text{t. j. } x \text{ neexistuje}\end{matrix}\,\middle|\,\begin{matrix} x\in\langle 1;\infty) \\ \text{t. j. } 1\leq x\end{matrix}\right]$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{\frac{x-1}{x+1}} \\ v'=1\end{matrix}\,\middle|\,\begin{matrix} u'=\frac{1}{\sqrt{1-\frac{x-1}{x+1}}}\cdot\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}\cdot\frac{(x+1)-(x-1)}{(x+1)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}}\sqrt{\frac{x-1}{x+1}}\sqrt{(x+1)^4}}=\frac{1}{\sqrt{\frac{2}{x+1}\cdot\frac{x-1}{x+1}\cdot(x+1)^4}} \\ =\frac{1}{\sqrt{2(x+1)\cdot(x+1)^2}}=\frac{1}{\sqrt{2}\,|x+1|\,\sqrt{x-1}}=\frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \\ v=x\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x+1)\sqrt{x-1}}$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{c|l|c} 0\le\frac{x-1}{x+1}\le 1 & x+1>0 \Leftrightarrow x>-1:\quad 0\le x-1\le x+1,\ \text{t. j. } 1\le x,\ -1\le 1,\ \text{t. j. } x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\ne -1 & x+1<0 \Leftrightarrow x<-1:\quad 0\ge x-1\ge x+1,\ \text{t. j. } 1\ge x,\ -1\ge 1,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 1\le x \end{array}\right]$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{x-1}{x+1}} & u'=\frac{1}{\sqrt{1-\frac{x-1}{x+1}}}\cdot\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}\cdot\frac{(x+1)-(x-1)}{(x+1)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}}\sqrt{\frac{x-1}{x+1}}\sqrt{(x+1)^4}}=\frac{1}{\sqrt{\frac{2}{x+1}\cdot\frac{x-1}{x+1}\cdot(x+1)^4}} \\ & \qquad=\frac{1}{\sqrt{2(x+1)\cdot(x+1)^2}}=\frac{1}{\sqrt{2}\,|x+1|\,\sqrt{x-1}}=\frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \\ v'=1 & v=x \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x+1)\sqrt{x-1}}=\left[\begin{array}{r|c|c} \text{Subst. } t=\sqrt{x-1} & t^2=x-1 & x\in(1;\infty) \\ x=t^2+1 & dx=2t\,dt & t\in(0;\infty) \end{array}\right]$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{c|l|l} 0\leq\frac{x-1}{x+1}\leq 1 & x+1>0 \Leftrightarrow x>-1:\ \ 0\leq x-1\leq x+1,\ \text{t. j. } 1\leq x,\ -1\leq 1,\ \text{t. j. } x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\neq -1 & x+1<0 \Leftrightarrow x<-1:\ \ 0\geq x-1\geq x+1,\ \text{t. j. } 1\geq x,\ -1\geq 1,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 1\leq x \end{array}\right]$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{x-1}{x+1}} & u'=\frac{1}{\sqrt{1-\frac{x-1}{x+1}}}\cdot\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}\cdot\frac{(x+1)-(x-1)}{(x+1)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}}\sqrt{\frac{x-1}{x+1}}\sqrt{(x+1)^4}}=\frac{1}{\sqrt{\frac{2}{x+1}\cdot\frac{x-1}{x+1}\cdot(x+1)^4}} \\ & \quad =\frac{1}{\sqrt{2(x+1)\cdot(x+1)^2}}=\frac{1}{\sqrt{2}\,|x+1|\,\sqrt{x-1}}=\frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \\ v'=1 & v=x \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x+1)\sqrt{x-1}}=\left[\begin{array}{r|l|l} \text{Subst. } t=\sqrt{x-1} & t^2=x-1 & x\in(1;\infty) \\ x=t^2+1 & dx=2t\,dt & t\in(0;\infty) \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\frac{\sqrt{2}}{2}\int\frac{(t^2+1)\cdot 2t\,dt}{(t^2+2)t}$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{x-1}{x+1}\leq 1 \\ x\neq -1\end{matrix}\,\middle|\,\begin{matrix} x+1>0 \Leftrightarrow x>-1:\ \ 0\leq x-1\leq x+1,\ \text{t. j. } 1\leq x,\ -1\leq 1,\ \text{t. j. } x\in\langle 1;\infty) \\ x+1<0 \Leftrightarrow x<-1:\ \ 0\geq x-1\geq x+1,\ \text{t. j. } 1\geq x,\ -1\geq 1,\ \text{t. j. } x \text{ neexistuje}\end{matrix}\,\middle|\,\begin{matrix} x\in\langle 1;\infty) \\ \text{t. j. } 1\leq x\end{matrix}\right]$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{\frac{x-1}{x+1}} \\ v'=1\end{matrix}\,\middle|\,\begin{matrix} u'=\frac{1}{\sqrt{1-\frac{x-1}{x+1}}}\cdot\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}\cdot\frac{(x+1)-(x-1)}{(x+1)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}}\sqrt{\frac{x-1}{x+1}}\sqrt{(x+1)^4}}=\frac{1}{\sqrt{\frac{2}{x+1}\cdot\frac{x-1}{x+1}\cdot(x+1)^4}} \\ =\frac{1}{\sqrt{2(x+1)\cdot(x+1)^2}}=\frac{1}{\sqrt{2}\,|x+1|\,\sqrt{x-1}}=\frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \\ v=x\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x+1)\sqrt{x-1}}=\left[\begin{matrix}\text{Subst. } t=\sqrt{x-1} \\ x=t^2+1\end{matrix}\,\middle|\,\begin{matrix} t^2=x-1 \\ dx=2t\,dt\end{matrix}\,\middle|\,\begin{matrix} x\in(1;\infty) \\ t\in(0;\infty)\end{matrix}\right]$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\frac{\sqrt{2}}{2}\int\frac{(t^2+1)\cdot 2t\,dt}{(t^2+2)t}=x\arcsin\sqrt{\frac{x-1}{x+1}}-\sqrt{2}\int\frac{(t^2+1)\,dt}{t^2+2}$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{c|l|l} 0\le\frac{x-1}{x+1}\le 1 & x+1>0 \Leftrightarrow x>-1:\ \ 0\le x-1\le x+1,\ \text{t. j. } 1\le x,\ -1\le 1,\ \text{t. j. } x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\ne -1 & x+1<0 \Leftrightarrow x<-1:\ \ 0\ge x-1\ge x+1,\ \text{t. j. } 1\ge x,\ -1\ge 1,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 1\le x \end{array}\right]$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{x-1}{x+1}} & u'=\frac{1}{\sqrt{1-\frac{x-1}{x+1}}}\cdot\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}\cdot\frac{(x+1)-(x-1)}{(x+1)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}}\sqrt{\frac{x-1}{x+1}}\sqrt{(x+1)^4}}=\frac{1}{\sqrt{\frac{2}{x+1}\cdot\frac{x-1}{x+1}\cdot(x+1)^4}} \\ & \qquad =\frac{1}{\sqrt{2(x+1)\cdot(x+1)^2}}=\frac{1}{\sqrt{2}\,|x+1|\,\sqrt{x-1}}=\frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \\ v'=1 & v=x \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x+1)\sqrt{x-1}}=\left[\begin{array}{r|l|l} \text{Subst. } t=\sqrt{x-1} & t^2=x-1 & x\in(1;\infty) \\ x=t^2+1 & dx=2t\,dt & t\in(0;\infty) \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\frac{\sqrt{2}}{2}\int\frac{(t^2+1)\cdot 2t\,dt}{(t^2+2)t}=x\arcsin\sqrt{\frac{x-1}{x+1}}-\sqrt{2}\int\frac{(t^2+1)\,dt}{t^2+2}$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\sqrt{2}\int\frac{(t^2+2-1)\,dt}{t^2+2}$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{c|l|c} 0\le\frac{x-1}{x+1}\le 1 & x+1>0 \Leftrightarrow x>-1:\quad 0\le x-1\le x+1,\ \text{t. j. } 1\le x,\ -1\le 1,\ \text{t. j. } x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\ne -1 & x+1<0 \Leftrightarrow x<-1:\quad 0\ge x-1\ge x+1,\ \text{t. j. } 1\ge x,\ -1\ge 1,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 1\le x \end{array}\right]$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{x-1}{x+1}} & u'=\frac{1}{\sqrt{1-\frac{x-1}{x+1}}}\cdot\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}\cdot\frac{(x+1)-(x-1)}{(x+1)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}}\sqrt{\frac{x-1}{x+1}}\sqrt{(x+1)^4}}=\frac{1}{\sqrt{\frac{2}{x+1}\cdot\frac{x-1}{x+1}\cdot(x+1)^4}} \\ & \quad =\frac{1}{\sqrt{2(x+1)\cdot(x+1)^2}}=\frac{1}{\sqrt{2}\,|x+1|\,\sqrt{x-1}}=\frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \\ v'=1 & v=x \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x+1)\sqrt{x-1}}=\left[\begin{array}{r|l|l} \text{Subst. } t=\sqrt{x-1} & t^2=x-1 & x\in(1;\infty) \\ x=t^2+1 & dx=2t\,dt & t\in(0;\infty) \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\frac{\sqrt{2}}{2}\int\frac{(t^2+1)\cdot 2t\,dt}{(t^2+2)t}=x\arcsin\sqrt{\frac{x-1}{x+1}}-\sqrt{2}\int\frac{(t^2+1)\,dt}{t^2+2}$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\sqrt{2}\int\frac{(t^2+2-1)\,dt}{t^2+2}=x\arcsin\sqrt{\frac{x-1}{x+1}}-\sqrt{2}\int dt+\sqrt{2}\int\frac{dt}{t^2+(\sqrt{2})^2}$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{c|l|c} 0\leq\frac{x-1}{x+1}\leq 1 & x+1>0 \Leftrightarrow x>-1:\quad 0\leq x-1\leq x+1,\ \text{t. j. } 1\leq x,\ -1\leq 1,\ \text{t. j. } x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\neq -1 & x+1<0 \Leftrightarrow x<-1:\quad 0\geq x-1\geq x+1,\ \text{t. j. } 1\geq x,\ -1\geq 1,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 1\leq x \end{array}\right]$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{\frac{x-1}{x+1}} & u'=\frac{1}{\sqrt{1-\frac{x-1}{x+1}}}\cdot\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}\cdot\frac{(x+1)-(x-1)}{(x+1)^2}=\frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}}\sqrt{\frac{x-1}{x+1}}\sqrt{(x+1)^4}}=\frac{1}{\sqrt{\frac{2}{x+1}\cdot\frac{x-1}{x+1}\cdot(x+1)^4}} \\ & \qquad =\frac{1}{\sqrt{2(x+1)\cdot(x+1)^2}}=\frac{1}{\sqrt{2}\,|x+1|\,\sqrt{x-1}}=\frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \\ v'=1 & v=x \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x+1)\sqrt{x-1}}=\left[\begin{array}{r|c|c} \text{Subst. } t=\sqrt{x-1} & t^2=x-1 & x\in(1;\infty) \\ x=t^2+1 & dx=2t\,dt & t\in(0;\infty) \end{array}\right]$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\frac{\sqrt{2}}{2}\int\frac{(t^2+1)\cdot 2t\,dt}{(t^2+2)t}=x\arcsin\sqrt{\frac{x-1}{x+1}}-\sqrt{2}\int\frac{(t^2+1)\,dt}{t^2+2}$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\sqrt{2}\int\frac{(t^2+2-1)\,dt}{t^2+2}=x\arcsin\sqrt{\frac{x-1}{x+1}}-\sqrt{2}\int dt+\sqrt{2}\int\frac{dt}{t^2+(\sqrt{2})^2}$$

$$=x\arcsin\sqrt{\frac{x-1}{x+1}}-\sqrt{2}t+\frac{\sqrt{2}}{\sqrt{2}}\operatorname{arctg}\frac{t}{\sqrt{2}}+c_1$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx = x\arcsin\sqrt{\frac{x-1}{x+1}} - \sqrt{2(x-1)} + \operatorname{arctg}\sqrt{\frac{x-1}{2}} + c_1$$

$$= \left[\begin{matrix} 0 \le \frac{x-1}{x+1} \le 1 \\ x \ne -1 \end{matrix}\ \middle|\ \begin{matrix} x+1>0 \Leftrightarrow x>-1\colon\ \ 0\le x-1 \le x+1,\ \text{t. j. } 1\le x,\ -1\le 1,\ \text{t. j. } x\in\langle 1;\infty) \\ x+1<0 \Leftrightarrow x<-1\colon\ \ 0\ge x-1 \ge x+1,\ \text{t. j. } 1\ge x,\ -1\ge 1,\ \text{t. j. } x \text{ neexistuje} \end{matrix}\ \middle|\ \begin{matrix} x\in\langle 1;\infty) \\ \text{t. j. } 1\le x \end{matrix}\right]$$

$$= \left[\begin{matrix} u = \arcsin\sqrt{\frac{x-1}{x+1}} \\ v' = 1 \end{matrix}\ \middle|\ \begin{matrix} u' = \frac{1}{\sqrt{1-\frac{x-1}{x+1}}}\cdot\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}\cdot\frac{(x+1)-(x-1)}{(x+1)^2} = \frac{\frac{1}{2}\cdot 2}{\sqrt{\frac{x+1-(x-1)}{x+1}}\sqrt{\frac{x-1}{x+1}}\sqrt{(x+1)^4}} = \frac{1}{\sqrt{\frac{2}{x+1}\cdot\frac{x-1}{x+1}\cdot(x+1)^4}} \\ = \frac{1}{\sqrt{2(x+1)\cdot(x+1)^2}} = \frac{1}{\sqrt{2}\,|x+1|\,\sqrt{x-1}} = \frac{\sqrt{2}}{2(x+1)\sqrt{x-1}} \\ v = x \end{matrix}\right]$$

$$= x\arcsin\sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2}\int\frac{x\,dx}{(x+1)\sqrt{x-1}} = \left[\begin{matrix} \text{Subst. } t=\sqrt{x-1} \\ x=t^2+1 \end{matrix}\ \middle|\ \begin{matrix} t^2=x-1 \\ dx = 2t\,dt \end{matrix}\ \middle|\ \begin{matrix} x\in(1;\infty) \\ t\in(0;\infty) \end{matrix}\right]$$

$$= x\arcsin\sqrt{\frac{x-1}{x+1}} - \frac{\sqrt{2}}{2}\int\frac{(t^2+1)\cdot 2t\,dt}{(t^2+2)t} = x\arcsin\sqrt{\frac{x-1}{x+1}} - \sqrt{2}\int\frac{(t^2+1)\,dt}{t^2+2}$$

$$= x\arcsin\sqrt{\frac{x-1}{x+1}} - \sqrt{2}\int\frac{(t^2+2-1)\,dt}{t^2+2} = x\arcsin\sqrt{\frac{x-1}{x+1}} - \sqrt{2}\int dt + \sqrt{2}\int\frac{dt}{t^2+(\sqrt{2})^2}$$

$$= x\arcsin\sqrt{\frac{x-1}{x+1}} - \sqrt{2}\,t + \frac{\sqrt{2}}{\sqrt{2}}\operatorname{arctg}\frac{t}{\sqrt{2}} + c_1$$

$$= x\arcsin\sqrt{\frac{x-1}{x+1}} - \sqrt{2(x-1)} + \operatorname{arctg}\sqrt{\frac{x-1}{2}} + c_1,\ x\in(-\infty;-1),\ c_1\in R.$$

# Riešené príklady – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}}\, dx$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{c|l|c} 0\leq\frac{x-1}{x+1}\leq 1 & x+1>0 \Leftrightarrow x>-1:\quad 0\leq x-1\leq x+1,\ \text{t. j. } 1\leq x,\ -1\leq 1,\ \text{t. j. }\ x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\neq -1 & x+1<0 \Leftrightarrow x<-1:\quad 0\geq x-1\geq x+1,\ \text{t. j. } 1\geq x,\ -1\geq 1,\ \text{t. j. }\ x \text{ neexistuje} & \text{t. j. }\ 1\leq x \end{array}\right]$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{c|l|c} 0\le\frac{x-1}{x+1}\le 1 & x+1>0 \Leftrightarrow x>-1\colon\ \ 0\le x-1\le x+1,\ \text{t. j. } 1\le x,\ -1\le 1,\ \text{t. j. } x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\ne -1 & x+1<0 \Leftrightarrow x<-1\colon\ \ 0\ge x-1\ge x+1,\ \text{t. j. } 1\ge x,\ -1\ge 1,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 1\le x \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=x+1 & x\in\langle 1;\infty) \\ dt=dx & t\in\langle 2;\infty) \end{array}\right]$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{matrix} 0\le\frac{x-1}{x+1}\le 1 \\ x\neq -1\end{matrix}\,\middle|\,\begin{matrix} x+1>0 \Leftrightarrow x>-1\colon & 0\le x-1\le x+1, \text{ t. j. } 1\le x,\ -1\le 1, \text{ t. j. } x\in\langle 1;\infty) \\ x+1<0 \Leftrightarrow x<-1\colon & 0\ge x-1\ge x+1, \text{ t. j. } 1\ge x,\ -1\ge 1, \text{ t. j. } x \text{ neexistuje}\end{matrix}\,\middle|\,\begin{matrix} x\in\langle 1;\infty) \\ \text{t. j. } 1\le x\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst.} & t=x+1 \\ & dt=dx\end{matrix}\,\middle|\,\begin{matrix} x\in\langle 1;\infty) \\ t\in\langle 2;\infty)\end{matrix}\right]=\int \arcsin\sqrt{\frac{t-2}{t}}\,dt=\int \arcsin\sqrt{1-\frac{2}{t}}\,dt$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{c|l|c} 0\leq\frac{x-1}{x+1}\leq 1 & x+1>0 \Leftrightarrow x>-1:\ \ 0\leq x-1\leq x+1,\ \text{t. j. } 1\leq x,\ -1\leq 1,\ \text{t. j. } x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\neq -1 & x+1<0 \Leftrightarrow x<-1:\ \ 0\geq x-1\geq x+1,\ \text{t. j. } 1\geq x,\ -1\geq 1,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 1\leq x \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=x+1 & x\in\langle 1;\infty) \\ dt=dx & t\in\langle 2;\infty) \end{array}\right]=\int \arcsin\sqrt{\frac{t-2}{t}}\,dt=\int \arcsin\sqrt{1-\frac{2}{t}}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{1-\frac{2}{t}} & u'=\frac{1}{\sqrt{1-(1-\frac{2}{t})}}\cdot\frac{1}{2}\left(1-\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-1)\cdot(-2t^{-2})=\frac{1}{\sqrt{\frac{2}{t}}\sqrt{\frac{t-2}{t}}\sqrt{t^4}}=\frac{1}{\sqrt{2(t-2)t^2}}=\frac{1}{\sqrt{2}\,|t|\,\sqrt{t-2}}=\frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v'=1 & v=t \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{t-2}}$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{c|l|c} 0\leq\frac{x-1}{x+1}\leq 1 & x+1>0 \Leftrightarrow x>-1:\ \ 0\leq x-1\leq x+1,\ \text{t. j. } 1\leq x,\ -1\leq 1,\ \text{t. j. } x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\neq -1 & x+1<0 \Leftrightarrow x<-1:\ \ 0\geq x-1\geq x+1,\ \text{t. j. } 1\geq x,\ -1\geq 1,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 1\leq x \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=x+1 & x\in\langle 1;\infty) \\ dt=dx & t\in\langle 2;\infty) \end{array}\right] = \int \arcsin\sqrt{\frac{t-2}{t}}\,dt = \int \arcsin\sqrt{1-\frac{2}{t}}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{1-\frac{2}{t}} & u'=\frac{1}{\sqrt{1-(1-\frac{2}{t})}}\cdot\frac{1}{2}\left(1-\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-1)\cdot(-2t^{-2}) = \frac{1}{\sqrt{\frac{2}{t}}\sqrt{\frac{t-2}{t}}\sqrt{t^4}} = \frac{1}{\sqrt{2(t-2)t^2}} = \frac{1}{\sqrt{2}\,|t|\,\sqrt{t-2}} = \frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v'=1 & v=t \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{t-2}} = \left[\begin{array}{r|l} \text{Subst. } z=t-2 & t\in(2;\infty) \\ dz=dt & z\in(0;\infty) \end{array}\right]$$

# Riešené príklady – 175

$$\int \arcsin \sqrt{\frac{x-1}{x+1}}\, dx$$

$$= \left[ \begin{array}{c|l|c} 0 \le \frac{x-1}{x+1} \le 1 & x+1>0 \Leftrightarrow x>-1:\ \ 0\le x-1 \le x+1,\ \text{t. j. } 1\le x,\ -1\le 1,\ \text{t. j. } x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\neq -1 & x+1<0 \Leftrightarrow x<-1:\ \ 0\ge x-1\ge x+1,\ \text{t. j. } 1\ge x,\ -1\ge 1,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 1\le x \end{array} \right]$$

$$= \left[ \begin{array}{r|l} \text{Subst. } t=x+1 & x\in\langle 1;\infty) \\ dt=dx & t\in\langle 2;\infty) \end{array} \right] = \int \arcsin \sqrt{\frac{t-2}{t}}\, dt = \int \arcsin \sqrt{1-\frac{2}{t}}\, dt$$

$$= \left[ \begin{array}{l|l} u=\arcsin\sqrt{1-\frac{2}{t}} & u'=\frac{1}{\sqrt{1-(1-\frac{2}{t})}}\cdot\frac{1}{2}\left(1-\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-1)\cdot(-2t^{-2}) = \frac{1}{\sqrt{\frac{2}{t}}\sqrt{\frac{t-2}{t}}\sqrt{t^4}} = \frac{1}{\sqrt{2(t-2)t^2}} = \frac{1}{\sqrt{2}\,|t|\,\sqrt{t-2}} = \frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v'=1 & v=t \end{array} \right]$$

$$= t\arcsin\sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2}\int \frac{t\,dt}{t\sqrt{t-2}} = \left[ \begin{array}{r|l} \text{Subst. } z=t-2 & t\in(2;\infty) \\ dz=dt & z\in(0;\infty) \end{array} \right]$$

$$= t\arcsin\sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2}\int \frac{dz}{\sqrt{z}}$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{c|l|c} 0\leq\frac{x-1}{x+1}\leq 1 & x+1>0 \Leftrightarrow x>-1:\ \ 0\leq x-1\leq x+1,\ \text{t. j. } 1\leq x,\ -1\leq 1,\ \text{t. j. } x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\neq -1 & x+1<0 \Leftrightarrow x<-1:\ \ 0\geq x-1\geq x+1,\ \text{t. j. } 1\geq x,\ -1\geq 1,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 1\leq x \end{array}\right]$$

$$=\left[\begin{array}{l|l} \text{Subst. } t=x+1 & x\in\langle 1;\infty) \\ dt=dx & t\in\langle 2;\infty) \end{array}\right]=\int \arcsin\sqrt{\frac{t-2}{t}}\,dt=\int \arcsin\sqrt{1-\frac{2}{t}}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{1-\frac{2}{t}} & u'=\frac{1}{\sqrt{1-(1-\frac{2}{t})}}\cdot\frac{1}{2}\left(1-\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-1)\cdot(-2t^{-2})=\frac{1}{\sqrt{\frac{2}{t}}\sqrt{\frac{t-2}{t}}\sqrt{t^4}}=\frac{1}{\sqrt{2(t-2)t^2}}=\frac{1}{\sqrt{2}\,|t|\,\sqrt{t-2}}=\frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v'=1 & v=t \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{t-2}}=\left[\begin{array}{l|l} \text{Subst. } z=t-2 & t\in(2;\infty) \\ dz=dt & z\in(0;\infty) \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\int\frac{dz}{\sqrt{z}}=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\int z^{-\frac{1}{2}}\,dz$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{array}{c|l|r} 0\leq\frac{x-1}{x+1}\leq 1 & x+1>0 \Leftrightarrow x>-1:\quad 0\leq x-1\leq x+1,\ \text{t. j. } 1\leq x,\ -1\leq 1,\ \text{t. j. } x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\neq -1 & x+1<0 \Leftrightarrow x<-1:\quad 0\geq x-1\geq x+1,\ \text{t. j. } 1\geq x,\ -1\geq 1,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 1\leq x \end{array}\right]$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=x+1 & x\in\langle 1;\infty) \\ dt=dx & t\in\langle 2;\infty) \end{array}\right]=\int \arcsin\sqrt{\frac{t-2}{t}}\,dt=\int \arcsin\sqrt{1-\frac{2}{t}}\,dt$$

$$=\left[\begin{array}{l|l} u=\arcsin\sqrt{1-\frac{2}{t}} & u'=\frac{1}{\sqrt{1-(1-\frac{2}{t})}}\cdot\frac{1}{2}\left(1-\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-1)\cdot(-2t^{-2})=\frac{1}{\sqrt{\frac{2}{t}}\sqrt{\frac{t-2}{t}}\sqrt{t^4}}=\frac{1}{\sqrt{2(t-2)t^2}}=\frac{1}{\sqrt{2}\,|t|\,\sqrt{t-2}}=\frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v'=1 & v=t \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{t-2}}=\left[\begin{array}{r|l} \text{Subst. } z=t-2 & t\in(2;\infty) \\ dz=dt & z\in(0;\infty) \end{array}\right]$$

$$=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\int\frac{dz}{\sqrt{z}}=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\int z^{-\frac{1}{2}}\,dz$$

$$=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\cdot\frac{z^{\frac{1}{2}}}{\frac{1}{2}}+c_2$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx$$

$$=\left[\begin{matrix} 0\leq\frac{x-1}{x+1}\leq 1 \\ x\neq -1\end{matrix}\;\middle|\; \begin{matrix} x+1>0 \Leftrightarrow x>-1\colon\; 0\leq x-1\leq x+1,\ \text{t. j. } 1\leq x,\ -1\leq 1,\ \text{t. j. } x\in\langle 1;\infty) \\ x+1<0 \Leftrightarrow x<-1\colon\; 0\geq x-1\geq x+1,\ \text{t. j. } 1\geq x,\ -1\geq 1,\ \text{t. j. } x \text{ neexistuje}\end{matrix}\;\middle|\; \begin{matrix} x\in\langle 1;\infty) \\ \text{t. j. } 1\leq x\end{matrix}\right]$$

$$=\left[\begin{matrix}\text{Subst.}\quad t=x+1 \\ dt=dx\end{matrix}\;\middle|\;\begin{matrix} x\in\langle 1;\infty) \\ t\in\langle 2;\infty)\end{matrix}\right]=\int \arcsin\sqrt{\frac{t-2}{t}}\,dt=\int \arcsin\sqrt{1-\frac{2}{t}}\,dt$$

$$=\left[\begin{matrix} u=\arcsin\sqrt{1-\frac{2}{t}} \\ v'=1\end{matrix}\;\middle|\;\begin{matrix} u'=\frac{1}{\sqrt{1-(1-\frac{2}{t})}}\cdot\frac{1}{2}\left(1-\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-1)\cdot(-2t^{-2})=\frac{1}{\sqrt{\frac{2}{t}}\sqrt{\frac{t-2}{t}}\sqrt{t^4}}=\frac{1}{\sqrt{2(t-2)t^2}}=\frac{1}{\sqrt{2}\,|t|\,\sqrt{t-2}}=\frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v=t\end{matrix}\right]$$

$$=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{t-2}}=\left[\begin{matrix}\text{Subst.}\quad z=t-2 \\ dz=dt\end{matrix}\;\middle|\;\begin{matrix} t\in(2;\infty) \\ z\in(0;\infty)\end{matrix}\right]$$

$$=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\int\frac{dz}{\sqrt{z}}=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\int z^{-\frac{1}{2}}\,dz$$

$$=t\arcsin\sqrt{\frac{t-2}{t}}-\frac{\sqrt{2}}{2}\cdot\frac{z^{\frac{1}{2}}}{\frac{1}{2}}+c_2=t\arcsin\sqrt{\frac{t-2}{t}}-\sqrt{2z}+c_2$$

# Riešené príklady – 175

$$\int \arcsin\sqrt{\frac{x-1}{x+1}}\,dx = (x+1)\arcsin\sqrt{\frac{x-1}{x+1}} - \sqrt{2(x+1)} + c_2$$

$$= \left[\begin{array}{c|l|r} 0\leq \frac{x-1}{x+1}\leq 1 & x+1>0 \Leftrightarrow x>-1:\ \ 0\leq x-1\leq x+1,\ \text{t. j. } 1\leq x,\ -1\leq 1,\ \text{t. j. } x\in\langle 1;\infty) & x\in\langle 1;\infty) \\ x\neq -1 & x+1<0 \Leftrightarrow x<-1:\ \ 0\geq x-1\geq x+1,\ \text{t. j. } 1\geq x,\ -1\geq 1,\ \text{t. j. } x \text{ neexistuje} & \text{t. j. } 1\leq x \end{array}\right]$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x+1 & x\in\langle 1;\infty) \\ dt=dx & t\in\langle 2;\infty) \end{array}\right] = \int \arcsin\sqrt{\frac{t-2}{t}}\,dt = \int \arcsin\sqrt{1-\frac{2}{t}}\,dt$$

$$= \left[\begin{array}{l|l} u=\arcsin\sqrt{1-\frac{2}{t}} & u'=\frac{1}{\sqrt{1-(1-\frac{2}{t})}}\cdot\frac{1}{2}\left(1-\frac{2}{t}\right)^{-\frac{1}{2}}\cdot(-1)\cdot(-2t^{-2}) = \frac{1}{\sqrt{\frac{2}{t}}\sqrt{\frac{t-2}{t}}\sqrt{t^4}} = \frac{1}{\sqrt{2(t-2)t^2}} = \frac{1}{\sqrt{2}\,|t|\,\sqrt{t-2}} = \frac{\sqrt{2}}{2t\sqrt{t-2}} \\ v'=1 & v=t \end{array}\right]$$

$$= t\arcsin\sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2}\int\frac{t\,dt}{t\sqrt{t-2}} = \left[\begin{array}{r|l} \text{Subst. } z=t-2=x+1 & t\in(2;\infty) \\ dz=dt & z\in(0;\infty) \end{array}\right]$$

$$= t\arcsin\sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2}\int\frac{dz}{\sqrt{z}} = t\arcsin\sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2}\int z^{-\frac{1}{2}}\,dz$$

$$= t\arcsin\sqrt{\frac{t-2}{t}} - \frac{\sqrt{2}}{2}\cdot\frac{z^{\frac{1}{2}}}{\frac{1}{2}} + c_2 = t\arcsin\sqrt{\frac{t-2}{t}} - \sqrt{2z} + c_2$$

$$= (x+1)\arcsin\sqrt{\frac{x-1}{x+1}} - \sqrt{2(x+1)} + c_2,\ x\in(1;\infty),\ c_2\in R.$$

# Riešené príklady – 176, 177

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{x-5}}$$

$$\int \frac{dx}{\sqrt{x-3}-\sqrt{x-5}}$$

# Riešené príklady – 176, 177

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{x-5}}$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{x-5}} \cdot \frac{\sqrt{x-3}-\sqrt{x-5}}{\sqrt{x-3}-\sqrt{x-5}}\, dx$$

$$\int \frac{dx}{\sqrt{x-3}-\sqrt{x-5}}$$

$$= \int \frac{1}{\sqrt{x-3}-\sqrt{x-5}} \cdot \frac{\sqrt{x-3}+\sqrt{x-5}}{\sqrt{x-3}+\sqrt{x-5}}\, dx$$

# Riešené príklady – 176, 177

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{x-5}}$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{x-5}} \cdot \frac{\sqrt{x-3}-\sqrt{x-5}}{\sqrt{x-3}-\sqrt{x-5}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{x-5}}{x-3-(x-5)}\, dx$$

$$\int \frac{dx}{\sqrt{x-3}-\sqrt{x-5}}$$

$$= \int \frac{1}{\sqrt{x-3}-\sqrt{x-5}} \cdot \frac{\sqrt{x-3}+\sqrt{x-5}}{\sqrt{x-3}+\sqrt{x-5}}\, dx = \int \frac{\sqrt{x-3}+\sqrt{x-5}}{x-3-(x-5)}\, dx$$

# Riešené príklady – 176, 177

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{x-5}}$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{x-5}} \cdot \frac{\sqrt{x-3}-\sqrt{x-5}}{\sqrt{x-3}-\sqrt{x-5}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{x-5}}{x-3-(x-5)}\, dx = \int \frac{\sqrt{x-3}-\sqrt{x-5}}{2}\, dx$$

$$= \frac{1}{2} \int (x-3)^{\frac{1}{2}}\, dx - \frac{1}{2} \int (x-5)^{\frac{1}{2}}\, dx$$

$$\int \frac{dx}{\sqrt{x-3}-\sqrt{x-5}}$$

$$= \int \frac{1}{\sqrt{x-3}-\sqrt{x-5}} \cdot \frac{\sqrt{x-3}+\sqrt{x-5}}{\sqrt{x-3}+\sqrt{x-5}}\, dx = \int \frac{\sqrt{x-3}+\sqrt{x-5}}{x-3-(x-5)}\, dx = \int \frac{\sqrt{x-3}+\sqrt{x-5}}{2}\, dx$$

$$= \frac{1}{2} \int (x-3)^{\frac{1}{2}}\, dx + \frac{1}{2} \int (x-5)^{\frac{1}{2}}\, dx$$

# Riešené príklady – 176, 177

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{x-5}}$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{x-5}} \cdot \frac{\sqrt{x-3}-\sqrt{x-5}}{\sqrt{x-3}-\sqrt{x-5}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{x-5}}{x-3-(x-5)}\, dx = \int \frac{\sqrt{x-3}-\sqrt{x-5}}{2}\, dx$$

$$= \frac{1}{2}\int (x-3)^{\frac{1}{2}}\, dx - \frac{1}{2}\int (x-5)^{\frac{1}{2}}\, dx = \frac{1}{2} \cdot \frac{(x-3)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2} \cdot \frac{(x-5)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\int \frac{dx}{\sqrt{x-3}-\sqrt{x-5}}$$

$$= \int \frac{1}{\sqrt{x-3}-\sqrt{x-5}} \cdot \frac{\sqrt{x-3}+\sqrt{x-5}}{\sqrt{x-3}+\sqrt{x-5}}\, dx = \int \frac{\sqrt{x-3}+\sqrt{x-5}}{x-3-(x-5)}\, dx = \int \frac{\sqrt{x-3}+\sqrt{x-5}}{2}\, dx$$

$$= \frac{1}{2}\int (x-3)^{\frac{1}{2}}\, dx + \frac{1}{2}\int (x-5)^{\frac{1}{2}}\, dx = \frac{1}{2} \cdot \frac{(x-3)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2} \cdot \frac{(x-5)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

# Riešené príklady – 176, 177

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{x-5}} = \frac{\sqrt{(x-3)^3}}{3} - \frac{\sqrt{(x-5)^3}}{3} + c$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{x-5}} \cdot \frac{\sqrt{x-3}-\sqrt{x-5}}{\sqrt{x-3}-\sqrt{x-5}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{x-5}}{x-3-(x-5)}\, dx = \int \frac{\sqrt{x-3}-\sqrt{x-5}}{2}\, dx$$

$$= \frac{1}{2}\int (x-3)^{\frac{1}{2}}\, dx - \frac{1}{2}\int (x-5)^{\frac{1}{2}}\, dx = \frac{1}{2}\cdot\frac{(x-3)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\cdot\frac{(x-5)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{\sqrt{(x-3)^3}}{3} - \frac{\sqrt{(x-5)^3}}{3} + c,\ x\in\langle 5;\infty),\ c\in R.$$

$$\int \frac{dx}{\sqrt{x-3}-\sqrt{x-5}} = \frac{\sqrt{(x-3)^3}}{3} + \frac{\sqrt{(x-5)^3}}{3} + c$$

$$= \int \frac{1}{\sqrt{x-3}-\sqrt{x-5}} \cdot \frac{\sqrt{x-3}+\sqrt{x-5}}{\sqrt{x-3}+\sqrt{x-5}}\, dx = \int \frac{\sqrt{x-3}+\sqrt{x-5}}{x-3-(x-5)}\, dx = \int \frac{\sqrt{x-3}+\sqrt{x-5}}{2}\, dx$$

$$= \frac{1}{2}\int (x-3)^{\frac{1}{2}}\, dx + \frac{1}{2}\int (x-5)^{\frac{1}{2}}\, dx = \frac{1}{2}\cdot\frac{(x-3)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2}\cdot\frac{(x-5)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{\sqrt{(x-3)^3}}{3} + \frac{\sqrt{(x-5)^3}}{3} + c,\ x\in\langle 5;\infty),\ c\in R.$$

# Riešené príklady – 178

$$\int \frac{\mathrm{d}x}{\sqrt{x-3}+\sqrt{5-x}}$$

# Riešené príklady – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}}\, dx$$

# Riešené príklady – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{5-x}}{x-3-(5-x)}\, dx$$

# Riešené príklady – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{5-x}}{x-3-(5-x)}\, dx = \int \frac{\sqrt{x-3}\, dx}{2x-8} - \int \frac{\sqrt{5-x}\, dx}{2x-8}$$

# Riešené príklady – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{5-x}}{x-3-(5-x)}\, dx = \int \frac{\sqrt{x-3}\, dx}{2x-8} - \int \frac{\sqrt{5-x}\, dx}{2x-8}$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=\sqrt{x-3} & u^2=x-3 & x=u^2+3 & dx=2u\,du & x\in\langle 3;4),\ u\in\langle 0;1),\ v\in(1;\sqrt{2}\rangle \\ \text{Subst. } v=\sqrt{5-x} & v^2=5-x & x=5-v^2 & dx=-2v\,dv & x\in(4;5\rangle,\ u\in(1;\sqrt{2}\rangle,\ v\in\langle 0;1) \end{array} \right]$$

# Riešené príklady – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$$

$$=\int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{5-x}}{x-3-(5-x)}\, dx = \int \frac{\sqrt{x-3}\, dx}{2x-8} - \int \frac{\sqrt{5-x}\, dx}{2x-8}$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=\sqrt{x-3} & u^2=x-3 & x=u^2+3 & dx=2u\,du & x\in\langle 3;4),\ u\in\langle 0;1),\ v\in(1;\sqrt{2}\rangle \\ \text{Subst. } v=\sqrt{5-x} & v^2=5-x & x=5-v^2 & dx=-2v\,dv & x\in(4;5\rangle,\ u\in(1;\sqrt{2}\rangle,\ v\in\langle 0;1) \end{array} \right]$$

$$=\int \frac{2u^2\, du}{2u^2+6-8} + \int \frac{2v^2\, dv}{-2v^2+10-8}$$

# Riešené príklady – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{5-x}}{x-3-(5-x)}\, dx = \int \frac{\sqrt{x-3}\, dx}{2x-8} - \int \frac{\sqrt{5-x}\, dx}{2x-8}$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=\sqrt{x-3} & u^2=x-3 & x=u^2+3 & dx=2u\, du & x\in\langle 3;4),\ u\in\langle 0;1),\ v\in(1;\sqrt{2}\rangle \\ \text{Subst. } v=\sqrt{5-x} & v^2=5-x & x=5-v^2 & dx=-2v\, dv & x\in(4;5\rangle,\ u\in(1;\sqrt{2}\rangle,\ v\in\langle 0;1) \end{array} \right]$$

$$= \int \frac{2u^2\, du}{2u^2+6-8} + \int \frac{2v^2\, dv}{-2v^2+10-8} = \int \frac{u^2\, du}{u^2-1} - \int \frac{v^2\, dv}{v^2-1}$$

# Riešené príklady – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$$

$$=\int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{5-x}}{x-3-(5-x)}\, dx = \int \frac{\sqrt{x-3}\, dx}{2x-8} - \int \frac{\sqrt{5-x}\, dx}{2x-8}$$

$$=\left[\begin{array}{l|l|l|l|l} \text{Subst. } u=\sqrt{x-3} & u^2=x-3 & x=u^2+3 & dx=2u\,du & x\in\langle 3;4),\ u\in\langle 0;1),\ v\in(1;\sqrt{2}\rangle \\ \text{Subst. } v=\sqrt{5-x} & v^2=5-x & x=5-v^2 & dx=-2v\,dv & x\in(4;5\rangle,\ u\in(1;\sqrt{2}\rangle,\ v\in\langle 0;1) \end{array}\right]$$

$$=\int \frac{2u^2\, du}{2u^2+6-8} + \int \frac{2v^2\, dv}{-2v^2+10-8} = \int \frac{u^2\, du}{u^2-1} - \int \frac{v^2\, dv}{v^2-1} = \int \frac{u^2-1+1}{u^2-1}\, du - \int \frac{v^2-1+1}{v^2-1}\, dv$$

# Riešené príklady – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{5-x}}{x-3-(5-x)}\, dx = \int \frac{\sqrt{x-3}\, dx}{2x-8} - \int \frac{\sqrt{5-x}\, dx}{2x-8}$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=\sqrt{x-3} & u^2=x-3 & x=u^2+3 & dx=2u\, du & x\in\langle 3;4),\ u\in\langle 0;1),\ v\in(1;\sqrt{2}\rangle \\ \text{Subst. } v=\sqrt{5-x} & v^2=5-x & x=5-v^2 & dx=-2v\, dv & x\in(4;5\rangle,\ u\in(1;\sqrt{2}\rangle,\ v\in\langle 0;1) \end{array} \right]$$

$$= \int \frac{2u^2\, du}{2u^2+6-8} + \int \frac{2v^2\, dv}{-2v^2+10-8} = \int \frac{u^2\, du}{u^2-1} - \int \frac{v^2\, dv}{v^2-1} = \int \frac{u^2-1+1}{u^2-1}\, du - \int \frac{v^2-1+1}{v^2-1}\, dv$$

$$= \int du + \int \frac{du}{u^2-1} - \int dv - \int \frac{dv}{v^2-1}$$

# Riešené príklady – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$$

$$=\int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{5-x}}{x-3-(5-x)}\, dx = \int \frac{\sqrt{x-3}\, dx}{2x-8} - \int \frac{\sqrt{5-x}\, dx}{2x-8}$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=\sqrt{x-3} & u^2=x-3 & x=u^2+3 & dx=2u\,du & x\in\langle 3;4),\ u\in\langle 0;1),\ v\in(1;\sqrt{2}\rangle \\ \text{Subst. } v=\sqrt{5-x} & v^2=5-x & x=5-v^2 & dx=-2v\,dv & x\in(4;5\rangle,\ u\in(1;\sqrt{2}\rangle,\ v\in\langle 0;1) \end{array} \right]$$

$$=\int \frac{2u^2\, du}{2u^2+6-8} + \int \frac{2v^2\, dv}{-2v^2+10-8} = \int \frac{u^2\, du}{u^2-1} - \int \frac{v^2\, dv}{v^2-1} = \int \frac{u^2-1+1}{u^2-1}\, du - \int \frac{v^2-1+1}{v^2-1}\, dv$$

$$=\int du + \int \frac{du}{u^2-1} - \int dv - \int \frac{dv}{v^2-1} = u + \frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| - v - \frac{1}{2}\ln\left|\frac{v-1}{v+1}\right| + c$$

# Riešené príklady – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$$

$$=\int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{5-x}}{x-3-(5-x)}\, dx = \int \frac{\sqrt{x-3}\, dx}{2x-8} - \int \frac{\sqrt{5-x}\, dx}{2x-8}$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=\sqrt{x-3} & u^2=x-3 & x=u^2+3 & dx=2u\, du & x\in\langle 3;4),\ u\in\langle 0;1),\ v\in(1;\sqrt{2}\rangle \\ \text{Subst. } v=\sqrt{5-x} & v^2=5-x & x=5-v^2 & dx=-2v\, dv & x\in(4;5\rangle,\ u\in(1;\sqrt{2}\rangle,\ v\in\langle 0;1) \end{array} \right]$$

$$=\int \frac{2u^2\, du}{2u^2+6-8} + \int \frac{2v^2\, dv}{-2v^2+10-8} = \int \frac{u^2\, du}{u^2-1} - \int \frac{v^2\, dv}{v^2-1} = \int \frac{u^2-1+1}{u^2-1}\, du - \int \frac{v^2-1+1}{v^2-1}\, dv$$

$$=\int du + \int \frac{du}{u^2-1} - \int dv - \int \frac{dv}{v^2-1} = u + \frac{1}{2} \ln \left| \frac{u-1}{u+1} \right| - v - \frac{1}{2} \ln \left| \frac{v-1}{v+1} \right| + c$$

$$=\sqrt{x-3} + \frac{1}{2} \ln \left| \frac{\sqrt{x-3}-1}{\sqrt{x-3}+1} \right| - \sqrt{5-x} - \frac{1}{2} \ln \left| \frac{\sqrt{5-x}-1}{\sqrt{5-x}+1} \right| + c$$

# Riešené príklady – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}}$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}}\, dx = \int \frac{\sqrt{x-3}-\sqrt{5-x}}{x-3-(5-x)}\, dx = \int \frac{\sqrt{x-3}\, dx}{2x-8} - \int \frac{\sqrt{5-x}\, dx}{2x-8}$$

$$= \left[ \begin{array}{l|l|l|l|l} \text{Subst. } u=\sqrt{x-3} & u^2=x-3 & x=u^2+3 & dx=2u\,du & x\in\langle 3;4),\ u\in\langle 0;1),\ v\in(1;\sqrt{2}\rangle \\ \text{Subst. } v=\sqrt{5-x} & v^2=5-x & x=5-v^2 & dx=-2v\,dv & x\in(4;5\rangle,\ u\in(1;\sqrt{2}\rangle,\ v\in\langle 0;1) \end{array} \right]$$

$$= \int \frac{2u^2\, du}{2u^2+6-8} + \int \frac{2v^2\, dv}{-2v^2+10-8} = \int \frac{u^2\, du}{u^2-1} - \int \frac{v^2\, dv}{v^2-1} = \int \frac{u^2-1+1}{u^2-1}\, du - \int \frac{v^2-1+1}{v^2-1}\, dv$$

$$= \int du + \int \frac{du}{u^2-1} - \int dv - \int \frac{dv}{v^2-1} = u + \frac{1}{2} \ln \left| \frac{u-1}{u+1} \right| - v - \frac{1}{2} \ln \left| \frac{v-1}{v+1} \right| + c$$

$$= \sqrt{x-3} + \frac{1}{2} \ln \left| \frac{\sqrt{x-3}-1}{\sqrt{x-3}+1} \right| - \sqrt{5-x} - \frac{1}{2} \ln \left| \frac{\sqrt{5-x}-1}{\sqrt{5-x}+1} \right| + c$$

$$= \left[ \begin{array}{l} \ln \left| \frac{\sqrt{x-3}-1}{\sqrt{x-3}+1} \right| - \ln \left| \frac{\sqrt{5-x}-1}{\sqrt{5-x}+1} \right| = \ln \left| \frac{\sqrt{x-3}-1}{\sqrt{x-3}+1} \cdot \frac{\sqrt{x-3}-1}{\sqrt{x-3}-1} \right| - \ln \left| \frac{\sqrt{5-x}-1}{\sqrt{5-x}+1} \cdot \frac{\sqrt{5-x}-1}{\sqrt{5-x}-1} \right| = \ln \left| \frac{x-3-2\sqrt{x-3}+1}{x-3-1} \right| - \ln \left| \frac{5-x-2\sqrt{5-x}+1}{5-x-1} \right| \\ = \ln \left| \frac{x-2-2\sqrt{x-3}}{x-4} \right| - \ln \left| \frac{6-x-2\sqrt{5-x}+1}{4-x} \right| = \ln \left| \frac{x-2-2\sqrt{x-3}}{x-4} \cdot \frac{4-x}{6-x-2\sqrt{5-x}} \right| = \ln \left| \frac{x-2-2\sqrt{x-3}}{6-x-2\sqrt{5-x}} \right| \end{array} \right]$$

# Riešené príklady – 178

$$\int \frac{dx}{\sqrt{x-3}+\sqrt{5-x}} = \sqrt{x-3} - \sqrt{5-x} + \frac{1}{2}\ln\left|\frac{x-2-2\sqrt{x-3}}{6-x-2\sqrt{5-x}}\right| + c$$

$$= \int \frac{1}{\sqrt{x-3}+\sqrt{5-x}} \cdot \frac{\sqrt{x-3}-\sqrt{5-x}}{\sqrt{x-3}-\sqrt{5-x}}\,dx = \int \frac{\sqrt{x-3}-\sqrt{5-x}}{x-3-(5-x)}\,dx = \int \frac{\sqrt{x-3}\,dx}{2x-8} - \int \frac{\sqrt{5-x}\,dx}{2x-8}$$

$$= \left[\begin{array}{l|l|l|l|l} \text{Subst. } u=\sqrt{x-3} & u^2=x-3 & x=u^2+3 & dx=2u\,du & x\in\langle 3;4),\ u\in\langle 0;1),\ v\in(1;\sqrt{2}\rangle \\ \text{Subst. } v=\sqrt{5-x} & v^2=5-x & x=5-v^2 & dx=-2v\,dv & x\in(4;5\rangle,\ u\in(1;\sqrt{2}\rangle,\ v\in\langle 0;1) \end{array}\right]$$

$$= \int \frac{2u^2\,du}{2u^2+6-8} + \int \frac{2v^2\,dv}{-2v^2+10-8} = \int \frac{u^2\,du}{u^2-1} - \int \frac{v^2\,dv}{v^2-1} = \int \frac{u^2-1+1}{u^2-1}\,du - \int \frac{v^2-1+1}{v^2-1}\,dv$$

$$= \int du + \int \frac{du}{u^2-1} - \int dv - \int \frac{dv}{v^2-1} = u + \frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| - v - \frac{1}{2}\ln\left|\frac{v-1}{v+1}\right| + c$$

$$= \sqrt{x-3} + \frac{1}{2}\ln\left|\frac{\sqrt{x-3}-1}{\sqrt{x-3}+1}\right| - \sqrt{5-x} - \frac{1}{2}\ln\left|\frac{\sqrt{5-x}-1}{\sqrt{5-x}+1}\right| + c$$

$$= \left[\begin{array}{l} \ln\left|\frac{\sqrt{x-3}-1}{\sqrt{x-3}+1}\right| - \ln\left|\frac{\sqrt{5-x}-1}{\sqrt{5-x}+1}\right| = \ln\left|\frac{\sqrt{x-3}-1}{\sqrt{x-3}+1}\cdot\frac{\sqrt{x-3}-1}{\sqrt{x-3}-1}\right| - \ln\left|\frac{\sqrt{5-x}-1}{\sqrt{5-x}+1}\cdot\frac{\sqrt{5-x}-1}{\sqrt{5-x}-1}\right| = \ln\left|\frac{x-3-2\sqrt{x-3}+1}{x-3-1}\right| - \ln\left|\frac{5-x-2\sqrt{5-x}+1}{5-x-1}\right| \\ = \ln\left|\frac{x-2-2\sqrt{x-3}}{x-4}\right| - \ln\left|\frac{6-x-2\sqrt{5-x}+1}{4-x}\right| = \ln\left|\frac{x-2-2\sqrt{x-3}}{x-4}\cdot\frac{4-x}{6-x-2\sqrt{5-x}}\right| = \ln\left|\frac{x-2-2\sqrt{x-3}}{6-x-2\sqrt{5-x}}\right| \end{array}\right]$$

$$= \sqrt{x-3} - \sqrt{5-x} + \frac{1}{2}\ln\left|\frac{x-2-2\sqrt{x-3}}{6-x-2\sqrt{5-x}}\right| + c,\ x\in\langle 3;4)\cup(4;5\rangle,\ c\in R.$$

# Riešené príklady – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,dx$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}}\,dx = \int \frac{\sqrt{x}\,dx}{\sqrt{1-x\sqrt{x}}}$$

# Riešené príklady – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,\mathrm{d}x$$

$$= \int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\,\mathrm{d}x$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}}\,\mathrm{d}x = \int \frac{\sqrt{x}\,\mathrm{d}x}{\sqrt{1-x\sqrt{x}}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{x} & x\in\langle 0;1) \\ x=t^2, & \mathrm{d}x=2t\,\mathrm{d}t & t\in\langle 0;1) \end{array}\right]$$

# Riešené príklady – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,dx$$

$$=\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\cdot\frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\,dx=\int \frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)}\,dx$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}}\,dx=\int \frac{\sqrt{x}\,dx}{\sqrt{1-x\sqrt{x}}}$$

$$=\left[\begin{array}{cc|c} \text{Subst.} & t=\sqrt{x} & x\in\langle 0;1) \\ x=t^2, & dx=2t\,dt & t\in\langle 0;1) \end{array}\right]=\int \frac{2t^2\,dt}{\sqrt{1-t^3}}$$

# Riešené príklady – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,dx$$

$$=\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\cdot\frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\,dx=\int \frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)}\,dx=\int \frac{2-x^2+2\sqrt{1-x^2}}{x^2}\,dx$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}}\,dx = \int \frac{\sqrt{x}\,dx}{\sqrt{1-x\sqrt{x}}}$$

$$=\left[\begin{array}{cc|c} \text{Subst.} & t=\sqrt{x} & x\in\langle 0;1) \\ x=t^2, & dx=2t\,dt & t\in\langle 0;1) \end{array}\right]=\int \frac{2t^2\,dt}{\sqrt{1-t^3}}=\left[\begin{array}{c|c} \text{Subst.}\ u=1-t^3 & t\in\langle 0;1) \\ du=-3t^2\,dt & u\in(0;1\rangle \end{array}\right]$$

# Riešené príklady – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,dx$$

$$=\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\cdot\frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\,dx=\int\frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)}\,dx=\int\frac{2-x^2+2\sqrt{1-x^2}}{x^2}\,dx$$

$$=2\int x^{-2}\,dx-\int dx+2\int\frac{\sqrt{1-x^2}\,dx}{x^2}$$

$$\int\sqrt{\frac{x}{1-x\sqrt{x}}}\,dx=\int\frac{\sqrt{x}\,dx}{\sqrt{1-x\sqrt{x}}}$$

$$=\left[\begin{array}{cc|c}\text{Subst.} & t=\sqrt{x} & x\in\langle 0;1)\\ x=t^2, & dx=2t\,dt & t\in\langle 0;1)\end{array}\right]=\int\frac{2t^2\,dt}{\sqrt{1-t^3}}=\left[\begin{array}{cc|c}\text{Subst.} & u=1-t^3 & t\in\langle 0;1)\\ & du=-3t^2\,dt & u\in(0;1\rangle\end{array}\right]=-\int\frac{2\,du}{3\sqrt{u}}=-\frac{2}{3}\int u^{-\frac{1}{2}}\,du$$

# Riešené príklady – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,dx$$

$$=\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\cdot\frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\,dx=\int \frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)}\,dx=\int \frac{2-x^2+2\sqrt{1-x^2}}{x^2}\,dx$$

$$=2\int x^{-2}\,dx-\int dx+2\int \frac{\sqrt{1-x^2}\,dx}{x^2}=\left[\begin{array}{l|l} u=\sqrt{1-x^2} & u'=\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)=\frac{-x}{\sqrt{1-x^2}} \\ v'=\frac{1}{x^2}=x^{-2} & v=\frac{x^{-1}}{-1}=-\frac{1}{x} \end{array}\right]$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}}\,dx=\int \frac{\sqrt{x}\,dx}{\sqrt{1-x\sqrt{x}}}$$

$$=\left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{x} & x\in\langle 0;1) \\ x=t^2, & dx=2t\,dt & t\in\langle 0;1) \end{array}\right]=\int \frac{2t^2\,dt}{\sqrt{1-t^3}}=\left[\begin{array}{l|l} \text{Subst. } u=1-t^3 & t\in\langle 0;1) \\ du=-3t^2\,dt & u\in(0;1\rangle \end{array}\right]=-\int \frac{2\,du}{3\sqrt{u}}=-\frac{2}{3}\int u^{-\frac{1}{2}}\,du$$

$$=-\frac{2}{3}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}}+c$$

# Riešené príklady – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,dx$$

$$=\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\cdot\frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\,dx=\int\frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)}\,dx=\int\frac{2-x^2+2\sqrt{1-x^2}}{x^2}\,dx$$

$$=2\int x^{-2}\,dx-\int dx+2\int\frac{\sqrt{1-x^2}\,dx}{x^2}=\left[\begin{array}{l|l} u=\sqrt{1-x^2} & u'=\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)=\frac{-x}{\sqrt{1-x^2}} \\ v'=\frac{1}{x^2}=x^{-2} & v=\frac{x^{-1}}{-1}=-\frac{1}{x}\end{array}\right]$$

$$=2\frac{x^{-1}}{-1}-x+2\left[-\frac{\sqrt{1-x^2}}{x}-\int\frac{dx}{\sqrt{1-x^2}}\right]$$

$$\int\sqrt{\frac{x}{1-x\sqrt{x}}}\,dx=\int\frac{\sqrt{x}\,dx}{\sqrt{1-x\sqrt{x}}}$$

$$=\left[\begin{array}{ll|l}\text{Subst.} & t=\sqrt{x} & x\in\langle 0;1)\\ x=t^2, & dx=2t\,dt & t\in\langle 0;1)\end{array}\right]=\int\frac{2t^2\,dt}{\sqrt{1-t^3}}=\left[\begin{array}{l|l}\text{Subst. } u=1-t^3 & t\in\langle 0;1)\\ du=-3t^2\,dt & u\in(0;1\rangle\end{array}\right]=-\int\frac{2\,du}{3\sqrt{u}}=-\frac{2}{3}\int u^{-\frac{1}{2}}\,du$$

$$=-\frac{2}{3}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}}+c=-\frac{4\sqrt{u}}{3}+c$$

# Riešené príklady – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,dx = -\frac{2}{x} - x - \frac{2\sqrt{1-x^2}}{x} - 2\arcsin x + c$$

$$= \int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\,dx = \int \frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)}\,dx = \int \frac{2-x^2+2\sqrt{1-x^2}}{x^2}\,dx$$

$$= 2\int x^{-2}\,dx - \int dx + 2\int \frac{\sqrt{1-x^2}\,dx}{x^2} = \left[ \begin{array}{l|l} u=\sqrt{1-x^2} & u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{1-x^2}} \\ v' = \frac{1}{x^2} = x^{-2} & v = \frac{x^{-1}}{-1} = -\frac{1}{x} \end{array} \right]$$

$$= 2\frac{x^{-1}}{-1} - x + 2\left[ -\frac{\sqrt{1-x^2}}{x} - \int \frac{dx}{\sqrt{1-x^2}} \right] = -\frac{2}{x} - x - \frac{2\sqrt{1-x^2}}{x} - 2\arcsin x + c,$$

$$x \in \langle -1; 0) \cup (0; 1\rangle,\ c \in R.$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}}\,dx = \int \frac{\sqrt{x}\,dx}{\sqrt{1-x\sqrt{x}}}$$

$$= \left[ \begin{array}{l|l} \text{Subst.} \quad t=\sqrt{x} & x \in \langle 0;1) \\ x=t^2,\ dx = 2t\,dt & t \in \langle 0;1) \end{array} \right] = \int \frac{2t^2\,dt}{\sqrt{1-t^3}} = \left[ \begin{array}{l|l} \text{Subst.}\ u=1-t^3 & t \in \langle 0;1) \\ du = -3t^2\,dt & u \in (0;1\rangle \end{array} \right] = -\int \frac{2\,du}{3\sqrt{u}} = -\frac{2}{3}\int u^{-\frac{1}{2}}\,du$$

$$= -\frac{2}{3} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = -\frac{4\sqrt{u}}{3} + c = -\frac{4\sqrt{1-t^3}}{3} + c$$

# Riešené príklady – 179, 180

$$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,dx = -\frac{2}{x} - x - \frac{2\sqrt{1-x^2}}{x} - 2\arcsin x + c$$

$$= \int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\,dx = \int \frac{1+2\sqrt{1-x^2}+1-x^2}{1-(1-x^2)}\,dx = \int \frac{2-x^2+2\sqrt{1-x^2}}{x^2}\,dx$$

$$= 2\int x^{-2}\,dx - \int dx + 2\int \frac{\sqrt{1-x^2}\,dx}{x^2} = \left[\begin{array}{c|c} u=\sqrt{1-x^2} & u'=\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)=\frac{-x}{\sqrt{1-x^2}} \\ v'=\frac{1}{x^2}=x^{-2} & v=\frac{x^{-1}}{-1}=-\frac{1}{x} \end{array}\right]$$

$$= 2\frac{x^{-1}}{-1} - x + 2\left[-\frac{\sqrt{1-x^2}}{x} - \int \frac{dx}{\sqrt{1-x^2}}\right] = -\frac{2}{x} - x - \frac{2\sqrt{1-x^2}}{x} - 2\arcsin x + c,$$

$$x \in \langle -1; 0) \cup (0; 1\rangle,\ c \in R.$$

$$\int \sqrt{\frac{x}{1-x\sqrt{x}}}\,dx = \int \frac{\sqrt{x}\,dx}{\sqrt{1-x\sqrt{x}}} = -\frac{4\sqrt{1-x\sqrt{x}}}{3} + c$$

$$= \left[\begin{array}{cc|c} \text{Subst.} & t=\sqrt{x} & x\in\langle 0;1) \\ x=t^2, & dx=2t\,dt & t\in\langle 0;1) \end{array}\right] = \int \frac{2t^2\,dt}{\sqrt{1-t^3}} = \left[\begin{array}{c|c} \text{Subst. } u=1-t^3 & t\in\langle 0;1) \\ du=-3t^2\,dt & u\in(0;1\rangle \end{array}\right] = -\int \frac{2\,du}{3\sqrt{u}} = -\frac{2}{3}\int u^{-\frac{1}{2}}\,du$$

$$= -\frac{2}{3}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = -\frac{4\sqrt{u}}{3} + c = -\frac{4\sqrt{1-t^3}}{3} + c = -\frac{4\sqrt{1-x\sqrt{x}}}{3} + c,\ x\in\langle 0;1),\ c\in R.$$

# Riešené príklady – 181

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$$

# Riešené príklady – 181

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right]$$

# Riešené príklady – 181

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,dt}{t^4+t^3}$$

# Riešené príklady – 181

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,dt}{t^4+t^3} = 12\int \frac{t^8\,dt}{t+1}$$

# Riešené príklady – 181

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,dt}{t^4+t^3} = 12\int \frac{t^8\,dt}{t+1}$$

$$= 12\int \frac{t^8+(t^7-t^7)-(t^6-t^6)+(t^5-t^5)-(t^4-t^4)+(t^3-t^3)-(t^2-t^2)+(t-t)-(1-1)}{t+1}\,dt$$

# Riešené príklady – 181

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,dt}{t^4+t^3} = 12\int \frac{t^8\,dt}{t+1}$$

$$= 12\int \frac{t^8+(t^7-t^7)-(t^6-t^6)+(t^5-t^5)-(t^4-t^4)+(t^3-t^3)-(t^2-t^2)+(t-t)-(1-1)}{t+1}\,dt$$

$$= 12\int \frac{(t^8+t^7)-(t^7+t^6)+(t^6+t^5)-(t^5+t^4)+(t^4+t^3)-(t^3+t^2)+(t^2+t)-(t+1)+1}{t+1}\,dt$$

# Riešené príklady – 181

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,dt}{t^4+t^3} = 12\int \frac{t^8\,dt}{t+1}$$

$$= 12\int \frac{t^8+(t^7-t^7)-(t^6-t^6)+(t^5-t^5)-(t^4-t^4)+(t^3-t^3)-(t^2-t^2)+(t-t)-(1-1)}{t+1}\,dt$$

$$= 12\int \frac{(t^8+t^7)-(t^7+t^6)+(t^6+t^5)-(t^5+t^4)+(t^4+t^3)-(t^3+t^2)+(t^2+t)-(t+1)+1}{t+1}\,dt$$

$$= 12\int \left[ t^7 - t^6 + t^5 - t^4 + t^3 - t^2 + t - 1 + \frac{1}{t+1} \right] dt$$

# Riešené príklady – 181

$$\int \frac{\mathrm{d}x}{\sqrt[3]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 & x\in(0;\infty) \\ \mathrm{d}x=12t^{11}\,\mathrm{d}t & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,\mathrm{d}t}{t^4+t^3} = 12\int \frac{t^8\,\mathrm{d}t}{t+1}$$

$$= 12\int \frac{t^8+(t^7-t^7)-(t^6-t^6)+(t^5-t^5)-(t^4-t^4)+(t^3-t^3)-(t^2-t^2)+(t-t)-(1-1)}{t+1}\,\mathrm{d}t$$

$$= 12\int \frac{(t^8+t^7)-(t^7+t^6)+(t^6+t^5)-(t^5+t^4)+(t^4+t^3)-(t^3+t^2)+(t^2+t)-(t+1)+1}{t+1}\,\mathrm{d}t$$

$$= 12\int \left[ t^7 - t^6 + t^5 - t^4 + t^3 - t^2 + t - 1 + \frac{1}{t+1} \right] \mathrm{d}t$$

$$= 12\left[ \frac{t^8}{8} - \frac{t^7}{7} + \frac{t^6}{6} - \frac{t^5}{5} + \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} - t + \ln|t+1| \right] + c$$

# Riešené príklady – 181

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|c|c} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,dt}{t^4+t^3} = 12\int \frac{t^8\,dt}{t+1}$$

$$= 12\int \frac{t^8+(t^7-t^7)-(t^6-t^6)+(t^5-t^5)-(t^4-t^4)+(t^3-t^3)-(t^2-t^2)+(t-t)-(1-1)}{t+1}\,dt$$

$$= 12\int \frac{(t^8+t^7)-(t^7+t^6)+(t^6+t^5)-(t^5+t^4)+(t^4+t^3)-(t^3+t^2)+(t^2+t)-(t+1)+1}{t+1}\,dt$$

$$= 12\int \left[ t^7 - t^6 + t^5 - t^4 + t^3 - t^2 + t - 1 + \frac{1}{t+1} \right] dt$$

$$= 12\left[ \frac{t^8}{8} - \frac{t^7}{7} + \frac{t^6}{6} - \frac{t^5}{5} + \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} - t + \ln|t+1| \right] + c$$

$$= 12\left[ \frac{\sqrt[12]{x^8}}{8} - \frac{\sqrt[12]{x^7}}{7} + \frac{\sqrt[12]{x^6}}{6} - \frac{\sqrt[12]{x^5}}{5} + \frac{\sqrt[12]{x^4}}{4} - \frac{\sqrt[12]{x^3}}{3} + \frac{\sqrt[12]{x^2}}{2} - \sqrt[12]{x} + \ln\left|\sqrt[12]{x}+1\right| \right] + c$$

# Riešené príklady – 181

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}} = \frac{3\sqrt[3]{x^2}}{2} - \frac{12\sqrt[12]{x^7}}{7} + 2\sqrt{x} - \frac{12\sqrt[12]{x^5}}{5} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12\ln\left(\sqrt[12]{x}+1\right) + c$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{12}}=t^4 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array}\right] = \int \frac{12t^{11}\,dt}{t^4+t^3} = 12\int \frac{t^8\,dt}{t+1}$$

$$= 12\int \frac{t^8+(t^7-t^7)-(t^6-t^6)+(t^5-t^5)-(t^4-t^4)+(t^3-t^3)-(t^2-t^2)+(t-t)-(1-1)}{t+1}\,dt$$

$$= 12\int \frac{(t^8+t^7)-(t^7+t^6)+(t^6+t^5)-(t^5+t^4)+(t^4+t^3)-(t^3+t^2)+(t^2+t)-(t+1)+1}{t+1}\,dt$$

$$= 12\int \left[t^7 - t^6 + t^5 - t^4 + t^3 - t^2 + t - 1 + \frac{1}{t+1}\right] dt$$

$$= 12\left[\frac{t^8}{8} - \frac{t^7}{7} + \frac{t^6}{6} - \frac{t^5}{5} + \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} - t + \ln|t+1|\right] + c$$

$$= 12\left[\frac{\sqrt[12]{x^8}}{8} - \frac{\sqrt[12]{x^7}}{7} + \frac{\sqrt[12]{x^6}}{6} - \frac{\sqrt[12]{x^5}}{5} + \frac{\sqrt[12]{x^4}}{4} - \frac{\sqrt[12]{x^3}}{3} + \frac{\sqrt[12]{x^2}}{2} - \sqrt[12]{x} + \ln\left|\sqrt[12]{x}+1\right|\right] + c$$

$$= \frac{3\sqrt[3]{x^2}}{2} - \frac{12\sqrt[12]{x^7}}{7} + 2\sqrt{x} - \frac{12\sqrt[12]{x^5}}{5} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12\ln\left(\sqrt[12]{x}+1\right) + c,\ x\in(0;\infty),\ c\in R.$$

# Riešené príklady – 182

$$\int \frac{dx}{\sqrt[6]{x}+\sqrt[4]{x}}$$

# Riešené príklady – 182

$$\int \frac{dx}{\sqrt[6]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right]$$

# Riešené príklady – 182

$$\int \frac{dx}{\sqrt[6]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,dt}{t^2+t^3}$$

# Riešené príklady – 182

$$\int \frac{dx}{\sqrt[6]{x}+\sqrt[4]{x}}$$

$$=\left[\begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array}\right]=\int\frac{12t^{11}\,dt}{t^2+t^3}=12\int\frac{t^9\,dt}{1+t}$$

# Riešené príklady – 182

$$\int \frac{dx}{\sqrt[6]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,dt}{t^2+t^3} = 12\int \frac{t^9\,dt}{1+t}$$

$$= 12\int \frac{t^9+(t^8-t^8)-(t^7-t^7)+(t^6-t^6)-(t^5-t^5)+(t^4-t^4)-(t^3-t^3)+(t^2-t^2)-(t-t)+(1-1)}{t+1}\,dt$$

# Riešené príklady – 182

$$\int \frac{dx}{\sqrt[6]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,dt}{t^2+t^3} = 12\int \frac{t^9\,dt}{1+t}$$

$$= 12\int \frac{t^9+(t^8-t^8)-(t^7-t^7)+(t^6-t^6)-(t^5-t^5)+(t^4-t^4)-(t^3-t^3)+(t^2-t^2)-(t-t)+(1-1)}{t+1}\,dt$$

$$= 12\int \frac{(t^9+t^8)-(t^8+t^7)+(t^7+t^6)-(t^6+t^5)+(t^5+t^4)-(t^4+t^3)+(t^3+t^2)-(t^2+t)+(t+1)-1}{t+1}\,dt$$

# Riešené príklady – 182

$$\int \frac{dx}{\sqrt[6]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,dt}{t^2+t^3} = 12\int \frac{t^9\,dt}{1+t}$$

$$=12\int \frac{t^9+(t^8-t^8)-(t^7-t^7)+(t^6-t^6)-(t^5-t^5)+(t^4-t^4)-(t^3-t^3)+(t^2-t^2)-(t-t)+(1-1)}{t+1}\,dt$$

$$=12\int \frac{(t^9+t^8)-(t^8+t^7)+(t^7+t^6)-(t^6+t^5)+(t^5+t^4)-(t^4+t^3)+(t^3+t^2)-(t^2+t)+(t+1)-1}{t+1}\,dt$$

$$=12\int \left[ t^8-t^7+t^6-t^5+t^4-t^3+t^2-t+1-\frac{1}{t+1}\right] dt$$

# Riešené príklady – 182

$$\int \frac{dx}{\sqrt[6]{x}+\sqrt[4]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,dt}{t^2+t^3} = 12\int \frac{t^9\,dt}{1+t}$$

$$= 12\int \frac{t^9+(t^8-t^8)-(t^7-t^7)+(t^6-t^6)-(t^5-t^5)+(t^4-t^4)-(t^3-t^3)+(t^2-t^2)-(t-t)+(1-1)}{t+1}\,dt$$

$$= 12\int \frac{(t^9+t^8)-(t^8+t^7)+(t^7+t^6)-(t^6+t^5)+(t^5+t^4)-(t^4+t^3)+(t^3+t^2)-(t^2+t)+(t+1)-1}{t+1}\,dt$$

$$= 12\int \left[ t^8 - t^7 + t^6 - t^5 + t^4 - t^3 + t^2 - t + 1 - \frac{1}{t+1} \right] dt$$

$$= 12\left[ \frac{t^9}{9} - \frac{t^8}{8} + \frac{t^7}{7} - \frac{t^6}{6} + \frac{t^5}{5} - \frac{t^4}{4} + \frac{t^3}{3} - \frac{t^2}{2} + t - \ln|t+1| \right] + c$$

# Riešené príklady – 182

$$\int \frac{dx}{\sqrt[6]{x}+\sqrt[4]{x}}$$

$$=\left[\begin{array}{r|l|l} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array}\right]=\int\frac{12t^{11}\,dt}{t^2+t^3}=12\int\frac{t^9\,dt}{1+t}$$

$$=12\int\frac{t^9+(t^8-t^8)-(t^7-t^7)+(t^6-t^6)-(t^5-t^5)+(t^4-t^4)-(t^3-t^3)+(t^2-t^2)-(t-t)+(1-1)}{t+1}\,dt$$

$$=12\int\frac{(t^9+t^8)-(t^8+t^7)+(t^7+t^6)-(t^6+t^5)+(t^5+t^4)-(t^4+t^3)+(t^3+t^2)-(t^2+t)+(t+1)-1}{t+1}\,dt$$

$$=12\int\left[t^8-t^7+t^6-t^5+t^4-t^3+t^2-t+1-\frac{1}{t+1}\right]dt$$

$$=12\left[\frac{t^9}{9}-\frac{t^8}{8}+\frac{t^7}{7}-\frac{t^6}{6}+\frac{t^5}{5}-\frac{t^4}{4}+\frac{t^3}{3}-\frac{t^2}{2}+t-\ln|t+1|\right]+c$$

$$=12\left[\frac{\sqrt[12]{x^9}}{9}-\frac{\sqrt[12]{x^8}}{8}+\frac{\sqrt[12]{x^7}}{7}-\frac{\sqrt[12]{x^6}}{6}+\frac{\sqrt[12]{x^5}}{5}-\frac{\sqrt[12]{x^4}}{4}+\frac{\sqrt[12]{x^3}}{3}-\frac{\sqrt[12]{x^2}}{2}+\sqrt[12]{x}-\ln\left|\sqrt[12]{x}+1\right|\right]+c$$

# Riešené príklady – 182

$$\int \frac{dx}{\sqrt[6]{x}+\sqrt[4]{x}} = \frac{4\sqrt[4]{x^3}}{3} - \frac{3\sqrt[3]{x^2}}{2} + \frac{12\sqrt[12]{x^7}}{7} - 2\sqrt{x} + \frac{12\sqrt[12]{x^5}}{5} - 3\sqrt[3]{x} + 4\sqrt[4]{x} - 6\sqrt[6]{x} + 12\sqrt[12]{x} - 12\ln\left(\sqrt[12]{x}+1\right) + c$$

$$= \left[ \begin{array}{r|c|c} \text{Subst. } x=t^{12},\ t=\sqrt[12]{x} & \sqrt[6]{x}=\sqrt[6]{t^{12}}=t^2 & x\in(0;\infty) \\ dx=12t^{11}\,dt & \sqrt[4]{x}=\sqrt[4]{t^{12}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{12t^{11}\,dt}{t^2+t^3} = 12\int \frac{t^9\,dt}{1+t}$$

$$= 12\int \frac{t^9+(t^8-t^8)-(t^7-t^7)+(t^6-t^6)-(t^5-t^5)+(t^4-t^4)-(t^3-t^3)+(t^2-t^2)-(t-t)+(1-1)}{t+1}\,dt$$

$$= 12\int \frac{(t^9+t^8)-(t^8+t^7)+(t^7+t^6)-(t^6+t^5)+(t^5+t^4)-(t^4+t^3)+(t^3+t^2)-(t^2+t)+(t+1)-1}{t+1}\,dt$$

$$= 12\int \left[ t^8 - t^7 + t^6 - t^5 + t^4 - t^3 + t^2 - t + 1 - \frac{1}{t+1} \right] dt$$

$$= 12\left[ \frac{t^9}{9} - \frac{t^8}{8} + \frac{t^7}{7} - \frac{t^6}{6} + \frac{t^5}{5} - \frac{t^4}{4} + \frac{t^3}{3} - \frac{t^2}{2} + t - \ln|t+1| \right] + c$$

$$= 12\left[ \frac{\sqrt[12]{x^9}}{9} - \frac{\sqrt[12]{x^8}}{8} + \frac{\sqrt[12]{x^7}}{7} - \frac{\sqrt[12]{x^6}}{6} + \frac{\sqrt[12]{x^5}}{5} - \frac{\sqrt[12]{x^4}}{4} + \frac{\sqrt[12]{x^3}}{3} - \frac{\sqrt[12]{x^2}}{2} + \sqrt[12]{x} - \ln\left|\sqrt[12]{x}+1\right| \right] + c$$

$$= \frac{4\sqrt[4]{x^3}}{3} - \frac{3\sqrt[3]{x^2}}{2} + \frac{12\sqrt[12]{x^7}}{7} - 2\sqrt{x} + \frac{12\sqrt[12]{x^5}}{5} - 3\sqrt[3]{x} + 4\sqrt[4]{x} - 6\sqrt[6]{x} + 12\sqrt[12]{x} - 12\ln\left(\sqrt[12]{x}+1\right) + c,\ x\in(0;\infty),\ c\in R.$$

# Riešené príklady – 183

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}}$$

# Riešené príklady – 183

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{15},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{15}}=t^5 & x\in(0;\infty) \\ dx=15t^{14}\,dt & \sqrt[5]{x}=\sqrt[5]{t^{15}}=t^3 & t\in(0;\infty) \end{array} \right]$$

# Riešené príklady – 183

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{15},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{15}}=t^5 & x\in(0;\infty) \\ dx=15t^{14}\,dt & \sqrt[5]{x}=\sqrt[5]{t^{15}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{15t^{14}\,dt}{t^5+t^3}$$

# Riešené príklady – 183

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}}$$

$$=\left[\begin{array}{r|l|l} \text{Subst. } x=t^{15},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{15}}=t^5 & x\in(0;\infty) \\ dx=15t^{14}\,dt & \sqrt[5]{x}=\sqrt[5]{t^{15}}=t^3 & t\in(0;\infty) \end{array}\right]=\int\frac{15t^{14}\,dt}{t^5+t^3}=15\int\frac{t^{11}\,dt}{t^2+1}$$

# Riešené príklady – 183

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{15},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{15}}=t^5 & x\in(0;\infty) \\ dx=15t^{14}\,dt & \sqrt[5]{x}=\sqrt[5]{t^{15}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{15t^{14}\,dt}{t^5+t^3} = 15\int \frac{t^{11}\,dt}{t^2+1}$$

$$= 15\int \frac{t^{11}+(t^9-t^9)-(t^7-t^7)+(t^5-t^5)-(t^3-t^3)+(t-t)}{t^2+1}\,dt$$

# Riešené príklady – 183

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{15},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{15}}=t^5 & x\in(0;\infty) \\ dx=15t^{14}\,dt & \sqrt[5]{x}=\sqrt[5]{t^{15}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{15t^{14}\,dt}{t^5+t^3} = 15\int \frac{t^{11}\,dt}{t^2+1}$$

$$= 15\int \frac{t^{11}+(t^9-t^9)-(t^7-t^7)+(t^5-t^5)-(t^3-t^3)+(t-t)}{t^2+1}\,dt$$

$$= 15\int \frac{(t^{11}+t^9)-(t^9+t^7)+(t^7+t^5)-(t^5+t^3)+(t^3+t)-t}{t^2+1}\,dt$$

# Riešené príklady – 183

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{15},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{15}}=t^5 & x\in(0;\infty) \\ dx=15t^{14}\,dt & \sqrt[5]{x}=\sqrt[5]{t^{15}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{15t^{14}\,dt}{t^5+t^3} = 15\int \frac{t^{11}\,dt}{t^2+1}$$

$$= 15\int \frac{t^{11}+(t^9-t^9)-(t^7-t^7)+(t^5-t^5)-(t^3-t^3)+(t-t)}{t^2+1}\,dt$$

$$= 15\int \frac{(t^{11}+t^9)-(t^9+t^7)+(t^7+t^5)-(t^5+t^3)+(t^3+t)-t}{t^2+1}\,dt$$

$$= 15\int \left[ t^9-t^7+t^5-t^3+t-\frac{t}{t^2+1} \right] dt$$

# Riešené príklady – 183

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{15},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{15}}=t^5 & x\in(0;\infty) \\ dx=15t^{14}\,dt & \sqrt[5]{x}=\sqrt[5]{t^{15}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{15t^{14}\,dt}{t^5+t^3} = 15\int \frac{t^{11}\,dt}{t^2+1}$$

$$=15\int \frac{t^{11}+(t^9-t^9)-(t^7-t^7)+(t^5-t^5)-(t^3-t^3)+(t-t)}{t^2+1}\,dt$$

$$=15\int \frac{(t^{11}+t^9)-(t^9+t^7)+(t^7+t^5)-(t^5+t^3)+(t^3+t)-t}{t^2+1}\,dt$$

$$=15\int \left[t^9-t^7+t^5-t^3+t-\frac{t}{t^2+1}\right] dt = 15\int \left[t^9-t^7+t^5-t^3+t-\frac{1}{2}\cdot\frac{2t}{t^2+1}\right] dt$$

# Riešené príklady – 183

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{15},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{15}}=t^5 & x\in(0;\infty) \\ dx=15t^{14}\,dt & \sqrt[5]{x}=\sqrt[5]{t^{15}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{15t^{14}\,dt}{t^5+t^3} = 15\int \frac{t^{11}\,dt}{t^2+1}$$

$$= 15\int \frac{t^{11}+(t^9-t^9)-(t^7-t^7)+(t^5-t^5)-(t^3-t^3)+(t-t)}{t^2+1}\,dt$$

$$= 15\int \frac{(t^{11}+t^9)-(t^9+t^7)+(t^7+t^5)-(t^5+t^3)+(t^3+t)-t}{t^2+1}\,dt$$

$$= 15\int \left[ t^9-t^7+t^5-t^3+t-\frac{t}{t^2+1} \right] dt = 15\int \left[ t^9-t^7+t^5-t^3+t-\frac{1}{2}\cdot\frac{2t}{t^2+1} \right] dt$$

$$= 15\left[ \frac{t^{10}}{10} - \frac{t^8}{8} + \frac{t^6}{6} - \frac{t^4}{4} + \frac{t^2}{2} - \frac{1}{2}\ln\left|t^2+1\right| \right] + c$$

# Riešené príklady – 183

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}}$$

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x=t^{15},\ t=\sqrt[15]{x} & \sqrt[3]{x}=\sqrt[3]{t^{15}}=t^5 & x\in(0;\infty) \\ dx=15t^{14}\,dt & \sqrt[5]{x}=\sqrt[5]{t^{15}}=t^3 & t\in(0;\infty) \end{array} \right] = \int \frac{15t^{14}\,dt}{t^5+t^3} = 15\int \frac{t^{11}\,dt}{t^2+1}$$

$$= 15\int \frac{t^{11}+(t^9-t^9)-(t^7-t^7)+(t^5-t^5)-(t^3-t^3)+(t-t)}{t^2+1}\,dt$$

$$= 15\int \frac{(t^{11}+t^9)-(t^9+t^7)+(t^7+t^5)-(t^5+t^3)+(t^3+t)-t}{t^2+1}\,dt$$

$$= 15\int \left[ t^9-t^7+t^5-t^3+t-\frac{t}{t^2+1} \right] dt = 15\int \left[ t^9-t^7+t^5-t^3+t-\frac{1}{2}\cdot\frac{2t}{t^2+1} \right] dt$$

$$= 15\left[ \frac{t^{10}}{10}-\frac{t^8}{8}+\frac{t^6}{6}-\frac{t^4}{4}+\frac{t^2}{2}-\frac{1}{2}\ln\left|t^2+1\right| \right]+c$$

$$= 15\left[ \frac{\sqrt[15]{x^{10}}}{10}-\frac{\sqrt[15]{x^8}}{8}+\frac{\sqrt[15]{x^6}}{6}-\frac{\sqrt[15]{x^4}}{4}+\frac{\sqrt[15]{x^2}}{2}-\frac{1}{2}\ln\left|\sqrt[15]{x^2}+1\right| \right]+c$$

# Riešené príklady – 183

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt[5]{x}} = \frac{3\sqrt[3]{x^2}}{2} - \frac{15\sqrt[15]{x^8}}{8} + \frac{5\sqrt[5]{x^2}}{2} - \frac{15\sqrt[15]{x^4}}{4} + \frac{15\sqrt[15]{x^2}}{2} - \frac{15}{2}\ln\left(\sqrt[15]{x^2}+1\right)\Big] + c$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } x=t^{15},\ t=\sqrt[12]{x} & \sqrt[3]{x}=\sqrt[3]{t^{15}}=t^5 & x\in(0;\infty) \\ dx=15t^{14}\,dt & \sqrt[5]{x}=\sqrt[5]{t^{15}}=t^3 & t\in(0;\infty) \end{array}\right] = \int \frac{15t^{14}\,dt}{t^5+t^3} = 15\int \frac{t^{11}\,dt}{t^2+1}$$

$$= 15\int \frac{t^{11}+(t^9-t^9)-(t^7-t^7)+(t^5-t^5)-(t^3-t^3)+(t-t)}{t^2+1}\,dt$$

$$= 15\int \frac{(t^{11}+t^9)-(t^9+t^7)+(t^7+t^5)-(t^5+t^3)+(t^3+t)-t}{t^2+1}\,dt$$

$$= 15\int \left[t^9-t^7+t^5-t^3+t-\frac{t}{t^2+1}\right]dt = 15\int \left[t^9-t^7+t^5-t^3+t-\frac{1}{2}\cdot\frac{2t}{t^2+1}\right]dt$$

$$= 15\left[\frac{t^{10}}{10} - \frac{t^8}{8} + \frac{t^6}{6} - \frac{t^4}{4} + \frac{t^2}{2} - \frac{1}{2}\ln\left|t^2+1\right|\right] + c$$

$$= 15\left[\frac{\sqrt[15]{x^{10}}}{10} - \frac{\sqrt[15]{x^8}}{8} + \frac{\sqrt[15]{x^6}}{6} - \frac{\sqrt[15]{x^4}}{4} + \frac{\sqrt[15]{x^2}}{2} - \frac{1}{2}\ln\left|\sqrt[15]{x^2}+1\right|\right] + c$$

$$= \frac{3\sqrt[3]{x^2}}{2} - \frac{15\sqrt[15]{x^8}}{8} + \frac{5\sqrt[5]{x^2}}{2} - \frac{15\sqrt[15]{x^4}}{4} + \frac{15\sqrt[15]{x^2}}{2} - \frac{15}{2}\ln\left(\sqrt[15]{x^2}+1\right)\Big] + c,$$

$$x\in(0;\infty),\ c\in R.$$

# Riešené príklady – 184, 185

$$\int \frac{dx}{\sqrt[3]{x}+1}$$

$$\int \left[ \sqrt{x^3} - \frac{1}{\sqrt{x}} \right] dx$$

# Riešené príklady – 184, 185

$$\int \frac{\mathrm{d}x}{\sqrt[3]{x}+1}$$

$$=\left[\begin{array}{c|c} \text{Subst. } x=t^3 & x\in\langle 0;\infty) \\ \mathrm{d}x=3t^2\,\mathrm{d}x & t\in\langle 0;\infty) \end{array}\right]$$

$$\int\left[\sqrt{x^3}-\frac{1}{\sqrt{x}}\right]\mathrm{d}x$$

$$=\int\left[x^{\frac{3}{2}}-x^{-\frac{1}{2}}\right]\mathrm{d}x$$

# Riešené príklady – 184, 185

$$\int \frac{dx}{\sqrt[3]{x}+1}$$

$$= \left[ \begin{array}{c|c} \text{Subst. } x=t^3 & x\in\langle 0;\infty) \\ dx=3t^2\,dt & t\in\langle 0;\infty) \end{array} \right] = \int \frac{3t^2\,dt}{t+1}$$

$$\int \left[ \sqrt{x^3} - \frac{1}{\sqrt{x}} \right] dx$$

$$= \int \left[ x^{\frac{3}{2}} - x^{-\frac{1}{2}} \right] dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

# Riešené príklady – 184, 185

$\int \frac{dx}{\sqrt[3]{x}+1}$

$$=\left[\begin{array}{c|c} \text{Subst. } x=t^3 & x\in\langle 0;\infty) \\ dx=3t^2\,dx & t\in\langle 0;\infty) \end{array}\right]=\int \frac{3t^2\,dt}{t+1}=3\int \frac{t^2+t-t-1+1}{t+1}\,dt$$

$\int \left[\sqrt{x^3}-\frac{1}{\sqrt{x}}\right] dx$

$$=\int \left[x^{\frac{3}{2}}-x^{-\frac{1}{2}}\right] dx=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}-\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+c=\frac{2}{5}x^{\frac{5}{2}}-2x^{\frac{1}{2}}+c$$

# Riešené príklady – 184, 185

$$\int \frac{dx}{\sqrt[3]{x}+1}$$

$$= \left[ \begin{array}{c|c} \text{Subst. } x = t^3 & x \in (0;\infty) \\ dx = 3t^2\,dx & t \in (0;\infty) \end{array} \right] = \int \frac{3t^2\,dt}{t+1} = 3\int \frac{t^2+t-t-1+1}{t+1}\,dt = 3\int \left[ t-1+\frac{1}{t+1} \right] dt$$

$$\int \left[ \sqrt{x^3} - \frac{1}{\sqrt{x}} \right] dx = \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c$$

$$= \int \left[ x^{\frac{3}{2}} - x^{-\frac{1}{2}} \right] dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2}{5}x^{\frac{5}{2}} - 2x^{\frac{1}{2}} + c$$

$$= \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c,\; x \in (0;\infty),\; c \in R.$$

# Riešené príklady – 184, 185

$$\int \frac{dx}{\sqrt[3]{x}+1}$$

$$= \left[\begin{array}{c|c} \text{Subst. } x=t^3 & x\in\langle 0;\infty) \\ dx=3t^2\,dx & t\in\langle 0;\infty) \end{array}\right] = \int \frac{3t^2\,dt}{t+1} = 3\int \frac{t^2+t-t-1+1}{t+1}\,dt = 3\int \left[t-1+\frac{1}{t+1}\right]dt$$

$$= \frac{3t^2}{2} - 3t + 3\ln(t+1) + c$$

$$\int \left[\sqrt{x^3} - \frac{1}{\sqrt{x}}\right] dx = \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c$$

$$= \int \left[x^{\frac{3}{2}} - x^{-\frac{1}{2}}\right] dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2}{5}x^{\frac{5}{2}} - 2x^{\frac{1}{2}} + c$$

$$= \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c,\ x\in(0;\infty),\ c\in R.$$

# Riešené príklady – 184, 185

$$\int \frac{dx}{\sqrt[3]{x}+1} = \frac{3\sqrt[3]{x^2}}{2} - 3\sqrt[3]{x} + 3\ln\left(\sqrt[3]{x}+1\right) + c$$

$$= \left[\begin{array}{c|c} \text{Subst. } x=t^3 & x\in\langle 0;\infty) \\ dx=3t^2\,dx & t\in\langle 0;\infty) \end{array}\right] = \int \frac{3t^2\,dt}{t+1} = 3\int \frac{t^2+t-t-1+1}{t+1}\,dt = 3\int\left[t-1+\frac{1}{t+1}\right]dt$$

$$= \frac{3t^2}{2} - 3t + 3\ln(t+1) + c$$

$$= \frac{3\sqrt[3]{x^2}}{2} - 3\sqrt[3]{x} + 3\ln\left(\sqrt[3]{x}+1\right) + c,\ x\in\langle 0;\infty),\ c\in R.$$

$$\int\left[\sqrt{x^3} - \frac{1}{\sqrt{x}}\right]dx = \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c$$

$$= \int\left[x^{\frac{3}{2}} - x^{-\frac{1}{2}}\right]dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2}{5}x^{\frac{5}{2}} - 2x^{\frac{1}{2}} + c$$

$$= \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c,\ x\in(0;\infty),\ c\in R.$$

# Riešené príklady – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,dx$$

# Riešené príklady – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,\mathrm{d}x$$

$$= \left[\begin{array}{ll|l|l} \text{Subst.} & t=\sqrt[6]{x} & \sqrt{x}=\sqrt{t^6}=t^3 & x\in(0;\infty) \\ x=t^6, & \mathrm{d}x=6t^5\,\mathrm{d}t & \sqrt[3]{x^2}=\sqrt[3]{t^{12}}=t^4 & t\in(0;\infty) \end{array}\right]$$

# Riešené príklady – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,dx$$

$$= \left[\begin{array}{ll|l|l} \text{Subst.} & t=\sqrt[6]{x} & \sqrt{x}=\sqrt{t^6}=t^3 & x\in(0;\infty) \\ x=t^6, & dx=6t^5\,dt & \sqrt[3]{x^2}=\sqrt[3]{t^{12}}=t^4 & t\in(0;\infty) \end{array}\right] = \int \frac{(t^6-1)6t^5\,dt}{(t^3+t^4)t^6}$$

# Riešené príklady – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,dx$$

$$= \left[\begin{array}{ll|l|l} \text{Subst.} & t=\sqrt[6]{x} & \sqrt{x}=\sqrt{t^6}=t^3 & x\in(0;\infty) \\ x=t^6, & dx=6t^5\,dt & \sqrt[3]{x^2}=\sqrt[3]{t^{12}}=t^4 & t\in(0;\infty) \end{array}\right] = \int \frac{(t^6-1)6t^5\,dt}{(t^3+t^4)t^6} = 6\int \frac{(t^6-1)\,dt}{t^4+t^5}$$

# Riešené príklady – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,dx$$

$$= \left[\begin{array}{ll|l|l} \text{Subst.} & t=\sqrt[6]{x} & \sqrt{x}=\sqrt{t^6}=t^3 & x\in(0;\infty) \\ x=t^6, & dx=6t^5\,dt & \sqrt[3]{x^2}=\sqrt[3]{t^{12}}=t^4 & t\in(0;\infty) \end{array}\right] = \int \frac{(t^6-1)6t^5\,dt}{(t^3+t^4)t^6} = 6\int \frac{(t^6-1)\,dt}{t^4+t^5}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1+\frac{t^4-1}{t^5+t^4} = t-1+\frac{(t^2-1)(t^2+1)}{t^4(t+1)} \\ = t-1+\frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1+\frac{(t-1)(t^2+1)}{t^4} = t-1+\frac{t^3-t^2+t-1}{t^4} = t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4} \end{array}\right]$$

# Riešené príklady – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,dx$$

$$= \left[ \begin{array}{ll|l|l} \text{Subst.} & t=\sqrt[6]{x} & \sqrt{x}=\sqrt{t^6}=t^3 & x\in(0;\infty) \\ x=t^6, & dx=6t^5\,dt & \sqrt[3]{x^2}=\sqrt[3]{t^{12}}=t^4 & t\in(0;\infty) \end{array} \right] = \int \frac{(t^6-1)6t^5\,dt}{(t^3+t^4)t^6} = 6\int \frac{(t^6-1)\,dt}{t^4+t^5}$$

$$= \left[ \begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1+\frac{t^4-1}{t^5+t^4} = t-1+\frac{(t^2-1)(t^2+1)}{t^4(t+1)} \\ = t-1+\frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1+\frac{(t-1)(t^2+1)}{t^4} = t-1+\frac{t^3-t^2+t-1}{t^4} = t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4} \end{array} \right]$$

$$= 6\int\left(t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4}\right)dt$$

# Riešené príklady – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,dx$$

$$= \left[\begin{array}{ll|l|l} \text{Subst.} & t=\sqrt[6]{x} & \sqrt{x}=\sqrt{t^6}=t^3 & x\in(0;\infty) \\ x=t^6, & dx=6t^5\,dt & \sqrt[3]{x^2}=\sqrt[3]{t^{12}}=t^4 & t\in(0;\infty) \end{array}\right] = \int \frac{(t^6-1)6t^5\,dt}{(t^3+t^4)t^6} = 6\int \frac{(t^6-1)\,dt}{t^4+t^5}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1+\frac{t^4-1}{t^5+t^4} = t-1+\frac{(t^2-1)(t^2+1)}{t^4(t+1)} \\ = t-1+\frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1+\frac{(t-1)(t^2+1)}{t^4} = t-1+\frac{t^3-t^2+t-1}{t^4} = t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4} \end{array}\right]$$

$$= 6\int\left(t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4}\right)dt = 6\int\left(t-1+\frac{1}{t}-t^{-2}+t^{-3}-t^{-4}\right)dt$$

# Riešené príklady – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad t=\sqrt[6]{x} & \sqrt{x}=\sqrt{t^6}=t^3 & x\in(0;\infty) \\ x=t^6,\ dx=6t^5\,dt & \sqrt[3]{x^2}=\sqrt[3]{t^{12}}=t^4 & t\in(0;\infty) \end{array}\right] = \int \frac{(t^6-1)6t^5\,dt}{(t^3+t^4)t^6} = 6\int \frac{(t^6-1)\,dt}{t^4+t^5}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky:} \quad \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1+\frac{t^4-1}{t^5+t^4} = t-1+\frac{(t^2-1)(t^2+1)}{t^4(t+1)} \\ = t-1+\frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1+\frac{(t-1)(t^2+1)}{t^4} = t-1+\frac{t^3-t^2+t-1}{t^4} = t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4} \end{array}\right]$$

$$= 6\int\Big(t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4}\Big)dt = 6\int\Big(t-1+\frac{1}{t}-t^{-2}+t^{-3}-t^{-4}\Big)dt$$

$$= 6\Big(\frac{t^2}{2}-t+\ln|t|-\frac{t^{-1}}{-1}+\frac{t^{-2}}{-2}+\frac{t^{-3}}{-3}\Big)+c$$

# Riešené príklady – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,dx$$

$$= \left[\begin{array}{ll|l|l} \text{Subst.} & t=\sqrt[6]{x} & \sqrt{x}=\sqrt{t^6}=t^3 & x\in(0;\infty) \\ x=t^6, & dx=6t^5\,dt & \sqrt[3]{x^2}=\sqrt[3]{t^{12}}=t^4 & t\in(0;\infty) \end{array}\right] = \int \frac{(t^6-1)6t^5\,dt}{(t^3+t^4)t^6} = 6\int \frac{(t^6-1)\,dt}{t^4+t^5}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1+\frac{t^4-1}{t^5+t^4} = t-1+\frac{(t^2-1)(t^2+1)}{t^4(t+1)} \\ = t-1+\frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1+\frac{(t-1)(t^2+1)}{t^4} = t-1+\frac{t^3-t^2+t-1}{t^4} = t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4} \end{array}\right]$$

$$= 6\int\Big(t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4}\Big)dt = 6\int\Big(t-1+\frac{1}{t}-t^{-2}+t^{-3}-t^{-4}\Big)dt$$

$$= 6\Big(\frac{t^2}{2}-t+\ln|t|-\frac{t^{-1}}{-1}+\frac{t^{-2}}{-2}+\frac{t^{-3}}{-3}\Big)+c$$

$$= 3t^2-6t+6\ln t++\frac{6}{t}-\frac{12}{t^2}+\frac{18}{t^3}+c$$

# Riešené príklady – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,dx$$

$$= \left[\begin{array}{ll|l|l} \text{Subst.} & t=\sqrt[6]{x} & \sqrt{x}=\sqrt{t^6}=t^3 & x\in(0;\infty) \\ x=t^6, & dx=6t^5\,dt & \sqrt[3]{x^2}=\sqrt[3]{t^{12}}=t^4 & t\in(0;\infty) \end{array}\right] = \int \frac{(t^6-1)6t^5\,dt}{(t^3+t^4)t^6} = 6\int \frac{(t^6-1)\,dt}{t^4+t^5}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1+\frac{t^4-1}{t^5+t^4} = t-1+\frac{(t^2-1)(t^2+1)}{t^4(t+1)} \\ = t-1+\frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1+\frac{(t-1)(t^2+1)}{t^4} = t-1+\frac{t^3-t^2+t-1}{t^4} = t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4} \end{array}\right]$$

$$= 6\int\left(t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4}\right)dt = 6\int\left(t-1+\frac{1}{t}-t^{-2}+t^{-3}-t^{-4}\right)dt$$

$$= 6\left(\frac{t^2}{2}-t+\ln|t|-\frac{t^{-1}}{-1}+\frac{t^{-2}}{-2}+\frac{t^{-3}}{-3}\right)+c$$

$$= 3t^2-6t+6\ln t++\frac{6}{t}-\frac{12}{t^2}+\frac{18}{t^3}+c = 3t^2-6t+\ln t^6+\frac{6}{t}-\frac{12}{t^2}+\frac{18}{t^3}+c$$

# Riešené príklady – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad t=\sqrt[6]{x} & \sqrt{x}=\sqrt{t^6}=t^3 & x\in(0;\infty) \\ x=t^6,\ dx=6t^5\,dt & \sqrt[3]{x^2}=\sqrt[3]{t^{12}}=t^4 & t\in(0;\infty) \end{array}\right] = \int \frac{(t^6-1)6t^5\,dt}{(t^3+t^4)t^6} = 6\int \frac{(t^6-1)\,dt}{t^4+t^5}$$

$$= \left[\begin{array}{l} \text{Rozklad na parciálne zlomky: } \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1+\frac{t^4-1}{t^5+t^4} = t-1+\frac{(t^2-1)(t^2+1)}{t^4(t+1)} \\ \quad = t-1+\frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1+\frac{(t-1)(t^2+1)}{t^4} = t-1+\frac{t^3-t^2+t-1}{t^4} = t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4} \end{array}\right]$$

$$= 6\int\Big(t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4}\Big)dt = 6\int\Big(t-1+\frac{1}{t}-t^{-2}+t^{-3}-t^{-4}\Big)dt$$

$$= 6\Big(\frac{t^2}{2}-t+\ln|t|-\frac{t^{-1}}{-1}+\frac{t^{-2}}{-2}+\frac{t^{-3}}{-3}\Big)+c$$

$$= 3t^2-6t+6\ln t++\frac{6}{t}-\frac{12}{t^2}+\frac{18}{t^3}+c = 3t^2-6t+\ln t^6+\frac{6}{t}-\frac{12}{t^2}+\frac{18}{t^3}+c$$

$$= 3\sqrt[6]{x^2}-6\sqrt[6]{x}+\ln\sqrt[6]{x^6}+\frac{6}{\sqrt[6]{x}}-\frac{12}{\sqrt[6]{x^2}}+\frac{18}{\sqrt[6]{x^3}}+c$$

# Riešené príklady – 186

$$\int \frac{x-1}{(\sqrt{x}+\sqrt[3]{x^2})x}\,dx = 3\sqrt[3]{x} - 6\sqrt[6]{x} + \ln x + \frac{6}{\sqrt[6]{x}} - \frac{12}{\sqrt[3]{x}} + \frac{18}{\sqrt{x}} + c$$

$$= \left[\begin{matrix} \text{Subst.} & t=\sqrt[6]{x} \\ x=t^6, & dx=6t^5\,dt \end{matrix}\;\middle|\; \begin{matrix} \sqrt{x}=\sqrt{t^6}=t^3 \\ \sqrt[3]{x^2}=\sqrt[3]{t^{12}}=t^4 \end{matrix}\;\middle|\; \begin{matrix} x\in(0;\infty) \\ t\in(0;\infty) \end{matrix}\right] = \int \frac{(t^6-1)6t^5\,dt}{(t^3+t^4)t^6} = 6\int \frac{(t^6-1)\,dt}{t^4+t^5}$$

$$= \left[ \begin{matrix} \text{Rozklad na parciálne zlomky:} \quad \frac{t^6-1}{t^5+t^4} = \frac{t^6+t^5-t^5-t^4+t^4-1}{t^5+t^4} = t-1+\frac{t^4-1}{t^5+t^4} = t-1+\frac{(t^2-1)(t^2+1)}{t^4(t+1)} \\ = t-1+\frac{(t-1)(t+1)(t^2+1)}{t^4(t+1)} = t-1+\frac{(t-1)(t^2+1)}{t^4} = t-1+\frac{t^3-t^2+t-1}{t^4} = t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4} \end{matrix} \right]$$

$$= 6\int\left(t-1+\frac{1}{t}-\frac{1}{t^2}+\frac{1}{t^3}-\frac{1}{t^4}\right)dt = 6\int\left(t-1+\frac{1}{t}-t^{-2}+t^{-3}-t^{-4}\right)dt$$

$$= 6\left(\frac{t^2}{2}-t+\ln|t|-\frac{t^{-1}}{-1}+\frac{t^{-2}}{-2}+\frac{t^{-3}}{-3}\right)+c$$

$$= 3t^2-6t+6\ln t++\frac{6}{t}-\frac{12}{t^2}+\frac{18}{t^3}+c = 3t^2-6t+\ln t^6+\frac{6}{t}-\frac{12}{t^2}+\frac{18}{t^3}+c$$

$$= 3\sqrt[6]{x^2}-6\sqrt[6]{x}+\ln\sqrt[6]{x^6}+\frac{6}{\sqrt[6]{x}}-\frac{12}{\sqrt[6]{x^2}}+\frac{18}{\sqrt[6]{x^3}}+c$$

$$= 3\sqrt[3]{x}-6\sqrt[6]{x}+\ln x+\frac{6}{\sqrt[6]{x}}-\frac{12}{\sqrt[3]{x}}+\frac{18}{\sqrt{x}}+c, \quad x\in(0;\infty),\ c\in R.$$

# Riešené príklady – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}}$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}}$$

# Riešené príklady – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}}$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array} \right]$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}}$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array} \right]$$

# Riešené príklady – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}}$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array} \right] = \int \frac{2t\,dt}{(t^2+1)t}$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}}$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array} \right] = \int \frac{-2t\,dt}{(2-t^2)t}$$

# Riešené príklady – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}}$$

$$= \left[\begin{array}{c|c} \text{Subst.} \quad t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2,\ dx=2\,dt & t\in(0;\infty) \end{array}\right] = \int \frac{2t\,dt}{(t^2+1)t} = \int \frac{2\,dt}{t^2+1}$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}}$$

$$= \left[\begin{array}{c|c} \text{Subst.} \quad t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2,\ dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array}\right] = \int \frac{-2t\,dt}{(2-t^2)t} = 2\int \frac{dt}{t^2-2}$$

# Riešené príklady – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}}$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array} \right] = \int \frac{2t\,dt}{(t^2+1)t} = \int \frac{2\,dt}{t^2+1} = 2\,\text{arctg}\,t + c$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}}$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array} \right] = \int \frac{-2t\,dt}{(2-t^2)t} = 2\int \frac{dt}{t^2-2} = \frac{2}{2\sqrt{2}} \ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + c$$

# Riešené príklady – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}} = 2 \operatorname{arctg} \sqrt{x-2} + c$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array} \right] = \int \frac{2t\,dt}{(t^2+1)t} = \int \frac{2\,dt}{t^2+1} = 2 \operatorname{arctg} t + c$$

$$= 2 \operatorname{arctg} \sqrt{x-2} + c,\ x\in(2;\infty),\ c\in R.$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}} = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array} \right] = \int \frac{-2t\,dt}{(2-t^2)t} = 2\int \frac{dt}{t^2-2} = \frac{2}{2\sqrt{2}} \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| + c$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \right| + c$$

# Riešené príklady – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}} = 2\operatorname{arctg}\sqrt{x-2} + c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array}\right] = \int \frac{2t\,dt}{(t^2+1)t} = \int \frac{2\,dt}{t^2+1} = 2\operatorname{arctg} t + c$$

$$= 2\operatorname{arctg}\sqrt{x-2} + c,\ x\in(2;\infty),\ c\in R.$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}} = \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}}\right| + c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array}\right] = \int \frac{-2t\,dt}{(2-t^2)t} = 2\int \frac{dt}{t^2-2} = \frac{2}{2\sqrt{2}} \ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + c$$

$$= \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}}\right| + c = \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \cdot \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}-\sqrt{2}}\right| + c$$

# Riešené príklady – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}} = 2\operatorname{arctg}\sqrt{x-2}+c$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2,\ dx=2\,dt & t\in(0;\infty) \end{array}\right] = \int \frac{2t\,dt}{(t^2+1)t} = \int \frac{2\,dt}{t^2+1} = 2\operatorname{arctg} t + c$$

$$= 2\operatorname{arctg}\sqrt{x-2}+c,\ x\in(2;\infty),\ c\in R.$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}} = \frac{1}{\sqrt{2}}\ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}}\right|+c$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2,\ dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array}\right] = \int \frac{-2t\,dt}{(2-t^2)t} = 2\int \frac{dt}{t^2-2} = \frac{2}{2\sqrt{2}}\ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right|+c$$

$$= \frac{1}{\sqrt{2}}\ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}}\right|+c = \frac{1}{\sqrt{2}}\ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}}\cdot\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}-\sqrt{2}}\right|+c$$

$$= \frac{1}{\sqrt{2}}\ln\left|\frac{1-x-\sqrt{2}\cdot\sqrt{1-x}+2}{1-x-2}\right|+c$$

# Riešené príklady – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}} = 2\,\mathrm{arctg}\,\sqrt{x-2} + c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array}\right] = \int \frac{2t\,dt}{(t^2+1)t} = \int \frac{2\,dt}{t^2+1} = 2\,\mathrm{arctg}\,t + c$$

$$= 2\,\mathrm{arctg}\,\sqrt{x-2} + c,\ x\in(2;\infty),\ c\in R.$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}} = \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}}\right| + c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array}\right] = \int \frac{-2t\,dt}{(2-t^2)t} = 2\int \frac{dt}{t^2-2} = \frac{2}{2\sqrt{2}} \ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + c$$

$$= \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}}\right| + c = \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \cdot \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}-\sqrt{2}}\right| + c$$

$$= \frac{1}{\sqrt{2}} \ln\left|\frac{1-x-\sqrt{2}\cdot\sqrt{1-x}+2}{1-x-2}\right| + c = \frac{1}{\sqrt{2}} \ln\left|\frac{3-x-\sqrt{2(1-x)}}{-x-1}\right| + c$$

# Riešené príklady – 187, 188

$$\int \frac{dx}{(x-1)\sqrt{x-2}} = 2 \operatorname{arctg} \sqrt{x-2} + c$$

$$= \left[\begin{array}{cc|c} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array}\right] = \int \frac{2t\,dt}{(t^2+1)t} = \int \frac{2\,dt}{t^2+1} = 2 \operatorname{arctg} t + c$$

$$= 2 \operatorname{arctg} \sqrt{x-2} + c,\ x\in(2;\infty),\ c\in R.$$

$$\int \frac{dx}{(x+1)\sqrt{1-x}} = \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}}\right| + c$$

$$= \left[\begin{array}{cc|c} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array}\right] = \int \frac{-2t\,dt}{(2-t^2)t} = 2\int \frac{dt}{t^2-2} = \frac{2}{2\sqrt{2}} \ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + c$$

$$= \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}}\right| + c = \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}} \cdot \frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}-\sqrt{2}}\right| + c$$

$$= \frac{1}{\sqrt{2}} \ln\left|\frac{1-x-\sqrt{2}\cdot\sqrt{1-x}+2}{1-x-2}\right| + c = \frac{1}{\sqrt{2}} \ln\left|\frac{3-x-\sqrt{2(1-x)}}{-x-1}\right| + c$$

$$= \frac{1}{\sqrt{2}} \ln\left|3-x-\sqrt{2(1-x)}\right| - \frac{1}{\sqrt{2}} \ln|x+1| + c,\ x\in(-\infty;1)-\{-1\},\ c\in R.$$

# Riešené príklady – 189, 190

$$\int (x-1)\sqrt{x-2}\,dx$$

$$\int \frac{\sqrt{1-x}}{x+1}\,dx$$

# Riešené príklady – 189, 190

$$\int (x-1)\sqrt{x-2}\,dx$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array} \right]$$

$$\int \frac{\sqrt{1-x}}{x+1}\,dx$$

$$= \left[ \begin{array}{ll|l} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array} \right]$$

# Riešené príklady – 189, 190

$$\int (x-1)\sqrt{x-2}\,dx$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array}\right] = \int (t^2+1)t\cdot 2t\,dt$$

$$\int \frac{\sqrt{1-x}}{x+1}\,dx$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array}\right] = \int \frac{-2t^2\,dt}{2-t^2}$$

# Riešené príklady – 189, 190

$$\int (x-1)\sqrt{x-2}\,dx$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array}\right] = \int (t^2+1)t\cdot 2t\,dt = \int (2t^4+2t^2)\,dt$$

$$\int \frac{\sqrt{1-x}}{x+1}\,dx$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array}\right] = \int \frac{-2t^2\,dt}{2-t^2} = 2\int \frac{t^2\,dt}{t^2-2}$$

# Riešené príklady – 189, 190

$$\int (x-1)\sqrt{x-2}\,dx$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2,\ dx=2\,dt & t\in(0;\infty) \end{array}\right] = \int (t^2+1)t\cdot 2t\,dt = \int (2t^4+2t^2)\,dt$$

$$= \frac{2t^5}{5} + \frac{2t^3}{3} + c$$

$$\int \frac{\sqrt{1-x}}{x+1}\,dx$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2,\ dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array}\right] = \int \frac{-2t^2\,dt}{2-t^2} = 2\int \frac{t^2\,dt}{t^2-2} = 2\int \frac{t^2-2+2\,dt}{t^2-2}$$

# Riešené príklady – 189, 190

$$\int (x-1)\sqrt{x-2}\,dx = \frac{2\sqrt{(x-2)^5}}{5} + \frac{2\sqrt{(x-2)^3}}{3} + c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array}\right] = \int (t^2+1)t\cdot 2t\,dt = \int (2t^4+2t^2)\,dt$$

$$= \frac{2t^5}{5} + \frac{2t^3}{3} + c = \frac{2\sqrt{(x-2)^5}}{5} + \frac{2\sqrt{(x-2)^3}}{3} + c,\ x\in(2;\infty),\ c\in R.$$

$$\int \frac{\sqrt{1-x}}{x+1}\,dx$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array}\right] = \int \frac{-2t^2\,dt}{2-t^2} = 2\int \frac{t^2\,dt}{t^2-2} = 2\int \frac{t^2-2+2\,dt}{t^2-2}$$

$$= 2\int dt + 4\int \frac{dt}{t^2-2}$$

# Riešené príklady – 189, 190

$$\int (x-1)\sqrt{x-2}\,dx = \frac{2\sqrt{(x-2)^5}}{5} + \frac{2\sqrt{(x-2)^3}}{3} + c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array}\right] = \int (t^2+1)t\cdot 2t\,dt = \int (2t^4+2t^2)\,dt$$

$$= \frac{2t^5}{5} + \frac{2t^3}{3} + c = \frac{2\sqrt{(x-2)^5}}{5} + \frac{2\sqrt{(x-2)^3}}{3} + c,\ x\in(2;\infty),\ c\in R.$$

$$\int \frac{\sqrt{1-x}}{x+1}\,dx$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array}\right] = \int \frac{-2t^2\,dt}{2-t^2} = 2\int \frac{t^2\,dt}{t^2-2} = 2\int \frac{t^2-2+2\,dt}{t^2-2}$$

$$= 2\int dt + 4\int \frac{dt}{t^2-2} = 2t + \frac{4}{2\sqrt{2}} \ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + c$$

# Riešené príklady – 189, 190

$$\int (x-1)\sqrt{x-2}\,dx = \frac{2\sqrt{(x-2)^5}}{5} + \frac{2\sqrt{(x-2)^3}}{3} + c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{x-2} & x\in(2;\infty) \\ x=t^2+2, & dx=2\,dt & t\in(0;\infty) \end{array}\right] = \int (t^2+1)t\cdot 2t\,dt = \int (2t^4+2t^2)\,dt$$

$$= \frac{2t^5}{5} + \frac{2t^3}{3} + c = \frac{2\sqrt{(x-2)^5}}{5} + \frac{2\sqrt{(x-2)^3}}{3} + c,\ x\in(2;\infty),\ c\in R.$$

$$\int \frac{\sqrt{1-x}}{x+1}\,dx = 2\sqrt{1-x} + \sqrt{2}\ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}}\right| + c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\sqrt{1-x} & x\in(-\infty;1)-\{-1\} \\ x=1-t^2, & dx=-2\,dt & t\in(0;\infty)-\{\sqrt{2}\} \end{array}\right] = \int \frac{-2t^2\,dt}{2-t^2} = 2\int \frac{t^2\,dt}{t^2-2} = 2\int \frac{t^2-2+2\,dt}{t^2-2}$$

$$= 2\int dt + 4\int \frac{dt}{t^2-2} = 2t + \frac{4}{2\sqrt{2}}\ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + c$$

$$= 2\sqrt{1-x} + \sqrt{2}\ln\left|\frac{\sqrt{1-x}-\sqrt{2}}{\sqrt{1-x}+\sqrt{2}}\right| + c,\ x\in(-\infty;1)-\{-1\},\ c\in R.$$

# Riešené príklady – 191

$$\int \frac{dx}{\left(x-\sqrt{x^2-1}\right)^2}$$

# Riešené príklady – 191

$$\int \frac{\mathrm{d}x}{\left(x-\sqrt{x^2-1}\right)^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 \mathrm{d}x}{\left(x-\sqrt{x^2-1}\right)^2 \cdot \left(x+\sqrt{x^2-1}\right)^2}$$

# Riešené príklady – 191

$$\int \frac{dx}{\left(x-\sqrt{x^2-1}\right)^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left(x-\sqrt{x^2-1}\right)^2 \cdot \left(x+\sqrt{x^2-1}\right)^2} = \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[\left(x-\sqrt{x^2-1}\right) \cdot \left(x+\sqrt{x^2-1}\right)\right]^2}$$

# Riešené príklady – 191

$$\int \frac{\mathrm{d}x}{\left(x-\sqrt{x^2-1}\right)^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 \mathrm{d}x}{\left(x-\sqrt{x^2-1}\right)^2 \cdot \left(x+\sqrt{x^2-1}\right)^2} = \int \frac{\left(x+\sqrt{x^2-1}\right)^2 \mathrm{d}x}{\left[\left(x-\sqrt{x^2-1}\right) \cdot \left(x+\sqrt{x^2-1}\right)\right]^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 \mathrm{d}x}{\left[x^2-(x^2-1)\right]^2}$$

# Riešené príklady – 191

$$\int \frac{dx}{\left(x-\sqrt{x^2-1}\right)^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left(x-\sqrt{x^2-1}\right)^2 \cdot \left(x+\sqrt{x^2-1}\right)^2} = \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[\left(x-\sqrt{x^2-1}\right) \cdot \left(x+\sqrt{x^2-1}\right)\right]^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[x^2-(x^2-1)\right]^2} = \int \frac{\left(x^2+2x\sqrt{x^2-1}+x^2-1\right) dx}{1}$$

# Riešené príklady – 191

$$\int \frac{dx}{\left(x-\sqrt{x^2-1}\right)^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left(x-\sqrt{x^2-1}\right)^2 \cdot \left(x+\sqrt{x^2-1}\right)^2} = \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[\left(x-\sqrt{x^2-1}\right) \cdot \left(x+\sqrt{x^2-1}\right)\right]^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[x^2-(x^2-1)\right]^2} = \int \frac{\left(x^2+2x\sqrt{x^2-1}+x^2-1\right) dx}{1}$$

$$= \int \left(2x^2-1+2x\sqrt{x^2-1}\right) dx$$

# Riešené príklady – 191

$$\int \frac{dx}{\left(x-\sqrt{x^2-1}\right)^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left(x-\sqrt{x^2-1}\right)^2 \cdot \left(x+\sqrt{x^2-1}\right)^2} = \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[\left(x-\sqrt{x^2-1}\right) \cdot \left(x+\sqrt{x^2-1}\right)\right]^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[x^2-(x^2-1)\right]^2} = \int \frac{\left(x^2+2x\sqrt{x^2-1}+x^2-1\right) dx}{1}$$

$$= \int \left(2x^2-1+2x\sqrt{x^2-1}\right) dx = 2\int x^2\, dx - \int dx + \int 2x\sqrt{x^2-1}\, dx$$

# Riešené príklady – 191

$$\int \frac{dx}{\left(x-\sqrt{x^2-1}\right)^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left(x-\sqrt{x^2-1}\right)^2 \cdot \left(x+\sqrt{x^2-1}\right)^2} = \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[\left(x-\sqrt{x^2-1}\right)\cdot\left(x+\sqrt{x^2-1}\right)\right]^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[x^2-(x^2-1)\right]^2} = \int \frac{\left(x^2+2x\sqrt{x^2-1}+x^2-1\right) dx}{1}$$

$$= \int\left(2x^2-1+2x\sqrt{x^2-1}\right)dx = 2\int x^2\, dx - \int dx + \int 2x\sqrt{x^2-1}\, dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2-1 & x\in(-\infty;-1\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 1;\infty),\ t\in\langle 0;\infty) \end{array}\right]$$

# Riešené príklady – 191

$$\int \frac{dx}{\left(x-\sqrt{x^2-1}\right)^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left(x-\sqrt{x^2-1}\right)^2 \cdot \left(x+\sqrt{x^2-1}\right)^2} = \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[\left(x-\sqrt{x^2-1}\right)\cdot\left(x+\sqrt{x^2-1}\right)\right]^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[x^2-(x^2-1)\right]^2} = \int \frac{\left(x^2+2x\sqrt{x^2-1}+x^2-1\right) dx}{1}$$

$$= \int \left(2x^2-1+2x\sqrt{x^2-1}\right) dx = 2\int x^2\,dx - \int dx + \int 2x\sqrt{x^2-1}\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2-1 & x\in(-\infty;-1\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 1;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{2x^3}{3} - x + \int \sqrt{t}\,dt$$

# Riešené príklady – 191

$$\int \frac{dx}{\left(x-\sqrt{x^2-1}\right)^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left(x-\sqrt{x^2-1}\right)^2 \cdot \left(x+\sqrt{x^2-1}\right)^2} = \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[\left(x-\sqrt{x^2-1}\right) \cdot \left(x+\sqrt{x^2-1}\right)\right]^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[x^2-(x^2-1)\right]^2} = \int \frac{\left(x^2+2x\sqrt{x^2-1}+x^2-1\right) dx}{1}$$

$$= \int \left(2x^2-1+2x\sqrt{x^2-1}\right) dx = 2\int x^2\, dx - \int dx + \int 2x\sqrt{x^2-1}\, dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2-1 & x\in(-\infty;-1),\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 1;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{2x^3}{3} - x + \int \sqrt{t}\, dt$$

$$= \frac{2x^3}{3} - x + \int t^{\frac{1}{2}}\, dt$$

# Riešené príklady – 191

$$\int \frac{dx}{\left(x-\sqrt{x^2-1}\right)^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left(x-\sqrt{x^2-1}\right)^2 \cdot \left(x+\sqrt{x^2-1}\right)^2} = \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[\left(x-\sqrt{x^2-1}\right) \cdot \left(x+\sqrt{x^2-1}\right)\right]^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[x^2-(x^2-1)\right]^2} = \int \frac{\left(x^2+2x\sqrt{x^2-1}+x^2-1\right) dx}{1}$$

$$= \int \left(2x^2-1+2x\sqrt{x^2-1}\right) dx = 2\int x^2\, dx - \int dx + \int 2x\sqrt{x^2-1}\, dx$$

$$= \left[\begin{array}{c|l} \text{Subst. } t=x^2-1 & x\in(-\infty;-1\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 1;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{2x^3}{3} - x + \int \sqrt{t}\, dt$$

$$= \frac{2x^3}{3} - x + \int t^{\frac{1}{2}}\, dt = \frac{2x^3}{3} - x + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

# Riešené príklady – 191

$$\int \frac{dx}{\left(x-\sqrt{x^2-1}\right)^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left(x-\sqrt{x^2-1}\right)^2 \cdot \left(x+\sqrt{x^2-1}\right)^2} = \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[\left(x-\sqrt{x^2-1}\right) \cdot \left(x+\sqrt{x^2-1}\right)\right]^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[x^2-(x^2-1)\right]^2} = \int \frac{\left(x^2+2x\sqrt{x^2-1}+x^2-1\right) dx}{1}$$

$$= \int \left(2x^2-1+2x\sqrt{x^2-1}\right) dx = 2\int x^2\,dx - \int dx + \int 2x\sqrt{x^2-1}\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2-1 & x\in(-\infty;-1\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 1;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{2x^3}{3} - x + \int \sqrt{t}\,dt$$

$$= \frac{2x^3}{3} - x + \int t^{\frac{1}{2}}\,dt = \frac{2x^3}{3} - x + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2x^3}{3} - x + \frac{2\sqrt{t^3}}{3} + c$$

# Riešené príklady – 191

$$\int \frac{dx}{\left(x-\sqrt{x^2-1}\right)^2} = \frac{2x^3}{3} - x + \frac{2\sqrt{(x^2-1)^3}}{3} + c$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left(x-\sqrt{x^2-1}\right)^2 \cdot \left(x+\sqrt{x^2-1}\right)^2} = \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[\left(x-\sqrt{x^2-1}\right)\cdot\left(x+\sqrt{x^2-1}\right)\right]^2}$$

$$= \int \frac{\left(x+\sqrt{x^2-1}\right)^2 dx}{\left[x^2-(x^2-1)\right]^2} = \int \frac{\left(x^2+2x\sqrt{x^2-1}+x^2-1\right) dx}{1}$$

$$= \int \left(2x^2-1+2x\sqrt{x^2-1}\right) dx = 2\int x^2\,dx - \int dx + \int 2x\sqrt{x^2-1}\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x^2-1 & x\in(-\infty;-1\rangle,\ t\in\langle 0;\infty) \\ dt=2x\,dx & x\in\langle 1;\infty),\ t\in\langle 0;\infty) \end{array}\right] = \frac{2x^3}{3} - x + \int \sqrt{t}\,dt$$

$$= \frac{2x^3}{3} - x + \int t^{\frac{1}{2}}\,dt = \frac{2x^3}{3} - x + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2x^3}{3} - x + \frac{2\sqrt{t^3}}{3} + c$$

$$= \frac{2x^3}{3} - x + \frac{2\sqrt{(x^2-1)^3}}{3} + c,\ x\in(-\infty;-1\rangle \cup \langle 1;\infty),\ c\in R.$$

# Riešené príklady – 192, 193

$$\int \frac{x^5\,dx}{\sqrt{x^2+1}}$$

$$\int \frac{x^5\,dx}{\sqrt{x^3+1}}$$

# Riešené príklady – 192, 193

$$\int \frac{x^5\,dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{x^2+1}}$$

$$\int \frac{x^5\,dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{x^3+1}}$$

# Riešené príklady – 192, 193

$$\int \frac{x^5\,dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{x^2+1}} = \left[ \begin{array}{r|l} \text{Subst. } t = x^2+1 & x \in \langle 0;\infty),\ t \in \langle 1;\infty) \\ dt = 2x\,dx & x \in (-\infty;0\rangle,\ t \in \langle 1;\infty) \end{array} \right]$$

$$\int \frac{x^5\,dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{x^3+1}} = \left[ \begin{array}{r|l} \text{Subst. } t = x^3+1 & x \in (-1;\infty) \\ dt = 3x^2\,dx & x \in (0;\infty) \end{array} \right]$$

# Riešené príklady – 192, 193

$$\int \frac{x^5\,dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2}\int \frac{x^4\cdot 2x\,dx}{\sqrt{x^2+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^2+1 & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \\ dt=2x\,dx & x\in(-\infty;0\rangle,\ t\in\langle 1;\infty) \end{array}\right] = \frac{1}{2}\int \frac{(t-1)^2\,dt}{\sqrt{t}}$$

$$\int \frac{x^5\,dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3}\int \frac{x^3\cdot 3x^2\,dx}{\sqrt{x^3+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^3+1 & x\in(-1;\infty) \\ dt=3x^2\,dx & x\in(0;\infty) \end{array}\right] = \frac{1}{3}\int \frac{(t-1)\,dt}{\sqrt{t}}$$

# Riešené príklady – 192, 193

$$\int \frac{x^5\,dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2}\int \frac{x^4\cdot 2x\,dx}{\sqrt{x^2+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^2+1 & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \\ dt=2x\,dx & x\in(-\infty;0\rangle,\ t\in\langle 1;\infty) \end{array}\right] = \frac{1}{2}\int \frac{(t-1)^2\,dt}{\sqrt{t}} = \frac{1}{2}\int \frac{t^2-2t+1}{t^{\frac{1}{2}}}\,dt$$

$$\int \frac{x^5\,dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3}\int \frac{x^3\cdot 3x^2\,dx}{\sqrt{x^3+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^3+1 & x\in(-1;\infty) \\ dt=3x^2\,dx & x\in(0;\infty) \end{array}\right] = \frac{1}{3}\int \frac{(t-1)\,dt}{\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}}\,dt$$

# Riešené príklady – 192, 193

$$\int \frac{x^5\,dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2}\int \frac{x^4\cdot 2x\,dx}{\sqrt{x^2+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^2+1 & x\in(0;\infty),\ t\in(1;\infty) \\ dt=2x\,dx & x\in(-\infty;0),\ t\in(1;\infty) \end{array}\right] = \frac{1}{2}\int \frac{(t-1)^2\,dt}{\sqrt{t}} = \frac{1}{2}\int \frac{t^2-2t+1}{t^{\frac{1}{2}}}\,dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{2} - t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{2}\right] dt$$

$$\int \frac{x^5\,dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3}\int \frac{x^3\cdot 3x^2\,dx}{\sqrt{x^3+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^3+1 & x\in(-1;\infty) \\ dt=3x^2\,dx & x\in(0;\infty) \end{array}\right] = \frac{1}{3}\int \frac{(t-1)\,dt}{\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}}\,dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3}\right] dt$$

# Riešené príklady – 192, 193

$$\int \frac{x^5\,dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2}\int \frac{x^4\cdot 2x\,dx}{\sqrt{x^2+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^2+1 & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \\ dt=2x\,dx & x\in(-\infty;0\rangle,\ t\in\langle 1;\infty) \end{array}\right] = \frac{1}{2}\int \frac{(t-1)^2\,dt}{\sqrt{t}} = \frac{1}{2}\int \frac{t^2-2t+1}{t^{\frac{1}{2}}}\,dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{2} - t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{2}\right] dt = \frac{t^{\frac{5}{2}}}{2\cdot\frac{5}{2}} - \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{2\cdot\frac{1}{2}} + c$$

$$\int \frac{x^5\,dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3}\int \frac{x^3\cdot 3x^2\,dx}{\sqrt{x^3+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^3+1 & x\in(-1;\infty) \\ dt=3x^2\,dx & x\in(0;\infty) \end{array}\right] = \frac{1}{3}\int \frac{(t-1)\,dt}{\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}}\,dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3}\right] dt$$

$$= \frac{t^{\frac{3}{2}}}{3\cdot\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{3\cdot\frac{1}{2}} + c$$

# Riešené príklady – 192, 193

$$\int \frac{x^5\,dx}{\sqrt{x^2+1}}$$

$$= \frac{1}{2}\int \frac{x^4\cdot 2x\,dx}{\sqrt{x^2+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^2+1 & x\in\langle 0;\infty), \ t\in\langle 1;\infty) \\ dt=2x\,dx & x\in(-\infty;0\rangle, \ t\in\langle 1;\infty) \end{array}\right] = \frac{1}{2}\int \frac{(t-1)^2\,dt}{\sqrt{t}} = \frac{1}{2}\int \frac{t^2-2t+1}{t^{\frac{1}{2}}}\,dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{2} - t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{2}\right] dt = \frac{t^{\frac{5}{2}}}{2\cdot\frac{5}{2}} - \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{2\cdot\frac{1}{2}} + c = \frac{\sqrt{t^5}}{5} - \frac{2\sqrt{t^3}}{3} + \sqrt{t} + c$$

$$\int \frac{x^5\,dx}{\sqrt{x^3+1}}$$

$$= \frac{1}{3}\int \frac{x^3\cdot 3x^2\,dx}{\sqrt{x^3+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^3+1 & x\in(-1;\infty) \\ dt=3x^2\,dx & x\in(0;\infty) \end{array}\right] = \frac{1}{3}\int \frac{(t-1)\,dt}{\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}}\,dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3}\right] dt$$

$$= \frac{t^{\frac{3}{2}}}{3\cdot\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{3\cdot\frac{1}{2}} + c = \frac{2\sqrt{t^3}}{9} - \frac{2\sqrt{t}}{3} + c$$

# Riešené príklady – 192, 193

$$\int \frac{x^5\,dx}{\sqrt{x^2+1}} = \frac{\sqrt{(x^2+1)^5}}{5} - \frac{2\sqrt{(x^2+1)^3}}{3} + \sqrt{x^2+1} + c$$

$$= \frac{1}{2}\int \frac{x^4\cdot 2x\,dx}{\sqrt{x^2+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^2+1 & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \\ dt=2x\,dx & x\in(-\infty;0\rangle,\ t\in\langle 1;\infty) \end{array}\right] = \frac{1}{2}\int \frac{(t-1)^2\,dt}{\sqrt{t}} = \frac{1}{2}\int \frac{t^2-2t+1}{t^{\frac{1}{2}}}\,dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{2} - t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{2}\right] dt = \frac{t^{\frac{5}{2}}}{2\cdot\frac{5}{2}} - \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{2\cdot\frac{1}{2}} + c = \frac{\sqrt{t^5}}{5} - \frac{2\sqrt{t^3}}{3} + \sqrt{t} + c$$

$$= \frac{\sqrt{(x^2+1)^5}}{5} - \frac{2\sqrt{(x^2+1)^3}}{3} + \sqrt{x^2+1} + c,\ x\in R,\ c\in R.$$

$$\int \frac{x^5\,dx}{\sqrt{x^3+1}} = \frac{2\sqrt{(x^3+1)^3}}{9} - \frac{2\sqrt{x^3+1}}{3} + c$$

$$= \frac{1}{3}\int \frac{x^3\cdot 3x^2\,dx}{\sqrt{x^3+1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^3+1 & x\in(-1;\infty) \\ dt=3x^2\,dx & x\in(0;\infty) \end{array}\right] = \frac{1}{3}\int \frac{(t-1)\,dt}{\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}}\,dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3}\right] dt$$

$$= \frac{t^{\frac{3}{2}}}{3\cdot\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{3\cdot\frac{1}{2}} + c = \frac{2\sqrt{t^3}}{9} - \frac{2\sqrt{t}}{3} + c = \frac{2\sqrt{(x^3+1)^3}}{9} - \frac{2\sqrt{x^3+1}}{3} + c,$$

$$x\in(-1;\infty),\ c\in R.$$

# Riešené príklady – 194, 195

$$\int \frac{x^5\,dx}{\sqrt{x^2-1}}$$

$$\int \frac{x^5\,dx}{\sqrt{x^3-1}}$$

# Riešené príklady – 194, 195

$$\int \frac{x^5\,dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{x^2-1}}$$

$$\int \frac{x^5\,dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{x^3-1}}$$

# Riešené príklady – 194, 195

$$\int \frac{x^5\,dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{x^2-1}} = \left[ \begin{array}{r|l} \text{Subst. } t = x^2-1 & x \in (1;\infty),\ t \in (0;\infty) \\ dt = 2x\,dx & x \in (-\infty;-1),\ t \in (0;\infty) \end{array} \right]$$

$$\int \frac{x^5\,dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{x^3-1}} = \left[ \begin{array}{r|l} \text{Subst. } t = x^3-1 & x \in (1;\infty) \\ dt = 3x^2\,dx & x \in (0;\infty) \end{array} \right]$$

# Riešené príklady – 194, 195

$$\int \frac{x^5\,dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{x^2-1}} = \left[ \begin{array}{r|l} \text{Subst. } t=x^2-1 & x\in(1;\infty),\ t\in(0;\infty) \\ dt=2x\,dx & x\in(-\infty;-1),\ t\in(0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{(t+1)^2\,dt}{\sqrt{t}}$$

$$\int \frac{x^5\,dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{x^3-1}} = \left[ \begin{array}{r|l} \text{Subst. } t=x^3-1 & x\in(1;\infty) \\ dt=3x^2\,dx & x\in(0;\infty) \end{array} \right] = \frac{1}{3}\int \frac{(t+1)\,dt}{\sqrt{t}}$$

# Riešené príklady – 194, 195

$$\int \frac{x^5\,dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{x^2-1}} = \left[ \begin{array}{r|l} \text{Subst. } t = x^2-1 & x \in (1;\infty),\ t \in (0;\infty) \\ dt = 2x\,dx & x \in (-\infty;-1),\ t \in (0;\infty) \end{array} \right] = \frac{1}{2}\int \frac{(t+1)^2\,dt}{\sqrt{t}} = \frac{1}{2}\int \frac{t^2+2t+1}{t^{\frac{1}{2}}}\,dt$$

$$\int \frac{x^5\,dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{x^3-1}} = \left[ \begin{array}{r|l} \text{Subst. } t = x^3-1 & x \in (1;\infty) \\ dt = 3x^2\,dx & x \in (0;\infty) \end{array} \right] = \frac{1}{3}\int \frac{(t+1)\,dt}{\sqrt{t}} = \int \frac{t+1}{3t^{\frac{1}{2}}}\,dt$$

# Riešené príklady – 194, 195

$$\int \frac{x^5\,dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{x^2-1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^2-1 & x\in(1;\infty),\ t\in(0;\infty) \\ dt=2x\,dx & x\in(-\infty;-1),\ t\in(0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{(t+1)^2\,dt}{\sqrt{t}} = \frac{1}{2}\int \frac{t^2+2t+1}{t^{\frac{1}{2}}}\,dt$$

$$= \int \left[ \frac{t^{\frac{3}{2}}}{2} + t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{2} \right] dt$$

$$\int \frac{x^5\,dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{x^3-1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^3-1 & x\in(1;\infty) \\ dt=3x^2\,dx & x\in(0;\infty) \end{array}\right] = \frac{1}{3}\int \frac{(t+1)\,dt}{\sqrt{t}} = \int \frac{t+1}{3t^{\frac{1}{2}}}\,dt = \int \left[ \frac{t^{\frac{1}{2}}}{3} + \frac{t^{-\frac{1}{2}}}{3} \right] dt$$

# Riešené príklady – 194, 195

$$\int \frac{x^5\,dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2}\int \frac{x^4\cdot 2x\,dx}{\sqrt{x^2-1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^2-1 & x\in(1;\infty),\ t\in(0;\infty) \\ dt=2x\,dx & x\in(-\infty;-1),\ t\in(0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{(t+1)^2\,dt}{\sqrt{t}} = \frac{1}{2}\int \frac{t^2+2t+1}{t^{\frac{1}{2}}}\,dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{2} + t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{2}\right] dt = \frac{t^{\frac{5}{2}}}{2\cdot\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{2\cdot\frac{1}{2}} + c$$

$$\int \frac{x^5\,dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3}\int \frac{x^3\cdot 3x^2\,dx}{\sqrt{x^3-1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^3-1 & x\in(1;\infty) \\ dt=3x^2\,dx & x\in(0;\infty) \end{array}\right] = \frac{1}{3}\int \frac{(t+1)\,dt}{\sqrt{t}} = \int \frac{t+1}{3t^{\frac{1}{2}}}\,dt = \int \left[\frac{t^{\frac{1}{2}}}{3} + \frac{t^{-\frac{1}{2}}}{3}\right] dt$$

$$= \frac{t^{\frac{3}{2}}}{3\cdot\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{3\cdot\frac{1}{2}} + c$$

# Riešené príklady – 194, 195

$$\int \frac{x^5\,dx}{\sqrt{x^2-1}}$$

$$= \frac{1}{2}\int \frac{x^4\cdot 2x\,dx}{\sqrt{x^2-1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^2-1 & x\in(1;\infty),\ t\in(0;\infty) \\ dt=2x\,dx & x\in(-\infty;-1),\ t\in(0;\infty) \end{array}\right] = \frac{1}{2}\int \frac{(t+1)^2\,dt}{\sqrt{t}} = \frac{1}{2}\int \frac{t^2+2t+1}{t^{\frac{1}{2}}}\,dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{2} + t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{2}\right] dt = \frac{t^{\frac{5}{2}}}{2\cdot\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{2\cdot\frac{1}{2}} + c = \frac{\sqrt{t^5}}{5} + \frac{2\sqrt{t^3}}{3} + \sqrt{t} + c$$

$$\int \frac{x^5\,dx}{\sqrt{x^3-1}}$$

$$= \frac{1}{3}\int \frac{x^3\cdot 3x^2\,dx}{\sqrt{x^3-1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^3-1 & x\in(1;\infty) \\ dt=3x^2\,dx & x\in(0;\infty) \end{array}\right] = \frac{1}{3}\int \frac{(t+1)\,dt}{\sqrt{t}} = \int \frac{t+1}{3t^{\frac{1}{2}}}\,dt = \int \left[\frac{t^{\frac{1}{2}}}{3} + \frac{t^{-\frac{1}{2}}}{3}\right] dt$$

$$= \frac{t^{\frac{3}{2}}}{3\cdot\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{3\cdot\frac{1}{2}} + c = \frac{2\sqrt{t^3}}{9} + \frac{2\sqrt{t}}{3} + c$$

# Riešené príklady – 194, 195

$$\int \frac{x^5\,dx}{\sqrt{x^2-1}} = \frac{\sqrt{(x^2-1)^5}}{5} + \frac{2\sqrt{(x^2-1)^3}}{3} + \sqrt{x^2-1} + c$$

$$= \frac{1}{2}\int \frac{x^4\cdot 2x\,dx}{\sqrt{x^2-1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^2-1 & x\in(1;\infty),\ t\in(0;\infty) \\ dt=2x\,dx & x\in(-\infty;-1),\ t\in(0;\infty)\end{array}\right] = \frac{1}{2}\int \frac{(t+1)^2\,dt}{\sqrt{t}} = \frac{1}{2}\int \frac{t^2+2t+1}{t^{\frac{1}{2}}}\,dt$$

$$= \int \left[\frac{t^{\frac{3}{2}}}{2} + t^{\frac{1}{2}} + \frac{t^{-\frac{1}{2}}}{2}\right]dt = \frac{t^{\frac{5}{2}}}{2\cdot\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{2\cdot\frac{1}{2}} + c = \frac{\sqrt{t^5}}{5} + \frac{2\sqrt{t^3}}{3} + \sqrt{t} + c$$

$$= \frac{\sqrt{(x^2-1)^5}}{5} + \frac{2\sqrt{(x^2-1)^3}}{3} + \sqrt{x^2-1} + c,\ x\in(-\infty;-1)\cup(1;\infty),\ c\in R.$$

$$\int \frac{x^5\,dx}{\sqrt{x^3-1}} = \frac{2\sqrt{(x^3-1)^3}}{9} + \frac{2\sqrt{x^3-1}}{3} + c$$

$$= \frac{1}{3}\int \frac{x^3\cdot 3x^2\,dx}{\sqrt{x^3-1}} = \left[\begin{array}{r|l} \text{Subst. } t=x^3-1 & x\in(1;\infty) \\ dt=3x^2\,dx & x\in(0;\infty)\end{array}\right] = \frac{1}{3}\int \frac{(t+1)\,dt}{\sqrt{t}} = \int \frac{t+1}{3t^{\frac{1}{2}}}\,dt = \int \left[\frac{t^{\frac{1}{2}}}{3} + \frac{t^{-\frac{1}{2}}}{3}\right]dt$$

$$= \frac{t^{\frac{3}{2}}}{3\cdot\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{3\cdot\frac{1}{2}} + c = \frac{2\sqrt{t^3}}{9} + \frac{2\sqrt{t}}{3} + c = \frac{2\sqrt{(x^3-1)^3}}{9} + \frac{2\sqrt{x^3-1}}{3} + c,$$

$$x\in(1;\infty),\ c\in R.$$

# Riešené príklady – 196, 197

$$\int \frac{x^5\,dx}{\sqrt{1-x^2}}$$

$$\int \frac{x^5\,dx}{\sqrt{1-x^3}}$$

# Riešené príklady – 196, 197

$$\int \frac{x^5\,dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{1-x^2}}$$

$$\int \frac{x^5\,dx}{\sqrt{1-x^3}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{1-x^3}}$$

# Riešené príklady – 196, 197

$$\int \frac{x^5\,dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{1-x^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=1-x^2 & x\in\langle 0;1), \; t\in(0;1\rangle \\ dt=-2x\,dx & x\in(-1;0\rangle, \; t\in(0;1\rangle \end{array} \right]$$

$$\int \frac{x^5\,dx}{\sqrt{1-x^3}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{1-x^3}} = \left[ \begin{array}{r|l} \text{Subst. } t=1-x^3 & x\in(-\infty;1) \\ dt=-3x^2\,dx & x\in(0;\infty) \end{array} \right]$$

# Riešené príklady – 196, 197

$$\int \frac{x^5\,dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{1-x^2}} = \left[\begin{array}{r|l} \text{Subst. } t=1-x^2 & x\in\langle 0;1),\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in(-1;0\rangle,\ t\in(0;1\rangle \end{array}\right] = -\frac{1}{2}\int \frac{(1-t)^2\,dt}{\sqrt{t}}$$

$$\int \frac{x^5\,dx}{\sqrt{1-x^3}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{1-x^3}} = \left[\begin{array}{r|l} \text{Subst. } t=1-x^3 & x\in(-\infty;1) \\ dt=-3x^2\,dx & x\in(0;\infty) \end{array}\right] = -\int \frac{(1-t)\,dt}{3\sqrt{t}}$$

# Riešené príklady – 196, 197

$$\int \frac{x^5\,dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{1-x^2}} = \left[\begin{array}{r|l} \text{Subst. } t=1-x^2 & x\in\langle 0;1),\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in(-1;0\rangle,\ t\in(0;1\rangle \end{array}\right] = -\frac{1}{2}\int \frac{(1-t)^2\,dt}{\sqrt{t}} = -\frac{1}{2}\int \frac{1-2t+t^2}{t^{\frac{1}{2}}}\,dt$$

$$\int \frac{x^5\,dx}{\sqrt{1-x^3}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{1-x^3}} = \left[\begin{array}{r|l} \text{Subst. } t=1-x^3 & x\in(-\infty;1) \\ dt=-3x^2\,dx & x\in(0;\infty) \end{array}\right] = -\int \frac{(1-t)\,dt}{3\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}}\,dt$$

# Riešené príklady – 196, 197

$$\int \frac{x^5\,dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{1-x^2}} = \left[\begin{array}{c|c} \text{Subst. } t=1-x^2 & x\in\langle 0;1),\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in(-1;0\rangle,\ t\in(0;1\rangle \end{array}\right] = -\frac{1}{2}\int \frac{(1-t)^2\,dt}{\sqrt{t}} = -\frac{1}{2}\int \frac{1-2t+t^2}{t^{\frac{1}{2}}}\,dt$$

$$= -\int \left[\frac{t^{-\frac{1}{2}}}{2} - t^{\frac{1}{2}} + \frac{t^{\frac{3}{2}}}{2}\right] dt$$

$$\int \frac{x^5\,dx}{\sqrt{1-x^3}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{1-x^3}} = \left[\begin{array}{c|c} \text{Subst. } t=1-x^3 & x\in(-\infty;1) \\ dt=-3x^2\,dx & x\in(0;\infty) \end{array}\right] = -\int \frac{(1-t)\,dt}{3\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}}\,dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3}\right] dt$$

# Riešené príklady – 196, 197

$$\int \frac{x^5\,dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{1-x^2}} = \left[\begin{array}{r|l} \text{Subst. } t=1-x^2 & x\in\langle 0;1)\,,\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in(-1;0\rangle\,,\ t\in(0;1\rangle \end{array}\right] = -\frac{1}{2}\int \frac{(1-t)^2\,dt}{\sqrt{t}} = -\frac{1}{2}\int \frac{1-2t+t^2}{t^{\frac{1}{2}}}\,dt$$

$$= -\int \left[\frac{t^{-\frac{1}{2}}}{2} - t^{\frac{1}{2}} + \frac{t^{\frac{3}{2}}}{2}\right] dt = -\frac{t^{\frac{1}{2}}}{2\cdot\frac{1}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{2\cdot\frac{5}{2}} + c$$

$$\int \frac{x^5\,dx}{\sqrt{1-x^3}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{1-x^3}} = \left[\begin{array}{r|l} \text{Subst. } t=1-x^3 & x\in(-\infty;1) \\ dt=-3x^2\,dx & x\in(0;\infty) \end{array}\right] = -\int \frac{(1-t)\,dt}{3\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}}\,dt = \int \left[\frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3}\right] dt$$

$$= \frac{t^{\frac{3}{2}}}{3\cdot\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{3\cdot\frac{1}{2}} + c$$

# Riešené príklady – 196, 197

$$\int \frac{x^5\,dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{1-x^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=1-x^2 & x\in\langle 0;1),\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in(-1;0\rangle,\ t\in(0;1\rangle \end{array} \right] = -\frac{1}{2}\int \frac{(1-t)^2\,dt}{\sqrt{t}} = -\frac{1}{2}\int \frac{1-2t+t^2}{t^{\frac{1}{2}}}\,dt$$

$$= -\int \left[ \frac{t^{-\frac{1}{2}}}{2} - t^{\frac{1}{2}} + \frac{t^{\frac{3}{2}}}{2} \right] dt = -\frac{t^{\frac{1}{2}}}{2\cdot\frac{1}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{2\cdot\frac{5}{2}} + c = -\sqrt{t} + \frac{2\sqrt{t^3}}{3} - \frac{\sqrt{t^5}}{5} + c$$

$$\int \frac{x^5\,dx}{\sqrt{1-x^3}}$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{1-x^3}} = \left[ \begin{array}{r|l} \text{Subst. } t=1-x^3 & x\in(-\infty;1) \\ dt=-3x^2\,dx & x\in(0;\infty) \end{array} \right] = -\int \frac{(1-t)\,dt}{3\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}}\,dt = \int \left[ \frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3} \right] dt$$

$$= \frac{t^{\frac{3}{2}}}{3\cdot\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{3\cdot\frac{1}{2}} + c = \frac{2\sqrt{t^3}}{9} - \frac{2\sqrt{t}}{3} + c$$

# Riešené príklady – 196, 197

$$\int \frac{x^5\,dx}{\sqrt{1-x^2}} = -\frac{\sqrt{(1-x^2)^5}}{5} + \frac{2\sqrt{(1-x^2)^3}}{3} - \sqrt{1-x^2} + c$$

$$= \frac{1}{2}\int \frac{x^4 \cdot 2x\,dx}{\sqrt{1-x^2}} = \left[ \begin{array}{r|l} \text{Subst. } t=1-x^2 & x\in\langle 0;1),\ t\in(0;1\rangle \\ dt=-2x\,dx & x\in(-1;0\rangle,\ t\in(0;1\rangle \end{array} \right] = -\frac{1}{2}\int \frac{(1-t)^2\,dt}{\sqrt{t}} = -\frac{1}{2}\int \frac{1-2t+t^2}{t^{\frac{1}{2}}}\,dt$$

$$= -\int \left[ \frac{t^{-\frac{1}{2}}}{2} - t^{\frac{1}{2}} + \frac{t^{\frac{3}{2}}}{2} \right] dt = -\frac{t^{\frac{1}{2}}}{2\cdot\frac{1}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{2\cdot\frac{5}{2}} + c = -\sqrt{t} + \frac{2\sqrt{t^3}}{3} - \frac{\sqrt{t^5}}{5} + c$$

$$= -\frac{\sqrt{(1-x^2)^5}}{5} + \frac{2\sqrt{(1-x^2)^3}}{3} - \sqrt{1-x^2} + c,\ x\in(-1;1),\ c\in R.$$

$$\int \frac{x^5\,dx}{\sqrt{1-x^3}} = \frac{2\sqrt{(1-x^3)^3}}{9} - \frac{2\sqrt{1-x^3}}{3} + c$$

$$= \frac{1}{3}\int \frac{x^3 \cdot 3x^2\,dx}{\sqrt{1-x^3}} = \left[ \begin{array}{r|l} \text{Subst. } t=1-x^3 & x\in(-\infty;1) \\ dt=-3x^2\,dx & x\in(0;\infty) \end{array} \right] = -\int \frac{(1-t)\,dt}{3\sqrt{t}} = \int \frac{t-1}{3t^{\frac{1}{2}}}\,dt = \int \left[ \frac{t^{\frac{1}{2}}}{3} - \frac{t^{-\frac{1}{2}}}{3} \right] dt$$

$$= \frac{t^{\frac{3}{2}}}{3\cdot\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{3\cdot\frac{1}{2}} + c = \frac{2\sqrt{t^3}}{9} - \frac{2\sqrt{t}}{3} + c = \frac{2\sqrt{(1-x^3)^3}}{9} - \frac{2\sqrt{1-x^3}}{3} + c,$$

$$x\in(-\infty;1),\ c\in R.$$

# Riešené príklady – 198

$$\int \frac{\sqrt{1-x^2}}{x^2}\,dx$$

# Riešené príklady – 198

$$\int \frac{\sqrt{1-x^2}}{x^2}\,dx$$

$$= \left[ \begin{array}{l|l} u = \sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}} & u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}} \\ v' = \frac{1}{x^2} = x^{-2} & v = \frac{x^{-1}}{-1} = -\frac{1}{x} \end{array} \right]$$

---

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x = \sin t & x \in \langle -1; 0), \ t \in \langle -\frac{\pi}{2}; 0) & t = \arcsin x, \ \cos t \geq 0 \\ dx = \cos t\,dt & x \in (0; 1\rangle, \ t \in (0; \frac{\pi}{2}\rangle & \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \end{array} \right]$$

---

$$= \left[ \begin{array}{r|l|l} \text{Subst. } x = \cos t & x \in \langle -1; 0), \ t \in (\frac{\pi}{2}; \pi\rangle & t = \arccos x, \ \sin t \geq 0 \\ dx = -\sin t\,dt & x \in (0; 1\rangle, \ t \in \langle 0; \frac{\pi}{2}) & \sqrt{1-x^2} = \sqrt{1-\cos^2 t} = \sqrt{\sin^2 t} = |\sin t| = \sin t \end{array} \right]$$

# Riešené príklady – 198

$$\int \frac{\sqrt{1-x^2}}{x^2}\,dx$$

$$=\left[\begin{array}{l|l} u=\sqrt{1-x^2}=(1-x^2)^{\frac{1}{2}} & u'=\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot(-2x)=\frac{-x}{\sqrt{1-x^2}} \\ v'=\frac{1}{x^2}=x^{-2} & v=\frac{x^{-1}}{-1}=-\frac{1}{x} \end{array}\right]=-\frac{\sqrt{1-x^2}}{x}-\int\frac{dx}{\sqrt{1-x^2}}$$

---

$$=\left[\begin{array}{r|l|l} \text{Subst. } x=\sin t & x\in\langle -1;0),\ t\in\langle -\frac{\pi}{2};0) & t=\arcsin x,\ \cos t\geq 0 \\ dx=\cos t\,dt & x\in(0;1\rangle,\ t\in(0;\frac{\pi}{2}\rangle & \sqrt{1-x^2}=\sqrt{1-\sin^2 t}=\sqrt{\cos^2 t}=|\cos t|=\cos t \end{array}\right]=\int\frac{\cos t\cdot\cos t\,dt}{\sin^2 t}$$

---

$$=\left[\begin{array}{r|l|l} \text{Subst. } x=\cos t & x\in\langle -1;0),\ t\in(\frac{\pi}{2};\pi\rangle & t=\arccos x,\ \sin t\geq 0 \\ dx=-\sin t\,dt & x\in(0;1\rangle,\ t\in\langle 0;\frac{\pi}{2}) & \sqrt{1-x^2}=\sqrt{1-\cos^2 t}=\sqrt{\sin^2 t}=|\sin t|=\sin t \end{array}\right]=-\int\frac{\sin t\cdot\sin t\,dt}{\cos^2 t}$$

# Riešené príklady – 198

$$\int \frac{\sqrt{1-x^2}}{x^2}\,dx$$

$$=\left[\begin{array}{l|l} u=\sqrt{1-x^2}=(1-x^2)^{\frac{1}{2}} & u'=\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot(-2x)=\frac{-x}{\sqrt{1-x^2}} \\ v'=\frac{1}{x^2}=x^{-2} & v=\frac{x^{-1}}{-1}=-\frac{1}{x} \end{array}\right]=-\frac{\sqrt{1-x^2}}{x}-\int\frac{dx}{\sqrt{1-x^2}}$$

$$=-\frac{\sqrt{1-x^2}}{x}-\arcsin x+c_1=-\frac{\sqrt{1-x^2}}{x}+\arccos x+c_2,\quad x\in(-1;1)-\{0\},\ c_1,c_2\in R.$$

---

$$=\left[\begin{array}{l|l|l} \text{Subst. } x=\sin t & x\in\langle-1;0),\ t\in\langle-\frac{\pi}{2};0) & t=\arcsin x,\ \cos t\geq 0 \\ dx=\cos t\,dt & x\in(0;1\rangle,\ t\in(0;\frac{\pi}{2}\rangle & \sqrt{1-x^2}=\sqrt{1-\sin^2 t}=\sqrt{\cos^2 t}=|\cos t|=\cos t \end{array}\right]=\int\frac{\cos t\cdot\cos t\,dt}{\sin^2 t}$$

$$=\int\frac{(1-\sin^2 t)\,dt}{\sin^2 t}$$

---

$$=\left[\begin{array}{l|l|l} \text{Subst. } x=\cos t & x\in\langle-1;0),\ t\in(\frac{\pi}{2};\pi\rangle & t=\arccos x,\ \sin t\geq 0 \\ dx=-\sin t\,dt & x\in(0;1\rangle,\ t\in\langle 0;\frac{\pi}{2}) & \sqrt{1-x^2}=\sqrt{1-\cos^2 t}=\sqrt{\sin^2 t}=|\sin t|=\sin t \end{array}\right]=-\int\frac{\sin t\cdot\sin t\,dt}{\cos^2 t}$$

$$=-\int\frac{(1-\cos^2 t)\,dt}{\sin^2 t}$$

# Riešené príklady – 198

$$\int \frac{\sqrt{1-x^2}}{x^2}\,dx = -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = \frac{\sqrt{1-x^2}}{x} + \arccos x + c_2$$

$$= \left[ \begin{array}{l|l} u=\sqrt{1-x^2}=(1-x^2)^{\frac{1}{2}} & u'=\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot(-2x)=\frac{-x}{\sqrt{1-x^2}} \\ v'=\frac{1}{x^2}=x^{-2} & v=\frac{x^{-1}}{-1}=-\frac{1}{x} \end{array} \right] = -\frac{\sqrt{1-x^2}}{x} - \int \frac{dx}{\sqrt{1-x^2}}$$

$$= -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = -\frac{\sqrt{1-x^2}}{x} + \arccos x + c_2, \quad x\in(-1;1)-\{0\},\ c_1,c_2\in R.$$

---

$$= \left[ \begin{array}{l|l|l} \text{Subst. } x=\sin t & x\in\langle-1;0),\ t\in\langle-\frac{\pi}{2};0) & t=\arcsin x,\ \cos t\geq 0 \\ dx=\cos t\,dt & x\in(0;1\rangle,\ t\in(0;\frac{\pi}{2}\rangle & \sqrt{1-x^2}=\sqrt{1-\sin^2 t}=\sqrt{\cos^2 t}=|\cos t|=\cos t \end{array} \right] = \int \frac{\cos t\cdot\cos t\,dt}{\sin^2 t}$$

$$= \int \frac{(1-\sin^2 t)\,dt}{\sin^2 t} = \int \left[\frac{1}{\sin^2 t} - 1\right] dt$$

---

$$= \left[ \begin{array}{l|l|l} \text{Subst. } x=\cos t & x\in\langle-1;0),\ t\in(\frac{\pi}{2};\pi\rangle & t=\arccos x,\ \sin t\geq 0 \\ dx=-\sin t\,dt & x\in(0;1\rangle,\ t\in\langle 0;\frac{\pi}{2}) & \sqrt{1-x^2}=\sqrt{1-\cos^2 t}=\sqrt{\sin^2 t}=|\sin t|=\sin t \end{array} \right] = -\int \frac{\sin t\cdot\sin t\,dt}{\cos^2 t}$$

$$= -\int \frac{(1-\cos^2 t)\,dt}{\sin^2 t} = \int \left[1 - \frac{1}{\cos^2 t}\right] dt$$

# Riešené príklady – 198

$$\int \frac{\sqrt{1-x^2}}{x^2}\,dx = -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = \frac{\sqrt{1-x^2}}{x} + \arccos x + c_2$$

$$= \left[ \begin{array}{l|l} u = \sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}} & u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}} \\ v' = \frac{1}{x^2} = x^{-2} & v = \frac{x^{-1}}{-1} = -\frac{1}{x} \end{array} \right] = -\frac{\sqrt{1-x^2}}{x} - \int \frac{dx}{\sqrt{1-x^2}}$$

$$= -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = -\frac{\sqrt{1-x^2}}{x} + \arccos x + c_2,$$

$$x \in (-1;1) - \{0\},\ c_1, c_2 \in R.$$

---

$$= \left[ \begin{array}{l|l|l} \text{Subst. } x = \sin t & x \in \langle -1;0),\ t \in \langle -\frac{\pi}{2};0) & t = \arcsin x,\ \cos t \geq 0, \quad \operatorname{cotg} t = \frac{\cos t}{\sin t} = \frac{\sqrt{1-x^2}}{x} \\ dx = \cos t\,dt & x \in (0;1\rangle,\ t \in (0;\frac{\pi}{2}\rangle & \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \end{array} \right] = \int \frac{\cos t \cdot \cos t\,dt}{\sin^2 t}$$

$$= \int \frac{(1-\sin^2 t)\,dt}{\sin^2 t} = \int \left[ \frac{1}{\sin^2 t} - 1 \right] dt = -\operatorname{cotg} t - t + c_1$$

---

$$= \left[ \begin{array}{l|l|l} \text{Subst. } x = \cos t & x \in \langle -1;0),\ t \in (\frac{\pi}{2};\pi\rangle & t = \arccos x,\ \sin t \geq 0, \quad \operatorname{tg} t = \frac{\sin t}{\cos t} = \frac{\sqrt{1-x^2}}{x} \\ dx = -\sin t\,dt & x \in (0;1\rangle,\ t \in \langle 0;\frac{\pi}{2}) & \sqrt{1-x^2} = \sqrt{1-\cos^2 t} = \sqrt{\sin^2 t} = |\sin t| = \sin t \end{array} \right] = -\int \frac{\sin t \cdot \sin t\,dt}{\cos^2 t}$$

$$= -\int \frac{(1-\cos^2 t)\,dt}{\sin^2 t} = \int \left[ 1 - \frac{1}{\cos^2 t} \right] dt = t - \operatorname{tg} t + c_2$$

# Riešené príklady – 198

$$\int \frac{\sqrt{1-x^2}}{x^2}\,\mathrm{d}x = -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = \frac{\sqrt{1-x^2}}{x} + \arccos x + c_2$$

$$= \left[\begin{array}{l|l} u = \sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}} & u' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot(-2x) = \frac{-x}{\sqrt{1-x^2}} \\ v' = \frac{1}{x^2} = x^{-2} & v = \frac{x^{-1}}{-1} = -\frac{1}{x} \end{array}\right] = -\frac{\sqrt{1-x^2}}{x} - \int \frac{\mathrm{d}x}{\sqrt{1-x^2}}$$

$$= -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1 = -\frac{\sqrt{1-x^2}}{x} + \arccos x + c_2,$$

$$x \in (-1;1) - \{0\},\ c_1, c_2 \in R.$$

---

$$= \left[\begin{array}{r|l|l} \text{Subst. } x = \sin t & x \in \langle -1; 0),\ t \in \langle -\frac{\pi}{2}; 0) & t = \arcsin x,\ \cos t \geq 0, \quad \operatorname{cotg} t = \frac{\cos t}{\sin t} = \frac{\sqrt{1-x^2}}{x} \\ \mathrm{d}x = \cos t\,\mathrm{d}t & x \in (0; 1\rangle,\ t \in (0; \frac{\pi}{2}\rangle & \sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t \end{array}\right] = \int \frac{\cos t \cdot \cos t\,\mathrm{d}t}{\sin^2 t}$$

$$= \int \frac{(1-\sin^2 t)\,\mathrm{d}t}{\sin^2 t} = \int \left[\frac{1}{\sin^2 t} - 1\right]\mathrm{d}t = -\operatorname{cotg} t - t + c_1 = -\frac{\sqrt{1-x^2}}{x} - \arcsin x + c_1,$$

$$x \in \langle -1; 0) \cup (0; 1\rangle,\ c_1 \in R.$$

---

$$= \left[\begin{array}{r|l|l} \text{Subst. } x = \cos t & x \in \langle -1; 0),\ t \in (\frac{\pi}{2}; \pi\rangle & t = \arccos x,\ \sin t \geq 0, \quad \operatorname{tg} t = \frac{\sin t}{\cos t} = \frac{\sqrt{1-x^2}}{x} \\ \mathrm{d}x = -\sin t\,\mathrm{d}t & x \in (0; 1\rangle,\ t \in \langle 0; \frac{\pi}{2}) & \sqrt{1-x^2} = \sqrt{1-\cos^2 t} = \sqrt{\sin^2 t} = |\sin t| = \sin t \end{array}\right] = -\int \frac{\sin t \cdot \sin t\,\mathrm{d}t}{\cos^2 t}$$

$$= -\int \frac{(1-\cos^2 t)\,\mathrm{d}t}{\sin^2 t} = \int \left[1 - \frac{1}{\cos^2 t}\right]\mathrm{d}t = t - \operatorname{tg} t + c_2 = \arccos x - \frac{\sqrt{1-x^2}}{x} + c_2,$$

$$x \in \langle -1; 0) \cup (0; 1\rangle,\ c_2 \in R.$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}}$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}} = \left[ \begin{array}{r|l|l} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x\,dx & x\in(0;\infty),\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right]$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}} = \left[ \begin{array}{r|l|l} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x\,dx & x\in(0;\infty),\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$=\int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}}=\left[\begin{array}{r|l|l} \text{Subst. } t=x^2 & x\in(-\infty;0)\,,\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x\,dx & x\in(0;\infty)\,,\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right]=\frac{1}{2}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$=\left[\begin{array}{l|l|l} \text{Subst. } \quad t=\frac{1}{u}=u^{-1} & t\in(0;\infty) & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in(0;\infty) & \end{array}\right]$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}} = \left[ \begin{array}{c|c|c} \text{Subst. } t=x^2 & x\in(-\infty;0)\,,\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x\,dx & x\in(0;\infty)\,,\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[ \begin{array}{l|l|l} \text{Subst. } \quad t=\frac{1}{u}=u^{-1} & t\in(0;\infty) & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in(0;\infty) & \end{array} \right]$$

$$= \frac{1}{2}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u}}$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}} = \left[ \begin{array}{c|c|c} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x\,dx & x\in(0;\infty),\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[ \begin{array}{c|c|c} \text{Subst. } t=\frac{1}{u}=u^{-1} & t\in(0;\infty) & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in(0;\infty) & \end{array} \right]$$

$$= \frac{1}{2}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2}\int \frac{du}{\sqrt{u^2+u+1}}$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}} = \left[ \begin{array}{c|c|c} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x\,dx & x\in(0;\infty),\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[ \begin{array}{c|c|c} \text{Subst. } \quad t=\frac{1}{u}=u^{-1} & t\in(0;\infty) & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in(0;\infty) & \end{array} \right]$$

$$= \frac{1}{2}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2}\int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2}\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}} = \left[ \begin{array}{c|c|c} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x\,dx & x\in(0;\infty),\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[ \begin{array}{c|c|c} \text{Subst. } \quad t=\frac{1}{u}=u^{-1} & t\in(0;\infty) & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in(0;\infty) & \end{array} \right]$$

$$= \frac{1}{2}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2}\int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2}\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= -\frac{1}{2}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right| + c_1$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$=\int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}} = \left[ \begin{array}{c|c|c} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x\,dx & x\in(0;\infty),\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[ \begin{array}{c|c|c} \text{Subst. } t=\frac{1}{u}=u^{-1} & t\in(0;\infty) & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in(0;\infty) & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t}=\frac{\sqrt{x^4+x^2+1}}{x^2} \end{array} \right]$$

$$=\frac{1}{2}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2}\int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2}\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$=-\frac{1}{2}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right|+c_1 = \left[ u=\frac{1}{t}=\frac{1}{x^2} \right]$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$=\int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}}=\left[\begin{array}{c|c|c} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x\,dx & x\in(0;\infty),\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right]=\frac{1}{2}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$=\left[\begin{array}{c|c|c} \text{Subst. } t=\frac{1}{u}=u^{-1} & t\in(0;\infty) & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in(0;\infty) & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t}=\frac{\sqrt{x^4+x^2+1}}{x^2} \end{array}\right]$$

$$=\frac{1}{2}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u}}=-\frac{1}{2}\int \frac{du}{\sqrt{u^2+u+1}}=-\frac{1}{2}\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$=-\frac{1}{2}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right|+c_1=\left[u=\frac{1}{t}=\frac{1}{x^2}\right]$$

$$=-\frac{1}{2}\ln\left|\frac{1}{x^2}+\frac{1}{2}+\frac{\sqrt{x^4+x^2+1}}{x^2}\right|+c_1$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}} = \left[ \begin{array}{c|c|c} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x\,dx & x\in(0;\infty),\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[ \begin{array}{c|c|c} \text{Subst. } t=\frac{1}{u}=u^{-1} & t\in(0;\infty) & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in(0;\infty) & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t}=\frac{\sqrt{x^4+x^2+1}}{x^2} \end{array} \right]$$

$$= \frac{1}{2}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2}\int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2}\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= -\frac{1}{2}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right| + c_1 = \left[ u=\frac{1}{t}=\frac{1}{x^2} \right]$$

$$= -\frac{1}{2}\ln\left|\frac{1}{x^2}+\frac{1}{2}+\frac{\sqrt{x^4+x^2+1}}{x^2}\right| + c_1 = c_1 - \frac{1}{2}\ln\left|\frac{2+x^2+2\sqrt{x^4+x^2+1}}{2x^2}\right|$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$= \int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}} = \left[\begin{array}{r|l|l} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x\,dx & x\in(0;\infty),\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right] = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } t=\frac{1}{u}=u^{-1} & t\in(0;\infty) & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in(0;\infty) & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t}=\frac{\sqrt{x^4+x^2+1}}{x^2} \end{array}\right]$$

$$= \frac{1}{2}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u}} = -\frac{1}{2}\int \frac{du}{\sqrt{u^2+u+1}} = -\frac{1}{2}\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= -\frac{1}{2}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right| + c_1 = \left[u=\frac{1}{t}=\frac{1}{x^2}\right]$$

$$= -\frac{1}{2}\ln\left|\frac{1}{x^2}+\frac{1}{2}+\frac{\sqrt{x^4+x^2+1}}{x^2}\right| + c_1 = c_1 - \frac{1}{2}\ln\left|\frac{2+x^2+2\sqrt{x^4+x^2+1}}{2x^2}\right|$$

$$= c_1 + \frac{\ln 2}{3} - \frac{1}{2}\ln\left|\frac{2+x^2+2\sqrt{x^4+x^2+1}}{x^2}\right|$$

# Riešené príklady – 199

$$\int \frac{dx}{x\sqrt{x^4+x^2+1}}$$

$$=\int \frac{x\,dx}{x^2\sqrt{x^4+x^2+1}}=\left[\begin{array}{c|c|c} \text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=2x\,dx & x\in(0;\infty),\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right]=\frac{1}{2}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$=\left[\begin{array}{c|c|c} \text{Subst. } \quad t=\frac{1}{u}=u^{-1} & t\in(0;\infty) & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in(0;\infty) & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t}=\frac{\sqrt{x^4+x^2+1}}{x^2} \end{array}\right]$$

$$=\frac{1}{2}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u}}=-\frac{1}{2}\int \frac{du}{\sqrt{u^2+u+1}}=-\frac{1}{2}\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$=-\frac{1}{2}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right|+c_1=\left[u=\frac{1}{t}=\frac{1}{x^2}\right]$$

$$=-\frac{1}{2}\ln\left|\frac{1}{x^2}+\frac{1}{2}+\frac{\sqrt{x^4+x^2+1}}{x^2}\right|+c_1=c_1-\frac{1}{2}\ln\left|\frac{2+x^2+2\sqrt{x^4+x^2+1}}{2x^2}\right|$$

$$=c_1+\frac{\ln 2}{3}-\frac{1}{2}\ln\left|\frac{2+x^2+2\sqrt{x^4+x^2+1}}{x^2}\right|=\left[\begin{array}{c} c=c_1+\frac{\ln 2}{3} \\ c_1\in R,\ c\in R \end{array}\right]$$

# Riešené príklady – 199

$$\int\frac{dx}{x\sqrt{x^4+x^2+1}} = c-\frac{1}{2}\ln\left|\frac{2+x^2+2\sqrt{x^4+x^2+1}}{x^2}\right|$$

$$=\int\frac{x\,dx}{x^2\sqrt{x^4+x^2+1}}=\left[\begin{array}{c|c|c}\text{Subst. } t=x^2 & x\in(-\infty;0),\ t\in(0;\infty) & x^4+x^2+1=(x^2+\frac{1}{2})^2+\frac{3}{4}>0\\ dt=2x\,dx & x\in(0;\infty),\ t\in(0;\infty) & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0\end{array}\right]=\frac{1}{2}\int\frac{dt}{t\sqrt{t^2+t+1}}$$

$$=\left[\begin{array}{c|c|c}\text{Subst. } t=\frac{1}{u}=u^{-1} & t\in(0;\infty) & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u}\\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in(0;\infty) & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t}=\frac{\sqrt{x^4+x^2+1}}{x^2}\end{array}\right]$$

$$=\frac{1}{2}\int\frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u}}=-\frac{1}{2}\int\frac{du}{\sqrt{u^2+u+1}}=-\frac{1}{2}\int\frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

$$=-\frac{1}{2}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right|+c_1=\left[u=\frac{1}{t}=\frac{1}{x^2}\right]$$

$$=-\frac{1}{2}\ln\left|\frac{1}{x^2}+\frac{1}{2}+\frac{\sqrt{x^4+x^2+1}}{x^2}\right|+c_1=c_1-\frac{1}{2}\ln\left|\frac{2+x^2+2\sqrt{x^4+x^2+1}}{2x^2}\right|$$

$$=c_1+\frac{\ln 2}{3}-\frac{1}{2}\ln\left|\frac{2+x^2+2\sqrt{x^4+x^2+1}}{x^2}\right|=\left[\begin{array}{c}c=c_1+\frac{\ln 2}{3}\\ c_1\in R,\ c\in R\end{array}\right]$$

$$=c-\frac{1}{2}\ln\left|\frac{2+x^2+2\sqrt{x^4+x^2+1}}{x^2}\right|,\ x\in R-\{0\},\ c\in R.$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}}$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{c|c|c} \text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right]$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{c|c|l} \text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{3}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[\begin{array}{c|c|c} \text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right] = \frac{1}{3}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } \quad t=\frac{1}{u}=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u\cdot\operatorname{sgn} u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in R-\{0\} & \end{array}\right]$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$=\int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[\begin{array}{c|c|c} \text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right] = \frac{1}{3}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$=\left[\begin{array}{l|l|l} \text{Subst. } \quad t=\frac{1}{u}=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u\cdot\operatorname{sgn} u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in R-\{0\} & \end{array}\right]$$

$$=\frac{1}{3}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}}$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{l|l|l} \text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{3}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[ \begin{array}{l|l|l} \text{Subst. } \quad t=\frac{1}{u}=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u\cdot \operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u\cdot \operatorname{sgn} u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in R-\{0\} & \end{array} \right]$$

$$= \frac{1}{3}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u\cdot \operatorname{sgn} u}} = \frac{1}{3}\int \frac{-\operatorname{sgn} u\,du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

# Riešené príklady – 200

$$\int\frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$=\int\frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}}=\left[\begin{array}{l|l|l}\text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0\\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0\end{array}\right]=\frac{1}{3}\int\frac{dt}{t\sqrt{t^2+t+1}}$$

$$=\left[\begin{array}{l|l|l}\text{Subst. } t=\frac{1}{u}=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn}u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u\cdot\operatorname{sgn}u}\\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in R-\{0\} & \end{array}\right]$$

$$=\frac{1}{3}\int\frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn}u}}=\frac{1}{3}\int\frac{-\operatorname{sgn}u\,du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}=\frac{-\operatorname{sgn}u}{3}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right|+c_1$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{r|l|l} \text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{3}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[ \begin{array}{l|l|l} \text{Subst.} \quad t=\frac{1}{u}=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u\cdot \operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u\cdot \operatorname{sgn} u} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in R-\{0\} & \end{array} \right]$$

$$= \frac{1}{3}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u\cdot \operatorname{sgn} u}} = \frac{1}{3}\int \frac{-\operatorname{sgn} u\,du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} u}{3}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right|+c_1 = \left[ \begin{array}{l} \operatorname{sgn} u \\ = \operatorname{sgn} x \end{array} \right]$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[\begin{array}{c|c|c} \text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right] = \frac{1}{3}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{c|c|c} \text{Subst. } t=\frac{1}{u}=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u\cdot\operatorname{sgn} u}, \quad u=\frac{1}{t}=\frac{1}{x^3} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in R-\{0\} & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array}\right]$$

$$= \frac{1}{3}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}} = \frac{1}{3}\int \frac{-\operatorname{sgn} u\,du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} u}{3}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right| + c_1 = \left[\begin{array}{l}\operatorname{sgn} u \\ \quad =\operatorname{sgn} x\end{array}\right]$$

$$= -\frac{\operatorname{sgn} x}{3}\ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right| + c_1$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{l|l|l} \text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{3}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[ \begin{array}{l|l|l} \text{Subst. } t=\frac{1}{u}=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u\cdot \operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u\cdot \operatorname{sgn} u}, \quad u=\frac{1}{t}=\frac{1}{x^3} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in R-\{0\} & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot \operatorname{sgn} t}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot \operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array} \right]$$

$$= \frac{1}{3}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u\cdot \operatorname{sgn} u}} = \frac{1}{3}\int \frac{-\operatorname{sgn} u\,du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} u}{3}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right| + c_1 = \left[ \begin{array}{l} \operatorname{sgn} u \\ \quad = \operatorname{sgn} x \end{array} \right]$$

$$= -\frac{\operatorname{sgn} x}{3}\ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right| + c_1 = c_1 - \frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[\begin{array}{l|l|l} \text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right] = \frac{1}{3}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } t=\frac{1}{u}=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u\cdot\operatorname{sgn} u}, \quad u=\frac{1}{t}=\frac{1}{x^3} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in R-\{0\} & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array}\right]$$

$$= \frac{1}{3}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}} = \frac{1}{3}\int \frac{-\operatorname{sgn} u\,du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} u}{3}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right|+c_1 = \left[\begin{array}{l} \operatorname{sgn} u \\ \quad =\operatorname{sgn} x \end{array}\right]$$

$$= -\frac{\operatorname{sgn} x}{3}\ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+c_1 = c_1-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

$$= c_1-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+\frac{\operatorname{sgn} x\cdot\ln 2}{3}$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{l|l|l} \text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right] = \frac{1}{3}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[ \begin{array}{l|l|l} \text{Subst. } \ t=\frac{1}{u}=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u\cdot\operatorname{sgn} u}, \quad u=\frac{1}{t}=\frac{1}{x^3} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in R-\{0\} & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array} \right]$$

$$= \frac{1}{3}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}} = \frac{1}{3}\int \frac{-\operatorname{sgn} u\,du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} u}{3}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right|+c_1 = \left[ \begin{array}{l} \operatorname{sgn} u \\ \quad =\operatorname{sgn} x \end{array} \right]$$

$$= -\frac{\operatorname{sgn} x}{3}\ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+c_1 = c_1-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

$$= c_1-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+\frac{\operatorname{sgn} x\cdot\ln 2}{3} = \left[ \begin{array}{l} c_2=c_1+\frac{\operatorname{sgn} x\cdot\ln 2}{3} \\ c_1\in R,\ \ c_2\in R \end{array} \right]$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}} = c_2 - \frac{\operatorname{sgn} x}{3} \ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[\begin{array}{l|l|l} \text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right] = \frac{1}{3}\int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } t=\frac{1}{u}=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u\cdot\operatorname{sgn} u}, \quad u=\frac{1}{t}=\frac{1}{x^3} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in R-\{0\} & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array}\right]$$

$$= \frac{1}{3}\int \frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}} = \frac{1}{3}\int \frac{-\operatorname{sgn} u\,du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} u}{3}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right|+c_1 = \left[\begin{array}{l} \operatorname{sgn} u \\ \quad =\operatorname{sgn} x \end{array}\right]$$

$$= -\frac{\operatorname{sgn} x}{3}\ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+c_1 = c_1-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

$$= c_1-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+\frac{\operatorname{sgn} x\cdot\ln 2}{3} = \left[\begin{array}{l} c_2=c_1+\frac{\operatorname{sgn} x\cdot\ln 2}{3} \\ c_1\in R,\ c_2\in R \end{array}\right]$$

$$= c_2-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|,\ x\in R-\{0\},\ c_2\in R.$$

# Riešené príklady – 200

$$\int\frac{dx}{x\sqrt{x^6+x^3+1}}=c_2-\frac{\operatorname{sgn}x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn}x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|$$

$$=\int\frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}}=\left[\begin{array}{l|l|l}\text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\tfrac12)^2+\tfrac34>0\\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\tfrac12)^2+\tfrac34>0\end{array}\right]=\frac13\int\frac{dt}{t\sqrt{t^2+t+1}}$$

$$=\left[\begin{array}{l|l|l}\text{Subst. } t=\tfrac1u=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\tfrac1{u^2}+\tfrac1u+1}=\sqrt{\tfrac{1+u+u^2}{u^2}}=\tfrac{\sqrt{u^2+u+1}}{|u|}=\tfrac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn}u}=\tfrac{\sqrt{(u+\frac12)^2+\frac34}}{u\cdot\operatorname{sgn}u},\quad u=\tfrac1t=\tfrac1{x^3}\\ dt=-u^{-2}\,du=-\tfrac{du}{u^2} & u\in R-\{0\} & \sqrt{u^2+u+1}=\sqrt{\tfrac1{t^2}+\tfrac1t+1}=\sqrt{\tfrac{1+t+t^2}{t^2}}=\tfrac{\sqrt{t^2+t+1}}{|t|}=\tfrac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn}t}=\tfrac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn}x^3}=\tfrac{\operatorname{sgn}x\cdot\sqrt{x^6+x^3+1}}{x^3}\end{array}\right]$$

$$=\frac13\int\frac{-\frac{du}{u^2}}{\frac1u\cdot\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn}u}}=\frac13\int\frac{-\operatorname{sgn}u\,du}{\sqrt{(u+\frac12)^2+\frac34}}=\frac{-\operatorname{sgn}u}{3}\ln\left|u+\frac12+\sqrt{u^2+u+1}\right|+c_1=\left[\begin{array}{l}\operatorname{sgn}u\\ \quad=\operatorname{sgn}x\end{array}\right]$$

$$=-\frac{\operatorname{sgn}x}{3}\ln\left|\frac1{x^3}+\frac12+\frac{\operatorname{sgn}x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+c_1=c_1-\frac{\operatorname{sgn}x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn}x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

$$=c_1-\frac{\operatorname{sgn}x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn}x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+\frac{\operatorname{sgn}x\cdot\ln 2}{3}=\left[\begin{array}{l}c_2=c_1+\frac{\operatorname{sgn}x\cdot\ln 2}{3}\\ c_1\in R,\ c_2\in R\end{array}\right]$$

$$=c_2-\frac{\operatorname{sgn}x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn}x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|,\ x\in R-\{0\},\ c_2\in R.$$

$$\boxed{x\in(0;\infty)}=c_2-\frac13\ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right|$$

$$\boxed{x\in(-\infty;0)}=c_2+\frac13\ln\left|\frac{2+x^3-2\sqrt{x^6+x^3+1}}{x^3}\right|$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}} = c_2 - \frac{\operatorname{sgn} x}{3} \ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[\begin{array}{l|l|l} \text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right] = \frac{1}{3}\int\frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } t=\frac{1}{u}=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u\cdot\operatorname{sgn} u},\quad u=\frac{1}{t}=\frac{1}{x^3} \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in R-\{0\} & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array}\right]$$

$$= \frac{1}{3}\int\frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn} u}} = \frac{1}{3}\int\frac{-\operatorname{sgn} u\,du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} u}{3}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right|+c_1 = \left[\begin{array}{l}\operatorname{sgn} u \\ =\operatorname{sgn} x\end{array}\right]$$

$$= -\frac{\operatorname{sgn} x}{3}\ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+c_1 = c_1 - \frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

$$= c_1 - \frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right| + \frac{\operatorname{sgn} x\cdot\ln 2}{3} = \left[\begin{array}{c} c_2=c_1+\frac{\operatorname{sgn} x\cdot\ln 2}{3} \\ c_1\in R,\ c_2\in R\end{array}\right]$$

$$= c_2 - \frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|,\ x\in R-\{0\},\ c_2\in R.$$

---

$$\boxed{x\in(0;\infty)} = c_2 - \frac{1}{3}\ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right| = \left[\begin{array}{c} c=c_2 \\ c\in R,\ c_2\in R\end{array}\right]$$

$$\boxed{x\in(-\infty;0)} = c_2 + \frac{1}{3}\ln\left|\frac{2+x^3-2\sqrt{x^6+x^3+1}}{x^3}\right| = c+\frac{\ln 3}{3}-\frac{1}{3}\ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right| = \left[\begin{array}{c} c=c_2+\frac{\ln 3}{3} \\ c\in R,\ c_2\in R\end{array}\right]$$

# Riešené príklady – 200

$$\int\frac{dx}{x\sqrt{x^6+x^3+1}}=c_2-\frac{\operatorname{sgn}x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn}x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|=c-\frac{1}{3}\ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right|$$

$$=\int\frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}}=\left[\begin{array}{l|l|l}\text{Subst. } t=x^3 & x\in R-\{0\} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0\\ dt=3x^2\,dx & t\in R-\{0\} & t^2+t+1=(t+\frac{1}{2})^2+\frac{3}{4}>0\end{array}\right]=\frac{1}{3}\int\frac{dt}{t\sqrt{t^2+t+1}}$$

$$=\left[\begin{array}{l|l|l}\text{Subst. } t=\frac{1}{u}=u^{-1} & t\in R-\{0\} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\sqrt{\frac{1+u+u^2}{u^2}}=\frac{\sqrt{u^2+u+1}}{|u|}=\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn}u}=\frac{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}{u\cdot\operatorname{sgn}u},\quad u=\frac{1}{t}=\frac{1}{x^3}\\ dt=-u^{-2}\,du=-\frac{du}{u^2} & u\in R-\{0\} & \sqrt{u^2+u+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn}t}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn}x^3}=\frac{\operatorname{sgn}x\cdot\sqrt{x^6+x^3+1}}{x^3}\end{array}\right]$$

$$=\frac{1}{3}\int\frac{-\frac{du}{u^2}}{\frac{1}{u}\cdot\frac{\sqrt{u^2+u+1}}{u\cdot\operatorname{sgn}u}}=\frac{1}{3}\int\frac{-\operatorname{sgn}u\,du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}=\frac{-\operatorname{sgn}u}{3}\ln\left|u+\frac{1}{2}+\sqrt{u^2+u+1}\right|+c_1=\left[\begin{array}{l}\operatorname{sgn}u\\ \quad=\operatorname{sgn}x\end{array}\right]$$

$$=-\frac{\operatorname{sgn}x}{3}\ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn}x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+c_1=c_1-\frac{\operatorname{sgn}x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn}x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

$$=c_1-\frac{\operatorname{sgn}x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn}x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+\frac{\operatorname{sgn}x\cdot\ln 2}{3}=\left[\begin{array}{c}c_2=c_1+\frac{\operatorname{sgn}x\cdot\ln 2}{3}\\ c_1\in R,\ c_2\in R\end{array}\right]$$

$$=c_2-\frac{\operatorname{sgn}x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn}x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|,\ x\in R-\{0\},\ c_2\in R.$$

$$\boxed{x\in(0;\infty)}=c_2-\frac{1}{3}\ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right|=\left[\begin{array}{c}c=c_2\\ c\in R,\ c_2\in R\end{array}\right]=c-\frac{1}{3}\ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right|.$$

$$\boxed{x\in(-\infty;0)}=c_2+\frac{1}{3}\ln\left|\frac{2+x^3-2\sqrt{x^6+x^3+1}}{x^3}\right|=\left[\begin{array}{c}c=c_2+\frac{\ln 3}{3}\\ c\in R,\ c_2\in R\end{array}\right]=c-\frac{1}{3}\ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right|.$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}}$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{c|c} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right]$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{l|l} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right]$$

$$= \left[ \sqrt{x^6+x^3+1} = \sqrt{\frac{1}{t^2}+\frac{1}{t}+1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t} = \frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}} \right]$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$=\int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}}=\left[\begin{array}{r|l} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right]$$

$$=\left[\sqrt{x^6+x^3+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot \operatorname{sgn} t}=\frac{1}{t\cdot \operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}\right]=\int \frac{-\frac{dt}{3t^2}}{\frac{1}{t}\cdot\frac{1}{t\cdot \operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{l|l} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ \quad 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right]$$

$$= \left[ \sqrt{x^6+x^3+1}=\sqrt{\tfrac{1}{t^2}+\tfrac{1}{t}+1}=\sqrt{\tfrac{1+t+t^2}{t^2}}=\tfrac{\sqrt{t^2+t+1}}{|t|}=\tfrac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\tfrac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\tfrac{1}{2})^2+\tfrac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t}\cdot\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= \frac{1}{3}\int \frac{-\operatorname{sgn} t\,dt}{\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{l|l} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right]$$

$$= \left[ \sqrt{x^6+x^3+1} = \sqrt{\frac{1}{t^2}+\frac{1}{t}+1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t\cdot \operatorname{sgn} t} = \frac{1}{t\cdot \operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t}\cdot\frac{1}{t\cdot \operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= \frac{1}{3}\int \frac{-\operatorname{sgn} t\,dt}{\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3}\ln\left|t+\frac{1}{2}+\sqrt{t^2+t+1}\right| + c_1$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{c|c} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right]$$

$$= \left[ \sqrt{x^6+x^3+1} = \sqrt{\frac{1}{t^2}+\frac{1}{t}+1} = \sqrt{\frac{1+t+t^2}{t^2}} = \frac{\sqrt{t^2+t+1}}{|t|} = \frac{\sqrt{t^2+t+1}}{t\cdot \operatorname{sgn} t} = \frac{1}{t\cdot \operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t}\cdot\frac{1}{t\cdot \operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= \frac{1}{3}\int \frac{-\operatorname{sgn} t\,dt}{\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3}\ln\left|t+\frac{1}{2}+\sqrt{t^2+t+1}\right| + c_1 = \left[\operatorname{sgn} u = \operatorname{sgn} x\right]$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{c|c} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right]$$

$$= \left[ \begin{array}{l} \sqrt{x^6+x^3+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}} \\ \sqrt{t^2+t+1}=\sqrt{\frac{1}{x^6}+\frac{1}{x^3}+1}=\sqrt{\frac{1+x^3+x^6}{x^6}}=\frac{\sqrt{x^6+x^3+1}}{|x^3|}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t}\cdot\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= \frac{1}{3}\int \frac{-\operatorname{sgn} t\,dt}{\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3}\ln\left|t+\frac{1}{2}+\sqrt{t^2+t+1}\right|+c_1 = \left[\operatorname{sgn} u=\operatorname{sgn} x\right]$$

$$= -\frac{\operatorname{sgn} x}{3}\ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+c_1$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[\begin{array}{l|l} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right]$$

$$= \left[\begin{array}{l} \sqrt{x^6+x^3+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}} \\ \sqrt{t^2+t+1}=\sqrt{\frac{1}{x^6}+\frac{1}{x^3}+1}=\sqrt{\frac{1+x^3+x^6}{x^6}}=\frac{\sqrt{x^6+x^3+1}}{|x^3|}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array}\right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t}\cdot\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= \frac{1}{3}\int \frac{-\operatorname{sgn} t\,dt}{\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3}\ln\left|t+\frac{1}{2}+\sqrt{t^2+t+1}\right|+c_1 = \left[\operatorname{sgn} u = \operatorname{sgn} x\right]$$

$$= -\frac{\operatorname{sgn} x}{3}\ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+c_1 = c_1-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{c|l} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right]$$

$$= \left[ \begin{array}{l} \sqrt{x^6+x^3+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot \operatorname{sgn} t}=\frac{1}{t\cdot \operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}} \\ \sqrt{t^2+t+1}=\sqrt{\frac{1}{x^6}+\frac{1}{x^3}+1}=\sqrt{\frac{1+x^3+x^6}{x^6}}=\frac{\sqrt{x^6+x^3+1}}{|x^3|}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot \operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t}\cdot\frac{1}{t\cdot \operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= \frac{1}{3}\int \frac{-\operatorname{sgn} t\,dt}{\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3} \ln\left|t+\frac{1}{2}+\sqrt{t^2+t+1}\right| + c_1 = \left[ \operatorname{sgn} u = \operatorname{sgn} x \right]$$

$$= -\frac{\operatorname{sgn} x}{3} \ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right| + c_1 = c_1 - \frac{\operatorname{sgn} x}{3} \ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

$$= c_1 - \frac{\operatorname{sgn} x}{3} \ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right| + \frac{\operatorname{sgn} x\cdot\ln 2}{3}$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}}$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{l|l} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right]$$

$$= \left[ \begin{array}{l} \sqrt{x^6+x^3+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}} \\ \sqrt{t^2+t+1}=\sqrt{\frac{1}{x^6}+\frac{1}{x^3}+1}=\sqrt{\frac{1+x^3+x^6}{x^6}}=\frac{\sqrt{x^6+x^3+1}}{|x^3|}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t}\cdot\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= \frac{1}{3}\int \frac{-\operatorname{sgn} t\,dt}{\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3}\ln\left|t+\frac{1}{2}+\sqrt{t^2+t+1}\right|+c_1 = \left[\operatorname{sgn} u=\operatorname{sgn} x\right]$$

$$= -\frac{\operatorname{sgn} x}{3}\ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+c_1 = c_1-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

$$= c_1-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+\frac{\operatorname{sgn} x\cdot\ln 2}{3} = \left[ \begin{array}{l} c_2=c_1+\frac{\operatorname{sgn} x\cdot\ln 2}{3} \\ c_1\in R,\ c_2\in R \end{array} \right]$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}} = c_2 - \frac{\operatorname{sgn} x}{3} \ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[\begin{array}{l|l} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right]$$

$$= \left[\begin{array}{l} \sqrt{x^6+x^3+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}} \\ \sqrt{t^2+t+1}=\sqrt{\frac{1}{x^6}+\frac{1}{x^3}+1}=\sqrt{\frac{1+x^3+x^6}{x^6}}=\frac{\sqrt{x^6+x^3+1}}{|x^3|}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array}\right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t}\cdot\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= \frac{1}{3}\int \frac{-\operatorname{sgn} t\,dt}{\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3} \ln\left|t+\frac{1}{2}+\sqrt{t^2+t+1}\right|+c_1 = \left[\operatorname{sgn} u = \operatorname{sgn} x\right]$$

$$= -\frac{\operatorname{sgn} x}{3} \ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+c_1 = c_1 - \frac{\operatorname{sgn} x}{3} \ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

$$= c_1 - \frac{\operatorname{sgn} x}{3} \ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right| + \frac{\operatorname{sgn} x\cdot\ln 2}{3} = \left[\begin{array}{l} c_2=c_1+\frac{\operatorname{sgn} x\cdot\ln 2}{3} \\ c_1\in R,\ c_2\in R \end{array}\right]$$

$$= c_2 - \frac{\operatorname{sgn} x}{3} \ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|,\ x\in R-\{0\},\ c_2\in R.$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}} = c_2 - \frac{\operatorname{sgn} x}{3} \ln \left| \frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \right|$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[ \begin{array}{l|l} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array} \right]$$

$$= \left[ \begin{array}{l} \sqrt{x^6+x^3+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}} \\ \sqrt{t^2+t+1}=\sqrt{\frac{1}{x^6}+\frac{1}{x^3}+1}=\sqrt{\frac{1+x^3+x^6}{x^6}}=\frac{\sqrt{x^6+x^3+1}}{|x^3|}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array} \right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t}\cdot\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= \frac{1}{3}\int \frac{-\operatorname{sgn} t\,dt}{\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3} \ln\left| t+\frac{1}{2}+\sqrt{t^2+t+1}\right| + c_1 = \left[ \operatorname{sgn} u = \operatorname{sgn} x \right]$$

$$= -\frac{\operatorname{sgn} x}{3} \ln\left| \frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right| + c_1 = c_1 - \frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

$$= c_1 - \frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right| + \frac{\operatorname{sgn} x\cdot\ln 2}{3} = \left[ \begin{array}{l} c_2=c_1+\frac{\operatorname{sgn} x\cdot\ln 2}{3} \\ c_1\in R,\ c_2\in R \end{array}\right]$$

$$= c_2 - \frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|,\ x\in R-\{0\},\ c_2\in R.$$

$$\boxed{x\in(0;\infty)} = c_2 - \frac{1}{3}\ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right|$$

$$\boxed{x\in(-\infty;0)} = c_2 + \frac{1}{3}\ln\left|\frac{2+x^3-2\sqrt{x^6+x^3+1}}{x^3}\right|$$

# Riešené príklady – 200

$$\int \frac{dx}{x\sqrt{x^6+x^3+1}} = c_2 - \frac{\operatorname{sgn} x}{3} \ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|$$

$$= \int \frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}} = \left[\begin{array}{l|l} \text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\} \\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0 \end{array}\right]$$

$$= \left[\begin{array}{l} \sqrt{x^6+x^3+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}} \\ \sqrt{t^2+t+1}=\sqrt{\frac{1}{x^6}+\frac{1}{x^3}+1}=\sqrt{\frac{1+x^3+x^6}{x^6}}=\frac{\sqrt{x^6+x^3+1}}{|x^3|}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3} \end{array}\right] = \int \frac{-\frac{dt}{3t^2}}{\frac{1}{t}\cdot\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

$$= \frac{1}{3}\int \frac{-\operatorname{sgn} t\,dt}{\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}} = \frac{-\operatorname{sgn} t}{3} \ln\left|t+\frac{1}{2}+\sqrt{t^2+t+1}\right| + c_1 = \left[\operatorname{sgn} u = \operatorname{sgn} x\right]$$

$$= -\frac{\operatorname{sgn} x}{3} \ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right| + c_1 = c_1 - \frac{\operatorname{sgn} x}{3} \ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

$$= c_1 - \frac{\operatorname{sgn} x}{3} \ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right| + \frac{\operatorname{sgn} x\cdot\ln 2}{3} = \left[\begin{array}{c} c_2=c_1+\frac{\operatorname{sgn} x\cdot\ln 2}{3} \\ c_1\in R,\ c_2\in R \end{array}\right]$$

$$= c_2 - \frac{\operatorname{sgn} x}{3} \ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|,\ x\in R-\{0\},\ c_2\in R.$$

---

$$\boxed{x\in(0;\infty)} = c_2 - \frac{1}{3} \ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right| = \left[\begin{array}{c} c=c_2 \\ c\in R,\ c_2\in R \end{array}\right]$$

$$\boxed{x\in(-\infty;0)} = c_2 + \frac{1}{3} \ln\left|\frac{2+x^3-2\sqrt{x^6+x^3+1}}{x^3}\right| = c + \frac{\ln 3}{3} - \frac{1}{3} \ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right| = \left[\begin{array}{c} c=c_2+\frac{\ln 3}{3} \\ c\in R,\ c_2\in R \end{array}\right]$$

# Riešené príklady – 200

$$\int\frac{dx}{x\sqrt{x^6+x^3+1}} = c_2-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right| = c-\frac{1}{3}\ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right|$$

$$=\int\frac{x^2\,dx}{x^3\sqrt{x^6+x^3+1}}=\left[\begin{array}{l|l}\text{Subst. } t=\frac{1}{x^3},\ x^3=\frac{1}{t}=t^{-1} & x\in R-\{0\},\ t\in R-\{0\}\\ 3x^2\,dx=-t^{-2}\,dt=-\frac{dt}{t^2} & x^6+x^3+1=(x^3+\frac{1}{2})^2+\frac{3}{4}>0\end{array}\right]$$

$$=\left[\begin{array}{l}\sqrt{x^6+x^3+1}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}=\sqrt{\frac{1+t+t^2}{t^2}}=\frac{\sqrt{t^2+t+1}}{|t|}=\frac{\sqrt{t^2+t+1}}{t\cdot\operatorname{sgn} t}=\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}\\ \sqrt{t^2+t+1}=\sqrt{\frac{1}{x^6}+\frac{1}{x^3}+1}=\sqrt{\frac{1+x^3+x^6}{x^6}}=\frac{\sqrt{x^6+x^3+1}}{|x^3|}=\frac{\sqrt{x^6+x^3+1}}{x^3\cdot\operatorname{sgn} x^3}=\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\end{array}\right]=\int\frac{-\frac{dt}{3t^2}}{\frac{1}{t}\cdot\frac{1}{t\cdot\operatorname{sgn} t}\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}$$

$$=\frac{1}{3}\int\frac{-\operatorname{sgn} t\,dt}{\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}}=\frac{-\operatorname{sgn} t}{3}\ln\left|t+\frac{1}{2}+\sqrt{t^2+t+1}\right|+c_1=\left[\operatorname{sgn} u=\operatorname{sgn} x\right]$$

$$=-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{1}{x^3}+\frac{1}{2}+\frac{\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+c_1=c_1-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{2x^3}\right|$$

$$=c_1-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|+\frac{\operatorname{sgn} x\cdot\ln 2}{3}=\left[\begin{array}{c}c_2=c_1+\frac{\operatorname{sgn} x\cdot\ln 2}{3}\\ c_1\in R,\ c_2\in R\end{array}\right]$$

$$=c_2-\frac{\operatorname{sgn} x}{3}\ln\left|\frac{2+x^3+2\operatorname{sgn} x\cdot\sqrt{x^6+x^3+1}}{x^3}\right|,\ x\in R-\{0\},\ c_2\in R.$$

---

$x\in(0;\infty)$: $$=c_2-\frac{1}{3}\ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right|=\left[\begin{array}{c}c=c_2\\ c\in R,\ c_2\in R\end{array}\right]=c-\frac{1}{3}\ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right|.$$

$x\in(-\infty;0)$: $$=c_2+\frac{1}{3}\ln\left|\frac{2+x^3-2\sqrt{x^6+x^3+1}}{x^3}\right|=\left[\begin{array}{c}c=c_2+\frac{\ln 3}{3}\\ c\in R,\ c_2\in R\end{array}\right]=c-\frac{1}{3}\ln\left|\frac{2+x^3+2\sqrt{x^6+x^3+1}}{x^3}\right|.$$

# Koniec 10. časti príklady II

Ďakujem za pozornosť.