

Matematická analýza 1

2023/2024

10. Neurčitý integrál Riešené príklady I

Pre správne zobrazenie, fungovanie tooltipov, 2D a 3D animácií je nevyhnutné súbor otvoriť pomocou programu Adobe Reader (zásuvný modul Adobe PDF Plug-In webového prehliadača nestačí).

Kliknutím na text pred ikonou  získate nápomoc.

Kliknutím na skratku v modrej lište vpravo hore sa dostanete na príslušný slajd, druhým kliknutím sa dostanete na koniec tohto slajdu.

Obsah – príklady 001–100

- 1 Riešené príklady 001–010
- 2 Riešené príklady 011–020
- 3 Riešené príklady 021–030
- 4 Riešené príklady 031–040
- 5 Riešené príklady 041–050
- 6 Riešené príklady 051–060
- 7 Riešené príklady 061–070
- 8 Riešené príklady 071–080
- 9 Riešené príklady 081–090
- 10 Riešené príklady 091–100

Zoznam integrálov – príklady 001–100

001. $\int \frac{dx}{\sin x}$ 002. $\int \frac{dx}{\cos x}$ 003. $\int \frac{dx}{1+\sin x}$ 004. $\int \frac{dx}{1+\cos x}$ 005. $\int \frac{dx}{\sinh x}$ 006. $\int \frac{dx}{\cosh x}$ 007. $\int \frac{dx}{1+\sinh x}$ 008. $\int \frac{dx}{1+\cosh x}$ 009. $\int \sin^2 x \, dx$ 010. $\int \cos^2 x \, dx$ 011. $\int \sin^3 x \, dx$
 012. $\int \sin^{2n+1} x \, dx$ 013. $\int \sin^n x \, dx$ 014. $\int \cos^3 x \, dx$ 015. $\int \cos^{2n+1} x \, dx$ 016. $\int \cos^n x \, dx$ 017. $\int \sinh^2 x \, dx$ 018. $\int \cosh^2 x \, dx$ 019. $\int \sinh^3 x \, dx$
 020. $\int \sinh^{2n+1} x \, dx$ 021. $\int \sinh^n x \, dx$ 022. $\int \cosh^3 x \, dx$ 023. $\int \cosh^{2n+1} x \, dx$ 024. $\int \cosh^n x \, dx$ 025. $\int \operatorname{tg}^2 x \, dx$ 026. $\int \operatorname{tg}^3 x \, dx$ 027. $\int \operatorname{cotg}^2 x \, dx$
 028. $\int \operatorname{cotg}^3 x \, dx$ 029. $\int \operatorname{tgh}^2 x \, dx$ 030. $\int \operatorname{cotgh}^2 x \, dx$ 031. $\int \frac{\cos x}{4+3 \sin x} \, dx$ 032. $\int \frac{1+3 \sin x+2 \cos x}{\sin 2x} \, dx$ 033. $\int \frac{dx}{\sin^2 x \cos^2 x}$ 034. $\int [\operatorname{tg} x + \operatorname{cotg} x] \, dx$
 035. $\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ 036. $\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$ 037. $\int \frac{dx}{4 \cos^2 x + \sin^2 x}$ 038. $\int \frac{dx}{4 \cos^2 x - \sin^2 x}$ 039. $\int \frac{x^2 dx}{\sin x^3}$ 040. $\int x^2 \sin x^3 \, dx$ 041. $\int \frac{\cos x \, dx}{\sqrt{\sin^2 x}}$ 042. $\int \frac{\sin x \, dx}{\sqrt{\cos^5 x}}$
 043. $\int \frac{\cos x - \sin x}{\cos x + \sin x} \, dx$ 044. $\int \frac{\ln \cos x}{\sin^2 x} \, dx$ 045. $\int \frac{dx}{\sin x \cos x}$ 046. $\int \frac{dx}{\cos x + \sin x}$ 047. $\int \frac{(\sin x - \cos x) \, dx}{\sqrt{\sin x + \cos x}}$ 048. $\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} \, dx$ 049. $\int \cos^n ax \cdot \sin ax \, dx$ 050. $\int \frac{\sin ax \, dx}{\cos^n ax}$
 051. $\int \sin^n ax \cdot \cos ax \, dx$ 052. $\int \frac{\cos ax \, dx}{\sin^n ax}$ 053. $\int \sin ax \cdot \cos bx \, dx$ 054. $\int \cos ax \cdot \cos bx \, dx$ 055. $\int \sin ax \cdot \sin bx \, dx$ 056. $\int \sin ax \cdot \cos ax \, dx$
 057. $\int \cos^2 ax \, dx$ 058. $\int \sin^2 ax \, dx$ 059. $\int x \operatorname{tg}^2 x \, dx$ 060. $\int x \operatorname{cotg}^2 x \, dx$ 061. $\int x \sin ax \, dx$ 062. $\int x \cos ax \, dx$ 063. $\int x^2 \sinh ax \, dx$ 064. $\int x^2 \sin ax \, dx$
 065. $\int x^2 \cosh ax \, dx$ 066. $\int x^2 \cos ax \, dx$ 067. $\int x^n \sin ax \, dx$ 068. $\int x^n \cos ax \, dx$ 069. $\int x^3 \sin ax \, dx$ 070. $\int x^3 \cos ax \, dx$ 071. $\int \sqrt{1 + \frac{1}{\sin x}} \, dx$
 072. $\int \operatorname{arctg} x \, dx$ 073. $\int x \operatorname{arctg} x \, dx$ 074. $\int \ln x \, dx$ 075. $\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, dx$ 076. $\int \ln(x-1)^5 \, dx$ 077. $\int \frac{\ln x}{x} \, dx$ 078. $\int \frac{dx}{x \ln x}$ 079. $\int x^2 \ln \sqrt{1-x} \, dx$ 080. $\int \frac{\ln x}{\sqrt{x}} \, dx$
 081. $\int x \ln x \, dx$ 082. $\int x^2 \ln x \, dx$ 083. $\int x^n \ln x \, dx$ 084. $\int (x+1)^2 \ln(x-1)^5 \, dx$ 085. $\int x^x (\ln x + 1) \, dx$ 086. $\int x \ln^2 x \, dx$ 087. $\int \ln(x^2+1) \, dx$
 088. $\int \ln(\sqrt{1+x} + \sqrt{1-x}) \, dx$ 089. $\int \ln(x + \sqrt{x^2+1}) \, dx$ 090. $\int x(x-a)(x-b) \, dx$ 091. $\int |x| \, dx$ 092. $\int \min_{x \in (0; \infty)} \left\{ 1, \frac{1}{x} \right\} \, dx$ 093. $\int \frac{dx}{5+4e^x}$
 094. $\int \frac{dx}{\sqrt{5+4e^x}}$ 095. $\int \frac{dx}{\sqrt{e^{2x}+e^x+1}}$ 096. $\int \sqrt{\frac{1-e^x}{1+e^x}} \, dx$ 097. $\int (x+1)e^x \, dx$ 098. $\int x^2 e^{ax} \, dx$ 099. $\int x^8 e^{ax} \, dx$ 100. $\int x^n e^x \, dx$

Riešené príklady – 001

$$\int \frac{dx}{\sin x}$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x}$$

$$= \left[\text{UGS: } \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \left| \begin{array}{l} x \in (-\pi + 2k\pi; 0 + 2k\pi), t \in (-\infty; 0) \\ x \in (0 + 2k\pi; \pi + 2k\pi), t \in (0; \infty) \end{array} \right. \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right]$$

$$= \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$$

$$= \int \frac{\sin x dx}{\sin^2 x}$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x}$$

$$= \left[\text{UGS: } \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \quad \left| \begin{array}{l} x \in (-\pi + 2k\pi; 0 + 2k\pi), t \in (-\infty; 0) \\ \sin x \neq 0 \end{array} \right. \\ dx = \frac{2dt}{t^2+1} \quad \left| \begin{array}{l} \sin x = \frac{2t}{t^2+1} \\ x \in (0 + 2k\pi; \pi + 2k\pi), t \in (0; \infty) \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{2t}{t^2+1}}$$

$$= \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \left[\text{Subst. } \begin{array}{l} t = \frac{x}{2} \quad \left| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ \sin x \neq 0, k \in \mathbb{Z} \end{array} \right. \\ dt = \frac{dx}{2} \quad \left| \begin{array}{l} t \in (0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}) \\ x \neq k\pi, t \neq \frac{k\pi}{2} \end{array} \right. \end{array} \right]$$

$$= \int \frac{\sin x dx}{\sin^2 x} = \int \frac{\sin x dx}{1 - \cos^2 x}$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x}$$

$$= \left[\text{UGS: } \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \left| \begin{array}{l} x \in (-\pi + 2k\pi; 0 + 2k\pi), t \in (-\infty; 0) \\ x \in (0 + 2k\pi; \pi + 2k\pi), t \in (0; \infty) \end{array} \right. \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t}$$

$$= \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \left[\text{Subst. } \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \left| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ t \in (0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}) \end{array} \right. \begin{array}{l} \sin x \neq 0, k \in \mathbb{Z} \\ x \neq k\pi, t \neq \frac{k\pi}{2} \end{array} \right] = \int \frac{dt}{\sin t \cos t}$$

$$= \int \frac{\sin x dx}{\sin^2 x} = \int \frac{\sin x dx}{1 - \cos^2 x} = \left[\text{Subst. } \begin{array}{l} t = \cos x \\ dt = -\sin x dx \end{array} \left| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ t \in (-1; 1) \end{array} \right. \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right]$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x}$$

$$= \left[\text{UGS: } \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \left| \begin{array}{l} x \in (-\pi + 2k\pi; 0 + 2k\pi), t \in (-\infty; 0) \\ x \in (0 + 2k\pi; \pi + 2k\pi), t \in (0; \infty) \end{array} \right. \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t}$$

$$= \ln |t| + c_1$$

$$= \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \left[\text{Subst. } \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \left| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ t \in (0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}) \end{array} \right. \begin{array}{l} \sin x \neq 0, k \in \mathbb{Z} \\ x \neq k\pi, t \neq \frac{k\pi}{2} \end{array} \right] = \int \frac{dt}{\sin t \cos t}$$

$$= \int \frac{(\cos^2 t + \sin^2 t) dt}{\sin t \cos t}$$

$$= \int \frac{\sin x dx}{\sin^2 x} = \int \frac{\sin x dx}{1 - \cos^2 x} = \left[\text{Subst. } \begin{array}{l} t = \cos x \\ dt = -\sin x dx \end{array} \left| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ t \in (-1; 1) \end{array} \right. \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{-dt}{1-t^2}$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x} = \ln \left| \operatorname{tg} \frac{x}{2} \right| + c_1$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi + 2k\pi; 0 + 2k\pi), t \in (-\infty; 0) \\ x \in (0 + 2k\pi; \pi + 2k\pi), t \in (0; \infty) \end{array} \right| \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t} \\ = \ln |t| + c_1 = \ln \left| \operatorname{tg} \frac{x}{2} \right| + c_1, x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}. \end{array}$$

$$= \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \right| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ t \in (0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}) \end{array} \left| \begin{array}{l} \sin x \neq 0, k \in \mathbb{Z} \\ x \neq k\pi, t \neq \frac{k\pi}{2} \end{array} \right] = \int \frac{dt}{\sin t \cos t} \\ = \int \frac{(\cos^2 t + \sin^2 t) dt}{\sin t \cos t} = \int \frac{\cos t dt}{\sin t} - \int \frac{-\sin t dt}{\cos t}$$

$$= \int \frac{\sin x dx}{\sin^2 x} = \int \frac{\sin x dx}{1 - \cos^2 x} = \left[\begin{array}{l} \text{Subst. } t = \cos x \\ dt = -\sin x dx \end{array} \right| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ t \in (-1; 1) \end{array} \left| \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{-dt}{1-t^2} = \int \frac{dt}{t^2-1}$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x} = \ln \left| \operatorname{tg} \frac{x}{2} \right| + c_1$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi + 2k\pi; 0 + 2k\pi), t \in (-\infty; 0) \\ x \in (0 + 2k\pi; \pi + 2k\pi), t \in (0; \infty) \end{array} \right| \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t} \\ = \ln |t| + c_1 = \ln \left| \operatorname{tg} \frac{x}{2} \right| + c_1, x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}. \end{array}$$

$$= \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } \left. \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \right| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ t \in (0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}) \end{array} \right| \begin{array}{l} \sin x \neq 0, k \in \mathbb{Z} \\ x \neq k\pi, t \neq \frac{k\pi}{2} \end{array} \right] = \int \frac{dt}{\sin t \cos t} \\ = \int \frac{(\cos^2 t + \sin^2 t) dt}{\sin t \cos t} = \int \frac{\cos t dt}{\sin t} - \int \frac{-\sin t dt}{\cos t} = \ln |\sin t| - \ln |\cos t| + c_1$$

$$= \int \frac{\sin x dx}{\sin^2 x} = \int \frac{\sin x dx}{1 - \cos^2 x} = \left[\begin{array}{l} \text{Subst. } \left. \begin{array}{l} t = \cos x \\ dt = -\sin x dx \end{array} \right| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ t \in (-1; 1) \end{array} \right| \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{-dt}{1-t^2} = \int \frac{dt}{t^2-1} \\ = \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_2$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x} = \ln \left| \operatorname{tg} \frac{x}{2} \right| + c_1$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi + 2k\pi; 0 + 2k\pi), t \in (-\infty; 0) \\ x \in (0 + 2k\pi; \pi + 2k\pi), t \in (0; \infty) \end{array} \right| \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t} \\ = \ln |t| + c_1 = \ln \left| \operatorname{tg} \frac{x}{2} \right| + c_1, x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}. \end{array}$$

$$= \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } \left. \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \right| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ t \in (0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}) \end{array} \right| \begin{array}{l} \sin x \neq 0, k \in \mathbb{Z} \\ x \neq k\pi, t \neq \frac{k\pi}{2} \end{array} \right] = \int \frac{dt}{\sin t \cos t} \\ = \int \frac{(\cos^2 t + \sin^2 t) dt}{\sin t \cos t} = \int \frac{\cos t dt}{\sin t} - \int \frac{-\sin t dt}{\cos t} = \ln |\sin t| - \ln |\cos t| + c_1 \\ = \ln |\operatorname{tg} t| + c_1 \end{array}$$

$$= \int \frac{\sin x dx}{\sin^2 x} = \int \frac{\sin x dx}{1 - \cos^2 x} = \left[\begin{array}{l} \text{Subst. } \left. \begin{array}{l} t = \cos x \\ dt = -\sin x dx \end{array} \right| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ t \in (-1; 1) \end{array} \right| \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{-dt}{1-t^2} = \int \frac{dt}{t^2-1} \\ = \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_2 = \frac{1}{2} \ln \frac{1-t}{1+t} + c_2 \end{array}$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x} = \ln \left| \operatorname{tg} \frac{x}{2} \right| + c_1 = \frac{1}{2} \ln \frac{1-\cos x}{1+\cos x} + c_2$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi + 2k\pi; 0 + 2k\pi), t \in (-\infty; 0) \\ x \in (0 + 2k\pi; \pi + 2k\pi), t \in (0; \infty) \end{array} \right| \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t} \\ = \ln |t| + c_1 = \ln \left| \operatorname{tg} \frac{x}{2} \right| + c_1, x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \right| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ t \in (0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}) \end{array} \left| \begin{array}{l} \sin x \neq 0, k \in \mathbb{Z} \\ x \neq k\pi, t \neq \frac{k\pi}{2} \end{array} \right] = \int \frac{dt}{\sin t \cos t} \\ = \int \frac{(\cos^2 t + \sin^2 t) dt}{\sin t \cos t} = \int \frac{\cos t dt}{\sin t} - \int \frac{-\sin t dt}{\cos t} = \ln |\sin t| - \ln |\cos t| + c_1 \\ = \ln |\operatorname{tg} t| + c_1 = \ln \left| \operatorname{tg} \frac{x}{2} \right| + c_1, x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\sin x dx}{\sin^2 x} = \int \frac{\sin x dx}{1 - \cos^2 x} = \left[\begin{array}{l} \text{Subst. } t = \cos x \\ dt = -\sin x dx \end{array} \right| \begin{array}{l} x \in (0 + k\pi; \pi + k\pi) \\ t \in (-1; 1) \end{array} \left| \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{-dt}{1-t^2} = \int \frac{dt}{t^2-1} \\ = \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_2 = \frac{1}{2} \ln \frac{1-t}{1+t} + c_2 = \frac{1}{2} \ln \frac{1-\cos x}{1+\cos x} + c_2, \\ x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}, c_2 \in \mathbb{R}.$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x}$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x}$$

$$= \left[\text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; 1) \\ x \in (-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi), t \in (-1; 1) \end{array} \right| \begin{array}{l} x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (1; \infty) \\ \cos x \neq 0, x \neq \frac{\pi}{2}+k\pi, k \in Z, t \neq \pm 1 \end{array} \right]$$

$$= \int \frac{\cos x \, dx}{\cos^2 x}$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x}$$

$$= \left[\text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2 dt}{t^2 + 1} \end{array} \right| \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi), t \in (-\infty; 1) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), t \in (1; \infty) \end{array} \right] \\ \left. \begin{array}{l} \cos x = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi), t \in (-1; 1) \\ \cos x \neq 0, x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}, t \neq \pm 1 \end{array} \right]$$

$$= \int \frac{\frac{2 dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{\cos x dx}{\cos^2 x} = \int \frac{\cos x dx}{1 - \sin^2 x}$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x}$$

$$= \left[\text{UGS: } \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \left| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; 1) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (1; \infty) \\ x \in (-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi), t \in (-1; 1) \\ \cos x \neq 0, x \neq \frac{\pi}{2}+k\pi, k \in \mathbb{Z}, t \neq \pm 1 \end{array} \right. \right]$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = - \int \frac{2dt}{t^2-1}$$

$$= \int \frac{\cos x dx}{\cos^2 x} = \int \frac{\cos x dx}{1-\sin^2 x} = \left[\text{Subst. } \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \left| \begin{array}{l} x \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \\ t \in (-1; 1) \end{array} \right. \begin{array}{l} \cos x \neq 0 \\ x \neq \frac{\pi}{2}+k\pi, k \in \mathbb{Z} \end{array} \right]$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x}$$

$$= \left[\text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; 1) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (1; \infty) \end{array} \right] \\ \left. \begin{array}{l} \cos x = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi), t \in (-1; 1) \\ \cos x \neq 0, x \neq \frac{\pi}{2}+k\pi, k \in \mathbb{Z}, t \neq \pm 1 \end{array} \right]$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = - \int \frac{2dt}{t^2-1} = -\frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c_1$$

$$= \int \frac{\cos x \, dx}{\cos^2 x} = \int \frac{\cos x \, dx}{1-\sin^2 x} = \left[\text{Subst. } \left. \begin{array}{l} t = \sin x \\ dt = \cos x \, dx \end{array} \right| \begin{array}{l} x \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \\ t \in (-1; 1) \end{array} \right] \left. \begin{array}{l} \cos x \neq 0 \\ x \neq \frac{\pi}{2}+k\pi, k \in \mathbb{Z} \end{array} \right]$$

$$= \int \frac{dt}{1-t^2}$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x}$$

$$= \left[\text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; 1) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (1; \infty) \end{array} \right] \\ \left. \begin{array}{l} \cos x = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi), t \in (-1; 1) \\ \cos x \neq 0, x \neq \frac{\pi}{2}+k\pi, k \in \mathbb{Z}, t \neq \pm 1 \end{array} \right]$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = - \int \frac{2dt}{t^2-1} = -\frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c_1 = \ln \left| \frac{t+1}{t-1} \right| + c_1$$

$$= \int \frac{\cos x dx}{\cos^2 x} = \int \frac{\cos x dx}{1-\sin^2 x} = \left[\text{Subst. } \left. \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \right| \begin{array}{l} x \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \\ t \in (-1; 1) \end{array} \right] \left. \begin{array}{l} \cos x \neq 0 \\ x \neq \frac{\pi}{2}+k\pi, k \in \mathbb{Z} \end{array} \right]$$

$$= \int \frac{dt}{1-t^2} = - \int \frac{dt}{t^2-1}$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x} = \ln \left| \frac{\operatorname{tg} \frac{x}{2} + 1}{\operatorname{tg} \frac{x}{2} - 1} \right| + c_1$$

$$= \left[\begin{array}{l} \text{UGS: } \left| t = \operatorname{tg} \frac{x}{2} \right. \left. \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi), t \in (-\infty; 1) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), t \in (1; \infty) \end{array} \right. \\ \left. dx = \frac{2dt}{t^2+1} \right| \cos x = \frac{1-t^2}{t^2+1} \left. \begin{array}{l} x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi), t \in (-1; 1) \\ \cos x \neq 0, x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}, t \neq \pm 1 \end{array} \right. \end{array} \right]$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = - \int \frac{2dt}{t^2-1} = -\frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c_1 = \ln \left| \frac{t+1}{t-1} \right| + c_1 = \ln \left| \frac{\operatorname{tg} \frac{x}{2} + 1}{\operatorname{tg} \frac{x}{2} - 1} \right| + c_1,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, \pi + 2k\pi, k \in \mathbb{Z} \right\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\cos x \, dx}{\cos^2 x} = \int \frac{\cos x \, dx}{1 - \sin^2 x} = \left[\begin{array}{l} \text{Subst. } t = \sin x \left. \begin{array}{l} x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi) \\ dt = \cos x \, dx \end{array} \right| \cos x \neq 0 \\ \left. t \in (-1; 1) \right. \left. \begin{array}{l} \\ x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \end{array} \right. \end{array} \right]$$

$$= \int \frac{dt}{1-t^2} = - \int \frac{dt}{t^2-1} = -\frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_2$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x} = \ln \left| \frac{\operatorname{tg} \frac{x}{2} + 1}{\operatorname{tg} \frac{x}{2} - 1} \right| + c_1$$

$$= \left[\begin{array}{l} \text{UGS: } \left| t = \operatorname{tg} \frac{x}{2} \right. \left. \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi), t \in (-\infty; 1) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), t \in (1; \infty) \end{array} \right. \\ \left. dx = \frac{2dt}{t^2+1} \right| \cos x = \frac{1-t^2}{t^2+1} \left. \begin{array}{l} x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi), t \in (-1; 1) \\ \cos x \neq 0, x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}, t \neq \pm 1 \end{array} \right. \end{array} \right]$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = - \int \frac{2dt}{t^2-1} = -\frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c_1 = \ln \left| \frac{t+1}{t-1} \right| + c_1 = \ln \left| \frac{\operatorname{tg} \frac{x}{2} + 1}{\operatorname{tg} \frac{x}{2} - 1} \right| + c_1,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, \pi + 2k\pi, k \in \mathbb{Z} \right\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\cos x dx}{\cos^2 x} = \int \frac{\cos x dx}{1 - \sin^2 x} = \left[\begin{array}{l} \text{Subst. } t = \sin x \left. \begin{array}{l} x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi) \\ \cos x \neq 0 \end{array} \right. \\ dt = \cos x dx \left. \begin{array}{l} t \in (-1; 1) \\ x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \end{array} \right. \end{array} \right]$$

$$= \int \frac{dt}{1-t^2} = - \int \frac{dt}{t^2-1} = -\frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_2 = -\frac{1}{2} \ln \frac{1-t}{1+t} + c_2$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x} = \ln \left| \frac{\operatorname{tg} \frac{x}{2} + 1}{\operatorname{tg} \frac{x}{2} - 1} \right| + c_1$$

$$= \left[\begin{array}{l} \text{UGS: } \left| t = \operatorname{tg} \frac{x}{2} \right. \left. \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi), t \in (-\infty; 1) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), t \in (1; \infty) \end{array} \right. \\ \left. dx = \frac{2dt}{t^2+1} \right| \cos x = \frac{1-t^2}{t^2+1} \left. \begin{array}{l} x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi), t \in (-1; 1) \\ \cos x \neq 0, x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}, t \neq \pm 1 \end{array} \right. \end{array} \right]$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = - \int \frac{2dt}{t^2-1} = -\frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c_1 = \ln \left| \frac{t+1}{t-1} \right| + c_1 = \ln \left| \frac{\operatorname{tg} \frac{x}{2} + 1}{\operatorname{tg} \frac{x}{2} - 1} \right| + c_1,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, \pi + 2k\pi, k \in \mathbb{Z} \right\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\cos x dx}{\cos^2 x} = \int \frac{\cos x dx}{1 - \sin^2 x} = \left[\begin{array}{l} \text{Subst. } t = \sin x \left. \begin{array}{l} x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi) \\ dt = \cos x dx \end{array} \right| \cos x \neq 0 \\ \left. t \in (-1; 1) \right. \left. \begin{array}{l} x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \end{array} \right. \end{array} \right]$$

$$= \int \frac{dt}{1-t^2} = - \int \frac{dt}{t^2-1} = -\frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_2 = -\frac{1}{2} \ln \frac{1-t}{1+t} + c_2$$

$$= \frac{1}{2} \ln \frac{1+t}{1-t} + c_2$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x} = \ln \left| \frac{\operatorname{tg} \frac{x}{2} + 1}{\operatorname{tg} \frac{x}{2} - 1} \right| + c_1 = \frac{1}{2} \ln \frac{1 + \sin x}{1 - \sin x} + c_2$$

$$= \left[\text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi), t \in (-\infty; 1) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), t \in (1; \infty) \end{array} \right]$$

$$\left. \begin{array}{l} \cos x = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi), t \in (-1; 1) \end{array} \right| \cos x \neq 0, x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}, t \neq \pm 1$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = - \int \frac{2dt}{t^2-1} = -\frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c_1 = \ln \left| \frac{t+1}{t-1} \right| + c_1 = \ln \left| \frac{\operatorname{tg} \frac{x}{2} + 1}{\operatorname{tg} \frac{x}{2} - 1} \right| + c_1,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, \pi + 2k\pi, k \in \mathbb{Z} \right\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\cos x dx}{\cos^2 x} = \int \frac{\cos x dx}{1 - \sin^2 x} = \left[\text{Subst. } \left. \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \right| \begin{array}{l} x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi) \\ t \in (-1; 1) \end{array} \right] \begin{array}{l} \cos x \neq 0 \\ x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \end{array}$$

$$= \int \frac{dt}{1-t^2} = - \int \frac{dt}{t^2-1} = -\frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_2 = -\frac{1}{2} \ln \frac{1-t}{1+t} + c_2$$

$$= \frac{1}{2} \ln \frac{1+t}{1-t} + c_2 = \frac{1}{2} \ln \frac{1 + \sin x}{1 - \sin x} + c_2, x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, c_2 \in \mathbb{R}.$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l|l} \text{UGS:} & t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} \end{array} \left| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; -1) \\ x \in (-\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; \infty) \end{array} \right. \begin{array}{l} \sin x \neq -1, k \in \mathbb{Z} \\ x \neq -\frac{\pi}{2}+2k\pi \end{array} \right]$$

$$= \int \frac{(1-\sin x) dx}{(1-\sin x)(1+\sin x)}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l} \text{UGS:} \\ dx = \frac{2dt}{t^2+1} \end{array} \left| \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \sin x = \frac{2t}{t^2+1} \end{array} \right. \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; -1) \\ x \in (-\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; \infty) \end{array} \left| \begin{array}{l} \sin x \neq -1, k \in \mathbb{Z} \\ x \neq -\frac{\pi}{2}+2k\pi \end{array} \right. \right] = \int \frac{\frac{2dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{(1-\sin x) dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x) dx}{1-\sin^2 x}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l} \text{UGS:} \\ dx = \frac{2dt}{t^2+1} \end{array} \left| \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \sin x = \frac{2t}{t^2+1} \end{array} \right. \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; -1) \\ x \in (-\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; \infty) \end{array} \left. \begin{array}{l} \sin x \neq -1, k \in \mathbb{Z} \\ x \neq -\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{1 + \frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}}$$

$$= \int \frac{(1-\sin x) dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x) dx}{1-\sin^2 x} = \int \frac{(1-\sin x) dx}{\cos^2 x}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l} \text{UGS:} \\ dx = \frac{2dt}{t^2+1} \end{array} \left| \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \sin x = \frac{2t}{t^2+1} \end{array} \right. \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; -1) \\ x \in (-\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; \infty) \end{array} \left| \begin{array}{l} \sin x \neq -1, k \in \mathbb{Z} \\ x \neq -\frac{\pi}{2}+2k\pi \end{array} \right. \right] = \int \frac{\frac{2dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2dt}{(t+1)^2}$$

$$= \int \frac{(1-\sin x) dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x) dx}{1-\sin^2 x} = \int \frac{(1-\sin x) dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x dx}{\cos^2 x}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l} \text{UGS:} \\ dx = \frac{2dt}{t^2+1} \end{array} \left| \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \sin x = \frac{2t}{t^2+1} \end{array} \right. \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; -1) \\ x \in (-\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; \infty) \end{array} \left. \begin{array}{l} \sin x \neq -1, k \in \mathbb{Z} \\ x \neq -\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2dt}{(t+1)^2} = \left[\begin{array}{l} \text{Subst. } u = t+1 \\ du = dt \end{array} \left| \begin{array}{l} t \in (-\infty; -1), u \in (-\infty; 0) \\ x \in (-1; \infty), t \in (0; \infty) \end{array} \right. \right]$$

$$= \int \frac{(1-\sin x) dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x) dx}{1-\sin^2 x} = \int \frac{(1-\sin x) dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x dx}{\cos^2 x}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \\ dt = -\sin x dx \end{array} \left| \begin{array}{l} x \in (0+2k\pi; \frac{\pi}{2}+2k\pi), t \in (0; 1) \\ x \in (\pi+2k\pi; \frac{\pi}{2}+2k\pi), t \in (-1; 0) \end{array} \right. \begin{array}{l} x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; 0) \\ x \in (\frac{3\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \end{array} \left. \begin{array}{l} \cos x \neq 0 \\ x \neq \frac{\pi}{2}+k\pi, k \in \mathbb{Z} \end{array} \right]$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l} \text{UGS:} \\ dx = \frac{2dt}{t^2+1} \end{array} \left| \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \sin x = \frac{2t}{t^2+1} \end{array} \right. \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; -1) \\ x \in (-\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; \infty) \end{array} \right. \left. \begin{array}{l} \sin x \neq -1, k \in \mathbb{Z} \\ x \neq -\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2dt}{(t+1)^2} = \left[\begin{array}{l} \text{Subst. } u = t+1 \\ du = dt \end{array} \left| \begin{array}{l} t \in (-\infty; -1), u \in (-\infty; 0) \\ x \in (-1; \infty), t \in (0; \infty) \end{array} \right. \right] = 2 \int \frac{du}{u^2}$$

$$= \int \frac{(1-\sin x) dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x) dx}{1-\sin^2 x} = \int \frac{(1-\sin x) dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x dx}{\cos^2 x}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \\ dt = -\sin x dx \end{array} \left| \begin{array}{l} x \in (0+2k\pi; \frac{\pi}{2}+2k\pi), t \in (0; 1) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; 0) \end{array} \right. \left. \begin{array}{l} x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; 0) \\ x \in (\frac{3\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \end{array} \right. \right. \left. \begin{array}{l} \cos x \neq 0 \\ x \neq \frac{\pi}{2}+k\pi, k \in \mathbb{Z} \end{array} \right]$$

$$= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l} \text{UGS:} \\ dx = \frac{2dt}{t^2+1} \end{array} \left| \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \sin x = \frac{2t}{t^2+1} \end{array} \right. \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; -1) \\ x \in (-\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; \infty) \end{array} \left| \begin{array}{l} \sin x \neq -1, k \in \mathbb{Z} \\ x \neq -\frac{\pi}{2}+2k\pi \end{array} \right. \right] = \int \frac{\frac{2dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2dt}{(t+1)^2} = \left[\begin{array}{l} \text{Subst. } u = t+1 \\ du = dt \end{array} \left| \begin{array}{l} t \in (-\infty; -1), u \in (-\infty; 0) \\ x \in (-1; \infty), t \in (0; \infty) \end{array} \right. \right] = 2 \int \frac{du}{u^2} = 2 \int u^{-2} du$$

$$= \int \frac{(1-\sin x) dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x) dx}{1-\sin^2 x} = \int \frac{(1-\sin x) dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x dx}{\cos^2 x}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \\ dt = -\sin x dx \end{array} \left| \begin{array}{l} x \in (0+2k\pi; \frac{\pi}{2}+2k\pi), t \in (0; 1) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; 0) \end{array} \right. \left| \begin{array}{l} \cos x \neq 0 \\ x \in (\frac{3\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \\ x \neq \frac{\pi}{2}+k\pi, k \in \mathbb{Z} \end{array} \right. \right]$$

$$= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2} dt$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$\begin{aligned}
 &= \left[\text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2 dt}{t^2+1} \\ \sin x = \frac{2t}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; -1) \\ x \in (-\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; \infty) \end{array} \right| \begin{array}{l} \sin x \neq -1, k \in \mathbb{Z} \\ x \neq -\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{1+\frac{2t}{t^2+1}} \\
 &= \int \frac{\frac{2 dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2 dt}{(t+1)^2} = \left[\text{Subst. } \left. \begin{array}{l} u = t+1 \\ du = dt \end{array} \right| \begin{array}{l} t \in (-\infty; -1), u \in (-\infty; 0) \\ x \in (-1; \infty), t \in (0; \infty) \end{array} \right] = 2 \int \frac{du}{u^2} = 2 \int u^{-2} du \\
 &= 2 \frac{u^{-1}}{-1} + C_1
 \end{aligned}$$

$$\begin{aligned}
 &= \int \frac{(1-\sin x) dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x) dx}{1-\sin^2 x} = \int \frac{(1-\sin x) dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x dx}{\cos^2 x} \\
 &= \left[\text{Subst. } \left. \begin{array}{l} t = \cos x \\ dt = -\sin x dx \end{array} \right| \begin{array}{l} x \in (0+2k\pi; \frac{\pi}{2}+2k\pi), t \in (0; 1) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; 0) \\ x \in (\pi+2k\pi; \frac{3\pi}{2}+2k\pi), t \in (-1; 0) \\ x \in (\frac{3\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \end{array} \right| \begin{array}{l} \cos x \neq 0 \\ x \neq \frac{\pi}{2}+k\pi, k \in \mathbb{Z} \end{array} \right] \\
 &= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2} dt = \operatorname{tg} x + \frac{t^{-1}}{-1} + C_2
 \end{aligned}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$\begin{aligned}
 &= \left[\text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \\ \sin x = \frac{2t}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; -1) \\ x \in (-\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; \infty) \end{array} \right| \begin{array}{l} \sin x \neq -1, k \in \mathbb{Z} \\ x \neq -\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{1+\frac{2t}{t^2+1}} \\
 &= \int \frac{\frac{2dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2dt}{(t+1)^2} = \left[\text{Subst. } \left. \begin{array}{l} u = t+1 \\ du = dt \end{array} \right| \begin{array}{l} t \in (-\infty; -1), u \in (-\infty; 0) \\ x \in (-1; \infty), t \in (0; \infty) \end{array} \right] = 2 \int \frac{du}{u^2} = 2 \int u^{-2} du \\
 &= 2 \frac{u^{-1}}{-1} + C_1 = C_1 - \frac{2}{u}
 \end{aligned}$$

$$\begin{aligned}
 &= \int \frac{(1-\sin x) dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x) dx}{1-\sin^2 x} = \int \frac{(1-\sin x) dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x dx}{\cos^2 x} \\
 &= \left[\text{Subst. } \left. \begin{array}{l} t = \cos x \\ dt = -\sin x dx \end{array} \right| \begin{array}{l} x \in (0+2k\pi; \frac{\pi}{2}+2k\pi), t \in (0; 1) \\ x \in (\pi+2k\pi; \frac{\pi}{2}+2k\pi), t \in (-1; 0) \end{array} \right| \begin{array}{l} x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; 0) \\ x \in (\frac{3\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \end{array} \right| \begin{array}{l} \cos x \neq 0 \\ x \neq \frac{\pi}{2}+k\pi, k \in \mathbb{Z} \end{array} \right] \\
 &= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2} dt = \operatorname{tg} x + \frac{t^{-1}}{-1} + C_2 = \operatorname{tg} x - \frac{1}{t} + C_2
 \end{aligned}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$\begin{aligned}
 &= \left[\text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \\ \sin x = \frac{2t}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-\infty; -1) \\ x \in (-\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; \infty) \end{array} \right| \begin{array}{l} \sin x \neq -1, k \in \mathbb{Z} \\ x \neq -\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{1+\frac{2t}{t^2+1}} \\
 &= \int \frac{\frac{2dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2dt}{(t+1)^2} = \left[\text{Subst. } \left. \begin{array}{l} u = t+1 \\ du = dt \end{array} \right| \begin{array}{l} t \in (-\infty; -1), u \in (-\infty; 0) \\ x \in (-1; \infty), t \in (0; \infty) \end{array} \right] = 2 \int \frac{du}{u^2} = 2 \int u^{-2} du \\
 &= 2 \frac{u^{-1}}{-1} + C_1 = C_1 - \frac{2}{u} = C_1 - \frac{2}{t+1}
 \end{aligned}$$

$$\begin{aligned}
 &= \int \frac{(1-\sin x) dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x) dx}{1-\sin^2 x} = \int \frac{(1-\sin x) dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x dx}{\cos^2 x} \\
 &= \left[\text{Subst. } \left. \begin{array}{l} t = \cos x \\ dt = -\sin x dx \end{array} \right| \begin{array}{l} x \in (0+2k\pi; \frac{\pi}{2}+2k\pi), t \in (0; 1) \\ x \in (\pi+2k\pi; \frac{3\pi}{2}+2k\pi), t \in (-1; 0) \end{array} \right| \begin{array}{l} x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (-1; 0) \\ x \in (\frac{3\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \end{array} \right| \begin{array}{l} \cos x \neq 0 \\ x \neq \frac{\pi}{2}+k\pi, k \in \mathbb{Z} \end{array} \right] \\
 &= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2} dt = \operatorname{tg} x + \frac{t^{-1}}{-1} + C_2 = \operatorname{tg} x - \frac{1}{t} + C_2 \\
 &= \frac{\sin x}{\cos x} - \frac{1}{\cos x} + C_2
 \end{aligned}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x} = c_1 - \frac{2}{\operatorname{tg} \frac{x}{2} + 1} = \frac{\sin x - 1}{\cos x} + c_2$$

$$= \left[\begin{array}{l} \text{UGS:} \\ dx = \frac{2dt}{t^2+1} \end{array} \left| \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \sin x = \frac{2t}{t^2+1} \end{array} \right. \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi), t \in (-\infty; -1) \\ x \in (-\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), t \in (-1; \infty) \end{array} \right. \left. \begin{array}{l} \sin x \neq -1, k \in \mathbb{Z} \\ x \neq -\frac{\pi}{2} + 2k\pi \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{1 + \frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2dt}{(t+1)^2} = \left[\begin{array}{l} \text{Subst. } u = t+1 \\ du = dt \end{array} \left| \begin{array}{l} t \in (-\infty; -1), u \in (-\infty; 0) \\ x \in (-1; \infty), t \in (0; \infty) \end{array} \right. \right] = 2 \int \frac{du}{u^2} = 2 \int u^{-2} du$$

$$= 2 \frac{u^{-1}}{-1} + c_1 = c_1 - \frac{2}{u} = c_1 - \frac{2}{t+1} = c_1 - \frac{2}{\operatorname{tg} \frac{x}{2} + 1},$$

$$x \in \mathbb{R} - \left\{ -\frac{\pi}{2} + 2k\pi, \pi + 2k\pi, k \in \mathbb{Z} \right\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{(1-\sin x) dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x) dx}{1-\sin^2 x} = \int \frac{(1-\sin x) dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x dx}{\cos^2 x}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \\ dt = -\sin x dx \end{array} \left| \begin{array}{l} x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi), t \in (0; 1) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), t \in (-1; 0) \\ x \in (\pi + 2k\pi; \frac{3\pi}{2} + 2k\pi), t \in (-1; 0) \\ x \in (\frac{3\pi}{2} + 2k\pi; \pi + 2k\pi), t \in (0; 1) \end{array} \right. \left. \begin{array}{l} \cos x \neq 0 \\ x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2} dt = \operatorname{tg} x + \frac{t^{-1}}{-1} + c_2 = \operatorname{tg} x - \frac{1}{t} + c_2$$

$$= \frac{\sin x}{\cos x} - \frac{1}{\cos x} + c_2 = \frac{\sin x - 1}{\cos x} + c_2, x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, c_2 \in \mathbb{R}.$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x}$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x}$$

$$= \left[\text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2 dt}{t^2+1} \end{array} \right| \cos x = \frac{1-t^2}{t^2+1} \right. \left. \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos x \neq -1, x \neq \pi+2k\pi, k \in \mathbb{Z} \end{array} \right]$$

$$= \int \frac{dx}{2 \cos^2 \frac{x}{2}}$$

$$= \int \frac{(1-\cos x) dx}{(1-\cos x)(1+\cos x)}$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x}$$

$$= \left[\text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2 dt}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos x = \frac{1-t^2}{t^2+1} \\ \cos x \neq -1, x \neq \pi+2k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{1 + \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{dx}{2 \cos^2 \frac{x}{2}} = \left[\text{Subst. } \left. \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \right| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi) \\ t \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \\ \cos x \neq -1, k \in \mathbb{Z} \\ x \neq \pi+2k\pi \end{array} \right]$$

$$= \int \frac{(1-\cos x) dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x) dx}{1-\cos^2 x}$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x}$$

$$= \left[\text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2 dt}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos x = \frac{1-t^2}{t^2+1} \\ \cos x \neq -1, x \neq \pi+2k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{1 + \frac{1-t^2}{t^2+1}} = \int \frac{2 dt}{t^2+1}$$

$$= \int \frac{dx}{2 \cos^2 \frac{x}{2}} = \left[\text{Subst. } \left. \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \right| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi) \\ t \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \\ \cos x \neq -1, k \in \mathbb{Z} \\ x \neq \pi+2k\pi \end{array} \right] = \int \frac{dt}{\cos^2 t}$$

$$= \int \frac{(1-\cos x) dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x) dx}{1-\cos^2 x} = \int \frac{(1-\cos x) dx}{\sin^2 x}$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x}$$

$$= \left[\text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2 dt}{t^2+1} \end{array} \right| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos x = \frac{1-t^2}{t^2+1} \\ \cos x \neq -1, x \neq \pi+2k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{1 + \frac{1-t^2}{t^2+1}} = \int \frac{\frac{2 dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= \int \frac{dx}{2 \cos^2 \frac{x}{2}} = \left[\text{Subst. } \left. \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \right| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi) \\ t \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \\ \cos x \neq -1, k \in \mathbb{Z} \\ x \neq \pi+2k\pi \end{array} \right] = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \int \frac{(1-\cos x) dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x) dx}{1-\cos^2 x} = \int \frac{(1-\cos x) dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x dx}{\sin^2 x}$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg} \frac{x}{2} + c_1$$

$$= \left[\text{UGS: } \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2dt}{t^2+1} \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos x = \frac{1-t^2}{t^2+1} \\ \cos x \neq -1, x \neq \pi+2k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1$$

$$= \int \frac{dx}{2 \cos^2 \frac{x}{2}} = \left[\text{Subst. } \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi) \\ t \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \\ \cos x \neq -1, k \in \mathbb{Z} \\ x \neq \pi+2k\pi \end{array} \right] = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{(1-\cos x) dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x) dx}{1-\cos^2 x} = \int \frac{(1-\cos x) dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x dx}{\sin^2 x}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-1; 0) \\ x \in (0+2k\pi; \frac{\pi}{2}+2k\pi), t \in (0; 1) \\ x \in (-\frac{\pi}{2}+2k\pi; 0+2k\pi), t \in (-1; 0) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \\ \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right]$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg} \frac{x}{2} + c_1$$

$$= \left[\text{UGS: } \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \cos x = \frac{1-t^2}{t^2+1} \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos x \neq -1, x \neq \pi+2k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2 dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1 = \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{2 \cos^2 \frac{x}{2}} = \left[\text{Subst. } \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi) \\ t \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \end{array} \right] \begin{array}{l} \cos x \neq -1, k \in \mathbb{Z} \\ x \neq \pi+2k\pi \end{array} = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{(1-\cos x) dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x) dx}{1-\cos^2 x} = \int \frac{(1-\cos x) dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x dx}{\sin^2 x}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-1; 0) \\ x \in (0+2k\pi; \frac{\pi}{2}+2k\pi), t \in (0; 1) \end{array} \right] \begin{array}{l} x \in (-\frac{\pi}{2}+2k\pi; 0+2k\pi), t \in (-1; 0) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \end{array} \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array}$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2}$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg} \frac{x}{2} + c_1$$

$$= \left[\text{UGS: } \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \cos x = \frac{1-t^2}{t^2+1} \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos x \neq -1, x \neq \pi+2k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2 dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1 = \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{2 \cos^2 \frac{x}{2}} = \left[\text{Subst. } \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi) \\ t \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \end{array} \right] \begin{array}{l} \cos x \neq -1, k \in \mathbb{Z} \\ x \neq \pi+2k\pi \end{array} = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{(1-\cos x) dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x) dx}{1-\cos^2 x} = \int \frac{(1-\cos x) dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x dx}{\sin^2 x}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-1; 0) \\ x \in (0+2k\pi; \frac{\pi}{2}+2k\pi), t \in (0; 1) \end{array} \right] \begin{array}{l} x \in (-\frac{\pi}{2}+2k\pi; 0+2k\pi), t \in (-1; 0) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \end{array} \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array}$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2} dt$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg} \frac{x}{2} + c_1$$

$$= \left[\text{UGS: } \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \cos x = \frac{1-t^2}{t^2+1} \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos x \neq -1, x \neq \pi+2k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2 dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1 = \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{2 \cos^2 \frac{x}{2}} = \left[\text{Subst. } \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi) \\ t \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \end{array} \right] \begin{array}{l} \cos x \neq -1, k \in \mathbb{Z} \\ x \neq \pi+2k\pi \end{array} = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{(1-\cos x) dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x) dx}{1-\cos^2 x} = \int \frac{(1-\cos x) dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x dx}{\sin^2 x}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-1; 0) \\ x \in (0+2k\pi; \frac{\pi}{2}+2k\pi), t \in (0; 1) \end{array} \right] \begin{array}{l} x \in (-\frac{\pi}{2}+2k\pi; 0+2k\pi), t \in (-1; 0) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \end{array} \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array}$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2} dt = -\operatorname{cotg} x - \frac{t^{-1}}{-1} + c_2$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg} \frac{x}{2} + c_1$$

$$= \left[\text{UGS: } \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \cos x = \frac{1-t^2}{t^2+1} \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos x \neq -1, x \neq \pi+2k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2 dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1 = \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{2 \cos^2 \frac{x}{2}} = \left[\text{Subst. } \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi) \\ t \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \end{array} \right] \begin{array}{l} \cos x \neq -1, k \in \mathbb{Z} \\ x \neq \pi+2k\pi \end{array} = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{(1-\cos x) dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x) dx}{1-\cos^2 x} = \int \frac{(1-\cos x) dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x dx}{\sin^2 x}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \middle| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-1; 0) \\ x \in (0+2k\pi; \frac{\pi}{2}+2k\pi), t \in (0; 1) \end{array} \right] \begin{array}{l} x \in (-\frac{\pi}{2}+2k\pi; 0+2k\pi), t \in (-1; 0) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \end{array} \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array}$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2} dt = -\operatorname{cotg} x - \frac{t^{-1}}{-1} + c_2 = \frac{1}{t} - \operatorname{cotg} x + c_2$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg} \frac{x}{2} + c_1$$

$$= \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \cos x = \frac{1-t^2}{t^2+1} \end{array} \right. \left| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos x \neq -1, x \neq \pi+2k\pi, k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{2 dt}{t^2+1} = \int dt$$

$$= t + c_1 = \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{2 \cos^2 \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } t = \frac{x}{2} \left| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi) \\ \cos x \neq -1, k \in \mathbb{Z} \end{array} \right. \\ dt = \frac{dx}{2} \left| \begin{array}{l} t \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \\ x \neq \pi+2k\pi \end{array} \right. \end{array} \right] = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{(1-\cos x) dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x) dx}{1-\cos^2 x} = \int \frac{(1-\cos x) dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x dx}{\sin^2 x}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sin x \left| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-1; 0) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \end{array} \right. \\ dt = \cos x dx \left| \begin{array}{l} x \in (-\frac{\pi}{2}+2k\pi; 0+2k\pi), t \in (-1; 0) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \end{array} \right. \left| \begin{array}{l} \sin x \neq 0 \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right. \end{array} \right]$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2} dt = -\operatorname{cotg} x - \frac{t^{-1}}{-1} + c_2 = \frac{1}{t} - \operatorname{cotg} x + c_2$$

$$= \frac{1}{\sin x} - \frac{\cos x}{\sin x} + c_2$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg} \frac{x}{2} + c_1 = \frac{1-\cos x}{\sin x} + c_2$$

$$= \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{x}{2} \\ \cos x = \frac{1-t^2}{t^2+1} \end{array} \right. \left| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos x \neq -1, x \neq \pi+2k\pi, k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2 dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1 = \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{2 \cos^2 \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } t = \frac{x}{2} \left| \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi) \\ \cos x \neq -1, k \in \mathbb{Z} \end{array} \right. \\ dt = \frac{dx}{2} \left| \begin{array}{l} t \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \\ x \neq \pi+2k\pi \end{array} \right. \end{array} \right] = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg} \frac{x}{2} + c_1, x \in \mathbb{R} - \{\pi+2k\pi, k \in \mathbb{Z}\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{(1-\cos x) dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x) dx}{1-\cos^2 x} = \int \frac{(1-\cos x) dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x dx}{\sin^2 x}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sin x \left| \begin{array}{l} x \in (-\pi+2k\pi; -\frac{\pi}{2}+2k\pi), t \in (-1; 0) \\ x \in (-\frac{\pi}{2}+2k\pi; 0+2k\pi), t \in (-1; 0) \\ \sin x \neq 0 \end{array} \right. \\ dt = \cos x dx \left| \begin{array}{l} x \in (0+2k\pi; \frac{\pi}{2}+2k\pi), t \in (0; 1) \\ x \in (\frac{\pi}{2}+2k\pi; \pi+2k\pi), t \in (0; 1) \\ x \neq k\pi, k \in \mathbb{Z} \end{array} \right. \end{array} \right]$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2} dt = -\operatorname{cotg} x - \frac{t^{-1}}{-1} + c_2 = \frac{1}{t} - \operatorname{cotg} x + c_2$$

$$= \frac{1}{\sin x} - \frac{\cos x}{\sin x} + c_2 = \frac{1-\cos x}{\sin x} + c_2, x \in \mathbb{R} - \{\pi+k\pi, k \in \mathbb{Z}\}, c_2 \in \mathbb{R}.$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x}$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t = \operatorname{tgh} \frac{x}{2} & x \in (-\infty; 0), t \in (-1; 0) \mid \sinh x \neq 0 \\ dx = \frac{2dt}{1-t^2} & \sinh x = \frac{2t}{1-t^2} & x \in (0; \infty), t \in (0; 1) \mid x \neq 0 \end{array} \right]$$

$$= \int \frac{\sinh x \, dx}{\sinh^2 x}$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}}$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t = \operatorname{tgh} \frac{x}{2} & x \in (-\infty; 0), t \in (-1; 0) & \sinh x \neq 0 \\ \text{dx} = \frac{2dt}{1-t^2} & \sinh x = \frac{2t}{1-t^2} & x \in (0; \infty), t \in (0; 1) & x \neq 0 \end{array} \right] = \int \frac{\frac{2dt}{1-t^2}}{\frac{2t}{1-t^2}}$$

$$= \int \frac{\sinh x \, dx}{\sinh^2 x} = \int \frac{\sinh x \, dx}{\cosh^2 x - 1}$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}} = \int \frac{2 \, dx}{e^x - e^{-x}}$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \left. \begin{array}{l} x \in (-\infty; 0), t \in (-1; 0) \\ x \in (0; \infty), t \in (0; 1) \end{array} \right| \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right] = \int \frac{2dt}{1-t^2} = \int \frac{dt}{t}$$

$$= \int \frac{\sinh x dx}{\sinh^2 x} = \int \frac{\sinh x dx}{\cosh^2 x - 1} = \left[\begin{array}{l} \text{Subst. } t = \cosh x \\ dt = \sinh x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (1; \infty) \\ x \in (0; \infty), t \in (1; \infty) \end{array} \right| \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right]$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}} = \int \frac{2 dx}{e^x - e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; 1) \\ x \in (0; \infty), t \in (1; \infty) \end{array} \right| \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right]$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \left. \begin{array}{l} x \in (-\infty; 0), t \in (-1; 0) \\ x \in (0; \infty), t \in (0; 1) \end{array} \right| \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right] = \int \frac{2dt}{1-t^2} = \int \frac{dt}{t} = \ln t + c_1$$

$$= \int \frac{\sinh x \, dx}{\sinh^2 x} = \int \frac{\sinh x \, dx}{\cosh^2 x - 1} \left[\begin{array}{l} \text{Subst. } t = \cosh x \\ dt = \sinh x \, dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (1; \infty) \\ x \in (0; \infty), t \in (1; \infty) \end{array} \right. \left. \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right] = \int \frac{dt}{t^2 - 1}$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}} = \int \frac{2 \, dx}{e^x - e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; 1) \\ x \in (0; \infty), t \in (1; \infty) \end{array} \right. \left. \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right] = \int \frac{2 \, dt}{t(t - \frac{1}{t})}$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x} = \ln \left| \operatorname{tgh} \frac{x}{2} \right| + c_1$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \left. \begin{array}{l} x \in (-\infty; 0), t \in (-1; 0) \\ x \in (0; \infty), t \in (0; 1) \end{array} \right| \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right] = \int \frac{\frac{2dt}{1-t^2}}{\frac{2t}{1-t^2}} = \int \frac{dt}{t} = \ln t + c_1$$

$$= \ln \left| \operatorname{tgh} \frac{x}{2} \right| + c_1, x \in \mathbb{R} - \{0\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\sinh x dx}{\sinh^2 x} = \int \frac{\sinh x dx}{\cosh^2 x - 1} = \left[\begin{array}{l} \text{Subst. } t = \cosh x \\ dt = \sinh x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (1; \infty) \\ x \in (0; \infty), t \in (1; \infty) \end{array} \right. \left| \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right. \right] = \int \frac{dt}{t^2 - 1}$$

$$= \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_2$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}} = \int \frac{2 dx}{e^x - e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; 1) \\ x \in (0; \infty), t \in (1; \infty) \end{array} \right. \left| \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right. \right] = \int \frac{2 dt}{t(t - \frac{1}{t})}$$

$$= \int \frac{2 dt}{t^2 - 1}$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x} = \ln \left| \operatorname{tgh} \frac{x}{2} \right| + c_1$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; 0), t \in (-1; 0) \\ x \in (0; \infty), t \in (0; 1) \end{array} \left. \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right] = \int \frac{2dt}{1-t^2} = \int \frac{dt}{t} = \ln t + c_1$$

$$= \ln \left| \operatorname{tgh} \frac{x}{2} \right| + c_1, x \in \mathbb{R} - \{0\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\sinh x dx}{\sinh^2 x} = \int \frac{\sinh x dx}{\cosh^2 x - 1} = \left[\begin{array}{l} \text{Subst. } t = \cosh x \\ dt = \sinh x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (1; \infty) \\ x \in (0; \infty), t \in (1; \infty) \end{array} \right. \left. \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right] = \int \frac{dt}{t^2 - 1}$$

$$= \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_2 = \frac{1}{2} \ln \frac{t-1}{t+1} + c_2$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}} = \int \frac{2 dx}{e^x - e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; 1) \\ x \in (0; \infty), t \in (1; \infty) \end{array} \right. \left. \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right] = \int \frac{2 dt}{t(t - \frac{1}{t})}$$

$$= \int \frac{2 dt}{t^2 - 1} = \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_3$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x} = \ln \left| \operatorname{tgh} \frac{x}{2} \right| + c_1 = \frac{1}{2} \ln \frac{\cosh x - 1}{\cosh x + 1} + c_2 = \frac{1}{2} \ln \left| \frac{e^x - 1}{e^x + 1} \right| + c_3$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \left. \begin{array}{l} x \in (-\infty; 0), t \in (-1; 0) \\ x \in (0; \infty), t \in (0; 1) \end{array} \right| \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right] = \int \frac{2dt}{1-t^2} = \int \frac{dt}{t} = \ln t + c_1$$

$$= \ln \left| \operatorname{tgh} \frac{x}{2} \right| + c_1, x \in \mathbb{R} - \{0\}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\sinh x \, dx}{\sinh^2 x} = \int \frac{\sinh x \, dx}{\cosh^2 x - 1} = \left[\begin{array}{l} \text{Subst. } t = \cosh x \\ dt = \sinh x \, dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (1; \infty) \\ x \in (0; \infty), t \in (1; \infty) \end{array} \right. \left. \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right] = \int \frac{dt}{t^2 - 1}$$

$$= \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_2 = \frac{1}{2} \ln \frac{t-1}{t+1} + c_2 = \frac{1}{2} \ln \frac{\cosh x - 1}{\cosh x + 1} + c_2, x \in \mathbb{R} - \{0\}, c_2 \in \mathbb{R}.$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}} = \int \frac{2 \, dx}{e^x - e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (0; 1) \\ x \in (0; \infty), t \in (1; \infty) \end{array} \right. \left. \begin{array}{l} \sinh x \neq 0 \\ x \neq 0 \end{array} \right] = \int \frac{2 \, dt}{t(t - \frac{1}{t})}$$

$$= \int \frac{2 \, dt}{t^2 - 1} = \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_3 = \frac{1}{2} \ln \left| \frac{e^x - 1}{e^x + 1} \right| + c_3, x \in \mathbb{R} - \{0\}, c_3 \in \mathbb{R}.$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x}$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cos x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; 0), t \in (-1; 0) \\ x \in (0; \infty), t \in (0; 1) \end{array} \right]$$

$$= \int \frac{\cosh x \, dx}{\cosh^2 x}$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}}$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cos x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; 0), t \in (-1; 0) \\ x \in (0; \infty), t \in (0; 1) \end{array} \right] = \int \frac{2 dt}{\frac{1-t^2}{1+t^2}}$$

$$= \int \frac{\cosh x dx}{\cosh^2 x} = \int \frac{\cosh x dx}{\sinh^2 x + 1}$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}} = \int \frac{2 dx}{e^x + e^{-x}}$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cos x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; 0), t \in (-1; 0) \\ x \in (0; \infty), t \in (0; 1) \end{array} \right] = \int \frac{2 dt}{\frac{1-t^2}{1+t^2}} = \int \frac{2 dt}{t^2+1}$$

$$= \int \frac{\cosh x dx}{\cosh^2 x} = \int \frac{\cosh x dx}{\sinh^2 x + 1} \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x dx \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right. \right]$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}} = \int \frac{2 dx}{e^x + e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x = \ln t \\ t \in (0; \infty) \end{array} \right. \right]$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cos x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; 0), t \in (-1; 0) \\ x \in (0; \infty), t \in (0; 1) \end{array} \right] = \int \frac{2dt}{1-t^2} = \int \frac{2dt}{t^2+1} = 2 \operatorname{arctg} t + c_1$$

$$= \int \frac{\cosh x dx}{\cosh^2 x} = \int \frac{\cosh x dx}{\sinh^2 x + 1} \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x dx \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right. \right] = \int \frac{dt}{t^2+1}$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}} = \int \frac{2 dx}{e^x + e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{2 dt}{t(t + \frac{1}{t})}$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x} = 2 \operatorname{arctg} \operatorname{tgh} \frac{x}{2} + c_1$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cos x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; 0), t \in (-1; 0) \\ x \in (0; \infty), t \in (0; 1) \end{array} \right] = \int \frac{2dt}{1-t^2} = \int \frac{2dt}{t^2+1} = 2 \operatorname{arctg} t + c_1$$

$$= 2 \operatorname{arctg} \operatorname{tgh} \frac{x}{2} + c_1, x \in \mathbb{R}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\cosh x dx}{\cosh^2 x} = \int \frac{\cosh x dx}{\sinh^2 x + 1} = \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x dx \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right. \right] = \int \frac{dt}{t^2+1} = \operatorname{arctg} t + c_2$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}} = \int \frac{2 dx}{e^x + e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{2 dt}{t(t + \frac{1}{t})} = \int \frac{2 dt}{t^2+1}$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x} = 2 \operatorname{arctg} \operatorname{tgh} \frac{x}{2} + c_1 = \operatorname{arctg} \sinh x + c_2$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cos x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; 0), t \in (-1; 0) \\ x \in (0; \infty), t \in (0; 1) \end{array} \right] = \int \frac{2dt}{1-t^2} = \int \frac{2dt}{t^2+1} = 2 \operatorname{arctg} t + c_1$$

$$= 2 \operatorname{arctg} \operatorname{tgh} \frac{x}{2} + c_1, x \in R, c_1 \in R.$$

$$= \int \frac{\cosh x dx}{\cosh^2 x} = \int \frac{\cosh x dx}{\sinh^2 x + 1} = \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x dx \end{array} \left| \begin{array}{l} x \in R \\ t \in R \end{array} \right. \right] = \int \frac{dt}{t^2+1} = \operatorname{arctg} t + c_2$$

$$= \operatorname{arctg} \sinh x + c_2, x \in R, c_2 \in R.$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}} = \int \frac{2 dx}{e^x + e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in R \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{2 dt}{t(t + \frac{1}{t})} = \int \frac{2 dt}{t^2+1}$$

$$= 2 \operatorname{arctg} t + c_3$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x} = 2 \operatorname{arctg} \operatorname{tgh} \frac{x}{2} + c_1 = \operatorname{arctg} \sinh x + c_2 = 2 \operatorname{arctg} e^x + c_3$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cos x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; 0), t \in (-1; 0) \\ x \in (0; \infty), t \in (0; 1) \end{array} \right] = \int \frac{2dt}{1-t^2} = \int \frac{2dt}{t^2+1} = 2 \operatorname{arctg} t + c_1$$

$$= 2 \operatorname{arctg} \operatorname{tgh} \frac{x}{2} + c_1, x \in R, c_1 \in R.$$

$$= \int \frac{\cosh x dx}{\cosh^2 x} = \int \frac{\cosh x dx}{\sinh^2 x + 1} \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x dx \end{array} \left| \begin{array}{l} x \in R \\ t \in R \end{array} \right. \right] = \int \frac{dt}{t^2+1} = \operatorname{arctg} t + c_2$$

$$= \operatorname{arctg} \sinh x + c_2, x \in R, c_2 \in R.$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}} = \int \frac{2 dx}{e^x + e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in R \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{2 dt}{t(t + \frac{1}{t})} = \int \frac{2 dt}{t^2+1}$$

$$= 2 \operatorname{arctg} t + c_3 = 2 \operatorname{arctg} e^x + c_3, x \in R, c_3 \in R.$$

Riešené príklady – 007

$$\int \frac{dx}{1 + \sinh x}$$

Riešené príklady – 007

$$\int \frac{dx}{1 + \sinh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t = \operatorname{tgh} \frac{x}{2} & x \in (-\infty; \ln(\sqrt{2}-1)), t \in (-1; \sqrt{2}-1) \\ \text{dx} = \frac{2dt}{1-t^2} & \sinh x = \frac{2t}{1-t^2} & x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; 1) \end{array} \left| \begin{array}{l} \sinh x \neq -1 \\ x \neq \ln(\sqrt{2}-1) \end{array} \right. \right]$$

$$= \int \frac{dx}{1 + \frac{e^x - e^{-x}}{2}}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (-1; \sqrt{2}-1) \\ x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; 1) \end{array} \left. \begin{array}{l} \sinh x \neq -1 \\ x \neq \ln(\sqrt{2}-1) \end{array} \right] \stackrel{?}{=} \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{2t}{1-t^2}}$$

$$= \int \frac{dx}{1 + \frac{e^x - e^{-x}}{2}} = \int \frac{2 dx}{2 + e^x - e^{-x}}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (-1; \sqrt{2}-1) \\ x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; 1) \end{array} \left. \begin{array}{l} \sinh x \neq -1 \\ x \neq \ln(\sqrt{2}-1) \end{array} \right] \stackrel{!}{=} \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}}$$

$$= \int \frac{dx}{1 + \frac{e^x - e^{-x}}{2}} = \int \frac{2dx}{2 + e^x - e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \end{array} \left| \begin{array}{l} x = \ln t \\ dx = \frac{dt}{t} \end{array} \right. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (0; \sqrt{2}-1) \\ x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; \infty) \end{array} \left. \begin{array}{l} \sinh x \neq -1 \\ x \neq \ln(\sqrt{2}-1) \end{array} \right] \stackrel{!}{=} \int \frac{2 \frac{dt}{t}}{2 + t - \frac{1}{t}}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (-1; \sqrt{2}-1) \\ x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; 1) \end{array} \left. \begin{array}{l} \sinh x \neq -1 \\ x \neq \ln(\sqrt{2}-1) \end{array} \right] \stackrel{!}{=} \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2dt}{t^2-2t-1}$$

$$= \int \frac{dx}{1 + \frac{e^x - e^{-x}}{2}} = \int \frac{2dx}{2 + e^x - e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \end{array} \left| \begin{array}{l} x = \ln t \\ dx = \frac{dt}{t} \end{array} \right. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (0; \sqrt{2}-1) \\ x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; \infty) \end{array} \left. \begin{array}{l} \sinh x \neq -1 \\ x \neq \ln(\sqrt{2}-1) \end{array} \right] \stackrel{!}{=} \int \frac{2dt}{t(2+t-\frac{1}{t})}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l} \text{UHS: } \left| t = \operatorname{tgh} \frac{x}{2} \right. \left. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (-1; \sqrt{2}-1) \\ \sinh x \neq -1 \end{array} \right. \\ \left. \left. \begin{array}{l} dx = \frac{2dt}{1-t^2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \left. \begin{array}{l} x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; 1) \\ x \neq \ln(\sqrt{2}-1) \end{array} \right. \right] \stackrel{!}{=} \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2dt}{t^2-2t-1} = \int \frac{-2dt}{(t-1)^2-2}$$

$$= \int \frac{dx}{1 + \frac{e^x - e^{-x}}{2}} = \int \frac{2dx}{2 + e^x - e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \left| x = \ln t \right. \left. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (0; \sqrt{2}-1) \\ \sinh x \neq -1 \end{array} \right. \\ \left. \left. \begin{array}{l} e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \right. \left. \begin{array}{l} x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; \infty) \\ x \neq \ln(\sqrt{2}-1) \end{array} \right. \right] \stackrel{!}{=} \\ = \int \frac{2dt}{t(2+t-\frac{1}{t})} = \int \frac{2dt}{t^2+2t-1}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l} \text{UHS: } \left| t = \operatorname{tgh} \frac{x}{2} \right. \left. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (-1; \sqrt{2}-1) \\ \sinh x \neq -1 \end{array} \right. \\ \left. \begin{array}{l} dx = \frac{2dt}{1-t^2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \left. \begin{array}{l} x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; 1) \\ x \neq \ln(\sqrt{2}-1) \end{array} \right. \end{array} \right] \stackrel{!}{=} \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2dt}{t^2-2t-1} = \int \frac{-2dt}{(t-1)^2-2} = - \int \frac{2dt}{(t-1)^2-(\sqrt{2})^2}$$

$$= \int \frac{dx}{1 + \frac{e^x - e^{-x}}{2}} = \int \frac{2dx}{2 + e^x - e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \left| x = \ln t \right. \left. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (0; \sqrt{2}-1) \\ \sinh x \neq -1 \end{array} \right. \\ \left. \begin{array}{l} e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \right. \left. \begin{array}{l} x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; \infty) \\ x \neq \ln(\sqrt{2}-1) \end{array} \right. \end{array} \right] \stackrel{!}{=}$$

$$= \int \frac{2dt}{t(2+t-\frac{1}{t})} = \int \frac{2dt}{t^2+2t-1} = \int \frac{2dt}{(t+1)^2-2}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l} \text{UHS: } \left| t = \operatorname{tgh} \frac{x}{2} \right. \left. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (-1; \sqrt{2}-1) \\ \sinh x \neq -1 \end{array} \right. \\ \left. \begin{array}{l} dx = \frac{2dt}{1-t^2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \left. \begin{array}{l} x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; 1) \\ x \neq \ln(\sqrt{2}-1) \end{array} \right. \end{array} \right] \stackrel{!}{=} \int \frac{\frac{2dt}{1-t^2}}{1+\frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2dt}{t^2-2t-1} = \int \frac{-2dt}{(t-1)^2-2} = - \int \frac{2dt}{(t-1)^2-(\sqrt{2})^2} = -\frac{2}{2\sqrt{2}} \ln \left| \frac{t-1-\sqrt{2}}{t-1+\sqrt{2}} \right| + C_1$$

$$= \int \frac{dx}{1+\frac{e^x-e^{-x}}{2}} = \int \frac{2dx}{2+e^x-e^{-x}} = \left[\begin{array}{l} \text{Subst. } t=e^x \left| x=\ln t \right. \left. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (0; \sqrt{2}-1) \\ \sinh x \neq -1 \end{array} \right. \\ \left. \begin{array}{l} e^{-x}=t^{-1}=\frac{1}{t} \\ dx=\frac{dt}{t} \end{array} \right. \left. \begin{array}{l} x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; \infty) \\ x \neq \ln(\sqrt{2}-1) \end{array} \right. \end{array} \right] \stackrel{!}{=} \int \frac{2dt}{t(2+t-\frac{1}{t})} = \int \frac{2dt}{t^2+2t-1} = \int \frac{2dt}{(t+1)^2-2} = \int \frac{2dt}{(t+1)^2-(\sqrt{2})^2}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$\begin{aligned}
 &= \left[\text{UHS: } \left. \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ dx = \frac{2dt}{1-t^2} \end{array} \right| \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (-1; \sqrt{2}-1) \\ x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; 1) \end{array} \right| \begin{array}{l} \sinh x \neq -1 \\ x \neq \ln(\sqrt{2}-1) \end{array} \right] \stackrel{p}{=} \int \frac{\frac{2dt}{1-t^2}}{1+\frac{2t}{1-t^2}} \\
 &= \int \frac{\frac{2dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2dt}{t^2-2t-1} = \int \frac{-2dt}{(t-1)^2-2} = -\int \frac{2dt}{(t-1)^2-(\sqrt{2})^2} = -\frac{2}{2\sqrt{2}} \ln \left| \frac{t-1-\sqrt{2}}{t-1+\sqrt{2}} \right| + C_1 \\
 &= \frac{1}{\sqrt{2}} \ln \left| \frac{t-1+\sqrt{2}}{t-1-\sqrt{2}} \right| + C_1
 \end{aligned}$$

$$\begin{aligned}
 &= \int \frac{dx}{1+\frac{e^x-e^{-x}}{2}} = \int \frac{2dx}{2+e^x-e^{-x}} = \left[\text{Subst. } \left. \begin{array}{l} t = e^x \\ dx = \frac{dt}{t} \end{array} \right| \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (0; \sqrt{2}-1) \\ x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; \infty) \end{array} \right| \begin{array}{l} \sinh x \neq -1 \\ x \neq \ln(\sqrt{2}-1) \end{array} \right] \stackrel{p}{=} \\
 &= \int \frac{2dt}{t(2+t-\frac{1}{t})} = \int \frac{2dt}{t^2+2t-1} = \int \frac{2dt}{(t+1)^2-2} = \int \frac{2dt}{(t+1)^2-(\sqrt{2})^2} = \frac{2}{2\sqrt{2}} \ln \left| \frac{t+1-\sqrt{2}}{t+1+\sqrt{2}} \right| + C_2
 \end{aligned}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x} = \frac{\sqrt{2}}{2} \ln \left| \frac{\operatorname{tgh} \frac{x}{2} - 1 + \sqrt{2}}{\operatorname{tgh} \frac{x}{2} - 1 - \sqrt{2}} \right| + c_1$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (-1; \sqrt{2}-1) \\ x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; 1) \end{array} \left. \begin{array}{l} \sinh x \neq -1 \\ x \neq \ln(\sqrt{2}-1) \end{array} \right] \stackrel{2}{=} \int \frac{\frac{2dt}{1-t^2}}{1+\frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2dt}{t^2-2t-1} = \int \frac{-2dt}{(t-1)^2-2} = - \int \frac{2dt}{(t-1)^2-(\sqrt{2})^2} = -\frac{2}{2\sqrt{2}} \ln \left| \frac{t-1-\sqrt{2}}{t-1+\sqrt{2}} \right| + c_1$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{t-1+\sqrt{2}}{t-1-\sqrt{2}} \right| + c_1 = \frac{\sqrt{2}}{2} \ln \left| \frac{\operatorname{tgh} \frac{x}{2} - 1 + \sqrt{2}}{\operatorname{tgh} \frac{x}{2} - 1 - \sqrt{2}} \right| + c_1, \quad x \in \mathbb{R} - \{\ln(\sqrt{2}-1)\}, \quad c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{1+\frac{e^x-e^{-x}}{2}} = \int \frac{2dx}{2+e^x-e^{-x}} = \left[\begin{array}{l} \text{Subst. } t=e^x \\ e^{-x}=t^{-1}=\frac{1}{t} \end{array} \left| \begin{array}{l} x = \ln t \\ dx = \frac{dt}{t} \end{array} \right. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (0; \sqrt{2}-1) \\ x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; \infty) \end{array} \left. \begin{array}{l} \sinh x \neq -1 \\ x \neq \ln(\sqrt{2}-1) \end{array} \right] \stackrel{2}{=} \int \frac{2dt}{t(2+t-\frac{1}{t})} = \int \frac{2dt}{t^2+2t-1} = \int \frac{2dt}{(t+1)^2-2} = \int \frac{2dt}{(t+1)^2-(\sqrt{2})^2} = \frac{2}{2\sqrt{2}} \ln \left| \frac{t+1-\sqrt{2}}{t+1+\sqrt{2}} \right| + c_2$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{t+1-\sqrt{2}}{t+1+\sqrt{2}} \right| + c_2$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x} = \frac{\sqrt{2}}{2} \ln \left| \frac{\operatorname{tgh} \frac{x}{2} - 1 + \sqrt{2}}{\operatorname{tgh} \frac{x}{2} - 1 - \sqrt{2}} \right| + c_1 = \frac{\sqrt{2}}{2} \ln \left| \frac{e^x + 1 - \sqrt{2}}{e^x + 1 + \sqrt{2}} \right| + c_2$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \sinh x = \frac{2t}{1-t^2} \end{array} \right. \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (-1; \sqrt{2}-1) \\ x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; 1) \end{array} \left. \begin{array}{l} \sinh x \neq -1 \\ x \neq \ln(\sqrt{2}-1) \end{array} \right] \stackrel{2}{=} \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2dt}{t^2-2t-1} = \int \frac{-2dt}{(t-1)^2-2} = - \int \frac{2dt}{(t-1)^2-(\sqrt{2})^2} = -\frac{2}{2\sqrt{2}} \ln \left| \frac{t-1-\sqrt{2}}{t-1+\sqrt{2}} \right| + c_1$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{t-1+\sqrt{2}}{t-1-\sqrt{2}} \right| + c_1 = \frac{\sqrt{2}}{2} \ln \left| \frac{\operatorname{tgh} \frac{x}{2} - 1 + \sqrt{2}}{\operatorname{tgh} \frac{x}{2} - 1 - \sqrt{2}} \right| + c_1, \quad x \in \mathbb{R} - \{\ln(\sqrt{2}-1)\}, \quad c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{1 + \frac{e^x - e^{-x}}{2}} = \int \frac{2dx}{2 + e^x - e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in (-\infty; \ln(\sqrt{2}-1)), t \in (0; \sqrt{2}-1) \\ x \in (\ln(\sqrt{2}-1); \infty), t \in (\sqrt{2}-1; \infty) \end{array} \right. \left. \begin{array}{l} \sinh x \neq -1 \\ x \neq \ln(\sqrt{2}-1) \end{array} \right] \stackrel{2}{=} \int \frac{2dt}{t(2+t-\frac{1}{t})} = \int \frac{2dt}{t^2+2t-1} = \int \frac{2dt}{(t+1)^2-2} = \int \frac{2dt}{(t+1)^2-(\sqrt{2})^2} = \frac{2}{2\sqrt{2}} \ln \left| \frac{t+1-\sqrt{2}}{t+1+\sqrt{2}} \right| + c_2$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{t+1-\sqrt{2}}{t+1+\sqrt{2}} \right| + c_2 = \frac{\sqrt{2}}{2} \ln \left| \frac{e^x + 1 - \sqrt{2}}{e^x + 1 + \sqrt{2}} \right| + c_2, \quad x \in \mathbb{R} - \{\ln(\sqrt{2}-1)\}, \quad c_2 \in \mathbb{R}.$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x}$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t = \operatorname{tgh} \frac{x}{2} & x \in \mathbb{R} \\ \hline dx = \frac{2dt}{1-t^2} & \cosh x = \frac{1+t^2}{1-t^2} & t \in (-1; 1) \end{array} \right]$$

$$= \int \frac{dx}{2 \cosh^2 \frac{x}{2}}$$

$$= \int \frac{\cosh x \, dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x}$$

$$= \int \frac{dx}{1 + \frac{e^x + e^{-x}}{2}}$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cosh x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in (-1; 1) \end{array} \right] = \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{1+t^2}{1-t^2}}$$

$$= \int \frac{dx}{2 \cosh^2 \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right. \right]$$

$$= \int \frac{\cosh x \, dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x \, dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (-\infty; 0) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right]$$

$$= \int \frac{dx}{1 + \frac{e^x + e^{-x}}{2}} = \int \frac{2 \, dx}{2 + e^x + e^{-x}}$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cosh x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in (-1; 1) \end{array} \right] = \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{1+t^2}{1-t^2}} = \int \frac{2dt}{1-t^2}$$

$$= \int \frac{dx}{2 \cosh^2 \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right. \right] = \int \frac{dt}{\cosh^2 t}$$

$$= \int \frac{\cosh x \, dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} \stackrel{!}{=} \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x \, dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (-\infty; 0) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x}$$

$$= \int \frac{dx}{1 + \frac{e^x + e^{-x}}{2}} = \int \frac{2dx}{2 + e^x + e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x = \ln t \\ t \in (0; \infty) \end{array} \right. \right]$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cosh x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in (-1; 1) \end{array} \right] = \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{1+t^2}{1-t^2}} = \int \frac{2dt}{1-t^2} = \int dt$$

$$= \int \frac{dx}{2 \cosh^2 \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right. \right] = \int \frac{dt}{\cosh^2 t} = \operatorname{tgh} t + c_1$$

$$= \int \frac{\cosh x \, dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} \stackrel{!}{=} \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x \, dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (-\infty; 0) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x}$$

$$= \frac{1}{-t} - \operatorname{cotgh} x + c_2$$

$$= \int \frac{dx}{1 + \frac{e^x + e^{-x}}{2}} = \int \frac{2 \, dx}{2 + e^x + e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x = \ln t \\ x \in \mathbb{R} \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{2 \, dt}{t(2+t+\frac{1}{t})}$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x} = \operatorname{tgh} \frac{x}{2} + c_1$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cosh x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in (-1; 1) \end{array} \right] = \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{1+t^2}{1-t^2}} = \int \frac{2dt}{\frac{1-t^2}{1-t^2} + \frac{1+t^2}{1-t^2}} = \int dt = t + c_1$$

$$= \int \frac{dx}{2 \cosh^2 \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right. \right] = \int \frac{dt}{\cosh^2 t} = \operatorname{tgh} t + c_1 = \operatorname{tgh} \frac{x}{2} + c_1, \quad x \in \mathbb{R}, \quad c_1 \in \mathbb{R}.$$

$$= \int \frac{\cosh x \, dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} \stackrel{!}{=} \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x \, dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (-\infty; 0) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x} \\ = \frac{1}{-t} - \operatorname{cotgh} x + c_2 = c_2 - \frac{1}{\sinh x} - \frac{\cosh x}{\sinh x}$$

$$= \int \frac{dx}{1 + \frac{e^x + e^{-x}}{2}} = \int \frac{2 \, dx}{2 + e^x + e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{2 \, dt}{t(2 + t + \frac{1}{t})} = \int \frac{2 \, dt}{t^2 + 2t + 1}$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x} = \operatorname{tgh} \frac{x}{2} + c_1 = c_2 - \frac{1+\cosh x}{\sinh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cosh x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in (-1; 1) \end{array} \right] = \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{1+t^2}{1-t^2}} = \int \frac{\frac{2dt}{1-t^2}}{\frac{2}{1-t^2}} = \int dt = t + c_1 = \operatorname{tgh} \frac{x}{2} + c_1, \\ x \in \mathbb{R}, c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{2 \cosh^2 \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right. \right] = \int \frac{dt}{\cosh^2 t} = \operatorname{tgh} t + c_1 = \operatorname{tgh} \frac{x}{2} + c_1, x \in \mathbb{R}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\cosh x dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (-\infty; 0) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x} \\ = \frac{1}{-t} - \operatorname{cotgh} x + c_2 = c_2 - \frac{1}{\sinh x} - \frac{\cosh x}{\sinh x} = c_2 - \frac{1+\cosh x}{\sinh x}, x \in \mathbb{R} - \{0\}, c_2 \in \mathbb{R}.$$

$$= \int \frac{dx}{1 + \frac{e^x + e^{-x}}{2}} = \int \frac{2 dx}{2 + e^x + e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x = \ln t \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{2 dt}{t(2+t+\frac{1}{t})} = \int \frac{2 dt}{t^2+2t+1} \\ = \int \frac{2 dt}{(t+1)^2}$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x} = \operatorname{tgh} \frac{x}{2} + c_1 = c_2 - \frac{1+\cosh x}{\sinh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cosh x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in (-1; 1) \end{array} \right] = \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{1+t^2}{1-t^2}} = \int \frac{2dt}{2} = \int dt = t + c_1 = \operatorname{tgh} \frac{x}{2} + c_1, \\ x \in \mathbb{R}, c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{2 \cosh^2 \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right. \right] = \int \frac{dt}{\cosh^2 t} = \operatorname{tgh} t + c_1 = \operatorname{tgh} \frac{x}{2} + c_1, x \in \mathbb{R}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\cosh x dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (-\infty; 0) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x} \\ = \frac{1}{-t} - \operatorname{cotgh} x + c_2 = c_2 - \frac{1}{\sinh x} - \frac{\cosh x}{\sinh x} = c_2 - \frac{1+\cosh x}{\sinh x}, x \in \mathbb{R} - \{0\}, c_2 \in \mathbb{R}.$$

$$= \int \frac{dx}{1 + \frac{e^x + e^{-x}}{2}} = \int \frac{2 dx}{2 + e^x + e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{2 dt}{t(2+t+\frac{1}{t})} = \int \frac{2 dt}{t^2+2t+1} \\ = \int \frac{2 dt}{(t+1)^2} = \int 2(t+1)^{-2} dt$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x} = \operatorname{tgh} \frac{x}{2} + c_1 = c_2 - \frac{1+\cosh x}{\sinh x}$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \left| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cosh x = \frac{1+t^2}{1-t^2} \end{array} \right. \begin{array}{l} x \in \mathbb{R} \\ t \in (-1; 1) \end{array} \right] = \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{1+t^2}{1-t^2}} = \int \frac{2dt}{2} = \int dt = t + c_1 = \operatorname{tgh} \frac{x}{2} + c_1, \\ x \in \mathbb{R}, c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{2 \cosh^2 \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right. \right] = \int \frac{dt}{\cosh^2 t} = \operatorname{tgh} t + c_1 = \operatorname{tgh} \frac{x}{2} + c_1, x \in \mathbb{R}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\cosh x dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x dx \end{array} \left| \begin{array}{l} x \in (-\infty; 0), t \in (-\infty; 0) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right. \right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x} \\ = \frac{1}{-t} - \operatorname{cotgh} x + c_2 = c_2 - \frac{1}{\sinh x} - \frac{\cosh x}{\sinh x} = c_2 - \frac{1+\cosh x}{\sinh x}, x \in \mathbb{R} - \{0\}, c_2 \in \mathbb{R}.$$

$$= \int \frac{dx}{1 + \frac{e^x + e^{-x}}{2}} = \int \frac{2 dx}{2 + e^x + e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ t \in (0; \infty) \end{array} \right. \right] = \int \frac{2 dt}{t(2+t+\frac{1}{t})} = \int \frac{2 dt}{t^2+2t+1} \\ = \int \frac{2 dt}{(t+1)^2} = \int 2(t+1)^{-2} dt = \frac{2(t+1)^{-1}}{-1} + c_3$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x} = \operatorname{tgh} \frac{x}{2} + c_1 = c_2 - \frac{1+\cosh x}{\sinh x} = -\frac{2}{e^x+1} + c_3$$

$$= \left[\begin{array}{l} \text{UHS:} \\ dx = \frac{2dt}{1-t^2} \end{array} \middle| \begin{array}{l} t = \operatorname{tgh} \frac{x}{2} \\ \cosh x = \frac{1+t^2}{1-t^2} \end{array} \middle| \begin{array}{l} x \in \mathbb{R} \\ t \in (-1; 1) \end{array} \right] = \int \frac{\frac{2dt}{1-t^2}}{1 + \frac{1+t^2}{1-t^2}} = \int \frac{2dt}{2} = \int dt = t + c_1 = \operatorname{tgh} \frac{x}{2} + c_1, \\ x \in \mathbb{R}, c_1 \in \mathbb{R}.$$

$$= \int \frac{dx}{2 \cosh^2 \frac{x}{2}} = \left[\begin{array}{l} \text{Subst. } t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \middle| \begin{array}{l} x \in \mathbb{R} \\ t \in \mathbb{R} \end{array} \right] = \int \frac{dt}{\cosh^2 t} = \operatorname{tgh} t + c_1 = \operatorname{tgh} \frac{x}{2} + c_1, x \in \mathbb{R}, c_1 \in \mathbb{R}.$$

$$= \int \frac{\cosh x dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\begin{array}{l} \text{Subst. } t = \sinh x \\ dt = \cosh x dx \end{array} \middle| \begin{array}{l} x \in (-\infty; 0), t \in (-\infty; 0) \\ x \in (0; \infty), t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x} \\ = \frac{1}{-t} - \operatorname{cotgh} x + c_2 = c_2 - \frac{1}{\sinh x} - \frac{\cosh x}{\sinh x} = c_2 - \frac{1+\cosh x}{\sinh x}, x \in \mathbb{R} - \{0\}, c_2 \in \mathbb{R}.$$

$$= \int \frac{dx}{1 + \frac{e^x + e^{-x}}{2}} = \int \frac{2 dx}{2 + e^x + e^{-x}} = \left[\begin{array}{l} \text{Subst. } t = e^x \\ e^{-x} = t^{-1} = \frac{1}{t} \\ dx = \frac{dt}{t} \end{array} \middle| \begin{array}{l} x = \ln t \\ t \in (0; \infty) \end{array} \right] = \int \frac{2 dt}{t(2+t+\frac{1}{t})} = \int \frac{2 dt}{t^2+2t+1} \\ = \int \frac{2 dt}{(t+1)^2} = \int 2(t+1)^{-2} dt = \frac{2(t+1)^{-1}}{-1} + c_3 = -\frac{2}{e^x+1} + c_3, x \in \mathbb{R}, c_3 \in \mathbb{R}.$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx$$

$$= \int \frac{1 - \cos 2x}{2} \, dx$$

$$= \int \sin x \cdot \sin x \, dx$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \sin^2 x & v' = 2 \sin x \cos x = \sin 2x \end{array} \right]$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx$$

$$= \int \frac{1 - \cos 2x}{2} \, dx = \int \left[\frac{1}{2} - \frac{\cos 2x}{2} \right] \, dx$$

$$= \int \sin x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin x \quad u' = \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right]$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \sin^2 x \quad v' = 2 \sin x \cos x = \sin 2x \end{array} \right] = x \sin^2 x - \int x \sin 2x \, dx$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 - \cos 2x}{2} \, dx = \int \left[\frac{1}{2} - \frac{\cos 2x}{2} \right] \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \sin x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin x \quad u' = \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right] = -\sin x \cdot \cos x + \int \cos^2 x \, dx$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \sin^2 x \quad v' = 2 \sin x \cos x = \sin 2x \end{array} \right] = x \sin^2 x - \int x \sin 2x \, dx = \left[\begin{array}{l} u' = \sin 2x \quad u = -\frac{\cos 2x}{2} \\ v = x \quad v' = 1 \end{array} \right]$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 - \cos 2x}{2} \, dx = \int \left[\frac{1}{2} - \frac{\cos 2x}{2} \right] \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \sin x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin x \quad u' = \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right] = -\sin x \cdot \cos x + \int \cos^2 x \, dx$$

$$= -\frac{\sin 2x}{2} + \int (1 - \sin^2 x) \, dx$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \sin^2 x \quad v' = 2 \sin x \cos x = \sin 2x \end{array} \right] = x \sin^2 x - \int x \sin 2x \, dx = \left[\begin{array}{l} u' = \sin 2x \quad u = -\frac{\cos 2x}{2} \\ v = x \quad v' = 1 \end{array} \right]$$

$$= x \sin^2 x - \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right]$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 - \cos 2x}{2} \, dx = \int \left[\frac{1}{2} - \frac{\cos 2x}{2} \right] \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \sin x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin x \quad u' = \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right] = -\sin x \cdot \cos x + \int \cos^2 x \, dx$$

$$= -\frac{\sin 2x}{2} + \int (1 - \sin^2 x) \, dx = -\frac{\sin 2x}{2} + x - \int \sin^2 x \, dx$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \sin^2 x \quad v' = 2 \sin x \cos x = \sin 2x \end{array} \right] = x \sin^2 x - \int x \sin 2x \, dx = \left[\begin{array}{l} u' = \sin 2x \quad u = -\frac{\cos 2x}{2} \\ v = x \quad v' = 1 \end{array} \right]$$

$$= x \sin^2 x - \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right] = \frac{2x \sin^2 x}{2} + \frac{x(\cos^2 x - \sin^2 x)}{2} - \frac{\sin 2x}{4} + c$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 - \cos 2x}{2} \, dx = \int \left[\frac{1}{2} - \frac{\cos 2x}{2} \right] \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \sin x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin x \quad u' = \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right] = -\sin x \cdot \cos x + \int \cos^2 x \, dx$$

$$= -\frac{\sin 2x}{2} + \int (1 - \sin^2 x) \, dx = -\frac{\sin 2x}{2} + x - \int \sin^2 x \, dx = -\frac{\sin 2x}{2} + x - I$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \sin^2 x \quad v' = 2 \sin x \cos x = \sin 2x \end{array} \right] = x \sin^2 x - \int x \sin 2x \, dx = \left[\begin{array}{l} u' = \sin 2x \quad u = -\frac{\cos 2x}{2} \\ v = x \quad v' = 1 \end{array} \right]$$

$$= x \sin^2 x - \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right] = \frac{2x \sin^2 x}{2} + \frac{x(\cos^2 x - \sin^2 x)}{2} - \frac{\sin 2x}{4} + c$$

$$= \frac{x \cos^2 x + x \sin^2 x}{2} - \frac{\sin 2x}{4} + c$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 - \cos 2x}{2} \, dx = \int \left[\frac{1}{2} - \frac{\cos 2x}{2} \right] \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \sin x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin x \quad u' = \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right] = -\sin x \cdot \cos x + \int \cos^2 x \, dx$$

$$= -\frac{\sin 2x}{2} + \int (1 - \sin^2 x) \, dx = -\frac{\sin 2x}{2} + x - \int \sin^2 x \, dx = -\frac{\sin 2x}{2} + x - I$$

$$= \left[\text{Rovnica } I = -\frac{\sin 2x}{2} + x - I \Rightarrow 2I = x - \frac{\sin 2x}{2} \right]$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \sin^2 x \quad v' = 2 \sin x \cos x = \sin 2x \end{array} \right] = x \sin^2 x - \int x \sin 2x \, dx = \left[\begin{array}{l} u' = \sin 2x \quad u = -\frac{\cos 2x}{2} \\ v = x \quad v' = 1 \end{array} \right]$$

$$= x \sin^2 x - \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right] = \frac{2x \sin^2 x}{2} + \frac{x(\cos^2 x - \sin^2 x)}{2} - \frac{\sin 2x}{4} + c$$

$$= \frac{x \cos^2 x + x \sin^2 x}{2} - \frac{\sin 2x}{4} + c = \frac{x}{2} - \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 - \cos 2x}{2} \, dx = \int \left[\frac{1}{2} - \frac{\cos 2x}{2} \right] \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$\begin{aligned}
 &= \int \sin x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin x \quad u' = \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right] = -\sin x \cdot \cos x + \int \cos^2 x \, dx \\
 &= -\frac{\sin 2x}{2} + \int (1 - \sin^2 x) \, dx = -\frac{\sin 2x}{2} + x - \int \sin^2 x \, dx = -\frac{\sin 2x}{2} + x - I \\
 &= \left[\text{Rovnica } I = -\frac{\sin 2x}{2} + x - I \Rightarrow 2I = x - \frac{\sin 2x}{2} \right] = \frac{x}{2} - \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.
 \end{aligned}$$

$$\begin{aligned}
 &= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \sin^2 x \quad v' = 2 \sin x \cos x = \sin 2x \end{array} \right] = x \sin^2 x - \int x \sin 2x \, dx = \left[\begin{array}{l} u' = \sin 2x \quad u = -\frac{\cos 2x}{2} \\ v = x \quad v' = 1 \end{array} \right] \\
 &= x \sin^2 x - \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right] = \frac{2x \sin^2 x}{2} + \frac{x(\cos^2 x - \sin^2 x)}{2} - \frac{\sin 2x}{4} + c \\
 &= \frac{x \cos^2 x + x \sin^2 x}{2} - \frac{\sin 2x}{4} + c = \frac{x}{2} - \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.
 \end{aligned}$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx$$

$$= \int \frac{1 + \cos 2x}{2} \, dx$$

$$= \int \cos x \cdot \cos x \, dx$$

$$= \left[\begin{array}{l} u' = 1 \\ v = \cos^2 x \end{array} \middle| \begin{array}{l} u = x \\ v' = -2 \cos x \sin x = -\sin 2x \end{array} \right]$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx$$

$$= \int \frac{1 + \cos 2x}{2} \, dx = \int \left[\frac{1}{2} + \frac{\cos 2x}{2} \right] \, dx$$

$$= \int \cos x \cdot \cos x \, dx = \left[\begin{array}{l} u = \cos x \quad u' = -\sin x \\ v' = \cos x \quad v = \sin x \end{array} \right]$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \cos^2 x \quad v' = -2 \cos x \sin x = -\sin 2x \end{array} \right] = x \cos^2 x + \int x \sin 2x \, dx$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 + \cos 2x}{2} \, dx = \int \left[\frac{1}{2} + \frac{\cos 2x}{2} \right] \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \cos x \cdot \cos x \, dx = \left[\begin{array}{l} u = \cos x \quad u' = -\sin x \\ v' = \cos x \quad v = \sin x \end{array} \right] = \cos x \cdot \sin x + \int \sin^2 x \, dx$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \cos^2 x \quad v' = -2 \cos x \sin x = -\sin 2x \end{array} \right] = x \cos^2 x + \int x \sin 2x \, dx = \left[\begin{array}{l} u' = \sin 2x \quad u = -\frac{\cos 2x}{2} \\ v = x \quad v' = 1 \end{array} \right]$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 + \cos 2x}{2} \, dx = \int \left[\frac{1}{2} + \frac{\cos 2x}{2} \right] \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \cos x \cdot \cos x \, dx = \left[\begin{array}{l} u = \cos x \quad u' = -\sin x \\ v' = \cos x \quad v = \sin x \end{array} \right] = \cos x \cdot \sin x + \int \sin^2 x \, dx$$

$$= \frac{\sin 2x}{2} + \int (1 - \cos^2 x) \, dx$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \cos^2 x \quad v' = -2 \cos x \sin x = -\sin 2x \end{array} \right] = x \cos^2 x + \int x \sin 2x \, dx = \left[\begin{array}{l} u' = \sin 2x \quad u = -\frac{\cos 2x}{2} \\ v = x \quad v' = 1 \end{array} \right]$$

$$= x \cos^2 x + \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right]$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 + \cos 2x}{2} \, dx = \int \left[\frac{1}{2} + \frac{\cos 2x}{2} \right] \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \cos x \cdot \cos x \, dx = \left[\begin{array}{l} u = \cos x \quad u' = -\sin x \\ v' = \cos x \quad v = \sin x \end{array} \right] = \cos x \cdot \sin x + \int \sin^2 x \, dx$$

$$= \frac{\sin 2x}{2} + \int (1 - \cos^2 x) \, dx = \frac{\sin 2x}{2} + x - \int \cos^2 x \, dx$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \cos^2 x \quad v' = -2 \cos x \sin x = -\sin 2x \end{array} \right] = x \cos^2 x + \int x \sin 2x \, dx = \left[\begin{array}{l} u' = \sin 2x \quad u = -\frac{\cos 2x}{2} \\ v = x \quad v' = 1 \end{array} \right]$$

$$= x \cos^2 x + \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right] = \frac{2x \cos^2 x}{2} - \frac{x(\cos^2 x - \sin^2 x)}{2} + \frac{\sin 2x}{4} + c$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 + \cos 2x}{2} \, dx = \int \left[\frac{1}{2} + \frac{\cos 2x}{2} \right] \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \cos x \cdot \cos x \, dx = \left[\begin{array}{l} u = \cos x \quad u' = -\sin x \\ v' = \cos x \quad v = \sin x \end{array} \right] = \cos x \cdot \sin x + \int \sin^2 x \, dx$$

$$= \frac{\sin 2x}{2} + \int (1 - \cos^2 x) \, dx = \frac{\sin 2x}{2} + x - \int \cos^2 x \, dx = \frac{\sin 2x}{2} + x - I$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \cos^2 x \quad v' = -2 \cos x \sin x = -\sin 2x \end{array} \right] = x \cos^2 x + \int x \sin 2x \, dx = \left[\begin{array}{l} u' = \sin 2x \quad u = -\frac{\cos 2x}{2} \\ v = x \quad v' = 1 \end{array} \right]$$

$$= x \cos^2 x + \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right] = \frac{2x \cos^2 x}{2} - \frac{x(\cos^2 x - \sin^2 x)}{2} + \frac{\sin 2x}{4} + c$$

$$= \frac{x \cos^2 x + x \sin^2 x}{2} + \frac{\sin 2x}{4} + c$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 + \cos 2x}{2} \, dx = \int \left[\frac{1}{2} + \frac{\cos 2x}{2} \right] \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \cos x \cdot \cos x \, dx = \left[\begin{array}{l} u = \cos x \quad u' = -\sin x \\ v' = \cos x \quad v = \sin x \end{array} \right] = \cos x \cdot \sin x + \int \sin^2 x \, dx$$

$$= \frac{\sin 2x}{2} + \int (1 - \cos^2 x) \, dx = \frac{\sin 2x}{2} + x - \int \cos^2 x \, dx = \frac{\sin 2x}{2} + x - I$$

$$= \left[\text{Rovnica } I = \frac{\sin 2x}{2} + x - I \Rightarrow 2I = x + \frac{\sin 2x}{2} \right]$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \cos^2 x \quad v' = -2 \cos x \sin x = -\sin 2x \end{array} \right] = x \cos^2 x + \int x \sin 2x \, dx = \left[\begin{array}{l} u' = \sin 2x \quad u = -\frac{\cos 2x}{2} \\ v = x \quad v' = 1 \end{array} \right]$$

$$= x \cos^2 x + \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right] = \frac{2x \cos^2 x}{2} - \frac{x(\cos^2 x - \sin^2 x)}{2} + \frac{\sin 2x}{4} + c$$

$$= \frac{x \cos^2 x + x \sin^2 x}{2} + \frac{\sin 2x}{4} + c = \frac{x}{2} + \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 + \cos 2x}{2} \, dx = \int \left[\frac{1}{2} + \frac{\cos 2x}{2} \right] \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \cos x \cdot \cos x \, dx = \left[\begin{array}{l} u = \cos x \quad u' = -\sin x \\ v' = \cos x \quad v = \sin x \end{array} \right] = \cos x \cdot \sin x + \int \sin^2 x \, dx$$

$$= \frac{\sin 2x}{2} + \int (1 - \cos^2 x) \, dx = \frac{\sin 2x}{2} + x - \int \cos^2 x \, dx = \frac{\sin 2x}{2} + x - I$$

$$= \left[\text{Rovnica } I = \frac{\sin 2x}{2} + x - I \Rightarrow 2I = x + \frac{\sin 2x}{2} \right] = \frac{x}{2} + \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} u' = 1 \quad u = x \\ v = \cos^2 x \quad v' = -2 \cos x \sin x = -\sin 2x \end{array} \right] = x \cos^2 x + \int x \sin 2x \, dx = \left[\begin{array}{l} u' = \sin 2x \quad u = -\frac{\cos 2x}{2} \\ v = x \quad v' = 1 \end{array} \right]$$

$$= x \cos^2 x + \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right] = \frac{2x \cos^2 x}{2} - \frac{x(\cos^2 x - \sin^2 x)}{2} + \frac{\sin 2x}{4} + c$$

$$= \frac{x \cos^2 x + x \sin^2 x}{2} + \frac{\sin 2x}{4} + c = \frac{x}{2} + \frac{\sin 2x}{4} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$\int \sin^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$= \int \sin^2 x \cdot \sin x \, dx$$

$$\int \sin^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sin^{2n} x \cdot \sin x \, dx$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx$$

$$\int \sin^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right]$$

$$\int \sin^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1 - \cos^2 x)^n \cdot \sin x \, dx$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right]$$

$$= - \int (1 - t^2) \, dt$$

$$\int \sin^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1 - \cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right]$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right]$$

$$= - \int (1 - t^2) \, dt = \int (t^2 - 1) \, dt$$

$$\int \sin^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1 - \cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right] = - \int (1 - t^2)^n \, dt$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right]$$

$$= - \int (1 - t^2) \, dt = \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c$$

$$\int \sin^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1 - \cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right] = - \int (1 - t^2)^n \, dt = - \int \left[\sum_{i=0}^n \binom{n}{i} 1^{n-i} (-t^2)^i \right] dt$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx = \frac{\cos^3 x}{3} - \cos x + c$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right]$$

$$= - \int (1 - t^2) \, dt = \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c = \frac{\cos^3 x}{3} - \cos x + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \sin^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1 - \cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right] = - \int (1 - t^2)^n \, dt = - \int \left[\sum_{i=0}^n \binom{n}{i} 1^{n-i} (-t^2)^i \right] dt$$

$$= - \int \left[\sum_{i=0}^n \binom{n}{i} (-1)^i t^{2i} \right] dt$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx = \frac{\cos^3 x}{3} - \cos x + c$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right]$$

$$= - \int (1 - t^2) \, dt = \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c = \frac{\cos^3 x}{3} - \cos x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \sin^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1 - \cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right] = - \int (1 - t^2)^n \, dt = - \int \left[\sum_{i=0}^n \binom{n}{i} 1^{n-i} (-t^2)^i \right] dt$$

$$= - \int \left[\sum_{i=0}^n \binom{n}{i} (-1)^i t^{2i} \right] dt = - \sum_{i=0}^n \binom{n}{i} (-1)^i \int t^{2i} \, dt$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx = \frac{\cos^3 x}{3} - \cos x + c$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right]$$

$$= - \int (1 - t^2) \, dt = \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c = \frac{\cos^3 x}{3} - \cos x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \sin^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1 - \cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right] = - \int (1 - t^2)^n \, dt = - \int \left[\sum_{i=0}^n \binom{n}{i} 1^{n-i} (-t^2)^i \right] dt$$

$$= - \int \left[\sum_{i=0}^n \binom{n}{i} (-1)^i t^{2i} \right] dt = - \sum_{i=0}^n \binom{n}{i} (-1)^i \int t^{2i} \, dt = \sum_{i=0}^n \binom{n}{i} (-1)^{i+1} \frac{t^{2i+1}}{2i+1} + c$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx = \frac{\cos^3 x}{3} - \cos x + c$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right]$$

$$= - \int (1 - t^2) \, dt = \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c = \frac{\cos^3 x}{3} - \cos x + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \sin^{2n+1} x \, dx = \sum_{i=0}^n \binom{n}{i} (-1)^{i+1} \frac{\cos^{2i+1} x}{2i+1} + c \quad n \in \mathbb{N}$$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1 - \cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (0 + k\pi; \pi + k\pi) \\ dt = -\sin x \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right] = - \int (1 - t^2)^n \, dt = - \int \left[\sum_{i=0}^n \binom{n}{i} 1^{n-i} (-t^2)^i \right] dt$$

$$= - \int \left[\sum_{i=0}^n \binom{n}{i} (-1)^i t^{2i} \right] dt = - \sum_{i=0}^n \binom{n}{i} (-1)^i \int t^{2i} \, dt = \sum_{i=0}^n \binom{n}{i} (-1)^{i+1} \frac{t^{2i+1}}{2i+1} + c$$

$$= - \binom{n}{0} \cos x + \binom{n}{1} \frac{\cos^3 x}{3} - \binom{n}{2} \frac{\cos^5 x}{5} + \dots + (-1)^{n+1} \binom{n}{n} \frac{\cos^{2n+1} x}{2n+1} + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 013

$$I_n = \int \sin^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

Riešené príklady – 013

$$I_n = \int \sin^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sin^{n-1} x \cdot \sin x \, dx$$

Riešené príklady – 013

$$I_n = \int \sin^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin^{n-1} x \quad | \quad u' = (n-1) \sin^{n-2} x \cdot \cos x \\ v' = \sin x \quad | \quad v = -\cos x \end{array} \right]$$

Riešené príklady – 013

$$I_n = \int \sin^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$\begin{aligned} &= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin^{n-1} x \quad | \quad u' = (n-1) \sin^{n-2} x \cdot \cos x \\ v' = \sin x \quad | \quad v = -\cos x \end{array} \right] \\ &= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \, dx \end{aligned}$$

Riešené príklady – 013

$$I_n = \int \sin^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin^{n-1} x \quad u' = (n-1) \sin^{n-2} x \cdot \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right]$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot (1 - \sin^2 x) \, dx$$

Riešené príklady – 013

$$I_n = \int \sin^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin^{n-1} x \quad u' = (n-1) \sin^{n-2} x \cdot \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right]$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot (1 - \sin^2 x) \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx - (n-1) \int \sin^n x \, dx$$

Riešené príklady – 013

$$I_n = \int \sin^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin^{n-1} x \quad u' = (n-1) \sin^{n-2} x \cdot \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right]$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot (1 - \sin^2 x) \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx - (n-1) \int \sin^n x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

Riešené príklady – 013

$$I_n = \int \sin^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin^{n-1} x \quad u' = (n-1) \sin^{n-2} x \cdot \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right]$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot (1 - \sin^2 x) \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx - (n-1) \int \sin^n x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n \\ \Rightarrow n I_n = I_n + (n-1) I_n = -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} \Rightarrow I_n = -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

Riešené príklady – 013

$$I_n = \int \sin^n x \, dx = -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx \quad n = 2, 3, 4, 5, \dots$$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin^{n-1} x \quad u' = (n-1) \sin^{n-2} x \cdot \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right]$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot (1 - \sin^2 x) \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx - (n-1) \int \sin^n x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n \\ \Rightarrow n I_n = I_n + (n-1) I_n = -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} \Rightarrow I_n = -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \quad x \in \mathbb{R},$$

Riešené príklady – 013

$$I_n = \int \sin^n x \, dx = -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx \quad n = 2, 3, 4, 5, \dots$$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l} u = \sin^{n-1} x \quad u' = (n-1) \sin^{n-2} x \cdot \cos x \\ v' = \sin x \quad v = -\cos x \end{array} \right]$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot (1 - \sin^2 x) \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx - (n-1) \int \sin^n x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n \\ \Rightarrow n I_n = I_n + (n-1) I_{n-2} - (n-1) I_n \Rightarrow I_n = -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \quad x \in \mathbb{R},$$

príčom $I_1 = \int \sin x \, dx = -\cos x + c$, $I_0 = \int \sin^0 x \, dx = \int dx = x + c$, $x \in \mathbb{R}$, $c \in \mathbb{R}$.

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx$$

$$\int \cos^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx$$

$$= \int \cos^2 x \cdot \cos x \, dx$$

$$\int \cos^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \cos^{2n} x \cdot \cos x \, dx$$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx$$

$$= \int \cos^2 x \cdot \cos x \, dx = \int (1 - \sin^2 x) \cdot \cos x \, dx$$

$$\int \cos^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \cos^{2n} x \cdot \cos x \, dx = \int (\cos^2 x)^n \cdot \cos x \, dx$$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx$$

$$= \int \cos^2 x \cdot \cos x \, dx = \int (1 - \sin^2 x) \cdot \cos x \, dx = \left[\begin{array}{l} \text{Subst. } t = \sin x \mid x \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = \cos x \, dx \mid t \in \langle -1; 1 \rangle, k \in \mathbb{Z} \end{array} \right]$$

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Riešené príklady – 014, 015

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$$= \int (1 - t^2) \, dt$$

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 $n \in \mathbb{N}$

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Riešené príklady – 014, 015

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$$= \int (1 - t^2) \, dt = t - \frac{t^3}{3} + c$$

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 $n \in \mathbb{N}$

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Riešené príklady – 014, 015

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$$= \int (1 - t^2) \, dt = t - \frac{t^3}{3} + c = \sin x - \frac{\sin^3 x}{3} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

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Riešené príklady – 014, 015

$$\int \cos^3 x \, dx = \sin x - \frac{\sin^3 x}{3} + c$$

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Riešené príklady – 014, 015

$$\int \cos^3 x \, dx = \sin x - \frac{\sin^3 x}{3} + c$$

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$$= \int (1 - t^2) \, dt = t - \frac{t^3}{3} + c = \sin x - \frac{\sin^3 x}{3} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

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 $n \in \mathbb{N}$

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$$= \int \left[\sum_{i=0}^n \binom{n}{i} (-1)^i t^{2i} \right] dt = \sum_{i=0}^n \binom{n}{i} (-1)^i \int t^{2i} \, dt$$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx = \sin x - \frac{\sin^3 x}{3} + c$$

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 $n \in \mathbb{N}$

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$$= \int \left[\sum_{i=0}^n \binom{n}{i} (-1)^i t^{2i} \right] dt = \sum_{i=0}^n \binom{n}{i} (-1)^i \int t^{2i} \, dt = \sum_{i=0}^n \binom{n}{i} (-1)^i \frac{t^{2i+1}}{2i+1} + c$$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx = \sin x - \frac{\sin^3 x}{3} + c$$

$$= \int \cos^2 x \cdot \cos x \, dx = \int (1 - \sin^2 x) \cdot \cos x \, dx = \left[\begin{array}{l} \text{Subst. } t = \sin x \mid x \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = \cos x \, dx \mid t \in \langle -1; 1 \rangle, k \in \mathbb{Z} \end{array} \right]$$

$$= \int (1 - t^2) \, dt = t - \frac{t^3}{3} + c = \sin x - \frac{\sin^3 x}{3} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \cos^{2n+1} x \, dx = \sum_{i=0}^n \binom{n}{i} (-1)^i \frac{\sin^{2i+1} x}{2i+1} + c \quad n \in \mathbb{N}$$

$$= \int \cos^{2n} x \cdot \cos x \, dx = \int (\cos^2 x)^n \cdot \cos x \, dx = \int (1 - \sin^2 x)^n \cdot \cos x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sin x \mid x \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = \cos x \, dx \mid t \in \langle -1; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int (1 - t^2)^n \, dt = \int \left[\sum_{i=0}^n \binom{n}{i} 1^{n-i} (-t^2)^i \right] dt$$

$$= \int \left[\sum_{i=0}^n \binom{n}{i} (-1)^i t^{2i} \right] dt = \sum_{i=0}^n \binom{n}{i} (-1)^i \int t^{2i} \, dt = \sum_{i=0}^n \binom{n}{i} (-1)^i \frac{t^{2i+1}}{2i+1} + c$$

$$= \binom{n}{0} \sin x - \binom{n}{1} \frac{\sin^3 x}{3} + \binom{n}{2} \frac{\sin^5 x}{5} - \dots + (-1)^n \binom{n}{n} \frac{\sin^{2n+1} x}{2n+1} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 016

$$I_n = \int \cos^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

Riešené príklady – 016

$$I_n = \int \cos^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \cos^{n-1} x \cdot \cos x \, dx$$

Riešené príklady – 016

$$I_n = \int \cos^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \cos^{n-1} x \cdot \cos x \, dx = \left[\begin{array}{l} u = \cos^{n-1} x \quad | \quad u' = -(n-1) \cos^{n-2} x \cdot \sin x \\ v' = \cos x \quad | \quad v = \sin x \end{array} \right]$$

Riešené príklady – 016

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$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

Riešené príklady – 016

$$I_n = \int \cos^n x \, dx$$

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$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot (1 - \cos^2 x) \, dx$$

Riešené príklady – 016

$$I_n = \int \cos^n x \, dx$$

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$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot (1 - \cos^2 x) \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx - (n-1) \int \cos^n x \, dx$$

Riešené príklady – 016

$$I_n = \int \cos^n x \, dx$$

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$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx - (n-1) \int \cos^n x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

Riešené príklady – 016

$$I_n = \int \cos^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \cos^{n-1} x \cdot \cos x \, dx = \left[\begin{array}{l} u = \cos^{n-1} x \quad | \quad u' = -(n-1) \cos^{n-2} x \cdot \sin x \\ v' = \cos x \quad | \quad v = \sin x \end{array} \right]$$

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$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot (1 - \cos^2 x) \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx - (n-1) \int \cos^n x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} - (n-1) I_n \\ \Rightarrow n I_n = I_n + (n-1) I_n = \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} \Rightarrow I_n = \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

Riešené príklady – 016

$$I_n = \int \cos^n x \, dx = \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx \quad n = 2, 3, 4, 5, \dots$$

$$= \int \cos^{n-1} x \cdot \cos x \, dx = \left[\begin{array}{l} u = \cos^{n-1} x \quad u' = -(n-1) \cos^{n-2} x \cdot \sin x \\ v' = \cos x \quad v = \sin x \end{array} \right]$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot (1 - \cos^2 x) \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx - (n-1) \int \cos^n x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

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$$= \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \quad x \in \mathbb{R},$$

Riešené príklady – 016

$$I_n = \int \cos^n x \, dx = \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx \quad n = 2, 3, 4, 5, \dots$$

$$= \int \cos^{n-1} x \cdot \cos x \, dx = \left[\begin{array}{l} u = \cos^{n-1} x \quad u' = -(n-1) \cos^{n-2} x \cdot \sin x \\ v' = \cos x \quad v = \sin x \end{array} \right]$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot (1 - \cos^2 x) \, dx$$

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$$= \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \quad x \in \mathbb{R},$$

príčom $I_1 = \int \cos x \, dx = \sin x + c$, $I_0 = \int \cos^0 x \, dx = \int dx = x + c$, $x \in \mathbb{R}$, $c \in \mathbb{R}$.

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx$$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx$$

$$= \int \frac{\cosh 2x - 1}{2} \, dx$$

$$= \int \sinh x \cdot \sinh x \, dx$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^2 \, dx$$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx$$

$$= \int \frac{\cosh 2x - 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} - \frac{1}{2} \right] dx$$

$$= \int \sinh x \cdot \cosh x \, dx = \begin{cases} u = \sinh x & u' = \cosh x \\ v' = \cosh x & v = \sinh x \end{cases}$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^2 dx = \int \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4} dx$$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c$$

$$= \int \frac{\cosh 2x - 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} - \frac{1}{2} \right] \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \sinh x \cdot \sinh x \, dx = \left[\begin{array}{l} u = \sinh x \quad u' = \cosh x \\ v' = \sinh x \quad v = \cosh x \end{array} \right] = \sinh x \cdot \cosh x - \int \cosh^2 x \, dx$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^2 \, dx = \int \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int [e^{2x} - 2 + e^{-2x}] \, dx$$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c$$

$$= \int \frac{\cosh 2x - 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} - \frac{1}{2} \right] \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \sinh x \cdot \sinh x \, dx = \left[\begin{array}{l} u = \sinh x \quad u' = \cosh x \\ v' = \sinh x \quad v = \cosh x \end{array} \right] = \sinh x \cdot \cosh x - \int \cosh^2 x \, dx$$

$$= \frac{\sinh 2x}{2} - \int (\sinh^2 x + 1) \, dx$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^2 \, dx = \int \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int [e^{2x} - 2 + e^{-2x}] \, dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} - 2x + \frac{e^{-2x}}{-2} \right] + c$$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c = \frac{e^{2x} - e^{-2x} - 4x}{8} + c$$

$$= \int \frac{\cosh 2x - 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} - \frac{1}{2} \right] \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \sinh x \cdot \sinh x \, dx = \left[\begin{array}{l} u = \sinh x \quad u' = \cosh x \\ v' = \sinh x \quad v = \cosh x \end{array} \right] = \sinh x \cdot \cosh x - \int \cosh^2 x \, dx$$

$$= \frac{\sinh 2x}{2} - \int (\sinh^2 x + 1) \, dx = \frac{\sinh 2x}{2} - \int \sinh^2 x \, dx - x$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^2 \, dx = \int \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int [e^{2x} - 2 + e^{-2x}] \, dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} - 2x + \frac{e^{-2x}}{-2} \right] + c = \frac{e^{2x} - e^{-2x} - 4x}{8} + c$$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c = \frac{e^{2x} - e^{-2x} - 4x}{8} + c$$

$$= \int \frac{\cosh 2x - 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} - \frac{1}{2} \right] \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \sinh x \cdot \sinh x \, dx = \int \begin{matrix} u = \sinh x & u' = \cosh x \\ v' = \sinh x & v = \cosh x \end{matrix} = \sinh x \cdot \cosh x - \int \cosh^2 x \, dx$$

$$= \frac{\sinh 2x}{2} - \int (\sinh^2 x + 1) \, dx = \frac{\sinh 2x}{2} - \int \sinh^2 x \, dx - x = \frac{\sinh 2x}{2} - I - x$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^2 \, dx = \int \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int [e^{2x} - 2 + e^{-2x}] \, dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} - 2x + \frac{e^{-2x}}{-2} \right] + c = \frac{e^{2x} - e^{-2x} - 4x}{8} + c = \frac{1}{4} \cdot \frac{e^{2x} - e^{-2x}}{2} - \frac{x}{2} + c$$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c = \frac{e^{2x} - e^{-2x} - 4x}{8} + c$$

$$= \int \frac{\cosh 2x - 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} - \frac{1}{2} \right] \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \sinh x \cdot \cosh x \, dx = \left[\begin{array}{l} u = \sinh x \quad u' = \cosh x \\ v' = \sinh x \quad v = \cosh x \end{array} \right] = \sinh x \cdot \cosh x - \int \cosh^2 x \, dx$$

$$= \frac{\sinh 2x}{2} - \int (\sinh^2 x + 1) \, dx = \frac{\sinh 2x}{2} - \int \sinh^2 x \, dx - x = \frac{\sinh 2x}{2} - I - x$$

$$= \left[\text{Rovnica } I = \frac{\sinh 2x}{2} - I - x \Rightarrow 2I = \frac{\sinh 2x}{2} - x \right]$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^2 \, dx = \int \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int [e^{2x} - 2 + e^{-2x}] \, dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} - 2x + \frac{e^{-2x}}{-2} \right] + c = \frac{e^{2x} - e^{-2x} - 4x}{8} + c = \frac{1}{4} \cdot \frac{e^{2x} - e^{-2x}}{2} - \frac{x}{2} + c$$

$$= \frac{\sinh 2x}{4} - \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c = \frac{e^{2x} - e^{-2x} - 4x}{8} + c$$

$$= \int \frac{\cosh 2x - 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} - \frac{1}{2} \right] \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \sinh x \cdot \sinh x \, dx = \left[\begin{array}{l} u = \sinh x \quad u' = \cosh x \\ v' = \sinh x \quad v = \cosh x \end{array} \right] = \sinh x \cdot \cosh x - \int \cosh^2 x \, dx$$

$$= \frac{\sinh 2x}{2} - \int (\sinh^2 x + 1) \, dx = \frac{\sinh 2x}{2} - \int \sinh^2 x \, dx - x = \frac{\sinh 2x}{2} - I - x$$

$$= \left[\text{Rovnica } I = \frac{\sinh 2x}{2} - I - x \Rightarrow 2I = \frac{\sinh 2x}{2} - x \right] = \frac{\sinh 2x}{4} - \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^2 \, dx = \int \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int [e^{2x} - 2 + e^{-2x}] \, dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} - 2x + \frac{e^{-2x}}{-2} \right] + c = \frac{e^{2x} - e^{-2x} - 4x}{8} + c = \frac{1}{4} \cdot \frac{e^{2x} - e^{-2x}}{2} - \frac{x}{2} + c$$

$$= \frac{\sinh 2x}{4} - \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx$$

$$= \int \frac{\cosh 2x + 1}{2} \, dx$$

$$= \int \cosh x \cdot \cosh x \, dx$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 \, dx$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx$$

$$= \int \frac{\cosh 2x + 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} + \frac{1}{2} \right] dx$$

$$= \int \cosh x \cdot \cosh x \, dx = \left[\begin{array}{l} u = \cosh x \quad u' = \sinh x \\ v' = \cosh x \quad v = \sinh x \end{array} \right]$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 dx = \int \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} dx$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c$$

$$= \int \frac{\cosh 2x + 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} + \frac{1}{2} \right] \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \cosh x \cdot \cosh x \, dx = \left[\begin{array}{l} u = \cosh x \quad u' = \sinh x \\ v' = \cosh x \quad v = \sinh x \end{array} \right] = \cosh x \cdot \sinh x - \int \sinh^2 x \, dx$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 \, dx = \int \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int [e^{2x} + 2 + e^{-2x}] \, dx$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c$$

$$= \int \frac{\cosh 2x + 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} + \frac{1}{2} \right] \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \cosh x \cdot \cosh x \, dx = \left[\begin{array}{l} u = \cosh x \quad u' = \sinh x \\ v' = \cosh x \quad v = \sinh x \end{array} \right] = \cosh x \cdot \sinh x - \int \sinh^2 x \, dx$$

$$= \frac{\sinh 2x}{2} - \int (\cosh^2 x - 1) \, dx$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 \, dx = \int \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int [e^{2x} + 2 + e^{-2x}] \, dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2} \right] + c$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c = \frac{e^{2x} - e^{-2x} + 4x}{8} + c$$

$$= \int \frac{\cosh 2x + 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} + \frac{1}{2} \right] \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \cosh x \cdot \cosh x \, dx = \left[\begin{array}{l} u = \cosh x \quad u' = \sinh x \\ v' = \cosh x \quad v = \sinh x \end{array} \right] = \cosh x \cdot \sinh x - \int \sinh^2 x \, dx$$

$$= \frac{\sinh 2x}{2} - \int (\cosh^2 x - 1) \, dx = \frac{\sinh 2x}{2} - \int \cosh^2 x \, dx + x$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 \, dx = \int \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int [e^{2x} + 2 + e^{-2x}] \, dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2} \right] + c = \frac{e^{2x} - e^{-2x} + 4x}{8} + c$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c = \frac{e^{2x} - e^{-2x} + 4x}{8} + c$$

$$= \int \frac{\cosh 2x + 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} + \frac{1}{2} \right] \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \cosh x \cdot \cosh x \, dx = \left[\begin{array}{l} u = \cosh x \quad u' = \sinh x \\ v' = \cosh x \quad v = \sinh x \end{array} \right] = \cosh x \cdot \sinh x - \int \sinh^2 x \, dx$$

$$= \frac{\sinh 2x}{2} - \int (\cosh^2 x - 1) \, dx = \frac{\sinh 2x}{2} - \int \cosh^2 x \, dx + x = \frac{\sinh 2x}{2} - I + x$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 \, dx = \int \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int [e^{2x} + 2 + e^{-2x}] \, dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2} \right] + c = \frac{e^{2x} - e^{-2x} + 4x}{8} + c = \frac{1}{4} \cdot \frac{e^{2x} - e^{-2x}}{2} + \frac{x}{2} + c$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c = \frac{e^{2x} - e^{-2x} + 4x}{8} + c$$

$$= \int \frac{\cosh 2x + 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} + \frac{1}{2} \right] \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \cosh x \cdot \cosh x \, dx = \left[\begin{array}{l} u = \cosh x \quad u' = \sinh x \\ v' = \cosh x \quad v = \sinh x \end{array} \right] = \cosh x \cdot \sinh x - \int \sinh^2 x \, dx$$

$$= \frac{\sinh 2x}{2} - \int (\cosh^2 x - 1) \, dx = \frac{\sinh 2x}{2} - \int \cosh^2 x \, dx + x = \frac{\sinh 2x}{2} - I + x$$

$$= \left[\text{Rovnica } I = \frac{\sinh 2x}{2} - I + x \Rightarrow 2I = \frac{\sinh 2x}{2} + x \right]$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 \, dx = \int \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int [e^{2x} + 2 + e^{-2x}] \, dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2} \right] + c = \frac{e^{2x} - e^{-2x} + 4x}{8} + c = \frac{1}{4} \cdot \frac{e^{2x} - e^{-2x}}{2} + \frac{x}{2} + c$$

$$= \frac{\sinh 2x}{4} + \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c = \frac{e^{2x} - e^{-2x} + 4x}{8} + c$$

$$= \int \frac{\cosh 2x + 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} + \frac{1}{2} \right] \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \cosh x \cdot \cosh x \, dx = \left[\begin{array}{l} u = \cosh x \quad u' = \sinh x \\ v' = \cosh x \quad v = \sinh x \end{array} \right] = \cosh x \cdot \sinh x - \int \sinh^2 x \, dx$$

$$= \frac{\sinh 2x}{2} - \int (\cosh^2 x - 1) \, dx = \frac{\sinh 2x}{2} - \int \cosh^2 x \, dx + x = \frac{\sinh 2x}{2} - I + x$$

$$= \left[\text{Rovnica } I = \frac{\sinh 2x}{2} - I + x \Rightarrow 2I = \frac{\sinh 2x}{2} + x \right] = \frac{\sinh 2x}{4} + \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 \, dx = \int \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int [e^{2x} + 2 + e^{-2x}] \, dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2} \right] + c = \frac{e^{2x} - e^{-2x} + 4x}{8} + c = \frac{1}{4} \cdot \frac{e^{2x} - e^{-2x}}{2} + \frac{x}{2} + c$$

$$= \frac{\sinh 2x}{4} + \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx$$

$$= \int \sinh^2 x \cdot \sinh x \, dx$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^x e^{-2x} - e^{-3x}}{8} dx$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right]$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^xe^{-2x} - e^{-3x}}{8} dx$$
$$= \int \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8} dx$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right]$$

$$= \int (t^2 - 1) \, dt$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^x e^{-2x} - e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} - 3e^x + 3e^{-x} - e^{-3x}] dx$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right]$$

$$= \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^xe^{-2x} - e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} - 3e^x + 3e^{-x} - e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3} \right] + c$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx = \frac{\cosh^3 x}{3} - \cosh x + c$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right]$$

$$= \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c = \frac{\cosh^3 x}{3} - \cosh x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^xe^{-2x} - e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} - 3e^x + 3e^{-x} - e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3} \right] + c = \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x - 3e^{-x} + \frac{e^{-3x}}{3} \right] + c$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx = \frac{\cosh^3 x}{3} - \cosh x + c = \frac{e^{3x} - 9e^x - 9e^{-x} + e^{-3x}}{24} + c$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right]$$

$$= \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c = \frac{\cosh^3 x}{3} - \cosh x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^xe^{-2x} - e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} - 3e^x + 3e^{-x} - e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3} \right] + c = \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x - 3e^{-x} + \frac{e^{-3x}}{3} \right] + c$$

$$= \frac{e^{3x} - 9e^x - 9e^{-x} + e^{-3x}}{24} + c$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx = \frac{\cosh^3 x}{3} - \cosh x + c = \frac{e^{3x} - 9e^x - 9e^{-x} + e^{-3x}}{24} + c$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right]$$

$$= \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c = \frac{\cosh^3 x}{3} - \cosh x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^xe^{-2x} - e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} - 3e^x + 3e^{-x} - e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3} \right] + c = \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x - 3e^{-x} + \frac{e^{-3x}}{3} \right] + c$$

$$= \frac{e^{3x} - 9e^x - 9e^{-x} + e^{-3x}}{24} + c = \frac{e^{3x} + 3e^x - 12e^x + 3e^{-x} - 12e^{-x} + e^{-3x}}{24} + c$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx = \frac{\cosh^3 x}{3} - \cosh x + c = \frac{e^{3x} - 9e^x - 9e^{-x} + e^{-3x}}{24} + c$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right]$$

$$= \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c = \frac{\cosh^3 x}{3} - \cosh x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^xe^{-2x} - e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} - 3e^x + 3e^{-x} - e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3} \right] + c = \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x - 3e^{-x} + \frac{e^{-3x}}{3} \right] + c$$

$$= \frac{e^{3x} - 9e^x - 9e^{-x} + e^{-3x}}{24} + c = \frac{e^{3x} + 3e^x - 12e^x + 3e^{-x} - 12e^{-x} + e^{-3x}}{24} + c$$

$$= \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{3 \cdot 8} - \frac{12e^x + 12e^{-x}}{24} + c$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx = \frac{\cosh^3 x}{3} - \cosh x + c = \frac{e^{3x} - 9e^x - 9e^{-x} + e^{-3x}}{24} + c$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right]$$

$$= \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c = \frac{\cosh^3 x}{3} - \cosh x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^xe^{-2x} - e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} - 3e^x + 3e^{-x} - e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3} \right] + c = \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x - 3e^{-x} + \frac{e^{-3x}}{3} \right] + c$$

$$= \frac{e^{3x} - 9e^x - 9e^{-x} + e^{-3x}}{24} + c = \frac{e^{3x} + 3e^x - 12e^x + 3e^{-x} - 12e^{-x} + e^{-3x}}{24} + c$$

$$= \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{3 \cdot 8} - \frac{12e^x + 12e^{-x}}{24} + c = \frac{1}{3} \cdot \left[\frac{e^{2x} + e^{-2x}}{2} \right]^3 - \frac{e^x + e^{-x}}{2} + c$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx = \frac{\cosh^3 x}{3} - \cosh x + c = \frac{e^{3x} - 9e^x - 9e^{-x} + e^{-3x}}{24} + c$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right]$$

$$= \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c = \frac{\cosh^3 x}{3} - \cosh x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^xe^{-2x} - e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} - 3e^x + 3e^{-x} - e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3} \right] + c = \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x - 3e^{-x} + \frac{e^{-3x}}{3} \right] + c$$

$$= \frac{e^{3x} - 9e^x - 9e^{-x} + e^{-3x}}{24} + c = \frac{e^{3x} + 3e^x - 12e^x + 3e^{-x} - 12e^{-x} + e^{-3x}}{24} + c$$

$$= \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{3 \cdot 8} - \frac{12e^x + 12e^{-x}}{24} + c = \frac{1}{3} \cdot \left[\frac{e^{2x} + e^{-2x}}{2} \right]^3 - \frac{e^x + e^{-x}}{2} + c$$

$$= \frac{\cosh^3 x}{3} - \cosh x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right]$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \int (t^2 - 1)^n \, dt$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$\begin{aligned} &= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx \\ &= \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \int (t^2 - 1)^n \, dt = \int \left[\sum_{i=0}^n \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt \end{aligned}$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \int (t^2 - 1)^n dt = \int \left[\sum_{i=0}^n \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^n \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i} \right] dt$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \int (t^2 - 1)^n dt = \int \left[\sum_{i=0}^n \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^n \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i} \right] dt = \sum_{i=0}^n \binom{n}{i} \frac{(-1)^n}{(-1)^i} \int t^{2i} dt$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \int (t^2 - 1)^n dt = \int \left[\sum_{i=0}^n \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^n \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i} \right] dt = \sum_{i=0}^n \binom{n}{i} \frac{(-1)^n}{(-1)^i} \int t^{2i} dt = \sum_{i=0}^n \binom{n}{i} \frac{(-1)^n}{(-1)^i} \frac{t^{2i+1}}{2i+1} + c$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \int (t^2 - 1)^n dt = \int \left[\sum_{i=0}^n \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^n \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i} \right] dt = \sum_{i=0}^n \binom{n}{i} \frac{(-1)^n}{(-1)^i} \int t^{2i} dt = \sum_{i=0}^n \binom{n}{i} \frac{(-1)^n}{(-1)^i} \frac{t^{2i+1}}{2i+1} + c$$

$$= \left[\frac{1}{(-1)^i} = \frac{1}{(-1)^i} \cdot \frac{(-1)^i}{(-1)^i} = \frac{(-1)^i}{(-1)^{2i}} = (-1)^i \right]$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \int (t^2 - 1)^n dt = \int \left[\sum_{i=0}^n \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^n \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i} \right] dt = \sum_{i=0}^n \binom{n}{i} \frac{(-1)^n}{(-1)^i} \int t^{2i} dt = \sum_{i=0}^n \binom{n}{i} \frac{(-1)^n}{(-1)^i} \frac{t^{2i+1}}{2i+1} + c$$

$$= \left[\frac{1}{(-1)^i} = \frac{1}{(-1)^i} \cdot \frac{(-1)^i}{(-1)^i} = \frac{(-1)^i}{(-1)^{2i}} = (-1)^i \right] = (-1)^n \sum_{i=0}^n \binom{n}{i} (-1)^i \frac{t^{2i+1}}{2i+1} + c$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx = (-1)^n \sum_{i=0}^n \binom{n}{i} (-1)^i \frac{\cosh^{2i+1} x}{2i+1} + c \quad n \in \mathbb{N}$$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \int (t^2 - 1)^n dt = \int \left[\sum_{i=0}^n \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^n \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i} \right] dt = \sum_{i=0}^n \binom{n}{i} \frac{(-1)^n}{(-1)^i} \int t^{2i} dt = \sum_{i=0}^n \binom{n}{i} \frac{(-1)^n}{(-1)^i} \frac{t^{2i+1}}{2i+1} + c$$

$$= \left[\frac{1}{(-1)^i} = \frac{1}{(-1)^i} \cdot \frac{(-1)^i}{(-1)^i} = \frac{(-1)^i}{(-1)^{2i}} = (-1)^i \right] = (-1)^n \sum_{i=0}^n \binom{n}{i} (-1)^i \frac{t^{2i+1}}{2i+1} + c$$

$$= (-1)^n \sum_{i=0}^n \binom{n}{i} (-1)^i \frac{\cosh^{2i+1} x}{2i+1} + c$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx = (-1)^n \sum_{i=0}^n \binom{n}{i} (-1)^i \frac{\cosh^{2i+1} x}{2i+1} + c \quad n \in \mathbb{N}$$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cosh x \mid x \in (-\infty; 0), t \in (1; \infty) \\ dt = \sinh x \, dx \mid x \in (0; \infty), t \in (1; \infty) \end{array} \right] = \int (t^2 - 1)^n dt = \int \left[\sum_{i=0}^n \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^n \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i} \right] dt = \sum_{i=0}^n \binom{n}{i} \frac{(-1)^n}{(-1)^i} \int t^{2i} dt = \sum_{i=0}^n \binom{n}{i} \frac{(-1)^n}{(-1)^i} \frac{t^{2i+1}}{2i+1} + c$$

$$= \left[\frac{1}{(-1)^i} = \frac{1}{(-1)^i} \cdot \frac{(-1)^i}{(-1)^i} = \frac{(-1)^i}{(-1)^{2i}} = (-1)^i \right] = (-1)^n \sum_{i=0}^n \binom{n}{i} (-1)^i \frac{t^{2i+1}}{2i+1} + c$$

$$= (-1)^n \sum_{i=0}^n \binom{n}{i} (-1)^i \frac{\cosh^{2i+1} x}{2i+1} + c$$

$$= (-1)^n \left[\binom{n}{0} \cosh x - \binom{n}{1} \frac{\cosh^3 x}{3} + \binom{n}{2} \frac{\cosh^5 x}{5} + \dots + (-1)^n \binom{n}{n} \frac{\cosh^{2n+1} x}{2n+1} \right] + c,$$

$x \in \mathbb{R}, c \in \mathbb{R}.$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l} u = \sinh^{n-1} x \quad | \quad u' = (n-1) \sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x \quad | \quad v = \cosh x \end{array} \right]$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l} u = \sinh^{n-1} x \quad | \quad u' = (n-1) \sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x \quad | \quad v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x \, dx$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l} u = \sinh^{n-1} x \quad | \quad u' = (n-1) \sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x \quad | \quad v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot (\sinh^2 x + 1) \, dx$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l} u = \sinh^{n-1} x \quad u' = (n-1) \sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x \quad v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot (\sinh^2 x + 1) \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^n x \, dx - (n-1) \int \sinh^{n-2} x \, dx$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l} u = \sinh^{n-1} x \quad | \quad u' = (n-1) \sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x \quad | \quad v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot (\sinh^2 x + 1) \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^n x \, dx - (n-1) \int \sinh^{n-2} x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1)I_n - (n-1)I_{n-2}$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l} u = \sinh^{n-1} x \quad | \quad u' = (n-1) \sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x \quad | \quad v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot (\sinh^2 x + 1) \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^n x \, dx - (n-1) \int \sinh^{n-2} x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2}$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2} \\ \Rightarrow n I_n = I_n + (n-1) I_n = \cosh x \cdot \sinh^{n-1} x - (n-1) I_{n-2} \Rightarrow I_n = \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} I_{n-2} \end{array} \right]$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx = \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} \int \sinh^{n-2} x \, dx \quad n = 2, 3, 4, 5, \dots$$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l} u = \sinh^{n-1} x \quad | \quad u' = (n-1) \sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x \quad | \quad v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot (\sinh^2 x + 1) \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^n x \, dx - (n-1) \int \sinh^{n-2} x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2}$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2} \\ \Rightarrow n I_n = I_n + (n-1) I_n = \cosh x \cdot \sinh^{n-1} x - (n-1) I_{n-2} \Rightarrow I_n = \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} I_{n-2}, \quad x \in \mathbb{R},$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx = \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} \int \sinh^{n-2} x \, dx \quad n = 2, 3, 4, 5, \dots$$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l} u = \sinh^{n-1} x \quad | \quad u' = (n-1) \sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x \quad | \quad v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot (\sinh^2 x + 1) \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^n x \, dx - (n-1) \int \sinh^{n-2} x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2}$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2} \\ \Rightarrow n I_n = I_n + (n-1) I_n = \cosh x \cdot \sinh^{n-1} x - (n-1) I_{n-2} \Rightarrow I_n = \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} I_{n-2}, \quad x \in \mathbb{R},$$

pričom $I_1 = \int \sinh x \, dx = \cosh x + c$, $I_0 = \int \sinh^0 x \, dx = \int dx = x + c$, $x \in \mathbb{R}$, $c \in \mathbb{R}$.

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$



 $n \in \mathbb{N}$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

 $n \in \mathbb{N}$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx$$






Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

 $n \in \mathbb{N}$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} dx$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

 $n \in \mathbb{N}$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x - e^{-x})^n dx$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

 $n \in \mathbb{N}$

$$\begin{aligned} &= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x - e^{-x})^n dx \\ &= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx \end{aligned}$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

 $n \in \mathbb{N}$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x - e^{-x})^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-ix} (-1)^i e^{-ix} \right] dx$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx$$

 $n \in \mathbb{N}$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x - e^{-x})^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-ix} (-1)^i e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (-1)^i e^{nx-2ix} \right] dx$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^n \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right] \quad n \in \mathbb{N}$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x - e^{-x})^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-ix} (-1)^i e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (-1)^i e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^n \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right], \quad x \in \mathbb{R},$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^n \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right] \quad n \in \mathbb{N}$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x - e^{-x})^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-ix} (-1)^i e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (-1)^i e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^n \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right], \quad x \in \mathbb{R},$$

pričom $\int dx = x + c$ pre $i = \frac{n}{2}$, $\int e^{(n-2i)x} dx = \frac{e^{(n-2i)x}}{n-2i} + c$ pre $i \neq \frac{n}{2}$, $c \in \mathbb{R}$.

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^n \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right] \quad n \in \mathbb{N}$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x - e^{-x})^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-ix} (-1)^i e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (-1)^i e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^n \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right], \quad x \in \mathbb{R},$$

pričom $\int dx = x + c$ pre $i = \frac{n}{2}$, $\int e^{(n-2i)x} dx = \frac{e^{(n-2i)x}}{n-2i} + c$ pre $i \neq \frac{n}{2}$, $c \in \mathbb{R}$.

$$n = 2k+1, \quad k=0,1,2,\dots \text{ (nepárne)}$$

$$n = 2k, \quad k=1,2,3,\dots \text{ (párne)}$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^n \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right] \quad n \in \mathbb{N}$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x - e^{-x})^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-ix} (-1)^i e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (-1)^i e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^n \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right], \quad x \in \mathbb{R},$$

pričom $\int dx = x + c$ pre $i = \frac{n}{2}$, $\int e^{(n-2i)x} dx = \frac{e^{(n-2i)x}}{n-2i} + c$ pre $i \neq \frac{n}{2}$, $c \in \mathbb{R}$.

$$n = 2k+1, \quad k=0,1,2,\dots \text{ (nepárne)}$$

$$I_n = \frac{1}{2^n} \left[\frac{e^{nx}}{n} - n \frac{e^{(n-2)x}}{n-2} + \dots + (-1)^k \binom{n}{k} e^x + (-1)^{k+1} \binom{n}{k+1} \frac{e^{-x}}{-1} + \dots + n \frac{e^{(2-n)x}}{2-n} - \frac{e^{-nx}}{-n} \right] + c.$$

$$n = 2k, \quad k=1,2,3,\dots \text{ (párne)}$$

$$I_n = \frac{1}{2^n} \left[\frac{e^{nx}}{n} - n \frac{e^{(n-2)x}}{n-2} + \dots + (-1)^{k-1} \binom{n}{k-1} \frac{e^{2x}}{2} + (-1)^k \binom{n}{k} x + (-1)^{k+1} \binom{n}{k+1} \frac{e^{-2x}}{-2} + \dots - n \frac{e^{(2-n)x}}{2-n} + \frac{e^{-nx}}{-n} \right] + c.$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx$$

$$= \int \cosh^2 x \cdot \cosh x \, dx$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^xe^{-2x} + e^{-3x}}{8} dx$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx = \begin{matrix} \text{Subst. } t = \sinh x & | & x \in \mathbb{R} \\ dt = \cosh x \, dx & | & t \in \mathbb{R} \end{matrix}$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^xe^{-2x} + e^{-3x}}{8} dx$$
$$= \int \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8} dx$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \sinh x \mid x \in \mathbb{R} \\ dt = \cosh x \, dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int (t^2 + 1) \, dt$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^xe^{-2x} + e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} + 3e^x + 3e^{-x} + e^{-3x}] dx$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx = \begin{matrix} \text{Subst. } t = \sinh x & | & x \in \mathbb{R} \\ dt = \cosh x \, dx & | & t \in \mathbb{R} \end{matrix}$$

$$= \int (t^2 + 1) \, dt = \frac{t^3}{3} + t + c$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^xe^{-2x} + e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} + 3e^x + 3e^{-x} + e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x + \frac{3e^{-x}}{-1} + \frac{e^{-3x}}{-3} \right] + c$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx = \frac{\sinh^3 x}{3} + \sinh x + c$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \sinh x \mid x \in \mathbb{R} \\ dt = \cosh x \, dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int (t^2 + 1) \, dt = \frac{t^3}{3} + t + c = \frac{\sinh^3 x}{3} + \sinh x + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^xe^{-2x} + e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} + 3e^x + 3e^{-x} + e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x + \frac{3e^{-x}}{-1} + \frac{e^{-3x}}{-3} \right] + c = \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x - 3e^{-x} - \frac{e^{-3x}}{3} \right] + c$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx = \frac{\sinh^3 x}{3} + \sinh x + c = \frac{e^{3x} + 9e^x - 9e^{-x} - e^{-3x}}{24} + c$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \sinh x \mid x \in \mathbb{R} \\ dt = \cosh x \, dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int (t^2 + 1) \, dt = \frac{t^3}{3} + t + c = \frac{\sinh^3 x}{3} + \sinh x + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^xe^{-2x} + e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} + 3e^x + 3e^{-x} + e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x + \frac{3e^{-x}}{-1} + \frac{e^{-3x}}{-3} \right] + c = \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x - 3e^{-x} - \frac{e^{-3x}}{3} \right] + c$$

$$= \frac{e^{3x} + 9e^x - 9e^{-x} - e^{-3x}}{24} + c$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx = \frac{\sinh^3 x}{3} + \sinh x + c = \frac{e^{3x} + 9e^x - 9e^{-x} - e^{-3x}}{24} + c$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \sinh x \mid x \in \mathbb{R} \\ dt = \cosh x \, dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int (t^2 + 1) \, dt = \frac{t^3}{3} + t + c = \frac{\sinh^3 x}{3} + \sinh x + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^xe^{-2x} + e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} + 3e^x + 3e^{-x} + e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x + \frac{3e^{-x}}{-1} + \frac{e^{-3x}}{-3} \right] + c = \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x - 3e^{-x} - \frac{e^{-3x}}{3} \right] + c$$

$$= \frac{e^{3x} + 9e^x - 9e^{-x} - e^{-3x}}{24} + c = \frac{e^{3x} - 3e^x + 12e^x + 3e^{-x} - 12e^{-x} - e^{-3x}}{24} + c$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx = \frac{\sinh^3 x}{3} + \sinh x + c = \frac{e^{3x} + 9e^x - 9e^{-x} - e^{-3x}}{24} + c$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \sinh x \mid x \in \mathbb{R} \\ dt = \cosh x \, dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int (t^2 + 1) \, dt = \frac{t^3}{3} + t + c = \frac{\sinh^3 x}{3} + \sinh x + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^xe^{-2x} + e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} + 3e^x + 3e^{-x} + e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x + \frac{3e^{-x}}{-1} + \frac{e^{-3x}}{-3} \right] + c = \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x - 3e^{-x} - \frac{e^{-3x}}{3} \right] + c$$

$$= \frac{e^{3x} + 9e^x - 9e^{-x} - e^{-3x}}{24} + c = \frac{e^{3x} - 3e^x + 12e^x + 3e^{-x} - 12e^{-x} - e^{-3x}}{24} + c$$

$$= \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{3 \cdot 8} + \frac{12e^x - 12e^{-x}}{24} + c$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx = \frac{\sinh^3 x}{3} + \sinh x + c = \frac{e^{3x} + 9e^x - 9e^{-x} - e^{-3x}}{24} + c$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \sinh x \mid x \in \mathbb{R} \\ dt = \cosh x \, dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int (t^2 + 1) \, dt = \frac{t^3}{3} + t + c = \frac{\sinh^3 x}{3} + \sinh x + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^xe^{-2x} + e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} + 3e^x + 3e^{-x} + e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x + \frac{3e^{-x}}{-1} + \frac{e^{-3x}}{-3} \right] + c = \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x - 3e^{-x} - \frac{e^{-3x}}{3} \right] + c$$

$$= \frac{e^{3x} + 9e^x - 9e^{-x} - e^{-3x}}{24} + c = \frac{e^{3x} - 3e^x + 12e^x + 3e^{-x} - 12e^{-x} - e^{-3x}}{24} + c$$

$$= \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{3 \cdot 8} + \frac{12e^x - 12e^{-x}}{24} + c = \frac{1}{3} \cdot \left[\frac{e^{2x} - e^{-2x}}{2} \right]^3 + \frac{e^x - e^{-x}}{2} + c$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx = \frac{\sinh^3 x}{3} + \sinh x + c = \frac{e^{3x} + 9e^x - 9e^{-x} - e^{-3x}}{24} + c$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx = \left[\begin{array}{l} \text{Subst. } t = \sinh x \mid x \in \mathbb{R} \\ dt = \cosh x \, dx \mid t \in \mathbb{R} \end{array} \right]$$

$$= \int (t^2 + 1) \, dt = \frac{t^3}{3} + t + c = \frac{\sinh^3 x}{3} + \sinh x + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^xe^{-2x} + e^{-3x}}{8} dx$$

$$= \int \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8} dx = \frac{1}{8} \int [e^{3x} + 3e^x + 3e^{-x} + e^{-3x}] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x + \frac{3e^{-x}}{-1} + \frac{e^{-3x}}{-3} \right] + c = \frac{1}{8} \left[\frac{e^{3x}}{3} + 3e^x - 3e^{-x} - \frac{e^{-3x}}{3} \right] + c$$

$$= \frac{e^{3x} + 9e^x - 9e^{-x} - e^{-3x}}{24} + c = \frac{e^{3x} - 3e^x + 12e^x + 3e^{-x} - 12e^{-x} - e^{-3x}}{24} + c$$

$$= \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{3 \cdot 8} + \frac{12e^x - 12e^{-x}}{24} + c = \frac{1}{3} \cdot \left[\frac{e^{2x} - e^{-2x}}{2} \right]^3 + \frac{e^x - e^{-x}}{2} + c$$

$$= \frac{\sinh^3 x}{3} + \sinh x + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx$$

Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx$$

Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx = \int (\sinh^2 x + 1)^n \cdot \cosh x \, dx$$

Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx$$

 $n \in \mathbb{N}$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx = \int (\sinh^2 x + 1)^n \cdot \cosh x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sinh x \mid x \in \mathbb{R} \\ dt = \cosh x \, dx \mid t \in \mathbb{R} \end{array} \right]$$

Riešené príklady – 023

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Riešené príklady – 023

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Riešené príklady – 023

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Riešené príklady – 023

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Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx = \sum_{i=0}^n \binom{n}{i} \frac{\sinh^{2i+1} x}{2i+1} + c \quad n \in \mathbb{N}$$

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$$= \int \left[\sum_{i=0}^n \binom{n}{i} t^{2i} \right] dt = \sum_{i=0}^n \binom{n}{i} \int t^{2i} \, dt = \sum_{i=0}^n \binom{n}{i} \frac{t^{2i+1}}{2i+1} + c = \sum_{i=0}^n \binom{n}{i} \frac{\sinh^{2i+1} x}{2i+1} + c$$

$$= \binom{n}{0} \sinh x - \binom{n}{1} \frac{\sinh^3 x}{3} + \binom{n}{2} \frac{\sinh^5 x}{5} + \dots \\ + (-1)^{n-1} \binom{n}{n-1} \frac{\sinh^{2n-1} x}{2n-1} + (-1)^n \binom{n}{n} \frac{\sinh^{2n+1} x}{2n+1} + c$$

Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx = \sum_{i=0}^n \binom{n}{i} \frac{\sinh^{2i+1} x}{2i+1} + c \quad n \in \mathbb{N}$$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx = \int (\sinh^2 x + 1)^n \cdot \cosh x \, dx$$

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$$= \int \left[\sum_{i=0}^n \binom{n}{i} t^{2i} \right] \, dt = \sum_{i=0}^n \binom{n}{i} \int t^{2i} \, dt = \sum_{i=0}^n \binom{n}{i} \frac{t^{2i+1}}{2i+1} + c = \sum_{i=0}^n \binom{n}{i} \frac{\sinh^{2i+1} x}{2i+1} + c$$

$$= \binom{n}{0} \sinh x - \binom{n}{1} \frac{\sinh^3 x}{3} + \binom{n}{2} \frac{\sinh^5 x}{5} + \dots \\ + (-1)^{n-1} \binom{n}{n-1} \frac{\sinh^{2n-1} x}{2n-1} + (-1)^n \binom{n}{n} \frac{\sinh^{2n+1} x}{2n+1} + c$$

$$= \sinh x - \frac{n \sinh^3 x}{3} + \binom{n}{2} \frac{\sinh^5 x}{5} + \dots \\ + (-1)^{n-1} \cdot \frac{n \sinh^{2n-1} x}{2n-1} + (-1)^n \cdot \frac{\sinh^{2n+1} x}{2n+1} + c, \\ x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx = \left[\begin{array}{l} u = \cosh^{n-1} x \quad u' = (n-1) \cosh^{n-2} x \cdot \sinh x \\ v' = \cosh x \quad v = \sinh x \end{array} \right]$$

Riešené príklady – 024

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$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot \sinh^2 x \, dx$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

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$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot \sinh^2 x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot (\cosh^2 x - 1) \, dx$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

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Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

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$$= \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2}$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

$$n = 2, 3, 4, 5, \dots$$

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$$= \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2}$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2} \\ \Rightarrow n I_n = I_n + (n-1) I_n = \sinh x \cdot \cosh^{n-1} x + (n-1) I_{n-2} \Rightarrow I_n = \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx = \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} \int \cosh^{n-2} x \, dx \quad n = 2, 3, 4, 5, \dots$$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx = \left[\begin{array}{l} u = \cosh^{n-1} x \quad | \quad u' = (n-1) \cosh^{n-2} x \cdot \sinh x \\ v' = \cosh x \quad | \quad v = \sinh x \end{array} \right]$$

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$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot (\cosh^2 x - 1) \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^n x \, dx + (n-1) \int \cosh^{n-2} x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2}$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2} \\ \Rightarrow n I_n = I_n + (n-1) I_n = \sinh x \cdot \cosh^{n-1} x + (n-1) I_{n-2} \Rightarrow I_n = \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \quad x \in \mathbb{R},$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx = \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} \int \cosh^{n-2} x \, dx \quad n = 2, 3, 4, 5, \dots$$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx = \left[\begin{array}{l} u = \cosh^{n-1} x \quad | \quad u' = (n-1) \cosh^{n-2} x \cdot \sinh x \\ v' = \cosh x \quad | \quad v = \sinh x \end{array} \right]$$

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$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^n x \, dx + (n-1) \int \cosh^{n-2} x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2}$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2} \\ \Rightarrow n I_n = I_n + (n-1) I_n = \sinh x \cdot \cosh^{n-1} x + (n-1) I_{n-2} \Rightarrow I_n = \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \quad x \in \mathbb{R},$$

pričom $I_1 = \int \cosh x \, dx = \sinh x + c$, $I_0 = \int \cosh^0 x \, dx = \int dx = x + c$, $x \in \mathbb{R}$, $c \in \mathbb{R}$.

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

 $n \in \mathbb{N}$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

 $n \in \mathbb{N}$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

 $n \in \mathbb{N}$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} dx$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

 $n \in \mathbb{N}$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x + e^{-x})^n dx$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

 $n \in \mathbb{N}$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x + e^{-x})^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

 $n \in \mathbb{N}$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x + e^{-x})^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-ix} e^{-ix} \right] dx$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx$$

 $n \in \mathbb{N}$

$$\begin{aligned} &= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x + e^{-x})^n dx \\ &= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-ix} e^{-ix} \right] dx \\ &= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-2ix} \right] dx \end{aligned}$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^n \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right] \quad n \in \mathbb{N}$$

$$\begin{aligned} &= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x + e^{-x})^n dx \\ &= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-ix} e^{-ix} \right] dx \\ &= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^n \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right], \quad x \in \mathbb{R}, \end{aligned}$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^n \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right] \quad n \in \mathbb{N}$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x + e^{-x})^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-ix} e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^n \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right], \quad x \in \mathbb{R},$$

príčom $\int dx = x + c$ pre $i = \frac{n}{2}$, $\int e^{(n-2i)x} dx = \frac{e^{(n-2i)x}}{n-2i} + c$ pre $i \neq \frac{n}{2}$, $c \in \mathbb{R}$.

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^n \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right] \quad n \in \mathbb{N}$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x + e^{-x})^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-ix} e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^n \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right], \quad x \in \mathbb{R},$$

pričom $\int dx = x + c$ pre $i = \frac{n}{2}$, $\int e^{(n-2i)x} dx = \frac{e^{(n-2i)x}}{n-2i} + c$ pre $i \neq \frac{n}{2}$, $c \in \mathbb{R}$.

$$n = 2k+1, \quad k=0,1,2,\dots \text{ (nepárne)}$$

$$n = 2k, \quad k=1,2,3,\dots \text{ (párne)}$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^n \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right] \quad n \in \mathbb{N}$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int (e^x + e^{-x})^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-ix} e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^n \binom{n}{i} e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^n \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right], \quad x \in \mathbb{R},$$

pričom $\int dx = x + c$ pre $i = \frac{n}{2}$, $\int e^{(n-2i)x} \, dx = \frac{e^{(n-2i)x}}{n-2i} + c$ pre $i \neq \frac{n}{2}$, $c \in \mathbb{R}$.

$$n = 2k+1, \quad k=0,1,2,\dots \text{ (nepárne)}$$

$$I_n = \frac{1}{2^n} \left[\frac{e^{nx}}{n} + n \frac{e^{(n-2)x}}{n-2} + \dots + \binom{n}{k} e^x + \binom{n}{k+1} \frac{e^{-x}}{-1} + \dots + n \frac{e^{(2-n)x}}{2-n} + \frac{e^{-nx}}{-n} \right] + c.$$

$$n = 2k, \quad k=1,2,3,\dots \text{ (párne)}$$

$$I_n = \frac{1}{2^n} \left[\frac{e^{nx}}{n} + n \frac{e^{(n-2)x}}{n-2} + \dots + \binom{n}{k-1} \frac{e^{2x}}{2} + \binom{n}{k} x + \binom{n}{k+1} \frac{e^{-2x}}{-2} + \dots + n \frac{e^{(2-n)x}}{2-n} + \frac{e^{-nx}}{-n} \right] + c.$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx$$

$$\int \operatorname{tg}^3 x \, dx$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx$$

$$\int \operatorname{tg}^3 x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{tg} x \mid x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi) \\ x = \operatorname{arctg} t + 2k\pi \mid dx = \frac{dt}{t^2+1}, \quad t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right]$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx$$

$$\int \operatorname{tg}^3 x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{tg} x \mid x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi) \\ x = \operatorname{arctg} t + 2k\pi \mid dx = \frac{dt}{t^2+1}, \quad t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^3 \, dt}{t^2+1}$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx = \int \left[\frac{1}{\cos^2 x} - 1 \right] dx$$

$$\int \operatorname{tg}^3 x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{tg} x \mid x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) \\ x = \operatorname{arctg} t + 2k\pi \mid dx = \frac{dt}{t^2+1}, \quad t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3 + t - t \, dt}{t^2+1}$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx = \int \left[\frac{1}{\cos^2 x} - 1 \right] dx = \operatorname{tg} x - x + c,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

$$\int \operatorname{tg}^3 x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{tg} x \mid x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) \\ x = \operatorname{arctg} t + 2k\pi \mid dx = \frac{dt}{t^2+1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3 + t - t \, dt}{t^2+1} = \int t \, dt - \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx = \int \left[\frac{1}{\cos^2 x} - 1 \right] dx = \operatorname{tg} x - x + c,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

$$\int \operatorname{tg}^3 x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{tg} x \mid x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) \\ x = \operatorname{arctg} t + 2k\pi \mid dx = \frac{dt}{t^2+1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3 + t - t \, dt}{t^2+1} = \int t \, dt - \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \int t \, dt - \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1}$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx = \int \left[\frac{1}{\cos^2 x} - 1 \right] dx = \operatorname{tg} x - x + c,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

$$\int \operatorname{tg}^3 x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{tg} x \mid x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) \\ x = \operatorname{arctg} t + 2k\pi \mid dx = \frac{dt}{t^2+1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3 + t - t \, dt}{t^2+1} = \int t \, dt - \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \int t \, dt - \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} = \frac{t^2}{2} - \ln(t^2+1) + c$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx = \int \left[\frac{1}{\cos^2 x} - 1 \right] dx = \operatorname{tg} x - x + c,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

$$\int \operatorname{tg}^3 x \, dx = \frac{\operatorname{tg}^2 x}{2} - \ln(\operatorname{tg}^2 x + 1) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{tg} x \mid x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) \\ x = \operatorname{arctg} t + 2k\pi \mid dx = \frac{dt}{t^2+1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3 + t - t \, dt}{t^2+1} = \int t \, dt - \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \int t \, dt - \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} = \frac{t^2}{2} - \ln(t^2+1) + c = \frac{\operatorname{tg}^2 x}{2} - \ln(\operatorname{tg}^2 x + 1) + c$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx = \int \left[\frac{1}{\cos^2 x} - 1 \right] dx = \operatorname{tg} x - x + c,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

$$\int \operatorname{tg}^3 x \, dx = \frac{\operatorname{tg}^2 x}{2} - \ln(\operatorname{tg}^2 x + 1) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{tg} x \mid x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) \\ x = \operatorname{arctg} t + 2k\pi \mid dx = \frac{dt}{t^2+1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3 + t - t \, dt}{t^2+1} = \int t \, dt - \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \int t \, dt - \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} = \frac{t^2}{2} - \ln(t^2+1) + c = \frac{\operatorname{tg}^2 x}{2} - \ln(\operatorname{tg}^2 x + 1) + c$$

$$= \frac{\operatorname{tg}^2 x}{2} - \ln\left(\frac{\sin^2 x}{\cos^2 x} + 1\right) + c$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx = \int \left[\frac{1}{\cos^2 x} - 1 \right] dx = \operatorname{tg} x - x + c,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

$$\int \operatorname{tg}^3 x \, dx = \frac{\operatorname{tg}^2 x}{2} - \ln(\operatorname{tg}^2 x + 1) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{tg} x \mid x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) \\ x = \operatorname{arctg} t + 2k\pi \mid dx = \frac{dt}{t^2+1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3 + t - t \, dt}{t^2+1} = \int t \, dt - \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \int t \, dt - \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} = \frac{t^2}{2} - \ln(t^2+1) + c = \frac{\operatorname{tg}^2 x}{2} - \ln(\operatorname{tg}^2 x + 1) + c$$

$$= \frac{\operatorname{tg}^2 x}{2} - \ln\left(\frac{\sin^2 x}{\cos^2 x} + 1\right) + c = \frac{\operatorname{tg}^2 x}{2} - \ln \frac{\sin^2 x + \cos^2 x}{\cos^2 x} + c$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx = \int \left[\frac{1}{\cos^2 x} - 1 \right] dx = \operatorname{tg} x - x + c,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

$$\int \operatorname{tg}^3 x \, dx = \frac{\operatorname{tg}^2 x}{2} - \ln(\operatorname{tg}^2 x + 1) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{tg} x \mid x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) \\ x = \operatorname{arctg} t + 2k\pi \mid dx = \frac{dt}{t^2+1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3 + t - t \, dt}{t^2+1} = \int t \, dt - \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \int t \, dt - \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} = \frac{t^2}{2} - \ln(t^2+1) + c = \frac{\operatorname{tg}^2 x}{2} - \ln(\operatorname{tg}^2 x + 1) + c$$

$$= \frac{\operatorname{tg}^2 x}{2} - \ln\left(\frac{\sin^2 x}{\cos^2 x} + 1\right) + c = \frac{\operatorname{tg}^2 x}{2} - \ln \frac{\sin^2 x + \cos^2 x}{\cos^2 x} + c = \frac{\operatorname{tg}^2 x}{2} - \ln \frac{1}{\cos^2 x} + c$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx = \int \left[\frac{1}{\cos^2 x} - 1 \right] dx = \operatorname{tg} x - x + c,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

$$\int \operatorname{tg}^3 x \, dx = \frac{\operatorname{tg}^2 x}{2} - \ln(\operatorname{tg}^2 x + 1) + c = \frac{\operatorname{tg}^2 x}{2} + \ln \cos^2 x + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{tg} x \mid x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) \\ x = \operatorname{arctg} t + 2k\pi \mid dx = \frac{dt}{t^2+1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3 + t - t \, dt}{t^2+1} = \int t \, dt - \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \int t \, dt - \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} = \frac{t^2}{2} - \ln(t^2+1) + c = \frac{\operatorname{tg}^2 x}{2} - \ln(\operatorname{tg}^2 x + 1) + c$$

$$= \frac{\operatorname{tg}^2 x}{2} - \ln\left(\frac{\sin^2 x}{\cos^2 x} + 1\right) + c = \frac{\operatorname{tg}^2 x}{2} - \ln \frac{\sin^2 x + \cos^2 x}{\cos^2 x} + c = \frac{\operatorname{tg}^2 x}{2} - \ln \frac{1}{\cos^2 x} + c$$

$$= \frac{\operatorname{tg}^2 x}{2} + \ln \cos^2 x + c, x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 027, 028

$$\int \cotg^2 x \, dx$$

$$\int \cotg^3 x \, dx$$

Riešené príklady – 027, 028

$$\int \cotg^2 x \, dx$$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx$$

$$\int \cotg^3 x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cotg x \mid x \in (0 + 2k\pi; \pi + 2k\pi) \\ x = \operatorname{arccotg} t \mid dx = -\frac{dt}{t^2+1}, t \in R, k \in Z \end{array} \right]$$

Riešené príklady – 027, 028

$$\int \cotg^2 x \, dx$$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1 - \sin^2 x}{\sin^2 x} \, dx$$

$$\int \cotg^3 x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cotg x \mid x \in (0 + 2k\pi; \pi + 2k\pi) \\ x = \operatorname{arccotg} t \mid dx = -\frac{dt}{t^2 + 1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = -\int \frac{t^3 \, dt}{t^2 + 1}$$

Riešené príklady – 027, 028

$$\int \cotg^2 x \, dx$$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1 - \sin^2 x}{\sin^2 x} \, dx = \int \left[\frac{1}{\sin^2 x} - 1 \right] dx$$

$$\int \cotg^3 x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cotg x \mid x \in (0 + 2k\pi; \pi + 2k\pi) \\ x = \operatorname{arccotg} t \mid dx = -\frac{dt}{t^2 + 1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = -\int \frac{t^3 \, dt}{t^2 + 1} = -\int \frac{t^3 + t - t \, dt}{t^2 + 1}$$

Riešené príklady – 027, 028

$$\int \cotg^2 x \, dx = -\cotg x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1 - \sin^2 x}{\sin^2 x} \, dx = \int \left[\frac{1}{\sin^2 x} - 1 \right] dx = -\cotg x - x + c,$$

$x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}, c \in \mathbb{R}.$

$$\int \cotg^3 x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cotg x \mid x \in (0 + 2k\pi; \pi + 2k\pi) \\ x = \operatorname{arccotg} t \mid dx = -\frac{dt}{t^2 + 1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = -\int \frac{t^3 \, dt}{t^2 + 1} = -\int \frac{t^3 + t - t \, dt}{t^2 + 1} = -\int t \, dt + \frac{1}{2} \int \frac{2t \, dt}{t^2 + 1}$$

Riešené príklady – 027, 028

$$\int \cotg^2 x \, dx = -\cotg x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1 - \sin^2 x}{\sin^2 x} \, dx = \int \left[\frac{1}{\sin^2 x} - 1 \right] dx = -\cotg x - x + c,$$

$x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}, c \in \mathbb{R}.$

$$\int \cotg^3 x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cotg x \mid x \in (0 + 2k\pi; \pi + 2k\pi) \\ x = \operatorname{arccotg} t \mid dx = -\frac{dt}{t^2+1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = -\int \frac{t^3 \, dt}{t^2+1} = -\int \frac{t^3 + t - t \, dt}{t^2+1} = -\int t \, dt + \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} - \int t \, dt$$

Riešené príklady – 027, 028

$$\int \cotg^2 x \, dx = -\cotg x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1 - \sin^2 x}{\sin^2 x} \, dx = \int \left[\frac{1}{\sin^2 x} - 1 \right] dx = -\cotg x - x + c,$$

$x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}, c \in \mathbb{R}.$

$$\int \cotg^3 x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cotg x \mid x \in (0 + 2k\pi; \pi + 2k\pi) \\ x = \operatorname{arccotg} t \mid dx = -\frac{dt}{t^2+1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = -\int \frac{t^3 \, dt}{t^2+1} = -\int \frac{t^3+t-t \, dt}{t^2+1} = -\int t \, dt + \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} - \int t \, dt = \ln(t^2+1) - \frac{t^2}{2} + c$$

Riešené príklady – 027, 028

$$\int \cotg^2 x \, dx = -\cotg x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1 - \sin^2 x}{\sin^2 x} \, dx = \int \left[\frac{1}{\sin^2 x} - 1 \right] dx = -\cotg x - x + c,$$

$$x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}, c \in \mathbb{R}.$$

$$\int \cotg^3 x \, dx = \ln(\cotg^2 x + 1) - \frac{\cotg^2 x}{2} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cotg x \mid x \in (0 + 2k\pi; \pi + 2k\pi) \\ x = \operatorname{arccotg} t \mid dx = -\frac{dt}{t^2 + 1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = -\int \frac{t^3 \, dt}{t^2 + 1} = -\int \frac{t^3 + t - t \, dt}{t^2 + 1} = -\int t \, dt + \frac{1}{2} \int \frac{2t \, dt}{t^2 + 1}$$

$$= \frac{1}{2} \int \frac{(t^2 + 1)' \, dt}{t^2 + 1} - \int t \, dt = \ln(t^2 + 1) - \frac{t^2}{2} + c = \ln(\cotg^2 x + 1) - \frac{\cotg^2 x}{2} + c$$

Riešené príklady – 027, 028

$$\int \cotg^2 x \, dx = -\cotg x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1 - \sin^2 x}{\sin^2 x} \, dx = \int \left[\frac{1}{\sin^2 x} - 1 \right] dx = -\cotg x - x + c,$$

$$x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}, c \in \mathbb{R}.$$

$$\int \cotg^3 x \, dx = \ln(\cotg^2 x + 1) - \frac{\cotg^2 x}{2} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cotg x \mid x \in (0 + 2k\pi; \pi + 2k\pi) \\ x = \operatorname{arccotg} t \mid dx = -\frac{dt}{t^2+1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = -\int \frac{t^3 \, dt}{t^2+1} = -\int \frac{t^3 + t - t \, dt}{t^2+1} = -\int t \, dt + \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} - \int t \, dt = \ln(t^2+1) - \frac{t^2}{2} + c = \ln(\cotg^2 x + 1) - \frac{\cotg^2 x}{2} + c$$

$$= \ln\left(\frac{\cos^2 x}{\sin^2 x} + 1\right) - \frac{\cotg^2 x}{2} + c$$

Riešené príklady – 027, 028

$$\int \cotg^2 x \, dx = -\cotg x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1 - \sin^2 x}{\sin^2 x} \, dx = \int \left[\frac{1}{\sin^2 x} - 1 \right] \, dx = -\cotg x - x + c,$$

$$x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}, c \in \mathbb{R}.$$

$$\int \cotg^3 x \, dx = \ln(\cotg^2 x + 1) - \frac{\cotg^2 x}{2} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cotg x \mid x \in (0 + 2k\pi; \pi + 2k\pi) \\ x = \operatorname{arccotg} t \mid dx = -\frac{dt}{t^2 + 1}, t \in \mathbb{R}, k \in \mathbb{Z} \end{array} \right] = -\int \frac{t^3 \, dt}{t^2 + 1} = -\int \frac{t^3 + t - t \, dt}{t^2 + 1} = -\int t \, dt + \frac{1}{2} \int \frac{2t \, dt}{t^2 + 1}$$

$$= \frac{1}{2} \int \frac{(t^2 + 1)' \, dt}{t^2 + 1} - \int t \, dt = \ln(t^2 + 1) - \frac{t^2}{2} + c = \ln(\cotg^2 x + 1) - \frac{\cotg^2 x}{2} + c$$

$$= \ln\left(\frac{\cos^2 x}{\sin^2 x} + 1\right) - \frac{\cotg^2 x}{2} + c = \ln \frac{\cos^2 x + \sin^2 x}{\sin^2 x} - \frac{\cotg^2 x}{2} + c$$

Riešené príklady – 027, 028

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Riešené príklady – 027, 028

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$$x \in R - \{k\pi, k \in Z\}, c \in R.$$

$$\int \cotg^3 x \, dx = \ln(\cotg^2 x + 1) - \frac{\cotg^2 x}{2} + c = -\ln \sin^2 x - \frac{\cotg^2 x}{2} + c$$

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$$= -\ln \sin^2 x - \frac{\cotg^2 x}{2} + c, x \in R - \{k\pi, k \in Z\}, c \in R.$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx$$

$$= \int \left(\frac{e^{2x}-1}{e^{2x}+1} \right)^2 \, dx$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx$$

$$= \int \left(\frac{e^{2x} - 1}{e^{2x} + 1} \right)^2 dx = \left[\begin{array}{l} \text{Subst. } t = e^{2x} \mid 2x = \ln t \mid x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} \mid t \in (0; \infty) \mid x \in \mathbb{R} \end{array} \right]$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \left[1 - \frac{1}{\cosh^2 x} \right] dx$$

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Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \left[1 - \frac{1}{\cosh^2 x} \right] \, dx = x - \operatorname{tgh} x + c_1,$$

$x \in R, c_1 \in R.$

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Riešené príklady – 029

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$$= \int \frac{(t^2 + 2t + 1 - 4t) \, dt}{2t(t^2 + 2t + 1)}$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

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Riešené príklady – 029

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Riešené príklady – 029

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Riešené príklady – 029

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$$= \int \frac{dt}{2t} - \int \frac{2 \, dt}{(t+1)^2} = \frac{1}{2} \int \frac{dt}{t} - 2 \int (t+1)^{-2} \, dt$$

Riešené príklady – 029

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Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

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$$= \frac{1}{2} \ln t + \frac{2}{t+1} + c_2$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

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$$x \in \mathbb{R}, c_1 \in \mathbb{R}.$$

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$$= \int \frac{dt}{2t} - \int \frac{2 dt}{(t+1)^2} = \frac{1}{2} \int \frac{dt}{t} - 2 \int (t+1)^{-2} dt = \frac{1}{2} \ln t - 2 \frac{(t+1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t + \frac{2}{t+1} + c_2 = \frac{1}{2} \ln e^{2x} + \frac{2}{e^{2x} + 1} + c_2$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \left[1 - \frac{1}{\cosh^2 x} \right] dx = x - \operatorname{tgh} x + c_1,$$

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$$= \int \frac{(t^2 + 2t + 1 - 4t) dt}{2t(t^2 + 2t + 1)} = \int \frac{(t^2 + 2t + 1) dt}{2t(t^2 + 2t + 1)} - \int \frac{4t dt}{2t(t^2 + 2t + 1)} = \int \frac{dt}{2t} - \int \frac{2 dt}{t^2 + 2t + 1}$$

$$= \int \frac{dt}{2t} - \int \frac{2 dt}{(t+1)^2} = \frac{1}{2} \int \frac{dt}{t} - 2 \int (t+1)^{-2} dt = \frac{1}{2} \ln t - 2 \frac{(t+1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t + \frac{2}{t+1} + c_2 = \frac{1}{2} \ln e^{2x} + \frac{2}{e^{2x} + 1} + c_2 = \frac{2x}{2} \ln e + \frac{2}{e^{2x} + 1} + c_2$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1 = x + \frac{2}{e^{2x}+1} + c_2$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \left[1 - \frac{1}{\cosh^2 x} \right] dx = x - \operatorname{tgh} x + c_1,$$

$x \in \mathbb{R}, c_1 \in \mathbb{R}.$

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$$= \int \frac{dt}{2t} - \int \frac{2 dt}{(t+1)^2} = \frac{1}{2} \int \frac{dt}{t} - 2 \int (t+1)^{-2} dt = \frac{1}{2} \ln t - 2 \frac{(t+1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t + \frac{2}{t+1} + c_2 = \frac{1}{2} \ln e^{2x} + \frac{2}{e^{2x}+1} + c_2 = \frac{2x}{2} \ln e + \frac{2}{e^{2x}+1} + c_2$$

$$= x + \frac{2}{e^{2x}+1} + c_2$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1 = x + \frac{2}{e^{2x}+1} + c_2$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \left[1 - \frac{1}{\cosh^2 x} \right] dx = x - \operatorname{tgh} x + c_1,$$

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$$= \int \left(\frac{e^{2x}-1}{e^{2x}+1} \right)^2 dx = \left[\begin{array}{l} \text{Subst. } t=e^{2x} \mid 2x=\ln t \mid x=\frac{\ln t}{2} \\ dx=\frac{dt}{2t} \mid t \in (0; \infty) \mid x \in \mathbb{R} \end{array} \right] = \int \left(\frac{t-1}{t+1} \right)^2 \frac{dt}{2t} = \int \frac{(t^2-2t+1) dt}{2t(t^2+2t+1)}$$

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$$= \int \frac{dt}{2t} - \int \frac{2 dt}{(t+1)^2} = \frac{1}{2} \int \frac{dt}{t} - 2 \int (t+1)^{-2} dt = \frac{1}{2} \ln t - 2 \frac{(t+1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t + \frac{2}{t+1} + c_2 = \frac{1}{2} \ln e^{2x} + \frac{2}{e^{2x}+1} + c_2 = \frac{2x}{2} \ln e + \frac{2}{e^{2x}+1} + c_2$$

$$= x + \frac{2}{e^{2x}+1} + c_2 = x + \frac{2}{e^{2x}+1} - 1 + c_2 + 1$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1 = x + \frac{2}{e^{2x}+1} + c_2$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \left[1 - \frac{1}{\cosh^2 x} \right] dx = x - \operatorname{tgh} x + c_1,$$

$x \in \mathbb{R}, c_1 \in \mathbb{R}.$

$$= \int \left(\frac{e^{2x}-1}{e^{2x}+1} \right)^2 dx = \left[\begin{array}{l} \text{Subst. } t=e^{2x} \mid 2x=\ln t \mid x=\frac{\ln t}{2} \\ dx=\frac{dt}{2t} \mid t \in (0; \infty) \mid x \in \mathbb{R} \end{array} \right] = \int \left(\frac{t-1}{t+1} \right)^2 \frac{dt}{2t} = \int \frac{(t^2-2t+1) dt}{2t(t^2+2t+1)}$$

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$$= x + \frac{2}{e^{2x}+1} + c_2 = x + \frac{2}{e^{2x}+1} - 1 + c_2 + 1 = x + \frac{2-e^{2x}-1}{e^{2x}+1} + c_2 + 1$$

Riešené príklady – 029

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$$= \frac{1}{2} \ln t + \frac{2}{t+1} + c_2 = \frac{1}{2} \ln e^{2x} + \frac{2}{e^{2x}+1} + c_2 = \frac{2x}{2} \ln e + \frac{2}{e^{2x}+1} + c_2$$

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$$= \left[\begin{array}{l} c_1 = c_2 + 1 \\ c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right]$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1 = x + \frac{2}{e^{2x}+1} + c_2$$

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$$= \int \frac{dt}{2t} - \int \frac{2 dt}{(t+1)^2} = \frac{1}{2} \int \frac{dt}{t} - 2 \int (t+1)^{-2} dt = \frac{1}{2} \ln t - 2 \frac{(t+1)^{-1}}{-1} + c_2$$

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$$= x + \frac{2}{e^{2x}+1} + c_2 = x + \frac{2}{e^{2x}+1} - 1 + c_2 + 1 = x + \frac{2-e^{2x}-1}{e^{2x}+1} + c_2 + 1$$

$$= \left[\begin{array}{l} c_1 = c_2 + 1 \\ c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right] = x - \frac{e^{2x}-1}{e^{2x}+1} + c_1$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1 = x + \frac{2}{e^{2x}+1} + c_2$$

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$$= x + \frac{2}{e^{2x}+1} + c_2 = x + \frac{2}{e^{2x}+1} - 1 + c_2 + 1 = x + \frac{2-e^{2x}-1}{e^{2x}+1} + c_2 + 1$$

$$= \left[\begin{array}{l} c_1 = c_2 + 1 \\ c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right] = x - \frac{e^{2x}-1}{e^{2x}+1} + c_1 = x - \operatorname{tgh} x + c_1, \quad x \in \mathbb{R}, c_1, c_2 \in \mathbb{R}.$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx$$

$$= \int \left(\frac{e^{2x}+1}{e^{2x}-1} \right)^2 dx$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx$$

$$= \int \left(\frac{e^{2x} + 1}{e^{2x} - 1} \right)^2 dx = \left[\begin{array}{l} \text{Subst. } t = e^{2x} \mid 2x = \ln t \mid x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} \mid t \in (0; \infty) \mid x \in \mathbb{R} \end{array} \right]$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x} \right] dx$$

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Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x} \right] dx = x - \operatorname{cotgh} x + c_1,$$

$x \in R, c_1 \in R.$

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Riešené príklady – 030

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Riešené príklady – 030

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$$= \int \frac{(t^2 - 2t + 1 + 4t) dt}{2t(t^2 - 2t + 1)} = \int \frac{(t^2 - 2t + 1) dt}{2t(t^2 - 2t + 1)} + \int \frac{4t dt}{2t(t^2 - 2t + 1)}$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1$$

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Riešené príklady – 030

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$$= \int \frac{dt}{2t} + \int \frac{2 dt}{(t-1)^2}$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1$$

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$$= \int \frac{dt}{2t} + \int \frac{2 dt}{(t-1)^2} = \frac{1}{2} \int \frac{dt}{t} + 2 \int (t-1)^{-2} dt$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1$$

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$$= \int \frac{dt}{2t} + \int \frac{2 dt}{(t-1)^2} = \frac{1}{2} \int \frac{dt}{t} + 2 \int (t-1)^{-2} dt = \frac{1}{2} \ln t + 2 \frac{(t-1)^{-1}}{-1} + c_2$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1$$

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$$= \int \frac{dt}{2t} + \int \frac{2 dt}{(t-1)^2} = \frac{1}{2} \int \frac{dt}{t} + 2 \int (t-1)^{-2} dt = \frac{1}{2} \ln t + 2 \frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t - \frac{2}{t-1} + c_2$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x} \right] dx = x - \operatorname{cotgh} x + c_1,$$

$x \in R, c_1 \in R.$

$$= \int \left(\frac{e^{2x} + 1}{e^{2x} - 1} \right)^2 dx = \left[\begin{array}{l} \text{Subst. } t = e^{2x} \mid 2x = \ln t \mid x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} \mid t \in (0; \infty) \mid x \in R \end{array} \right] = \int \left(\frac{t+1}{t-1} \right)^2 \frac{dt}{2t} = \int \frac{(t^2 + 2t + 1) dt}{2t(t^2 - 2t + 1)}$$

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$$= \frac{1}{2} \ln t - \frac{2}{t-1} + c_2 = \frac{1}{2} \ln e^{2x} - \frac{2}{e^{2x} - 1} + c_2$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x} \right] dx = x - \operatorname{cotgh} x + c_1,$$

$x \in R, c_1 \in R.$

$$= \int \left(\frac{e^{2x} + 1}{e^{2x} - 1} \right)^2 dx = \left[\begin{array}{l} \text{Subst. } t = e^{2x} \mid 2x = \ln t \mid x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} \mid t \in (0; \infty) \mid x \in R \end{array} \right] = \int \left(\frac{t+1}{t-1} \right)^2 \frac{dt}{2t} = \int \frac{(t^2 + 2t + 1) dt}{2t(t^2 - 2t + 1)}$$

$$= \int \frac{(t^2 - 2t + 1 + 4t) dt}{2t(t^2 - 2t + 1)} = \int \frac{(t^2 - 2t + 1) dt}{2t(t^2 - 2t + 1)} + \int \frac{4t dt}{2t(t^2 - 2t + 1)} = \int \frac{dt}{2t} + \int \frac{2 dt}{t^2 - 2t + 1}$$

$$= \int \frac{dt}{2t} + \int \frac{2 dt}{(t-1)^2} = \frac{1}{2} \int \frac{dt}{t} + 2 \int (t-1)^{-2} dt = \frac{1}{2} \ln t + 2 \frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t - \frac{2}{t-1} + c_2 = \frac{1}{2} \ln e^{2x} - \frac{2}{e^{2x} - 1} + c_2 = \frac{2x}{2} \ln e - \frac{2}{e^{2x} - 1} + c_2$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x} \right] dx = x - \operatorname{cotgh} x + c_1,$$

$x \in \mathbb{R}, c_1 \in \mathbb{R}.$

$$= \int \left(\frac{e^{2x}+1}{e^{2x}-1} \right)^2 dx = \left[\begin{array}{l} \text{Subst. } t=e^{2x} \mid 2x=\ln t \mid x=\frac{\ln t}{2} \\ dx=\frac{dt}{2t} \mid t \in (0; \infty) \mid x \in \mathbb{R} \end{array} \right] = \int \left(\frac{t+1}{t-1} \right)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1) dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t) dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1) dt}{2t(t^2-2t+1)} + \int \frac{4t dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2 dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2 dt}{(t-1)^2} = \frac{1}{2} \int \frac{dt}{t} + 2 \int (t-1)^{-2} dt = \frac{1}{2} \ln t + 2 \frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t - \frac{2}{t-1} + c_2 = \frac{1}{2} \ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2} \ln e - \frac{2}{e^{2x}-1} + c_2$$

$$= x - \frac{2}{e^{2x}-1} + c_2$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x} \right] dx = x - \operatorname{cotgh} x + c_1,$$

$x \in \mathbb{R}, c_1 \in \mathbb{R}.$

$$= \int \left(\frac{e^{2x}+1}{e^{2x}-1} \right)^2 dx = \left[\begin{array}{l} \text{Subst. } t=e^{2x} \mid 2x=\ln t \mid x=\frac{\ln t}{2} \\ dx=\frac{dt}{2t} \mid t \in (0; \infty) \mid x \in \mathbb{R} \end{array} \right] = \int \left(\frac{t+1}{t-1} \right)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1) dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t) dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1) dt}{2t(t^2-2t+1)} + \int \frac{4t dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2 dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2 dt}{(t-1)^2} = \frac{1}{2} \int \frac{dt}{t} + 2 \int (t-1)^{-2} dt = \frac{1}{2} \ln t + 2 \frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t - \frac{2}{t-1} + c_2 = \frac{1}{2} \ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2} \ln e - \frac{2}{e^{2x}-1} + c_2$$

$$= x - \frac{2}{e^{2x}-1} + c_2 = x + \frac{-2}{e^{2x}-1} - 1 + c_2 + 1$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x} \right] dx = x - \operatorname{cotgh} x + c_1,$$

$x \in \mathbb{R}, c_1 \in \mathbb{R}.$

$$= \int \left(\frac{e^{2x}+1}{e^{2x}-1} \right)^2 dx = \left[\begin{array}{l} \text{Subst. } t=e^{2x} \mid 2x=\ln t \mid x=\frac{\ln t}{2} \\ dx=\frac{dt}{2t} \mid t \in (0; \infty) \mid x \in \mathbb{R} \end{array} \right] = \int \left(\frac{t+1}{t-1} \right)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1) dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t) dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1) dt}{2t(t^2-2t+1)} + \int \frac{4t dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2 dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2 dt}{(t-1)^2} = \frac{1}{2} \int \frac{dt}{t} + 2 \int (t-1)^{-2} dt = \frac{1}{2} \ln t + 2 \frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t - \frac{2}{t-1} + c_2 = \frac{1}{2} \ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2} \ln e - \frac{2}{e^{2x}-1} + c_2$$

$$= x - \frac{2}{e^{2x}-1} + c_2 = x + \frac{-2}{e^{2x}-1} - 1 + c_2 + 1 = x + \frac{-2-e^{2x}+1}{e^{2x}-1} + c_2 + 1$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x} \right] dx = x - \operatorname{cotgh} x + c_1,$$

$x \in \mathbb{R}, c_1 \in \mathbb{R}.$

$$= \int \left(\frac{e^{2x}+1}{e^{2x}-1} \right)^2 dx = \left[\begin{array}{l} \text{Subst. } t=e^{2x} \mid 2x=\ln t \mid x=\frac{\ln t}{2} \\ dx=\frac{dt}{2t} \mid t \in (0; \infty) \mid x \in \mathbb{R} \end{array} \right] = \int \left(\frac{t+1}{t-1} \right)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1) dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t) dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1) dt}{2t(t^2-2t+1)} + \int \frac{4t dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2 dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2 dt}{(t-1)^2} = \frac{1}{2} \int \frac{dt}{t} + 2 \int (t-1)^{-2} dt = \frac{1}{2} \ln t + 2 \frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t - \frac{2}{t-1} + c_2 = \frac{1}{2} \ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2} \ln e - \frac{2}{e^{2x}-1} + c_2$$

$$= x - \frac{2}{e^{2x}-1} + c_2 = x + \frac{-2}{e^{2x}-1} - 1 + c_2 + 1 = x + \frac{-2-e^{2x}+1}{e^{2x}-1} + c_2 + 1$$

$$= \left[\begin{array}{l} c_1 = c_2 + 1 \\ c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right]$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x} \right] dx = x - \operatorname{cotgh} x + c_1,$$

$x \in \mathbb{R}, c_1 \in \mathbb{R}.$

$$= \int \left(\frac{e^{2x}+1}{e^{2x}-1} \right)^2 dx = \left[\begin{array}{l} \text{Subst. } t=e^{2x} \mid 2x=\ln t \mid x=\frac{\ln t}{2} \\ dx=\frac{dt}{2t} \mid t \in (0; \infty) \mid x \in \mathbb{R} \end{array} \right] = \int \left(\frac{t+1}{t-1} \right)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1) dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t) dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1) dt}{2t(t^2-2t+1)} + \int \frac{4t dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2 dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2 dt}{(t-1)^2} = \frac{1}{2} \int \frac{dt}{t} + 2 \int (t-1)^{-2} dt = \frac{1}{2} \ln t + 2 \frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t - \frac{2}{t-1} + c_2 = \frac{1}{2} \ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2} \ln e - \frac{2}{e^{2x}-1} + c_2$$

$$= x - \frac{2}{e^{2x}-1} + c_2 = x + \frac{-2}{e^{2x}-1} - 1 + c_2 + 1 = x + \frac{-2-e^{2x}+1}{e^{2x}-1} + c_2 + 1$$

$$= \left[\begin{array}{l} c_1 = c_2 + 1 \\ c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right] = x - \frac{e^{2x}+1}{e^{2x}-1} + c_1$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x} \right] dx = x - \operatorname{cotgh} x + c_1,$$

$x \in \mathbb{R}, c_1 \in \mathbb{R}.$

$$= \int \left(\frac{e^{2x}+1}{e^{2x}-1} \right)^2 dx = \left[\begin{array}{l} \text{Subst. } t=e^{2x} \mid 2x=\ln t \mid x=\frac{\ln t}{2} \\ dx=\frac{dt}{2t} \mid t \in (0; \infty) \mid x \in \mathbb{R} \end{array} \right] = \int \left(\frac{t+1}{t-1} \right)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1) dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t) dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1) dt}{2t(t^2-2t+1)} + \int \frac{4t dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2 dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2 dt}{(t-1)^2} = \frac{1}{2} \int \frac{dt}{t} + 2 \int (t-1)^{-2} dt = \frac{1}{2} \ln t + 2 \frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t - \frac{2}{t-1} + c_2 = \frac{1}{2} \ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2} \ln e - \frac{2}{e^{2x}-1} + c_2$$

$$= x - \frac{2}{e^{2x}-1} + c_2 = x + \frac{-2}{e^{2x}-1} - 1 + c_2 + 1 = x + \frac{-2-e^{2x}+1}{e^{2x}-1} + c_2 + 1$$

$$= \left[\begin{array}{l} c_1 = c_2 + 1 \\ c_1 \in \mathbb{R}, c_2 \in \mathbb{R} \end{array} \right] = x - \frac{e^{2x}+1}{e^{2x}-1} + c_1 = x - \operatorname{cotgh} x + c_1, \quad x \in \mathbb{R}, c_1, c_2 \in \mathbb{R}.$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3 \sin x} dx$$

$$\int \frac{1+3 \sin x+2 \cos x}{\sin 2x} dx$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3 \sin x} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=4+3 \sin x \mid x \in \left\langle -\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi \right\rangle \\ dt=3 \cos x dx \mid t \in (1; 7), k \in \mathbb{Z} \end{array} \right]$$

$$\int \frac{1+3 \sin x+2 \cos x}{\sin 2x} dx$$

$$= \int \frac{dx}{\sin 2x} + \int \frac{3 \sin x+2 \cos x}{2 \sin x \cos x} dx$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3 \sin x} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t=4+3 \sin x \mid x \in \left(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi\right) \\ dt=3 \cos x dx \mid t \in (1; 7), k \in \mathbb{Z} \end{array} \right] = \frac{1}{3} \int \frac{dt}{t}$$

$$\int \frac{1+3 \sin x+2 \cos x}{\sin 2x} dx$$

$$= \int \frac{dx}{\sin 2x} + \int \frac{3 \sin x+2 \cos x}{2 \sin x \cos x} dx = \left[\text{Subst. } \begin{array}{l} u=2x \mid x \in \left(0+\frac{k\pi}{2}; \frac{\pi}{2}+\frac{k\pi}{2}\right) = \left(\frac{k\pi}{2}; (k+1)\frac{\pi}{2}\right) \mid x \neq \frac{k\pi}{2} \\ du=2 dx \mid u \in (0+k\pi; \pi+k\pi) = (k\pi; (k+1)\pi) \mid k \in \mathbb{Z} \end{array} \right]$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3 \sin x} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=4+3 \sin x \mid x \in \left\langle -\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi \right\rangle \\ dt=3 \cos x dx \mid t \in (1; 7), k \in \mathbb{Z} \end{array} \right] = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln t + c$$

$$\int \frac{1+3 \sin x+2 \cos x}{\sin 2x} dx$$

$$= \int \frac{dx}{\sin 2x} + \int \frac{3 \sin x+2 \cos x}{2 \sin x \cos x} dx = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \left(0+\frac{k\pi}{2}; \frac{\pi}{2}+\frac{k\pi}{2} \right) = \left(\frac{k\pi}{2}; (k+1)\frac{\pi}{2} \right) \mid x \neq \frac{k\pi}{2} \\ du=2 dx \mid u \in (0+k\pi; \pi+k\pi) = (k\pi; (k+1)\pi) \mid k \in \mathbb{Z} \end{array} \right]$$

$$= \frac{1}{2} \int \frac{du}{\sin u} + \frac{3}{2} \int \frac{dx}{\cos x} + \frac{dx}{\sin x}$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3 \sin x} dx = \frac{1}{3} \ln(4+3 \sin x) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=4+3 \sin x \mid x \in \left(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi\right) \\ dt=3 \cos x dx \mid t \in (1; 7), k \in \mathbb{Z} \end{array} \right] = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln t + c$$

$$= \frac{1}{3} \ln(4+3 \sin x) + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \frac{1+3 \sin x+2 \cos x}{\sin 2x} dx = \frac{1}{2} \ln |\operatorname{tg} x| - \frac{3}{2} \ln \left| \operatorname{tg} \frac{x}{2} - 1 \right| + \frac{3}{2} \ln \left| \operatorname{tg} \frac{x}{2} + 1 \right| + \ln \left| \operatorname{tg} \frac{x}{2} \right| + c$$

$$= \int \frac{dx}{\sin 2x} + \int \frac{3 \sin x+2 \cos x}{2 \sin x \cos x} dx = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \left(0+\frac{k\pi}{2}; \frac{\pi}{2}+\frac{k\pi}{2}\right) = \left(\frac{k\pi}{2}; (k+1)\frac{\pi}{2}\right) \mid x \neq \frac{k\pi}{2} \\ du=2 dx \mid u \in (0+k\pi; \pi+k\pi) = (k\pi; (k+1)\pi) \mid k \in \mathbb{Z} \end{array} \right]$$

$$= \frac{1}{2} \int \frac{du}{\sin u} + \frac{3}{2} \int \frac{dx}{\cos x} + \frac{dx}{\sin x} = \left[\begin{array}{l} \text{Pr. 01} \\ \text{Pr. 02} \end{array} \right]$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3 \sin x} dx = \frac{1}{3} \ln(4+3 \sin x) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=4+3 \sin x \mid x \in \langle -\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi \rangle \\ dt=3 \cos x dx \mid t \in (1; 7), k \in \mathbb{Z} \end{array} \right] = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln t + c$$

$$= \frac{1}{3} \ln(4+3 \sin x) + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \frac{1+3 \sin x+2 \cos x}{\sin 2x} dx = \frac{1}{2} \ln |\operatorname{tg} x| - \frac{3}{2} \ln \left| \operatorname{tg} \frac{x}{2} - 1 \right| + \frac{3}{2} \ln \left| \operatorname{tg} \frac{x}{2} + 1 \right| + \ln \left| \operatorname{tg} \frac{x}{2} \right| + c$$

$$= \int \frac{dx}{\sin 2x} + \int \frac{3 \sin x+2 \cos x}{2 \sin x \cos x} dx = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in (0+\frac{k\pi}{2}; \frac{\pi}{2}+\frac{k\pi}{2}) = (\frac{k\pi}{2}; (k+1)\frac{\pi}{2}) \mid x \neq \frac{k\pi}{2} \\ du=2 dx \mid u \in (0+k\pi; \pi+k\pi) = (k\pi; (k+1)\pi) \mid k \in \mathbb{Z} \end{array} \right]$$

$$= \frac{1}{2} \int \frac{du}{\sin u} + \frac{3}{2} \int \frac{dx}{\cos x} + \frac{dx}{\sin x} = \left[\begin{array}{l} \text{Pr. 01} \\ \text{Pr. 02} \end{array} \right] = \frac{1}{2} \ln \left| \operatorname{tg} \frac{u}{2} \right| - \frac{3}{2} \ln \left| \frac{\operatorname{tg} \frac{x}{2} - 1}{\operatorname{tg} \frac{x}{2} + 1} \right| + \ln \left| \operatorname{tg} \frac{x}{2} \right| + c$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3 \sin x} dx = \frac{1}{3} \ln(4+3 \sin x) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=4+3 \sin x \mid x \in \langle -\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi \rangle \\ dt=3 \cos x dx \mid t \in (1; 7), k \in \mathbb{Z} \end{array} \right] = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln t + c$$

$$= \frac{1}{3} \ln(4+3 \sin x) + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \frac{1+3 \sin x+2 \cos x}{\sin 2x} dx = \frac{1}{2} \ln |\operatorname{tg} x| - \frac{3}{2} \ln \left| \operatorname{tg} \frac{x}{2} - 1 \right| + \frac{3}{2} \ln \left| \operatorname{tg} \frac{x}{2} + 1 \right| + \ln \left| \operatorname{tg} \frac{x}{2} \right| + c$$

$$= \int \frac{dx}{\sin 2x} + \int \frac{3 \sin x+2 \cos x}{2 \sin x \cos x} dx = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in (0+\frac{k\pi}{2}; \frac{\pi}{2}+\frac{k\pi}{2}) = (\frac{k\pi}{2}; (k+1)\frac{\pi}{2}) \mid x \neq \frac{k\pi}{2} \\ du=2 dx \mid u \in (0+k\pi; \pi+k\pi) = (k\pi; (k+1)\pi) \mid k \in \mathbb{Z} \end{array} \right]$$

$$= \frac{1}{2} \int \frac{du}{\sin u} + \frac{3}{2} \int \frac{dx}{\cos x} + \frac{dx}{\sin x} = \left[\begin{array}{l} \text{Pr. 01} \\ \text{Pr. 02} \end{array} \right] = \frac{1}{2} \ln \left| \operatorname{tg} \frac{u}{2} \right| - \frac{3}{2} \ln \left| \frac{\operatorname{tg} \frac{x}{2} - 1}{\operatorname{tg} \frac{x}{2} + 1} \right| + \ln \left| \operatorname{tg} \frac{x}{2} \right| + c$$

$$= \frac{1}{2} \ln |\operatorname{tg} x| - \frac{3}{2} \ln \left| \frac{\operatorname{tg} \frac{x}{2} - 1}{\operatorname{tg} \frac{x}{2} + 1} \right| + \ln \left| \operatorname{tg} \frac{x}{2} \right| + c$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3 \sin x} dx = \frac{1}{3} \ln(4+3 \sin x) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=4+3 \sin x \mid x \in \langle -\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi \rangle \\ dt=3 \cos x dx \mid t \in (1; 7), k \in \mathbb{Z} \end{array} \right] = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln t + c$$

$$= \frac{1}{3} \ln(4+3 \sin x) + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \frac{1+3 \sin x+2 \cos x}{\sin 2x} dx = \frac{1}{2} \ln |\operatorname{tg} x| - \frac{3}{2} \ln \left| \operatorname{tg} \frac{x}{2} - 1 \right| + \frac{3}{2} \ln \left| \operatorname{tg} \frac{x}{2} + 1 \right| + \ln \left| \operatorname{tg} \frac{x}{2} \right| + c$$

$$= \int \frac{dx}{\sin 2x} + \int \frac{3 \sin x+2 \cos x}{2 \sin x \cos x} dx = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in (0+\frac{k\pi}{2}; \frac{\pi}{2}+\frac{k\pi}{2}) = (\frac{k\pi}{2}; (k+1)\frac{\pi}{2}) \mid x \neq \frac{k\pi}{2} \\ du=2 dx \mid u \in (0+k\pi; \pi+k\pi) = (k\pi; (k+1)\pi) \mid k \in \mathbb{Z} \end{array} \right]$$

$$= \frac{1}{2} \int \frac{du}{\sin u} + \frac{3}{2} \int \frac{dx}{\cos x} + \frac{dx}{\sin x} = \left[\begin{array}{l} \text{Pr. 01} \\ \text{Pr. 02} \end{array} \right] = \frac{1}{2} \ln \left| \operatorname{tg} \frac{u}{2} \right| - \frac{3}{2} \ln \left| \frac{\operatorname{tg} \frac{x}{2} - 1}{\operatorname{tg} \frac{x}{2} + 1} \right| + \ln \left| \operatorname{tg} \frac{x}{2} \right| + c$$

$$= \frac{1}{2} \ln |\operatorname{tg} x| - \frac{3}{2} \ln \left| \frac{\operatorname{tg} \frac{x}{2} - 1}{\operatorname{tg} \frac{x}{2} + 1} \right| + \ln \left| \operatorname{tg} \frac{x}{2} \right| + c$$

$$= \frac{1}{2} \ln |\operatorname{tg} x| - \frac{3}{2} \ln \left| \operatorname{tg} \frac{x}{2} - 1 \right| + \frac{3}{2} \ln \left| \operatorname{tg} \frac{x}{2} + 1 \right| + \ln \left| \operatorname{tg} \frac{x}{2} \right| + c,$$

$$x \in \mathbb{R} - \left\{ \frac{k\pi}{2}, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x}$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x] dx$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x}$$

$$= \int \frac{4 dx}{(2 \sin x \cos x)^2}$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x] dx$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) dx$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x}$$

$$= \int \frac{4 dx}{(2 \sin x \cos x)^2} = \int \frac{2 \cdot 2 dx}{\sin^2 2x}$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \frac{dx}{\sin^2 x} + \int \frac{dx}{\cos^2 x}$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x] dx$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) dx = \int \left(\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x} \right) dx$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c$$

$$= \int \frac{4 dx}{(2 \sin x \cos x)^2} = \int \frac{2 \cdot 2 dx}{\sin^2 2x} = \left[\begin{array}{l} \text{Subst. } t=2x \mid x \in (0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}) = (\frac{k\pi}{2}; (k+1)\frac{\pi}{2}) \mid x \neq \frac{k\pi}{2} \\ dt = 2 dx \mid t \in (0 + k\pi; \pi + k\pi) = (k\pi; (k+1)\pi) \mid k \in \mathbb{Z} \end{array} \right]$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \frac{dx}{\sin^2 x} + \int \frac{dx}{\cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x] dx$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) dx = \int \left(\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x} \right) dx = \ln |\sin x| - \ln |\cos x| + c$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c$$

$$= \int \frac{4 dx}{(2 \sin x \cos x)^2} = \int \frac{2 \cdot 2 dx}{\sin^2 2x} = \left[\text{Subst. } t=2x \mid x \in \left(0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}\right) = \left(\frac{k\pi}{2}; (k+1)\frac{\pi}{2}\right) \mid x \neq \frac{k\pi}{2} \right. \\ \left. dt = 2 dx \mid t \in (0+k\pi; \pi+k\pi) = (k\pi; (k+1)\pi) \mid k \in \mathbb{Z} \right]$$

$$= \int \frac{2 dt}{\sin^2 t}$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \frac{dx}{\sin^2 x} + \int \frac{dx}{\cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c = \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} + c$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x] dx$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) dx = \int \left(\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x} \right) dx = \ln |\sin x| - \ln |\cos x| + c$$

$$= \ln \left| \frac{\sin x}{\cos x} \right| + c$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c$$

$$= \int \frac{4 dx}{(2 \sin x \cos x)^2} = \int \frac{2 \cdot 2 dx}{\sin^2 2x} = \left[\text{Subst. } t=2x \mid x \in \left(0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}\right) = \left(\frac{k\pi}{2}; (k+1)\frac{\pi}{2}\right) \mid x \neq \frac{k\pi}{2} \right. \\ \left. dt = 2 dx \mid t \in (0+k\pi; \pi+k\pi) = (k\pi; (k+1)\pi) \mid k \in \mathbb{Z} \right]$$

$$= \int \frac{2 dt}{\sin^2 t} = -2 \operatorname{cotg} t + c$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \frac{dx}{\sin^2 x} + \int \frac{dx}{\cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c = \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} + c \\ = 2 \frac{\sin^2 x - \cos^2 x}{2 \sin x \cos x} + c$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x] dx = \ln |\operatorname{tg} x| + c$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) dx = \int \left(\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x} \right) dx = \ln |\sin x| - \ln |\cos x| + c$$

$$= \ln \left| \frac{\sin x}{\cos x} \right| + c = \ln |\operatorname{tg} x| + c, x \in \mathbb{R} - \left\{ \frac{k\pi}{2}, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c = -2 \operatorname{cotg} 2x + c$$

$$= \int \frac{4 dx}{(2 \sin x \cos x)^2} = \int \frac{2 \cdot 2 dx}{\sin^2 2x} = \left[\text{Subst. } t=2x \mid x \in (0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}) = (\frac{k\pi}{2}; (k+1)\frac{\pi}{2}) \mid x \neq \frac{k\pi}{2} \right]$$

$$dt = 2 dx \mid t \in (0 + k\pi; \pi + k\pi) = (k\pi; (k+1)\pi) \mid k \in \mathbb{Z}$$

$$= \int \frac{2 dt}{\sin^2 t} = -2 \operatorname{cotg} t + c = -2 \operatorname{cotg} 2x + c, x \in \mathbb{R} - \left\{ \frac{k\pi}{2}, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \frac{dx}{\sin^2 x} + \int \frac{dx}{\cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c = \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} + c$$

$$= 2 \frac{\sin^2 x - \cos^2 x}{2 \sin x \cos x} + c = 2 \frac{-\cos 2x}{\sin 2x} + c$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x] dx = \ln |\operatorname{tg} x| + c$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) dx = \int \left(\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x} \right) dx = \ln |\sin x| - \ln |\cos x| + c$$

$$= \ln \left| \frac{\sin x}{\cos x} \right| + c = \ln |\operatorname{tg} x| + c, x \in \mathbb{R} - \left\{ \frac{k\pi}{2}, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c = -2 \operatorname{cotg} 2x + c$$

$$= \int \frac{4 dx}{(2 \sin x \cos x)^2} = \int \frac{2 \cdot 2 dx}{\sin^2 2x} = \left[\text{Subst. } t=2x \mid x \in (0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}) = (\frac{k\pi}{2}; (k+1)\frac{\pi}{2}) \mid x \neq \frac{k\pi}{2} \right. \\ \left. dt = 2 dx \mid t \in (0 + k\pi; \pi + k\pi) = (k\pi; (k+1)\pi) \mid k \in \mathbb{Z} \right]$$

$$= \int \frac{2 dt}{\sin^2 t} = -2 \operatorname{cotg} t + c = -2 \operatorname{cotg} 2x + c, x \in \mathbb{R} - \left\{ \frac{k\pi}{2}, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \frac{dx}{\sin^2 x} + \int \frac{dx}{\cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c = \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} + c$$

$$= 2 \frac{\sin^2 x - \cos^2 x}{2 \sin x \cos x} + c = 2 \frac{-\cos 2x}{\sin 2x} + c = c - 2 \operatorname{cotg} 2x, x \in \mathbb{R} - \left\{ \frac{k\pi}{2}, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x] dx = \ln |\operatorname{tg} x| + c$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) dx = \int \left(\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x} \right) dx = \ln |\sin x| - \ln |\cos x| + c$$

$$= \ln \left| \frac{\sin x}{\cos x} \right| + c = \ln |\operatorname{tg} x| + c, x \in \mathbb{R} - \left\{ \frac{k\pi}{2}, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] \text{ ☞}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] \stackrel{!}{=} \int \frac{2 dx}{(a^2 + b^2) + (a^2 - b^2) \cos 2x}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] \stackrel{?}{=} \int \frac{2 dx}{(a^2 + b^2) + (a^2 - b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] = \int \frac{2 dx}{(a^2 + b^2) + (a^2 - b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 + b^2) + (a^2 - b^2) \cos u}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] \stackrel{!}{=} \int \frac{2 dx}{(a^2 + b^2) + (a^2 - b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 + b^2) + (a^2 - b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ du = \frac{2 dt}{t^2 + 1} \mid \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right]$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] = \int \frac{2 dx}{(a^2 + b^2) + (a^2 - b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 + b^2) + (a^2 - b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ \cos u = \frac{1-t^2}{t^2+1} \end{array} \right| \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2 + b^2) + (a^2 - b^2) \frac{1-t^2}{t^2+1}}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] = \int \frac{2 dx}{(a^2 + b^2) + (a^2 - b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 + b^2) + (a^2 - b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \tan \frac{u}{2} = \tan x \\ du = \frac{2 dt}{t^2 + 1} \end{array} \right| \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2 + 1}}{(a^2 + b^2) + (a^2 - b^2) \frac{1-t^2}{t^2 + 1}}$$

$$= \int \frac{2 dt}{(a^2 + b^2)(t^2 + 1) + (a^2 - b^2)(1 - t^2)}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] = \int \frac{2 dx}{(a^2 + b^2) + (a^2 - b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 + b^2) + (a^2 - b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ du = \frac{2 dt}{t^2 + 1} \end{array} \right| \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2 + 1}}{(a^2 + b^2) + (a^2 - b^2) \frac{1-t^2}{t^2 + 1}}$$

$$= \int \frac{2 dt}{(a^2 + b^2)(t^2 + 1) + (a^2 - b^2)(1 - t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 + b^2 t^2 + b^2) + (a^2 - a^2 t^2 - b^2 + b^2 t^2)}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] = \int \frac{2 dx}{(a^2 + b^2) + (a^2 - b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 + b^2) + (a^2 - b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ du = \frac{2 dt}{t^2 + 1} \end{array} \right| \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ \cos u = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2 + b^2) + (a^2 - b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2 + b^2)(t^2 + 1) + (a^2 - b^2)(1 - t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 + b^2 t^2 + b^2) + (a^2 - a^2 t^2 - b^2 + b^2 t^2)}$$

$$= \int \frac{2 dt}{2b^2 t^2 + 2a^2}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] = \int \frac{2 dx}{(a^2 + b^2) + (a^2 - b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 + b^2) + (a^2 - b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ du = \frac{2 dt}{t^2 + 1} \end{array} \right| \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2 + 1}}{(a^2 + b^2) + (a^2 - b^2) \frac{1-t^2}{t^2 + 1}}$$

$$= \int \frac{2 dt}{(a^2 + b^2)(t^2 + 1) + (a^2 - b^2)(1 - t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 + b^2 t^2 + b^2) + (a^2 - a^2 t^2 - b^2 + b^2 t^2)}$$

$$= \int \frac{2 dt}{2b^2 t^2 + 2a^2} = \int \frac{dt}{b^2 t^2 + a^2}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] = \int \frac{2 dx}{(a^2 + b^2) + (a^2 - b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 + b^2) + (a^2 - b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ du = \frac{2 dt}{t^2 + 1} \end{array} \right| \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ \cos u = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2 + b^2) + (a^2 - b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2 + b^2)(t^2 + 1) + (a^2 - b^2)(1 - t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 + b^2 t^2 + b^2) + (a^2 - a^2 t^2 - b^2 + b^2 t^2)}$$

$$= \int \frac{2 dt}{2b^2 t^2 + 2a^2} = \int \frac{dt}{b^2 t^2 + a^2} = \frac{1}{b^2} \int \frac{dt}{t^2 + \frac{a^2}{b^2}}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] = \int \frac{2 dx}{(a^2 + b^2) + (a^2 - b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 + b^2) + (a^2 - b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ du = \frac{2 dt}{t^2 + 1} \end{array} \right| \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ \cos u = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2 + b^2) + (a^2 - b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2 + b^2)(t^2 + 1) + (a^2 - b^2)(1 - t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 + b^2 t^2 + b^2) + (a^2 - a^2 t^2 - b^2 + b^2 t^2)}$$

$$= \int \frac{2 dt}{2b^2 t^2 + 2a^2} = \int \frac{dt}{b^2 t^2 + a^2} = \frac{1}{b^2} \int \frac{dt}{t^2 + \frac{a^2}{b^2}} = \frac{1}{b^2} \int \frac{dt}{t^2 + \left(\frac{a}{b}\right)^2}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] = \int \frac{2 dx}{(a^2+b^2) + (a^2-b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2+b^2) + (a^2-b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ du = \frac{2 dt}{t^2+1} \end{array} \right| \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ \cos u = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2+b^2) + (a^2-b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2+b^2)(t^2+1) + (a^2-b^2)(1-t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 + b^2 t^2 + b^2) + (a^2 - a^2 t^2 - b^2 + b^2 t^2)}$$

$$= \int \frac{2 dt}{2b^2 t^2 + 2a^2} = \int \frac{dt}{b^2 t^2 + a^2} = \frac{1}{b^2} \int \frac{dt}{t^2 + \frac{a^2}{b^2}} = \frac{1}{b^2} \int \frac{dt}{t^2 + \left(\frac{a}{b}\right)^2}$$

$$= \frac{1}{b^2} \cdot \frac{b}{a} \operatorname{arctg} \frac{t}{\frac{a}{b}} + c$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] = \int \frac{2 dx}{(a^2+b^2) + (a^2-b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2+b^2) + (a^2-b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ du = \frac{2 dt}{t^2+1} \end{array} \right| \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ \cos u = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2+b^2) + (a^2-b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2+b^2)(t^2+1) + (a^2-b^2)(1-t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 + b^2 t^2 + b^2) + (a^2 - a^2 t^2 - b^2 + b^2 t^2)}$$

$$= \int \frac{2 dt}{2b^2 t^2 + 2a^2} = \int \frac{dt}{b^2 t^2 + a^2} = \frac{1}{b^2} \int \frac{dt}{t^2 + \frac{a^2}{b^2}} = \frac{1}{b^2} \int \frac{dt}{t^2 + \left(\frac{a}{b}\right)^2}$$

$$= \frac{1}{b^2} \cdot \frac{b}{a} \operatorname{arctg} \frac{t}{\frac{a}{b}} + c = \frac{1}{ab} \operatorname{arctg} \left(\frac{b}{a} t\right) + c$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in \mathbb{R} \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] = \int \frac{2 dx}{(a^2+b^2) + (a^2-b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2+b^2) + (a^2-b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left. \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ du = \frac{2 dt}{t^2+1} \end{array} \right| \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2+b^2) + (a^2-b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2+b^2)(t^2+1) + (a^2-b^2)(1-t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 + b^2 t^2 + b^2) + (a^2 - a^2 t^2 - b^2 + b^2 t^2)}$$

$$= \int \frac{2 dt}{2b^2 t^2 + 2a^2} = \int \frac{dt}{b^2 t^2 + a^2} = \frac{1}{b^2} \int \frac{dt}{t^2 + \frac{a^2}{b^2}} = \frac{1}{b^2} \int \frac{dt}{t^2 + \left(\frac{a}{b}\right)^2}$$

$$= \frac{1}{b^2} \cdot \frac{b}{a} \operatorname{arctg} \frac{t}{\frac{a}{b}} + c = \frac{1}{ab} \operatorname{arctg} \left(\frac{b}{a} t\right) + c = \frac{1}{ab} \operatorname{arctg} \left(\frac{b}{a} \operatorname{tg} \frac{u}{2}\right) + c$$

Riešené príklady – 035

$$\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} = \frac{1}{ab} \operatorname{arctg} \left(\frac{b}{a} \operatorname{tg} x \right) + c \quad a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} \text{Pre všetky } x \in R \\ a^2 \cos^2 x + b^2 \sin^2 x > 0 \end{array} \right] = \int \frac{2 dx}{(a^2+b^2) + (a^2-b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in R \\ du=2dx \mid u \in R \end{array} \right] = \int \frac{du}{(a^2+b^2) + (a^2-b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in R \right. \\ \left. du = \frac{2dt}{t^2+1} \mid \cos u = \frac{1-t^2}{t^2+1} \mid x \in \left(-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi\right), k \in Z \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2+b^2) + (a^2-b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2+b^2)(t^2+1) + (a^2-b^2)(1-t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 + b^2 t^2 + b^2) + (a^2 - a^2 t^2 - b^2 + b^2 t^2)}$$

$$= \int \frac{2 dt}{2b^2 t^2 + 2a^2} = \int \frac{dt}{b^2 t^2 + a^2} = \frac{1}{b^2} \int \frac{dt}{t^2 + \frac{a^2}{b^2}} = \frac{1}{b^2} \int \frac{dt}{t^2 + \left(\frac{a}{b}\right)^2}$$

$$= \frac{1}{b^2} \cdot \frac{b}{a} \operatorname{arctg} \frac{t}{\frac{a}{b}} + c = \frac{1}{ab} \operatorname{arctg} \left(\frac{b}{a} t \right) + c = \frac{1}{ab} \operatorname{arctg} \left(\frac{b}{a} \operatorname{tg} \frac{u}{2} \right) + c$$

$$= \frac{1}{ab} \operatorname{arctg} \left(\frac{b}{a} \operatorname{tg} x \right) + c, \quad x \in R - \left\{ \frac{\pi}{2} + k\pi, k \in Z \right\}, \quad c \in R.$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$



Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg} \frac{a}{b} \right]$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

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$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

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Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

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$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 - b^2) + (a^2 + b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS:} \\ du = \frac{2 dt}{t^2 + 1} \mid \cos u = \frac{1-t^2}{t^2+1} \end{array} \left| \begin{array}{l} t = \tg \frac{u}{2} = \tg x \\ u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \right]$$

Riešené príklady – 036

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$$= \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \tg \frac{u}{2} = \tg x \\ \cos u = \frac{1-t^2}{t^2+1} \end{array} \right. \mid \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2 - b^2) + (a^2 + b^2) \frac{1-t^2}{t^2+1}}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

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$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 - b^2) + (a^2 + b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| t = \tg \frac{u}{2} = \tg x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \right. \\ \left. du = \frac{2 dt}{t^2 + 1} \mid \cos u = \frac{1-t^2}{t^2 + 1} \mid x \in \left(-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi\right), k \in \mathbb{Z} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2 + 1}}{(a^2 - b^2) + (a^2 + b^2) \frac{1-t^2}{t^2 + 1}}$$

$$= \int \frac{2 dt}{(a^2 - b^2)(t^2 + 1) + (a^2 + b^2)(1 - t^2)}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

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$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 - b^2) + (a^2 + b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ \cos u = \frac{1-t^2}{t^2+1} \end{array} \right. \mid \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2 - b^2) + (a^2 + b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2 - b^2)(t^2+1) + (a^2 + b^2)(1-t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 - b^2 t^2 - b^2) + (a^2 - a^2 t^2 + b^2 - b^2 t^2)}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 - b^2) + (a^2 + b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| t = \tg \frac{u}{2} = \tg x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \right. \\ \left. du = \frac{2 dt}{t^2 + 1} \mid \cos u = \frac{1-t^2}{t^2 + 1} \mid x \in \left(-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi\right), k \in \mathbb{Z} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2 + 1}}{(a^2 - b^2) + (a^2 + b^2) \frac{1-t^2}{t^2 + 1}}$$

$$= \int \frac{2 dt}{(a^2 - b^2)(t^2 + 1) + (a^2 + b^2)(1 - t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 - b^2 t^2 - b^2) + (a^2 - a^2 t^2 + b^2 - b^2 t^2)}$$

$$= \int \frac{2 dt}{-2b^2 t^2 + 2a^2}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 - b^2) + (a^2 + b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ du = \frac{2dt}{t^2+1} \mid \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{(a^2 - b^2) + (a^2 + b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2 - b^2)(t^2+1) + (a^2 + b^2)(1-t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 - b^2 t^2 - b^2) + (a^2 - a^2 t^2 + b^2 - b^2 t^2)}$$

$$= \int \frac{2 dt}{-2b^2 t^2 + 2a^2} = \int \frac{dt}{-b^2 t^2 + a^2}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 - b^2) + (a^2 + b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ du = \frac{2 dt}{t^2 + 1} \mid \cos u = \frac{1-t^2}{t^2 + 1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2 + 1}}{(a^2 - b^2) + (a^2 + b^2) \frac{1-t^2}{t^2 + 1}}$$

$$= \int \frac{2 dt}{(a^2 - b^2)(t^2 + 1) + (a^2 + b^2)(1 - t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 - b^2 t^2 - b^2) + (a^2 - a^2 t^2 + b^2 - b^2 t^2)}$$

$$= \int \frac{2 dt}{-2b^2 t^2 + 2a^2} = \int \frac{dt}{-b^2 t^2 + a^2} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \frac{a^2}{b^2}}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 - b^2) + (a^2 + b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \tg \frac{u}{2} = \tg x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ du = \frac{2 dt}{t^2 + 1} \mid \cos u = \frac{1-t^2}{t^2 + 1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2 + 1}}{(a^2 - b^2) + (a^2 + b^2) \frac{1-t^2}{t^2 + 1}}$$

$$= \int \frac{2 dt}{(a^2 - b^2)(t^2 + 1) + (a^2 + b^2)(1 - t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 - b^2 t^2 - b^2) + (a^2 - a^2 t^2 + b^2 - b^2 t^2)}$$

$$= \int \frac{2 dt}{-2b^2 t^2 + 2a^2} = \int \frac{dt}{-b^2 t^2 + a^2} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \frac{a^2}{b^2}} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \left(\frac{a}{b}\right)^2}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 - b^2) + (a^2 + b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \tg \frac{u}{2} = \tg x \\ \cos u = \frac{1-t^2}{t^2+1} \end{array} \right. \mid \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2 - b^2) + (a^2 + b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2 - b^2)(t^2+1) + (a^2 + b^2)(1-t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 - b^2 t^2 - b^2) + (a^2 - a^2 t^2 + b^2 - b^2 t^2)}$$

$$= \int \frac{2 dt}{-2b^2 t^2 + 2a^2} = \int \frac{dt}{-b^2 t^2 + a^2} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \frac{a^2}{b^2}} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \left(\frac{a}{b}\right)^2} = -\frac{1}{b^2} \cdot \frac{b}{2a} \ln \left| \frac{t - \frac{a}{b}}{t + \frac{a}{b}} \right| + c$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 - b^2) + (a^2 + b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ du = \frac{2 dt}{t^2 + 1} \mid \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2 - b^2) + (a^2 + b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2 - b^2)(t^2+1) + (a^2 + b^2)(1-t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 - b^2 t^2 - b^2) + (a^2 - a^2 t^2 + b^2 - b^2 t^2)}$$

$$= \int \frac{2 dt}{-2b^2 t^2 + 2a^2} = \int \frac{dt}{-b^2 t^2 + a^2} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \frac{a^2}{b^2}} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \left(\frac{a}{b}\right)^2} = -\frac{1}{b^2} \cdot \frac{b}{2a} \ln \left| \frac{t - \frac{a}{b}}{t + \frac{a}{b}} \right| + c$$

$$= -\frac{1}{2ab} \ln \left| \frac{bt-a}{bt+a} \right| + c$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 - b^2) + (a^2 + b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ du = \frac{2 dt}{t^2 + 1} \mid \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2 - b^2) + (a^2 + b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2 - b^2)(t^2+1) + (a^2 + b^2)(1-t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 - b^2 t^2 - b^2) + (a^2 - a^2 t^2 + b^2 - b^2 t^2)}$$

$$= \int \frac{2 dt}{-2b^2 t^2 + 2a^2} = \int \frac{dt}{-b^2 t^2 + a^2} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \frac{a^2}{b^2}} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \left(\frac{a}{b}\right)^2} = -\frac{1}{b^2} \cdot \frac{b}{2a} \ln \left| \frac{t - \frac{a}{b}}{t + \frac{a}{b}} \right| + c$$

$$= -\frac{1}{2ab} \ln \left| \frac{bt-a}{bt+a} \right| + c = \frac{1}{2ab} \ln \left| \frac{bt+a}{bt-a} \right| + c$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 - b^2) + (a^2 + b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ du = \frac{2 dt}{t^2 + 1} \mid \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2 - b^2) + (a^2 + b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2 - b^2)(t^2+1) + (a^2 + b^2)(1-t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 - b^2 t^2 - b^2) + (a^2 - a^2 t^2 + b^2 - b^2 t^2)}$$

$$= \int \frac{2 dt}{-2b^2 t^2 + 2a^2} = \int \frac{dt}{-b^2 t^2 + a^2} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \frac{a^2}{b^2}} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \left(\frac{a}{b}\right)^2} = -\frac{1}{b^2} \cdot \frac{b}{2a} \ln \left| \frac{t - \frac{a}{b}}{t + \frac{a}{b}} \right| + c$$

$$= -\frac{1}{2ab} \ln \left| \frac{bt-a}{bt+a} \right| + c = \frac{1}{2ab} \ln \left| \frac{bt+a}{bt-a} \right| + c = \frac{1}{2ab} \ln \left| \frac{b \operatorname{tg} \frac{u}{2} + a}{b \operatorname{tg} \frac{u}{2} - a} \right| + c$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x} = \frac{1}{2ab} \ln \left| \frac{b \operatorname{tg} x + a}{b \operatorname{tg} x - a} \right| + c \quad a > 0, b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \arctg \frac{a}{b} \right] = \int \frac{2 dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

$$= \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2dx \mid u \in \mathbb{R} \end{array} \right] = \int \frac{du}{(a^2 - b^2) + (a^2 + b^2) \cos u}$$

$$= \left[\begin{array}{l} \text{UGS: } \left| t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \right. \\ \left. du = \frac{2 dt}{t^2 + 1} \mid \cos u = \frac{1-t^2}{t^2+1} \mid x \in \left(-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi\right), k \in \mathbb{Z} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{(a^2 - b^2) + (a^2 + b^2) \frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{(a^2 - b^2)(t^2 + 1) + (a^2 + b^2)(1 - t^2)} = \int \frac{2 dt}{(a^2 t^2 + a^2 - b^2 t^2 - b^2) + (a^2 - a^2 t^2 + b^2 - b^2 t^2)}$$

$$= \int \frac{2 dt}{-2b^2 t^2 + 2a^2} = \int \frac{dt}{-b^2 t^2 + a^2} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \frac{a^2}{b^2}} = -\frac{1}{b^2} \int \frac{dt}{t^2 - \left(\frac{a}{b}\right)^2} = -\frac{1}{b^2} \cdot \frac{b}{2a} \ln \left| \frac{t - \frac{a}{b}}{t + \frac{a}{b}} \right| + c$$

$$= -\frac{1}{2ab} \ln \left| \frac{bt - a}{bt + a} \right| + c = \frac{1}{2ab} \ln \left| \frac{bt + a}{bt - a} \right| + c = \frac{1}{2ab} \ln \left| \frac{b \operatorname{tg} \frac{u}{2} + a}{b \operatorname{tg} \frac{u}{2} - a} \right| + c$$

$$= \frac{1}{2ab} \ln \left| \frac{b \operatorname{tg} x + a}{b \operatorname{tg} x - a} \right| + c, \quad x \in \mathbb{R} - \left\{ \pm \arctg \frac{a}{b}, \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}}$$

$$= \int \frac{dx}{4 \cos^2 x + (1 - \cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1}$$

$$= \int \frac{dx}{4(1 - \sin^2 x) + \sin^2 x} = \int \frac{dx}{4 - 3 \sin^2 x}$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \cos^2 x + \sin^2 x \\ = 3 \cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x + (1 - \cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1} = \int \frac{dx}{3 \frac{1+\cos 2x}{2} + \frac{2}{2}}$$

$$= \int \frac{dx}{4(1 - \sin^2 x) + \sin^2 x} = \int \frac{dx}{4 - 3 \sin^2 x} = \int \frac{dx}{\frac{8}{2} - 3 \frac{1 - \cos 2x}{2}}$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \cos^2 x + \sin^2 x \\ = 3 \cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right] = \int \frac{2 dx}{5+3 \cos 2x}$$

$$= \int \frac{dx}{4 \cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1} = \int \frac{dx}{3 \frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2 dx}{5+3 \cos 2x}$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-3 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{5+3 \cos 2x}$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \cos^2 x + \sin^2 x \\ = 3 \cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right] = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1} = \int \frac{dx}{3 \frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-3 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \cos^2 x + \sin^2 x \\ = 3 \cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right] = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1} = \int \frac{dx}{3 \frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-3 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{5+3 \cos u}$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \cos^2 x + \sin^2 x \\ = 3 \cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right] = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1} = \int \frac{dx}{3 \frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-3 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{5+3 \cos u} = \left[\begin{array}{l} \text{UGS:} \quad \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ du = \frac{2dt}{t^2+1} \mid \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right]$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \cos^2 x + \sin^2 x \\ = 3 \cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right] = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1} = \int \frac{dx}{3 \frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-3 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{5+3 \cos u} = \left[\begin{array}{l} \text{UGS:} \quad \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi), k \in \mathbb{Z} \end{array} \right. \\ du = \frac{2 dt}{t^2+1} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{5+3 \frac{1-t^2}{t^2+1}}$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \cos^2 x + \sin^2 x \\ = 3 \cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right] = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1} = \int \frac{dx}{3 \frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-3 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{5+3 \cos u} = \left[\begin{array}{l} \text{UGS:} \quad \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{5+3 \frac{1-t^2}{t^2+1}} = \int \frac{\frac{2 dt}{t^2+1}}{\frac{5t^2+5+3-3t^2}{t^2+1}}$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \cos^2 x + \sin^2 x \\ = 3 \cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right] = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1} = \int \frac{dx}{3 \frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-3 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{5+3 \cos u} = \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ u \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \end{array} \right. \\ \left. \begin{array}{l} du = \frac{2 dt}{t^2+1} \\ \cos u = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{5+3 \frac{1-t^2}{t^2+1}} = \int \frac{\frac{2 dt}{t^2+1}}{\frac{5t^2+5+3-3t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{2t^2+8}$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \cos^2 x + \sin^2 x \\ = 3 \cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right] = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1} = \int \frac{dx}{3 \frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-3 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{5+3 \cos u} = \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ \cos u = \frac{1-t^2}{t^2+1} \end{array} \right. \mid \begin{array}{l} u \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \\ x \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{5+3 \frac{1-t^2}{t^2+1}} = \int \frac{\frac{2 dt}{t^2+1}}{\frac{5t^2+5+3-3t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{2t^2+8} = \int \frac{dt}{t^2+4}$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \cos^2 x + \sin^2 x \\ = 3 \cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right] = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1} = \int \frac{dx}{3 \frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-3 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{5+3 \cos u} = \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ u \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \end{array} \right. \\ \left. \begin{array}{l} du = \frac{2dt}{t^2+1} \\ \cos u = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{5+3 \frac{1-t^2}{t^2+1}} = \int \frac{\frac{2 dt}{t^2+1}}{\frac{5t^2+5+3-3t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{2t^2+8} = \int \frac{dt}{t^2+4} = \frac{1}{2} \operatorname{arctg} \frac{t}{2} + c$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \cos^2 x + \sin^2 x \\ = 3 \cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right] = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1} = \int \frac{dx}{3 \frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-3 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{5+3 \cos u} = \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ u \in (-\pi+2k\pi; \pi+2k\pi), t \in \mathbb{R} \end{array} \right. \\ \left. \begin{array}{l} du = \frac{2dt}{t^2+1} \\ \cos u = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{5+3 \frac{1-t^2}{t^2+1}} = \int \frac{\frac{2 dt}{t^2+1}}{\frac{5t^2+5+3-3t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{2t^2+8} = \int \frac{dt}{t^2+4} = \frac{1}{2} \operatorname{arctg} \frac{t}{2} + c = \frac{1}{2} \operatorname{arctg} \frac{\operatorname{tg} \frac{u}{2}}{2} + c$$

Riešené príklady – 037

$$\int \frac{dx}{4 \cos^2 x + \sin^2 x} = \frac{1}{2} \operatorname{arctg} \frac{\operatorname{tg} x}{2} + c$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \cos^2 x + \sin^2 x \\ = 3 \cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right] = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3 \cos^2 x + 1} = \int \frac{dx}{3 \frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-3 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{5+3 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{5+3 \cos u} = \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \end{array} \right. \\ \left. \begin{array}{l} du = \frac{2 dt}{t^2+1} \\ \cos u = \frac{1-t^2}{t^2+1} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{5+3 \frac{1-t^2}{t^2+1}} = \int \frac{\frac{2 dt}{t^2+1}}{\frac{5t^2+5+3-3t^2}{t^2+1}}$$

$$= \int \frac{2 dt}{2t^2+8} = \int \frac{dt}{t^2+4} = \frac{1}{2} \operatorname{arctg} \frac{t}{2} + c = \frac{1}{2} \operatorname{arctg} \frac{\operatorname{tg} \frac{u}{2}}{2} + c = \frac{1}{2} \operatorname{arctg} \frac{\operatorname{tg} x}{2} + c,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}}$$

$$= \int \frac{dx}{4 \cos^2 x - (1 - \cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1}$$

$$= \int \frac{dx}{4(1 - \sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5 \sin^2 x}$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \arctg 2 \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x - (1 - \cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1} = \int \frac{dx}{5 \frac{1+\cos 2x}{2} - \frac{2}{2}}$$

$$= \int \frac{dx}{4(1 - \sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5 \sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5 \frac{1 - \cos 2x}{2}}$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \arctg 2 \end{array} \right] = \int \frac{2 dx}{3+5 \cos 2x}$$

$$= \int \frac{dx}{4 \cos^2 x - (1 - \cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1} = \int \frac{dx}{5 \frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2 dx}{3+5 \cos 2x}$$

$$= \int \frac{dx}{4(1 - \sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5 \sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5 \frac{1 - \cos 2x}{2}} = \int \frac{2 dx}{3+5 \cos 2x}$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \arctg 2 \end{array} \right] = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x - (1 - \cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1} = \int \frac{dx}{5 \frac{1+\cos 2x}{2} - \frac{1}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1 - \sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5 \sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5 \frac{1 - \cos 2x}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \arctg 2 \end{array} \right] = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x - (1 - \cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1} = \int \frac{dx}{5 \frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1 - \sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5 \sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5 \frac{1 - \cos 2x}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{3+5 \cos u}$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \arctg 2 \end{array} \right] = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x - (1 - \cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1} = \int \frac{dx}{5 \frac{1+\cos 2x}{2} - \frac{1}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1 - \sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5 \sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5 \frac{1 - \cos 2x}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{3+5 \cos u} = \left[\begin{array}{l} \text{UGS:} \\ du = \frac{2 dt}{t^2+1} \end{array} \left| \begin{array}{l} t = \tg \frac{u}{2} = \tg x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \right]$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \arctg 2 \end{array} \right] = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x - (1 - \cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1} = \int \frac{dx}{5 \frac{1+\cos 2x}{2} - \frac{1}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1 - \sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5 \sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5 \frac{1 - \cos 2x}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{3+5 \cos u} = \left[\begin{array}{l} \text{UGS:} \\ du = \frac{2 dt}{t^2+1} \end{array} \right] \left[\begin{array}{l} t = \tg \frac{u}{2} = \tg x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{3+5 \frac{1-t^2}{t^2+1}}$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \arctg 2 \end{array} \right] = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x - (1 - \cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1} = \int \frac{dx}{5 \frac{1+\cos 2x}{2} - \frac{1}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1 - \sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5 \sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5 \frac{1 - \cos 2x}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{3+5 \cos u} = \left[\begin{array}{l} \text{UGS:} \\ du = \frac{2 dt}{t^2+1} \end{array} \right] \left[\begin{array}{l} t = \tg \frac{u}{2} = \tg x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{3+5 \frac{1-t^2}{t^2+1}} = \int \frac{2 dt}{8-2t^2}$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \arctg 2 \end{array} \right] = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x - (1 - \cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1} = \int \frac{dx}{5 \frac{1+\cos 2x}{2} - \frac{1}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1 - \sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5 \sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5 \frac{1 - \cos 2x}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{3+5 \cos u} = \left[\begin{array}{l} \text{UGS:} \\ du = \frac{2 dt}{t^2+1} \end{array} \left| \begin{array}{l} t = \tg \frac{u}{2} = \tg x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \right] = \int \frac{\frac{2 dt}{t^2+1}}{3+5 \frac{1-t^2}{t^2+1}} = \int \frac{2 dt}{8-2t^2}$$

$$= - \int \frac{dt}{t^2-4}$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \arctg 2 \end{array} \right] = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x - (1 - \cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1} = \int \frac{dx}{5 \frac{1+\cos 2x}{2} - \frac{1}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1 - \sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5 \sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5 \frac{1 - \cos 2x}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{3+5 \cos u} = \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \tg \frac{u}{2} = \tg x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ du = \frac{2 dt}{t^2+1} \mid \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \right] = \int \frac{\frac{2 dt}{t^2+1}}{3+5 \frac{1-t^2}{t^2+1}} = \int \frac{2 dt}{8-2t^2}$$

$$= - \int \frac{dt}{t^2-4} = - \frac{1}{2 \cdot 2} \ln \left| \frac{t-2}{t+2} \right| + c$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \arctg 2 \end{array} \right] = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1} = \int \frac{dx}{5 \frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4-5 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-5 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{3+5 \cos u} = \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \\ \cos u = \frac{1-t^2}{t^2+1} \end{array} \right. \mid \begin{array}{l} u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{3+5 \frac{1-t^2}{t^2+1}} = \int \frac{2 dt}{8-2t^2}$$

$$= - \int \frac{dt}{t^2-4} = -\frac{1}{2 \cdot 2} \ln \left| \frac{t-2}{t+2} \right| + C = \frac{1}{4} \ln \left| \frac{t+2}{t-2} \right| + C$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \arctg 2 \end{array} \right] = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1} = \int \frac{dx}{5 \frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4-5 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-5 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{3+5 \cos u} = \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \\ du = \frac{2 dt}{t^2+1} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{3+5 \frac{1-t^2}{t^2+1}} = \int \frac{2 dt}{8-2t^2}$$

$$= - \int \frac{dt}{t^2-4} = - \frac{1}{2 \cdot 2} \ln \left| \frac{t-2}{t+2} \right| + C = \frac{1}{4} \ln \left| \frac{t+2}{t-2} \right| + C = \frac{1}{4} \ln \left| \frac{\operatorname{tg} \frac{u}{2} + 2}{\operatorname{tg} \frac{u}{2} - 2} \right| + C$$

Riešené príklady – 038

$$\int \frac{dx}{4 \cos^2 x - \sin^2 x} = \frac{1}{4} \ln \left| \frac{\operatorname{tg} x + 2}{\operatorname{tg} x - 2} \right| + c$$

$$= \int \frac{dx}{4 \frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \arctg 2 \end{array} \right] = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4 \cos^2 x - (1 - \cos^2 x)} = \int \frac{dx}{5 \cos^2 x - 1} = \int \frac{dx}{5 \frac{1+\cos 2x}{2} - \frac{1}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4-5 \sin^2 x} = \int \frac{dx}{\frac{8}{2}-5 \frac{1-\cos 2x}{2}} = \int \frac{2 dx}{3+5 \cos 2x} = \left[\begin{array}{l} \text{Subst. } u=2x \mid x \in \mathbb{R} \\ du=2 dx \mid u \in \mathbb{R} \end{array} \right]$$

$$= \int \frac{du}{3+5 \cos u} = \left[\begin{array}{l} \text{UGS: } \left| \begin{array}{l} t = \operatorname{tg} \frac{u}{2} = \operatorname{tg} x \mid u \in (-\pi + 2k\pi; \pi + 2k\pi), t \in \mathbb{R} \\ du = \frac{2 dt}{t^2+1} \mid \cos u = \frac{1-t^2}{t^2+1} \mid x \in (-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \end{array} \right. \right] = \int \frac{\frac{2 dt}{t^2+1}}{3+5 \frac{1-t^2}{t^2+1}} = \int \frac{2 dt}{8-2t^2}$$

$$= - \int \frac{dt}{t^2-4} = - \frac{1}{2 \cdot 2} \ln \left| \frac{t-2}{t+2} \right| + c = \frac{1}{4} \ln \left| \frac{t+2}{t-2} \right| + c = \frac{1}{4} \ln \left| \frac{\operatorname{tg} \frac{u}{2} + 2}{\operatorname{tg} \frac{u}{2} - 2} \right| + c$$

$$= \frac{1}{4} \ln \left| \frac{\operatorname{tg} x + 2}{\operatorname{tg} x - 2} \right| + c, \quad x \in \mathbb{R} - \left\{ \pm \arctg 2, \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}, \quad c \in \mathbb{R}.$$

Riešené príklady – 039, 040

$$\int \frac{x^2 dx}{\sin x^3}$$

$$\int x^2 \sin x^3 dx$$

Riešené príklady – 039, 040

$$\int \frac{x^2 dx}{\sin x^3}$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \\ du=3x^2 dx \end{array} \left| \begin{array}{l} x \neq \pm\sqrt[3]{k\pi}, u \neq \pm k\pi \\ k=0, 1, 2, 3, 4, \dots \end{array} \right. \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \\ dt=3x^2 dx \end{array} \left| \begin{array}{l} x \neq \pm\sqrt[3]{k\pi}, t \neq \pm k\pi \\ k=0, 1, 2, 3, 4, \dots \end{array} \right. \right]$$

$$\int x^2 \sin x^3 dx$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \\ du=3x^2 dx \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ u \in \mathbb{R} \end{array} \right. \right]$$

Riešené príklady – 039, 040

$$\int \frac{x^2 dx}{\sin x^3}$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \\ du=3x^2 dx \end{array} \left| \begin{array}{l} x \neq \pm\sqrt[3]{k\pi}, u \neq \pm k\pi \\ k=0, 1, 2, 3, 4, \dots \end{array} \right. \right] = \frac{1}{3} \int \frac{du}{\sin u}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \\ dt=3x^2 dx \end{array} \left| \begin{array}{l} x \neq \pm\sqrt[3]{k\pi}, t \neq \pm k\pi \\ k=0, 1, 2, 3, 4, \dots \end{array} \right. \right] = \frac{1}{3} \int \frac{dt}{\sin t}$$

$$\int x^2 \sin x^3 dx$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \\ du=3x^2 dx \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ u \in \mathbb{R} \end{array} \right. \right] = \frac{1}{3} \int \sin u du$$

Riešené príklady – 039, 040

$$\int \frac{x^2 dx}{\sin x^3}$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \\ du=3x^2 dx \end{array} \left| \begin{array}{l} x \neq \pm\sqrt[3]{k\pi}, u \neq \pm k\pi \\ k=0, 1, 2, 3, 4, \dots \end{array} \right. \right] = \frac{1}{3} \int \frac{du}{\sin u} = \left[\begin{array}{l} \text{UGS:} \\ t=\text{tg } \frac{u}{2} \end{array} \left| \begin{array}{l} du = \frac{2dt}{t^2+1} \\ \sin u = \frac{2}{t^2+1} \end{array} \right. \left. \begin{array}{l} u \in (-\pi+2l\pi; 2l\pi), t \in (-\infty; 0) \\ u \in (2l\pi; \pi+2l\pi), t \in (0; \infty), l \in \mathbb{Z} \end{array} \right. \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \\ dt=3x^2 dx \end{array} \left| \begin{array}{l} x \neq \pm\sqrt[3]{k\pi}, t \neq \pm k\pi \\ k=0, 1, 2, 3, 4, \dots \end{array} \right. \right] = \frac{1}{3} \int \frac{dt}{\sin t} = \frac{1}{3} \int \frac{\sin t dt}{\sin^2 t}$$

$$\int x^2 \sin x^3 dx$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \\ du=3x^2 dx \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ u \in \mathbb{R} \end{array} \right. \right] = \frac{1}{3} \int \sin u du = -\frac{1}{3} \cos u + c$$

Riešené príklady – 039, 040

$$\int \frac{x^2 dx}{\sin x^3}$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \\ du=3x^2 dx \end{array} \left| \begin{array}{l} x \neq \pm\sqrt[3]{k\pi}, u \neq \pm k\pi \\ k=0, 1, 2, 3, 4, \dots \end{array} \right. \right] = \frac{1}{3} \int \frac{du}{\sin u} = \left[\begin{array}{l} \text{UGS:} \\ t=\text{tg } \frac{u}{2} \end{array} \left| \begin{array}{l} du = \frac{2dt}{t^2+1} \\ \sin u = \frac{2}{t^2+1} \end{array} \right. \left. \begin{array}{l} u \in (-\pi+2l\pi; 2l\pi), t \in (-\infty; 0) \\ u \in (2l\pi; \pi+2l\pi), t \in (0; \infty), l \in \mathbb{Z} \end{array} \right] \right.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \\ dt=3x^2 dx \end{array} \left| \begin{array}{l} x \neq \pm\sqrt[3]{k\pi}, t \neq \pm k\pi \\ k=0, 1, 2, 3, 4, \dots \end{array} \right. \right] = \frac{1}{3} \int \frac{dt}{\sin t} = \frac{1}{3} \int \frac{\sin t dt}{\sin^2 t} = \frac{1}{3} \int \frac{\sin t dt}{1-\cos^2 t}$$

$$\int x^2 \sin x^3 dx = -\frac{1}{3} \cos x^3 + c$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \\ du=3x^2 dx \end{array} \left| \begin{array}{l} x \in \mathbb{R} \\ u \in \mathbb{R} \end{array} \right. \right] = \frac{1}{3} \int \sin u du = -\frac{1}{3} \cos u + c = -\frac{1}{3} \cos x^3 + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 039, 040

$$\int \frac{x^2 dx}{\sin x^3}$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \mid x \neq \pm\sqrt[3]{k\pi}, u \neq \pm k\pi \\ du=3x^2 dx \mid k=0, 1, 2, 3, 4, \dots \end{array} \right] = \frac{1}{3} \int \frac{du}{\sin u} = \left[\begin{array}{l} \text{UGS: } \mid du = \frac{2dt}{t^2+1} \mid u \in (-\pi+2l\pi; 2l\pi), t \in (-\infty; 0) \\ t = \text{tg } \frac{u}{2} \mid \sin u = \frac{2}{t^2+1} \mid u \in (2l\pi; \pi+2l\pi), t \in (0; \infty), l \in \mathbb{Z} \end{array} \right]$$

$$= \frac{1}{3} \int \frac{1+t^2}{2t} \frac{2 du}{1+t^2}$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \neq \pm\sqrt[3]{k\pi}, t \neq \pm k\pi \\ dt=3x^2 dx \mid k=0, 1, 2, 3, 4, \dots \end{array} \right] = \frac{1}{3} \int \frac{dt}{\sin t} = \frac{1}{3} \int \frac{\sin t dt}{\sin^2 t} = \frac{1}{3} \int \frac{\sin t dt}{1-\cos^2 t} = \frac{1}{3} \int \frac{-\sin t dt}{\cos^2 t - 1}$$

$$\int x^2 \sin x^3 dx = -\frac{1}{3} \cos x^3 + c$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \mid x \in \mathbb{R} \\ du=3x^2 dx \mid u \in \mathbb{R} \end{array} \right] = \frac{1}{3} \int \sin u du = -\frac{1}{3} \cos u + c = -\frac{1}{3} \cos x^3 + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 039, 040

$$\int \frac{x^2 dx}{\sin x^3}$$

$$= \left[\text{Subst. } u=x^3 \left| \begin{array}{l} x \neq \pm\sqrt[3]{k\pi}, u \neq \pm k\pi \\ du=3x^2 dx \\ k=0, 1, 2, 3, 4, \dots \end{array} \right. \right] = \frac{1}{3} \int \frac{du}{\sin u} = \left[\text{UGS: } \left| \begin{array}{l} du = \frac{2dt}{t^2+1} \\ t = \text{tg } \frac{u}{2} \\ \sin u = \frac{2}{t^2+1} \\ u \in (-\pi+2l\pi; 2l\pi), t \in (-\infty; 0) \\ u \in (2l\pi; \pi+2l\pi), t \in (0; \infty), l \in \mathbb{Z} \end{array} \right. \right]$$

$$= \frac{1}{3} \int \frac{1+t^2}{2t} \frac{2 du}{1+t^2} = \frac{1}{3} \int \frac{dt}{t}$$

$$= \left[\text{Subst. } t=x^3 \left| \begin{array}{l} x \neq \pm\sqrt[3]{k\pi}, t \neq \pm k\pi \\ dt=3x^2 dx \\ k=0, 1, 2, 3, 4, \dots \end{array} \right. \right] = \frac{1}{3} \int \frac{dt}{\sin t} = \frac{1}{3} \int \frac{\sin t dt}{\sin^2 t} = \frac{1}{3} \int \frac{\sin t dt}{1-\cos^2 t} = \frac{1}{3} \int \frac{-\sin t dt}{\cos^2 t - 1}$$

$$= \left[\text{Subst. } u=\cos t \left| \begin{array}{l} t \neq \pm k\pi, u \neq \pm 1 \\ du = -\sin t dt \\ k=0, 1, 2, 3, \dots \end{array} \right. \right]$$

$$\int x^2 \sin x^3 dx = -\frac{1}{3} \cos x^3 + c$$

$$= \left[\text{Subst. } u=x^3 \left| \begin{array}{l} x \in \mathbb{R} \\ du=3x^2 dx \\ u \in \mathbb{R} \end{array} \right. \right] = \frac{1}{3} \int \sin u du = -\frac{1}{3} \cos u + c = -\frac{1}{3} \cos x^3 + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 039, 040

$$\int \frac{x^2 dx}{\sin x^3}$$

$$= \left[\text{Subst. } u=x^3 \left| \begin{array}{l} x \neq \pm\sqrt[3]{k\pi}, u \neq \pm k\pi \\ du=3x^2 dx \\ k=0, 1, 2, 3, 4, \dots \end{array} \right. \right] = \frac{1}{3} \int \frac{du}{\sin u} = \left[\text{UGS: } \left| \begin{array}{l} du = \frac{2dt}{t^2+1} \\ t = \text{tg } \frac{u}{2} \\ \sin u = \frac{2}{t^2+1} \\ u \in (-\pi+2l\pi; 2l\pi), t \in (-\infty; 0) \\ u \in (2l\pi; \pi+2l\pi), t \in (0; \infty), l \in \mathbb{Z} \end{array} \right. \right]$$

$$= \frac{1}{3} \int \frac{1+t^2}{2t} \frac{2du}{1+t^2} = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln |t| + c_1$$

$$= \left[\text{Subst. } t=x^3 \left| \begin{array}{l} x \neq \pm\sqrt[3]{k\pi}, t \neq \pm k\pi \\ dt=3x^2 dx \\ k=0, 1, 2, 3, 4, \dots \end{array} \right. \right] = \frac{1}{3} \int \frac{dt}{\sin t} = \frac{1}{3} \int \frac{\sin t dt}{\sin^2 t} = \frac{1}{3} \int \frac{\sin t dt}{1-\cos^2 t} = \frac{1}{3} \int \frac{-\sin t dt}{\cos^2 t - 1}$$

$$= \left[\text{Subst. } u=\cos t \left| \begin{array}{l} t \neq \pm k\pi, u \neq \pm 1 \\ du = -\sin t dt \\ k=0, 1, 2, 3, \dots \end{array} \right. \right] = \frac{1}{3} \int \frac{du}{u^2-1}$$

$$\int x^2 \sin x^3 dx = -\frac{1}{3} \cos x^3 + c$$

$$= \left[\text{Subst. } u=x^3 \left| \begin{array}{l} x \in \mathbb{R} \\ du=3x^2 dx \\ u \in \mathbb{R} \end{array} \right. \right] = \frac{1}{3} \int \sin u du = -\frac{1}{3} \cos u + c = -\frac{1}{3} \cos x^3 + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 039, 040

$$\int \frac{x^2 dx}{\sin x^3} = \frac{1}{3} \ln \left| \operatorname{tg} \frac{x^3}{2} \right| + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \mid x \neq \pm\sqrt[3]{k\pi}, u \neq \pm k\pi \\ du=3x^2 dx \mid k=0, 1, 2, 3, 4, \dots \end{array} \right] = \frac{1}{3} \int \frac{du}{\sin u} = \left[\begin{array}{l} \text{UGS: } \mid du = \frac{2dt}{t^2+1} \mid u \in (-\pi+2l\pi; 2l\pi), t \in (-\infty; 0) \\ t = \operatorname{tg} \frac{u}{2} \mid \sin u = \frac{2}{t^2+1} \mid u \in (2l\pi; \pi+2l\pi), t \in (0; \infty), l \in \mathbb{Z} \end{array} \right]$$

$$= \frac{1}{3} \int \frac{1+t^2}{2t} \frac{2du}{1+t^2} = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln |t| + c_1 = \frac{1}{3} \ln \left| \operatorname{tg} \frac{x^3}{2} \right| + c_1,$$

$$x \in \mathbb{R} - \{ \pm\sqrt[3]{k\pi}, k=0, 1, 2, 3, 4, \dots \}, c_1 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \neq \pm\sqrt[3]{k\pi}, t \neq \pm k\pi \\ dt=3x^2 dx \mid k=0, 1, 2, 3, 4, \dots \end{array} \right] = \frac{1}{3} \int \frac{dt}{\sin t} = \frac{1}{3} \int \frac{\sin t dt}{\sin^2 t} = \frac{1}{3} \int \frac{\sin t dt}{1-\cos^2 t} = \frac{1}{3} \int \frac{-\sin t dt}{\cos^2 t - 1}$$

$$= \left[\begin{array}{l} \text{Subst. } u=\cos t \mid t \neq \pm k\pi, u \neq \pm 1 \\ du = -\sin t dt \mid k=0, 1, 2, 3, \dots \end{array} \right] = \frac{1}{3} \int \frac{du}{u^2-1} = \frac{1}{3} \cdot \frac{1}{2} \ln \left| \frac{u-1}{u+1} \right| + c_2$$

$$\int x^2 \sin x^3 dx = -\frac{1}{3} \cos x^3 + c$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \mid x \in \mathbb{R} \\ du=3x^2 dx \mid u \in \mathbb{R} \end{array} \right] = \frac{1}{3} \int \sin u du = -\frac{1}{3} \cos u + c = -\frac{1}{3} \cos x^3 + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 039, 040

$$\int \frac{x^2 dx}{\sin x^3} = \frac{1}{3} \ln \left| \operatorname{tg} \frac{x^3}{2} \right| + c_1$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \mid x \neq \pm\sqrt[3]{k\pi}, u \neq \pm k\pi \\ du=3x^2 dx \mid k=0, 1, 2, 3, 4, \dots \end{array} \right] = \frac{1}{3} \int \frac{du}{\sin u} = \left[\begin{array}{l} \text{UGS: } \mid du = \frac{2dt}{t^2+1} \mid u \in (-\pi+2l\pi; 2l\pi), t \in (-\infty; 0) \\ t = \operatorname{tg} \frac{u}{2} \mid \sin u = \frac{2}{t^2+1} \mid u \in (2l\pi; \pi+2l\pi), t \in (0; \infty), l \in \mathbb{Z} \end{array} \right]$$

$$= \frac{1}{3} \int \frac{1+t^2}{2t} \frac{2du}{1+t^2} = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln |t| + c_1 = \frac{1}{3} \ln \left| \operatorname{tg} \frac{x^3}{2} \right| + c_1,$$

$$x \in \mathbb{R} - \{ \pm\sqrt[3]{k\pi}, k=0, 1, 2, 3, 4, \dots \}, c_1 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \neq \pm\sqrt[3]{k\pi}, t \neq \pm k\pi \\ dt=3x^2 dx \mid k=0, 1, 2, 3, 4, \dots \end{array} \right] = \frac{1}{3} \int \frac{dt}{\sin t} = \frac{1}{3} \int \frac{\sin t dt}{\sin^2 t} = \frac{1}{3} \int \frac{\sin t dt}{1-\cos^2 t} = \frac{1}{3} \int \frac{-\sin t dt}{\cos^2 t - 1}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \cos t \mid t \neq \pm k\pi, u \neq \pm 1 \\ du = -\sin t dt \mid k=0, 1, 2, 3, \dots \end{array} \right] = \frac{1}{3} \int \frac{du}{u^2-1} = \frac{1}{3} \cdot \frac{1}{2} \ln \left| \frac{u-1}{u+1} \right| + c_2 = \frac{1}{6} \ln \left| \frac{\cos t - 1}{\cos t + 1} \right| + c_2$$

$$\int x^2 \sin x^3 dx = -\frac{1}{3} \cos x^3 + c$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \mid x \in \mathbb{R} \\ du=3x^2 dx \mid u \in \mathbb{R} \end{array} \right] = \frac{1}{3} \int \sin u du = -\frac{1}{3} \cos u + c = -\frac{1}{3} \cos x^3 + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 039, 040

$$\int \frac{x^2 dx}{\sin x^3} = \frac{1}{3} \ln \left| \operatorname{tg} \frac{x^3}{2} \right| + c_1 = \frac{1}{6} \ln \left| \frac{\cos x^3 - 1}{\cos x^3 + 1} \right| + c_2$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \mid x \neq \pm\sqrt[3]{k\pi}, u \neq \pm k\pi \\ du=3x^2 dx \mid k=0, 1, 2, 3, 4, \dots \end{array} \right] = \frac{1}{3} \int \frac{du}{\sin u} = \left[\begin{array}{l} \text{UGS: } \mid du = \frac{2dt}{t^2+1} \mid u \in (-\pi+2l\pi; 2l\pi), t \in (-\infty; 0) \\ t = \operatorname{tg} \frac{u}{2} \mid \sin u = \frac{2}{t^2+1} \mid u \in (2l\pi; \pi+2l\pi), t \in (0; \infty), l \in \mathbb{Z} \end{array} \right]$$

$$= \frac{1}{3} \int \frac{1+t^2}{2t} \frac{2dt}{1+t^2} = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln |t| + c_1 = \frac{1}{3} \ln \left| \operatorname{tg} \frac{x^3}{2} \right| + c_1,$$

$$x \in \mathbb{R} - \{\pm\sqrt[3]{k\pi}, k=0, 1, 2, 3, 4, \dots\}, c_1 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x^3 \mid x \neq \pm\sqrt[3]{k\pi}, t \neq \pm k\pi \\ dt=3x^2 dx \mid k=0, 1, 2, 3, 4, \dots \end{array} \right] = \frac{1}{3} \int \frac{dt}{\sin t} = \frac{1}{3} \int \frac{\sin t dt}{\sin^2 t} = \frac{1}{3} \int \frac{\sin t dt}{1-\cos^2 t} = \frac{1}{3} \int \frac{-\sin t dt}{\cos^2 t - 1}$$

$$= \left[\begin{array}{l} \text{Subst. } u=\cos t \mid t \neq \pm k\pi, u \neq \pm 1 \\ du = -\sin t dt \mid k=0, 1, 2, 3, \dots \end{array} \right] = \frac{1}{3} \int \frac{du}{u^2-1} = \frac{1}{3} \cdot \frac{1}{2} \ln \left| \frac{u-1}{u+1} \right| + c_2 = \frac{1}{6} \ln \left| \frac{\cos t - 1}{\cos t + 1} \right| + c_2$$

$$= \frac{1}{6} \ln \left| \frac{\cos x^3 - 1}{\cos x^3 + 1} \right| + c_2, x \in \mathbb{R} - \{\pm\sqrt[3]{k\pi}, k=0, 1, 2, 3, 4, \dots\}, c_2 \in \mathbb{R}.$$

$$\int x^2 \sin x^3 dx = -\frac{1}{3} \cos x^3 + c$$

$$= \left[\begin{array}{l} \text{Subst. } u=x^3 \mid x \in \mathbb{R} \\ du=3x^2 dx \mid u \in \mathbb{R} \end{array} \right] = \frac{1}{3} \int \sin u du = -\frac{1}{3} \cos u + c = -\frac{1}{3} \cos x^3 + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 041, 042, 043

$$\int \frac{\sin x \, dx}{\sqrt{\cos^5 x}} = \int \cos^{-\frac{5}{2}} x \cdot \sin x \, dx$$

$$\int \frac{\cos x \, dx}{\sqrt[3]{\sin^2 x}} = \int \sin^{-\frac{2}{3}} x \cdot \cos x \, dx$$

$$\int \frac{\cos x - \sin x}{\cos x + \sin x} \, dx$$

Riešené príklady – 041, 042, 043

$$\int \frac{\sin x \, dx}{\sqrt{\cos^5 x}} = \int \cos^{-\frac{5}{2}} x \cdot \sin x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (-\frac{\pi}{2} + 2k\pi; 0 + 2k\pi), t \in (0; 1) \\ dt = -\sin x \, dx \mid x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi), t \in (0; 1), k \in \mathbb{Z} \end{array} \right]$$

$$\int \frac{\cos x \, dx}{\sqrt[3]{\sin^2 x}} = \int \sin^{-\frac{2}{3}} x \cdot \cos x \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sin x \mid x \in (2k\pi; \pi + 2k\pi), t \in (0; 1) \\ dt = \cos x \, dx \mid x \in (\pi + 2k\pi; 2\pi + 2k\pi), t \in (0; 1) \end{array} \right]$$

$$\int \frac{\cos x - \sin x}{\cos x + \sin x} \, dx$$

$$= \int \frac{(\sin x + \cos x)'}{\sin x + \cos x} \, dx$$

Riešené príklady – 041, 042, 043

$$\int \frac{\sin x \, dx}{\sqrt{\cos^5 x}} = \int \cos^{-\frac{5}{2}} x \cdot \sin x \, dx$$

$$= \left[\text{Subst. } t = \cos x \mid x \in \left(-\frac{\pi}{2} + 2k\pi; 0 + 2k\pi\right), t \in (0; 1) \right. \\ \left. dt = -\sin x \, dx \mid x \in \left(0 + 2k\pi; \frac{\pi}{2} + 2k\pi\right), t \in (0; 1), k \in \mathbb{Z} \right] = -\int t^{-\frac{5}{2}} \, dt$$

$$\int \frac{\cos x \, dx}{\sqrt[3]{\sin^2 x}} = \int \sin^{-\frac{2}{3}} x \cdot \cos x \, dx$$

$$= \left[\text{Subst. } t = \sin x \mid x \in (2k\pi; \pi + 2k\pi), t \in (0; 1) \right. \\ \left. dt = \cos x \, dx \mid x \in (\pi + 2k\pi; 2\pi + 2k\pi), t \in (0; 1) \right] = \int t^{-\frac{2}{3}} \, dt$$

$$\int \frac{\cos x - \sin x}{\cos x + \sin x} \, dx = \ln |\sin x + \cos x| + c$$

$$= \int \frac{(\sin x + \cos x)'}{\sin x + \cos x} \, dx = \ln |\sin x + \cos x| + c, x \in \mathbb{R} - \left\{ \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 041, 042, 043

$$\int \frac{\sin x \, dx}{\sqrt{\cos^5 x}} = \int \cos^{-\frac{5}{2}} x \cdot \sin x \, dx$$

$$= \left[\text{Subst. } t = \cos x \mid x \in \left(-\frac{\pi}{2} + 2k\pi; 0 + 2k\pi\right), t \in (0; 1) \right. \\ \left. dt = -\sin x \, dx \mid x \in \left(0 + 2k\pi; \frac{\pi}{2} + 2k\pi\right), t \in (0; 1), k \in \mathbb{Z} \right] = -\int t^{-\frac{5}{2}} dt = -\frac{t^{-\frac{3}{2}}}{-\frac{3}{2}} + c$$

$$\int \frac{\cos x \, dx}{\sqrt[3]{\sin^2 x}} = \int \sin^{-\frac{2}{3}} x \cdot \cos x \, dx$$

$$= \left[\text{Subst. } t = \sin x \mid x \in (2k\pi; \pi + 2k\pi), t \in (0; 1) \right. \\ \left. dt = \cos x \, dx \mid x \in (\pi + 2k\pi; 2\pi + 2k\pi), t \in (0; 1) \right] = \int t^{-\frac{2}{3}} dt = \frac{t^{\frac{1}{3}}}{\frac{1}{3}} + c$$

$$\int \frac{\cos x - \sin x}{\cos x + \sin x} dx = \ln |\sin x + \cos x| + c$$

$$= \int \frac{(\sin x + \cos x)'}{\sin x + \cos x} dx = \ln |\sin x + \cos x| + c, x \in \mathbb{R} - \left\{ \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 041, 042, 043

$$\int \frac{\sin x \, dx}{\sqrt{\cos^5 x}} = \int \cos^{-\frac{5}{2}} x \cdot \sin x \, dx$$

$$= \left[\text{Subst. } t = \cos x \mid x \in \left(-\frac{\pi}{2} + 2k\pi; 0 + 2k\pi\right), t \in (0; 1) \right. \\ \left. dt = -\sin x \, dx \mid x \in \left(0 + 2k\pi; \frac{\pi}{2} + 2k\pi\right), t \in (0; 1), k \in \mathbb{Z} \right] = -\int t^{-\frac{5}{2}} dt = -\frac{t^{-\frac{3}{2}}}{-\frac{3}{2}} + c = \frac{2}{3} \frac{1}{\sqrt{t^3}} + c$$

$$\int \frac{\cos x \, dx}{\sqrt[3]{\sin^2 x}} = \int \sin^{-\frac{2}{3}} x \cdot \cos x \, dx = 3\sqrt[3]{\sin x} + c$$

$$= \left[\text{Subst. } t = \sin x \mid x \in (2k\pi; \pi + 2k\pi), t \in (0; 1) \right. \\ \left. dt = \cos x \, dx \mid x \in (\pi + 2k\pi; 2\pi + 2k\pi), t \in (0; 1) \right] = \int t^{-\frac{2}{3}} dt = \frac{t^{\frac{1}{3}}}{\frac{1}{3}} + c = 3\sqrt[3]{t} + c$$

$$\int \frac{\cos x - \sin x}{\cos x + \sin x} dx = \ln |\sin x + \cos x| + c$$

$$= \int \frac{(\sin x + \cos x)'}{\sin x + \cos x} dx = \ln |\sin x + \cos x| + c, x \in \mathbb{R} - \left\{ \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 041, 042, 043

$$\int \frac{\sin x \, dx}{\sqrt{\cos^5 x}} = \int \cos^{-\frac{5}{2}} x \cdot \sin x \, dx = \frac{2}{3\sqrt{\cos^3 x}} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos x \mid x \in (-\frac{\pi}{2} + 2k\pi; 0 + 2k\pi), t \in (0; 1) \\ dt = -\sin x \, dx \mid x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi), t \in (0; 1), k \in \mathbb{Z} \end{array} \right] = -\int t^{-\frac{5}{2}} dt = -\frac{t^{-\frac{3}{2}}}{-\frac{3}{2}} + c = \frac{2}{3} \frac{1}{\sqrt{t^3}} + c$$

$$= \frac{2}{3\sqrt{\cos^3 x}} + c, \quad x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi), k \in \mathbb{Z}, c \in \mathbb{R}.$$

$$\int \frac{\cos x \, dx}{\sqrt[3]{\sin^2 x}} = \int \sin^{-\frac{2}{3}} x \cdot \cos x \, dx = 3\sqrt[3]{\sin x} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sin x \mid x \in (2k\pi; \pi + 2k\pi), t \in (0; 1) \\ dt = \cos x \, dx \mid x \in (\pi + 2k\pi; 2\pi + 2k\pi), t \in (0; 1) \end{array} \right] = \int t^{-\frac{2}{3}} dt = \frac{t^{\frac{1}{3}}}{\frac{1}{3}} + c = 3\sqrt[3]{t} + c = 3\sqrt[3]{\sin x} + c,$$

$$x \in \mathbb{R} - \{2k\pi, k \in \mathbb{Z}\}, c \in \mathbb{R}.$$

$$\int \frac{\cos x - \sin x}{\cos x + \sin x} dx = \ln |\sin x + \cos x| + c$$

$$= \int \frac{(\sin x + \cos x)'}{\sin x + \cos x} dx = \ln |\sin x + \cos x| + c, \quad x \in \mathbb{R} - \left\{ \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 044, 045

$$\int \frac{\ln \cos x}{\sin^2 x} dx$$

$$\int \frac{dx}{\sin x \cos x}$$

Riešené príklady – 044, 045

$$\int \frac{\ln \cos x}{\sin^2 x} dx$$

$$= \left[\begin{array}{l} \cos x > 0, \sin x \neq 0, k \in \mathbb{Z} \\ x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right), x \neq k\pi \end{array} \right]$$

$$\int \frac{dx}{\sin x \cos x}$$

$$= \int \frac{(\cos^2 x + \sin^2 x) dx}{\sin x \cos x}$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}}$$

Riešené príklady – 044, 045

$$\int \frac{\ln \cos x}{\sin^2 x} dx$$

$$= \left[\begin{array}{l} \cos x > 0, \sin x \neq 0, k \in \mathbb{Z} \\ x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right), x \neq k\pi \end{array} \right] = \left[\begin{array}{l} u = \ln \cos x \quad u' = \frac{-\sin x}{\cos x} \\ v' = \frac{1}{\sin^2 x} \quad v = -\cotg x = -\frac{\cos x}{\sin x} \end{array} \right]$$

$$\int \frac{dx}{\sin x \cos x}$$

$$= \int \frac{(\cos^2 x + \sin^2 x) dx}{\sin x \cos x} = \int \frac{\cos^2 x dx}{\sin x \cos x} + \int \frac{\sin^2 x dx}{\sin x \cos x}$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}} = \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x}{\cos x}}$$

Riešené príklady – 044, 045

$$\int \frac{\ln \cos x}{\sin^2 x} dx$$

$$= \left[\begin{array}{l} \cos x > 0, \sin x \neq 0, k \in \mathbb{Z} \\ x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right), x \neq k\pi \end{array} \right] = \left[\begin{array}{l} u = \ln \cos x \quad u' = \frac{-\sin x}{\cos x} \\ v' = \frac{1}{\sin^2 x} \quad v = -\cotg x = -\frac{\cos x}{\sin x} \end{array} \right] = -\cotg x \cdot \ln \cos x - \int dx$$

$$\int \frac{dx}{\sin x \cos x}$$

$$= \int \frac{(\cos^2 x + \sin^2 x) dx}{\sin x \cos x} = \int \frac{\cos^2 x dx}{\sin x \cos x} + \int \frac{\sin^2 x dx}{\sin x \cos x} = \int \frac{\cos x dx}{\sin x} - \int \frac{\sin x dx}{\cos x}$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}} = \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \int \frac{(\operatorname{tg} x)' dx}{\operatorname{tg} x}$$

Riešené príklady – 044, 045

$$\int \frac{\ln \cos x}{\sin^2 x} dx = -\cotg x \cdot \ln \cos x - x + c$$

$$= \left[\begin{array}{l} \cos x > 0, \sin x \neq 0, k \in \mathbb{Z} \\ x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right), x \neq k\pi \end{array} \right] = \left[\begin{array}{l} u = \ln \cos x \mid u' = \frac{-\sin x}{\cos x} \\ v' = \frac{1}{\sin^2 x} \mid v = -\cotg x = -\frac{\cos x}{\sin x} \end{array} \right] = -\cotg x \cdot \ln \cos x - \int dx$$

$$= -\cotg x \cdot \ln \cos x - x + c, x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) - \{2k\pi\}, k \in \mathbb{Z}, c \in \mathbb{R}.$$

$$\int \frac{dx}{\sin x \cos x} = \ln |\operatorname{tg} x| + c$$

$$= \int \frac{(\cos^2 x + \sin^2 x) dx}{\sin x \cos x} = \int \frac{\cos^2 x dx}{\sin x \cos x} + \int \frac{\sin^2 x dx}{\sin x \cos x} = \int \frac{\cos x dx}{\sin x} - \int \frac{\sin x dx}{\cos x}$$

$$= \int \frac{(\sin x)' dx}{\sin x} - \int \frac{(\cos x)' dx}{\cos x}$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}} = \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \int \frac{(\operatorname{tg} x)' dx}{\operatorname{tg} x} = \ln |\operatorname{tg} x| + c, x \in \mathbb{R} - \left\{\frac{k\pi}{2}, k \in \mathbb{Z}\right\}, c \in \mathbb{R}.$$

Riešené príklady – 044, 045

$$\int \frac{\ln \cos x}{\sin^2 x} dx = -\cotg x \cdot \ln \cos x - x + c$$

$$= \left[\begin{array}{l} \cos x > 0, \sin x \neq 0, k \in \mathbb{Z} \\ x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right), x \neq k\pi \end{array} \right] = \left[\begin{array}{l} u = \ln \cos x \quad u' = \frac{-\sin x}{\cos x} \\ v' = \frac{1}{\sin^2 x} \quad v = -\cotg x = -\frac{\cos x}{\sin x} \end{array} \right] = -\cotg x \cdot \ln \cos x - \int dx$$

$$= -\cotg x \cdot \ln \cos x - x + c, x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) - \{2k\pi\}, k \in \mathbb{Z}, c \in \mathbb{R}.$$

$$\int \frac{dx}{\sin x \cos x} = \ln |\operatorname{tg} x| + c$$

$$= \int \frac{(\cos^2 x + \sin^2 x) dx}{\sin x \cos x} = \int \frac{\cos^2 x dx}{\sin x \cos x} + \int \frac{\sin^2 x dx}{\sin x \cos x} = \int \frac{\cos x dx}{\sin x} - \int \frac{\sin x dx}{\cos x}$$

$$= \int \frac{(\sin x)' dx}{\sin x} - \int \frac{(\cos x)' dx}{\cos x} = \ln |\sin x| - \ln |\cos x| + c$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}} = \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \int \frac{(\operatorname{tg} x)' dx}{\operatorname{tg} x} = \ln |\operatorname{tg} x| + c, x \in \mathbb{R} - \left\{\frac{k\pi}{2}, k \in \mathbb{Z}\right\}, c \in \mathbb{R}.$$

Riešené príklady – 044, 045

$$\int \frac{\ln \cos x}{\sin^2 x} dx = -\cotg x \cdot \ln \cos x - x + c$$

$$= \left[\begin{array}{l} \cos x > 0, \sin x \neq 0, k \in \mathbb{Z} \\ x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right), x \neq k\pi \end{array} \right] = \left[\begin{array}{l} u = \ln \cos x \mid u' = \frac{-\sin x}{\cos x} \\ v' = \frac{1}{\sin^2 x} \mid v = -\cotg x = -\frac{\cos x}{\sin x} \end{array} \right] = -\cotg x \cdot \ln \cos x - \int dx$$

$$= -\cotg x \cdot \ln \cos x - x + c, x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) - \{2k\pi\}, k \in \mathbb{Z}, c \in \mathbb{R}.$$

$$\int \frac{dx}{\sin x \cos x} = \ln |\operatorname{tg} x| + c$$

$$= \int \frac{(\cos^2 x + \sin^2 x) dx}{\sin x \cos x} = \int \frac{\cos^2 x dx}{\sin x \cos x} + \int \frac{\sin^2 x dx}{\sin x \cos x} = \int \frac{\cos x dx}{\sin x} - \int \frac{\sin x dx}{\cos x}$$

$$= \int \frac{(\sin x)' dx}{\sin x} - \int \frac{(\cos x)' dx}{\cos x} = \ln |\sin x| - \ln |\cos x| + c = \ln \left| \frac{\sin x}{\cos x} \right| + c$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}} = \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \int \frac{(\operatorname{tg} x)' dx}{\operatorname{tg} x} = \ln |\operatorname{tg} x| + c, x \in \mathbb{R} - \left\{\frac{k\pi}{2}, k \in \mathbb{Z}\right\}, c \in \mathbb{R}.$$

Riešené príklady – 044, 045

$$\int \frac{\ln \cos x}{\sin^2 x} dx = -\cotg x \cdot \ln \cos x - x + c$$

$$= \left[\begin{array}{l} \cos x > 0, \sin x \neq 0, k \in \mathbb{Z} \\ x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right), x \neq k\pi \end{array} \right] = \left[\begin{array}{l} u = \ln \cos x \quad u' = \frac{-\sin x}{\cos x} \\ v' = \frac{1}{\sin^2 x} \quad v = -\cotg x = -\frac{\cos x}{\sin x} \end{array} \right] = -\cotg x \cdot \ln \cos x - \int dx$$

$$= -\cotg x \cdot \ln \cos x - x + c, x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) - \{2k\pi\}, k \in \mathbb{Z}, c \in \mathbb{R}.$$

$$\int \frac{dx}{\sin x \cos x} = \ln |\operatorname{tg} x| + c$$

$$= \int \frac{(\cos^2 x + \sin^2 x) dx}{\sin x \cos x} = \int \frac{\cos^2 x dx}{\sin x \cos x} + \int \frac{\sin^2 x dx}{\sin x \cos x} = \int \frac{\cos x dx}{\sin x} - \int \frac{\sin x dx}{\cos x}$$

$$= \int \frac{(\sin x)' dx}{\sin x} - \int \frac{(\cos x)' dx}{\cos x} = \ln |\sin x| - \ln |\cos x| + c = \ln \left| \frac{\sin x}{\cos x} \right| + c$$

$$= \ln |\operatorname{tg} x| + c, x \in \mathbb{R} - \left\{ \frac{k\pi}{2}, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}} = \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \int \frac{(\operatorname{tg} x)' dx}{\operatorname{tg} x} = \ln |\operatorname{tg} x| + c, x \in \mathbb{R} - \left\{ \frac{k\pi}{2}, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

$$= \int \frac{(\cos x - \sin x) dx}{\cos^2 x - \sin^2 x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} & \sin x = \frac{2t}{t^2+1} & \sin x \neq -\cos x & x \in (-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi), t \in (-\infty; 1 - \sqrt{2}) \\ dx = \frac{2dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & x \neq \frac{3\pi}{4} + k\pi & x \in (-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi), t \in (1 - \sqrt{2}; 1 + \sqrt{2}) \\ & & & x \in (\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi), t \in (1 + \sqrt{2}; \infty), k \in \mathbb{Z} \end{array} \right]$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

$$= \int \frac{(\cos x - \sin x) dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x dx}{(1 - \sin^2 x) - \sin^2 x} - \int \frac{\sin x dx}{\cos^2 x - (1 - \cos^2 x)}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} & \sin x = \frac{2t}{t^2+1} & \sin x \neq -\cos x & x \in (-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi), t \in (-\infty; 1 - \sqrt{2}) \\ dx = \frac{2dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & x \neq \frac{3\pi}{4} + k\pi & x \in (-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi), t \in (1 - \sqrt{2}; 1 + \sqrt{2}) \\ & & & x \in (\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi), t \in (1 + \sqrt{2}; \infty), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1} + \frac{2t}{t^2+1}}$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

$$= \int \frac{(\cos x - \sin x) dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x dx}{(1 - \sin^2 x) - \sin^2 x} - \int \frac{\sin x dx}{\cos^2 x - (1 - \cos^2 x)} = \int \frac{\cos x dx}{1 - 2 \sin^2 x} - \int \frac{\sin x dx}{2 \cos^2 x - 1}$$

$$= \left[\begin{array}{l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} \\ dx = \frac{2 dt}{t^2 + 1} \end{array} \left| \begin{array}{l} \sin x = \frac{2t}{t^2 + 1} \\ \cos x = \frac{1 - t^2}{t^2 + 1} \end{array} \right| \begin{array}{l} \sin x \neq -\cos x \\ x \neq \frac{3\pi}{4} + k\pi \end{array} \left| \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi), t \in (-\infty; 1 - \sqrt{2}) \\ x \in (-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi), t \in (1 - \sqrt{2}; 1 + \sqrt{2}) \\ x \in (\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi), t \in (1 + \sqrt{2}; \infty), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2 + 1}}{\frac{1 - t^2}{t^2 + 1} + \frac{2t}{t^2 + 1}}$$

$$= \int \frac{2 dt}{2t + 1 - t^2}$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

$$= \int \frac{(\cos x - \sin x) dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x dx}{(1 - \sin^2 x) - \sin^2 x} - \int \frac{\sin x dx}{\cos^2 x - (1 - \cos^2 x)} = \int \frac{\cos x dx}{1 - 2 \sin^2 x} - \int \frac{\sin x dx}{2 \cos^2 x - 1}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sin x \mid du = \cos x dx \mid \sin x \neq \pm \cos x \\ v = \cos x \mid dv = -\sin x dx \mid x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \end{array} \right]$$

$$= \left[\begin{array}{l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid \sin x \neq -\cos x \mid x \in (-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi), t \in (-\infty; 1 - \sqrt{2}) \\ dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid x \neq \frac{3\pi}{4} + k\pi \mid x \in (-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi), t \in (1 - \sqrt{2}; 1 + \sqrt{2}) \\ x \in (\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi), t \in (1 + \sqrt{2}; \infty), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2 dt}{t^2+1}}{\frac{1-t^2}{t^2+1} + \frac{2t}{t^2+1}}$$

$$= \int \frac{2 dt}{2t+1-t^2} = \int \frac{-2 dt}{t^2-2t+1-2}$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

$$= \int \frac{(\cos x - \sin x) dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x dx}{(1 - \sin^2 x) - \sin^2 x} - \int \frac{\sin x dx}{\cos^2 x - (1 - \cos^2 x)} = \int \frac{\cos x dx}{1 - 2\sin^2 x} - \int \frac{\sin x dx}{2\cos^2 x - 1}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sin x \mid du = \cos x dx \mid \sin x \neq \pm \cos x \\ v = \cos x \mid dv = -\sin x dx \mid x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{du}{1 - 2u^2} - \int \frac{-dv}{2v^2 - 1}$$

$$= \left[\begin{array}{l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid \sin x \neq -\cos x \mid \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi), t \in (-\infty; 1 - \sqrt{2}) \\ x \in (-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi), t \in (1 - \sqrt{2}; 1 + \sqrt{2}) \\ x \in (\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi), t \in (1 + \sqrt{2}; \infty), k \in \mathbb{Z} \end{array} \\ dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid x \neq \frac{3\pi}{4} + k\pi \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1} + \frac{2t}{t^2+1}}$$

$$= \int \frac{2dt}{2t+1-t^2} = \int \frac{-2dt}{t^2-2t+1-2} = \int \frac{-2dt}{(t-1)^2 - (\sqrt{2})^2}$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

$$= \int \frac{(\cos x - \sin x) dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x dx}{(1 - \sin^2 x) - \sin^2 x} - \int \frac{\sin x dx}{\cos^2 x - (1 - \cos^2 x)} = \int \frac{\cos x dx}{1 - 2\sin^2 x} - \int \frac{\sin x dx}{2\cos^2 x - 1}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sin x \mid du = \cos x dx \mid \sin x \neq \pm \cos x \\ v = \cos x \mid dv = -\sin x dx \mid x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{du}{1 - 2u^2} - \int \frac{-dv}{2v^2 - 1} = \int \frac{\frac{1}{2} dv}{v^2 - \frac{1}{4}} - \int \frac{\frac{1}{2} du}{u^2 - \frac{1}{4}}$$

$$= \left[\begin{array}{l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid \sin x \neq -\cos x \mid \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi), t \in (-\infty; 1 - \sqrt{2}) \\ x \in (-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi), t \in (1 - \sqrt{2}; 1 + \sqrt{2}) \\ x \in (\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi), t \in (1 + \sqrt{2}; \infty), k \in \mathbb{Z} \end{array} \\ dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid x \neq \frac{3\pi}{4} + k\pi \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1} + \frac{2t}{t^2+1}}$$

$$= \int \frac{2dt}{2t+1-t^2} = \int \frac{-2dt}{t^2-2t+1-2} = \int \frac{-2dt}{(t-1)^2 - (\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } u = t-1 \mid x \neq 1 \pm \sqrt{2} \\ du = dt \mid t \neq \pm \sqrt{2} \end{array} \right]$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

$$\begin{aligned} &= \int \frac{(\cos x - \sin x) dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x dx}{(1 - \sin^2 x) - \sin^2 x} - \int \frac{\sin x dx}{\cos^2 x - (1 - \cos^2 x)} = \int \frac{\cos x dx}{1 - 2\sin^2 x} - \int \frac{\sin x dx}{2\cos^2 x - 1} \\ &= \left[\begin{array}{l} \text{Subst. } u = \sin x \mid du = \cos x dx \mid \sin x \neq \pm \cos x \\ \quad \quad \quad v = \cos x \mid dv = -\sin x dx \mid x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{du}{1 - 2u^2} - \int \frac{-dv}{2v^2 - 1} = \int \frac{\frac{1}{2} dv}{v^2 - \frac{1}{4}} - \int \frac{\frac{1}{2} du}{u^2 - \frac{1}{4}} \\ &= \frac{1}{2} \frac{1}{\sqrt{2}} \ln \left| \frac{v - \frac{\sqrt{2}}{2}}{v + \frac{\sqrt{2}}{2}} \right| - \frac{1}{2} \frac{1}{\sqrt{2}} \ln \left| \frac{u - \frac{\sqrt{2}}{2}}{u + \frac{\sqrt{2}}{2}} \right| + C_1 \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid \sin x \neq -\cos x \mid \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi), t \in (-\infty; 1 - \sqrt{2}) \\ x \in (-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi), t \in (1 - \sqrt{2}; 1 + \sqrt{2}) \end{array} \\ dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid x \neq \frac{3\pi}{4} + k\pi \mid \begin{array}{l} x \in (\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi), t \in (1 + \sqrt{2}; \infty), k \in \mathbb{Z} \end{array} \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1} + \frac{2t}{t^2+1}} \\ &= \int \frac{2dt}{2t+1-t^2} = \int \frac{-2dt}{t^2-2t+1-2} = \int \frac{-2dt}{(t-1)^2 - (\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } u = t-1 \mid x \neq 1 \pm \sqrt{2} \\ \quad \quad \quad du = dt \mid t \neq \pm \sqrt{2} \end{array} \right] = \int \frac{-2du}{u^2 - (\sqrt{2})^2} \end{aligned}$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

$$\begin{aligned} &= \int \frac{(\cos x - \sin x) dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x dx}{(1 - \sin^2 x) - \sin^2 x} - \int \frac{\sin x dx}{\cos^2 x - (1 - \cos^2 x)} = \int \frac{\cos x dx}{1 - 2\sin^2 x} - \int \frac{\sin x dx}{2\cos^2 x - 1} \\ &= \left[\begin{array}{l} \text{Subst. } u = \sin x \mid du = \cos x dx \\ v = \cos x \mid dv = -\sin x dx \end{array} \mid \begin{array}{l} \sin x \neq \pm \cos x \\ x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{du}{1 - 2u^2} - \int \frac{-dv}{2v^2 - 1} = \int \frac{\frac{1}{2} dv}{v^2 - \frac{1}{4}} - \int \frac{\frac{1}{2} du}{u^2 - \frac{1}{4}} \\ &= \frac{1}{2} \frac{1}{\sqrt{2}} \ln \left| \frac{v - \frac{\sqrt{2}}{2}}{v + \frac{\sqrt{2}}{2}} \right| - \frac{1}{2} \frac{1}{\sqrt{2}} \ln \left| \frac{u - \frac{\sqrt{2}}{2}}{u + \frac{\sqrt{2}}{2}} \right| + c_1 = \frac{1}{2\sqrt{2}} \ln \left| \frac{2v - \sqrt{2}}{2v + \sqrt{2}} \right| - \frac{1}{2\sqrt{2}} \ln \left| \frac{2u - \sqrt{2}}{2u + \sqrt{2}} \right| + c_1 \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid \sin x \neq -\cos x \\ dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid x \neq \frac{3\pi}{4} + k\pi \end{array} \mid \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi), t \in (-\infty; 1 - \sqrt{2}) \\ x \in (-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi), t \in (1 - \sqrt{2}; 1 + \sqrt{2}) \\ x \in (\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi), t \in (1 + \sqrt{2}; \infty), k \in \mathbb{Z} \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1} + \frac{2t}{t^2+1}} \\ &= \int \frac{2dt}{2t+1-t^2} = \int \frac{-2dt}{t^2-2t+1-2} = \int \frac{-2dt}{(t-1)^2 - (\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } u = t-1 \mid x \neq 1 \pm \sqrt{2} \\ du = dt \mid t \neq \pm \sqrt{2} \end{array} \right] = \int \frac{-2du}{u^2 - (\sqrt{2})^2} \\ &= \frac{-2}{2\sqrt{2}} \ln \left| \frac{u - \sqrt{2}}{u + \sqrt{2}} \right| + c_2 \end{aligned}$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x} = \frac{\sqrt{2}}{4} \ln \left| \frac{2 \cos x - \sqrt{2}}{2 \cos x + \sqrt{2}} \right| - \frac{\sqrt{2}}{4} \ln \left| \frac{2 \sin x - \sqrt{2}}{2 \sin x + \sqrt{2}} \right| + C_1$$

$$\begin{aligned} &= \int \frac{(\cos x - \sin x) dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x dx}{(1 - \sin^2 x) - \sin^2 x} - \int \frac{\sin x dx}{\cos^2 x - (1 - \cos^2 x)} = \int \frac{\cos x dx}{1 - 2 \sin^2 x} - \int \frac{\sin x dx}{2 \cos^2 x - 1} \\ &= \left[\begin{array}{l} \text{Subst. } u = \sin x \mid du = \cos x dx \mid \sin x \neq \pm \cos x \\ v = \cos x \mid dv = -\sin x dx \mid x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{du}{1 - 2u^2} - \int \frac{-dv}{2v^2 - 1} = \int \frac{\frac{1}{2} dv}{v^2 - \frac{1}{4}} - \int \frac{\frac{1}{2} du}{u^2 - \frac{1}{4}} \\ &= \frac{1}{2 \cdot \frac{\sqrt{2}}{2}} \ln \left| \frac{v - \frac{\sqrt{2}}{2}}{v + \frac{\sqrt{2}}{2}} \right| - \frac{1}{2 \cdot \frac{\sqrt{2}}{2}} \ln \left| \frac{u - \frac{\sqrt{2}}{2}}{u + \frac{\sqrt{2}}{2}} \right| + C_1 = \frac{1}{2\sqrt{2}} \ln \left| \frac{2v - \sqrt{2}}{2v + \sqrt{2}} \right| - \frac{1}{2\sqrt{2}} \ln \left| \frac{2u - \sqrt{2}}{2u + \sqrt{2}} \right| + C_1 \\ &= \frac{\sqrt{2}}{4} \ln \left| \frac{2 \cos x - \sqrt{2}}{2 \cos x + \sqrt{2}} \right| - \frac{\sqrt{2}}{4} \ln \left| \frac{2 \sin x - \sqrt{2}}{2 \sin x + \sqrt{2}} \right| + C_1, \quad x \in \mathbb{R} - \left\{ \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \right\}, \quad C_1 \in \mathbb{R}. \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid \sin x \neq -\cos x \mid \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi), t \in (-\infty; 1 - \sqrt{2}) \\ x \in (-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi), t \in (1 - \sqrt{2}; 1 + \sqrt{2}) \\ x \in (\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi), t \in (1 + \sqrt{2}; \infty), k \in \mathbb{Z} \end{array} \\ dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid x \neq \frac{3\pi}{4} + k\pi \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1} + \frac{2t}{t^2+1}} \\ &= \int \frac{2dt}{2t+1-t^2} = \int \frac{-2dt}{t^2-2t+1-2} = \int \frac{-2dt}{(t-1)^2 - (\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } u = t-1 \mid x \neq 1 \pm \sqrt{2} \\ du = dt \mid t \neq \pm \sqrt{2} \end{array} \right] = \int \frac{-2du}{u^2 - (\sqrt{2})^2} \\ &= \frac{-2}{2\sqrt{2}} \ln \left| \frac{u - \sqrt{2}}{u + \sqrt{2}} \right| + C_2 = \frac{1}{\sqrt{2}} \ln \left| \frac{(t-1) + \sqrt{2}}{(t-1) - \sqrt{2}} \right| + C_2 \end{aligned}$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x} = \frac{\sqrt{2}}{4} \ln \left| \frac{2 \cos x - \sqrt{2}}{2 \cos x + \sqrt{2}} \right| - \frac{\sqrt{2}}{4} \ln \left| \frac{2 \sin x - \sqrt{2}}{2 \sin x + \sqrt{2}} \right| + c_1 = \frac{\sqrt{2}}{2} \ln \left| \frac{\operatorname{tg} \frac{x}{2} - 1 + \sqrt{2}}{\operatorname{tg} \frac{x}{2} - 1 - \sqrt{2}} \right| + c_2$$

$$= \int \frac{(\cos x - \sin x) dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x dx}{(1 - \sin^2 x) - \sin^2 x} - \int \frac{\sin x dx}{\cos^2 x - (1 - \cos^2 x)} = \int \frac{\cos x dx}{1 - 2 \sin^2 x} - \int \frac{\sin x dx}{2 \cos^2 x - 1}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sin x \mid du = \cos x dx \mid \sin x \neq \pm \cos x \\ v = \cos x \mid dv = -\sin x dx \mid x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \end{array} \right] = \int \frac{du}{1 - 2u^2} - \int \frac{-dv}{2v^2 - 1} = \int \frac{\frac{1}{2} dv}{v^2 - \frac{1}{4}} - \int \frac{\frac{1}{2} du}{u^2 - \frac{1}{4}}$$

$$= \frac{1}{2 \cdot \frac{\sqrt{2}}{2}} \ln \left| \frac{v - \frac{\sqrt{2}}{2}}{v + \frac{\sqrt{2}}{2}} \right| - \frac{1}{2 \cdot \frac{\sqrt{2}}{2}} \ln \left| \frac{u - \frac{\sqrt{2}}{2}}{u + \frac{\sqrt{2}}{2}} \right| + c_1 = \frac{1}{2\sqrt{2}} \ln \left| \frac{2v - \sqrt{2}}{2v + \sqrt{2}} \right| - \frac{1}{2\sqrt{2}} \ln \left| \frac{2u - \sqrt{2}}{2u + \sqrt{2}} \right| + c_1$$

$$= \frac{\sqrt{2}}{4} \ln \left| \frac{2 \cos x - \sqrt{2}}{2 \cos x + \sqrt{2}} \right| - \frac{\sqrt{2}}{4} \ln \left| \frac{2 \sin x - \sqrt{2}}{2 \sin x + \sqrt{2}} \right| + c_1, \quad x \in \mathbb{R} - \left\{ \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \right\}, \quad c_1 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid \sin x \neq -\cos x \mid \begin{array}{l} x \in (-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi), t \in (-\infty; 1 - \sqrt{2}) \\ x \in (-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi), t \in (1 - \sqrt{2}; 1 + \sqrt{2}) \end{array} \\ dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid x \neq \frac{3\pi}{4} + k\pi \mid \begin{array}{l} x \in (\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi), t \in (1 + \sqrt{2}; \infty), k \in \mathbb{Z} \end{array} \end{array} \right] = \int \frac{\frac{2dt}{t^2+1}}{\frac{1-t^2}{t^2+1} + \frac{2t}{t^2+1}}$$

$$= \int \frac{2dt}{2t+1-t^2} = \int \frac{-2dt}{t^2-2t+1-2} = \int \frac{-2dt}{(t-1)^2 - (\sqrt{2})^2} = \left[\begin{array}{l} \text{Subst. } u = t-1 \mid x \neq 1 \pm \sqrt{2} \\ du = dt \mid t \neq \pm \sqrt{2} \end{array} \right] = \int \frac{-2du}{u^2 - (\sqrt{2})^2}$$

$$= \frac{-2}{2\sqrt{2}} \ln \left| \frac{u - \sqrt{2}}{u + \sqrt{2}} \right| + c_2 = \frac{1}{\sqrt{2}} \ln \left| \frac{(t-1) + \sqrt{2}}{(t-1) - \sqrt{2}} \right| + c_2 = \frac{\sqrt{2}}{2} \ln \left| \frac{\operatorname{tg} \frac{x}{2} - 1 + \sqrt{2}}{\operatorname{tg} \frac{x}{2} - 1 - \sqrt{2}} \right| + c_2,$$

$$x \in \mathbb{R} - \left\{ \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \right\}, \quad c_2 \in \mathbb{R}.$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x) dx}{\sqrt[4]{\sin x + \cos x}}$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} dx$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x) dx}{\sqrt[4]{\sin x + \cos x}}$$

$$= \left[\begin{array}{l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid x \in (-\pi+2k\pi; \pi+2k\pi), \sin x \neq -\cos x, x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \\ dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid \sin x + \cos x > 0, x \in (-\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi), t \in (1-\sqrt{2}; 1+\sqrt{2}) \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sin x + \cos x \mid x \in (-\frac{\pi}{4}+2k\pi; \frac{\pi}{4}+2k\pi), t \in (0; \sqrt{2}) \\ dt = (\cos x - \sin x) dx \mid x \in (\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi), t \in (0; \sqrt{2}), k \in \mathbb{Z} \end{array} \right]$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} dx$$

$$= \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} dx$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x) dx}{\sqrt[4]{\sin x + \cos x}}$$

$$= \left[\text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid x \in (-\pi+2k\pi; \pi+2k\pi), \sin x \neq -\cos x, x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \right. \\ \left. dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid \sin x + \cos x > 0, x \in (-\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi), t \in (1-\sqrt{2}; 1+\sqrt{2}) \right]$$

$$= \int \frac{\left(\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} \right) \frac{2dt}{1+t^2}}{\sqrt[4]{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}}$$

$$= \left[\text{Subst. } t = \sin x + \cos x \mid x \in (-\frac{\pi}{4}+2k\pi; \frac{\pi}{4}+2k\pi), t \in (0; \sqrt{2}) \right. \\ \left. dt = (\cos x - \sin x) dx \mid x \in (\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi), t \in (0; \sqrt{2}), k \in \mathbb{Z} \right] \stackrel{!}{=} - \int \frac{dt}{\sqrt[4]{t}}$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} dx$$

$$= \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} dx = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x) dx}{\sqrt[4]{\sin x + \cos x}}$$

$$= \left[\text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid x \in (-\pi+2k\pi; \pi+2k\pi), \sin x \neq -\cos x, x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z}^{\infty} \right. \\ \left. dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid \sin x + \cos x > 0, x \in (-\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi), t \in (1-\sqrt{2}; 1+\sqrt{2})^{\infty} \right]$$

$$= \int \frac{\left(\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} \right) \frac{2dt}{1+t^2}}{\sqrt[4]{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}} = \int \frac{(t^2+2t-1) dt}{\sqrt[4]{1+2t-t^2} (1+t^2)^2}$$

$$= \left[\text{Subst. } t = \sin x + \cos x \mid x \in (-\frac{\pi}{4}+2k\pi; \frac{\pi}{4}+2k\pi), t \in (0; \sqrt{2}) \right. \\ \left. dt = (\cos x - \sin x) dx \mid x \in (\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi), t \in (0; \sqrt{2}), k \in \mathbb{Z} \right] \stackrel{!}{=} - \int \frac{dt}{\sqrt[4]{t}} = - \int t^{-\frac{1}{4}} dt$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} dx$$

$$= \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} dx = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx = \int \frac{(\sin x + \cos x)'}{\sin x + \cos x} dx$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x) dx}{\sqrt[4]{\sin x + \cos x}}$$

$$= \left[\text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid x \in (-\pi+2k\pi; \pi+2k\pi), \sin x \neq -\cos x, x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z}^{\infty} \right. \\ \left. dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid \sin x + \cos x > 0, x \in (-\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi), t \in (1-\sqrt{2}; 1+\sqrt{2})^{\infty} \right]$$

$$= \int \frac{\left(\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} \right) \frac{2dt}{1+t^2}}{\sqrt[4]{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}} = \int \frac{(t^2+2t-1) dt}{\sqrt[4]{\frac{1+2t-t^2}{1+t^2}} (1+t^2)^2} = \dots \quad [\text{Príliš veľa práce pre normálneho smrteľníka.}]$$

$$= \left[\text{Subst. } t = \sin x + \cos x \mid x \in (-\frac{\pi}{4}+2k\pi; \frac{\pi}{4}+2k\pi), t \in (0; \sqrt{2}) \right. \\ \left. dt = (\cos x - \sin x) dx \mid x \in (\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi), t \in (0; \sqrt{2}), k \in \mathbb{Z} \right] \stackrel{!}{=} - \int \frac{dt}{\sqrt[4]{t}} = - \int t^{-\frac{1}{4}} dt = -\frac{t^{\frac{3}{4}}}{\frac{3}{4}} + c$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} dx = \ln |\sin x + \cos x| + c$$

$$= \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} dx = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx = \int \frac{(\sin x + \cos x)'}{\sin x + \cos x} dx = \ln |\sin x + \cos x| + c,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, -\frac{\pi}{4} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x) dx}{\sqrt[4]{\sin x + \cos x}}$$

$$= \left[\text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid x \in (-\pi+2k\pi; \pi+2k\pi), \sin x \neq -\cos x, x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z}^{\infty} \right. \\ \left. dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid \sin x + \cos x > 0, x \in (-\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi), t \in (1-\sqrt{2}; 1+\sqrt{2})^{\infty} \right]$$

$$= \int \frac{\left(\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} \right) \frac{2dt}{1+t^2}}{\sqrt[4]{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}} = \int \frac{(t^2+2t-1) dt}{\sqrt[4]{\frac{1+2t-t^2}{1+t^2}} (1+t^2)^2} = \dots \quad [\text{Príliš veľa práce pre normálneho smrteľníka.}]$$

$$= \left[\text{Subst. } t = \sin x + \cos x \mid x \in (-\frac{\pi}{4}+2k\pi; \frac{\pi}{4}+2k\pi), t \in (0; \sqrt{2}) \right. \\ \left. dt = (\cos x - \sin x) dx \mid x \in (\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi), t \in (0; \sqrt{2}), k \in \mathbb{Z} \right] \stackrel{!}{=} - \int \frac{dt}{\sqrt[4]{t}} = - \int t^{-\frac{1}{4}} dt = -\frac{t^{\frac{3}{4}}}{\frac{3}{4}} + c$$

$$= c - \frac{4}{3} \sqrt[4]{t^3}$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} dx = \ln |\sin x + \cos x| + c$$

$$= \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} dx = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx = \int \frac{(\sin x + \cos x)'}{\sin x + \cos x} dx = \ln |\sin x + \cos x| + c,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, -\frac{\pi}{4} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x) dx}{\sqrt[4]{\sin x + \cos x}} = c - \frac{4}{3} \sqrt[4]{(\sin x + \cos x)^3}$$

$$= \left[\begin{array}{l} \text{UGS: } t = \operatorname{tg} \frac{x}{2} \mid \sin x = \frac{2t}{t^2+1} \mid x \in (-\pi+2k\pi; \pi+2k\pi), \sin x \neq -\cos x, x \neq \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \\ dx = \frac{2dt}{t^2+1} \mid \cos x = \frac{1-t^2}{t^2+1} \mid \sin x + \cos x > 0, x \in (-\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi), t \in (1-\sqrt{2}; 1+\sqrt{2}) \end{array} \right]$$

$$= \int \frac{\left(\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}\right) \frac{2dt}{1+t^2}}{\sqrt[4]{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}} = \int \frac{(t^2+2t-1) dt}{\sqrt[4]{\frac{1+2t-t^2}{1+t^2}} (1+t^2)^2} = \dots \quad [\text{Príliš veľa práce pre normálneho smrteľníka.}]$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sin x + \cos x \mid x \in (-\frac{\pi}{4}+2k\pi; \frac{\pi}{4}+2k\pi), t \in (0; \sqrt{2}) \\ dt = (\cos x - \sin x) dx \mid x \in (\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi), t \in (0; \sqrt{2}), k \in \mathbb{Z} \end{array} \right] \stackrel{!}{=} - \int \frac{dt}{\sqrt[4]{t}} = - \int t^{-\frac{1}{4}} dt = -\frac{t^{\frac{3}{4}}}{\frac{3}{4}} + c$$

$$= c - \frac{4}{3} \sqrt[4]{t^3} = c - \frac{4}{3} \sqrt[4]{(\sin x + \cos x)^3}, x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right), k \in \mathbb{Z}, c \in \mathbb{R}.$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} dx = \ln |\sin x + \cos x| + c$$

$$= \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} dx = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx = \int \frac{(\sin x + \cos x)'}{\sin x + \cos x} dx = \ln |\sin x + \cos x| + c,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, -\frac{\pi}{4} + k\pi, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 049, 050

$$\int \cos^n ax \cdot \sin ax \, dx$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$\int \frac{\sin ax \, dx}{\cos^n ax}$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

Riešené príklady – 049, 050

$$\int \cos^n ax \cdot \sin ax \, dx$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos ax \mid ax \in \langle 0 + k\pi; \pi + k\pi \rangle \\ dt = -a \sin ax \, dx \mid t \in \langle -1; 1 \rangle, k \in \mathbb{Z} \end{array} \right]$$

$$\int \frac{\sin ax \, dx}{\cos^n ax}$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \cos ax \mid ax \in \langle 0 + k\pi; \frac{\pi}{2} + k\pi \rangle \cup \langle \frac{\pi}{2} + k\pi; \pi + k\pi \rangle \\ dt = -a \sin ax \, dx \mid t \in \langle -1; 0 \rangle \cup \langle 0; 1 \rangle, k \in \mathbb{Z} \end{array} \right]$$

Riešené príklady – 049, 050

$$\int \cos^n ax \cdot \sin ax \, dx$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \cos ax \mid ax \in \langle 0 + k\pi; \pi + k\pi \rangle \\ dt = -a \sin ax \, dx \mid t \in \langle -1; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^n dt}{-a}$$

$$\int \frac{\sin ax \, dx}{\cos^n ax}$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \cos ax \mid ax \in \langle 0 + k\pi; \frac{\pi}{2} + k\pi \rangle \cup \langle \frac{\pi}{2} + k\pi; \pi + k\pi \rangle \\ dt = -a \sin ax \, dx \mid t \in \langle -1; 0 \rangle \cup \langle 0; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int \frac{dt}{-at^n}$$

Riešené príklady – 049, 050

$$\int \cos^n ax \cdot \sin ax \, dx$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \cos ax \mid ax \in \langle 0 + k\pi; \pi + k\pi \rangle \\ dt = -a \sin ax \, dx \mid t \in \langle -1; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^n dt}{-a} = -\frac{1}{a} \int t^n dt$$

$$\int \frac{\sin ax \, dx}{\cos^n ax}$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \cos ax \mid ax \in \langle 0 + k\pi; \frac{\pi}{2} + k\pi \rangle \cup \langle \frac{\pi}{2} + k\pi; \pi + k\pi \rangle \\ dt = -a \sin ax \, dx \mid t \in \langle -1; 0 \rangle \cup \langle 0; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int \frac{dt}{-at^n} = -\frac{1}{a} \int \frac{dt}{t^n}$$

$$\boxed{n=1} = -\frac{1}{a} \int \frac{dt}{t}$$

$$\boxed{n>1} = -\frac{1}{a} \int t^{-n} dt$$

Riešené príklady – 049, 050

$$\int \cos^n ax \cdot \sin ax \, dx$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \cos ax \mid ax \in \langle 0+k\pi; \pi+k\pi \rangle \\ dt = -a \sin ax \, dx \mid t \in \langle -1; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^n dt}{-a} = -\frac{1}{a} \int t^n dt = -\frac{1}{a} \cdot \frac{t^{n+1}}{n+1} + c$$

$$\int \frac{\sin ax \, dx}{\cos^n ax}$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \cos ax \mid ax \in \langle 0+k\pi; \frac{\pi}{2}+k\pi \rangle \cup \langle \frac{\pi}{2}+k\pi; \pi+k\pi \rangle \\ dt = -a \sin ax \, dx \mid t \in \langle -1; 0 \rangle \cup \langle 0; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int \frac{dt}{-at^n} = -\frac{1}{a} \int \frac{dt}{t^n}$$

$$\boxed{n=1} = -\frac{1}{a} \int \frac{dt}{t} = -\frac{1}{a} \ln |t| + c$$

$$\boxed{n>1} = -\frac{1}{a} \int t^{-n} dt = -\frac{1}{a} \cdot \frac{t^{-n+1}}{-n+1} + c$$

Riešené príklady – 049, 050

$$\int \cos^n ax \cdot \sin ax \, dx = -\frac{\cos^{n+1} ax}{a(n+1)} + c \quad a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \cos ax \mid ax \in \langle 0+k\pi; \pi+k\pi \rangle \\ dt = -a \sin ax \, dx \mid t \in \langle -1; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^n dt}{-a} = -\frac{1}{a} \int t^n dt = -\frac{1}{a} \cdot \frac{t^{n+1}}{n+1} + c$$

$$= -\frac{\cos^{n+1} ax}{a(n+1)} + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \frac{\sin ax \, dx}{\cos^n ax} = -\frac{1}{a} \ln |\cos ax| + c \quad \text{resp.} \quad \frac{1}{a(n-1) \cos^{n-1} ax} + c \quad a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \cos ax \mid ax \in \langle 0+k\pi; \frac{\pi}{2}+k\pi \rangle \cup \langle \frac{\pi}{2}+k\pi; \pi+k\pi \rangle \\ dt = -a \sin ax \, dx \mid t \in \langle -1; 0 \rangle \cup \langle 0; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int \frac{dt}{-at^n} = -\frac{1}{a} \int \frac{dt}{t^n}$$

$$\boxed{n=1} = -\frac{1}{a} \int \frac{dt}{t} = -\frac{1}{a} \ln |t| + c = -\frac{1}{a} \ln |\cos ax| + c, n=1,$$

$$\boxed{n>1} = -\frac{1}{a} \int t^{-n} dt = -\frac{1}{a} \cdot \frac{t^{-n+1}}{-n+1} + c = \frac{1}{a(n-1) \cos^{n-1} ax} + c, n=2,3,4, \dots,$$

$$x \in \mathbb{R} - \left\{ \frac{\pi+2k\pi}{2a}, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 051, 052

$$\int \sin^n ax \cdot \cos ax \, dx$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$I = \int \frac{\cos ax \, dx}{\sin^n ax}$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

Riešené príklady – 051, 052

$$\int \sin^n ax \cdot \cos ax \, dx$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sin ax \mid ax \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx \mid t \in \langle -1; 1 \rangle, k \in \mathbb{Z} \end{array} \right]$$

$$I = \int \frac{\cos ax \, dx}{\sin^n ax}$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sin ax \mid ax \in \langle -\frac{\pi}{2} + k\pi; 0 + k\pi \rangle \cup \langle 0 + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx \mid t \in \langle -1; 0 \rangle \cup \langle 0; 1 \rangle, k \in \mathbb{Z} \end{array} \right]$$

Riešené príklady – 051, 052

$$\int \sin^n ax \cdot \cos ax \, dx$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin ax \mid ax \in \left(-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi\right) \\ dt = a \cos ax \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right] = \int \frac{t^n \, dt}{a}$$

$$I = \int \frac{\cos ax \, dx}{\sin^n ax}$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin ax \mid ax \in \left(-\frac{\pi}{2} + k\pi; 0 + k\pi\right) \cup \left(0 + k\pi; \frac{\pi}{2} + k\pi\right) \\ dt = a \cos ax \, dx \mid t \in (-1; 0) \cup (0; 1), k \in \mathbb{Z} \end{array} \right] = \int \frac{dt}{at^n}$$

Riešené príklady – 051, 052

$$\int \sin^n ax \cdot \cos ax \, dx$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin ax \mid ax \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx \mid t \in \langle -1; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^n dt}{a} = \frac{1}{a} \int t^n dt$$

$$I = \int \frac{\cos ax \, dx}{\sin^n ax}$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin ax \mid ax \in \langle -\frac{\pi}{2} + k\pi; 0 + k\pi \rangle \cup \langle 0 + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx \mid t \in \langle -1; 0 \rangle \cup \langle 0; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int \frac{dt}{at^n} = \frac{1}{a} \int \frac{dt}{t^n}$$

$$\boxed{n=1} = \frac{1}{a} \int \frac{dt}{t}$$

$$\boxed{n>1} = \frac{1}{a} \int t^{-n} dt$$

Riešené príklady – 051, 052

$$\int \sin^n ax \cdot \cos ax \, dx$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin ax \mid ax \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx \mid t \in \langle -1; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int \frac{t^n dt}{a} = \frac{1}{a} \int t^n dt = \frac{1}{a} \cdot \frac{t^{n+1}}{n+1} + c$$

$$I = \int \frac{\cos ax \, dx}{\sin^n ax}$$

$$a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin ax \mid ax \in \langle -\frac{\pi}{2} + k\pi; 0 + k\pi \rangle \cup \langle 0 + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx \mid t \in \langle -1; 0 \rangle \cup \langle 0; 1 \rangle, k \in \mathbb{Z} \end{array} \right] = \int \frac{dt}{at^n} = \frac{1}{a} \int \frac{dt}{t^n}$$

$$\boxed{n=1} = \frac{1}{a} \int \frac{dt}{t} = \frac{1}{a} \ln |t| + c$$

$$\boxed{n>1} = \frac{1}{a} \int t^{-n} dt = \frac{1}{a} \cdot \frac{t^{-n+1}}{-n+1} + c$$

Riešené príklady – 051, 052

$$\int \sin^n ax \cdot \cos ax \, dx = \frac{\sin^{n+1} ax}{a(n+1)} + c \quad a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin ax \mid ax \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx \mid t \in (-1; 1), k \in \mathbb{Z} \end{array} \right] = \int \frac{t^n dt}{a} = \frac{1}{a} \int t^n dt = \frac{1}{a} \cdot \frac{t^{n+1}}{n+1} + c$$

$$= \frac{\sin^{n+1} ax}{a(n+1)} + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

$$I = \int \frac{\cos ax \, dx}{\sin^n ax} = \frac{1}{a} \ln |\sin ax| + c \text{ resp. } I = -\frac{1}{a(n-1) \sin^{n-1} ax} + c \quad a \in \mathbb{R} - \{0\}, n \in \mathbb{N}$$

$$= \left[\text{Subst. } \begin{array}{l} t = \sin ax \mid ax \in \langle -\frac{\pi}{2} + k\pi; 0 + k\pi \rangle \cup \langle 0 + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx \mid t \in (-1; 0) \cup (0; 1), k \in \mathbb{Z} \end{array} \right] = \int \frac{dt}{at^n} = \frac{1}{a} \int \frac{dt}{t^n}$$

$$\boxed{n=1} = \frac{1}{a} \int \frac{dt}{t} = \frac{1}{a} \ln |t| + c = \frac{1}{a} \ln |\sin ax| + c, n=1,$$

$$\boxed{n>1} = \frac{1}{a} \int t^{-n} dt = \frac{1}{a} \cdot \frac{t^{-n+1}}{-n+1} + c = -\frac{1}{a(n-1) \sin^{n-1} ax} + c, n=2,3,4,\dots,$$

$$x \in \mathbb{R} - \left\{ \frac{k\pi}{a}, k \in \mathbb{Z} \right\}, c \in \mathbb{R}.$$

Riešené príklady – 053

$$I = \int \sin ax \cdot \cos bx \, dx$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

Riešené príklady – 053

$$I = \int \sin ax \cdot \cos bx \, dx$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$= \left[\begin{array}{l} \sin \alpha \cdot \cos \beta \\ = \frac{\sin(\alpha - \beta) + \sin(\alpha + \beta)}{2} \end{array} \right]$$



$$= \left[\begin{array}{l} u = \sin ax \quad | \quad u' = a \cos ax \\ v' = \cos bx \quad | \quad v = \frac{1}{b} \sin bx \end{array} \right]$$



Riešené príklady – 053

$$I = \int \sin ax \cdot \cos bx \, dx$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$= \left[\begin{array}{l} \sin \alpha \cdot \cos \beta \\ = \frac{\sin(\alpha - \beta) + \sin(\alpha + \beta)}{2} \end{array} \right] = \int \frac{\sin(ax - bx) + \sin(ax + bx)}{2} \, dx$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = \frac{\sin ax \cdot \sin bx}{b} - \frac{a}{b} \int \cos ax \cdot \sin bx \, dx$$

Riešené príklady – 053

$$I = \int \sin ax \cdot \cos bx \, dx$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$= \left[\begin{array}{l} \sin \alpha \cdot \cos \beta \\ = \frac{\sin(\alpha - \beta) + \sin(\alpha + \beta)}{2} \end{array} \right] = \int \frac{\sin(ax - bx) + \sin(ax + bx)}{2} \, dx = \int \frac{\sin(a-b)x \, dx}{2} + \int \frac{\sin(a+b)x \, dx}{2}$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = \frac{\sin ax \cdot \sin bx}{b} - \frac{a}{b} \int \cos ax \cdot \sin bx \, dx$$

$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right]$$

Riešené príklady – 053

$$I = \int \sin ax \cdot \cos bx \, dx = c_1 - \frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$= \left[\begin{array}{l} \sin \alpha \cdot \cos \beta \\ = \frac{\sin(\alpha-\beta) + \sin(\alpha+\beta)}{2} \end{array} \right] = \int \frac{\sin(ax-bx) + \sin(ax+bx)}{2} \, dx = \int \frac{\sin(a-b)x \, dx}{2} + \int \frac{\sin(a+b)x \, dx}{2}$$

$$= -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + c_1, \quad x \in \mathbb{R}, c_1 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = \frac{\sin ax \cdot \sin bx}{b} - \frac{a}{b} \int \cos ax \cdot \sin bx \, dx$$

$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = \frac{\sin ax \cdot \sin bx}{b} - \frac{a}{b} \left[-\frac{\cos ax \cdot \cos bx}{b} - \frac{a}{b} \int \sin ax \cdot \cos bx \, dx \right]$$

Riešené príklady – 053

$$I = \int \sin ax \cdot \cos bx \, dx = c_1 - \frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$= \left[\begin{array}{l} \sin \alpha \cdot \cos \beta \\ = \frac{\sin(\alpha-\beta) + \sin(\alpha+\beta)}{2} \end{array} \right] = \int \frac{\sin(ax-bx) + \sin(ax+bx)}{2} \, dx = \int \frac{\sin(a-b)x \, dx}{2} + \int \frac{\sin(a+b)x \, dx}{2}$$

$$= -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + c_1, \quad x \in \mathbb{R}, \quad c_1 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = \frac{\sin ax \cdot \sin bx}{b} - \frac{a}{b} \int \cos ax \cdot \sin bx \, dx$$

$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = \frac{\sin ax \cdot \sin bx}{b} - \frac{a}{b} \left[-\frac{\cos ax \cdot \cos bx}{b} - \frac{a}{b} \int \sin ax \cdot \cos bx \, dx \right]$$

$$= \frac{b \sin ax \cdot \sin bx}{b^2} + \frac{a \cos ax \cdot \cos bx}{b^2} + \frac{a^2}{b^2} \int \sin ax \cdot \cos bx \, dx$$

Riešené príklady – 053

$$I = \int \sin ax \cdot \cos bx \, dx = c_1 - \frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$\begin{aligned} &= \left[\begin{array}{l} \sin \alpha \cdot \cos \beta \\ = \frac{\sin(\alpha-\beta) + \sin(\alpha+\beta)}{2} \end{array} \right] = \int \frac{\sin(ax-bx) + \sin(ax+bx)}{2} \, dx = \int \frac{\sin(a-b)x}{2} \, dx + \int \frac{\sin(a+b)x}{2} \, dx \\ &= -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + c_1, \quad x \in \mathbb{R}, c_1 \in \mathbb{R}. \end{aligned}$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = \frac{\sin ax \cdot \sin bx}{b} - \frac{a}{b} \int \cos ax \cdot \sin bx \, dx$$

$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = \frac{\sin ax \cdot \sin bx}{b} - \frac{a}{b} \left[-\frac{\cos ax \cdot \cos bx}{b} - \frac{a}{b} \int \sin ax \cdot \cos bx \, dx \right]$$

$$= \frac{b \sin ax \cdot \sin bx}{b^2} + \frac{a \cos ax \cdot \cos bx}{b^2} + \frac{a^2}{b^2} \int \sin ax \cdot \cos bx \, dx$$

$$= \left[\text{Rovnica } I = \frac{b \sin ax \cdot \sin bx}{b^2} + \frac{a \cos ax \cdot \cos bx}{b^2} + \frac{a^2}{b^2} I \Rightarrow \frac{b^2 - a^2}{b^2} I = (1 - \frac{a^2}{b^2}) I = \frac{b \sin ax \cdot \sin bx}{b^2} + \frac{a \cos ax \cdot \cos bx}{b^2} \right]$$

Riešené príklady – 053

$$I = \int \sin ax \cdot \cos bx \, dx = c_1 - \frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} = \frac{b \sin ax \cdot \sin bx + a \cos ax \cdot \cos bx}{b^2 - a^2} + c_2$$

$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$

$$= \left[\begin{array}{l} \sin \alpha \cdot \cos \beta \\ = \frac{\sin(\alpha - \beta) + \sin(\alpha + \beta)}{2} \end{array} \right] = \int \frac{\sin(ax - bx) + \sin(ax + bx)}{2} \, dx = \int \frac{\sin(a-b)x}{2} \, dx + \int \frac{\sin(a+b)x}{2} \, dx$$

$$= -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + c_1, \quad x \in \mathbb{R}, \quad c_1 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = \frac{\sin ax \cdot \sin bx}{b} - \frac{a}{b} \int \cos ax \cdot \sin bx \, dx$$

$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = \frac{\sin ax \cdot \sin bx}{b} - \frac{a}{b} \left[-\frac{\cos ax \cdot \cos bx}{b} - \frac{a}{b} \int \sin ax \cdot \cos bx \, dx \right]$$

$$= \frac{b \sin ax \cdot \sin bx}{b^2} + \frac{a \cos ax \cdot \cos bx}{b^2} + \frac{a^2}{b^2} \int \sin ax \cdot \cos bx \, dx$$

$$= \left[\text{Rovnica } I = \frac{b \sin ax \cdot \sin bx}{b^2} + \frac{a \cos ax \cdot \cos bx}{b^2} + \frac{a^2}{b^2} I \Rightarrow \frac{b^2 - a^2}{b^2} I = (1 - \frac{a^2}{b^2}) I = \frac{b \sin ax \cdot \sin bx}{b^2} + \frac{a \cos ax \cdot \cos bx}{b^2} \right]$$

$$= \frac{b}{b^2 - a^2} \sin ax \cdot \sin bx + \frac{a}{b^2 - a^2} \cos ax \cdot \cos bx + c_2, \quad x \in \mathbb{R}, \quad c_2 \in \mathbb{R}.$$

Riešené príklady – 054

$$I = \int \cos ax \cdot \cos bx \, dx$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

Riešené príklady – 054

$$I = \int \cos ax \cdot \cos bx \, dx$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$= \left[\begin{array}{l} \cos \alpha \cdot \cos \beta \\ = \frac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2} \end{array} \right]$$



$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right]$$



Riešené príklady – 054

$$I = \int \cos ax \cdot \cos bx \, dx$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$= \left[\begin{array}{l} \cos \alpha \cdot \cos \beta \\ = \frac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2} \end{array} \right] = \int \frac{\cos(ax - bx) + \cos(ax + bx)}{2} \, dx$$



$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = \frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \int \sin ax \cdot \sin bx \, dx$$



Riešené príklady – 054

$$I = \int \cos ax \cdot \cos bx \, dx$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$= \left[\begin{array}{l} \cos \alpha \cdot \cos \beta \\ = \frac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2} \end{array} \right] = \int \frac{\cos(ax - bx) + \cos(ax + bx)}{2} \, dx = \int \frac{\cos(a-b)x \, dx}{2} + \int \frac{\cos(a+b)x \, dx}{2}$$

$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = \frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \int \sin ax \cdot \sin bx \, dx$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right]$$

Riešené príklady – 054

$$I = \int \cos ax \cdot \cos bx \, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C_1$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$\begin{aligned} &= \left[\begin{array}{l} \cos \alpha \cdot \cos \beta \\ = \frac{\cos(\alpha-\beta) + \cos(\alpha+\beta)}{2} \end{array} \right] = \int \frac{\cos(ax-bx) + \cos(ax+bx)}{2} \, dx = \int \frac{\cos(a-b)x \, dx}{2} + \int \frac{\cos(a+b)x \, dx}{2} \\ &= \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C_1, \quad x \in \mathbb{R}, C_1 \in \mathbb{R}. \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = \frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \int \sin ax \cdot \sin bx \, dx \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = \frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \left[-\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \int \cos ax \cdot \cos bx \, dx \right] \end{aligned}$$

Riešené príklady – 054

$$I = \int \cos ax \cdot \cos bx \, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C_1$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$\begin{aligned} &= \left[\begin{array}{l} \cos \alpha \cdot \cos \beta \\ = \frac{\cos(\alpha-\beta) + \cos(\alpha+\beta)}{2} \end{array} \right] = \int \frac{\cos(ax-bx) + \cos(ax+bx)}{2} \, dx = \int \frac{\cos(a-b)x}{2} \, dx + \int \frac{\cos(a+b)x}{2} \, dx \\ &= \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C_1, \quad x \in \mathbb{R}, C_1 \in \mathbb{R}. \end{aligned}$$

$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = \frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \int \sin ax \cdot \sin bx \, dx$$

$$\begin{aligned} &= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = \frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \left[-\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \int \cos ax \cdot \cos bx \, dx \right] \\ &= \frac{b \cos ax \cdot \sin bx}{b^2} - \frac{a \sin ax \cdot \cos bx}{b^2} + \frac{a^2}{b^2} \int \cos ax \cdot \cos bx \, dx \end{aligned}$$

Riešené príklady – 054

$$I = \int \cos ax \cdot \cos bx \, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C_1$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$\begin{aligned} &= \left[\begin{array}{l} \cos \alpha \cdot \cos \beta \\ = \frac{\cos(\alpha-\beta) + \cos(\alpha+\beta)}{2} \end{array} \right] = \int \frac{\cos(ax-bx) + \cos(ax+bx)}{2} \, dx = \int \frac{\cos(a-b)x}{2} \, dx + \int \frac{\cos(a+b)x}{2} \, dx \\ &= \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C_1, \quad x \in \mathbb{R}, C_1 \in \mathbb{R}. \end{aligned}$$

$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = \frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \int \sin ax \cdot \sin bx \, dx$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = \frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \left[-\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \int \cos ax \cdot \cos bx \, dx \right]$$

$$= \frac{b \cos ax \cdot \sin bx}{b^2} - \frac{a \sin ax \cdot \cos bx}{b^2} + \frac{a^2}{b^2} \int \cos ax \cdot \cos bx \, dx$$

$$= \left[\text{Rovnica } I = \frac{b \cos ax \cdot \sin bx}{b^2} - \frac{a \sin ax \cdot \cos bx}{b^2} + \frac{a^2}{b^2} I \Rightarrow \frac{b^2 - a^2}{b^2} I = (1 - \frac{a^2}{b^2}) I = \frac{b \cos ax \cdot \sin bx}{b^2} - \frac{a \sin ax \cdot \cos bx}{b^2} \right]$$

Riešené príklady – 054

$$I = \int \cos ax \cdot \cos bx \, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C_1 = \frac{b \cos ax \cdot \sin bx - a \sin ax \cdot \cos bx}{b^2 - a^2} + C_2$$

$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$

$$= \left[\begin{array}{l} \cos \alpha \cdot \cos \beta \\ = \frac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2} \end{array} \right] = \int \frac{\cos(ax - bx) + \cos(ax + bx)}{2} \, dx = \int \frac{\cos(a-b)x \, dx}{2} + \int \frac{\cos(a+b)x \, dx}{2}$$

$$= \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C_1, \quad x \in \mathbb{R}, \quad C_1 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = \frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \int \sin ax \cdot \sin bx \, dx$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = \frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \left[-\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \int \cos ax \cdot \cos bx \, dx \right]$$

$$= \frac{b \cos ax \cdot \sin bx}{b^2} - \frac{a \sin ax \cdot \cos bx}{b^2} + \frac{a^2}{b^2} \int \cos ax \cdot \cos bx \, dx$$

$$= \left[\text{Rovnica } I = \frac{b \cos ax \cdot \sin bx}{b^2} - \frac{a \sin ax \cdot \cos bx}{b^2} + \frac{a^2}{b^2} I \Rightarrow \frac{b^2 - a^2}{b^2} I = (1 - \frac{a^2}{b^2}) I = \frac{b \cos ax \cdot \sin bx}{b^2} - \frac{a \sin ax \cdot \cos bx}{b^2} \right]$$

$$= \frac{b}{b^2 - a^2} \cos ax \cdot \sin bx - \frac{a}{b^2 - a^2} \sin ax \cdot \cos bx + C_2, \quad x \in \mathbb{R}, \quad C_2 \in \mathbb{R}.$$

Riešené príklady – 055

$$I = \int \sin ax \cdot \sin bx \, dx$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

Riešené príklady – 055

$$I = \int \sin ax \cdot \sin bx \, dx$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$= \left[\begin{array}{l} \sin \alpha \cdot \sin \beta \\ = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2} \end{array} \right]$$



$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right]$$



Riešené príklady – 055

$$I = \int \sin ax \cdot \sin bx \, dx$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$= \left[\begin{array}{l} \sin \alpha \cdot \sin \beta \\ = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2} \end{array} \right] = \int \frac{\cos(ax - bx) - \cos(ax + bx)}{2} \, dx$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = -\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \int \cos ax \cdot \cos bx \, dx$$

Riešené príklady – 055

$$I = \int \sin ax \cdot \sin bx \, dx$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$= \left[\begin{array}{l} \sin \alpha \cdot \sin \beta \\ = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2} \end{array} \right] = \int \frac{\cos(ax - bx) - \cos(ax + bx)}{2} \, dx = \int \frac{\cos(a-b)x \, dx}{2} - \int \frac{\cos(a+b)x \, dx}{2}$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = -\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \int \cos ax \cdot \cos bx \, dx$$

$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right]$$

Riešené príklady – 055

$$I = \int \sin ax \cdot \sin bx \, dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + c_1$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$\begin{aligned} &= \left[\begin{array}{l} \sin \alpha \cdot \sin \beta \\ = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2} \end{array} \right] = \int \frac{\cos(ax-bx) - \cos(ax+bx)}{2} \, dx = \int \frac{\cos(a-b)x \, dx}{2} - \int \frac{\cos(a+b)x \, dx}{2} \\ &= \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + c_1, \quad x \in \mathbb{R}, c_1 \in \mathbb{R}. \end{aligned}$$

$$\left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = -\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \int \cos ax \cdot \cos bx \, dx$$

$$\left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = -\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \left[\frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \int \sin ax \cdot \sin bx \, dx \right]$$

Riešené príklady – 055

$$I = \int \sin ax \cdot \sin bx \, dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + c_1$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$\begin{aligned} &= \left[\begin{array}{l} \sin \alpha \cdot \sin \beta \\ = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2} \end{array} \right] = \int \frac{\cos(ax-bx) - \cos(ax+bx)}{2} \, dx = \int \frac{\cos(a-b)x \, dx}{2} - \int \frac{\cos(a+b)x \, dx}{2} \\ &= \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + c_1, \quad x \in \mathbb{R}, c_1 \in \mathbb{R}. \end{aligned}$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = -\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \int \cos ax \cdot \cos bx \, dx$$

$$\begin{aligned} &= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = -\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \left[\frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \int \sin ax \cdot \sin bx \, dx \right] \\ &= -\frac{b \sin ax \cdot \cos bx}{b^2} + \frac{a \cos ax \cdot \sin bx}{b^2} + \frac{a^2}{b^2} \int \sin ax \cdot \sin bx \, dx \end{aligned}$$

Riešené príklady – 055

$$I = \int \sin ax \cdot \sin bx \, dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + c_1$$

$$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$$

$$\begin{aligned} &= \left[\begin{array}{l} \sin \alpha \cdot \sin \beta \\ = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2} \end{array} \right] = \int \frac{\cos(ax-bx) - \cos(ax+bx)}{2} \, dx = \int \frac{\cos(a-b)x \, dx}{2} - \int \frac{\cos(a+b)x \, dx}{2} \\ &= \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + c_1, \quad x \in \mathbb{R}, c_1 \in \mathbb{R}. \end{aligned}$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = -\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \int \cos ax \cdot \cos bx \, dx$$

$$\begin{aligned} &= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = -\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \left[\frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \int \sin ax \cdot \sin bx \, dx \right] \\ &= -\frac{b \sin ax \cdot \cos bx}{b^2} + \frac{a \cos ax \cdot \sin bx}{b^2} + \frac{a^2}{b^2} \int \sin ax \cdot \sin bx \, dx \end{aligned}$$

$$= \left[\text{Rovnica } I = -\frac{b \sin ax \cdot \cos bx}{b^2} + \frac{a \cos ax \cdot \sin bx}{b^2} + \frac{a^2}{b^2} I \Rightarrow \frac{b^2 - a^2}{b^2} I = \left(1 - \frac{a^2}{b^2}\right) I = -\frac{b \sin ax \cdot \cos bx}{b^2} + \frac{a \cos ax \cdot \sin bx}{b^2} \right]$$

Riešené príklady – 055

$$I = \int \sin ax \cdot \sin bx \, dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + c_1 = \frac{-b \sin ax \cdot \cos bx + a \cos ax \cdot \sin bx}{b^2 - a^2} + c_2$$

$a, b \in \mathbb{R} - \{0\}, a \neq \pm b$

$$= \left[\begin{array}{l} \sin \alpha \cdot \sin \beta \\ = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2} \end{array} \right] = \int \frac{\cos(ax - bx) - \cos(ax + bx)}{2} \, dx = \int \frac{\cos(a-b)x \, dx}{2} - \int \frac{\cos(a+b)x \, dx}{2}$$

$$= \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + c_1, \quad x \in \mathbb{R}, \quad c_1 \in \mathbb{R}.$$

$$= \left[\begin{array}{l} u = \sin ax \quad u' = a \cos ax \\ v' = \sin bx \quad v = -\frac{1}{b} \cos bx \end{array} \right] = -\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \int \cos ax \cdot \cos bx \, dx$$

$$= \left[\begin{array}{l} u = \cos ax \quad u' = -a \sin ax \\ v' = \cos bx \quad v = \frac{1}{b} \sin bx \end{array} \right] = -\frac{\sin ax \cdot \cos bx}{b} + \frac{a}{b} \left[\frac{\cos ax \cdot \sin bx}{b} + \frac{a}{b} \int \sin ax \cdot \sin bx \, dx \right]$$

$$= -\frac{b \sin ax \cdot \cos bx}{b^2} + \frac{a \cos ax \cdot \sin bx}{b^2} + \frac{a^2}{b^2} \int \sin ax \cdot \sin bx \, dx$$

$$= \left[\text{Rovnica } I = -\frac{b \sin ax \cdot \cos bx}{b^2} + \frac{a \cos ax \cdot \sin bx}{b^2} + \frac{a^2}{b^2} I \Rightarrow \frac{b^2 - a^2}{b^2} I = (1 - \frac{a^2}{b^2}) I = -\frac{b \sin ax \cdot \cos bx}{b^2} + \frac{a \cos ax \cdot \sin bx}{b^2} \right]$$

$$= -\frac{b}{b^2 - a^2} \sin ax \cdot \cos bx + \frac{a}{b^2 - a^2} \cos ax \cdot \sin bx + c_2, \quad x \in \mathbb{R}, \quad c_2 \in \mathbb{R}.$$

Riešené príklady – 056, 057, 058

$$\int \sin ax \cdot \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\int \cos^2 ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\int \sin^2 ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

Riešené príklady – 056, 057, 058

$$\int \sin ax \cdot \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \int \frac{2 \sin ax \cdot \cos ax \, dx}{2}$$

$$\int \cos^2 ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \int \frac{1 + \cos 2ax}{2} \, dx$$

$$\int \sin^2 ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \int \frac{1 - \cos 2ax}{2} \, dx$$

Riešené príklady – 056, 057, 058

$$\int \sin ax \cdot \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \int \frac{2 \sin ax \cdot \cos ax \, dx}{2} = \int \frac{\sin 2ax \, dx}{2}$$

$$\int \cos^2 ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \int \frac{1 + \cos 2ax}{2} \, dx = \int \frac{dx}{2} + \int \frac{\cos 2ax \, dx}{2}$$

$$\int \sin^2 ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \int \frac{1 - \cos 2ax}{2} \, dx = \int \frac{dx}{2} - \int \frac{\cos 2ax \, dx}{2}$$

Riešené príklady – 056, 057, 058

$$\int \sin ax \cdot \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \int \frac{2 \sin ax \cdot \cos ax \, dx}{2} = \int \frac{\sin 2ax \, dx}{2} = \frac{-\cos 2ax}{2 \cdot 2a} + c$$

$$\int \cos^2 ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \int \frac{1 + \cos 2ax}{2} \, dx = \int \frac{dx}{2} + \int \frac{\cos 2ax \, dx}{2} = \frac{x}{2} + \frac{\sin 2ax}{2 \cdot 2a} + c$$

$$\int \sin^2 ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \int \frac{1 - \cos 2ax}{2} \, dx = \int \frac{dx}{2} - \int \frac{\cos 2ax \, dx}{2} = \frac{x}{2} - \frac{\sin 2ax}{2 \cdot 2a} + c$$

Riešené príklady – 056, 057, 058

$$\int \sin ax \cdot \cos ax \, dx = -\frac{\cos 2ax}{4a} + c \quad a \in \mathbb{R}, a \neq 0$$

$$= \int \frac{2 \sin ax \cdot \cos ax \, dx}{2} = \int \frac{\sin 2ax \, dx}{2} = -\frac{\cos 2ax}{2 \cdot 2a} + c = -\frac{\cos 2ax}{4a} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + c \quad a \in \mathbb{R}, a \neq 0$$

$$= \int \frac{1 + \cos 2ax}{2} \, dx = \int \frac{dx}{2} + \int \frac{\cos 2ax \, dx}{2} = \frac{x}{2} + \frac{\sin 2ax}{2 \cdot 2a} + c = \frac{x}{2} + \frac{\sin 2ax}{4a} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + c \quad a \in \mathbb{R}, a \neq 0$$

$$= \int \frac{1 - \cos 2ax}{2} \, dx = \int \frac{dx}{2} - \int \frac{\cos 2ax \, dx}{2} = \frac{x}{2} - \frac{\sin 2ax}{2 \cdot 2a} + c = \frac{x}{2} - \frac{\sin 2ax}{4a} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 059, 060

$$\int x \operatorname{tg}^2 x \, dx$$

$$\int x \operatorname{cotg}^2 x \, dx$$

Riešené príklady – 059, 060

$$\int x \operatorname{tg}^2 x \, dx$$

$$= \int \frac{x \sin^2 x \, dx}{\cos^2 x}$$

$$\int x \operatorname{cotg}^2 x \, dx$$

$$= \int \frac{x \cos^2 x \, dx}{\sin^2 x}$$

Riešené príklady – 059, 060

$$\int x \operatorname{tg}^2 x \, dx$$

$$= \int \frac{x \sin^2 x \, dx}{\cos^2 x} = \int \frac{x(1 - \cos^2 x) \, dx}{\cos^2 x}$$

$$\int x \operatorname{cotg}^2 x \, dx$$

$$= \int \frac{x \cos^2 x \, dx}{\sin^2 x} = \int \frac{x(1 - \sin^2 x) \, dx}{\sin^2 x}$$

Riešené príklady – 059, 060

$$\int x \operatorname{tg}^2 x \, dx$$

$$= \int \frac{x \sin^2 x \, dx}{\cos^2 x} = \int \frac{x(1 - \cos^2 x) \, dx}{\cos^2 x} = \int \frac{x \, dx}{\cos^2 x} - \int x \, dx$$

$$\int x \operatorname{cotg}^2 x \, dx$$

$$= \int \frac{x \cos^2 x \, dx}{\sin^2 x} = \int \frac{x(1 - \sin^2 x) \, dx}{\sin^2 x} = \int \frac{x \, dx}{\sin^2 x} - \int x \, dx$$

Riešené príklady – 059, 060

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$$= \int \frac{x \cos^2 x \, dx}{\sin^2 x} = \int \frac{x(1 - \sin^2 x) \, dx}{\sin^2 x} = \int \frac{x \, dx}{\sin^2 x} - \int x \, dx = \left[\begin{array}{l} u = x \quad | \quad u' = 1 \\ v' = \frac{1}{\sin^2 x} \quad | \quad v = -\operatorname{cotg} x = -\frac{\cos x}{\sin x} \end{array} \right]$$

$$= -x \operatorname{cotg} x + \int \frac{\cos x \, dx}{\sin x} - \int x \, dx$$

Riešené príklady – 059, 060

$$\int x \operatorname{tg}^2 x \, dx$$

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$$= x \operatorname{tg} x - \int \frac{\sin x \, dx}{\cos x} - \int x \, dx = x \operatorname{tg} x + \int \frac{-\sin x \, dx}{\cos x} - \frac{x^2}{2}$$

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$$= \int \frac{x \cos^2 x \, dx}{\sin^2 x} = \int \frac{x(1 - \sin^2 x) \, dx}{\sin^2 x} = \int \frac{x \, dx}{\sin^2 x} - \int x \, dx = \left[\begin{array}{l} u = x \quad | \quad u' = 1 \\ v' = \frac{1}{\sin^2 x} \quad | \quad v = -\operatorname{cotg} x = -\frac{\cos x}{\sin x} \end{array} \right]$$

$$= -x \operatorname{cotg} x + \int \frac{\cos x \, dx}{\sin x} - \int x \, dx = -x \operatorname{cotg} x + \ln |\sin x| - \frac{x^2}{2} + c,$$

$$x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}.$$

Riešené príklady – 059, 060

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$$= x \operatorname{tg} x + \ln |\cos x| - \frac{x^2}{2} + c, \quad x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}.$$

$$\int x \operatorname{cotg}^2 x \, dx = -x \operatorname{cotg} x + \ln |\sin x| - \frac{x^2}{2} + c$$

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Riešené príklady – 061

$$S_1 = \int x \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

Riešené príklady – 061

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[Odhad riešenia metódou neurčitých koeficientov]

$$S_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$$

Riešené príklady – 061

$$S_1 = \int x \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

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[Odhad riešenia metódou neurčitých koeficientov]

$$S_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$$

[Derivácia rovnosti $\int x \sin ax \, dx = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$]

$$x \sin ax = [\beta \sin ax + (\alpha + \beta x)a \cos ax] + [\delta \cos ax - (\gamma + \delta x)a \sin ax].$$

Riešené príklady – 061

$$S_1 = \int x \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

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$$[0 + 1 \cdot x] \sin ax + [0 + 0 \cdot x] \cos ax = [\beta - \gamma a - \delta ax] \sin ax + [\alpha a + \delta + \beta ax] \cos ax.$$

Riešené príklady – 061

$$S_1 = \int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c \quad a \in R, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{1}{a} \cos ax \end{array} \right] = -\frac{x \cos ax}{a} + \int \frac{\cos ax}{a} \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a \cdot a} + c \\ &= \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c, \quad x \in R, c \in R. \end{aligned}$$

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$$0 = \beta - \gamma a, \quad 1 = -\delta a \quad \text{pre } \sin ax,$$

Riešené príklady – 061

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$$= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{1}{a} \cos ax \end{array} \right] = -\frac{x \cos ax}{a} + \int \frac{\cos ax}{a} \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a \cdot a} + c$$

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$$x \sin ax = [\beta \sin ax + (\alpha + \beta x)a \cos ax] + [\delta \cos ax - (\gamma + \delta x)a \sin ax].$$

$$[0 + 1 \cdot x] \sin ax + [0 + 0 \cdot x] \cos ax = [\beta - \gamma a - \delta a x] \sin ax + [\alpha a + \delta + \beta a x] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$$0 = \beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \delta + \alpha a, \quad 0 = \beta a \text{ pre } \cos ax.$$

Riešené príklady – 061

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$$[0 + 1 \cdot x] \sin ax + [0 + 0 \cdot x] \cos ax = [\beta - \gamma a - \delta ax] \sin ax + [\alpha a + \delta + \beta ax] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$$0 = \beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \delta + \alpha a, \quad 0 = \beta a \text{ pre } \cos ax.$$

$$\Rightarrow \beta = \frac{0}{a},$$

Riešené príklady – 061

$$S_1 = \int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c \quad a \in R, a \neq 0$$

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$$0 = \beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \delta + \alpha a, \quad 0 = \beta a \text{ pre } \cos ax.$$

$$\Rightarrow \beta = \frac{0}{a}, \quad \gamma = \frac{\beta}{a},$$

Riešené príklady – 061

$$S_1 = \int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c \quad a \in \mathbb{R}, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{1}{a} \cos ax \end{array} \right] = -\frac{x \cos ax}{a} + \int \frac{\cos ax}{a} \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a \cdot a} + c \\ &= \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$$0 = \beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \delta + \alpha a, \quad 0 = \beta a \text{ pre } \cos ax.$$

$$\Rightarrow \beta = \frac{0}{a}, \quad \gamma = \frac{\beta}{a}, \quad \delta = \frac{1}{-a},$$

Riešené príklady – 061

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

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$$S_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$$

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$$x \sin ax = [\beta \sin ax + (\alpha + \beta x)a \cos ax] + [\delta \cos ax - (\gamma + \delta x)a \sin ax].$$

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$$0 = \beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \delta + \alpha a, \quad 0 = \beta a \text{ pre } \cos ax.$$

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[Derivácia rovnosti $\int x \sin ax \, dx = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$]

$$x \sin ax = [\beta \sin ax + (\alpha + \beta x)a \cos ax] + [\delta \cos ax - (\gamma + \delta x)a \sin ax].$$

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$$0 = \beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \delta + \alpha a, \quad 0 = \beta a \text{ pre } \cos ax.$$

$$\Rightarrow \beta = \frac{0}{a}, \quad \gamma = \frac{\beta}{a}, \quad \delta = \frac{1}{-a}, \quad \alpha = \frac{\delta}{-a} \Rightarrow \beta = 0, \quad \gamma = 0, \quad \delta = -\frac{1}{a}, \quad \alpha = \frac{1}{a^2}.$$

Riešené príklady – 061

$$S_1 = \int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c \quad a \in R, a \neq 0$$

$$= \left[\begin{array}{l} u = x \\ v' = \sin ax \end{array} \left| \begin{array}{l} u' = 1 \\ v = -\frac{1}{a} \cos ax \end{array} \right. \right] = -\frac{x \cos ax}{a} + \int \frac{\cos ax}{a} \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a \cdot a} + c$$

$$= \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c, \quad x \in R, c \in R.$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c, \quad x \in R, c \in R.$$

[Derivácia rovnosti $\int x \sin ax \, dx = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$]

$$x \sin ax = [\beta \sin ax + (\alpha + \beta x)a \cos ax] + [\delta \cos ax - (\gamma + \delta x)a \sin ax].$$

$$[0 + 1 \cdot x] \sin ax + [0 + 0 \cdot x] \cos ax = [\beta - \gamma a - \delta ax] \sin ax + [\alpha a + \delta + \beta ax] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$$0 = \beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \delta + \alpha a, \quad 0 = \beta a \text{ pre } \cos ax.$$

$$\Rightarrow \beta = \frac{0}{a}, \quad \gamma = \frac{\beta}{a}, \quad \delta = \frac{1}{-a}, \quad \alpha = \frac{\delta}{-a} \Rightarrow \beta = 0, \quad \gamma = 0, \quad \delta = -\frac{1}{a}, \quad \alpha = \frac{1}{a^2}.$$

Riešené príklady – 062

$$C_1 = \int x \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

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[Odhad riešenia metódou neurčitých koeficientov]

$$C_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$$

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$$x \cos ax = [\beta \sin ax + (\alpha + \beta x)a \cos ax] + [\delta \cos ax - (\gamma + \delta x)a \sin ax].$$

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Riešené príklady – 062

$$C_1 = \int x \cos ax \, dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a} + c \quad a \in R, a \neq 0$$

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$$0 = \beta - \gamma a, \quad 0 = -\delta a \text{ pre } \sin ax, \quad 0 = \delta + \alpha a, \quad 1 = \beta a \text{ pre } \cos ax.$$

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$$0 = \beta - \gamma a, \quad 0 = -\delta a \text{ pre } \sin ax, \quad 0 = \delta + \alpha a, \quad 1 = \beta a \text{ pre } \cos ax.$$

$$\Rightarrow \beta = \frac{1}{a},$$

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Riešené príklady – 062

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[Odhad riešenia metódou neurčitých koeficientov]

$$C_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c = \frac{x \sin ax}{a} + \frac{\cos ax}{a^2} + c, \quad x \in R, c \in R.$$

[Derivácia rovnosti $\int x \cos ax \, dx = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$]

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Riešené príklady – 063, 064

$$\int x^2 \sinh ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

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Riešené príklady – 063, 064

$$\int x^2 \sinh ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sinh ax & v = \frac{\cosh ax}{a} \end{array} \right]$$

$$\int x^2 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right]$$

Riešené príklady – 063, 064

$$\int x^2 \sinh ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sinh ax & v = \frac{\cosh ax}{a} \end{array} \right] = \frac{x^2 \cosh ax}{a} - \frac{2}{a} \int x \cosh ax \, dx$$

$$\int x^2 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = -\frac{x^2 \cos ax}{a} + \frac{2}{a} \int x \cos ax \, dx$$

Riešené príklady – 063, 064

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$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sinh ax & v = \frac{\cosh ax}{a} \end{array} \right] = \frac{x^2 \cosh ax}{a} - \frac{2}{a} \int x \cosh ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cosh ax & v = \frac{\sinh ax}{a} \end{array} \right]$$

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Riešené príklady – 063, 064

$$\int x^2 \sinh ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \quad | \quad u' = 2x \\ v' = \sinh ax \quad | \quad v = \frac{\cosh ax}{a} \end{array} \right] = \frac{x^2 \cosh ax}{a} - \frac{2}{a} \int x \cosh ax \, dx = \left[\begin{array}{l} u = x \quad | \quad u' = 1 \\ v' = \cosh ax \quad | \quad v = \frac{\sinh ax}{a} \end{array} \right] \\ &= \frac{x^2 \cosh ax}{a} - \frac{2}{a} \left[\frac{x \sinh ax}{a} - \frac{1}{a} \int \sinh ax \, dx \right] \end{aligned}$$

$$\int x^2 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \quad | \quad u' = 2x \\ v' = \sin ax \quad | \quad v = -\frac{\cos ax}{a} \end{array} \right] = -\frac{x^2 \cos ax}{a} + \frac{2}{a} \int x \cos ax \, dx = \left[\begin{array}{l} u = x \quad | \quad u' = 1 \\ v' = \cos ax \quad | \quad v = \frac{\sin ax}{a} \end{array} \right] \\ &= -\frac{x^2 \cos ax}{a} + \frac{2}{a} \left[\frac{x \sin ax}{a} - \frac{1}{a} \int \sin ax \, dx \right] \end{aligned}$$

Riešené príklady – 063, 064

$$\int x^2 \sinh ax \, dx$$

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$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \quad | \quad u' = 2x \\ v = \sinh ax \quad | \quad v = \frac{\cosh ax}{a} \end{array} \right] = \frac{x^2 \cosh ax}{a} - \frac{2}{a} \int x \cosh ax \, dx = \left[\begin{array}{l} u = x \quad | \quad u' = 1 \\ v = \cosh ax \quad | \quad v = \frac{\sinh ax}{a} \end{array} \right] \\ &= \frac{x^2 \cosh ax}{a} - \frac{2}{a} \left[\frac{x \sinh ax}{a} - \frac{1}{a} \int \sinh ax \, dx \right] = \frac{x^2 \cosh ax}{a} - \frac{2}{a} \left[\frac{x \sinh ax}{a} - \frac{\cosh ax}{a \cdot a} \right] \end{aligned}$$

$$\int x^2 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \quad | \quad u' = 2x \\ v = \sin ax \quad | \quad v = -\frac{\cos ax}{a} \end{array} \right] = -\frac{x^2 \cos ax}{a} + \frac{2}{a} \int x \cos ax \, dx = \left[\begin{array}{l} u = x \quad | \quad u' = 1 \\ v = \cos ax \quad | \quad v = \frac{\sin ax}{a} \end{array} \right] \\ &= -\frac{x^2 \cos ax}{a} + \frac{2}{a} \left[\frac{x \sin ax}{a} - \frac{1}{a} \int \sin ax \, dx \right] = -\frac{x^2 \cos ax}{a} + \frac{2}{a} \left[\frac{x \sin ax}{a} - \frac{-\cos ax}{a \cdot a} \right] \end{aligned}$$

Riešené príklady – 063, 064

$$\int x^2 \sinh ax \, dx = \frac{2 \cosh ax}{a^3} - \frac{2x \sinh ax}{a^2} + \frac{x^2 \cosh ax}{a} + c \quad a \in \mathbb{R}, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \quad | \quad u' = 2x \\ v' = \sinh ax \quad | \quad v = \frac{\cosh ax}{a} \end{array} \right] = \frac{x^2 \cosh ax}{a} - \frac{2}{a} \int x \cosh ax \, dx = \left[\begin{array}{l} u = x \quad | \quad u' = 1 \\ v' = \cosh ax \quad | \quad v = \frac{\sinh ax}{a} \end{array} \right] \\ &= \frac{x^2 \cosh ax}{a} - \frac{2}{a} \left[\frac{x \sinh ax}{a} - \frac{1}{a} \int \sinh ax \, dx \right] = \frac{x^2 \cosh ax}{a} - \frac{2}{a} \left[\frac{x \sinh ax}{a} - \frac{\cosh ax}{a \cdot a} \right] \\ &= \frac{2 \cosh ax}{a^3} - \frac{2x \sinh ax}{a^2} + \frac{x^2 \cosh ax}{a} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

$$\int x^2 \sin ax \, dx = \frac{2 \cos ax}{a^3} + \frac{2x \sin ax}{a^2} - \frac{x^2 \cos ax}{a} + c \quad a \in \mathbb{R}, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \quad | \quad u' = 2x \\ v' = \sin ax \quad | \quad v = -\frac{\cos ax}{a} \end{array} \right] = -\frac{x^2 \cos ax}{a} + \frac{2}{a} \int x \cos ax \, dx = \left[\begin{array}{l} u = x \quad | \quad u' = 1 \\ v' = \cos ax \quad | \quad v = \frac{\sin ax}{a} \end{array} \right] \\ &= -\frac{x^2 \cos ax}{a} + \frac{2}{a} \left[\frac{x \sin ax}{a} - \frac{1}{a} \int \sin ax \, dx \right] = -\frac{x^2 \cos ax}{a} + \frac{2}{a} \left[\frac{x \sin ax}{a} - \frac{-\cos ax}{a \cdot a} \right] \\ &= \frac{2 \cos ax}{a^3} + \frac{2x \sin ax}{a^2} - \frac{x^2 \cos ax}{a} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx$$

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[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$\begin{aligned} x^2 \sin ax = & [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ & + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax]. \end{aligned}$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \sin ax = [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax].$$

$$[0 + 0 \cdot x + 1 \cdot x^2] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2] \sin ax + [\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2] \cos ax.$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \sin ax = [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax].$$

$$[0 + 0 \cdot x + 1 \cdot x^2] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2] \sin ax + [\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \sin ax = [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax].$$

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[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 1 = -\varphi a \text{ pre } \sin ax,$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx$$

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$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

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$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 1 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 0 = \gamma a \text{ pre } \cos ax.$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx$$

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[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 1 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 0 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \gamma = \frac{0}{a},$$

Riešené príklady – 064

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[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 1 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 0 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \gamma = \frac{0}{a}, \quad \psi = \frac{2\gamma}{a},$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx$$

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[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 1 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 0 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \gamma = \frac{0}{a}, \quad \psi = \frac{2\gamma}{a}, \quad \alpha = \frac{\psi}{-a},$$

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[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 1 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 0 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \gamma = \frac{0}{a}, \quad \psi = \frac{2\gamma}{a}, \quad \alpha = \frac{\psi}{-a}, \quad \varphi = \frac{1}{-a},$$

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[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 1 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 0 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \gamma = \frac{0}{a}, \quad \psi = \frac{2\gamma}{a}, \quad \alpha = \frac{\psi}{-a}, \quad \varphi = \frac{1}{-a}, \quad \beta = \frac{2\varphi}{-a},$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx$$

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[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 1 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 0 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \gamma = \frac{0}{a}, \quad \psi = \frac{2\gamma}{a}, \quad \alpha = \frac{\psi}{-a}, \quad \varphi = \frac{1}{-a}, \quad \beta = \frac{2\varphi}{-a}, \quad \delta = \frac{\beta}{a}$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

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$$[0 + 0 \cdot x + 1 \cdot x^2] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2] \sin ax + [\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 1 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 0 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \gamma = \frac{0}{a}, \quad \psi = \frac{2\gamma}{a}, \quad \alpha = \frac{\psi}{-a}, \quad \varphi = \frac{1}{-a}, \quad \beta = \frac{2\varphi}{-a}, \quad \delta = \frac{\beta}{a} \\ \Rightarrow \gamma = 0, \quad \psi = 0, \quad \alpha = 0,$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx$$

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[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \sin ax = [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax].$$

$$[0 + 0 \cdot x + 1 \cdot x^2] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2] \sin ax + [\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 1 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 0 = \gamma a \text{ pre } \cos ax.$$

$$\Rightarrow \gamma = \frac{0}{a}, \quad \psi = \frac{2\gamma}{a}, \quad \alpha = \frac{\psi}{-a}, \quad \varphi = \frac{1}{-a}, \quad \beta = \frac{2\varphi}{-a}, \quad \delta = \frac{\beta}{a}$$

$$\Rightarrow \gamma = 0, \quad \psi = 0, \quad \alpha = 0, \quad \varphi = -\frac{1}{a}, \quad \beta = \frac{2}{a^2}, \quad \delta = \frac{2}{a^3}.$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax \, dx = \frac{2 \cos ax}{a^3} + \frac{2x \sin ax}{a^2} - \frac{x^2 \cos ax}{a} + c \quad a \in \mathbb{R}, a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov]

$$\begin{aligned} S_2 &= (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c \\ &= \frac{2x \sin ax}{a^2} + \frac{2 \cos ax}{a^3} - \frac{x^2 \cos ax}{a} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

[Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$\begin{aligned} x^2 \sin ax &= [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ &\quad + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax]. \end{aligned}$$

$$\begin{aligned} [0 + 0 \cdot x + 1 \cdot x^2] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2] \sin ax + [\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2] \cos ax. \end{aligned}$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$\begin{aligned} 0 &= \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 1 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 0 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \quad \gamma &= \frac{0}{a}, \quad \psi = \frac{2\gamma}{a}, \quad \alpha = \frac{\psi}{-a}, \quad \varphi = \frac{1}{-a}, \quad \beta = \frac{2\varphi}{-a}, \quad \delta = \frac{\beta}{a} \\ &\Rightarrow \quad \gamma = 0, \quad \psi = 0, \quad \alpha = 0, \quad \varphi = -\frac{1}{a}, \quad \beta = \frac{2}{a^2}, \quad \delta = \frac{2}{a^3}. \end{aligned}$$

Riešené príklady – 065, 066

$$\int x^2 \cosh ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\int x^2 \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

Riešené príklady – 065, 066

$$\int x^2 \cosh ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cosh ax & v = \frac{\sinh ax}{a} \end{array} \right]$$

$$\int x^2 \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right]$$

Riešené príklady – 065, 066

$$\int x^2 \cosh ax \, dx$$

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$$= \left[\begin{array}{l} u = x^2 \\ v' = \cosh ax \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{\sinh ax}{a} \end{array} \right] = \frac{x^2 \sinh ax}{a} - \frac{2}{a} \int x \sinh ax \, dx$$

$$\int x^2 \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l} u = x^2 \\ v' = \cos ax \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{\sin ax}{a} \end{array} \right] = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx$$

Riešené príklady – 065, 066

$$\int x^2 \cosh ax \, dx$$

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$$\int x^2 \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l} u = x^2 \\ v' = \cos ax \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{\sin ax}{a} \end{array} \right] = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx = \left[\begin{array}{l} u = x \\ v' = \sin ax \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{\cos ax}{a} \end{array} \right]$$

Riešené príklady – 065, 066

$$\int x^2 \cosh ax \, dx$$

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$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \\ v' = \cosh ax \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{\sinh ax}{a} \end{array} \right] = \frac{x^2 \sinh ax}{a} - \frac{2}{a} \int x \sinh ax \, dx = \left[\begin{array}{l} u = x \\ v' = \sinh ax \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{\cosh ax}{a} \end{array} \right] \\ &= \frac{x^2 \sinh ax}{a} - \frac{2}{a} \left[\frac{x \cosh ax}{a} - \frac{1}{a} \int \cosh ax \, dx \right] \end{aligned}$$

$$\int x^2 \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \\ v' = \cos ax \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{\sin ax}{a} \end{array} \right] = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx = \left[\begin{array}{l} u = x \\ v' = \sin ax \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{\cos ax}{a} \end{array} \right] \\ &= \frac{x^2 \sin ax}{a} - \frac{2}{a} \left[-\frac{x \cos ax}{a} + \frac{1}{a} \int \cos ax \, dx \right] \end{aligned}$$

Riešené príklady – 065, 066

$$\int x^2 \cosh ax \, dx$$

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$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \\ v' = \cosh ax \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{\sinh ax}{a} \end{array} \right] = \frac{x^2 \sinh ax}{a} - \frac{2}{a} \int x \sinh ax \, dx = \left[\begin{array}{l} u = x \\ v' = \sinh ax \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{\cosh ax}{a} \end{array} \right] \\ &= \frac{x^2 \sinh ax}{a} - \frac{2}{a} \left[\frac{x \cosh ax}{a} - \frac{1}{a} \int \cosh ax \, dx \right] = \frac{x^2 \sinh ax}{a} - \frac{2}{a} \left[\frac{x \cosh ax}{a} - \frac{\sinh ax}{a \cdot a} \right] \end{aligned}$$

$$\int x^2 \cos ax \, dx$$

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$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \\ v' = \cos ax \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{\sin ax}{a} \end{array} \right] = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx = \left[\begin{array}{l} u = x \\ v' = \sin ax \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{\cos ax}{a} \end{array} \right] \\ &= \frac{x^2 \sin ax}{a} - \frac{2}{a} \left[-\frac{x \cos ax}{a} + \frac{1}{a} \int \cos ax \, dx \right] = \frac{x^2 \sin ax}{a} - \frac{2}{a} \left[-\frac{x \cos ax}{a} + \frac{\sin ax}{a \cdot a} \right] \end{aligned}$$

Riešené príklady – 065, 066

$$\int x^2 \cosh ax \, dx = \frac{2 \sinh ax}{a^3} - \frac{2x \cosh ax}{a^2} + \frac{x^2 \sinh ax}{a} + c \quad a \in R, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \\ v' = \cosh ax \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{\sinh ax}{a} \end{array} \right] = \frac{x^2 \sinh ax}{a} - \frac{2}{a} \int x \sinh ax \, dx = \left[\begin{array}{l} u = x \\ v' = \sinh ax \end{array} \middle| \begin{array}{l} u' = 1 \\ v = \frac{\cosh ax}{a} \end{array} \right] \\ &= \frac{x^2 \sinh ax}{a} - \frac{2}{a} \left[\frac{x \cosh ax}{a} - \frac{1}{a} \int \cosh ax \, dx \right] = \frac{x^2 \sinh ax}{a} - \frac{2}{a} \left[\frac{x \cosh ax}{a} - \frac{\sinh ax}{a \cdot a} \right] \\ &= \frac{2 \sinh ax}{a^3} - \frac{2x \cosh ax}{a^2} + \frac{x^2 \sinh ax}{a} + c, \quad x \in R, c \in R. \end{aligned}$$

$$\int x^2 \cos ax \, dx = -\frac{2 \sin ax}{a^3} + \frac{2x \cos ax}{a^2} + \frac{x^2 \sin ax}{a} + c \quad a \in R, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \\ v' = \cos ax \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{\sin ax}{a} \end{array} \right] = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx = \left[\begin{array}{l} u = x \\ v' = \sin ax \end{array} \middle| \begin{array}{l} u' = 1 \\ v = -\frac{\cos ax}{a} \end{array} \right] \\ &= \frac{x^2 \sin ax}{a} - \frac{2}{a} \left[-\frac{x \cos ax}{a} + \frac{1}{a} \int \cos ax \, dx \right] = \frac{x^2 \sin ax}{a} - \frac{2}{a} \left[-\frac{x \cos ax}{a} + \frac{\sin ax}{a \cdot a} \right] \\ &= -\frac{2 \sin ax}{a^3} + \frac{2x \cos ax}{a^2} + \frac{x^2 \sin ax}{a} + c, \quad x \in R, c \in R. \end{aligned}$$

Riešené príklady – 066

$$C_2 = \int x^2 \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

Riešené príklady – 066

$$C_2 = \int x^2 \cos ax \, dx$$

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[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Riešené príklady – 066

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[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$\begin{aligned} x^2 \cos ax = & [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ & + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax]. \end{aligned}$$

Riešené príklady – 066

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$$[0 + 0 \cdot x + 0 \cdot x^2] \sin ax + [0 + 0 \cdot x + 1 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2] \sin ax + [\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2] \cos ax.$$

Riešené príklady – 066

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[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

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[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax].$$

$$[0 + 0 \cdot x + 0 \cdot x^2] \sin ax + [0 + 0 \cdot x + 1 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2] \sin ax + [\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 0 = -\varphi a \text{ pre } \sin ax,$$

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$$x^2 \cos ax = [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax].$$

$$[0 + 0 \cdot x + 0 \cdot x^2] \sin ax + [0 + 0 \cdot x + 1 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2] \sin ax + [\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 0 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 1 = \gamma a \text{ pre } \cos ax.$$

Riešené príklady – 066

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[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax].$$

$$[0 + 0 \cdot x + 0 \cdot x^2] \sin ax + [0 + 0 \cdot x + 1 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2] \sin ax + [\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 0 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 1 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \varphi = \frac{0}{-a},$$

Riešené príklady – 066

$$C_2 = \int x^2 \cos ax \, dx$$

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[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax].$$

$$[0 + 0 \cdot x + 0 \cdot x^2] \sin ax + [0 + 0 \cdot x + 1 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2] \sin ax + [\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 0 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 1 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \varphi = \frac{0}{-a}, \quad \beta = \frac{2\varphi}{-a},$$

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[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

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$$[0 + 0 \cdot x + 0 \cdot x^2] \sin ax + [0 + 0 \cdot x + 1 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2] \sin ax + [\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 0 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 1 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \varphi = \frac{0}{-a}, \quad \beta = \frac{2\varphi}{-a}, \quad \delta = \frac{\beta}{a},$$

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[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 0 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 1 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \varphi = \frac{0}{-a}, \quad \beta = \frac{2\varphi}{-a}, \quad \delta = \frac{\beta}{a}, \quad \gamma = \frac{1}{a},$$

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$$[0 + 0 \cdot x + 0 \cdot x^2] \sin ax + [0 + 0 \cdot x + 1 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2] \sin ax + [\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2] \cos ax.$$

[Riešenie 6 rovnic so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 0 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 1 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \varphi = \frac{0}{-a}, \quad \beta = \frac{2\varphi}{-a}, \quad \delta = \frac{\beta}{a}, \quad \gamma = \frac{1}{a}, \quad \psi = \frac{2\gamma}{a},$$

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$$[0 + 0 \cdot x + 0 \cdot x^2] \sin ax + [0 + 0 \cdot x + 1 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2] \sin ax + [\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2] \cos ax.$$

[Riešenie 6 rovnic so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 0 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 1 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \varphi = \frac{0}{-a}, \quad \beta = \frac{2\varphi}{-a}, \quad \delta = \frac{\beta}{a}, \quad \gamma = \frac{1}{a}, \quad \psi = \frac{2\gamma}{a}, \quad \alpha = \frac{\psi}{-a}$$

Riešené príklady – 066

$$C_2 = \int x^2 \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax].$$

$$[0 + 0 \cdot x + 0 \cdot x^2] \sin ax + [0 + 0 \cdot x + 1 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2] \sin ax + [\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2] \cos ax.$$

[Riešenie 6 rovnic so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 0 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 1 = \gamma a \text{ pre } \cos ax.$$

$$\Rightarrow \varphi = \frac{0}{-a}, \quad \beta = \frac{2\varphi}{-a}, \quad \delta = \frac{\beta}{a}, \quad \gamma = \frac{1}{a}, \quad \psi = \frac{2\gamma}{a}, \quad \alpha = \frac{\psi}{-a}$$

$$\Rightarrow \varphi = 0, \quad \beta = 0, \quad \delta = 0,$$

Riešené príklady – 066

$$C_2 = \int x^2 \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

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$$x^2 \cos ax = [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax].$$

$$[0 + 0 \cdot x + 0 \cdot x^2] \sin ax + [0 + 0 \cdot x + 1 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2] \sin ax + [\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2] \cos ax.$$

[Riešenie 6 rovnic so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$0 = \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 0 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 1 = \gamma a \text{ pre } \cos ax.$$

$$\Rightarrow \varphi = \frac{0}{-a}, \quad \beta = \frac{2\varphi}{-a}, \quad \delta = \frac{\beta}{a}, \quad \gamma = \frac{1}{a}, \quad \psi = \frac{2\gamma}{a}, \quad \alpha = \frac{\psi}{-a}$$

$$\Rightarrow \varphi = 0, \quad \beta = 0, \quad \delta = 0, \quad \gamma = \frac{1}{a}, \quad \psi = \frac{2}{a^2}, \quad \alpha = -\frac{2}{a^3}.$$

Riešené príklady – 066

$$C_2 = \int x^2 \cos ax \, dx = \frac{2x \cos ax}{a^2} - \frac{2 \sin ax}{a^3} + \frac{x^2 \sin ax}{a} + c \quad a \in R, a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov]

$$\begin{aligned} C_2 &= (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c \\ &= \frac{2x \cos ax}{a^2} - \frac{2 \sin ax}{a^3} + \frac{x^2 \sin ax}{a} + c, \quad x \in R, c \in R. \end{aligned}$$

[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$\begin{aligned} x^2 \cos ax &= [(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax] \\ &\quad + [(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax]. \end{aligned}$$

$$\begin{aligned} [0 + 0 \cdot x + 0 \cdot x^2] \sin ax + [0 + 0 \cdot x + 1 \cdot x^2] \cos ax \\ = [\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2] \sin ax + [\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2] \cos ax. \end{aligned}$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$$\begin{aligned} 0 &= \beta - \delta a, \quad 0 = 2\gamma - \psi a, \quad 0 = -\varphi a \text{ pre } \sin ax, \quad 0 = \alpha a + \psi, \quad 0 = \beta a + 2\varphi, \quad 1 = \gamma a \text{ pre } \cos ax. \\ \Rightarrow \varphi &= \frac{0}{-a}, \quad \beta = \frac{2\varphi}{-a}, \quad \delta = \frac{\beta}{a}, \quad \gamma = \frac{1}{a}, \quad \psi = \frac{2\gamma}{a}, \quad \alpha = \frac{\psi}{-a} \\ \Rightarrow \varphi &= 0, \quad \beta = 0, \quad \delta = 0, \quad \gamma = \frac{1}{a}, \quad \psi = \frac{2}{a^2}, \quad \alpha = -\frac{2}{a^3}. \end{aligned}$$

Riešené príklady – 067, 068

$$S_n = \int x^n \sin ax \, dx$$

$$n \in \mathbb{N}, a \in \mathbb{R}, a \neq 0$$

$$C_n = \int x^n \cos ax \, dx$$

$$n \in \mathbb{N}, a \in \mathbb{R}, a \neq 0$$

Riešené príklady – 067, 068

$$S_n = \int x^n \sin ax \, dx$$

$$n \in \mathbb{N}, a \in \mathbb{R}, a \neq 0$$

$$S_n = \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right]$$

$$S_1 = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right]$$

$$C_n = \int x^n \cos ax \, dx$$

$$n \in \mathbb{N}, a \in \mathbb{R}, a \neq 0$$

$$C_n = \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right]$$

$$C_1 = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right]$$

Riešené príklady – 067, 068

$$S_n = \int x^n \sin ax \, dx$$

$$n \in \mathbb{N}, a \in \mathbb{R}, a \neq 0$$

$$S_n = \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = \frac{-x^n \cos ax}{a} + \frac{n}{a} \int x^{n-1} \cos ax \, dx$$

$$S_1 = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = \frac{-x \cos ax}{a} + \int \frac{\cos ax}{a} \, dx$$

$$C_n = \int x^n \cos ax \, dx$$

$$n \in \mathbb{N}, a \in \mathbb{R}, a \neq 0$$

$$C_n = \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right] = \frac{x^n \sin ax}{a} - \frac{n}{a} \int x^{n-1} \sin ax \, dx$$

$$C_1 = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right] = \frac{x \sin ax}{a} - \int \frac{\sin ax}{a} \, dx$$

Riešené príklady – 067, 068

$$S_n = \int x^n \sin ax \, dx = -\frac{x^n \cos ax}{a} + \frac{n}{a} C_{n-1} \quad n \in \mathbb{N}, a \in \mathbb{R}, a \neq 0$$

$$S_n = \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = -\frac{x^n \cos ax}{a} + \frac{n}{a} \int x^{n-1} \cos ax \, dx = -\frac{x^n \cos ax}{a} + \frac{n}{a} C_{n-1},$$

$$S_1 = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = -\frac{x \cos ax}{a} + \int \frac{\cos ax}{a} \, dx = -\frac{x \cos ax}{a} + \frac{C_0}{a} = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} + c, \\ x \in \mathbb{R}, c \in \mathbb{R}.$$

$$C_n = \int x^n \cos ax \, dx = \frac{x^n \sin ax}{a} - \frac{n}{a} S_{n-1} \quad n \in \mathbb{N}, a \in \mathbb{R}, a \neq 0$$

$$C_n = \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right] = \frac{x^n \sin ax}{a} - \frac{n}{a} \int x^{n-1} \sin ax \, dx = \frac{x^n \sin ax}{a} - \frac{n}{a} S_{n-1},$$

$$C_1 = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right] = \frac{x \sin ax}{a} - \int \frac{\sin ax}{a} \, dx = \frac{x \sin ax}{a} - \frac{S_0}{a} = \frac{x \sin ax}{a} + \frac{\cos ax}{a^2} + c, \\ x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$



Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\left[\begin{array}{l|l} \text{Odhad riešenia metódou} & \text{Pr. 67} \\ \text{neurčitých koeficientov} & \text{Pr. 68} \end{array} \left| \int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx \right. \left. \int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx \right]$$

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\left[\begin{array}{l} \text{Odhad riešenia metódou} \\ \text{neurčitých koeficientov} \end{array} \left| \begin{array}{l} \text{Pr. 67} \\ \text{Pr. 68} \end{array} \right. \int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx \left| \int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx \right. \right]$$

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

$$\left[\text{Derivácia rovnosti } \int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c \right]$$

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha + \beta x^2) a \cos ax \right] + \left[(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3) a \sin ax \right].$$

Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\left[\begin{array}{l} \text{Odhad riešenia metódou} \\ \text{neurčitých koeficientov} \end{array} \left| \begin{array}{l} \text{Pr. 67} \\ \text{Pr. 68} \end{array} \right. \int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx \quad \left| \int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx \right. \right]$$

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

$$\left[\text{Derivácia rovnosti } \int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c \right]$$

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha + \beta x^2) a \cos ax \right] + \left[(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3) a \sin ax \right].$$

$$\begin{aligned} & [0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3] \cos ax \\ & = [(2\beta - \gamma a)x - \delta a x^3] \sin ax + [(\alpha a + \gamma) + (\beta a + 3\delta)x^2] \cos ax. \end{aligned}$$

Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\left[\begin{array}{l} \text{Odhad riešenia metódou} \\ \text{neurčitých koeficientov} \end{array} \left| \begin{array}{l} \text{Pr. 67} \\ \text{Pr. 68} \end{array} \right. \int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx \left| \int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx \right. \right]$$

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

$$\left[\text{Derivácia rovnosti } \int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c \right]$$

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha + \beta x^2) a \cos ax \right] + \left[(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3) a \sin ax \right].$$

$$\begin{aligned} & [0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3] \cos ax \\ & = [(2\beta - \gamma a)x - \delta a x^3] \sin ax + [(\alpha a + \gamma) + (\beta a + 3\delta)x^2] \cos ax. \end{aligned}$$

$$\left[\text{Riešenie 4 rovníc so štyrmi neznámymi } \alpha, \beta, \gamma, \delta \right]$$

Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\left[\text{Odhad riešenia metódou} \mid \text{Pr. 67} \mid \int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx \mid \int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx \right]$$

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

$$\left[\text{Derivácia rovnosti} \mid \int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c \right]$$

$$x^3 \sin ax = [2\beta x \sin ax + (\alpha + \beta x^2)a \cos ax] + [(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3)a \sin ax].$$

$$[0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3] \cos ax = [(2\beta - \gamma a)x - \delta a x^3] \sin ax + [(\alpha a + \gamma) + (\beta a + 3\delta)x^2] \cos ax.$$

$$\left[\text{Riešenie 4 rovníc so štyrmi neznámymi } \alpha, \beta, \gamma, \delta \right]$$

$$0 = 2\beta - \gamma a, \quad 1 = -\delta a \quad \text{pre } \sin ax,$$

Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\left[\begin{array}{l} \text{Odhad riešenia metódou} \\ \text{neurčitých koeficientov} \end{array} \left| \begin{array}{l} \text{Pr. 67} \\ \text{Pr. 68} \end{array} \right. \int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx \left| \int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx \right. \right]$$

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

$$\left[\text{Derivácia rovnosti } \int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c \right]$$

$$x^3 \sin ax = [2\beta x \sin ax + (\alpha + \beta x^2)a \cos ax] + [(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3)a \sin ax].$$

$$[0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3] \cos ax = [(2\beta - \gamma a)x - \delta a x^3] \sin ax + [(\alpha a + \gamma) + (\beta a + 3\delta)x^2] \cos ax.$$

$$\left[\text{Riešenie 4 rovníc so štyrmi neznámymi } \alpha, \beta, \gamma, \delta \right]$$

$$0 = 2\beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \alpha a + \gamma, \quad 0 = \beta a + 3\delta \text{ pre } \cos ax.$$

Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 | $\int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx$ | Pr. 68 | $\int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx$]

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

[Derivácia rovnosti $\int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$]

$$x^3 \sin ax = [2\beta x \sin ax + (\alpha + \beta x^2)a \cos ax] + [(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3)a \sin ax].$$

$$[0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3] \cos ax = [(2\beta - \gamma a)x - \delta a x^3] \sin ax + [(\alpha a + \gamma) + (\beta a + 3\delta)x^2] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$$0 = 2\beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \alpha a + \gamma, \quad 0 = \beta a + 3\delta \text{ pre } \cos ax.$$

$$\Rightarrow \delta = \frac{1}{-a},$$

Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\left[\begin{array}{l} \text{Odhad riešenia metódou} \\ \text{neurčitých koeficientov} \end{array} \left| \begin{array}{l} \text{Pr. 67} \\ \text{Pr. 68} \end{array} \right. \int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx \left| \int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx \right. \right]$$

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

$$\left[\text{Derivácia rovnosti } \int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c \right]$$

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha + \beta x^2) a \cos ax \right] + \left[(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3) a \sin ax \right].$$

$$\begin{aligned} & [0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3] \cos ax \\ & = [(2\beta - \gamma a)x - \delta a x^3] \sin ax + [(\alpha a + \gamma) + (\beta a + 3\delta)x^2] \cos ax. \end{aligned}$$

$$\left[\text{Riešenie 4 rovníc so štyrmi neznámymi } \alpha, \beta, \gamma, \delta \right]$$

$$0 = 2\beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \alpha a + \gamma, \quad 0 = \beta a + 3\delta \text{ pre } \cos ax.$$

$$\Rightarrow \delta = \frac{1}{-a}, \quad \beta = \frac{3\delta}{-a},$$

Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\left[\text{Odhad riešenia metódou} \mid \text{Pr. 67} \mid \int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx \mid \int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx \right]$$

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

$$\left[\text{Derivácia rovnosti} \mid \int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c \right]$$

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha + \beta x^2) a \cos ax \right] + \left[(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3) a \sin ax \right].$$

$$\left[0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3 \right] \sin ax + \left[0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3 \right] \cos ax \\ = \left[(2\beta - \gamma a)x - \delta a x^3 \right] \sin ax + \left[(\alpha a + \gamma) + (\beta a + 3\delta)x^2 \right] \cos ax.$$

$$\left[\text{Riešenie 4 rovníc so štyrmi neznámymi } \alpha, \beta, \gamma, \delta \right]$$

$$0 = 2\beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \alpha a + \gamma, \quad 0 = \beta a + 3\delta \text{ pre } \cos ax.$$

$$\Rightarrow \delta = \frac{1}{-a}, \quad \beta = \frac{3\delta}{-a}, \quad \gamma = \frac{2\beta}{a},$$

Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\left[\begin{array}{l} \text{Odhad riešenia metódou} \\ \text{neurčitých koeficientov} \end{array} \left| \begin{array}{l} \text{Pr. 67} \\ \text{Pr. 68} \end{array} \right. \int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx \left| \int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx \right. \right]$$

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

$$\left[\text{Derivácia rovnosti } \int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c \right]$$

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha + \beta x^2) a \cos ax \right] + \left[(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3) a \sin ax \right].$$

$$\begin{aligned} & [0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3] \cos ax \\ & = [(2\beta - \gamma a)x - \delta a x^3] \sin ax + [(\alpha a + \gamma) + (\beta a + 3\delta)x^2] \cos ax. \end{aligned}$$

$$\left[\text{Riešenie 4 rovníc so štyrmi neznámymi } \alpha, \beta, \gamma, \delta \right]$$

$$0 = 2\beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \alpha a + \gamma, \quad 0 = \beta a + 3\delta \text{ pre } \cos ax.$$

$$\Rightarrow \delta = \frac{1}{-a}, \quad \beta = \frac{3\delta}{-a}, \quad \gamma = \frac{2\beta}{a}, \quad \alpha = \frac{\gamma}{-a}$$

Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\left[\text{Odhad riešenia metódou} \mid \text{Pr. 67} \mid \int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx \mid \int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx \right]$$

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

$$\left[\text{Derivácia rovnosti} \mid \int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c \right]$$

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha + \beta x^2) a \cos ax \right] + \left[(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3) a \sin ax \right].$$

$$\begin{aligned} & [0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3] \cos ax \\ & = [(2\beta - \gamma a)x - \delta a x^3] \sin ax + [(\alpha a + \gamma) + (\beta a + 3\delta)x^2] \cos ax. \end{aligned}$$

$$\left[\text{Riešenie 4 rovníc so štyrmi neznámymi } \alpha, \beta, \gamma, \delta \right]$$

$$0 = 2\beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \alpha a + \gamma, \quad 0 = \beta a + 3\delta \text{ pre } \cos ax.$$

$$\Rightarrow \delta = \frac{1}{-a}, \quad \beta = \frac{3\delta}{-a}, \quad \gamma = \frac{2\beta}{a}, \quad \alpha = \frac{\gamma}{-a} \Rightarrow \delta = -\frac{1}{a}, \quad \beta = \frac{3}{a^2}, \quad \gamma = \frac{6}{a^3}, \quad \alpha = -\frac{6}{a^4}.$$

Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx = -\frac{6 \sin ax}{a^4} + \frac{3x^2 \sin ax}{a^2} + \frac{6x \cos ax}{a^3} - \frac{x^3 \cos ax}{a} + c \quad a \in \mathbb{R}, a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 | $\int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx$ | Pr. 68 | $\int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx$]

$$\begin{aligned} S_3 &= (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c \\ &= -\frac{6 \sin ax}{a^4} + \frac{3x^2 \sin ax}{a^2} + \frac{6x \cos ax}{a^3} - \frac{x^3 \cos ax}{a} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

[Derivácia rovnosti $\int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$]

$$\begin{aligned} x^3 \sin ax &= [2\beta x \sin ax + (\alpha + \beta x^2)a \cos ax] \\ &\quad + [(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3)a \sin ax]. \end{aligned}$$

$$\begin{aligned} [0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3] \sin ax &+ [0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3] \cos ax \\ &= [(2\beta - \gamma a)x - \delta a x^3] \sin ax + [(\alpha a + \gamma) + (\beta a + 3\delta)x^2] \cos ax. \end{aligned}$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$$0 = 2\beta - \gamma a, \quad 1 = -\delta a \text{ pre } \sin ax, \quad 0 = \alpha a + \gamma, \quad 0 = \beta a + 3\delta \text{ pre } \cos ax.$$

$$\Rightarrow \delta = \frac{1}{-a}, \quad \beta = \frac{3\delta}{-a}, \quad \gamma = \frac{2\beta}{a}, \quad \alpha = \frac{\gamma}{-a} \Rightarrow \delta = -\frac{1}{a}, \quad \beta = \frac{3}{a^2}, \quad \gamma = \frac{6}{a^3}, \quad \alpha = -\frac{6}{a^4}.$$

Riešené príklady – 070

$$C_3 = \int x^3 \cos ax \, dx$$

$$a \in \mathbb{R}, a \neq 0$$



Riešené príklady – 070

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$$C_3 = (\alpha x + \beta x^3) \sin ax + (\gamma + \delta x^2) \cos ax + c$$

Riešené príklady – 070

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$$\left[\text{Derivácia rovnosti } \int x^3 \cos ax \, dx = (\alpha x + \beta x^3) \sin ax + (\gamma + \delta x^2) \cos ax + c \right]$$

$$x^3 \cos ax = [(\alpha + 3\beta x^2) \sin ax + (\alpha x + \beta x^3) a \cos ax] + [2\delta x \cos ax - (\gamma + \delta x^2) a \sin ax].$$

Riešené príklady – 070

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$$[0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3] \cos ax \\ = [(\alpha - \gamma a) + (3\beta - \delta a)x^2] \sin ax + [(\alpha a + 2\delta)x + \beta a x^3] \cos ax.$$

Riešené príklady – 070

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$$\left[\text{Riešenie 4 rovníc so štyrmi neznámymi } \alpha, \beta, \gamma, \delta \right]$$

Riešené príklady – 070

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$$\begin{aligned} & [0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3] \sin ax + [0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3] \cos ax \\ & = [(\alpha - \gamma a) + (3\beta - \delta a)x^2] \sin ax + [(\alpha a + 2\delta)x + \beta a x^3] \cos ax. \end{aligned}$$

$$\left[\text{Riešenie 4 rovníc so štyrmi neznámymi } \alpha, \beta, \gamma, \delta \right]$$

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Riešené príklady – 070

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$$\left[\text{Riešenie 4 rovníc so štyrmi neznámymi } \alpha, \beta, \gamma, \delta \right]$$

$$0 = \alpha - \gamma a, \quad 0 = 3\beta - \delta a \text{ pre } \sin ax, \quad 0 = \alpha a + 2\delta, \quad 1 = \beta a \text{ pre } \cos ax.$$

Riešené príklady – 070

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$$0 = \alpha - \gamma a, \quad 0 = 3\beta - \delta a \text{ pre } \sin ax, \quad 0 = \alpha a + 2\delta, \quad 1 = \beta a \text{ pre } \cos ax.$$

$$\Rightarrow \beta = \frac{1}{a},$$

Riešené príklady – 070

$$C_3 = \int x^3 \cos ax \, dx$$

$$a \in R, a \neq 0$$

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Riešené príklady – 070

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$$\left[\text{Riešenie 4 rovníc so štyrmi neznámymi } \alpha, \beta, \gamma, \delta \right]$$

$$0 = \alpha - \gamma a, \quad 0 = 3\beta - \delta a \quad \text{pre } \sin ax, \quad 0 = \alpha a + 2\delta, \quad 1 = \beta a \quad \text{pre } \cos ax.$$

$$\Rightarrow \beta = \frac{1}{a}, \quad \delta = \frac{3\beta}{a}, \quad \alpha = \frac{2\delta}{-a}, \quad \gamma = \frac{\alpha}{a}$$

Riešené príklady – 070

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$$\left[\text{Riešenie 4 rovníc so štyrmi neznámymi } \alpha, \beta, \gamma, \delta \right]$$

$$0 = \alpha - \gamma a, \quad 0 = 3\beta - \delta a \quad \text{pre } \sin ax, \quad 0 = \alpha a + 2\delta, \quad 1 = \beta a \quad \text{pre } \cos ax.$$

$$\Rightarrow \beta = \frac{1}{a}, \quad \delta = \frac{3\beta}{a}, \quad \alpha = \frac{2\delta}{-a}, \quad \gamma = \frac{\alpha}{a} \Rightarrow \beta = \frac{1}{a}, \quad \delta = \frac{3}{a^2}, \quad \alpha = -\frac{6}{a^3}, \quad \gamma = -\frac{6}{a^4}.$$

Riešené príklady – 070

$$C_3 = \int x^3 \cos ax \, dx = -\frac{6x \sin ax}{a^3} + \frac{x^3 \sin ax}{a} - \frac{6 \cos ax}{a^4} + \frac{3x^2 \cos ax}{a^2} + c \quad a \in \mathbb{R}, a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 | $\int x^3 \cos ax \, dx = \frac{x^3 \sin ax}{a} - \frac{3}{a} \int x^2 \sin ax \, dx$ | Pr. 68 | $\int x^2 \sin ax \, dx = -\frac{x^2 \cos ax}{a} + \frac{2}{a} \int x \cos ax \, dx$]

$$\begin{aligned} C_3 &= (\alpha x + \beta x^3) \sin ax + (\gamma + \delta x^2) \cos ax + c \\ &= -\frac{6x \sin ax}{a^3} + \frac{x^3 \sin ax}{a} - \frac{6 \cos ax}{a^4} + \frac{3x^2 \cos ax}{a^2} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

[Derivácia rovnosti $\int x^3 \cos ax \, dx = (\alpha x + \beta x^3) \sin ax + (\gamma + \delta x^2) \cos ax + c$]

$$\begin{aligned} x^3 \cos ax &= [(\alpha + 3\beta x^2) \sin ax + (\alpha x + \beta x^3) a \cos ax] \\ &\quad + [2\delta x \cos ax - (\gamma + \delta x^2) a \sin ax]. \end{aligned}$$

$$\begin{aligned} [0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3] \sin ax &+ [0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3] \cos ax \\ &= [(\alpha - \gamma a) + (3\beta - \delta a)x^2] \sin ax + [(\alpha a + 2\delta)x + \beta a x^3] \cos ax. \end{aligned}$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$$0 = \alpha - \gamma a, \quad 0 = 3\beta - \delta a \text{ pre } \sin ax, \quad 0 = \alpha a + 2\delta, \quad 1 = \beta a \text{ pre } \cos ax.$$

$$\Rightarrow \beta = \frac{1}{a}, \quad \delta = \frac{3\beta}{a}, \quad \alpha = \frac{2\delta}{-a}, \quad \gamma = \frac{\alpha}{a} \Rightarrow \beta = \frac{1}{a}, \quad \delta = \frac{3}{a^2}, \quad \alpha = -\frac{6}{a^3}, \quad \gamma = -\frac{6}{a^4}.$$

Riešené príklady – 071

$$\int \sqrt{1 + \frac{1}{\sin x}} dx$$



Riešené príklady – 071

$$\int \sqrt{1 + \frac{1}{\sin x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1 + \frac{1}{\sin x}, \sin x \in (0; 1) \mid x \in (0 + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z} \\ \sin x = \frac{1}{t-1} = (t-1)^{-1}, t \in (2; \infty) \mid \sin x \in (-1; 0) \Rightarrow 1 + \frac{1}{\sin x} < 0 \\ \cos x dx = -(t-1)^{-2} dt = \frac{-dt}{(t-1)^2} \Rightarrow dx = \frac{-dt}{(t-1)^2} \cdot \frac{1}{\cos x}, \cos x \neq 0 \\ \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{(t-1)^2} = \frac{(t-1)^2 - 1}{(t-1)^2} = \frac{t^2 - 2t + 1 - 1}{(t-1)^2} = \frac{t^2 - 2t}{(t-1)^2} = \frac{t(t-2)}{(t-1)^2} \end{array} \right. \left. \begin{array}{l} x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi): 0 < \cos x = \frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{t-1}{\sqrt{t(t-2)}} = \frac{-dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi): 0 > \cos x = -\frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{-(t-1)}{\sqrt{t(t-2)}} = \frac{dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \end{array} \right.$$

Riešené príklady – 071

$$\int \sqrt{1 + \frac{1}{\sin x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1 + \frac{1}{\sin x}, \sin x \in (0; 1) \mid x \in (0 + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z} \\ \sin x = \frac{1}{t-1} = (t-1)^{-1}, t \in (2; \infty) \mid \sin x \in (-1; 0) \Rightarrow 1 + \frac{1}{\sin x} < 0 \\ \cos x dx = -(t-1)^{-2} dt = \frac{-dt}{(t-1)^2} \Rightarrow dx = \frac{-dt}{(t-1)^2} \cdot \frac{1}{\cos x}, \cos x \neq 0 \\ \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{(t-1)^2} = \frac{(t-1)^2 - 1}{(t-1)^2} = \frac{t^2 - 2t + 1 - 1}{(t-1)^2} = \frac{t^2 - 2t}{(t-1)^2} = \frac{t(t-2)}{(t-1)^2} \end{array} \right. \left. \begin{array}{l} x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi): 0 < \cos x = \frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{t-1}{\sqrt{t(t-2)}} = \frac{-dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi): 0 > \cos x = -\frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{-(t-1)}{\sqrt{t(t-2)}} = \frac{dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \end{array} \right.$$

$$\boxed{x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi), k \in \mathbb{Z}} = \int \frac{-\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}}$$

$$\boxed{x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z}} = \int \frac{\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}}$$

Riešené príklady – 071

$$\int \sqrt{1 + \frac{1}{\sin x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1 + \frac{1}{\sin x}, \sin x \in (0; 1) \mid x \in (0 + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z} \\ \sin x = \frac{1}{t-1} = (t-1)^{-1}, t \in (2; \infty) \mid \sin x \in (-1; 0) \Rightarrow 1 + \frac{1}{\sin x} < 0 \\ \cos x dx = -(t-1)^{-2} dt = \frac{-dt}{(t-1)^2} \Rightarrow dx = \frac{-dt}{(t-1)^2} \cdot \frac{1}{\cos x}, \cos x \neq 0 \\ \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{(t-1)^2} = \frac{(t-1)^2 - 1}{(t-1)^2} = \frac{t^2 - 2t + 1 - 1}{(t-1)^2} = \frac{t^2 - 2t}{(t-1)^2} = \frac{t(t-2)}{(t-1)^2} \end{array} \right. \left. \begin{array}{l} x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi): 0 < \cos x = \frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{t-1}{\sqrt{t(t-2)}} = \frac{-dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi): 0 > \cos x = -\frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{-(t-1)}{\sqrt{t(t-2)}} = \frac{dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \end{array} \right.$$

$$\boxed{x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi), k \in \mathbb{Z}} = \int \frac{-\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}} = \int \frac{-dt}{(t-1)\sqrt{t-2}}$$

$$\boxed{x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z}} = \int \frac{\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}} = \int \frac{dt}{(t-1)\sqrt{t-2}}$$

Riešené príklady – 071

$$\int \sqrt{1 + \frac{1}{\sin x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1 + \frac{1}{\sin x}, \sin x \in (0; 1) \mid x \in (0 + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z} \\ \sin x = \frac{1}{t-1} = (t-1)^{-1}, t \in (2; \infty) \mid \sin x \in (-1; 0) \Rightarrow 1 + \frac{1}{\sin x} < 0 \\ \cos x dx = -(t-1)^{-2} dt = \frac{-dt}{(t-1)^2} \Rightarrow dx = \frac{-dt}{(t-1)^2} \cdot \frac{1}{\cos x}, \cos x \neq 0 \\ \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{(t-1)^2} = \frac{(t-1)^2 - 1}{(t-1)^2} = \frac{t^2 - 2t + 1 - 1}{(t-1)^2} = \frac{t^2 - 2t}{(t-1)^2} = \frac{t(t-2)}{(t-1)^2} \end{array} \right. \left. \begin{array}{l} x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi): 0 < \cos x = \frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{t-1}{\sqrt{t(t-2)}} = \frac{-dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi): 0 > \cos x = -\frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{-(t-1)}{\sqrt{t(t-2)}} = \frac{dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \end{array} \right]$$

$$\boxed{x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi), k \in \mathbb{Z}} = \int \frac{-\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}} = \int \frac{-dt}{(t-1)\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{t-2} = \sqrt{\frac{1}{\sin x} - 1} \mid t \in (2; \infty) \\ u^2 = t-2, 2u du = dt \mid u \in (0; \infty) \end{array} \right]$$

$$\boxed{x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z}} = \int \frac{\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}} = \int \frac{dt}{(t-1)\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{t-2} = \sqrt{\frac{1}{\sin x} - 1} \mid t \in (2; \infty) \\ u^2 = t-2, 2u du = dt \mid u \in (0; \infty) \end{array} \right]$$

Riešené príklady – 071

$$\int \sqrt{1 + \frac{1}{\sin x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1 + \frac{1}{\sin x}, \sin x \in (0; 1) \mid x \in (0 + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z} \\ \sin x = \frac{1}{t-1} = (t-1)^{-1}, t \in (2; \infty) \mid \sin x \in (-1; 0) \Rightarrow 1 + \frac{1}{\sin x} < 0 \\ \cos x dx = -(t-1)^{-2} dt = \frac{-dt}{(t-1)^2} \Rightarrow dx = \frac{-dt}{(t-1)^2} \cdot \frac{1}{\cos x}, \cos x \neq 0 \\ \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{(t-1)^2} = \frac{(t-1)^2 - 1}{(t-1)^2} = \frac{t^2 - 2t + 1 - 1}{(t-1)^2} = \frac{t^2 - 2t}{(t-1)^2} = \frac{t(t-2)}{(t-1)^2} \end{array} \right. \left. \begin{array}{l} x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi): 0 < \cos x = \frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{t-1}{\sqrt{t(t-2)}} = \frac{-dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi): 0 > \cos x = -\frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{-(t-1)}{\sqrt{t(t-2)}} = \frac{dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \end{array} \right]$$

$$\boxed{x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi), k \in \mathbb{Z}} = \int \frac{-\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}} = \int \frac{-dt}{(t-1)\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{t-2} = \sqrt{\frac{1}{\sin x} - 1} \mid t \in (2; \infty) \\ u^2 = t-2, 2u du = dt \mid u \in (0; \infty) \end{array} \right]$$

$$= \int \frac{-2u du}{(u^2+1)u}$$

$$\boxed{x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z}} = \int \frac{\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}} = \int \frac{dt}{(t-1)\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{t-2} = \sqrt{\frac{1}{\sin x} - 1} \mid t \in (2; \infty) \\ u^2 = t-2, 2u du = dt \mid u \in (0; \infty) \end{array} \right]$$

$$= \int \frac{2u du}{(u^2+1)u}$$

Riešené príklady – 071

$$\int \sqrt{1 + \frac{1}{\sin x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1 + \frac{1}{\sin x}, \sin x \in (0; 1) \mid x \in (0 + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z} \\ \sin x = \frac{1}{t-1} = (t-1)^{-1}, t \in (2; \infty) \mid \sin x \in (-1; 0) \Rightarrow 1 + \frac{1}{\sin x} < 0 \\ \cos x dx = -(t-1)^{-2} dt = \frac{-dt}{(t-1)^2} \Rightarrow dx = \frac{-dt}{(t-1)^2} \cdot \frac{1}{\cos x}, \cos x \neq 0 \\ \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{(t-1)^2} = \frac{(t-1)^2 - 1}{(t-1)^2} = \frac{t^2 - 2t + 1 - 1}{(t-1)^2} = \frac{t^2 - 2t}{(t-1)^2} = \frac{t(t-2)}{(t-1)^2} \end{array} \right. \left. \begin{array}{l} x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi): 0 < \cos x = \frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{t-1}{\sqrt{t(t-2)}} = \frac{-dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi): 0 > \cos x = -\frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{-(t-1)}{\sqrt{t(t-2)}} = \frac{dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \end{array} \right.$$

$$\boxed{x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi), k \in \mathbb{Z}} = \int \frac{-\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}} = \int \frac{-dt}{(t-1)\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{t-2} = \sqrt{\frac{1}{\sin x} - 1} \mid t \in (2; \infty) \\ u^2 = t-2, 2u du = dt \mid u \in (0; \infty) \end{array} \right]$$

$$= \int \frac{-2u du}{(u^2+1)u} = -2 \int \frac{du}{u^2+1}$$

$$\boxed{x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z}} = \int \frac{\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}} = \int \frac{dt}{(t-1)\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{t-2} = \sqrt{\frac{1}{\sin x} - 1} \mid t \in (2; \infty) \\ u^2 = t-2, 2u du = dt \mid u \in (0; \infty) \end{array} \right]$$

$$= \int \frac{2u du}{(u^2+1)u} = 2 \int \frac{du}{u^2+1}$$

Riešené príklady – 071

$$\int \sqrt{1 + \frac{1}{\sin x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1 + \frac{1}{\sin x}, \sin x \in (0; 1) \mid x \in (0 + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z} \\ \sin x = \frac{1}{t-1} = (t-1)^{-1}, t \in (2; \infty) \mid \sin x \in (-1; 0) \Rightarrow 1 + \frac{1}{\sin x} < 0 \\ \cos x dx = -(t-1)^{-2} dt = \frac{-dt}{(t-1)^2} \Rightarrow dx = \frac{-dt}{(t-1)^2} \cdot \frac{1}{\cos x}, \cos x \neq 0 \\ \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{(t-1)^2} = \frac{(t-1)^2 - 1}{(t-1)^2} = \frac{t^2 - 2t + 1 - 1}{(t-1)^2} = \frac{t^2 - 2t}{(t-1)^2} = \frac{t(t-2)}{(t-1)^2} \end{array} \right. \left. \begin{array}{l} x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi): 0 < \cos x = \frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{t-1}{\sqrt{t(t-2)}} = \frac{-dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi): 0 > \cos x = -\frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{-(t-1)}{\sqrt{t(t-2)}} = \frac{dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \end{array} \right.$$

$$\boxed{x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi), k \in \mathbb{Z}} = \int \frac{-\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}} = \int \frac{-dt}{(t-1)\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{t-2} = \sqrt{\frac{1}{\sin x} - 1} \mid t \in (2; \infty) \\ u^2 = t-2, 2u du = dt \mid u \in (0; \infty) \end{array} \right]$$

$$= \int \frac{-2u du}{(u^2+1)u} = -2 \int \frac{du}{u^2+1} = -2 \operatorname{arctg} u + c$$

$$\boxed{x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z}} = \int \frac{\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}} = \int \frac{dt}{(t-1)\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{t-2} = \sqrt{\frac{1}{\sin x} - 1} \mid t \in (2; \infty) \\ u^2 = t-2, 2u du = dt \mid u \in (0; \infty) \end{array} \right]$$

$$= \int \frac{2u du}{(u^2+1)u} = 2 \int \frac{du}{u^2+1} = 2 \operatorname{arctg} u + c$$

Riešené príklady – 071

$$\int \sqrt{1 + \frac{1}{\sin x}} dx = \mp 2 \operatorname{arctg} \sqrt{\frac{1}{\sin x} - 1} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = 1 + \frac{1}{\sin x}, \sin x \in (0; 1) \mid x \in (0 + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z} \\ \sin x = \frac{1}{t-1} = (t-1)^{-1}, t \in (2; \infty) \mid \sin x \in (-1; 0) \Rightarrow 1 + \frac{1}{\sin x} < 0 \\ \cos x dx = -(t-1)^{-2} dt = \frac{-dt}{(t-1)^2} \Rightarrow dx = \frac{-dt}{(t-1)^2} \cdot \frac{1}{\cos x}, \cos x \neq 0 \\ \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{(t-1)^2} = \frac{(t-1)^2 - 1}{(t-1)^2} = \frac{t^2 - 2t + 1 - 1}{(t-1)^2} = \frac{t^2 - 2t}{(t-1)^2} = \frac{t(t-2)}{(t-1)^2} \end{array} \right. \left. \begin{array}{l} x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi): 0 < \cos x = \frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{t-1}{\sqrt{t(t-2)}} = \frac{-dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \\ x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi): 0 > \cos x = -\frac{\sqrt{t(t-2)}}{t-1} \\ dx = \frac{-dt}{(t-1)^2} \cdot \frac{-(t-1)}{\sqrt{t(t-2)}} = \frac{dt}{(t-1)\sqrt{t(t-2)}}, t \in (2; \infty) \end{array} \right.$$

$$\boxed{x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi), k \in \mathbb{Z}} = \int \frac{-\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}} = \int \frac{-dt}{(t-1)\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{t-2} = \sqrt{\frac{1}{\sin x} - 1} \mid t \in (2; \infty) \\ u^2 = t-2, 2u du = dt \mid u \in (0; \infty) \end{array} \right]$$

$$= \int \frac{-2u du}{(u^2+1)u} = -2 \int \frac{du}{u^2+1} = -2 \operatorname{arctg} u + c = -2 \operatorname{arctg} \sqrt{\frac{1}{\sin x} - 1} + c,$$

$$x \in (0 + 2k\pi; \frac{\pi}{2} + 2k\pi), k \in \mathbb{Z}, c \in \mathbb{R}.$$

$$\boxed{x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z}} = \int \frac{\sqrt{t} dt}{(t-1)\sqrt{t}\sqrt{t-2}} = \int \frac{dt}{(t-1)\sqrt{t-2}} = \left[\begin{array}{l} \text{Subst. } u = \sqrt{t-2} = \sqrt{\frac{1}{\sin x} - 1} \mid t \in (2; \infty) \\ u^2 = t-2, 2u du = dt \mid u \in (0; \infty) \end{array} \right]$$

$$= \int \frac{2u du}{(u^2+1)u} = 2 \int \frac{du}{u^2+1} = 2 \operatorname{arctg} u + c = +2 \operatorname{arctg} \sqrt{\frac{1}{\sin x} - 1} + c,$$

$$x \in (\frac{\pi}{2} + 2k\pi; \pi + 2k\pi), k \in \mathbb{Z}, c \in \mathbb{R}.$$

Riešené príklady – 072, 073

$$\int \operatorname{arctg} x \, dx$$

$$\int x \operatorname{arctg} x \, dx$$

Riešené príklady – 072, 073

$$\int \operatorname{arctg} x \, dx$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right]$$

$$\int x \operatorname{arctg} x \, dx$$

$$= \left[\begin{array}{l|l} u' = x & u = \frac{x^2}{2} \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right]$$

Riešené príklady – 072, 073

$$\int \operatorname{arctg} x \, dx$$

$$= \left[\begin{array}{l} u' = 1 \\ v = \operatorname{arctg} x \end{array} \middle| \begin{array}{l} u = x \\ v' = \frac{1}{1+x^2} \end{array} \right] = x \operatorname{arctg} x - \int \frac{x \, dx}{1+x^2}$$

$$\int x \operatorname{arctg} x \, dx$$

$$= \left[\begin{array}{l} u' = x \\ v = \operatorname{arctg} x \end{array} \middle| \begin{array}{l} u = \frac{x^2}{2} \\ v' = \frac{1}{1+x^2} \end{array} \right] = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2 \, dx}{1+x^2} = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{(1+x^2-1) \, dx}{1+x^2}$$

Riešené príklady – 072, 073

$$\int \operatorname{arctg} x \, dx$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right] = x \operatorname{arctg} x - \int \frac{x \, dx}{1+x^2} = x \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx$$

$$\int x \operatorname{arctg} x \, dx$$

$$= \left[\begin{array}{l|l} u' = x & u = \frac{x^2}{2} \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right] = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2 \, dx}{1+x^2} = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{(1+x^2-1) \, dx}{1+x^2}$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int dx + \frac{1}{2} \int \frac{dx}{1+x^2}$$

Riešené príklady – 072, 073

$$\int \operatorname{arctg} x \, dx$$

$$= \left[\begin{array}{l} u' = 1 \\ v = \operatorname{arctg} x \end{array} \middle| \begin{array}{l} u = x \\ v' = \frac{1}{1+x^2} \end{array} \right] = x \operatorname{arctg} x - \int \frac{x \, dx}{1+x^2} = x \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx$$

$$= x \operatorname{arctg} x - \frac{1}{2} \ln |1+x^2| + c$$

$$\int x \operatorname{arctg} x \, dx$$

$$= \left[\begin{array}{l} u' = x \\ v = \operatorname{arctg} x \end{array} \middle| \begin{array}{l} u = \frac{x^2}{2} \\ v' = \frac{1}{1+x^2} \end{array} \right] = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2 \, dx}{1+x^2} = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{(1+x^2-1) \, dx}{1+x^2}$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int dx + \frac{1}{2} \int \frac{dx}{1+x^2} = \frac{x^2}{2} \operatorname{arctg} x - \frac{x}{2} + \frac{1}{2} \operatorname{arctg} x + c$$

Riešené príklady – 072, 073

$$\int \operatorname{arctg} x \, dx = x \operatorname{arctg} x - \ln \sqrt{1+x^2} + c$$

$$\begin{aligned}
 &= \left[\begin{array}{l} u' = 1 \\ v = \operatorname{arctg} x \end{array} \middle| \begin{array}{l} u = x \\ v' = \frac{1}{1+x^2} \end{array} \right] = x \operatorname{arctg} x - \int \frac{x \, dx}{1+x^2} = x \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx \\
 &= x \operatorname{arctg} x - \frac{1}{2} \ln |1+x^2| + c = x \operatorname{arctg} x - \frac{1}{2} \ln (1+x^2) + c \\
 &= x \operatorname{arctg} x - \ln \sqrt{1+x^2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.
 \end{aligned}$$

$$\int x \operatorname{arctg} x \, dx = \frac{1+x^2}{2} \operatorname{arctg} x - \frac{x}{2} + c$$

$$\begin{aligned}
 &= \left[\begin{array}{l} u' = x \\ v = \operatorname{arctg} x \end{array} \middle| \begin{array}{l} u = \frac{x^2}{2} \\ v' = \frac{1}{1+x^2} \end{array} \right] = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2 \, dx}{1+x^2} = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{(1+x^2-1) \, dx}{1+x^2} \\
 &= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int dx + \frac{1}{2} \int \frac{dx}{1+x^2} = \frac{x^2}{2} \operatorname{arctg} x - \frac{x}{2} + \frac{1}{2} \operatorname{arctg} x + c \\
 &= \frac{1+x^2}{2} \operatorname{arctg} x - \frac{x}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.
 \end{aligned}$$

Riešené príklady – 074, 075

$$\int \ln x \, dx$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, dx$$

Riešené príklady – 074, 075

$$\int \ln x \, dx$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = 1 \quad v = x \end{array} \right]$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{arctg} x \quad x \in (0; \infty) \\ dt = \frac{dx}{x^2+1} \quad t \in (0; \frac{\pi}{2}) \end{array} \right]$$

$$= \left[\begin{array}{l} u = \ln \operatorname{arctg} x \quad u' = \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{x^2+1} \\ v' = \frac{1}{x^2+1} \quad v = \operatorname{arctg} x \end{array} \right]$$

Riešené príklady – 074, 075

$$\int \ln x \, dx$$

$$= \left[\begin{array}{l} u = \ln x \quad | \quad u' = \frac{1}{x} \\ v' = 1 \quad | \quad v = x \end{array} \right] = x \ln x - \int \frac{x \, dx}{x}$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{arctg} x \quad | \quad x \in (0; \infty) \\ dt = \frac{dx}{x^2+1} \quad | \quad t \in (0; \frac{\pi}{2}) \end{array} \right] = \int \ln t \, dt$$

$$= \left[\begin{array}{l} u = \ln \operatorname{arctg} x \quad | \quad u' = \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{x^2+1} \\ v' = \frac{1}{x^2+1} \quad | \quad v = \operatorname{arctg} x \end{array} \right] = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \int \frac{dx}{x^2+1}$$

Riešené príklady – 074, 075

$$\int \ln x \, dx$$

$$= \left[\begin{array}{l} u = \ln x \quad | \quad u' = \frac{1}{x} \\ v' = 1 \quad | \quad v = x \end{array} \right] = x \ln x - \int \frac{x \, dx}{x} = x \ln x - \int dx$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, dx = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{arctg} x \quad | \quad x \in (0; \infty) \\ dt = \frac{dx}{x^2+1} \quad | \quad t \in (0; \frac{\pi}{2}) \end{array} \right] = \int \ln t \, dt = \left[\begin{array}{l} u = \ln t \quad | \quad u' = \frac{1}{t} \\ v' = 1 \quad | \quad v = t \end{array} \right]$$

$$= \left[\begin{array}{l} u = \ln \operatorname{arctg} x \quad | \quad u' = \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{x^2+1} \\ v' = \frac{1}{x^2+1} \quad | \quad v = \operatorname{arctg} x \end{array} \right] = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \int \frac{dx}{x^2+1}$$

$$= \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c, \quad x \in (0; \infty), \quad c \in \mathbb{R}.$$

Riešené príklady – 074, 075

$$\int \ln x \, dx = x \ln x - x + c$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = 1 \quad v = x \end{array} \right] = x \ln x - \int \frac{x \, dx}{x} = x \ln x - \int dx = x \ln x - x + c,$$

$$x \in (0; \infty), c \in \mathbb{R}.$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, dx = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{arctg} x \quad x \in (0; \infty) \\ dt = \frac{dx}{x^2+1} \quad t \in (0; \frac{\pi}{2}) \end{array} \right] = \int \ln t \, dt = \left[\begin{array}{l} u = \ln t \quad u' = \frac{1}{t} \\ v' = 1 \quad v = t \end{array} \right] = t \ln t - \int dt$$

$$= \left[\begin{array}{l} u = \ln \operatorname{arctg} x \quad u' = \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{x^2+1} \\ v' = \frac{1}{x^2+1} \quad v = \operatorname{arctg} x \end{array} \right] = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \int \frac{dx}{x^2+1}$$

$$= \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c, x \in (0; \infty), c \in \mathbb{R}.$$

Riešené príklady – 074, 075

$$\int \ln x \, dx = x \ln x - x + c$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = 1 \quad v = x \end{array} \right] = x \ln x - \int \frac{x \, dx}{x} = x \ln x - \int dx = x \ln x - x + c,$$

$$x \in (0; \infty), c \in \mathbb{R}.$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, dx = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{arctg} x \quad x \in (0; \infty) \\ dt = \frac{dx}{x^2+1} \quad t \in (0; \frac{\pi}{2}) \end{array} \right] = \int \ln t \, dt = \left[\begin{array}{l} u = \ln t \quad u' = \frac{1}{t} \\ v' = 1 \quad v = t \end{array} \right] = t \ln t - \int dt$$

$$= t \ln t - t + c$$

$$= \left[\begin{array}{l} u = \ln \operatorname{arctg} x \quad u' = \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{x^2+1} \\ v' = \frac{1}{x^2+1} \quad v = \operatorname{arctg} x \end{array} \right] = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \int \frac{dx}{x^2+1}$$

$$= \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c, x \in (0; \infty), c \in \mathbb{R}.$$

Riešené príklady – 074, 075

$$\int \ln x \, dx = x \ln x - x + c$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = 1 \quad v = x \end{array} \right] = x \ln x - \int \frac{x \, dx}{x} = x \ln x - \int dx = x \ln x - x + c,$$

$$x \in (0; \infty), c \in \mathbb{R}.$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, dx = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \operatorname{arctg} x \quad x \in (0; \infty) \\ dt = \frac{dx}{x^2+1} \quad t \in (0; \frac{\pi}{2}) \end{array} \right] = \int \ln t \, dt = \left[\begin{array}{l} u = \ln t \quad u' = \frac{1}{t} \\ v' = 1 \quad v = t \end{array} \right] = t \ln t - \int dt$$

$$= t \ln t - t + c = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c, \quad x \in (0; \infty), c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} u = \ln \operatorname{arctg} x \quad u' = \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{x^2+1} \\ v' = \frac{1}{x^2+1} \quad v = \operatorname{arctg} x \end{array} \right] = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \int \frac{dx}{x^2+1}$$

$$= \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c, \quad x \in (0; \infty), c \in \mathbb{R}.$$

Riešené príklady – 076

$$\int \ln(x-1)^5 dx = 5 \int \ln(x-1) dx$$

Riešené príklady – 076

$$\int \ln(x-1)^5 dx = 5 \int \ln(x-1) dx$$

$$= \left[\begin{array}{l} u = \ln(x-1) \quad u' = \frac{1}{x-1} \\ v' = 1 \quad v = x \end{array} \right]$$

$$= \left[\begin{array}{l} u = \ln(x-1) \quad u' = \frac{1}{x-1} \\ v' = 1 \quad v = x-1 \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \quad x \in (1; \infty) \\ dt = dx \quad t \in (0; \infty) \end{array} \right]$$

Riešené príklady – 076

$$\int \ln(x-1)^5 dx = 5 \int \ln(x-1) dx$$

$$= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x \end{array} \right] = 5x \ln(x-1) - 5 \int \frac{x dx}{x-1}$$

$$= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x-1 \end{array} \right] = 5(x-1) \ln(x-1) - 5 \int \frac{(x-1) dx}{x-1}$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \middle| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right] = 5 \int \ln t dt$$

Riešené príklady – 076

$$\int \ln(x-1)^5 dx = 5 \int \ln(x-1) dx$$

$$= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x \end{array} \right] = 5x \ln(x-1) - 5 \int \frac{x dx}{x-1} = 5x \ln(x-1) - 5 \int \frac{(x-1+1) dx}{x-1}$$

$$= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x-1 \end{array} \right] = 5(x-1) \ln(x-1) - 5 \int \frac{(x-1) dx}{x-1}$$

$$= 5(x-1) \ln(x-1) - 5 \int dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \middle| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right] = 5 \int \ln t dt = \left[\begin{array}{l} u = \ln t \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{t} \\ v = t \end{array} \right]$$

Riešené príklady – 076

$$\int \ln(x-1)^5 dx = 5 \int \ln(x-1) dx = 5(x-1) \ln(x-1) - 5x + c$$

$$= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x \end{array} \right] = 5x \ln(x-1) - 5 \int \frac{x dx}{x-1} = 5x \ln(x-1) - 5 \int \frac{(x-1+1) dx}{x-1}$$

$$= 5x \ln(x-1) - 5 \int dx - 5 \int \frac{dx}{x-1}$$

$$= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x-1 \end{array} \right] = 5(x-1) \ln(x-1) - 5 \int \frac{(x-1) dx}{x-1}$$

$$= 5(x-1) \ln(x-1) - 5 \int dx = 5(x-1) \ln(x-1) - 5x + c, \quad x \in (1; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \middle| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right] = 5 \int \ln t dt = \left[\begin{array}{l} u = \ln t \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{t} \\ v = t \end{array} \right] = 5t \ln t - 5 \int dt$$

Riešené príklady – 076

$$\int \ln(x-1)^5 dx = 5 \int \ln(x-1) dx = 5(x-1) \ln(x-1) - 5x + c$$

$$= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x \end{array} \right] = 5x \ln(x-1) - 5 \int \frac{x dx}{x-1} = 5x \ln(x-1) - 5 \int \frac{(x-1+1) dx}{x-1}$$

$$= 5x \ln(x-1) - 5 \int dx - 5 \int \frac{dx}{x-1} = 5x \ln(x-1) - 5x - 5 \ln(x-1) + c$$

$$= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x-1 \end{array} \right] = 5(x-1) \ln(x-1) - 5 \int \frac{(x-1) dx}{x-1}$$

$$= 5(x-1) \ln(x-1) - 5 \int dx = 5(x-1) \ln(x-1) - 5x + c, \quad x \in (1; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \middle| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right] = 5 \int \ln t dt = \left[\begin{array}{l} u = \ln t \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{t} \\ v = t \end{array} \right] = 5t \ln t - 5 \int dt$$

$$= 5t \ln t - 5t + c_1$$

Riešené príklady – 076

$$\int \ln(x-1)^5 dx = 5 \int \ln(x-1) dx = 5(x-1) \ln(x-1) - 5x + c$$

$$= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x \end{array} \right] = 5x \ln(x-1) - 5 \int \frac{x dx}{x-1} = 5x \ln(x-1) - 5 \int \frac{(x-1+1) dx}{x-1}$$

$$= 5x \ln(x-1) - 5 \int dx - 5 \int \frac{dx}{x-1} = 5x \ln(x-1) - 5x - 5 \ln(x-1) + c$$

$$= 5(x-1) \ln(x-1) - 5x + c = (x-1) \ln(x-1)^5 - 5x + c, \quad x \in (1; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x-1 \end{array} \right] = 5(x-1) \ln(x-1) - 5 \int \frac{(x-1) dx}{x-1}$$

$$= 5(x-1) \ln(x-1) - 5 \int dx = 5(x-1) \ln(x-1) - 5x + c, \quad x \in (1; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = x-1 \\ dt = dx \end{array} \middle| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right] = 5 \int \ln t dt = \left[\begin{array}{l} u = \ln t \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{t} \\ v = t \end{array} \right] = 5t \ln t - 5 \int dt$$

$$= 5t \ln t - 5t + c_1 = 5(x-1) \ln(x-1) - 5(x-1) + c_1$$

Riešené príklady – 076

$$\int \ln(x-1)^5 dx = 5 \int \ln(x-1) dx = 5(x-1) \ln(x-1) - 5x + c$$

$$\begin{aligned} &= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x \end{array} \right] = 5x \ln(x-1) - 5 \int \frac{x dx}{x-1} = 5x \ln(x-1) - 5 \int \frac{(x-1+1) dx}{x-1} \\ &= 5x \ln(x-1) - 5 \int dx - 5 \int \frac{dx}{x-1} = 5x \ln(x-1) - 5x - 5 \ln(x-1) + c \\ &= 5(x-1) \ln(x-1) - 5x + c = (x-1) \ln(x-1)^5 - 5x + c, \quad x \in (1; \infty), \quad c \in \mathbb{R}. \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x-1 \end{array} \right] = 5(x-1) \ln(x-1) - 5 \int \frac{(x-1) dx}{x-1} \\ &= 5(x-1) \ln(x-1) - 5 \int dx = 5(x-1) \ln(x-1) - 5x + c, \quad x \in (1; \infty), \quad c \in \mathbb{R}. \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \middle| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right] = 5 \int \ln t dt = \left[\begin{array}{l} u = \ln t \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{t} \\ v = t \end{array} \right] = 5t \ln t - 5 \int dt \\ &= 5t \ln t - 5t + c_1 = 5(x-1) \ln(x-1) - 5(x-1) + c_1 = \left[\begin{array}{l} c = c_1 + 5 \\ c \in \mathbb{R}, c_1 \in \mathbb{R} \end{array} \right] \end{aligned}$$

Riešené príklady – 076

$$\int \ln(x-1)^5 dx = 5 \int \ln(x-1) dx = 5(x-1) \ln(x-1) - 5x + c$$

$$= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x \end{array} \right] = 5x \ln(x-1) - 5 \int \frac{x dx}{x-1} = 5x \ln(x-1) - 5 \int \frac{(x-1+1) dx}{x-1}$$

$$= 5x \ln(x-1) - 5 \int dx - 5 \int \frac{dx}{x-1} = 5x \ln(x-1) - 5x - 5 \ln(x-1) + c$$

$$= 5(x-1) \ln(x-1) - 5x + c = (x-1) \ln(x-1)^5 - 5x + c, \quad x \in (1; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} u = \ln(x-1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x-1} \\ v = x-1 \end{array} \right] = 5(x-1) \ln(x-1) - 5 \int \frac{(x-1) dx}{x-1}$$

$$= 5(x-1) \ln(x-1) - 5 \int dx = 5(x-1) \ln(x-1) - 5x + c, \quad x \in (1; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \middle| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right] = 5 \int \ln t dt = \left[\begin{array}{l} u = \ln t \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{t} \\ v = t \end{array} \right] = 5t \ln t - 5 \int dt$$

$$= 5t \ln t - 5t + c_1 = 5(x-1) \ln(x-1) - 5(x-1) + c_1 = \left[\begin{array}{l} c = c_1 + 5 \\ c \in \mathbb{R}, c_1 \in \mathbb{R} \end{array} \right]$$

$$= 5(x-1) \ln(x-1) - 5x + c, \quad x \in (1; \infty), \quad c \in \mathbb{R}.$$

Riešené príklady – 077, 078

$$I = \int \frac{\ln x}{x} dx$$

$$\int \frac{dx}{x \ln x}$$

Riešené príklady – 077, 078

$$I = \int \frac{\ln x}{x} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \ln x \mid x \in (0; \infty) \\ dt = \frac{dx}{x} \mid t \in \mathbb{R} \end{array} \right]$$

$$= \left[\begin{array}{l} u = \ln x \mid u' = \frac{1}{x} \\ v' = \frac{1}{x} \mid v = \ln x \end{array} \right]$$

$$\int \frac{dx}{x \ln x}$$

$$= \int \frac{\frac{1}{x}}{\ln x} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \ln x \mid x \in (0; 1) \cup (1; \infty) \\ dt = \frac{dx}{x} \mid t \in (-\infty; 0) \cup (0; \infty) \end{array} \right]$$

Riešené príklady – 077, 078

$$I = \int \frac{\ln x}{x} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \ln x \mid x \in (0; \infty) \\ dt = \frac{dx}{x} \mid t \in \mathbb{R} \end{array} \right] = \int dt$$

$$= \left[\begin{array}{l} u = \ln x \mid u' = \frac{1}{x} \\ v' = \frac{1}{x} \mid v = \ln x \end{array} \right] = \ln^2 x - \int \frac{\ln x}{x} dx$$

$$\int \frac{dx}{x \ln x}$$

$$= \int \frac{\frac{1}{x}}{\ln x} dx = \int \frac{[\ln x]'}{\ln x} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \ln x \mid x \in (0; 1) \cup (1; \infty) \\ dt = \frac{dx}{x} \mid t \in (-\infty; 0) \cup (0; \infty) \end{array} \right] = \int \frac{dt}{t}$$

Riešené príklady – 077, 078

$$I = \int \frac{\ln x}{x} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \ln x \mid x \in (0; \infty) \\ dt = \frac{dx}{x} \mid t \in \mathbb{R} \end{array} \right] = \int dt = \frac{t^2}{2} + c$$

$$= \left[\begin{array}{l} u = \ln x \mid u' = \frac{1}{x} \\ v' = \frac{1}{x} \mid v = \ln x \end{array} \right] = \ln^2 x - \int \frac{\ln x}{x} dx = \ln^2 x - I$$

$$\int \frac{dx}{x \ln x} = \ln |\ln x| + c$$

$$= \int \frac{1}{\ln x} dx = \int \frac{[\ln x]'}{\ln x} dx = \ln |\ln x| + c, \quad x \in (0; 1) \cup (1; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = \ln x \mid x \in (0; 1) \cup (1; \infty) \\ dt = \frac{dx}{x} \mid t \in (-\infty; 0) \cup (0; \infty) \end{array} \right] = \int \frac{dt}{t} = \ln |t| + c$$

Riešené príklady – 077, 078

$$I = \int \frac{\ln x}{x} dx = \frac{\ln^2 x}{2} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \ln x \mid x \in (0; \infty) \\ dt = \frac{dx}{x} \mid t \in \mathbb{R} \end{array} \right] = \int dt = \frac{t^2}{2} + c = \frac{\ln^2 x}{2} + c, \quad x \in (0; \infty), \quad c \in \mathbb{R}.$$

$$I = \left[\begin{array}{l} u = \ln x \mid u' = \frac{1}{x} \\ v' = \frac{1}{x} \mid v = \ln x \end{array} \right] = \ln^2 x - \int \frac{\ln x}{x} dx = \ln^2 x - I = \left[\text{Rovnica } I = \ln^2 x - I \right]$$

$$\int \frac{dx}{x \ln x} = \ln |\ln x| + c$$

$$= \int \frac{1}{\ln x} dx = \int \frac{[\ln x]'}{\ln x} dx = \ln |\ln x| + c, \quad x \in (0; 1) \cup (1; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = \ln x \mid x \in (0; 1) \cup (1; \infty) \\ dt = \frac{dx}{x} \mid t \in (-\infty; 0) \cup (0; \infty) \end{array} \right] = \int \frac{dt}{t} = \ln |t| + c = \ln |\ln x| + c, \\ x \in (0; 1) \cup (1; \infty), \quad c \in \mathbb{R}.$$

Riešené príklady – 077, 078

$$I = \int \frac{\ln x}{x} dx = \frac{\ln^2 x}{2} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \ln x \mid x \in (0; \infty) \\ dt = \frac{dx}{x} \mid t \in \mathbb{R} \end{array} \right] = \int dt = \frac{t^2}{2} + c = \frac{\ln^2 x}{2} + c, \quad x \in (0; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} u = \ln x \mid u' = \frac{1}{x} \\ v' = \frac{1}{x} \mid v = \ln x \end{array} \right] = \ln^2 x - \int \frac{\ln x}{x} dx = \ln^2 x - I = \left[\begin{array}{l} \text{Rovnica } I = \ln^2 x - I \end{array} \right] = \frac{\ln^2 x}{2} + c, \\ x \in (0; \infty), \quad c \in \mathbb{R}.$$

$$\int \frac{dx}{x \ln x} = \ln |\ln x| + c$$

$$= \int \frac{1}{\ln x} dx = \int \frac{[\ln x]'}{\ln x} dx = \ln |\ln x| + c, \quad x \in (0; 1) \cup (1; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = \ln x \mid x \in (0; 1) \cup (1; \infty) \\ dt = \frac{dx}{x} \mid t \in (-\infty; 0) \cup (0; \infty) \end{array} \right] = \int \frac{dt}{t} = \ln |t| + c = \ln |\ln x| + c, \\ x \in (0; 1) \cup (1; \infty), \quad c \in \mathbb{R}.$$

Riešené príklady – 079, 080

$$\int x^2 \ln \sqrt{1-x} dx = \int x^2 \ln (1-x)^{\frac{1}{2}} dx$$

$$\int \frac{\ln x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} \ln x dx$$

Riešené príklady – 079, 080

$$\int x^2 \ln \sqrt{1-x} dx = \int x^2 \ln (1-x)^{\frac{1}{2}} dx$$

$$= \int \frac{x^2}{2} \ln (1-x) dx$$

$$\int \frac{\ln x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} \ln x dx$$

$$= \left[\begin{array}{l} u = \ln x \quad | \quad u' = \frac{1}{x} \\ v' = x^{-\frac{1}{2}} \quad | \quad v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} \end{array} \right]$$

Riešené príklady – 079, 080

$$\int x^2 \ln \sqrt{1-x} dx = \int x^2 \ln (1-x)^{\frac{1}{2}} dx$$

$$= \int \frac{x^2}{2} \ln (1-x) dx = \left[\begin{array}{l} u = \ln(1-x) \quad u' = \frac{-1}{1-x} = \frac{1}{x-1} \\ v' = \frac{x^2}{2} \quad v = \frac{x^3}{2 \cdot 3} = \frac{x^3}{6} \end{array} \right]$$

$$\int \frac{\ln x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} \ln x dx$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = x^{-\frac{1}{2}} \quad v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} \end{array} \right] = 2x^{\frac{1}{2}} \ln x - 2 \int x^{-\frac{1}{2}} dx$$

Riešené príklady – 079, 080

$$\int x^2 \ln \sqrt{1-x} dx = \int x^2 \ln (1-x)^{\frac{1}{2}} dx$$

$$= \int \frac{x^2}{2} \ln (1-x) dx = \left[\begin{array}{l} u = \ln(1-x) \quad u' = \frac{-1}{1-x} = \frac{1}{x-1} \\ v' = \frac{x^2}{2} \quad v = \frac{x^3}{2 \cdot 3} = \frac{x^3}{6} \end{array} \right] = \frac{x^3 \ln(1-x)}{6} - \int \frac{x^3 dx}{6(x-1)}$$

$$\int \frac{\ln x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} \ln x dx$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = x^{-\frac{1}{2}} \quad v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} \end{array} \right] = 2x^{\frac{1}{2}} \ln x - 2 \int x^{-\frac{1}{2}} dx = 2\sqrt{x} \cdot \ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

Riešené príklady – 079, 080

$$\int x^2 \ln \sqrt{1-x} dx = \int x^2 \ln (1-x)^{\frac{1}{2}} dx$$

$$= \int \frac{x^2}{2} \ln (1-x) dx = \left[\begin{array}{l} u = \ln(1-x) \quad u' = \frac{-1}{1-x} = \frac{1}{x-1} \\ v' = \frac{x^2}{2} \quad v = \frac{x^3}{2 \cdot 3} = \frac{x^3}{6} \end{array} \right] = \frac{x^3 \ln(1-x)}{6} - \int \frac{x^3 dx}{6(x-1)}$$

$$= \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^3 - 1 + 1) dx}{6(x-1)}$$

$$\int \frac{\ln x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} \ln x dx = 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + c$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = x^{-\frac{1}{2}} \quad v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} \end{array} \right] = 2x^{\frac{1}{2}} \ln x - 2 \int x^{-\frac{1}{2}} dx = 2\sqrt{x} \cdot \ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + c, \quad x \in (0; \infty), \quad c \in \mathbb{R}.$$

Riešené príklady – 079, 080

$$\int x^2 \ln \sqrt{1-x} dx = \int x^2 \ln (1-x)^{\frac{1}{2}} dx$$

$$= \int \frac{x^2}{2} \ln (1-x) dx = \left[\begin{array}{l} u = \ln(1-x) \quad u' = \frac{-1}{1-x} = \frac{1}{x-1} \\ v' = \frac{x^2}{2} \quad v = \frac{x^3}{2 \cdot 3} = \frac{x^3}{6} \end{array} \right] = \frac{x^3 \ln(1-x)}{6} - \int \frac{x^3 dx}{6(x-1)}$$

$$= \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^3-1+1) dx}{6(x-1)} = \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^3-1) dx}{6(x-1)} - \int \frac{dx}{6(x-1)}$$

$$\int \frac{\ln x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} \ln x dx = 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + c$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = x^{-\frac{1}{2}} \quad v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} \end{array} \right] = 2x^{\frac{1}{2}} \ln x - 2 \int x^{-\frac{1}{2}} dx = 2\sqrt{x} \cdot \ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + c, \quad x \in (0; \infty), \quad c \in \mathbb{R}.$$

Riešené príklady – 079, 080

$$\int x^2 \ln \sqrt{1-x} dx = \int x^2 \ln (1-x)^{\frac{1}{2}} dx$$

$$\begin{aligned}
 &= \int \frac{x^2}{2} \ln (1-x) dx = \left[\begin{array}{l} u = \ln(1-x) \quad u' = \frac{-1}{1-x} = \frac{1}{x-1} \\ v' = \frac{x^2}{2} \quad v = \frac{x^3}{2 \cdot 3} = \frac{x^3}{6} \end{array} \right] = \frac{x^3 \ln(1-x)}{6} - \int \frac{x^3 dx}{6(x-1)} \\
 &= \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^3-1+1) dx}{6(x-1)} = \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^3-1) dx}{6(x-1)} - \int \frac{dx}{6(x-1)} \\
 &= \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^2+x+1) dx}{6} - \int \frac{dx}{6(x-1)}
 \end{aligned}$$

$$\int \frac{\ln x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} \ln x dx = 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + c$$

$$\begin{aligned}
 &= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = x^{-\frac{1}{2}} \quad v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} \end{array} \right] = 2x^{\frac{1}{2}} \ln x - 2 \int x^{-\frac{1}{2}} dx = 2\sqrt{x} \cdot \ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c \\
 &= 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + c, \quad x \in (0; \infty), \quad c \in R.
 \end{aligned}$$

Riešené príklady – 079, 080

$$\int x^2 \ln \sqrt{1-x} dx = \int x^2 \ln (1-x)^{\frac{1}{2}} dx$$

$$= \int \frac{x^2}{2} \ln (1-x) dx = \left[\begin{array}{l} u = \ln(1-x) \quad u' = \frac{-1}{1-x} = \frac{1}{x-1} \\ v' = \frac{x^2}{2} \quad v = \frac{x^3}{2 \cdot 3} = \frac{x^3}{6} \end{array} \right] = \frac{x^3 \ln(1-x)}{6} - \int \frac{x^3 dx}{6(x-1)}$$

$$= \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^3-1+1) dx}{6(x-1)} = \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^3-1) dx}{6(x-1)} - \int \frac{dx}{6(x-1)}$$

$$= \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^2+x+1) dx}{6} - \int \frac{dx}{6(x-1)} = \frac{x^3 \ln(1-x)}{6} - \frac{\frac{x^3}{3} + \frac{x^2}{2} + x}{6} - \frac{\ln|x-1|}{6} + c$$

$$\int \frac{\ln x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} \ln x dx = 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + c$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = x^{-\frac{1}{2}} \quad v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} \end{array} \right] = 2x^{\frac{1}{2}} \ln x - 2 \int x^{-\frac{1}{2}} dx = 2\sqrt{x} \cdot \ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + c, \quad x \in (0; \infty), \quad c \in \mathbb{R}.$$

Riešené príklady – 079, 080

$$\int x^2 \ln \sqrt{1-x} dx = \int x^2 \ln (1-x)^{\frac{1}{2}} dx$$

$$= \int \frac{x^2}{2} \ln (1-x) dx = \left[\begin{array}{l} u = \ln(1-x) \quad u' = \frac{-1}{1-x} = \frac{1}{x-1} \\ v' = \frac{x^2}{2} \quad v = \frac{x^3}{2 \cdot 3} = \frac{x^3}{6} \end{array} \right] = \frac{x^3 \ln(1-x)}{6} - \int \frac{x^3 dx}{6(x-1)}$$

$$= \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^3-1+1) dx}{6(x-1)} = \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^3-1) dx}{6(x-1)} - \int \frac{dx}{6(x-1)}$$

$$= \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^2+x+1) dx}{6} - \int \frac{dx}{6(x-1)} = \frac{x^3 \ln(1-x)}{6} - \frac{\frac{x^3}{3} + \frac{x^2}{2} + x}{6} - \frac{\ln|x-1|}{6} + c$$

$$= \frac{x^3 \ln(1-x)}{6} - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} - \frac{\ln(1-x)}{6} + c$$

$$\int \frac{\ln x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} \ln x dx = 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + c$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = x^{-\frac{1}{2}} \quad v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} \end{array} \right] = 2x^{\frac{1}{2}} \ln x - 2 \int x^{-\frac{1}{2}} dx = 2\sqrt{x} \cdot \ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + c, \quad x \in (0; \infty), \quad c \in \mathbb{R}.$$

Riešené príklady – 079, 080

$$\int x^2 \ln \sqrt{1-x} dx = \int x^2 \ln (1-x)^{\frac{1}{2}} dx = (x^3-1) \ln \sqrt[6]{1-x} - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + c$$

$$\begin{aligned} &= \int \frac{x^2}{2} \ln (1-x) dx = \left[\begin{array}{l} u = \ln(1-x) \quad u' = \frac{-1}{1-x} = \frac{1}{x-1} \\ v' = \frac{x^2}{2} \quad v = \frac{x^3}{2 \cdot 3} = \frac{x^3}{6} \end{array} \right] = \frac{x^3 \ln(1-x)}{6} - \int \frac{x^3 dx}{6(x-1)} \\ &= \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^3-1+1) dx}{6(x-1)} = \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^3-1) dx}{6(x-1)} - \int \frac{dx}{6(x-1)} \\ &= \frac{x^3 \ln(1-x)}{6} - \int \frac{(x^2+x+1) dx}{6} - \int \frac{dx}{6(x-1)} = \frac{x^3 \ln(1-x)}{6} - \frac{\frac{x^3}{3} + \frac{x^2}{2} + x}{6} - \frac{\ln|x-1|}{6} + c \\ &= \frac{x^3 \ln(1-x)}{6} - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} - \frac{\ln(1-x)}{6} + c = \frac{x^3-1}{6} \ln(1-x) - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + c, \\ & \qquad \qquad \qquad x \in (-\infty; 1), c \in R. \end{aligned}$$

$$\int \frac{\ln x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} \ln x dx = 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + c$$

$$\begin{aligned} &= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = x^{-\frac{1}{2}} \quad v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} \end{array} \right] = 2x^{\frac{1}{2}} \ln x - 2 \int x^{-\frac{1}{2}} dx = 2\sqrt{x} \cdot \ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c \\ &= 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + c, \quad x \in (0; \infty), c \in R. \end{aligned}$$

Riešené príklady – 081, 082, 083

$$\int x \ln x \, dx$$

$$\int x^2 \ln x \, dx$$

$$\int x^n \ln x \, dx$$

Riešené príklady – 081, 082, 083

$$\int x \ln x \, dx$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x & v = \frac{x^2}{2} \end{array} \right]$$

$$\int x^2 \ln x \, dx$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x^2 & v = \frac{x^3}{3} \end{array} \right]$$

$$\int x^n \ln x \, dx$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x^n & v = \frac{x^{n+1}}{n+1} \end{array} \right]$$

Riešené príklady – 081, 082, 083

$$\int x \ln x \, dx$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = x \quad v = \frac{x^2}{2} \end{array} \right] = \frac{x^2}{2} \ln x - \int \frac{x \, dx}{2}$$

$$\int x^2 \ln x \, dx$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = x^2 \quad v = \frac{x^3}{3} \end{array} \right] = \frac{x^3}{3} \ln x - \int \frac{x^2 \, dx}{3}$$

$$\int x^n \ln x \, dx$$

$$= \left[\begin{array}{l} u = \ln x \quad u' = \frac{1}{x} \\ v' = x^n \quad v = \frac{x^{n+1}}{n+1} \end{array} \right] = \frac{x^{n+1}}{n+1} \ln x - \int \frac{x^n \, dx}{n+1}$$

Riešené príklady – 081, 082, 083

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$$

$$= \left[\begin{array}{l} u = \ln x \\ v' = x \end{array} \middle| \begin{array}{l} u' = \frac{1}{x} \\ v = \frac{x^2}{2} \end{array} \right] = \frac{x^2}{2} \ln x - \int \frac{x \, dx}{2} = \frac{x^2}{2} \ln x - \frac{x^2}{2 \cdot 2} \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c, \\ x \in (0; \infty), c \in \mathbb{R}.$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + c$$

$$= \left[\begin{array}{l} u = \ln x \\ v' = x^2 \end{array} \middle| \begin{array}{l} u' = \frac{1}{x} \\ v = \frac{x^3}{3} \end{array} \right] = \frac{x^3}{3} \ln x - \int \frac{x^2 \, dx}{3} = \frac{x^3}{3} \ln x - \frac{x^3}{3 \cdot 3} \, dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + c, \\ x \in (0; \infty), c \in \mathbb{R}.$$

$$\int x^n \ln x \, dx = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + c$$

$$= \left[\begin{array}{l} u = \ln x \\ v' = x^n \end{array} \middle| \begin{array}{l} u' = \frac{1}{x} \\ v = \frac{x^{n+1}}{n+1} \end{array} \right] = \frac{x^{n+1}}{n+1} \ln x - \int \frac{x^n \, dx}{n+1} = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + c, x \in (0; \infty), c \in \mathbb{R}.$$

Riešené príklady – 084

$$\int (x+1)^2 \ln(x-1)^5 dx$$

Riešené príklady – 084

$$\int (x+1)^2 \ln(x-1)^5 dx$$

$$= 5 \int (x+1)^2 \ln(x-1) dx$$

Riešené príklady – 084

$$\int (x+1)^2 \ln(x-1)^5 dx$$

$$= 5 \int (x+1)^2 \ln(x-1) dx = \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \left| \begin{array}{l} x=t+1 \\ x+1=t+2 \end{array} \right| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right]$$

Riešené príklady – 084

$$\int (x+1)^2 \ln(x-1)^5 dx$$

$$= 5 \int (x+1)^2 \ln(x-1) dx = \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x=t+1 \\ dt=dx \mid x+1=t+2 \end{array} \left| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = 5 \int (t+2)^2 \ln t dt$$

Riešené príklady – 084

$$\int (x+1)^2 \ln(x-1)^5 dx$$

$$= 5 \int (x+1)^2 \ln(x-1) dx = \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \middle| \begin{array}{l} x=t+1 \\ x+1=t+2 \end{array} \middle| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right] = 5 \int (t+2)^2 \ln t dt$$

$$= \left[\begin{array}{l} u = \ln t \\ v' = (t+2)^2 \end{array} \middle| \begin{array}{l} u' = \frac{1}{t} \\ v = \frac{(t+2)^3}{3} \end{array} \right]$$

Riešené príklady – 084

$$\int (x+1)^2 \ln(x-1)^5 dx$$

$$= 5 \int (x+1)^2 \ln(x-1) dx = \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \left| \begin{array}{l} x=t+1 \\ x+1=t+2 \end{array} \right. \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right] = 5 \int (t+2)^2 \ln t dt$$

$$= \left[\begin{array}{l} u = \ln t \\ v' = (t+2)^2 \end{array} \left| \begin{array}{l} u' = \frac{1}{t} \\ v = \frac{(t+2)^3}{3} \end{array} \right. \right] = \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{(t+2)^3}{t} dt$$

Riešené príklady – 084

$$\int (x+1)^2 \ln(x-1)^5 dx$$

$$= 5 \int (x+1)^2 \ln(x-1) dx = \left[\begin{array}{l} \text{Subst. } t=x-1 \mid x=t+1 \\ dt=dx \mid x+1=t+2 \end{array} \left| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = 5 \int (t+2)^2 \ln t dt$$

$$= \left[\begin{array}{l} u = \ln t \mid u' = \frac{1}{t} \\ v' = (t+2)^2 \mid v = \frac{(t+2)^3}{3} \end{array} \right] = \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{(t+2)^3}{t} dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{t^3 + 6t^2 + 12t + 8}{t} dt$$

Riešené príklady – 084

$$\int (x+1)^2 \ln(x-1)^5 dx$$

$$= 5 \int (x+1)^2 \ln(x-1) dx = \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \left| \begin{array}{l} x=t+1 \\ x+1=t+2 \end{array} \right. \left| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = 5 \int (t+2)^2 \ln t dt$$

$$= \left[\begin{array}{l} u = \ln t \\ v' = (t+2)^2 \end{array} \left| \begin{array}{l} u' = \frac{1}{t} \\ v = \frac{(t+2)^3}{3} \end{array} \right. \right] = \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{(t+2)^3}{t} dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{t^3 + 6t^2 + 12t + 8}{t} dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \left[t^2 + 6t + 12 + \frac{8}{t} \right] dt$$

Riešené príklady – 084

$$\int (x+1)^2 \ln(x-1)^5 dx$$

$$= 5 \int (x+1)^2 \ln(x-1) dx = \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \left| \begin{array}{l} x=t+1 \\ x+1=t+2 \end{array} \right| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right] = 5 \int (t+2)^2 \ln t dt$$

$$= \left[\begin{array}{l} u = \ln t \\ v' = (t+2)^2 \end{array} \left| \begin{array}{l} u' = \frac{1}{t} \\ v = \frac{(t+2)^3}{3} \end{array} \right. \right] = \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{(t+2)^3}{t} dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{t^3 + 6t^2 + 12t + 8}{t} dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \left[t^2 + 6t + 12 + \frac{8}{t} \right] dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \left[\frac{t^3}{3} + \frac{6t^2}{2} + 12t + 8 \ln t \right] + c$$

Riešené príklady – 084

$$\int (x+1)^2 \ln(x-1)^5 dx$$

$$= 5 \int (x+1)^2 \ln(x-1) dx = \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \left| \begin{array}{l} x=t+1 \\ x+1=t+2 \end{array} \right. \left| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = 5 \int (t+2)^2 \ln t dt$$

$$= \left[\begin{array}{l} u = \ln t \\ v' = (t+2)^2 \end{array} \left| \begin{array}{l} u' = \frac{1}{t} \\ v = \frac{(t+2)^3}{3} \end{array} \right. \right] = \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{(t+2)^3}{t} dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{t^3 + 6t^2 + 12t + 8}{t} dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \left[t^2 + 6t + 12 + \frac{8}{t} \right] dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \left[\frac{t^3}{3} + \frac{6t^2}{2} + 12t + 8 \ln t \right] + c$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5t^3}{9} - 5t^2 - 20t - \frac{40}{3} \ln t + c$$

Riešené príklady – 084

$$\int (x+1)^2 \ln(x-1)^5 dx$$

$$= 5 \int (x+1)^2 \ln(x-1) dx = \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \left| \begin{array}{l} x=t+1 \\ x+1=t+2 \end{array} \right. \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right] = 5 \int (t+2)^2 \ln t dt$$

$$= \left[\begin{array}{l} u = \ln t \\ v' = (t+2)^2 \end{array} \left| \begin{array}{l} u' = \frac{1}{t} \\ v = \frac{(t+2)^3}{3} \end{array} \right. \right] = \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{(t+2)^3}{t} dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{t^3 + 6t^2 + 12t + 8}{t} dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \left[t^2 + 6t + 12 + \frac{8}{t} \right] dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \left[\frac{t^3}{3} + \frac{6t^2}{2} + 12t + 8 \ln t \right] + c$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5t^3}{9} - 5t^2 - 20t - \frac{40}{3} \ln t + c$$

$$= \frac{5(t+2)^3 - 40}{3} \ln t - \frac{5t^3}{9} - 5t^2 - 20t + c$$

Riešené príklady – 084

$$\int (x+1)^2 \ln(x-1)^5 dx = \frac{5(x+1)^3 - 40}{3} \ln(x-1) - \frac{5(x-1)^3}{9} - 5(x-1)^2 - 20(x-1) + c$$

$$= 5 \int (x+1)^2 \ln(x-1) dx = \left[\begin{array}{l} \text{Subst. } t=x-1 \\ dt=dx \end{array} \left| \begin{array}{l} x=t+1 \\ x+1=t+2 \end{array} \right. \left| \begin{array}{l} x \in (1; \infty) \\ t \in (0; \infty) \end{array} \right. \right] = 5 \int (t+2)^2 \ln t dt$$

$$= \left[\begin{array}{l} u = \ln t \\ v' = (t+2)^2 \end{array} \left| \begin{array}{l} u' = \frac{1}{t} \\ v = \frac{(t+2)^3}{3} \end{array} \right. \right] = \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{(t+2)^3}{t} dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \frac{t^3 + 6t^2 + 12t + 8}{t} dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \int \left[t^2 + 6t + 12 + \frac{8}{t} \right] dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3} \left[\frac{t^3}{3} + \frac{6t^2}{2} + 12t + 8 \ln t \right] + c$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5t^3}{9} - 5t^2 - 20t - \frac{40}{3} \ln t + c$$

$$= \frac{5(t+2)^3 - 40}{3} \ln t - \frac{5t^3}{9} - 5t^2 - 20t + c$$

$$= \frac{5(x+1)^3 - 40}{3} \ln(x-1) - \frac{5(x-1)^3}{9} - 5(x-1)^2 - 20(x-1) + c, \quad x \in (1; \infty), \quad c \in \mathbb{R}.$$

Riešené príklady – 085, 086

$$\int x^x (\ln x + 1) dx$$

$$\int x \ln^2 x dx$$

Riešené príklady – 085, 086

$$\int x^x (\ln x + 1) dx$$

$$= \left[\begin{array}{l} u = x^x \quad | \quad u' = (e^{\ln x})' = (e^{x \ln x})' = e^{x \ln x} (\ln x + \frac{x}{x}) = x^x (\ln x + 1) \\ v' = \ln x + 1 \quad | \quad v = \dots \text{ nie je nutné, pretože } u' = [x^x]' = x^x (\ln x + 1) \end{array} \right]$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^x \quad | \quad dt = x^x (\ln x + 1) dx \quad | \quad x^x \text{ má min pre } x = \frac{1}{e}, \text{ t. j. } \ln x + 1 = 0 \quad | \quad x \in (0; \frac{1}{e}) \quad | \quad x \in (\frac{1}{e}; \infty) \\ (x^x)' = (e^{\ln x})' = (e^{x \ln x})' = e^{x \ln x} (\ln x + \frac{x}{x}) = x^x (\ln x + 1) \quad | \quad t \in (e^{-\frac{1}{e}}; 1) \quad | \quad t \in (e^{-\frac{1}{e}}; \infty) \end{array} \right]$$

$$\int x \ln^2 x dx$$

$$= \left[\begin{array}{l} u = \ln^2 x \quad | \quad u' = \frac{2 \ln x}{x} \\ v' = x \quad | \quad v = \frac{x^2}{2} \end{array} \right]$$

Riešené príklady – 085, 086

$$\int x^x (\ln x + 1) dx = x^x + c$$

$$= \left[\begin{array}{l} u = x^x \\ v' = \ln x + 1 \end{array} \middle| \begin{array}{l} u' = (e^{\ln x})' = (e^{x \ln x})' = e^{x \ln x} (\ln x + \frac{x}{x}) = x^x (\ln x + 1) \\ v = \dots \text{nie je nutné, pretože } u' = [x^x]' = x^x (\ln x + 1) \end{array} \right] = x^x + c, \quad x \in (0; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^x \\ (x^x)' = (e^{\ln x})' = (e^{x \ln x})' = e^{x \ln x} (\ln x + \frac{x}{x}) = x^x (\ln x + 1) \end{array} \middle| \begin{array}{l} dt = x^x (\ln x + 1) dx \\ x^x \text{ má min pre } x = \frac{1}{e}, \text{ t. j. } \ln x + 1 = 0 \\ x \in (0; \frac{1}{e}) \\ t \in (e^{-\frac{1}{e}}; 1) \end{array} \middle| \begin{array}{l} x \in (\frac{1}{e}; \infty) \\ t \in (e^{-\frac{1}{e}}; \infty) \end{array} \right] = \int dt$$

$$\int x \ln^2 x dx$$

$$= \left[\begin{array}{l} u = \ln^2 x \\ v' = x \end{array} \middle| \begin{array}{l} u' = \frac{2 \ln x}{x} \\ v = \frac{x^2}{2} \end{array} \right] = \frac{x^2 \ln^2 x}{2} - \int x \ln x dx$$

Riešené príklady – 085, 086

$$\int x^x (\ln x + 1) dx = x^x + c$$

$$= \left[\begin{array}{l} u = x^x \\ v' = \ln x + 1 \end{array} \middle| \begin{array}{l} u' = (e^{\ln x})' = (e^{x \ln x})' = e^{x \ln x} (\ln x + \frac{x}{x}) = x^x (\ln x + 1) \\ v = \dots \text{nie je nutné, pretože } u' = [x^x]' = x^x (\ln x + 1) \end{array} \right] = x^x + c, \quad x \in (0; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^x \\ (x^x)' = (e^{\ln x})' = (e^{x \ln x})' = e^{x \ln x} (\ln x + \frac{x}{x}) = x^x (\ln x + 1) \end{array} \middle| \begin{array}{l} dt = x^x (\ln x + 1) dx \\ x^x \text{ má min pre } x = \frac{1}{e}, \text{ t. j. } \ln x + 1 = 0 \\ x \in (0; \frac{1}{e}) \quad \left| \quad x \in (\frac{1}{e}; \infty) \right. \\ t \in (e^{-\frac{1}{e}}; 1) \quad \left| \quad t \in (e^{-\frac{1}{e}}; \infty) \right. \end{array} \right] = \int dt$$

$$= t + c$$

$$\int x \ln^2 x dx$$

$$= \left[\begin{array}{l} u = \ln^2 x \\ v' = x \end{array} \middle| \begin{array}{l} u' = \frac{2 \ln x}{x} \\ v = \frac{x^2}{2} \end{array} \right] = \frac{x^2 \ln^2 x}{2} - \int x \ln x dx = \left[\begin{array}{l} u = \ln x \\ v' = x \end{array} \middle| \begin{array}{l} u' = \frac{1}{x} \\ v = \frac{x^2}{2} \end{array} \right]$$

Riešené príklady – 085, 086

$$\int x^x (\ln x + 1) dx = x^x + c$$

$$= \left[\begin{array}{l} u = x^x \\ v' = \ln x + 1 \end{array} \middle| \begin{array}{l} u' = (e^{\ln x})' = (e^{x \ln x})' = e^{x \ln x} (\ln x + \frac{x}{x}) = x^x (\ln x + 1) \\ v = \dots \text{nie je nutné, pretože } u' = [x^x]' = x^x (\ln x + 1) \end{array} \right] = x^x + c, \quad x \in (0; \infty), \quad c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^x \mid dt = x^x (\ln x + 1) dx \mid x^x \text{ má min pre } x = \frac{1}{e}, \text{ t. j. } \ln x + 1 = 0 \mid x \in (0; \frac{1}{e}) \mid x \in (\frac{1}{e}; \infty) \\ (x^x)' = (e^{\ln x})' = (e^{x \ln x})' = e^{x \ln x} (\ln x + \frac{x}{x}) = x^x (\ln x + 1) \mid t \in (e^{-\frac{1}{e}}; 1) \mid t \in (e^{-\frac{1}{e}}; \infty) \end{array} \right] = \int dt$$

$$= t + c = x^x + c, \quad x \in (0; \infty), \quad c \in \mathbb{R}.$$

$$\int x \ln^2 x dx$$

$$= \left[\begin{array}{l} u = \ln^2 x \\ v' = x \end{array} \middle| \begin{array}{l} u' = \frac{2 \ln x}{x} \\ v = \frac{x^2}{2} \end{array} \right] = \frac{x^2 \ln^2 x}{2} - \int x \ln x dx = \left[\begin{array}{l} u = \ln x \\ v' = x \end{array} \middle| \begin{array}{l} u' = \frac{1}{x} \\ v = \frac{x^2}{2} \end{array} \right]$$

$$= \frac{x^2 \ln^2 x}{2} - \left[\frac{x^2 \ln x}{2} - \int \frac{x dx}{2} \right]$$

Riešené príklady – 085, 086

$$\int x^x (\ln x + 1) dx = x^x + c$$

$$= \left[\begin{array}{l} u = x^x \\ v' = \ln x + 1 \end{array} \middle| \begin{array}{l} u' = (e^{\ln x})' = (e^{x \ln x})' = e^{x \ln x} (\ln x + \frac{x}{x}) = x^x (\ln x + 1) \\ v = \dots \text{nie je nutné, pretože } u' = [x^x]' = x^x (\ln x + 1) \end{array} \right] = x^x + c, x \in (0; \infty), c \in \mathbb{R}.$$

$$= \left[\begin{array}{l} \text{Subst. } t = x^x \mid dt = x^x (\ln x + 1) dx \mid x^x \text{ má min pre } x = \frac{1}{e}, \text{ t. j. } \ln x + 1 = 0 \mid x \in (0; \frac{1}{e}) \mid x \in (\frac{1}{e}; \infty) \\ (x^x)' = (e^{\ln x})' = (e^{x \ln x})' = e^{x \ln x} (\ln x + \frac{x}{x}) = x^x (\ln x + 1) \mid t \in (e^{-\frac{1}{e}}; 1) \mid t \in (e^{-\frac{1}{e}}; \infty) \end{array} \right] = \int dt$$

$$= t + c = x^x + c, x \in (0; \infty), c \in \mathbb{R}.$$

$$\int x \ln^2 x dx = \frac{x^2 \ln^2 x}{2} - \frac{x^2 \ln x}{2} + \frac{x^2}{4} + c$$

$$= \left[\begin{array}{l} u = \ln^2 x \\ v' = x \end{array} \middle| \begin{array}{l} u' = \frac{2 \ln x}{x} \\ v = \frac{x^2}{2} \end{array} \right] = \frac{x^2 \ln^2 x}{2} - \int x \ln x dx = \left[\begin{array}{l} u = \ln x \\ v' = x \end{array} \middle| \begin{array}{l} u' = \frac{1}{x} \\ v = \frac{x^2}{2} \end{array} \right]$$

$$= \frac{x^2 \ln^2 x}{2} - \left[\frac{x^2 \ln x}{2} - \int \frac{x dx}{2} \right] = \frac{x^2 \ln^2 x}{2} - \frac{x^2 \ln x}{2} + \frac{x^2}{4} + c, x \in (0; \infty), c \in \mathbb{R}.$$

Riešené príklady – 087, 088

$$\int \ln(x^2+1) dx$$

$$\int \ln(\sqrt{1+x} + \sqrt{1-x}) dx$$

Riešené príklady – 087, 088

$$\int \ln(x^2+1) dx$$

$$= \left[\begin{array}{l} u = \ln(x^2+1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{2x}{x^2+1} \\ v = x \end{array} \right]$$

$$\int \ln(\sqrt{1+x} + \sqrt{1-x}) dx$$

$$= \left[\begin{array}{l} u = \ln(\sqrt{1+x} + \sqrt{1-x}) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{2x} - \frac{1}{2x\sqrt{1-x^2}} \\ v = x \end{array} \right]$$

$$\left[\begin{array}{l} [\ln(\sqrt{1+x} + \sqrt{1-x})]' = [\ln((1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}})]' = \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}} - \frac{1}{2}(1-x)^{-\frac{1}{2}}}{(1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}}} = \frac{\frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{1-x}}}{\sqrt{1+x} + \sqrt{1-x}} = \frac{\frac{\sqrt{1-x} - \sqrt{1+x}}{2\sqrt{1+x}\sqrt{1-x}}}{\sqrt{1+x} + \sqrt{1-x}} \\ = \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1+x}\sqrt{1-x}} \cdot \frac{1}{2\sqrt{1+x}\sqrt{1-x}} = \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x} + \sqrt{1+x}} \cdot \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x} - \sqrt{1+x}} \cdot \frac{1}{2\sqrt{1-x^2}} = \frac{1-x - 2\sqrt{1-x}\sqrt{1+x} + 1+x}{[(1-x) - (1+x)] \cdot 2\sqrt{1-x^2}} = \frac{2 - 2\sqrt{1-x^2}}{-2x \cdot 2\sqrt{1-x^2}} = \frac{\sqrt{1-x^2} - 1}{2x\sqrt{1-x^2}} \end{array} \right]$$

Riešené príklady – 087, 088

$$\int \ln(x^2+1) dx$$

$$= \left[\begin{array}{l} u = \ln(x^2+1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{2x}{x^2+1} \\ v = x \end{array} \right] = x \ln(x^2+1) - \int \frac{2x^2 dx}{x^2+1}$$

$$\int \ln(\sqrt{1+x} + \sqrt{1-x}) dx$$

$$= \left[\begin{array}{l} u = \ln(\sqrt{1+x} + \sqrt{1-x}) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{2x} - \frac{1}{2x\sqrt{1-x^2}} \\ v = x \end{array} \right] = x \ln(\sqrt{1+x} + \sqrt{1-x}) - \int \left[\frac{1}{2} - \frac{1}{2\sqrt{1-x^2}} \right] dx$$

$$\left[\begin{array}{l} [\ln(\sqrt{1+x} + \sqrt{1-x})]' = [\ln((1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}})]' = \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}} - \frac{1}{2}(1-x)^{-\frac{1}{2}}}{(1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}}} = \frac{\frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{1-x}}}{\sqrt{1+x} + \sqrt{1-x}} = \frac{\frac{\sqrt{1-x} - \sqrt{1+x}}{2\sqrt{1+x}\sqrt{1-x}}}{\sqrt{1+x} + \sqrt{1-x}} \\ = \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1+x}\sqrt{1-x}} \cdot \frac{1}{2\sqrt{1+x}\sqrt{1-x}} = \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x} + \sqrt{1+x}} \cdot \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x} - \sqrt{1+x}} \cdot \frac{1}{2\sqrt{1-x^2}} = \frac{1-x - 2\sqrt{1-x}\sqrt{1+x} + 1+x}{[(1-x) - (1+x)] \cdot 2\sqrt{1-x^2}} = \frac{2 - 2\sqrt{1-x^2}}{-2x \cdot 2\sqrt{1-x^2}} = \frac{\sqrt{1-x^2} - 1}{2x\sqrt{1-x^2}} \end{array} \right]$$

Riešené príklady – 087, 088

$$\int \ln(x^2+1) dx$$

$$= \left[\begin{array}{l} u = \ln(x^2+1) \\ v' = 1 \end{array} \right] \left[\begin{array}{l} u' = \frac{2x}{x^2+1} \\ v = x \end{array} \right] = x \ln(x^2+1) - \int \frac{2x^2 dx}{x^2+1} = x \ln(x^2+1) - 2 \int \frac{x^2+1-1}{x^2+1} dx$$

$$\int \ln(\sqrt{1+x} + \sqrt{1-x}) dx = x \ln(\sqrt{1+x} + \sqrt{1-x}) - \frac{x}{2} + \frac{\arcsin x}{2} + c$$

$$= \left[\begin{array}{l} u = \ln(\sqrt{1+x} + \sqrt{1-x}) \\ v' = 1 \end{array} \right] \left[\begin{array}{l} u' = \frac{1}{2x} - \frac{1}{2x\sqrt{1-x^2}} \\ v = x \end{array} \right] = x \ln(\sqrt{1+x} + \sqrt{1-x}) - \int \left[\frac{1}{2} - \frac{1}{2\sqrt{1-x^2}} \right] dx$$

$$\left[\begin{array}{l} [\ln(\sqrt{1+x} + \sqrt{1-x})]' = [\ln((1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}})]' = \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}} - \frac{1}{2}(1-x)^{-\frac{1}{2}}}{(1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}}} = \frac{\frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{1-x}}}{\sqrt{1+x} + \sqrt{1-x}} = \frac{\frac{\sqrt{1-x} - \sqrt{1+x}}{2\sqrt{1+x}\sqrt{1-x}}}{\sqrt{1+x} + \sqrt{1-x}} \\ = \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1+x}\sqrt{1-x}} \cdot \frac{1}{2\sqrt{1+x}\sqrt{1-x}} = \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x} + \sqrt{1+x}} \cdot \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x} - \sqrt{1+x}} \cdot \frac{1}{2\sqrt{1-x^2}} = \frac{1-x-2\sqrt{1-x}\sqrt{1+x}+1+x}{[(1-x)-(1+x)] \cdot 2\sqrt{1-x^2}} = \frac{2-2\sqrt{1-x^2}}{-2x \cdot 2\sqrt{1-x^2}} = \frac{\sqrt{1-x^2}-1}{2x\sqrt{1-x^2}} \end{array} \right]$$

$$= x \ln(\sqrt{1+x} + \sqrt{1-x}) - \frac{x}{2} + \frac{\arcsin x}{2} + c, \quad x \in (-1; 0) \cup (0; 1), \quad c \in \mathbb{R}.$$

Riešené príklady – 087, 088

$$\int \ln(x^2+1) dx$$

$$= \left[\begin{array}{l} u = \ln(x^2+1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{2x}{x^2+1} \\ v = x \end{array} \right] = x \ln(x^2+1) - \int \frac{2x^2 dx}{x^2+1} = x \ln(x^2+1) - 2 \int \frac{x^2+1-1}{x^2+1} dx$$

$$= x \ln(x^2+1) - 2 \int dx + \int \frac{2 dx}{x^2+1}$$

$$\int \ln(\sqrt{1+x} + \sqrt{1-x}) dx = x \ln(\sqrt{1+x} + \sqrt{1-x}) - \frac{x}{2} + \frac{\arcsin x}{2} + c$$

$$= \left[\begin{array}{l} u = \ln(\sqrt{1+x} + \sqrt{1-x}) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{2x} - \frac{1}{2x\sqrt{1-x^2}} \\ v = x \end{array} \right] = x \ln(\sqrt{1+x} + \sqrt{1-x}) - \int \left[\frac{1}{2} - \frac{1}{2\sqrt{1-x^2}} \right] dx$$

$$\left[\begin{array}{l} [\ln(\sqrt{1+x} + \sqrt{1-x})]' = [\ln((1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}})]' = \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}} - \frac{1}{2}(1-x)^{-\frac{1}{2}}}{(1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}}} = \frac{\frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{1-x}}}{\sqrt{1+x} + \sqrt{1-x}} = \frac{\frac{\sqrt{1-x} - \sqrt{1+x}}{2\sqrt{1+x}\sqrt{1-x}}}{\sqrt{1+x} + \sqrt{1-x}} \\ = \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1+x}\sqrt{1-x}} \cdot \frac{1}{2\sqrt{1+x}\sqrt{1-x}} = \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x} + \sqrt{1+x}} \cdot \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x} - \sqrt{1+x}} \cdot \frac{1}{2\sqrt{1-x^2}} = \frac{1-x-2\sqrt{1-x}\sqrt{1+x}+1+x}{[(1-x)-(1+x)] \cdot 2\sqrt{1-x^2}} = \frac{2-2\sqrt{1-x^2}}{-2x \cdot 2\sqrt{1-x^2}} = \frac{\sqrt{1-x^2}-1}{2x\sqrt{1-x^2}} \end{array} \right]$$

$$= x \ln(\sqrt{1+x} + \sqrt{1-x}) - \frac{x}{2} + \frac{\arcsin x}{2} + c, \quad x \in (-1; 0) \cup (0; 1), \quad c \in \mathbb{R}.$$

Riešené príklady – 087, 088

$$\int \ln(x^2+1) dx = x \ln(x^2+1) - 2x + 2 \operatorname{arctg} x + c$$

$$= \left[\begin{array}{l} u = \ln(x^2+1) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{2x}{x^2+1} \\ v = x \end{array} \right] = x \ln(x^2+1) - \int \frac{2x^2 dx}{x^2+1} = x \ln(x^2+1) - 2 \int \frac{x^2+1-1}{x^2+1} dx$$

$$= x \ln(x^2+1) - 2 \int dx + \int \frac{2 dx}{x^2+1} = x \ln(x^2+1) - 2x + 2 \operatorname{arctg} x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$\int \ln(\sqrt{1+x} + \sqrt{1-x}) dx = x \ln(\sqrt{1+x} + \sqrt{1-x}) - \frac{x}{2} + \frac{\arcsin x}{2} + c$$

$$= \left[\begin{array}{l} u = \ln(\sqrt{1+x} + \sqrt{1-x}) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = \frac{1}{2x} - \frac{1}{2x\sqrt{1-x^2}} \\ v = x \end{array} \right] = x \ln(\sqrt{1+x} + \sqrt{1-x}) - \int \left[\frac{1}{2} - \frac{1}{2\sqrt{1-x^2}} \right] dx$$

$$\left[\begin{array}{l} [\ln(\sqrt{1+x} + \sqrt{1-x})]' = [\ln((1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}})]' = \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}} - \frac{1}{2}(1-x)^{-\frac{1}{2}}}{(1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}}} = \frac{\frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{1-x}}}{\sqrt{1+x} + \sqrt{1-x}} = \frac{\frac{\sqrt{1-x} - \sqrt{1+x}}{2\sqrt{1+x}\sqrt{1-x}}}{\sqrt{1+x} + \sqrt{1-x}} \\ = \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1+x}\sqrt{1-x}} \cdot \frac{1}{2\sqrt{1+x}\sqrt{1-x}} = \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x} + \sqrt{1+x}} \cdot \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x} - \sqrt{1+x}} \cdot \frac{1}{2\sqrt{1-x^2}} = \frac{1-x - 2\sqrt{1-x}\sqrt{1+x} + 1+x}{[(1-x) - (1+x)] \cdot 2\sqrt{1-x^2}} = \frac{2-2\sqrt{1-x^2}}{-2x \cdot 2\sqrt{1-x^2}} = \frac{\sqrt{1-x^2}-1}{2x\sqrt{1-x^2}} \end{array} \right]$$

$$= x \ln(\sqrt{1+x} + \sqrt{1-x}) - \frac{x}{2} + \frac{\arcsin x}{2} + c, \quad x \in (-1; 0) \cup (0; 1), c \in \mathbb{R}.$$

Riešené príklady – 089, 090

$$\int \ln(x + \sqrt{x^2 + 1}) dx$$

$$\int x(x-a)(x-b) dx$$

 $a, b \in \mathbb{R}$

Riešené príklady – 089, 090

$$\int \ln(x + \sqrt{x^2 + 1}) dx$$

$$= \left[\begin{array}{l} u = \ln(x + \sqrt{x^2 + 1}) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = [\ln(x + (x^2 + 1)^{\frac{1}{2}})]' = \frac{1 + \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} \cdot 2x}{x + \sqrt{x^2 + 1}} = \frac{1 + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{\sqrt{x^2 + 1} + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}} \end{array} \right]$$

$$\int x(x-a)(x-b) dx$$

 $a, b \in \mathbb{R}$

$$= \int [x(x^2 - ax - bx + ab)] dx$$

Riešené príklady – 089, 090

$$\int \ln(x + \sqrt{x^2 + 1}) dx$$

$$= \left[\begin{array}{l} u = \ln(x + \sqrt{x^2 + 1}) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = [\ln(x + (x^2 + 1)^{\frac{1}{2}})]' = \frac{1 + \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} \cdot 2x}{x + \sqrt{x^2 + 1}} = \frac{1 + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{\sqrt{x^2 + 1} + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}} \end{array} \right]$$

$$= x \ln(x + \sqrt{x^2 + 1}) - \int \frac{x dx}{\sqrt{x^2 + 1}}$$

$$\int x(x-a)(x-b) dx$$

 $a, b \in \mathbb{R}$

$$= \int [x(x^2 - ax - bx + ab)] dx = \int [x^3 - (a+b)x^2 + abx] dx$$

Riešené príklady – 089, 090

$$\int \ln(x + \sqrt{x^2 + 1}) dx$$

$$= \left[\begin{array}{l} u = \ln(x + \sqrt{x^2 + 1}) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = [\ln(x + (x^2 + 1)^{\frac{1}{2}})]' = \frac{1 + \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} \cdot 2x}{x + \sqrt{x^2 + 1}} = \frac{1 + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{\sqrt{x^2 + 1} + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}} \end{array} \right]$$

$$= x \ln(x + \sqrt{x^2 + 1}) - \int \frac{x dx}{\sqrt{x^2 + 1}} = \left[\begin{array}{l} \text{Subst. } t = x^2 + 1 \\ dt = 2x dx \end{array} \middle| \begin{array}{l} x \in (-\infty; 0) \\ t \in (1; \infty) \end{array} \right] \quad \left[\begin{array}{l} x \in (0; \infty) \\ t \in (1; \infty) \end{array} \right]$$

$$\int x(x-a)(x-b) dx = \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c$$

 $a, b \in R$

$$= \int [x(x^2 - ax - bx + ab)] dx = \int [x^3 - (a+b)x^2 + abx] dx$$

$$= \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c, \quad x \in R, c \in R.$$

Riešené príklady – 089, 090

$$\int \ln(x + \sqrt{x^2 + 1}) dx$$

$$= \left[\begin{array}{l} u = \ln(x + \sqrt{x^2 + 1}) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = [\ln(x + (x^2 + 1)^{\frac{1}{2}})]' = \frac{1 + \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} \cdot 2x}{x + \sqrt{x^2 + 1}} = \frac{1 + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{\sqrt{x^2 + 1} + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}} \end{array} \right]$$

$$= x \ln(x + \sqrt{x^2 + 1}) - \int \frac{x dx}{\sqrt{x^2 + 1}} = \left[\begin{array}{l} \text{Subst. } t = x^2 + 1 \mid x \in (-\infty; 0) \mid x \in (0; \infty) \\ dt = 2x dx \mid t \in (1; \infty) \mid t \in (1; \infty) \end{array} \right]$$

$$= x \ln(x + \sqrt{x^2 + 1}) - \frac{1}{2} \int \frac{dt}{\sqrt{t}} = x \ln(x + \sqrt{x^2 + 1}) - \frac{1}{2} \int t^{-\frac{1}{2}} dt$$

$$\int x(x-a)(x-b) dx = \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c$$

 $a, b \in R$

$$= \int [x(x^2 - ax - bx + ab)] dx = \int [x^3 - (a+b)x^2 + abx] dx$$

$$= \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c, \quad x \in R, \quad c \in R.$$

Riešené príklady – 089, 090

$$\int \ln(x + \sqrt{x^2 + 1}) dx$$

$$= \left[\begin{array}{l} u = \ln(x + \sqrt{x^2 + 1}) \\ v' = 1 \end{array} \middle| \begin{array}{l} u' = [\ln(x + (x^2 + 1)^{\frac{1}{2}})]' = \frac{1 + \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} \cdot 2x}{x + \sqrt{x^2 + 1}} = \frac{1 + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{\sqrt{x^2 + 1} + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}} \end{array} \right]$$

$$= x \ln(x + \sqrt{x^2 + 1}) - \int \frac{x dx}{\sqrt{x^2 + 1}} = \left[\begin{array}{l} \text{Subst. } t = x^2 + 1 \mid x \in (-\infty; 0) \mid x \in (0; \infty) \\ dt = 2x dx \mid t \in (1; \infty) \mid t \in (1; \infty) \end{array} \right]$$

$$= x \ln(x + \sqrt{x^2 + 1}) - \frac{1}{2} \int \frac{dt}{\sqrt{t}} = x \ln(x + \sqrt{x^2 + 1}) - \frac{1}{2} \int t^{-\frac{1}{2}} dt$$

$$= x \ln(x + \sqrt{x^2 + 1}) - \frac{1}{2} \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\int x(x-a)(x-b) dx = \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c$$

 $a, b \in \mathbb{R}$

$$= \int [x(x^2 - ax - bx + ab)] dx = \int [x^3 - (a+b)x^2 + abx] dx$$

$$= \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

Riešené príklady – 089, 090

$$\int \ln(x + \sqrt{x^2 + 1}) dx = x \ln(x + \sqrt{x^2 + 1}) - \sqrt{x^2 + 1} + c$$

$$= \left[\begin{array}{l} u = \ln(x + \sqrt{x^2 + 1}) \\ v' = 1 \end{array} \right] \left[\begin{array}{l} u' = [\ln(x + (x^2 + 1)^{\frac{1}{2}})]' = \frac{1 + \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} \cdot 2x}{x + \sqrt{x^2 + 1}} = \frac{1 + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{\sqrt{x^2 + 1} + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}} \end{array} \right]$$

$$= x \ln(x + \sqrt{x^2 + 1}) - \int \frac{x dx}{\sqrt{x^2 + 1}} = \left[\begin{array}{l} \text{Subst. } t = x^2 + 1 \mid x \in (-\infty; 0) \mid x \in (0; \infty) \\ dt = 2x dx \mid t \in (1; \infty) \mid t \in (1; \infty) \end{array} \right]$$

$$= x \ln(x + \sqrt{x^2 + 1}) - \frac{1}{2} \int \frac{dt}{\sqrt{t}} = x \ln(x + \sqrt{x^2 + 1}) - \frac{1}{2} \int t^{-\frac{1}{2}} dt$$

$$= x \ln(x + \sqrt{x^2 + 1}) - \frac{1}{2} t^{\frac{1}{2}} + c = x \ln(x + \sqrt{x^2 + 1}) - \sqrt{x^2 + 1} + c, x \in R, c \in R.$$

$$\int x(x-a)(x-b) dx = \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c \quad a, b \in R$$

$$= \int [x(x^2 - ax - bx + ab)] dx = \int [x^3 - (a+b)x^2 + abx] dx$$

$$= \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c, x \in R, c \in R.$$

Riešené príklady – 091, 092

$$\int |x| dx$$

$$\int_{x \in (0; \infty)} \min \left\{ 1, \frac{1}{x} \right\} dx \quad x \in (0; 1) \quad x \in (1; \infty)$$

Riešené príklady – 091, 092

$$\int |x| dx$$

$$\boxed{x \geq 0} = \int x dx$$

$$\boxed{x \leq 0} = \int (-x) dx$$

$$\int_{x \in (0; \infty)} \min \left\{ 1, \frac{1}{x} \right\} dx$$

$$x \in (0; 1)$$

$$x \in (1; \infty)$$

$$\boxed{x \in (0; 1)} = \int dx$$

$$\boxed{x \in (1; \infty)} = \int \frac{dx}{x}$$

Riešené príklady – 091, 092

$$\int |x| dx$$

$$\boxed{x \geq 0} = \int x dx = \frac{x^2}{2} + c$$

$$\boxed{x \leq 0} = \int (-x) dx = -\frac{x^2}{2} + c$$

$$\int_{x \in (0; \infty)} \min \left\{ 1, \frac{1}{x} \right\} dx$$

$$x \in (0; 1)$$

$$x \in (1; \infty)$$

$$\boxed{x \in (0; 1)} = \int dx = x + c$$

$$\boxed{x \in (1; \infty)} = \int \frac{dx}{x} = \ln x + c_1$$

Riešené príklady – 091, 092

$$\int |x| dx$$

$$\boxed{x \geq 0} = \int x dx = \frac{x^2}{2} + c = \frac{x \cdot x}{2} + c$$

$$\boxed{x \leq 0} = \int (-x) dx = -\frac{x^2}{2} + c = \frac{x \cdot (-x)}{2} + c$$

$$\int_{x \in (0; \infty)} \min \left\{ 1, \frac{1}{x} \right\} dx \quad x \in (0; 1) \quad x \in (1; \infty)$$

$$\boxed{x \in (0; 1)} = \int dx = x + c$$

$$\boxed{x \in (1; \infty)} = \int \frac{dx}{x} = \ln x + c_1$$

Neurčitý integrál na intervale tvoria spojité primitívne funkcie.

Riešené príklady – 091, 092

$$\int |x| dx = \frac{x \cdot |x|}{2} + c$$

$$\boxed{x \geq 0} = \int x dx = \frac{x^2}{2} + c = \frac{x \cdot x}{2} + c = \frac{x \cdot |x|}{2} + c, \quad x \in \langle 0; \infty \rangle, \quad c \in \mathbb{R}.$$

$$\boxed{x \leq 0} = \int (-x) dx = -\frac{x^2}{2} + c = \frac{x \cdot (-x)}{2} + c = \frac{x \cdot |x|}{2} + c, \quad x \in \langle -\infty; 0 \rangle, \quad c \in \mathbb{R}.$$

$$\int_{x \in (0; \infty)} \min \left\{ 1, \frac{1}{x} \right\} dx \quad x \in (0; 1) \quad x \in \langle 1; \infty \rangle$$

$$\boxed{x \in (0; 1)} = \int dx = x + c$$

$$\boxed{x \in \langle 1; \infty \rangle} = \int \frac{dx}{x} = \ln x + c_1$$

Neurčitý integrál na intervale tvoria spojité primitívne funkcie. Aby uvedené funkcie tvorili neurčitý integrál danej funkcie, musia byť spojité na celom $(0; \infty)$, t. j. aj v bode $x=1$.

Riešené príklady – 091, 092

$$\int |x| dx = \frac{x \cdot |x|}{2} + c$$

$$\boxed{x \geq 0} = \int x dx = \frac{x^2}{2} + c = \frac{x \cdot x}{2} + c = \frac{x \cdot |x|}{2} + c, \quad x \in \langle 0; \infty \rangle, \quad c \in \mathbb{R}.$$

$$\boxed{x \leq 0} = \int (-x) dx = -\frac{x^2}{2} + c = \frac{x \cdot (-x)}{2} + c = \frac{x \cdot |x|}{2} + c, \quad x \in \langle -\infty; 0 \rangle, \quad c \in \mathbb{R}.$$

$$\int_{x \in (0; \infty)} \min \left\{ 1, \frac{1}{x} \right\} dx = x + c \text{ pre } x \in (0; 1), \text{ resp. } 1 + \ln x + c \text{ pre } x \in \langle 1; \infty \rangle.$$

$$\boxed{x \in (0; 1)} = \int dx = x + c = x + c, \quad x \in (0; 1), \quad c \in \mathbb{R}.$$

$$\boxed{x \in (1; \infty)} = \int \frac{dx}{x} = \ln x + c_1 = 1 + \ln x + c, \quad x \in \langle 1; \infty \rangle, \quad c, c_1 \in \mathbb{R}.$$

Neurčitý integrál na intervale tvoria spojité primitívne funkcie. Aby uvedené funkcie tvorili neurčitý integrál danej funkcie, musia byť spojité na celom $(0; \infty)$, t. j. aj v bode $x=1$. \Rightarrow Musí platiť $1+c = \ln 1 + c_1$ pre všetky $c, c_1 \in \mathbb{R}$, t. j. $c_1 = 1+c$.

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x = \ln t, dx = \frac{dt}{t} \mid t \in (0; \infty) \end{array} \right]$$

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x = \ln t, dx = \frac{dt}{t} \mid t \in (0; \infty) \end{array} \right]$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t}$$

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + \left(\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2}$$

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right]$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + \left(\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} \\
 &= \frac{1}{4} \int \frac{dt}{\left(t+\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2}
 \end{aligned}$$

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right] \\
 &= \int \frac{2u du}{(u^2-5)u}
 \end{aligned}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x = \ln t, dx = \frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + \left(\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2}$$

$$= \frac{1}{4} \int \frac{dt}{\left(t+\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} = \left[\begin{array}{l} \text{Subst. } u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in \left(\frac{5}{8}; \infty\right) \end{array} \right]$$

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x = \ln t, dx = \frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\begin{array}{l} \text{Subst. } u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right]$$

$$= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + \left(\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2}$$

$$= \frac{1}{4} \int \frac{dt}{\left(t+\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} = \left[\begin{array}{l} \text{Subst. } u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in \left(\frac{5}{8}; \infty\right) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - \left(\frac{5}{8}\right)^2}$$

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\begin{array}{l} \text{Subst. } u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right]$$

$$= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln \left| \frac{u-\sqrt{5}}{u+\sqrt{5}} \right| + C$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + (\frac{5}{8})^2 - (\frac{5}{8})^2}$$

$$= \frac{1}{4} \int \frac{dt}{(t+\frac{5}{8})^2 - (\frac{5}{8})^2} = \left[\begin{array}{l} \text{Subst. } u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in (\frac{5}{8}; \infty) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - (\frac{5}{8})^2} = \frac{1}{4} \cdot \frac{1}{2 \cdot \frac{5}{8}} \ln \frac{u - \frac{5}{8}}{u + \frac{5}{8}} + C_1$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\begin{array}{l} \text{Subst. } u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right]$$

$$= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln \left| \frac{u-\sqrt{5}}{u+\sqrt{5}} \right| + c = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + \left(\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} \\
 &= \frac{1}{4} \int \frac{dt}{\left(t+\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} = \left[\text{Subst. } \begin{array}{l} u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in \left(\frac{5}{8}; \infty\right) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - \left(\frac{5}{8}\right)^2} = \frac{1}{4} \cdot \frac{1}{2 \cdot \frac{5}{8}} \ln \frac{u - \frac{5}{8}}{u + \frac{5}{8}} + C_1 \\
 &= \frac{1}{5} \ln \frac{t + \frac{5}{8} - \frac{5}{8}}{t + \frac{5}{8} + \frac{5}{8}} + C_1
 \end{aligned}$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right] \\
 &= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln \left| \frac{u-\sqrt{5}}{u+\sqrt{5}} \right| + c = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c, x \in \mathbb{R}, c \in \mathbb{R}.
 \end{aligned}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + \left(\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} \\
 &= \frac{1}{4} \int \frac{dt}{\left(t+\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} = \left[\text{Subst. } \begin{array}{l} u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in \left(\frac{5}{8}; \infty\right) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - \left(\frac{5}{8}\right)^2} = \frac{1}{4} \cdot \frac{1}{2 \cdot \frac{5}{8}} \ln \frac{u - \frac{5}{8}}{u + \frac{5}{8}} + C_1 \\
 &= \frac{1}{5} \ln \frac{t + \frac{5}{8} - \frac{5}{8}}{t + \frac{5}{8} + \frac{5}{8}} + C_1 = \frac{1}{5} \ln \frac{t}{t + \frac{5}{4}} + C_1
 \end{aligned}$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right] \\
 &= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln \left| \frac{u-\sqrt{5}}{u+\sqrt{5}} \right| + c = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c, x \in \mathbb{R}, c \in \mathbb{R}.
 \end{aligned}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + \left(\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} \\
 &= \frac{1}{4} \int \frac{dt}{\left(t+\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} = \left[\text{Subst. } \begin{array}{l} u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in \left(\frac{5}{8}; \infty\right) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - \left(\frac{5}{8}\right)^2} = \frac{1}{4} \cdot \frac{1}{2 \cdot \frac{5}{8}} \ln \frac{u - \frac{5}{8}}{u + \frac{5}{8}} + C_1 \\
 &= \frac{1}{5} \ln \frac{t + \frac{5}{8} - \frac{5}{8}}{t + \frac{5}{8} + \frac{5}{8}} + C_1 = \frac{1}{5} \ln \frac{t}{t + \frac{5}{4}} + C_1 = \frac{1}{5} \ln \frac{4t}{4t+5} + C_1
 \end{aligned}$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right] \\
 &= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln \left| \frac{u-\sqrt{5}}{u+\sqrt{5}} \right| + c = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c, x \in \mathbb{R}, c \in \mathbb{R}.
 \end{aligned}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + \left(\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} \\
 &= \frac{1}{4} \int \frac{dt}{\left(t+\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} = \left[\text{Subst. } \begin{array}{l} u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in \left(\frac{5}{8}; \infty\right) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - \left(\frac{5}{8}\right)^2} = \frac{1}{4} \cdot \frac{1}{2 \cdot \frac{5}{8}} \ln \frac{u - \frac{5}{8}}{u + \frac{5}{8}} + C_1 \\
 &= \frac{1}{5} \ln \frac{t + \frac{5}{8} - \frac{5}{8}}{t + \frac{5}{8} + \frac{5}{8}} + C_1 = \frac{1}{5} \ln \frac{t}{t + \frac{5}{4}} + C_1 = \frac{1}{5} \ln \frac{4t}{4t+5} + C_1 = \frac{\ln 4}{5} + \frac{\ln t}{5} - \frac{1}{5} \ln(4t+5) + C_1
 \end{aligned}$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right] \\
 &= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln \left| \frac{u-\sqrt{5}}{u+\sqrt{5}} \right| + c = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c, x \in \mathbb{R}, c \in \mathbb{R}.
 \end{aligned}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + \left(\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} \\
 &= \frac{1}{4} \int \frac{dt}{\left(t+\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2} = \left[\text{Subst. } \begin{array}{l} u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in \left(\frac{5}{8}; \infty\right) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - \left(\frac{5}{8}\right)^2} = \frac{1}{4} \cdot \frac{1}{2 \cdot \frac{5}{8}} \ln \frac{u - \frac{5}{8}}{u + \frac{5}{8}} + C_1 \\
 &= \frac{1}{5} \ln \frac{t + \frac{5}{8} - \frac{5}{8}}{t + \frac{5}{8} + \frac{5}{8}} + C_1 = \frac{1}{5} \ln \frac{t}{t + \frac{5}{4}} + C_1 = \frac{1}{5} \ln \frac{4t}{4t+5} + C_1 = \frac{\ln 4}{5} + \frac{\ln t}{5} - \frac{1}{5} \ln(4t+5) + C_1 \\
 &= \left[c = \frac{\ln 4}{5} + C_1, c \in \mathbb{R}, C_1 \in \mathbb{R} \right]
 \end{aligned}$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right] \\
 &= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln \left| \frac{u-\sqrt{5}}{u+\sqrt{5}} \right| + c = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c, x \in \mathbb{R}, c \in \mathbb{R}.
 \end{aligned}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x} = \frac{x}{5} - \frac{1}{5} \ln(4e^x + 5) + c$$

$$\begin{aligned} &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + (\frac{5}{8})^2 - (\frac{5}{8})^2} \\ &= \frac{1}{4} \int \frac{dt}{(t+\frac{5}{8})^2 - (\frac{5}{8})^2} = \left[\text{Subst. } \begin{array}{l} u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in (\frac{5}{8}; \infty) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - (\frac{5}{8})^2} = \frac{1}{4} \cdot \frac{1}{2 \cdot \frac{5}{8}} \ln \frac{u - \frac{5}{8}}{u + \frac{5}{8}} + c_1 \\ &= \frac{1}{5} \ln \frac{t + \frac{5}{8} - \frac{5}{8}}{t + \frac{5}{8} + \frac{5}{8}} + c_1 = \frac{1}{5} \ln \frac{t}{t + \frac{5}{4}} + c_1 = \frac{1}{5} \ln \frac{4t}{4t+5} + c_1 = \frac{\ln 4}{5} + \frac{\ln t}{5} - \frac{1}{5} \ln(4t+5) + c_1 \\ &= \left[c = \frac{\ln 4}{5} + c_1, c \in \mathbb{R}, c_1 \in \mathbb{R} \right] = \frac{x}{5} - \frac{1}{5} \ln(4e^x + 5) + c, x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c$$

$$\begin{aligned} &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right] \\ &= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln \left| \frac{u-\sqrt{5}}{u+\sqrt{5}} \right| + c = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c, x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x} = \frac{x}{5} - \frac{1}{5} \ln(4e^x + 5) + c$$

$$\begin{aligned} &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + (\frac{5}{8})^2 - (\frac{5}{8})^2} \\ &= \frac{1}{4} \int \frac{dt}{(t+\frac{5}{8})^2 - (\frac{5}{8})^2} = \left[\text{Subst. } \begin{array}{l} u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in (\frac{5}{8}; \infty) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - (\frac{5}{8})^2} = \frac{1}{4} \cdot \frac{1}{2 \cdot \frac{5}{8}} \ln \frac{u - \frac{5}{8}}{u + \frac{5}{8}} + c_1 \\ &= \frac{1}{5} \ln \frac{t + \frac{5}{8} - \frac{5}{8}}{t + \frac{5}{8} + \frac{5}{8}} + c_1 = \frac{1}{5} \ln \frac{t}{t + \frac{5}{4}} + c_1 = \frac{1}{5} \ln \frac{4t}{4t+5} + c_1 = \frac{\ln 4}{5} + \frac{\ln t}{5} - \frac{1}{5} \ln(4t+5) + c_1 \\ &= \left[c = \frac{\ln 4}{5} + c_1, c \in \mathbb{R}, c_1 \in \mathbb{R} \right] = \frac{x}{5} - \frac{1}{5} \ln(4e^x + 5) + c, x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c$$

$$\begin{aligned} &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right] \\ &= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln \left| \frac{u-\sqrt{5}}{u+\sqrt{5}} \right| + c = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c, x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

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$$\int \frac{dx}{5+4e^x} = \frac{x}{5} - \frac{1}{5} \ln(4e^x + 5) + c$$

$$\begin{aligned} &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + (\frac{5}{8})^2 - (\frac{5}{8})^2} \\ &= \frac{1}{4} \int \frac{dt}{(t+\frac{5}{8})^2 - (\frac{5}{8})^2} = \left[\text{Subst. } \begin{array}{l} u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in (\frac{5}{8}; \infty) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - (\frac{5}{8})^2} = \frac{1}{4} \cdot \frac{1}{2 \cdot \frac{5}{8}} \ln \frac{u - \frac{5}{8}}{u + \frac{5}{8}} + c_1 \\ &= \frac{1}{5} \ln \frac{t + \frac{5}{8} - \frac{5}{8}}{t + \frac{5}{8} + \frac{5}{8}} + c_1 = \frac{1}{5} \ln \frac{t}{t + \frac{5}{4}} + c_1 = \frac{1}{5} \ln \frac{4t}{4t+5} + c_1 = \frac{\ln 4}{5} + \frac{\ln t}{5} - \frac{1}{5} \ln(4t+5) + c_1 \\ &= \left[c = \frac{\ln 4}{5} + c_1, c \in \mathbb{R}, c_1 \in \mathbb{R} \right] = \frac{x}{5} - \frac{1}{5} \ln(4e^x + 5) + c, x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c$$

$$\begin{aligned} &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right] \\ &= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln \left| \frac{u-\sqrt{5}}{u+\sqrt{5}} \right| + c = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c, x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x} = \frac{x}{5} - \frac{1}{5} \ln(4e^x + 5) + c$$

$$\begin{aligned} &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + (\frac{5}{8})^2 - (\frac{5}{8})^2} \\ &= \frac{1}{4} \int \frac{dt}{(t+\frac{5}{8})^2 - (\frac{5}{8})^2} = \left[\text{Subst. } \begin{array}{l} u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in (\frac{5}{8}; \infty) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - (\frac{5}{8})^2} = \frac{1}{4} \cdot \frac{1}{2 \cdot \frac{5}{8}} \ln \frac{u - \frac{5}{8}}{u + \frac{5}{8}} + c_1 \\ &= \frac{1}{5} \ln \frac{t + \frac{5}{8} - \frac{5}{8}}{t + \frac{5}{8} + \frac{5}{8}} + c_1 = \frac{1}{5} \ln \frac{t}{t + \frac{5}{4}} + c_1 = \frac{1}{5} \ln \frac{4t}{4t+5} + c_1 = \frac{\ln 4}{5} + \frac{\ln t}{5} - \frac{1}{5} \ln(4t+5) + c_1 \\ &= \left[c = \frac{\ln 4}{5} + c_1, c \in \mathbb{R}, c_1 \in \mathbb{R} \right] = \frac{x}{5} - \frac{1}{5} \ln(4e^x + 5) + c, x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c$$

$$\begin{aligned} &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right] \\ &= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln \left| \frac{u-\sqrt{5}}{u+\sqrt{5}} \right| + c = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c, x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x} = \frac{x}{5} - \frac{1}{5} \ln(4e^x + 5) + c$$

$$\begin{aligned} &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + (\frac{5}{8})^2 - (\frac{5}{8})^2} \\ &= \frac{1}{4} \int \frac{dt}{(t+\frac{5}{8})^2 - (\frac{5}{8})^2} = \left[\text{Subst. } \begin{array}{l} u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in (\frac{5}{8}; \infty) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - (\frac{5}{8})^2} = \frac{1}{4} \cdot \frac{1}{2 \cdot \frac{5}{8}} \ln \frac{u - \frac{5}{8}}{u + \frac{5}{8}} + c_1 \\ &= \frac{1}{5} \ln \frac{t + \frac{5}{8} - \frac{5}{8}}{t + \frac{5}{8} + \frac{5}{8}} + c_1 = \frac{1}{5} \ln \frac{t}{t + \frac{5}{4}} + c_1 = \frac{1}{5} \ln \frac{4t}{4t+5} + c_1 = \frac{\ln 4}{5} + \frac{\ln t}{5} - \frac{1}{5} \ln(4t+5) + c_1 \\ &= \left[c = \frac{\ln 4}{5} + c_1, c \in \mathbb{R}, c_1 \in \mathbb{R} \right] = \frac{x}{5} - \frac{1}{5} \ln(4e^x + 5) + c, x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c$$

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Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x} = \frac{x}{5} - \frac{1}{5} \ln(4e^x + 5) + c$$

$$\begin{aligned} &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4} \int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4} \int \frac{dt}{t^2+2 \cdot \frac{5t}{8} + (\frac{5}{8})^2 - (\frac{5}{8})^2} \\ &= \frac{1}{4} \int \frac{dt}{(t+\frac{5}{8})^2 - (\frac{5}{8})^2} = \left[\text{Subst. } \begin{array}{l} u=t+\frac{5}{8} \mid t \in (0; \infty) \\ du=dt \mid u \in (\frac{5}{8}; \infty) \end{array} \right] = \frac{1}{4} \int \frac{du}{u^2 - (\frac{5}{8})^2} = \frac{1}{4} \cdot \frac{1}{2 \cdot \frac{5}{8}} \ln \frac{u - \frac{5}{8}}{u + \frac{5}{8}} + c_1 \\ &= \frac{1}{5} \ln \frac{t + \frac{5}{8} - \frac{5}{8}}{t + \frac{5}{8} + \frac{5}{8}} + c_1 = \frac{1}{5} \ln \frac{t}{t + \frac{5}{4}} + c_1 = \frac{1}{5} \ln \frac{4t}{4t+5} + c_1 = \frac{\ln 4}{5} + \frac{\ln t}{5} - \frac{1}{5} \ln(4t+5) + c_1 \\ &= \left[c = \frac{\ln 4}{5} + c_1, c \in \mathbb{R}, c_1 \in \mathbb{R} \right] = \frac{x}{5} - \frac{1}{5} \ln(4e^x + 5) + c, x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c$$

$$\begin{aligned} &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4 dt}{4t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u=\sqrt{5+4t}=\sqrt{5+4e^x} \mid t \in (0; \infty) \\ u^2=5+4t, 2u du=4 dt \mid u \in (\sqrt{5}; \infty) \end{array} \right] \\ &= \int \frac{2u du}{(u^2-5)u} = \int \frac{2 du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln \left| \frac{u-\sqrt{5}}{u+\sqrt{5}} \right| + c = \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5+4e^x} - \sqrt{5}}{\sqrt{5+4e^x} + \sqrt{5}} \right| + c, x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x = \ln t, dx = \frac{dt}{t} \mid t \in (0; \infty) \end{array} \right]$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l} \text{Subst. } u^2=4t \mid t \in (0; \infty) \\ 2u du=4 dt \mid u \in (0; \infty) \end{array} \right]$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u^2=4t \mid t \in (0; \infty) \\ 2u du=4 dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4} \sqrt{5+u^2}}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l} \text{Subst. } u^2=4t \mid t \in (0; \infty) \\ 2u du=4 dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4} \sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l} \text{Subst. } u^2=4t \mid t \in (0; \infty) \\ 2u du=4 dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4} \sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l} \text{Subst. } v=\frac{1}{u}, u=\frac{1}{v}=v^{-1} \mid \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} \mid u \in (0; \infty) \\ du=-v^{-2} dv=-\frac{dv}{v^2} \mid v \in (0; \infty) \end{array} \right]$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l} \text{Subst. } u^2=4t \mid t \in (0; \infty) \\ 2udu=4dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l} \text{Subst. } v=\frac{1}{u}, u=\frac{1}{v}=v^{-1} \\ du=-v^{-2}dv=-\frac{dv}{v^2} \end{array} \right] \left[\begin{array}{l} \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} \mid u \in (0; \infty) \\ v \in (0; \infty) \end{array} \right] = \int \frac{-\frac{2 dv}{v^2}}{\frac{1}{v} \frac{\sqrt{5v^2+1}}{v}}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u^2=4t \mid t \in (0; \infty) \\ 2udu=4dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}} \\
 &= \left[\text{Subst. } \begin{array}{l} v=\frac{1}{u}, u=\frac{1}{v}=v^{-1} \\ du=-v^{-2}dv=-\frac{dv}{v^2} \end{array} \mid \begin{array}{l} \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} \mid u \in (0; \infty) \\ v \in (0; \infty) \end{array} \right] = \int \frac{-\frac{2 dv}{v^2}}{\frac{1}{v} \frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2 dv}{\sqrt{5v^2+1}} \\
 &= -\frac{2}{\sqrt{5}} \int \frac{dv}{\sqrt{v^2+\frac{1}{5}}}
 \end{aligned}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u^2=4t \mid t \in (0; \infty) \\ 2udu=4dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}} \\
 &= \left[\text{Subst. } \begin{array}{l} v=\frac{1}{u}, u=\frac{1}{v}=v^{-1} \mid \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} \mid u \in (0; \infty) \\ du=-v^{-2}dv=-\frac{dv}{v^2} \mid v \in (0; \infty) \end{array} \right] = \int \frac{-\frac{2 dv}{v^2}}{\frac{1}{v}\frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2 dv}{\sqrt{5v^2+1}} \\
 &= -\frac{2}{\sqrt{5}} \int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}} \ln \left[v + \sqrt{v^2+\frac{1}{5}} \right] + C_1
 \end{aligned}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$\begin{aligned}
 &= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u^2=4t \mid t \in (0; \infty) \\ 2u du=4 dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4} \sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}} \\
 &= \left[\text{Subst. } \begin{array}{l} v=\frac{1}{u}, u=\frac{1}{v}=v^{-1} \mid \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} \mid u \in (0; \infty) \\ du=-v^{-2} dv=-\frac{dv}{v^2} \mid \sqrt{v^2+\frac{1}{5}}=\sqrt{\frac{1}{u^2}+\frac{1}{5}}=\frac{\sqrt{5+u^2}}{\sqrt{5}u} \mid v \in (0; \infty) \end{array} \right] = \int \frac{-\frac{2 dv}{v^2}}{\frac{1}{v} \frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2 dv}{\sqrt{5v^2+1}} \\
 &= -\frac{2}{\sqrt{5}} \int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}} \ln \left[v + \sqrt{v^2+\frac{1}{5}} \right] + C_1 = -\frac{2}{\sqrt{5}} \ln \left[\frac{1}{u} + \frac{\sqrt{5+u^2}}{\sqrt{5}u} \right] + C_1
 \end{aligned}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\text{Subst. } \begin{array}{l} t = e^x \mid x \in \mathbb{R} \\ x = \ln t, dx = \frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u^2 = 4t \mid t \in (0; \infty) \\ 2u du = 4 dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4} \sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}}$$

$$= \left[\text{Subst. } \begin{array}{l} v = \frac{1}{u}, u = \frac{1}{v} = v^{-1} \mid \sqrt{5+u^2} = \sqrt{5+\frac{1}{v^2}} = \frac{\sqrt{5v^2+1}}{v} \mid u \in (0; \infty) \\ du = -v^{-2} dv = -\frac{dv}{v^2} \mid \sqrt{v^2+\frac{1}{5}} = \sqrt{\frac{1}{u^2}+\frac{1}{5}} = \frac{\sqrt{5+u^2}}{\sqrt{5}u} \mid v \in (0; \infty) \end{array} \right] = \int \frac{-\frac{2 dv}{v^2}}{\frac{1}{v} \frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2 dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}} \int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}} \ln \left[v + \sqrt{v^2+\frac{1}{5}} \right] + C_1 = -\frac{2}{\sqrt{5}} \ln \left[\frac{1}{u} + \frac{\sqrt{5+u^2}}{\sqrt{5}u} \right] + C_1$$

$$= -\frac{2}{\sqrt{5}} \ln \frac{\sqrt{5+\sqrt{5+u^2}}}{\sqrt{5}u} + C_1$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u^2=4t \mid t \in (0; \infty) \\ 2udu=4dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}}$$

$$= \left[\text{Subst. } \begin{array}{l} v=\frac{1}{u}, u=\frac{1}{v}=v^{-1} \mid \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} \mid u \in (0; \infty) \\ du=-v^{-2}dv=-\frac{dv}{v^2} \mid \sqrt{v^2+\frac{1}{5}}=\sqrt{\frac{1}{u^2}+\frac{1}{5}}=\frac{\sqrt{5+u^2}}{\sqrt{5}u} \mid v \in (0; \infty) \end{array} \right] = \int \frac{-\frac{2 dv}{v^2}}{\frac{1}{v}\frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2 dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}} \int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}} \ln \left[v + \sqrt{v^2+\frac{1}{5}} \right] + c_1 = -\frac{2}{\sqrt{5}} \ln \left[\frac{1}{u} + \frac{\sqrt{5+u^2}}{\sqrt{5}u} \right] + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln \frac{\sqrt{5}+\sqrt{5+u^2}}{\sqrt{5}u} + c_1 = -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+u^2}) + \frac{2}{\sqrt{5}} \ln \sqrt{5} + \frac{2}{\sqrt{5}} \ln u + c_1$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u^2=4t \mid t \in (0; \infty) \\ 2u du=4 dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4} \sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}}$$

$$= \left[\text{Subst. } \begin{array}{l} v=\frac{1}{u}, u=\frac{1}{v}=v^{-1} \mid \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} \mid u \in (0; \infty) \\ du=-v^{-2} dv=-\frac{dv}{v^2} \mid \sqrt{v^2+\frac{1}{5}}=\sqrt{\frac{1}{u^2}+\frac{1}{5}}=\frac{\sqrt{5+u^2}}{\sqrt{5}u} \mid v \in (0; \infty) \end{array} \right] = \int \frac{-\frac{2 dv}{v^2}}{\frac{1}{v} \frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2 dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}} \int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}} \ln \left[v + \sqrt{v^2+\frac{1}{5}} \right] + c_1 = -\frac{2}{\sqrt{5}} \ln \left[\frac{1}{u} + \frac{\sqrt{5+u^2}}{\sqrt{5}u} \right] + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln \frac{\sqrt{5}+\sqrt{5+u^2}}{\sqrt{5}u} + c_1 = -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+u^2}) + \frac{2}{\sqrt{5}} \ln \sqrt{5} + \frac{2}{\sqrt{5}} \ln u + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+u^2}) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln u^2}{\sqrt{5}} + c_1$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid x \in \mathbb{R} \\ x=\ln t, dx=\frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u^2=4t \mid t \in (0; \infty) \\ 2u du=4 dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4} \sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}}$$

$$= \left[\text{Subst. } \begin{array}{l} v=\frac{1}{u}, u=\frac{1}{v}=v^{-1} \mid \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} \mid u \in (0; \infty) \\ du=-v^{-2} dv=-\frac{dv}{v^2} \mid \sqrt{v^2+\frac{1}{5}}=\sqrt{\frac{1}{u^2}+\frac{1}{5}}=\frac{\sqrt{5+u^2}}{\sqrt{5}u} \mid v \in (0; \infty) \end{array} \right] = \int \frac{-\frac{2 dv}{v^2}}{\frac{1}{v} \frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2 dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}} \int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}} \ln \left[v + \sqrt{v^2+\frac{1}{5}} \right] + c_1 = -\frac{2}{\sqrt{5}} \ln \left[\frac{1}{u} + \frac{\sqrt{5+u^2}}{\sqrt{5}u} \right] + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln \frac{\sqrt{5}+\sqrt{5+u^2}}{\sqrt{5}u} + c_1 = -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+u^2}) + \frac{2}{\sqrt{5}} \ln \sqrt{5} + \frac{2}{\sqrt{5}} \ln u + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+u^2}) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln u^2}{\sqrt{5}} + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+4t}) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln(4t)}{\sqrt{5}} + c_1$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\text{Subst. } \begin{array}{l} t = e^x \mid x \in \mathbb{R} \\ x = \ln t, dx = \frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u^2 = 4t \mid t \in (0; \infty) \\ 2u du = 4 dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4} \sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}}$$

$$= \left[\text{Subst. } \begin{array}{l} v = \frac{1}{u}, u = \frac{1}{v} = v^{-1} \mid \sqrt{5+u^2} = \sqrt{5+\frac{1}{v^2}} = \frac{\sqrt{5v^2+1}}{v} \mid u \in (0; \infty) \\ du = -v^{-2} dv = -\frac{dv}{v^2} \mid \sqrt{v^2+\frac{1}{5}} = \sqrt{\frac{1}{u^2}+\frac{1}{5}} = \frac{\sqrt{5+u^2}}{\sqrt{5}u} \mid v \in (0; \infty) \end{array} \right] = \int \frac{-\frac{2 dv}{v^2}}{\frac{1}{v} \frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2 dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}} \int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}} \ln \left[v + \sqrt{v^2+\frac{1}{5}} \right] + c_1 = -\frac{2}{\sqrt{5}} \ln \left[\frac{1}{u} + \frac{\sqrt{5+u^2}}{\sqrt{5}u} \right] + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln \frac{\sqrt{5}+\sqrt{5+u^2}}{\sqrt{5}u} + c_1 = -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+u^2}) + \frac{2}{\sqrt{5}} \ln \sqrt{5} + \frac{2}{\sqrt{5}} \ln u + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+u^2}) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln u^2}{\sqrt{5}} + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+4t}) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln(4t)}{\sqrt{5}} + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+4t}) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln 4}{\sqrt{5}} + \frac{\ln t}{\sqrt{5}} + c_1$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}}$$

$$= \left[\text{Subst. } \begin{array}{l} t = e^x \mid x \in \mathbb{R} \\ x = \ln t, dx = \frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u^2 = 4t \mid t \in (0; \infty) \\ 2u du = 4 dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4} \sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}}$$

$$= \left[\text{Subst. } \begin{array}{l} v = \frac{1}{u}, u = \frac{1}{v} = v^{-1} \mid \sqrt{5+u^2} = \sqrt{5+\frac{1}{v^2}} = \frac{\sqrt{5v^2+1}}{v} \mid u \in (0; \infty) \\ du = -v^{-2} dv = -\frac{dv}{v^2} \mid \sqrt{v^2+\frac{1}{5}} = \sqrt{\frac{1}{u^2}+\frac{1}{5}} = \frac{\sqrt{5+u^2}}{\sqrt{5}u} \mid v \in (0; \infty) \end{array} \right] = \int \frac{-\frac{2 dv}{v^2}}{\frac{1}{v} \frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2 dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}} \int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}} \ln \left[v + \sqrt{v^2+\frac{1}{5}} \right] + c_1 = -\frac{2}{\sqrt{5}} \ln \left[\frac{1}{u} + \frac{\sqrt{5+u^2}}{\sqrt{5}u} \right] + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln \frac{\sqrt{5}+\sqrt{5+u^2}}{\sqrt{5}u} + c_1 = -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+u^2}) + \frac{2}{\sqrt{5}} \ln \sqrt{5} + \frac{2}{\sqrt{5}} \ln u + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+u^2}) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln u^2}{\sqrt{5}} + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+4t}) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln(4t)}{\sqrt{5}} + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln (\sqrt{5}+\sqrt{5+4t}) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln 4}{\sqrt{5}} + \frac{\ln t}{\sqrt{5}} + c_1 = \left[c = \frac{\ln 5}{\sqrt{5}} + \frac{\ln 4}{\sqrt{5}} + c_1, c \in \mathbb{R}, c_1 \in \mathbb{R} \right]$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{x}{\sqrt{5}} - \frac{2}{\sqrt{5}} \ln(\sqrt{5} + \sqrt{5+4e^x}) + c$$

$$= \left[\text{Subst. } \begin{array}{l} t = e^x \mid x \in \mathbb{R} \\ x = \ln t, dx = \frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\text{Subst. } \begin{array}{l} u^2 = 4t \mid t \in (0; \infty) \\ 2u du = 4dt \mid u \in (0; \infty) \end{array} \right] = \int \frac{\frac{u du}{2}}{\frac{u^2}{4} \sqrt{5+u^2}} = \int \frac{2 du}{u\sqrt{5+u^2}}$$

$$= \left[\text{Subst. } \begin{array}{l} v = \frac{1}{u}, u = \frac{1}{v} = v^{-1} \mid \sqrt{5+u^2} = \sqrt{5+\frac{1}{v^2}} = \frac{\sqrt{5v^2+1}}{v} \mid u \in (0; \infty) \\ du = -v^{-2} dv = -\frac{dv}{v^2} \mid \sqrt{v^2+\frac{1}{5}} = \sqrt{\frac{1}{u^2}+\frac{1}{5}} = \frac{\sqrt{5+u^2}}{\sqrt{5}u} \mid v \in (0; \infty) \end{array} \right] = \int \frac{-\frac{2 dv}{v^2}}{\frac{1}{v} \frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2 dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}} \int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}} \ln \left[v + \sqrt{v^2+\frac{1}{5}} \right] + c_1 = -\frac{2}{\sqrt{5}} \ln \left[\frac{1}{u} + \frac{\sqrt{5+u^2}}{\sqrt{5}u} \right] + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln \frac{\sqrt{5} + \sqrt{5+u^2}}{\sqrt{5}u} + c_1 = -\frac{2}{\sqrt{5}} \ln(\sqrt{5} + \sqrt{5+u^2}) + \frac{2}{\sqrt{5}} \ln \sqrt{5} + \frac{2}{\sqrt{5}} \ln u + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln(\sqrt{5} + \sqrt{5+u^2}) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln u^2}{\sqrt{5}} + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln(\sqrt{5} + \sqrt{5+4t}) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln(4t)}{\sqrt{5}} + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln(\sqrt{5} + \sqrt{5+4t}) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln 4}{\sqrt{5}} + \frac{\ln t}{\sqrt{5}} + c_1 = \left[c = \frac{\ln 5}{\sqrt{5}} + \frac{\ln 4}{\sqrt{5}} + c_1, c \in \mathbb{R}, c_1 \in \mathbb{R} \right]$$

$$= -\frac{2}{\sqrt{5}} \ln(\sqrt{5} + \sqrt{5+4e^x}) + \frac{x}{\sqrt{5}} + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x} + e^x + 1}}$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x} + e^x + 1}}$$

$$= \left[\begin{array}{l} \text{Subst. } t = e^x \mid x \in \mathbb{R} \\ x = \ln t, dx = \frac{dt}{t} \mid t \in (0; \infty) \end{array} \right]$$

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Riešené príklady – 095

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Riešené príklady – 095

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$$= - \int \frac{du}{\sqrt{(u + \frac{1}{2})^2 + 1 - \frac{1}{4}}} = - \int \frac{du}{\sqrt{(u + \frac{1}{2})^2 + \frac{3}{4}}}$$

Riešené príklady – 095

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Riešené príklady – 095

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Riešené príklady – 095

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Riešené príklady – 095

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Riešené príklady – 095

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$$= - \ln \left(u + \frac{1}{2} + \sqrt{1 + u + u^2} \right) + c_1 = - \ln \left(\frac{1}{t} + \frac{1}{2} + \frac{\sqrt{t^2 + t + 1}}{t} \right) + c_1$$

Riešené príklady – 095

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$$= - \ln \left(u + \frac{1}{2} + \sqrt{1 + u + u^2} \right) + c_1 = - \ln \left(\frac{1}{t} + \frac{1}{2} + \frac{\sqrt{t^2 + t + 1}}{t} \right) + c_1$$

$$= - \ln \frac{2+t+2\sqrt{t^2+t+1}}{2t} + c_1$$

Riešené príklady – 095

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$$= - \ln \frac{2+t+2\sqrt{t^2+t+1}}{2t} + c_1 = - \ln(2+t+2\sqrt{t^2+t+1}) + \ln 2 + \ln t + c_1$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x} + e^x + 1}}$$

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$$= - \ln \left(u + \frac{1}{2} + \sqrt{1 + u + u^2} \right) + c_1 = - \ln \left(\frac{1}{t} + \frac{1}{2} + \frac{\sqrt{t^2 + t + 1}}{t} \right) + c_1$$

$$= - \ln \frac{2+t+2\sqrt{t^2+t+1}}{2t} + c_1 = - \ln(2+t+2\sqrt{t^2+t+1}) + \ln 2 + \ln t + c_1$$

$$= \left[\begin{array}{l} c = c_1 + \ln 2 \\ c \in \mathbb{R}, c_1 \in \mathbb{R} \end{array} \right]$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x} + e^x + 1}} = x - \ln(2 + e^x + 2\sqrt{e^{2x} + e^x + 1}) + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid x \in \mathbb{R} \\ x = \ln t, dx = \frac{dt}{t} \mid t \in (0; \infty) \end{array} \right] = \int \frac{\frac{dt}{t}}{\sqrt{t^2+t+1}} = \int \frac{dt}{t\sqrt{t^2+t+1}}$$

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$$= - \ln \left(v + \sqrt{v^2 + \frac{3}{4}} \right) + c_1 = - \ln \left(u + \frac{1}{2} + \sqrt{(u + \frac{1}{2})^2 + \frac{3}{4}} \right) + c_1$$

$$= - \ln \left(u + \frac{1}{2} + \sqrt{1 + u + u^2} \right) + c_1 = - \ln \left(\frac{1}{t} + \frac{1}{2} + \frac{\sqrt{t^2+t+1}}{t} \right) + c_1$$

$$= - \ln \frac{2+t+2\sqrt{t^2+t+1}}{2t} + c_1 = - \ln(2+t+2\sqrt{t^2+t+1}) + \ln 2 + \ln t + c_1$$

$$= \left[\begin{array}{l} c = c_1 + \ln 2 \\ c \in \mathbb{R}, c_1 \in \mathbb{R} \end{array} \right] = x - \ln(2 + e^x + 2\sqrt{e^{2x} + e^x + 1}) + c, x \in \mathbb{R}, c \in \mathbb{R}.$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ x = \ln t, dx = \frac{dt}{t} \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right]$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ x = \ln t, dx = \frac{dt}{t} \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}} \frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \frac{dt}{t}$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ x = \ln t, dx = \frac{dt}{t} \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}} \frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \frac{dt}{t}$$

$$= \left[\text{Subst. } u = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid u^2 = \frac{1-t}{1+t}, u^2 + u^2 t = 1-t, t = \frac{1-u^2}{1+u^2} \mid t \in (0; 1) \right. \\ \left. dt = \frac{-2u(1+u^2) - 2u(1-u^2)}{(1+u^2)^2} du = \frac{-4u du}{(1+u^2)^2} \mid u \in (0; 1) \right]$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ x=\ln t, dx=\frac{dt}{t} \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}} \frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \frac{dt}{t}$$

$$= \left[\text{Subst. } \begin{array}{l} u=\sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t}-1} \mid u^2 = \frac{1-t}{1+t}, u^2+u^2t=1-t, t=\frac{1-u^2}{1+u^2} \mid t \in (0; 1) \\ dt = \frac{-2u(1+u^2)-2u(1-u^2)}{(1+u^2)^2} du = \frac{-4u du}{(1+u^2)^2} \mid u \in (0; 1) \end{array} \right] = \int u \cdot \frac{-4u du}{\frac{(1+u^2)^2}{1+u^2}}$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ x = \ln t, dx = \frac{dt}{t} \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}} \frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \frac{dt}{t}$$

$$= \left[\text{Subst. } u = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid u^2 = \frac{1-t}{1+t}, u^2 + u^2 t = 1-t, t = \frac{1-u^2}{1+u^2} \mid t \in (0; 1) \right. \\ \left. dt = \frac{-2u(1+u^2) - 2u(1-u^2)}{(1+u^2)^2} du = \frac{-4u du}{(1+u^2)^2} \mid u \in (0; 1) \right] = \int u \cdot \frac{-4u du}{(1+u^2)^2} \frac{1}{1+u^2}$$

$$= \int \frac{-4u^2 du}{(1+u^2)(1-u^2)} = \int \frac{4u^2 du}{(1+u^2)(u^2-1)}$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ x = \ln t, dx = \frac{dt}{t} \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}} \frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \frac{dt}{t}$$

$$= \left[\text{Subst. } u = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid u^2 = \frac{1-t}{1+t}, u^2 + u^2 t = 1-t, t = \frac{1-u^2}{1+u^2} \mid t \in (0; 1) \right. \\ \left. dt = \frac{-2u(1+u^2) - 2u(1-u^2)}{(1+u^2)^2} du = \frac{-4u du}{(1+u^2)^2} \mid u \in (0; 1) \right] = \int u \cdot \frac{-4u du}{(1+u^2)^2} \frac{1}{1+u^2}$$

$$= \int \frac{-4u^2 du}{(1+u^2)(1-u^2)} = \int \frac{4u^2 du}{(1+u^2)(u^2-1)} = \int \frac{(2u^2-2+2u^2+2) du}{(u^2-1)(u^2+1)}$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ x = \ln t, dx = \frac{dt}{t} \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}} \frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \frac{dt}{t}$$

$$= \left[\text{Subst. } \begin{array}{l} u = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid u^2 = \frac{1-t}{1+t}, u^2 + u^2 t = 1-t, t = \frac{1-u^2}{1+u^2} \mid t \in (0; 1) \\ dt = \frac{-2u(1+u^2) - 2u(1-u^2)}{(1+u^2)^2} du = \frac{-4u du}{(1+u^2)^2} \mid u \in (0; 1) \end{array} \right] = \int u \cdot \frac{\frac{-4u du}{(1+u^2)^2}}{\frac{1-u^2}{1+u^2}}$$

$$= \int \frac{-4u^2 du}{(1+u^2)(1-u^2)} = \int \frac{4u^2 du}{(1+u^2)(u^2-1)} = \int \frac{(2u^2-2+2u^2+2) du}{(u^2-1)(u^2+1)} = \int \left[\frac{2}{u^2+1} + \frac{2}{u^2-1} \right] du$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ x = \ln t, dx = \frac{dt}{t} \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}} \frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \frac{dt}{t}$$

$$= \left[\text{Subst. } \begin{array}{l} u = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid u^2 = \frac{1-t}{1+t}, u^2 + u^2 t = 1-t, t = \frac{1-u^2}{1+u^2} \mid t \in (0; 1) \\ dt = \frac{-2u(1+u^2) - 2u(1-u^2)}{(1+u^2)^2} du = \frac{-4u du}{(1+u^2)^2} \mid u \in (0; 1) \end{array} \right] = \int u \cdot \frac{\frac{-4u du}{(1+u^2)^2}}{\frac{1-u^2}{1+u^2}}$$

$$= \int \frac{-4u^2 du}{(1+u^2)(1-u^2)} = \int \frac{4u^2 du}{(1+u^2)(u^2-1)} = \int \frac{(2u^2-2+2u^2+2) du}{(u^2-1)(u^2+1)} = \int \left[\frac{2}{u^2+1} + \frac{2}{u^2-1} \right] du$$

$$= 2 \operatorname{arctg} u + \frac{2}{2} \ln \left| \frac{u-1}{u+1} \right| + c$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ x=\ln t, dx=\frac{dt}{t} \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}} \frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \frac{dt}{t}$$

$$= \left[\text{Subst. } u = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid u^2 = \frac{1-t}{1+t}, u^2 + u^2 t = 1-t, t = \frac{1-u^2}{1+u^2} \mid t \in (0; 1) \right. \\ \left. dt = \frac{-2u(1+u^2) - 2u(1-u^2)}{(1+u^2)^2} du = \frac{-4u du}{(1+u^2)^2} \mid 1 \pm u = 1 \pm \frac{\sqrt{1-t}}{\sqrt{1+t}} = \frac{\sqrt{1+t} \pm \sqrt{1-t}}{\sqrt{1+t}} > 0 \mid u \in (0; 1) \right. \\ \left. \frac{1-u}{1+u} = \frac{\sqrt{1+t} - \sqrt{1-t}}{\sqrt{1+t} + \sqrt{1-t}} = \frac{\sqrt{1+t} - \sqrt{1-t}}{\sqrt{1+t} + \sqrt{1-t}} \cdot \frac{\sqrt{1+t} - \sqrt{1-t}}{\sqrt{1+t} - \sqrt{1-t}} = \frac{1+t - 2\sqrt{1-t^2} + 1-t}{1+t - (1-t)} = \frac{2 - 2\sqrt{1-t^2}}{2t} = \frac{1 - \sqrt{1-t^2}}{t} > 0 \right] = \int u \cdot \frac{-4u du}{\frac{(1+u^2)^2}{\frac{1-u^2}{1+u^2}}}$$

$$= \int \frac{-4u^2 du}{(1+u^2)(1-u^2)} = \int \frac{4u^2 du}{(1+u^2)(u^2-1)} = \int \frac{(2u^2-2+2u^2+2) du}{(u^2-1)(u^2+1)} = \int \left[\frac{2}{u^2+1} + \frac{2}{u^2-1} \right] du$$

$$= 2 \operatorname{arctg} u + \frac{2}{2} \ln \left| \frac{u-1}{u+1} \right| + c = 2 \operatorname{arctg} \sqrt{\frac{1-t}{1+t}} + \ln \frac{1-\sqrt{1-t^2}}{t} + c$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\text{Subst. } \begin{array}{l} t=e^x \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ x=\ln t, dx=\frac{dt}{t} \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}} \frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \frac{dt}{t}$$

$$= \left[\text{Subst. } \begin{array}{l} u = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid u^2 = \frac{1-t}{1+t}, u^2 + u^2 t = 1-t, t = \frac{1-u^2}{1+u^2} \mid t \in (0; 1) \\ dt = \frac{-2u(1+u^2) - 2u(1-u^2)}{(1+u^2)^2} du = \frac{-4u du}{(1+u^2)^2} \mid 1 \pm u = 1 \pm \frac{\sqrt{1-t}}{\sqrt{1+t}} = \frac{\sqrt{1+t} \pm \sqrt{1-t}}{\sqrt{1+t}} > 0 \mid u \in (0; 1) \\ \frac{1-u}{1+u} = \frac{\sqrt{1+t} - \sqrt{1-t}}{\sqrt{1+t} + \sqrt{1-t}} = \frac{\sqrt{1+t} - \sqrt{1-t}}{\sqrt{1+t} + \sqrt{1-t}} \cdot \frac{\sqrt{1+t} - \sqrt{1-t}}{\sqrt{1+t} - \sqrt{1-t}} = \frac{1+t - 2\sqrt{1-t^2} + 1-t}{1+t - (1-t)} = \frac{2 - 2\sqrt{1-t^2}}{2t} = \frac{1 - \sqrt{1-t^2}}{t} > 0 \end{array} \right] = \int u \cdot \frac{\frac{-4u du}{(1+u^2)^2}}{\frac{1-u^2}{1+u^2}}$$

$$= \int \frac{-4u^2 du}{(1+u^2)(1-u^2)} = \int \frac{4u^2 du}{(1+u^2)(u^2-1)} = \int \frac{(2u^2 - 2 + 2u^2 + 2) du}{(u^2-1)(u^2+1)} = \int \left[\frac{2}{u^2+1} + \frac{2}{u^2-1} \right] du$$

$$= 2 \operatorname{arctg} u + \frac{2}{2} \ln \left| \frac{u-1}{u+1} \right| + c = 2 \operatorname{arctg} \sqrt{\frac{1-t}{1+t}} + \ln \frac{1-\sqrt{1-t^2}}{t} + c$$

$$= 2 \operatorname{arctg} \sqrt{\frac{1-t}{1+t}} + \ln(1 - \sqrt{1-t^2}) - \ln t + c$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx = 2 \operatorname{arctg} \sqrt{\frac{1-e^x}{1+e^x}} + \ln(1 - \sqrt{1-e^{2x}}) - x + c$$

$$= \left[\begin{array}{l} \text{Subst. } t=e^x \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ x = \ln t, dx = \frac{dt}{t} \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}} \frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}} \frac{dt}{t}$$

$$= \left[\begin{array}{l} \text{Subst. } u = \sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{2-1-t}{1+t}} = \sqrt{\frac{2}{1+t} - 1} \mid u^2 = \frac{1-t}{1+t}, u^2 + u^2 t = 1-t, t = \frac{1-u^2}{1+u^2} \mid t \in (0; 1) \\ dt = \frac{-2u(1+u^2) - 2u(1-u^2)}{(1+u^2)^2} du = \frac{-4u du}{(1+u^2)^2} \mid 1 \pm u = 1 \pm \frac{\sqrt{1-t}}{\sqrt{1+t}} = \frac{\sqrt{1+t} \pm \sqrt{1-t}}{\sqrt{1+t}} > 0 \mid u \in (0; 1) \\ \frac{1-u}{1+u} = \frac{\sqrt{1+t} - \sqrt{1-t}}{\sqrt{1+t} + \sqrt{1-t}} = \frac{\sqrt{1+t} - \sqrt{1-t}}{\sqrt{1+t} + \sqrt{1-t}} \cdot \frac{\sqrt{1+t} - \sqrt{1-t}}{\sqrt{1+t} - \sqrt{1-t}} = \frac{1+t - 2\sqrt{1-t^2} + 1-t}{1+t - (1-t)} = \frac{2 - 2\sqrt{1-t^2}}{2t} = \frac{1 - \sqrt{1-t^2}}{t} > 0 \end{array} \right] = \int u \cdot \frac{-4u du}{(1+u^2)^2}$$

$$= \int \frac{-4u^2 du}{(1+u^2)(1-u^2)} = \int \frac{4u^2 du}{(1+u^2)(u^2-1)} = \int \frac{(2u^2-2+2u^2+2) du}{(u^2-1)(u^2+1)} = \int \left[\frac{2}{u^2+1} + \frac{2}{u^2-1} \right] du$$

$$= 2 \operatorname{arctg} u + \frac{2}{2} \ln \left| \frac{u-1}{u+1} \right| + c = 2 \operatorname{arctg} \sqrt{\frac{1-t}{1+t}} + \ln \frac{1 - \sqrt{1-t^2}}{t} + c$$

$$= 2 \operatorname{arctg} \sqrt{\frac{1-t}{1+t}} + \ln(1 - \sqrt{1-t^2}) - \ln t + c$$

$$= 2 \operatorname{arctg} \sqrt{\frac{1-e^x}{1+e^x}} + \ln(1 - \sqrt{1-e^{2x}}) - x + c, x \in (-\infty; 0), c \in \mathbb{R}.$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{\frac{1-e^x}{1+e^x}} = \sqrt{\frac{2-1-e^x}{1+e^x}} = \sqrt{\frac{2}{1+e^x}} - 1 \quad \left| \begin{array}{l} t^2(1+e^x) = 1-e^x \Rightarrow e^x = \frac{1-t^2}{1+t^2} \\ 1+e^x > 0 \text{ pre } x \in \mathbb{R} \end{array} \right. \quad \left. \begin{array}{l} x \in (-\infty; 0) \\ 1-e^x \geq 0 \text{ pre } x \leq 0 \end{array} \right| t \in (0; 1) \\ dx = \left[\frac{-2t}{1-t^2} - \frac{2t}{1+t^2} \right] dt = \left[\frac{2t}{t^2-1} - \frac{2t}{t^2+1} \right] dt \quad \left| x = \ln \frac{1-t^2}{1+t^2} = \ln(1-t^2) - \ln(1+t^2) \right. \end{array} \right]$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{\frac{1-e^x}{1+e^x}} = \sqrt{\frac{2-1-e^x}{1+e^x}} = \sqrt{\frac{2}{1+e^x}} - 1 \quad \left| \begin{array}{l} t^2(1+e^x) = 1-e^x \Rightarrow e^x = \frac{1-t^2}{1+t^2} \\ 1+e^x > 0 \text{ pre } x \in \mathbb{R} \end{array} \right. \quad \left. \begin{array}{l} x \in (-\infty; 0) \\ 1-e^x \geq 0 \text{ pre } x \leq 0 \end{array} \right| \\ dx = \left[\frac{-2t}{1-t^2} - \frac{2t}{1+t^2} \right] dt = \left[\frac{2t}{t^2-1} - \frac{2t}{t^2+1} \right] dt \quad \left| \begin{array}{l} x = \ln \frac{1-t^2}{1+t^2} = \ln(1-t^2) - \ln(1+t^2) \\ 1-e^x \geq 0 \text{ pre } x \leq 0 \end{array} \right. \quad \left. \begin{array}{l} t \in (0; 1) \end{array} \right] \end{array} \right.$$

$$= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1} \right] dt$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{\frac{1-e^x}{1+e^x}} = \sqrt{\frac{2-1-e^x}{1+e^x}} = \sqrt{\frac{2}{1+e^x} - 1} \mid t^2(1+e^x) = 1-e^x \Rightarrow e^x = \frac{1-t^2}{1+t^2} \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ dx = \left[\frac{-2t}{1-t^2} - \frac{2t}{1+t^2} \right] dt = \left[\frac{2t}{t^2-1} - \frac{2t}{t^2+1} \right] dt \mid x = \ln \frac{1-t^2}{1+t^2} = \ln(1-t^2) - \ln(1+t^2) \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right]$$

$$= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1} \right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1} \right] dt$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{\frac{1-e^x}{1+e^x}} = \sqrt{\frac{2-1-e^x}{1+e^x}} = \sqrt{\frac{2}{1+e^x}} - 1 \quad t^2(1+e^x) = 1-e^x \Rightarrow e^x = \frac{1-t^2}{1+t^2} \quad \left. \begin{array}{l} 1+e^x > 0 \text{ pre } x \in \mathbb{R} \\ x \in (-\infty; 0) \end{array} \right\} \\ dx = \left[\frac{-2t}{1-t^2} - \frac{2t}{1+t^2} \right] dt = \left[\frac{2t}{t^2-1} - \frac{2t}{t^2+1} \right] dt \quad \left. \begin{array}{l} x = \ln \frac{1-t^2}{1+t^2} = \ln(1-t^2) - \ln(1+t^2) \\ 1-e^x \geq 0 \text{ pre } x \leq 0 \\ t \in (0; 1) \end{array} \right\} \end{array} \right]$$

$$= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1} \right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1} \right] dt$$

$$= \int \left[2 + \frac{2}{t^2-1} - 2 + \frac{2}{t^2+1} \right] dt$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{\frac{1-e^x}{1+e^x}} = \sqrt{\frac{2-1-e^x}{1+e^x}} = \sqrt{\frac{2}{1+e^x} - 1} \mid t^2(1+e^x) = 1-e^x \Rightarrow e^x = \frac{1-t^2}{1+t^2} \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ dx = \left[\frac{-2t}{1-t^2} - \frac{2t}{1+t^2} \right] dt = \left[\frac{2t}{t^2-1} - \frac{2t}{t^2+1} \right] dt \mid x = \ln \frac{1-t^2}{1+t^2} = \ln(1-t^2) - \ln(1+t^2) \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right]$$

$$= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1} \right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1} \right] dt$$

$$= \int \left[2 + \frac{2}{t^2-1} - 2 + \frac{2}{t^2+1} \right] dt = 2 \int \frac{dt}{t^2-1} + 2 \int \frac{dt}{t^2+1}$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{\frac{1-e^x}{1+e^x}} = \sqrt{\frac{2-1-e^x}{1+e^x}} = \sqrt{\frac{2}{1+e^x} - 1} \mid t^2(1+e^x) = 1-e^x \Rightarrow e^x = \frac{1-t^2}{1+t^2} \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \\ dx = \left[\frac{-2t}{1-t^2} - \frac{2t}{1+t^2} \right] dt = \left[\frac{2t}{t^2-1} - \frac{2t}{t^2+1} \right] dt \mid x = \ln \frac{1-t^2}{1+t^2} = \ln(1-t^2) - \ln(1+t^2) \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \end{array} \right]$$

$$= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1} \right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1} \right] dt$$

$$= \int \left[2 + \frac{2}{t^2-1} - 2 + \frac{2}{t^2+1} \right] dt = 2 \int \frac{dt}{t^2-1} + 2 \int \frac{dt}{t^2+1}$$

$$= \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + 2 \operatorname{arctg} t + c = \frac{2}{2} \ln \frac{1-t}{1+t} + 2 \operatorname{arctg} t + c$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \sqrt{\frac{1-e^x}{1+e^x}} = \sqrt{\frac{2-1-e^x}{1+e^x}} = \sqrt{\frac{2}{1+e^x}} - 1 \mid t^2(1+e^x) = 1-e^x \Rightarrow e^x = \frac{1-t^2}{1+t^2} \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \right. \\
 &\quad \left. dx = \left[\frac{-2t}{1-t^2} - \frac{2t}{1+t^2} \right] dt = \left[\frac{2t}{t^2-1} - \frac{2t}{t^2+1} \right] dt \mid x = \ln \frac{1-t^2}{1+t^2} = \ln(1-t^2) - \ln(1+t^2) \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \right] \\
 &= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1} \right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1} \right] dt \\
 &= \int \left[2 + \frac{2}{t^2-1} - 2 + \frac{2}{t^2+1} \right] dt = 2 \int \frac{dt}{t^2-1} + 2 \int \frac{dt}{t^2+1} \\
 &= \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + 2 \operatorname{arctg} t + c = \frac{2}{2} \ln \frac{1-t}{1+t} + 2 \operatorname{arctg} t + c \\
 &= \left[\frac{1-t}{1+t} = \frac{1-\sqrt{\frac{1-e^x}{1+e^x}}}{1+\sqrt{\frac{1-e^x}{1+e^x}}} = \frac{1-\sqrt{1-e^x}}{\sqrt{1+e^x}} = \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}+\sqrt{1+e^x}} = \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}+\sqrt{1+e^x}} = \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}-\sqrt{1-e^x}} = \frac{(1+e^x)-2\sqrt{1+e^x}\sqrt{1-e^x}+(1-e^x)}{(1+e^x)-(1-e^x)} = \frac{2-2\sqrt{1-e^{2x}}}{2e^x} \right]
 \end{aligned}$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \sqrt{\frac{1-e^x}{1+e^x}} = \sqrt{\frac{2-1-e^x}{1+e^x}} = \sqrt{\frac{2}{1+e^x} - 1} \mid t^2(1+e^x) = 1-e^x \Rightarrow e^x = \frac{1-t^2}{1+t^2} \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \right. \\
 &\quad \left. dx = \left[\frac{-2t}{1-t^2} - \frac{2t}{1+t^2} \right] dt = \left[\frac{2t}{t^2-1} - \frac{2t}{t^2+1} \right] dt \mid x = \ln \frac{1-t^2}{1+t^2} = \ln(1-t^2) - \ln(1+t^2) \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \right] \\
 &= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1} \right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1} \right] dt \\
 &= \int \left[2 + \frac{2}{t^2-1} - 2 + \frac{2}{t^2+1} \right] dt = 2 \int \frac{dt}{t^2-1} + 2 \int \frac{dt}{t^2+1} \\
 &= \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + 2 \operatorname{arctg} t + c = \frac{2}{2} \ln \frac{1-t}{1+t} + 2 \operatorname{arctg} t + c \\
 &= \left[\frac{1-t}{1+t} = \frac{1-\sqrt{\frac{1-e^x}{1+e^x}}}{1+\sqrt{\frac{1-e^x}{1+e^x}}} = \frac{1-\sqrt{1-e^x}}{\sqrt{1+e^x}} = \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}+\sqrt{1-e^x}} = \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}+\sqrt{1-e^x}} \cdot \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}-\sqrt{1-e^x}} = \frac{(1+e^x)-2\sqrt{1+e^x}\sqrt{1-e^x}+(1-e^x)}{(1+e^x)-(1-e^x)} = \frac{2-2\sqrt{1-e^{2x}}}{2e^x} \right] \\
 &= \ln \frac{1-\sqrt{1-e^{2x}}}{e^x} + 2 \operatorname{arctg} \sqrt{\frac{1-e^x}{1+e^x}} + c
 \end{aligned}$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx$$

$$\begin{aligned}
 &= \left[\text{Subst. } t = \sqrt{\frac{1-e^x}{1+e^x}} = \sqrt{\frac{2-1-e^x}{1+e^x}} = \sqrt{\frac{2}{1+e^x} - 1} \mid t^2(1+e^x) = 1-e^x \Rightarrow e^x = \frac{1-t^2}{1+t^2} \mid 1+e^x > 0 \text{ pre } x \in \mathbb{R} \mid x \in (-\infty; 0) \right. \\
 &\quad \left. dx = \left[\frac{-2t}{1-t^2} - \frac{2t}{1+t^2} \right] dt = \left[\frac{2t}{t^2-1} - \frac{2t}{t^2+1} \right] dt \mid x = \ln \frac{1-t^2}{1+t^2} = \ln(1-t^2) - \ln(1+t^2) \mid 1-e^x \geq 0 \text{ pre } x \leq 0 \mid t \in (0; 1) \right] \\
 &= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1} \right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1} \right] dt \\
 &= \int \left[2 + \frac{2}{t^2-1} - 2 + \frac{2}{t^2+1} \right] dt = 2 \int \frac{dt}{t^2-1} + 2 \int \frac{dt}{t^2+1} \\
 &= \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + 2 \operatorname{arctg} t + c = \frac{2}{2} \ln \frac{1-t}{1+t} + 2 \operatorname{arctg} t + c \\
 &= \left[\frac{1-t}{1+t} = \frac{1-\sqrt{\frac{1-e^x}{1+e^x}}}{1+\sqrt{\frac{1-e^x}{1+e^x}}} = \frac{1-\sqrt{1-e^x}}{\sqrt{1+e^x}} = \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}+\sqrt{1-e^x}} = \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}+\sqrt{1-e^x}} \cdot \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}-\sqrt{1-e^x}} = \frac{(1+e^x)-2\sqrt{1+e^x}\sqrt{1-e^x}+(1-e^x)}{(1+e^x)-(1-e^x)} = \frac{2-2\sqrt{1-e^{2x}}}{2e^x} \right] \\
 &= \ln \frac{1-\sqrt{1-e^{2x}}}{e^x} + 2 \operatorname{arctg} \sqrt{\frac{1-e^x}{1+e^x}} + c \\
 &= \ln(1-\sqrt{1-e^{2x}}) - \ln e^x + 2 \operatorname{arctg} \sqrt{\frac{1-e^x}{1+e^x}} + c
 \end{aligned}$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}} dx = \ln(1-\sqrt{1-e^{2x}}) - x + 2 \operatorname{arctg} \sqrt{\frac{1-e^x}{1+e^x}} + c$$

$$= \left[\begin{array}{l} \text{Subst. } t = \sqrt{\frac{1-e^x}{1+e^x}} = \sqrt{\frac{2-1-e^x}{1+e^x}} = \sqrt{\frac{2}{1+e^x} - 1} \quad t^2(1+e^x) = 1-e^x \Rightarrow e^x = \frac{1-t^2}{1+t^2} \quad \left. \begin{array}{l} 1+e^x > 0 \text{ pre } x \in \mathbb{R} \\ x \in (-\infty; 0) \end{array} \right\} \\ dx = \left[\frac{-2t}{1-t^2} - \frac{2t}{1+t^2} \right] dt = \left[\frac{2t}{t^2-1} - \frac{2t}{t^2+1} \right] dt \quad \left. \begin{array}{l} x = \ln \frac{1-t^2}{1+t^2} = \ln(1-t^2) - \ln(1+t^2) \\ 1-e^x \geq 0 \text{ pre } x \leq 0 \end{array} \right\} t \in (0; 1) \end{array} \right]$$

$$= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1} \right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1} \right] dt$$

$$= \int \left[2 + \frac{2}{t^2-1} - 2 + \frac{2}{t^2+1} \right] dt = 2 \int \frac{dt}{t^2-1} + 2 \int \frac{dt}{t^2+1}$$

$$= \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + 2 \operatorname{arctg} t + c = \frac{2}{2} \ln \frac{1-t}{1+t} + 2 \operatorname{arctg} t + c$$

$$= \left[\frac{1-t}{1+t} = \frac{1-\sqrt{\frac{1-e^x}{1+e^x}}}{1+\sqrt{\frac{1-e^x}{1+e^x}}} = \frac{1-\sqrt{\frac{1-e^x}{1+e^x}}}{1+\sqrt{\frac{1-e^x}{1+e^x}}} = \frac{\frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}\sqrt{1+e^x}}}{\frac{\sqrt{1+e^x}+\sqrt{1-e^x}}{\sqrt{1+e^x}\sqrt{1+e^x}}} = \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}+\sqrt{1-e^x}} \cdot \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}-\sqrt{1-e^x}} = \frac{(1+e^x)-2\sqrt{1+e^x}\sqrt{1-e^x}+(1-e^x)}{(1+e^x)-(1-e^x)} = \frac{2-2\sqrt{1-e^{2x}}}{2e^x} \right]$$

$$= \ln \frac{1-\sqrt{1-e^{2x}}}{e^x} + 2 \operatorname{arctg} \sqrt{\frac{1-e^x}{1+e^x}} + c$$

$$= \ln(1-\sqrt{1-e^{2x}}) - \ln e^x + 2 \operatorname{arctg} \sqrt{\frac{1-e^x}{1+e^x}} + c$$

$$= \ln(1-\sqrt{1-e^{2x}}) - x + 2 \operatorname{arctg} \sqrt{\frac{1-e^x}{1+e^x}} + c, \quad x \in (-\infty; 0), \quad c \in \mathbb{R}.$$

Riešené príklady – 097, 098

$$\int (x + 1)e^x dx$$

$$I_2 = \int x^2 e^{ax} dx$$

$$a \in \mathbb{R}, a \neq 0$$

Riešené príklady – 097, 098

$$\int (x + 1)e^x dx$$

$$= \left[\begin{array}{l|l} u = x+1 & u' = 1 \\ v' = e^x & v = e^x \end{array} \right]$$

$$I_2 = \int x^2 e^{ax} dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right]$$

$$\left[\begin{array}{l} \text{Odhad riešenia} \\ \text{metóda neurčitých koeficientov} \end{array} \right] I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$$

Riešené príklady – 097, 098

$$\int (x+1)e^x dx$$

$$= \left[\begin{array}{l} u = x+1 \quad u' = 1 \\ v' = e^x \quad v = e^x \end{array} \right] = (x+1)e^x - \int e^x dx$$

$$I_2 = \int x^2 e^{ax} dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l} u = x^2 \quad u' = 2x \\ v' = e^{ax} \quad v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^2 e^{ax}}{a} - \int \frac{2x e^{ax}}{a} dx$$

[
Odhad riešenia
metóda neurčitých koeficientov
]
 $I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$

[
Derivácia rovnosti
 $I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$
]
 $x^2 e^{ax} = (\beta + 2\gamma x)e^{ax} + (\alpha + \beta x + \gamma x^2)ae^{ax}$

Riešené príklady – 097, 098

$$\int (x+1)e^x dx$$

$$= \left[\begin{array}{l} u = x+1 \quad u' = 1 \\ v' = e^x \quad v = e^x \end{array} \right] = (x+1)e^x - \int e^x dx = (x+1)e^x - e^x + c$$

$$I_2 = \int x^2 e^{ax} dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l} u = x^2 \quad u' = 2x \\ v' = e^{ax} \quad v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^2 e^{ax}}{a} - \int \frac{2xe^{ax}}{a} dx = \left[\begin{array}{l} u = \frac{2x}{a} \quad u' = \frac{2}{a} \\ v' = e^{ax} \quad v = \frac{e^{ax}}{a} \end{array} \right]$$

$$\left[\begin{array}{l} \text{Odhad riešenia} \\ \text{metóda neurčitých koeficientov} \end{array} \right] I_2 = (\alpha + \beta x + \gamma x^2) e^{ax} + c$$

$$\left[\begin{array}{l} \text{Derivácia rovnosti} \\ I_2 = (\alpha + \beta x + \gamma x^2) e^{ax} + c \end{array} \right] x^2 e^{ax} = (\beta + 2\gamma x) e^{ax} + (\alpha + \beta x + \gamma x^2) a e^{ax}$$

$$[0 + 0 \cdot x + 1 \cdot x^2] e^{ax} = [(\beta + \alpha a) + (2\gamma + \beta a)x + \gamma a x^2] e^{ax}.$$

Riešené príklady – 097, 098

$$\int (x+1)e^x dx = xe^x + c$$

$$= \left[\begin{array}{l} u = x+1 \\ v' = e^x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = e^x \end{array} \right] = (x+1)e^x - \int e^x dx = (x+1)e^x - e^x + c = xe^x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$I_2 = \int x^2 e^{ax} dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l} u = x^2 \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^2 e^{ax}}{a} - \int \frac{2xe^{ax}}{a} dx = \left[\begin{array}{l} u = \frac{2x}{a} \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = \frac{2}{a} \\ v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^2 e^{ax}}{a} - \left[\frac{2xe^{ax}}{a^2} - \int \frac{2e^{ax}}{a^2} dx \right]$$

[Odhad riešenia
metóda neurčitých koeficientov] $I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$

[Derivácia rovnosti
 $I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$] $x^2 e^{ax} = (\beta + 2\gamma x)e^{ax} + (\alpha + \beta x + \gamma x^2)ae^{ax}$

$$[0 + 0 \cdot x + 1 \cdot x^2]e^{ax} = [(\beta + \alpha a) + (2\gamma + \beta a)x + \gamma a x^2]e^{ax}.$$

[3 rovnice
3 neznáme] $0 = \beta + \alpha a, 0 = 2\gamma + \beta a, 1 = \gamma a$

Riešené príklady – 097, 098

$$\int (x+1)e^x dx = xe^x + c$$

$$= \left[\begin{array}{l} u = x+1 \\ v' = e^x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = e^x \end{array} \right] = (x+1)e^x - \int e^x dx = (x+1)e^x - e^x + c = xe^x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$I_2 = \int x^2 e^{ax} dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^2 e^{ax}}{a} - \int \frac{2xe^{ax}}{a} dx = \left[\begin{array}{l} u = \frac{2x}{a} \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = \frac{2}{a} \\ v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^2 e^{ax}}{a} - \left[\frac{2xe^{ax}}{a^2} - \int \frac{2e^{ax}}{a^2} dx \right] \\ &= \frac{x^2 e^{ax}}{a} - \frac{2xe^{ax}}{a^2} + \frac{2e^{ax}}{a \cdot a^2} + c \end{aligned}$$

[Odhad riešenia] $I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$
[metóda neurčitých koeficientov]

[Derivácia rovnosti] $x^2 e^{ax} = (\beta + 2\gamma x)e^{ax} + (\alpha + \beta x + \gamma x^2)ae^{ax}$
[$I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$]

$$[0 + 0 \cdot x + 1 \cdot x^2]e^{ax} = [(\beta + \alpha a) + (2\gamma + \beta a)x + \gamma a x^2]e^{ax}.$$

[3 rovnice] $0 = \beta + \alpha a, 0 = 2\gamma + \beta a, 1 = \gamma a$
[3 neznáme]

Riešené príklady – 097, 098

$$\int (x+1)e^x dx = xe^x + c$$

$$= \left[\begin{array}{l} u = x+1 \\ v' = e^x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = e^x \end{array} \right] = (x+1)e^x - \int e^x dx = (x+1)e^x - e^x + c = xe^x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$I_2 = \int x^2 e^{ax} dx = \frac{x^2 e^{ax}}{a} - \frac{2xe^{ax}}{a^2} + \frac{2e^{ax}}{a^3} + c \quad a \in \mathbb{R}, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^2 e^{ax}}{a} - \int \frac{2xe^{ax}}{a} dx = \left[\begin{array}{l} u = \frac{2x}{a} \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = \frac{2}{a} \\ v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^2 e^{ax}}{a} - \left[\frac{2xe^{ax}}{a^2} - \int \frac{2e^{ax}}{a^2} dx \right] \\ &= \frac{x^2 e^{ax}}{a} - \frac{2xe^{ax}}{a^2} + \frac{2e^{ax}}{a \cdot a^2} + c = \frac{x^2 e^{ax}}{a} - \frac{2xe^{ax}}{a^2} + \frac{2e^{ax}}{a^3} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

[Odhad riešenia] $I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$
[metóda neurčitých koeficientov]

[Derivácia rovnosti] $x^2 e^{ax} = (\beta + 2\gamma x)e^{ax} + (\alpha + \beta x + \gamma x^2)ae^{ax}$
[$I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$]

$$[0 + 0 \cdot x + 1 \cdot x^2]e^{ax} = [(\beta + \alpha a) + (2\gamma + \beta a)x + \gamma a x^2]e^{ax}.$$

[3 rovnice] $0 = \beta + \alpha a, 0 = 2\gamma + \beta a, 1 = \gamma a$
[3 neznáme]

Riešené príklady – 097, 098

$$\int (x+1)e^x dx = xe^x + c$$

$$= \left[\begin{array}{l} u = x+1 \\ v' = e^x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = e^x \end{array} \right] = (x+1)e^x - \int e^x dx = (x+1)e^x - e^x + c = xe^x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$I_2 = \int x^2 e^{ax} dx = \frac{x^2 e^{ax}}{a} - \frac{2xe^{ax}}{a^2} + \frac{2e^{ax}}{a^3} + c \quad a \in \mathbb{R}, a \neq 0$$

$$\begin{aligned} &= \left[\begin{array}{l} u = x^2 \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = 2x \\ v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^2 e^{ax}}{a} - \int \frac{2xe^{ax}}{a} dx = \left[\begin{array}{l} u = \frac{2x}{a} \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = \frac{2}{a} \\ v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^2 e^{ax}}{a} - \left[\frac{2xe^{ax}}{a^2} - \int \frac{2e^{ax}}{a^2} dx \right] \\ &= \frac{x^2 e^{ax}}{a} - \frac{2xe^{ax}}{a^2} + \frac{2e^{ax}}{a \cdot a^2} + c = \frac{x^2 e^{ax}}{a} - \frac{2xe^{ax}}{a^2} + \frac{2e^{ax}}{a^3} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}. \end{aligned}$$

[Odhad riešenia
metóda neurčitých koeficientov] $I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$

[Derivácia rovnosti
 $I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$] $x^2 e^{ax} = (\beta + 2\gamma x)e^{ax} + (\alpha + \beta x + \gamma x^2)ae^{ax}$

$$[0 + 0 \cdot x + 1 \cdot x^2]e^{ax} = [(\beta + \alpha a) + (2\gamma + \beta a)x + \gamma ax^2]e^{ax}.$$

[3 rovnice
3 neznáme] $0 = \beta + \alpha a, 0 = 2\gamma + \beta a, 1 = \gamma a$

Riešené príklady – 097, 098

$$\int (x+1)e^x dx = xe^x + c$$

$$= \left[\begin{array}{l} u = x+1 \\ v' = e^x \end{array} \middle| \begin{array}{l} u' = 1 \\ v = e^x \end{array} \right] = (x+1)e^x - \int e^x dx = (x+1)e^x - e^x + c = xe^x + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

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Odhad riešenia
[metóda neurčitých koeficientov]

$$I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$$

Derivácia rovnosti
[$I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$]

$$x^2 e^{ax} = (\beta + 2\gamma x)e^{ax} + (\alpha + \beta x + \gamma x^2)ae^{ax}$$

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[3 rovnice
3 neznáme]

$$0 = \beta + \alpha a, \quad 0 = 2\gamma + \beta a, \quad 1 = \gamma a \Rightarrow \gamma = \frac{1}{a},$$

Riešené príklady – 097, 098

$$\int (x+1)e^x dx = xe^x + c$$

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[Odhad riešenia] $I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$
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[3 neznáme]

Riešené príklady – 097, 098

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[Odhad riešenia] $I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$
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$$[0 + 0 \cdot x + 1 \cdot x^2]e^{ax} = [(\beta + \alpha a) + (2\gamma + \beta a)x + \gamma a x^2]e^{ax}.$$

[3 rovnice] $0 = \beta + \alpha a, 0 = 2\gamma + \beta a, 1 = \gamma a \Rightarrow \gamma = \frac{1}{a}, \beta = \frac{2\gamma}{-a} = -\frac{2}{a^2}, \alpha = \frac{\beta}{-a} = \frac{2}{a^3}.$
[3 neznáme]

Riešené príklady – 097, 098

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[Odhad riešenia] $I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c = \left(\frac{2}{a^3} - \frac{2x}{a^2} + \frac{x^2}{a} \right) e^{ax} + c, x \in R, c \in R.$
[metóda neurčitých koeficientov]

[Derivácia rovnosti] $x^2 e^{ax} = (\beta + 2\gamma x)e^{ax} + (\alpha + \beta x + \gamma x^2)ae^{ax}$
[$I_2 = (\alpha + \beta x + \gamma x^2)e^{ax} + c$]

$$[0 + 0 \cdot x + 1 \cdot x^2] e^{ax} = [(\beta + \alpha a) + (2\gamma + \beta a)x + \gamma a x^2] e^{ax}.$$

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[3 neznáme]

Riešené príklady – 099

$$I_8 = \int x^8 e^{ax} dx$$

$$a \in \mathbb{R}, a \neq 0$$

Riešené príklady – 099

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[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$$

Riešené príklady – 099

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[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$]

$$\begin{aligned} x^8 e^{ax} &= (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7) e^{ax} \\ &\quad + (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) a e^{ax}. \end{aligned}$$

Riešené príklady – 099

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[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$]

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Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$]

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[9 rovníc a 9 neznámych] $0 = \beta + \alpha a,$

Riešené príklady – 099

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Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$]

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[9 rovníc a 9 neznámych] $0 = \beta + \alpha a, 0 = 2\gamma + \beta a,$

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Riešené príklady – 099

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$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$$

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Riešené príklady – 099

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[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$$

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[9 rovníc a 9 neznámych] $0 = \beta + \alpha a, 0 = 2\gamma + \beta a, 0 = 3\delta + \gamma a, 0 = 4\psi + \delta a,$
 $0 = 5\varphi + \psi a,$

Riešené príklady – 099

$$I_8 = \int x^8 e^{ax} dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l} u = x^8 \quad u' = 8x^7 \\ v' = e^{ax} \quad v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^8 e^{ax}}{a} - \frac{8}{a} \int x^7 e^{ax} dx = \left[\begin{array}{l} u = x^7 \quad u' = 7x^6 \\ v' = e^{ax} \quad v = \frac{e^{ax}}{a} \end{array} \right] = \dots$$

Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$]

$$x^8 e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7) e^{ax} + (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) a e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a, 0 = 2\gamma + \beta a, 0 = 3\delta + \gamma a, 0 = 4\psi + \delta a,$
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Riešené príklady – 099

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$$\frac{-8!x^7}{7!a^2} + \frac{8!x^8}{8!a}$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$]

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$$\delta = \frac{4\psi}{-a} = -\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{a^6}, \gamma = \frac{3\delta}{-a} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{a^7},$$

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$$I_8 = \int x^8 e^{ax} dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l} u = x^8 \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = 8x^7 \\ v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^8 e^{ax}}{a} - \frac{8}{a} \int x^7 e^{ax} dx = \left[\begin{array}{l} u = x^7 \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = 7x^6 \\ v = \frac{e^{ax}}{a} \end{array} \right] = \dots$$

Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$$

$$- \frac{8!x}{1!a^8} + \frac{8!x^2}{2!a^7} - \frac{8!x^3}{3!a^6} + \frac{8!x^4}{4!a^5} - \frac{8!x^5}{5!a^4} + \frac{8!x^6}{6!a^3} - \frac{8!x^7}{7!a^2} + \frac{8!x^8}{8!a}$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$]

$$x^8 e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7) e^{ax}$$

$$+ (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) a e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a$, $0 = 2\gamma + \beta a$, $0 = 3\delta + \gamma a$, $0 = 4\psi + \delta a$,
 $0 = 5\varphi + \psi a$, $0 = 6\mu + \varphi a$, $0 = 7\nu + \mu a$, $0 = 8\theta + \nu a$, $1 = \theta a$.

$$\Rightarrow \theta = \frac{1}{a}, \nu = \frac{8\theta}{-a} = -\frac{8}{a^2}, \mu = \frac{7\nu}{-a} = \frac{8 \cdot 6}{a^3}, \varphi = \frac{6\mu}{-a} = -\frac{8 \cdot 7 \cdot 5}{a^4}, \psi = \frac{5\varphi}{-a} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{a^5},$$

$$\delta = \frac{4\psi}{-a} = -\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{a^6}, \gamma = \frac{3\delta}{-a} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{a^7}, \beta = \frac{2\gamma}{-a} = -\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{a^8},$$

Riešené príklady – 099

$$I_8 = \int x^8 e^{ax} dx$$

$$a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l} u = x^8 \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = 8x^7 \\ v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^8 e^{ax}}{a} - \frac{8}{a} \int x^7 e^{ax} dx = \left[\begin{array}{l} u = x^7 \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = 7x^6 \\ v = \frac{e^{ax}}{a} \end{array} \right] = \dots$$

Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$$

$$\frac{8!}{0!a^9} + \frac{8!x}{1!a^8} + \frac{8!x^2}{2!a^7} + \frac{8!x^3}{3!a^6} + \frac{8!x^4}{4!a^5} + \frac{8!x^5}{5!a^4} + \frac{8!x^6}{6!a^3} + \frac{8!x^7}{7!a^2} + \frac{8!x^8}{8!a}$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$]

$$x^8 e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7) e^{ax} + (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) a e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a$, $0 = 2\gamma + \beta a$, $0 = 3\delta + \gamma a$, $0 = 4\psi + \delta a$,

$$0 = 5\varphi + \psi a, 0 = 6\mu + \varphi a, 0 = 7\nu + \mu a, 0 = 8\theta + \nu a, 1 = \theta a.$$

$$\Rightarrow \theta = \frac{1}{a}, \nu = \frac{8\theta}{-a} = -\frac{8}{a^2}, \mu = \frac{7\nu}{-a} = \frac{8 \cdot 6}{a^3}, \varphi = \frac{6\mu}{-a} = -\frac{8 \cdot 7 \cdot 5}{a^4}, \psi = \frac{5\varphi}{-a} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{a^5},$$

$$\delta = \frac{4\psi}{-a} = -\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{a^6}, \gamma = \frac{3\delta}{-a} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{a^7}, \beta = \frac{2\gamma}{-a} = -\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{a^8}, \alpha = \frac{\beta}{-a} = \frac{8!}{a^9}.$$

Riešené príklady – 099

$$I_8 = \int x^8 e^{ax} dx = \sum_{i=0}^8 \frac{(-1)^i 8! \cdot x^i e^{ax}}{i! \cdot a^{9-i}} + c = 8! e^{ax} \sum_{i=0}^8 \frac{(-x)^i}{i! \cdot a^{9-i}} + c \quad a \in \mathbb{R}, a \neq 0$$

$$= \left[\begin{array}{l} u = x^8 \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = 8x^7 \\ v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^8 e^{ax}}{a} - \frac{8}{a} \int x^7 e^{ax} dx = \left[\begin{array}{l} u = x^7 \\ v' = e^{ax} \end{array} \middle| \begin{array}{l} u' = 7x^6 \\ v = \frac{e^{ax}}{a} \end{array} \right] = \dots$$

Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$$

$$= \left(\frac{8!}{0! a^9} + \frac{8! x}{1! a^8} + \frac{8! x^2}{2! a^7} + \frac{8! x^3}{3! a^6} + \frac{8! x^4}{4! a^5} + \frac{8! x^5}{5! a^4} + \frac{8! x^6}{6! a^3} + \frac{8! x^7}{7! a^2} + \frac{8! x^8}{8! a} \right) e^{ax} + c, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) e^{ax} + c$]

$$x^8 e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7) e^{ax}$$

$$+ (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) a e^{ax}.$$

[9 rovníc a 9 neznámych]

$$0 = \beta + \alpha a, \quad 0 = 2\gamma + \beta a, \quad 0 = 3\delta + \gamma a, \quad 0 = 4\psi + \delta a,$$

$$0 = 5\varphi + \psi a, \quad 0 = 6\mu + \varphi a, \quad 0 = 7\nu + \mu a, \quad 0 = 8\theta + \nu a, \quad 1 = \theta a.$$

$$\Rightarrow \theta = \frac{1}{a}, \quad \nu = \frac{8\theta}{-a} = -\frac{8}{a^2}, \quad \mu = \frac{7\nu}{-a} = \frac{8 \cdot 6}{a^3}, \quad \varphi = \frac{6\mu}{-a} = -\frac{8 \cdot 7 \cdot 5}{a^4}, \quad \psi = \frac{5\varphi}{-a} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{a^5},$$

$$\delta = \frac{4\psi}{-a} = -\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{a^6}, \quad \gamma = \frac{3\delta}{-a} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{a^7}, \quad \beta = \frac{2\gamma}{-a} = -\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{a^8}, \quad \alpha = \frac{\beta}{-a} = \frac{8!}{a^9}.$$

Riešené príklady – 100

$$I_n = \int x^n e^x dx$$

 $n \in \mathbb{N}$

Riešené príklady – 100

$$I_n = \int x^n e^x dx$$

 $n \in \mathbb{N}$

$$= \left[\begin{array}{l} u = x^n \quad u' = nx^{n-1} \\ v' = e^x \quad v = e^x \end{array} \right]$$

$$I_1 = \int x e^x dx = x e^x - 1 I_0 = x e^x - e^x + c$$

Riešené príklady – 100

$$I_n = \int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx \quad n \in \mathbb{N}$$

$$= \left[\begin{array}{l} u = x^n \quad u' = nx^{n-1} \\ v' = e^x \quad v = e^x \end{array} \right] = x^n e^x - n \int x^{n-1} e^x dx = x^n e^x - n I_{n-1}, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$I_1 = \int x e^x dx = x e^x - 1 I_0 = x e^x - e^x + c, \quad \text{pričom } I_0 = \int e^x dx = e^x + c.$$

Riešené príklady – 100

$$I_n = \int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx \quad n \in \mathbb{N}$$

$$= \left[\begin{array}{l} u = x^n \quad u' = nx^{n-1} \\ v' = e^x \quad v = e^x \end{array} \right] = x^n e^x - n \int x^{n-1} e^x dx = x^n e^x - n I_{n-1}, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$I_1 = \int x e^x dx = x e^x - 1 I_0 = x e^x - e^x + c, \quad \text{pričom } I_0 = \int e^x dx = e^x + c.$$

$$I_2 = \int x^2 e^x dx = x^2 e^x - 2 I_1 = x^2 e^x - 2(x e^x - e^x) + c = x^2 e^x - 2x e^x + 2 \cdot 1 e^x + c.$$

Riešené príklady – 100

$$I_n = \int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx \quad n \in \mathbb{N}$$

$$= \left[\begin{array}{l} u = x^n \quad u' = nx^{n-1} \\ v' = e^x \quad v = e^x \end{array} \right] = x^n e^x - n \int x^{n-1} e^x dx = x^n e^x - n I_{n-1}, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$I_1 = \int x e^x dx = x e^x - 1 I_0 = x e^x - e^x + c, \quad \text{pričom } I_0 = \int e^x dx = e^x + c.$$

$$I_2 = \int x^2 e^x dx = x^2 e^x - 2 I_1 = x^2 e^x - 2(x e^x - e^x) + c = x^2 e^x - 2x e^x + 2 \cdot 1 e^x + c.$$

$$I_3 = \int x^3 e^x dx = x^3 e^x - 3 I_2 = x^3 e^x - 3(x^2 e^x - 2x e^x + 2 \cdot 1 e^x) + c \\ = x^3 e^x - 3x^2 e^x + 3 \cdot 2x e^x - 3 \cdot 2 \cdot 1 e^x + c.$$

Riešené príklady – 100

$$I_n = \int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx \quad n \in \mathbb{N}$$

$$= \left[\begin{array}{l} u = x^n \quad u' = nx^{n-1} \\ v' = e^x \quad v = e^x \end{array} \right] = x^n e^x - n \int x^{n-1} e^x dx = x^n e^x - n I_{n-1}, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$I_1 = \int x e^x dx = x e^x - 1 I_0 = x e^x - e^x + c, \quad \text{pričom } I_0 = \int e^x dx = e^x + c.$$

$$I_2 = \int x^2 e^x dx = x^2 e^x - 2 I_1 = x^2 e^x - 2(x e^x - e^x) + c = x^2 e^x - 2x e^x + 2 \cdot 1 e^x + c.$$

$$I_3 = \int x^3 e^x dx = x^3 e^x - 3 I_2 = x^3 e^x - 3(x^2 e^x - 2x e^x + 2 \cdot 1 e^x) + c \\ = x^3 e^x - 3x^2 e^x + 3 \cdot 2x e^x - 3 \cdot 2 \cdot 1 e^x + c.$$

$$I_4 = \int x^4 e^x dx = x^4 e^x - 4 I_3 = x^4 e^x - 4(x^3 e^x - 3x^2 e^x + 3 \cdot 2x e^x - 3 \cdot 2 \cdot 1 e^x) + c \\ = x^4 e^x - 4x^3 e^x + 4 \cdot 3x^2 e^x - 4 \cdot 3 \cdot 2x e^x + 4 \cdot 3 \cdot 2 \cdot 1 e^x + c.$$

Riešené príklady – 100

$$I_n = \int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx \quad n \in \mathbb{N}$$

$$= \left[\begin{array}{l} u = x^n \quad u' = nx^{n-1} \\ v' = e^x \quad v = e^x \end{array} \right] = x^n e^x - n \int x^{n-1} e^x dx = x^n e^x - n I_{n-1}, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$I_1 = \int x e^x dx = x e^x - 1 I_0 = x e^x - e^x + c, \quad \text{pričom } I_0 = \int e^x dx = e^x + c.$$

$$I_2 = \int x^2 e^x dx = x^2 e^x - 2 I_1 = x^2 e^x - 2(x e^x - e^x) + c = x^2 e^x - 2x e^x + 2 \cdot 1 e^x + c.$$

$$I_3 = \int x^3 e^x dx = x^3 e^x - 3 I_2 = x^3 e^x - 3(x^2 e^x - 2x e^x + 2 \cdot 1 e^x) + c \\ = x^3 e^x - 3x^2 e^x + 3 \cdot 2x e^x - 3 \cdot 2 \cdot 1 e^x + c.$$

$$I_4 = \int x^4 e^x dx = x^4 e^x - 4 I_3 = x^4 e^x - 4(x^3 e^x - 3x^2 e^x + 3 \cdot 2x e^x - 3 \cdot 2 \cdot 1 e^x) + c \\ = x^4 e^x - 4x^3 e^x + 4 \cdot 3x^2 e^x - 4 \cdot 3 \cdot 2x e^x + 4 \cdot 3 \cdot 2 \cdot 1 e^x + c.$$

$$\dots \\ I_n = \int x^n e^x dx = x^n e^x - n I_{n-1} = e^x \sum_{i=0}^n (-1)^i n(n-1) \cdots (n-i+1) x^{n-i} + c$$

Riešené príklady – 100

$$I_n = \int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx \quad n \in \mathbb{N}$$

$$= \left[\begin{array}{l} u = x^n \quad u' = nx^{n-1} \\ v' = e^x \quad v = e^x \end{array} \right] = x^n e^x - n \int x^{n-1} e^x dx = x^n e^x - n I_{n-1}, \quad x \in \mathbb{R}, \quad c \in \mathbb{R}.$$

$$I_1 = \int x e^x dx = x e^x - 1 I_0 = x e^x - e^x + c, \quad \text{pričom } I_0 = \int e^x dx = e^x + c.$$

$$I_2 = \int x^2 e^x dx = x^2 e^x - 2 I_1 = x^2 e^x - 2(x e^x - e^x) + c = x^2 e^x - 2x e^x + 2 \cdot 1 e^x + c.$$

$$I_3 = \int x^3 e^x dx = x^3 e^x - 3 I_2 = x^3 e^x - 3(x^2 e^x - 2x e^x + 2 \cdot 1 e^x) + c \\ = x^3 e^x - 3x^2 e^x + 3 \cdot 2x e^x - 3 \cdot 2 \cdot 1 e^x + c.$$

$$I_4 = \int x^4 e^x dx = x^4 e^x - 4 I_3 = x^4 e^x - 4(x^3 e^x - 3x^2 e^x + 3 \cdot 2x e^x - 3 \cdot 2 \cdot 1 e^x) + c \\ = x^4 e^x - 4x^3 e^x + 4 \cdot 3x^2 e^x - 4 \cdot 3 \cdot 2x e^x + 4 \cdot 3 \cdot 2 \cdot 1 e^x + c.$$

$$\dots \\ I_n = \int x^n e^x dx = x^n e^x - n I_{n-1} = e^x \sum_{i=0}^n (-1)^i n(n-1) \cdots (n-i+1) x^{n-i} + c \\ = [x^n - n x^{n-1} + n(n-1) x^{n-2} - \dots + (-1)^n n!] e^x + c$$

Riešené príklady – 100

$$I_n = \int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx = n! e^x \sum_{i=0}^n \frac{(-1)^i x^{n-i}}{(n-i)!} + c \quad n \in \mathbb{N}$$

$$= \left[\begin{array}{l} u = x^n \quad u' = nx^{n-1} \\ v' = e^x \quad v = e^x \end{array} \right] = x^n e^x - n \int x^{n-1} e^x dx = x^n e^x - n I_{n-1}, \quad x \in \mathbb{R}, c \in \mathbb{R}.$$

$$I_1 = \int x e^x dx = x e^x - 1 I_0 = x e^x - e^x + c, \quad \text{pričom } I_0 = \int e^x dx = e^x + c.$$

$$I_2 = \int x^2 e^x dx = x^2 e^x - 2 I_1 = x^2 e^x - 2(x e^x - e^x) + c = x^2 e^x - 2x e^x + 2 \cdot 1 e^x + c.$$

$$I_3 = \int x^3 e^x dx = x^3 e^x - 3 I_2 = x^3 e^x - 3(x^2 e^x - 2x e^x + 2 \cdot 1 e^x) + c \\ = x^3 e^x - 3x^2 e^x + 3 \cdot 2x e^x - 3 \cdot 2 \cdot 1 e^x + c.$$

$$I_4 = \int x^4 e^x dx = x^4 e^x - 4 I_3 = x^4 e^x - 4(x^3 e^x - 3x^2 e^x + 3 \cdot 2x e^x - 3 \cdot 2 \cdot 1 e^x) + c \\ = x^4 e^x - 4x^3 e^x + 4 \cdot 3x^2 e^x - 4 \cdot 3 \cdot 2x e^x + 4 \cdot 3 \cdot 2 \cdot 1 e^x + c.$$

$$\dots \\ I_n = \int x^n e^x dx = x^n e^x - n I_{n-1} = e^x \sum_{i=0}^n (-1)^i n(n-1) \cdots (n-i+1) x^{n-i} + c \\ = [x^n - n x^{n-1} + n(n-1) x^{n-2} - \dots + (-1)^n n!] e^x + c = \sum_{i=0}^n \frac{(-1)^i n! x^{n-i} e^x}{(n-i)!} + c.$$

Koniec 10. časti príklady I

Ďakujem za pozornosť.